Ratios of partial wave amplitudes in the decays of $J = 1$ and $J = 2$ mesons†

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Plan

Introduction

Meson Phenomenology

Nonets	J^{PC}	Flavor states
Pseudoscalar (P)	0^{-+}	$\pi^{\pm,0}$, $K^{\pm,0,\bar{0}}$, η_N, η_S $(\eta,\ \eta')$
Vector (V_μ)	1^{--}	$\rho^{\pm,0}$, $K^{*\pm,0,\bar{0}}$, ω_N, ω_S $(\omega,\ \phi)$.
Axial-vector (A_μ)	1^{++}	$a_1^{\pm,0}$, $K_{1,A}^{\pm,0,\bar{0}}$, $f_{1,A}^N$, $f_{1,A}^S$ $(f_1,\ f_1')$.
Pseudovector (B_μ)	1^{+-}	$b_1^{\pm,0}$, $K_{1,B}^{\pm,0,\bar{0}}$, $f_{1,B}^N$, $f_{1,B}^S$ $(h_1,\ h_1')$.
Pseudotensor ($T_{\mu\nu}$)	2^{-+}	$\Pi_2^{\pm,0}$, $K_2^{\pm,0,\bar{0}}$, η_2^N, η_2^S $(\eta_2,\ \eta_2'*)$.
Tensor ($X_{\mu\nu}$)	2^{++}	$a_2^{\pm,0}$, $K_2^{*\pm,0,\bar{0}}$, f_2^N, f_2^S $(f_2,\ f_2')$.
Axial-tensor ($W_{\mu\nu}$)	2^{--}	???, $K_2'^{\pm,0,\bar{0}}$, ???.

Table 1: The nonets, their spin and parity, and the flavor states.

The decays of interest:

$$A_\mu \to V_\mu P,\ B_\mu \to V_\mu P,\ T_{\mu\nu} \to X_{\mu\nu} P, \text{ and } T_{\mu\nu} \to V_\mu P.$$

Formalism

Partial Waves

Scattering cross-section can be decomposed into (infinitely many) partial waves:

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\right|^2 \tag{1}$$

The angular information is lost when calculating the decay width.

- Decay width is a number, scattering cross-section is a function of angles and momenta.
- Decays *do* proceed through partial waves (number of $\ell-$channels are finite, restricted by the J^P values of the parent and daughters).
- Decompose the *amplitude*.
- Helicity formalism (Jacob & Wick, 1959); Tensor formalism (Zemach, 1965); Covariant helicity formalism (Chung, 1993 & 1997).

Helicity Amplitude

Spin states: $|J, M\rangle$ (parent), $|s, \lambda\rangle$, and $|\sigma, \nu\rangle$ (daughters).

$$|J, M\rangle = |\ell, m_\ell\rangle \oplus |S, \delta\rangle, \quad |S, \delta\rangle = |s, \lambda\rangle \oplus |\sigma, \nu\rangle \tag{2}$$

The amplitude,

$$i\mathcal{M}^J(\theta, \phi; M) \propto D^{J*}_{M\delta}(\phi, \theta, 0) F^J_{\lambda\nu}; \quad \delta = \lambda - \nu \tag{3}$$

In the frame of reference where momenta: $k_{0,\mu} = (M_p, \vec{0})$ (parent), $k_{1,\mu} = (E_{d,1}, 0, 0, -k)$, and $k_{2,\mu} = (E_{d,2}, 0, 0, k)$ (daughters) ($\theta = \phi = 0$),

$$i\mathcal{M}^J(0, 0; M) \propto F^J_{\lambda\nu} \tag{4}$$

The helicity amplitudes ($F^J_{\lambda\nu}$) are related to the ℓS coupling amplitudes ($G^J_{\ell S}$) as,

$$F^J_{\lambda\nu} = \sum_{\ell S} \sqrt{\frac{2\ell + 1}{2J + 1}} \langle \ell 0 S\delta | J\delta\rangle \langle s\lambda\sigma - \nu | S\delta\rangle G^J_{\ell S} \tag{5}$$

$a_1(1260) \to \rho\pi$

The Lagrangian:

$$\mathcal{L} = i g_0 \, a_{1\mu} \rho^\mu \pi \tag{6}$$

The amplitude:

$$i\mathcal{M} = ig_0 \, \epsilon_\mu(0, M)\epsilon^{\mu *}(\vec{k}, \lambda) = -ig_0 \begin{cases} 1 & M = \lambda = \pm 1 \\ \gamma & M = \lambda = 0 \end{cases} \tag{7}$$

