Light Front Wave Functions from AdS/QCD models

Alfredo Vega

In collaboration with
M. A. Martin Contreras

HADRON 2021, Virtually hosted by UNAM, México

July 29, 2021

Outline

Introduction

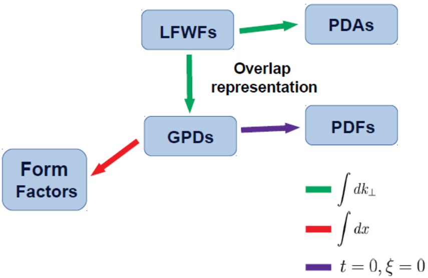
Basic Ideas behind holographic LFWF

Extensions in AdS side

Final Comments and Conclusions

Introduction

Introduction



The idea:
Compute ***everything*** from the **LFWF.**

(Picture from Khepani Raya talk)

Maybe the most simple way to obtain a LFWF is to use Brodsky-Huang-Lepage prescription [1]

$$\psi_{c.m}(r) \overset{TF}{\leftrightarrow} \psi_{c.m}(q) \overset{BHL}{\leftrightarrow} \psi_{LC}\left[\sqrt{\frac{k_\perp^2+m^2}{4x(1-x)}} - m^2\right].$$

[1] S. J. Brodsky, T. Huang, and G. P. Lepage, Proceedings of the Banff Summer Institute on Particles and Fields 2, Banff, Alberta, 1981, edited by A. Z. Capri and A. N. Kamal (Plenum, New York, 1983), p. 143; P. Lepage, S. J. Brodsky, T. Huang, and P. B. Mackenzie, ibid., p. 83; T. Huang, AIP Conf. Proc. 68, 1000 (1980).

⋆ Ideas about AdS / QCD models to follow this talk.

- Extensions of AdS / CFT to QCD are related at two approaches:
  - Top-Down approach.
    You start from a string theory on $AdS_{d+1}xC$, and try to get at low energies a theory similar to QCD in the border.
  - Bottom-Up approach.
    Starting from QCD in 4d we try to build a theory with higher dimensions (not necessarily a string theory).
- Exist a dictionary, which relate quantities at both sides in holographic duality.
- In bottom-up approach, and here with Asymptotically AdS metrics with a non-dynamical dilaton, it is possible reproduce some hadronic phenomenology.

$$S = \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\left(\mathcal{L}_{Part} + \mathcal{L}_{Int}\right),$$

Basic Ideas behind holographic LFWF

⋆ **Basic Idea** [2]

Comparison of Form Factors in light front and in AdS side, offer us a possibility to relate AdS modes that describe hadrons with LFWF.

- In Light Front (for hadrons with two partons)

$$F(Q^2) = 2\,\pi \int_0^1 dx\, \frac{1-x}{x} \int_0^\infty d\zeta\, \zeta\, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) \frac{\left|\widetilde{\psi}_{q_1 \bar{q}_2}(x,\zeta)\right|^2}{(1-x)^2},$$

- In AdS

$$F(Q^2) = \int_0^\infty dz\, \Phi(z) J(Q^2,z) \Phi(z)\,,$$

where $\phi(z)$ correspond to AdS modes that represent hadrons and $J(q^2,z)$ it is dual to electromagnetic current.

[2] S. Brodsky and G. de Teramond, Phys. Rev. Lett. 96, 201601 (2006); Phys. Rev. D 77, 056007 (2008).

⋆ **Trick to compare both form factors.**

$$J(Q,z) = z\,Q\,K_1(zQ) = \int_0^1 dx\,J_0\left(\zeta\,Q\sqrt{\tfrac{1-x}{x}}\right) \quad (Hard\ Wall),$$

$$J(Q^2,z) = \Gamma(1+\tfrac{Q^2}{4\kappa})U(\tfrac{Q^2}{4\kappa},0,\kappa^2 z^2) \xrightarrow[Q^2\to\infty]{} zQK_1(zQ) \quad (Soft\ Wall).$$

Using this,

$$F(Q^2) = 2\pi\int_0^1 dx\,\frac{1-x}{x}\int_0^\infty d\zeta\,\zeta\,J_0\left(\zeta Q\sqrt{\frac{1-x}{x}}\right)\frac{\left|\widetilde{\psi}_{q_1\bar{q}_2}(x,\zeta)\right|^2}{(1-x)^2}, \quad \text{(LF)}$$

$$F(Q^2) = \int_0^\infty dz\,\Phi(z)\left[\int_0^1 dx\,J_0\left(\zeta\,Q\sqrt{\frac{1-x}{x}}\right)\right]\Phi(z)\,, \quad \text{(AdS)}$$

If additionally considering which "x" at both sides are the same and $z^2 = \zeta^2 = x(1-x)b^2$, it is possible compare both and extract LFWF in term of AdS modes.