From Eq. (5),

$$F^1_{10} = \frac{1}{\sqrt{3}} G_0 + \frac{1}{\sqrt{6}} G_2 \tag{8}$$

$$F^1_{00} = \frac{1}{\sqrt{3}} G_0 - \sqrt{\frac{2}{3}} G_2 \tag{9}$$

The ratio of the PWAs:

$$\frac{G_2}{G_0} = \sqrt{2}\left(\frac{M_{d,1} - E_{d,1}}{2M_{d,1} + E_{d,1}}\right) = -0.045 \qquad (\text{expt.} = -0.062 \pm 0.02) \tag{10}$$

$\pi_2(1670) \to f_2(1270)\pi$

The Lagrangian:

$$\mathcal{L} = g_2\, \pi_{2,\mu\nu}\, f_2^{\mu\nu}\, \pi + g_3\, \pi_{2,\alpha\mu\nu}\, \mathfrak{f}_2^{\alpha\mu\nu}\, \pi \tag{11}$$

The amplitude:

$$i\mathcal{M}_T^{(J=2)}(0,0;M) = ig_2\, \epsilon_{\mu\nu}(\vec{0},M)\epsilon^{\mu\nu *}(\vec{k},\lambda) + i2g_3\, \Big(k_0 \cdot k_1 \epsilon_{\mu\nu}(\vec{0},M)\epsilon^{\mu\nu *}(\vec{k}_1,\lambda) - k_{0,\alpha} k_1^{\nu} \epsilon_{\mu\nu}(\vec{0},M)\epsilon^{\alpha\mu *}(\vec{k}_1,\lambda)\Big) \tag{12}$$

$$= i \begin{cases} \dfrac{(M_{d,1}^2 + 2E_{d,1}^2)}{3M_{d,1}^2} g_2 + 2g_3 \frac{M_p}{M_{d,1}^2} E_{d,1}^3 & M = \lambda = 0 \\ g_2 \dfrac{E_{d,1}}{M_{d,1}} + g_3 \frac{M_p}{M_{d,1}} (3k^2 + 2M_{d,1}^2) & M = \lambda = \pm 1 \\ g_2 + 2g_3 M_p E_{d,1} & M = \lambda = \pm 2 \end{cases} \tag{13}$$

$\pi_2(1670) \to f_2(1270)\pi$

From Eq. (5),

$$F^2_{20} = \frac{1}{\sqrt{5}}G_0 + \sqrt{\frac{2}{7}}G_2 + \frac{1}{\sqrt{70}}G_4$$
$$F^2_{10} = \frac{1}{\sqrt{5}}G_0 - \frac{1}{\sqrt{14}}G_2 + \sqrt{\frac{8}{35}}G_4$$
$$F^2_{00} = \frac{1}{\sqrt{5}}G_0 - \frac{2}{\sqrt{7}}G_2 + \frac{6}{\sqrt{35}}G_4 \qquad (14)$$

where, $G_0 = G^2_{02}$, $G_2 = G^2_{22}$ and $G_4 = G^2_{42}$.

$$\frac{G_2}{G_0} = \begin{cases} -0.023 & g_3 = 0 \\ f(g_3/g_2) & g_3 \neq 0 \end{cases} \qquad (\text{expt.} = -0.18 \pm 0.06) \qquad (15)$$

$g_2 = g_c^{PT}\sqrt{2}\cos\beta_t$ and $g_3 = g_d^{PT}\sqrt{2}\cos\beta_t$.