In Soft Wall case we obtain

$$\psi(x, b_\perp) = A\sqrt{x(1-x)}e^{-\frac{1}{2}\kappa^2 x(1-x) b_\perp^2},$$

which in momentum space

$$\psi(x, k_\perp) = \frac{4\pi A}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}.$$

⋆ Examples of extensions

$$\psi(x,k_\perp) = N\frac{4\pi}{\kappa\sqrt{x(1-x)}}g_1(x)e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}g_2(x)}.$$

- S. J. Brodsky and G. F. de Teramond, PRD 77 (2008) 056007. A. V, I. Schmidt, T. Branz, T. Gutsche and V. E. Lyubovitskij, PRD 80 (2009) 055014.

$$K_0 = \frac{k_\perp^2}{x(1-x)} \to K = K_0 + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}.$$

- Uses of 't Hooft equation. (S. Chabysheva and J. Hiller, Annals of Physics 337 (2013) 143 - 152; M. Ahmady, H. Dahiya, S. Kaur, Ch. Mondal, R. Sandapen, N. Sharma, 2105.01018 [hep-ph]).
- Use a shape that fulfill the following constraints. For $x \to 1$, the wave function must reproduce scaling of PDFs as $(1-x)^\tau$. And at large Q^2, the form factors scales as $1/(Q^2)^{\tau-1}$. (T. Gutsche, V. Lyubovitskij, I. Schmidt and A. V, PRD 87, 056001 (2013)).

In this cases changes are introduced in QCD side.

Extensions in AdS side

In Light Front (for hadrons with two partons)

$$F(Q^2) = 2\,\pi \int_0^1 dx\, \frac{1-x}{x} \int_0^\infty d\zeta\, \zeta\, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) \frac{\left|\widetilde{\psi}_{q_1 \bar{q}_2}(x,\zeta)\right|^2}{(1-x)^2}.$$

In AdS

$$F(Q^2) = \int_0^\infty dz\, \Phi(z) J(Q^2, z) \Phi(z)\,.$$

If

$$J(Q,z) = \int_0^1 dx\, g(Q^2,x) J_0\left(\zeta\, Q \sqrt{\frac{1-x}{x}}\right),$$

then, if we use $\zeta^2 = x(1-x)b^2$

$$\frac{\left|\widetilde{\psi}(x,\zeta)\right|^2}{g(Q,x)} = x(1-x)\,\frac{|\Phi(\zeta)|^2}{2\pi\zeta} \to \frac{\sqrt{x(1-x)}}{2\,\pi\, b}\,\left|\Phi\left(\sqrt{x(1-x)}\, b\right)\right|^2.$$

In the large Q^2 limit $g(Q,x)=1$, because equation for $J(Q^2,z)$ is the same that hard wall case.

TABLE I. Summary of the AdS/QCD models used to construct the LF wave function with their corresponding parameters and references.

Model	Dilaton	Deformation	Parameters	Ref.
1	$\phi_1(z)=\kappa^2 z^2$	$h_1(z)=0$	$\kappa=0.375$ GeV	[24]
2	$\phi_2(z)=\mu_G^2 z^2 \tanh\left(\frac{\mu_{G^2}^4}{\mu_G^2} z^2\right)$	$h_2(z)=0$	$\mu_G=0.370$ GeV and $\mu_{G^2}=0.368$	[25]
3	$\phi_3(z)=\kappa^2 z^2+Mz+\tanh\left(\frac{1}{Mz}-G\right)$	$h_3(z)=0$	$\kappa=0.358$ GeV, $M=0.083$ GeV and $G=0.082$ GeV	[26]
4	$\phi_4(z)=0$	$h_4(z)=\frac{1}{2}kz^2$	$k=-0.280^2$ GeV^2	[27]

$$ds^2=e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu-dz^2) \quad , \quad A(z)=ln\left(\frac{R}{z}\right)+h(z).$$

$$V(z)=\frac{15}{4\,z^2}+\frac{1}{4}\left[\phi'(z)^2+9\,h(z)^2\right]-\frac{3}{2}\phi'(z)\,h'(z)-\frac{1}{2}\left[\phi''(z)-3\,h''(z)\right]-\frac{3}{2\,z}\left[\phi'(z)-3\,h'(z)\right]+\frac{e^{2\,h(z)}M_5^2\,R^2}{z^2}.$$