Lagrangian

$J = 1$

$$\mathcal{L} = ig_c^A \mathrm{Tr}\{A_\mu [V^\mu, P]\} + ig_d^A \mathrm{Tr}\{\mathcal{A}_{\mu\nu} [\mathcal{V}^{\mu\nu}, P]\}$$
$$+ g_c^B \mathrm{Tr}\{B_\mu \{V^\mu, P\}\} + g_d^B \mathrm{Tr}\{\mathcal{B}_{\mu\nu} \{\mathcal{V}^{\mu\nu}, P\}\} \quad (16)$$

$J = 2$

$$\mathcal{L} = g_c^{PT} \mathrm{Tr}\{T_{\mu\nu} \{X^{\mu\nu}, P\}\} + g_d^{PT} \mathrm{Tr}\{\mathcal{T}_{\alpha\mu\nu} \{\mathcal{X}^{\alpha\mu\nu}, P\}\}$$
$$+ ig_c^T \mathrm{Tr}\{W_{\mu\nu} \{X^{\mu\nu}, P\}\} + ig_d^T \mathrm{Tr}\{\mathcal{W}_{\alpha\mu\nu} \{\mathcal{X}^{\alpha\mu\nu}, P\}\}$$
$$+ ig_v^{PT} \mathrm{Tr}\{T_{\mu\nu} [V^\mu, \partial^\nu P]\} + ig_t^{PT} \mathrm{Tr}\{\mathcal{T}_{\alpha\mu\nu} [\mathcal{V}^{\alpha\mu}, \partial^\nu P]\}$$
$$+ g_v^T \mathrm{Tr}\{W_{\mu\nu} [V^\mu, \partial^\nu P]\} + g_t^T \mathrm{Tr}\{\mathcal{W}_{\alpha\mu\nu} [\mathcal{V}^{\alpha\mu}, \partial^\nu P]\} \quad (17)$$

where, $\mathcal{A}^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $\mathcal{B}^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$, $\mathcal{V}^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$, $\mathcal{T}_{\alpha\mu\nu} = \partial_\alpha T_{\mu\nu} - \partial_\mu T_{\alpha\nu}$, $\mathcal{X}^{\alpha\mu\nu} = \partial^\alpha X^{\mu\nu} - \partial^\mu X^{\alpha\nu}$, and $\mathcal{W}_{\alpha\mu\nu} = \partial_\alpha W_{\mu\nu} - \partial_\mu W_{\alpha\nu}$.

Lagrangian

$J = 1$

$$\mathcal{L} = i g_c^A \mathrm{Tr}\{A_\mu [V^\mu, P]\} + i g_d^A \mathrm{Tr}\{\mathcal{A}_{\mu\nu} [\mathcal{V}^{\mu\nu}, P]\} + g_c^B \mathrm{Tr}\{B_\mu \{V^\mu, P\}\} + g_d^B \mathrm{Tr}\{\mathcal{B}_{\mu\nu} \{\mathcal{V}^{\mu\nu}, P\}\} \tag{18}$$

$J = 2$

$$\mathcal{L} = g_c^{PT} \mathrm{Tr}\{T_{\mu\nu} \{X^{\mu\nu}, P\}\} + g_d^{PT} \mathrm{Tr}\{\mathcal{T}_{\alpha\mu\nu} \{\mathcal{X}^{\alpha\mu\nu}, P\}\} + i g_c^{AT} \mathrm{Tr}\{W_{\mu\nu} \{X^{\mu\nu}, P\}\} + i g_d^{AT} \mathrm{Tr}\{\mathcal{W}_{\alpha\mu\nu} \{\mathcal{X}^{\alpha\mu\nu}, P\}\} + i g_v^{PT} \mathrm{Tr}\{T_{\mu\nu} [V^\mu, \partial^\nu P]\} + i g_t^{PT} \mathrm{Tr}\{\mathcal{T}_{\alpha\mu\nu} [\mathcal{V}^{\alpha\mu}, \partial^\nu P]\} + g_v^{AT} \mathrm{Tr}\{W_{\mu\nu} [V^\mu, \partial^\nu P]\} + g_t^{AT} \mathrm{Tr}\{\mathcal{W}_{\alpha\mu\nu} [\mathcal{V}^{\alpha\mu}, \partial^\nu P]\} \tag{19}$$

Contact interactions (No derivatives), Derivative interactions (2 derivatives),
Vector interactions (1 derivative), Tensor interactions (3 derivatives).