[23] E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998).
[24] A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, Phys. Rev. D 74, 015005 (2006).
[25] D. Li and M. Huang, J. High Energy Phys. 11 (2013) 088.
[26] N. R. Braga and L. F. Ferreira, Phys. Lett. B 795, 462 (2019).
[27] E. Folco Capossoli, M. A. M. Contreras, D. Li, A. Vega, and H. Boschi-Filho, Chin. Phys. C 44, 064104 (2020).

Extensions in AdS side

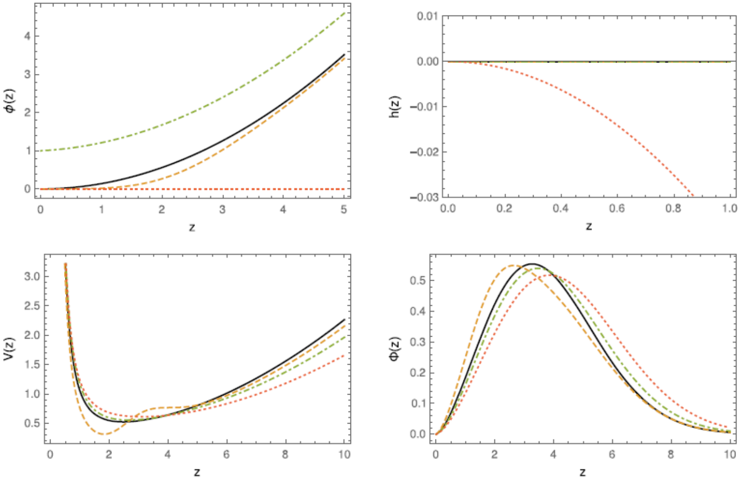


FIG. 1. The set of dilaton fields (left upper panel), the deformation function (right upper panel), holographic potentials (left lower panel), and AdS modes (right lower panel) considered in this work, where models 1, 2, 3, and 4 are denoted by black, dashed, dot-dashed, and dotted lines, respectively.

Extensions in AdS side

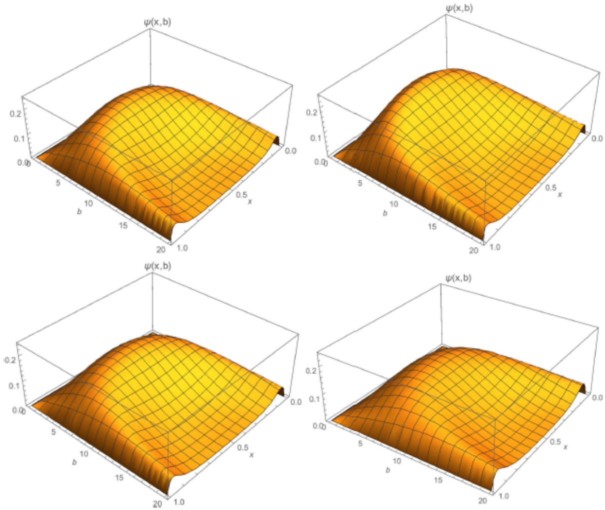


FIG. 2. The LFWF $\tilde{\psi}_{q_1\bar{q}_2}(x,b)$ for each model discussed: model 1 (left upper panel), model 2 (right upper panel), model 3 (left lower panel), and model 4 (right lower panel).

Final Comments and Conclusions

- The relation between AdS modes and the LFWF has been restricted to the hard-wall and soft-wall models with a quadratic dilaton, or phenomenological modifications on the QCD side.
- Exist plenty of AdS/QCD models that use different dilatons, or different asymptotically AdS geometries that try to catch aspects of hadronic phenomenology that the standard hard-wall or soft-wall models with quadratic dilaton do not address. By now, the LFWFs of these sorts of AdS/QCD models have not been studied
- Therefore, we believe the approach considered here could be interesting, because it allows to compute the LFWFs associated with a wide variety of AdS/QCD models.


That's all Folks!