Mixing

$$\begin{pmatrix} |f_1(1285)\rangle \\ |f_1(1420)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_a & -\sin\theta_a \\ \sin\theta_a & \cos\theta_a \end{pmatrix} \begin{pmatrix} |\bar{n}n\rangle_a \\ |\bar{s}s\rangle_a \end{pmatrix} \tag{20}$$

$$\begin{pmatrix} |h_1(1170)\rangle \\ |h_1(1415)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{pv} & -\sin\theta_{pv} \\ \sin\theta_{pv} & \cos\theta_{pv} \end{pmatrix} \begin{pmatrix} |\bar{n}n\rangle_{pv} \\ |\bar{s}s\rangle_{pv} \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} |K_2(1770)\rangle \\ |K_2(1820)\rangle \end{pmatrix} = \begin{pmatrix} \cos\beta_D & \pm i\sin\beta_D \\ \pm i\sin\beta_D & \cos\beta_D \end{pmatrix} \begin{pmatrix} |1^1D_2\rangle \\ |1^3D_2\rangle \end{pmatrix} \tag{22}$$

i needed because of mixing of opposite $C-$states; $\pm i$ needed as vector and tensor modes of decay have opposite $C-$behavior.

Amplitudes

$J = 1$

$$
\begin{aligned}
i\mathcal{M} &= ig_c\ \epsilon_\mu(0,M)\epsilon^{\mu*}(\vec{k},\lambda) + i2g_d\left(k_0\cdot k_1\ \epsilon^\mu(\vec{0},M)\epsilon^*_\mu(\vec{k}_1,\lambda)\right.\\
&\left. -k_0^\nu\ k_{1,\mu}\ \epsilon^\mu(\vec{0},M)\epsilon^*_\nu(\vec{k}_1,\lambda)\right)\\
&= -i\begin{cases} g_c + 2g_d M_p E_{d,1} & M=\lambda=\pm1\\ \gamma(g_c + 2g_d M_p E_{d,1} - 2g_d M_p\beta k) & M=\lambda=0\end{cases}
\end{aligned}
\tag{23}
$$

$g_c \in \{g_c^A, g_c^B\}$, $g_d \in \{g_d^A(=0), g_d^B\}$, $\gamma = \frac{E_{d,1}}{M_{d,1}}$, and $\beta = \frac{k}{E_{d,1}}$.

Amplitudes

$J = 2$, vector mode:

$$i\mathcal{M} = -g_v^{PT}\,\epsilon_{\mu\nu}(\vec{0}, M)\epsilon^{\mu *}(\vec{k}_1, \lambda)k_2^{\nu} - g_t^{PT}\Big[2k_0 \cdot k_1 \epsilon_{\mu\nu}(\vec{0}, M)\epsilon^{\mu *}(\vec{k}_1, \lambda) - 2k_{0,\mu}k_1^{\nu}\epsilon_{\alpha\nu}(\vec{0}, M)\epsilon^{\mu *}(\vec{k}_1, \lambda)k_2^{\nu}\Big] \quad (24)$$

$$= \frac{k}{\sqrt{2}}\begin{cases} g_v^{PT} + 2g_t^{PT} M_p E_{d,1} & M = \lambda = \pm 1 \\ \frac{2}{\sqrt{3}}\left(\frac{E_{d,1}}{M_{d,1}} g_v^{PT} + 2g_t^{PT}\frac{M_p}{M_{d,1}}(2k^2 + M_{d,1}^2)\right) & M = \lambda = 0 \end{cases} \quad (25)$$

PWA from amplitude

$J = 2$, vector mode:

$$F^2_{10} = \sqrt{\frac{3}{10}} G_1 + \frac{1}{\sqrt{5}} G_3$$
$$F^2_{00} = \sqrt{\frac{2}{5}} G_1 - \sqrt{\frac{3}{5}} G_3 \tag{26}$$

where, $G_1 = G^2_{01}$ and $G_3 = G^2_{31}$.

$$\frac{G_3}{G_1} = \begin{cases} -0.13 & g_t^{PT} = 0 \\ f(g_t^{PT}/g_v^{PT}) & g_t^{PT} \neq 0 \end{cases} \qquad (\text{expt.} = -0.72 \pm 0.2) \tag{27}$$

Decay Widths

The decays widths:

$$\Gamma_{1\to 10} = \frac{k}{24\pi M_p^2}\left(2(g_c + 2g_d M_p E_{d,1})^2 + \frac{1}{M_{d,1}^2}\left(g_c E_{d,1} + 2g_d M_p M_{d,1}^2\right)^2\right) \tag{28}$$

$$\Gamma_{2\to 20} = \frac{k}{40\pi M_p^2}\left[(g_c + 2g_d M_p E_{d,1})^2\left(\frac{4k^4}{9M_{d,1}^4} + \frac{10k^2}{3M_{d,1}^2} + 5\right) + 2g_d^2\frac{k^4 M_p^2}{9M_{d,1}^2}\left(\frac{8k^2}{M_{d,1}^2} + 17\right)\right] \tag{29}$$

$$\Gamma_{2\to 10} = \frac{k}{40\pi M_p^2}\left[(g_v^{PT} + 2g_t^{PT} M_p E_{d,1})^2\frac{k^3}{3}\left(\frac{2k^2}{M_{d,1}^2} + 5\right) + \frac{8}{3}(g_t^{PT})^2\frac{M_p^2}{M_{d,1}^2}k^6 - \frac{8}{3}g_t^{PT}(g_v^{PT} + 2g_t^{PT} M_p E_{d,1})k^4\frac{M_p E_{d,1}}{M_{d,1}^2}\right] \tag{30}$$

Results and Discussion

$J = 1$

g_c^A (GeV)	g_c^B (GeV)	g_d^B (GeV^{-1})	θ_a	θ_{pv}
-3.26 ± 0.14	-6.63 ± 0.73	4.37 ± 0.38	$(26.8 \pm 1.9)^\circ$	$(25.0 \pm 5.7)^\circ$

Table 2: The values of the parameters used in the Lagrangian.

- Partial widths of $a_1(1260) \to \rho\pi$ and $f_1'(1420) \to K^*K$ decays used to find g_c^A and θ_a.
- $D/S-$ratio and partial width of $b_1(1235) \to \omega\pi$ decay, and partial width of $h_1(1415) \to K^*K$ decay used to estimate the values of g_c^B, g_d^B, & θ_{pv}.
- The value mixing angle for 1^{++} singlets ($(26.8 \pm .19)^\circ$) matches the expt value ($(24^{+4.0}_{-3.4})^\circ$(LHCb, 2013)).
- Mixing angle for 1^{+-} singlets is considerably larger (at $(25.0 \pm 5.7)^\circ$ as opposed to the expt value ($(0.6 \pm 2.6)^\circ$ (BESIII, 2017)).

$J = 1$

Decay	$\lvert\vec{k}\rvert$ (MeV)	D/S	
		(Theor.)	(Expt.)
$a_1(1260) \to \rho\pi$	352	-0.045 ± 0.018	-0.062 ± 0.02
$f_1'(1420) \to K^{*\pm}K^{\pm}$	161	-0.0076 ± 0.0008	
$b_1(1235) \to \omega\pi$	347.2	0.277 ± 0.05	0.277 ± 0.027
$h_1(1170) \to \rho\pi$	303	0.28 ± 0.13	
$h_1'(1415) \to K^{*\pm}K^{\pm}$	139.1	0.021 ± 0.007	

Table 3: The ratios of PWA for some the axial- and pseudo-vector meson decays.

- Local interactions are sufficient to reproduce the experimental results for 1^{++}, non-kaonic states.
- Non-local interactions are essential for the 1^{+-} states.
- The value of the D/S–ratio decreases as the 3-momentum carried by the decay products decreases.

$J = 1$

Decay	Width (MeV)	
	(Theor.)	(Expt.)
$a_1(1260) \to \rho\pi$	420 ± 170	dominant (420 ± 35)
$f_1'(1420) \to K^*K$	44.5 ± 4.8	44.5 ± 4.2
$b_1(1235) \to \omega\pi$	110 ± 20	110 ± 7
$h_1(1170) \to \rho\pi$	133 ± 62	seen
$h_1'(1415) \to K^*K$	91 ± 30	seen ($\Gamma_{tot} = 90 \pm 20$)

Table 4: The decay widths of the axial-vector and pseudovector mesons.

$J = 2$, tensor mode

g_c^{PT} (GeV)	g_d^{PT} (GeV^{-1})	g_c^{T} (GeV)	g_d^{T} (GeV^{-1})	β_D
49.67 ± 18.79	-12.14 ± 4.23	14.73	-5.06 ± 0.38	$(35.85 \pm 0.46)^\circ$

Table 5: The values of the parameters used in the Lagrangian.

- $D/S-$ratio and width of $\pi_2(1670) \to f_2(1270)\pi$ (PDG, 2020), widths of $K_2(1770) \to K_2^*(1430)\pi$ and $f_2(1270)K$ (Koenigstein, 2016) decays used to estimate g_c^i and g_d^i, (i = PT, AT), and β_D.
- The kaon mixing angle has nearly the same magnitude as (($\beta_D \approx -39^\circ$) Barnes, 2003), but opposite sign.
- The ratio $g_c^i/g_d^i \sim M_P^2$.

$J = 2$, tensor mode

Decay	$\vert\vec{k}\vert$ (MeV)	$D/S\left(\frac{G_2}{G_0}\right)$ (Theor.)	$D/S\left(\frac{G_2}{G_0}\right)$ (Expt.)	$G/S\left(\frac{G_4}{G_0}\right)$ (Theor.)
$\pi_2(1670) \to f_2\pi$	325.3	-0.18 ± 0.1	-0.18 ± 0.06	0.091 ± 0.051
$\pi_2(1670) \to f_2'\pi$	58.8	-0.0021 ± 0.0012	$\times\times\times$	0.001 ± 0.0005
$K_2(1770) \to K_2^*\pi$	285.3	0.083 ± 0.045	$\times\times\times$	-0.041 ± 0.022
$K_2(1770) \to f_2K$	53.4	0.0036 ± 0.002	$\times\times\times$	-0.0018 ± 0.001

Table 6: The ratios of PWA for some the pseudo-tensor meson decays.

- Non-local interactions are essential to explain the D/S−ratio as well as the F/P−ratio of the decay of pseudotensors.
- G−waves ($\ell = 4$) are as important as the D−waves ($\ell = 2$).
- D−waves and G−waves have opposite phase.
- D−waves interfere with S−waves destructively in the iso-vector decay.
- D−waves interfere with S−waves constructively in the kaon decays.

$J = 2$, vector mode

g_v^{PT}	g_t^{PT} (GeV^{-2})	g_v^{T}	g_t^{T} (GeV^{-2})
1.798 ± 0.16	0.638 ± 0.057	8.23 ± 0.4	0.95 ± 0.14

Table 7: The values of the parameters used in the Lagrangian.

- $F/P-$ratio and width of $\pi_2(1670) \to \rho\pi$ (PDG, 2020), widths of $K_2(1770) \to K^*\pi$, $K^*\eta$ and ρK (Koenigstein, 2016) decays used to estimate g_v^i, and g_t^i (i = PT, AT).
- The ratio $g_v^i/g_t^i \sim M_P^2$.

$J = 2$, vector mode

Decay	$\lvert\vec{k}\rvert$ (MeV)	$F/P\left(\frac{G_3}{G_1}\right)$	
		(Set-1)	(Expt.)
$\pi_2(1670) \to \rho\pi$	646	-0.72 ± 0.2	-0.72 ± 0.16
$\pi_2(1670) \to K^*K$	452.4	-0.476 ± 0.098	$\times\times\times$
$K_2(1770) \to \rho K$	611.3	-0.113 ± 0.034	$\times\times\times$
$K_2(1770) \to \omega K$	607.3	-0.11 ± 0.025	$\times\times\times$
$K_2(1770) \to \phi K$	440.6	-0.038 ± 0.009	$\times\times\times$
$K_2(1770) \to K^*\pi$	652.8	-0.099 ± 0.022	$\times\times\times$
$K_2(1770) \to K^*\eta$	507.4	-0.064 ± 0.015	$\times\times\times$

Table 8: The $F/P-$ratios for some the pseudo-tensor meson decaying to vector and pseudoscalar mesons.

$J = 2$, decay widths

Decay	Width (MeV)		
	(Theor.)	(Expt.)	(Ref.)
$\pi_2(1670) \to f_2\pi$	146.4 ± 81.5	146.4 ± 9.7	146.4 ± 9.7
$\pi_2(1670) \to f_2'\pi$	8.64 ± 4.67		0.1 ± 0.1
$\pi_2(1670) \to \rho\pi$	80.6 ± 22.4	80.6 ± 10.8	80.6 ± 10.8
$\pi_2(1670) \to K^*K$	26.9 ± 5.5	10.9 ± 3.7	11.7 ± 1.6
$K_2(1770) \to K_2^*\pi$	84.5 ± 45.2	seen	84.5 ± 5.6
$K_2(1770) \to f_2K$	5.8 ± 3.19	seen	5.8 ± 0.4
$K_2(1770) \to \rho K$	14.5 ± 4.32		22.2 ± 3.0
$K_2(1770) \to \omega K$	7.02 ± 1.59	seen	8.3 ± 1.1
$K_2(1770) \to \phi K$	2.35 ± 0.53	seen	4.2 ± 0.6
$K_2(1770) \to K^*\pi$	27.2 ± 6.35	seen	25.5 ± 3.4
$K_2(1770) \to K^*\eta$	7.8 ± 1.8		10.5 ± 1.4

Table 9: The decay widths for some the pseudo-tensor meson decays.

Pseudotensor iso-singlets

- $\eta_2(1645)$ and $\eta_2(1870)^\dagger$ arise due to iso-singlet mixing.
- Sign and magnitude of the mixing angle (β_{pt}) is disputed.
- Large β_{pt} implies (unphysically) large $\eta_2(1870) \to a_2\pi$ decay width.
- $\beta_{pt} = 0$ implies $\eta_2(1870) \to a_2\pi$ decay is forbidden.
- β_{pt} must be small but non-zero.

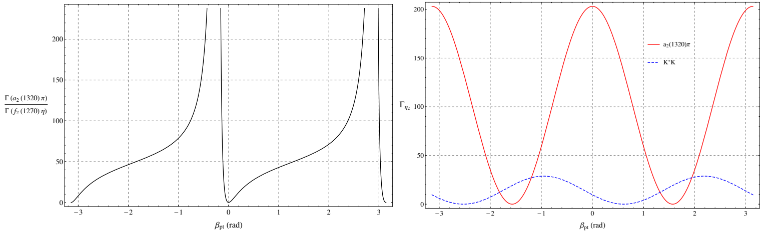

Figure 1: (L) Ratio of DW of $\eta_2(1870)$to $a_2\pi/f_2\eta$; (R) DW of $\eta_2(1645)$ to $a_2\pi, K^*K$.

† Status of $\eta_2(1870)$ is disputed (Klempt, 2007; Anisovich, 2010; Anisovich, 2000).

Conclusion

Conclusion

- Covariant helicity formalism has been used to analyse the decay of $J = 1, 2$ mesons.
- Non-local interactions are as important as the local interactions to explain the partial wave amplitudes of meson decays.
- In the decay of axial-vector (pseudovector) mesons, $D-$waves interfere destructively (constructively) with the $S-$waves.
- The mixing angle in the pseudovector iso-singlets is larger than the expt value.
- The $D-$waves interfere destructively with the $G-$waves in pseudotensor decays.
- Amplitudes of the higher partial waves become smaller as $|\vec{k}|$ decreases.
- Kaons (K_2) mix substantially; more data needed to estimate.
- Pseudotensor iso-singlet mixing is (probably) small but non-zero.

More details: arxiv:hep-ph/2107.13501

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Thank You!

Additional Info

Choice of ℓ

Example: $2^{-+} \to 2^{++}0^{-+}$

$$J = 2, s = 2, \sigma = 2$$

$$\Rightarrow S = 2$$

$$J \in |\ell - S| \ldots (\ell + S)$$

$$\text{For the given } J \text{ and } S, \qquad \ell = 0, 1, 2, 3, 4.$$

$$\text{Parity} \Rightarrow (-1)^{\ell+S+1} = -1$$

$$\Rightarrow (-1)^{\ell} = 1 \qquad \Rightarrow \ell \in \{\text{even}\}$$

$$\boxed{\ell = 0, 2, 4}$$

PWAs

PWAs for $J = 1$:

$$G_2 = \sqrt{\frac{2}{3}} \left(g_c \left(\frac{M_{d,1} - E_{d,1}}{M_{d,1}} \right) + 2 g_d M_p (E_{d,1} - M_{d,1}) \right) \tag{31}$$

$$G_0 = \frac{1}{\sqrt{3}} \left(g_c \left(\frac{2M_{d,1} + E_{d,1}}{M_{d,1}} \right) + 2 g_d M_p (2E_{d,1} + M_{d,1}) \right) \tag{32}$$

PWAs for $J = 2$, vector decay mode:

$$G_1 = \frac{1}{M_{d,1}} \sqrt{\frac{2}{15}} \Big(-2 g_t^{PT} M_p \left(3E_{d,1} M_{d,1} + 4k^2 + 2M_{d,1}^2 \right) + g_v^{PT} (2E_{d,1} + 3M_{d,1}) \Big) \tag{33}$$

$$G_3 = \frac{2}{\sqrt{5} M_{d,1}} \left(2 g_t^{PT} M_p \left(E_{d,1} M_{d,1} - 2k^2 - M_{d,1}^2 \right) + g_v^{PT} (E_{d,1} - M_{d,1}) \right) \tag{34}$$

PWAs

PWAs for $J = 2$, tensor decay mode:

$$G_0 = \frac{\sqrt{5}}{3\left(16+3\sqrt{2}\right)M_{d,1}^2}\Big[\Big(18\sqrt{2}E_{d,1}^3 g_d M_P + 6\sqrt{2}E_{d,1}^2 g_C + 4E_{d,1}M_{d,1}(5g_C + 14g_d M_{d,1}M_P)$$

$$+M_{d,1}\left(\left(28+3\sqrt{2}\right)g_C M_{d,1} + 20 g_d M_P\left(3k^2+2M_{d,1}^2\right)\right)\Big)\Big] \quad (35)$$

$$G_2 = -\frac{\sqrt{14}}{3\left(16+3\sqrt{2}\right)M_{d,1}^2}\Big[\Big(6\sqrt{2}E_{d,1}^3 g_d M_P + 2\sqrt{2}E_{d,1}^2 g_C + E_{d,1}M_{d,1}\Big(\left(12+\sqrt{2}\right)g_C$$

$$-8\left(3+\sqrt{2}\right)g_d M_{d,1}M_P\Big) + M_{d,1}\left(\left(12+\sqrt{2}\right)g_d M_P\left(3k^2+2M_{d,1}^2\right) - 3\left(4+\sqrt{2}\right)g_C M_{d,1}\right)\Big)\Big] \quad (36)$$

$$G_4 = \frac{2\sqrt{70}}{3\left(16+3\sqrt{2}\right)M_{d,1}^2}\Big[\Big(-3\sqrt{2}E_{d,1}^3 g_d M_P - \sqrt{2}E_{d,1}^2 g_C + E_{d,1}M_{d,1}\Big(\left(2+\sqrt{2}\right)g_C$$

$$+\left(\sqrt{2}-4\right)g_d M_{d,1}M_P\Big) + M_{d,1}\left(\left(2+\sqrt{2}\right)g_d M_P\left(3k^2+2M_{d,1}^2\right) - 2g_C M_{d,1}\right)\Big)\Big] \quad (37)$$

Ratios of PWAs for η_2 and η_2' decays

Decay	D/S	G/S	F/P
$\eta_2(1645) \to a_2(1320)\pi$	-0.122 ± 0.065	0.063 ± 0.034	
$\eta_2(1645) \to K^*K$			-0.51 ± 0.09
$\eta_2(1870) \to a_2(1320)\pi$	-0.125 ± 0.067	0.056 ± 0.030	
$\eta_2(1870) \to f_2(1270)\eta$	-0.017 ± 0.0095	0.0085 ± 0.0047	
$\eta_2(1870) \to K^*K$			-0.46 ± 0.09

Table 10: The ratios of the PWAs for some decays of $\eta_2(1645)$ and $\eta_2(1870)$.

Widths of η_2 and η_2' decays

Decay	Width (MeV)				
	$\beta_{pt} = 12.4^\circ$	$\beta_{pt} = -42^\circ$	$\beta_{pt} = 0.0^\circ$	$\beta_{pt} = -1.98^\circ$	$\beta_{pt} = 2.8^\circ$
$\eta_2(1645) \to a_2(1320)\pi$	194 ± 103	112.2 ± 59.8	203 ± 108	203 ± 108	202 ± 108
$\eta_2(1645) \to K^*K$	4.4 ± 0.8	0.39 ± 0.07	9.6 ± 1.7	10.5 ± 1.9	8.3 ± 1.5
$\eta_2(1870) \to a_2(1320)\pi$	241 ± 130	2347 ± 1266	0	6.3 ± 3.4	12.6 ± 6.8
$\eta_2(1870) \to f_2(1270)\eta$	18.2 ± 10.1	65.7 ± 36.3	5.1 ± 2.81	3.7 ± 2.0	7.4 ± 4.1
$\eta_2(1870) \to K^*K$	75.6 ± 14.8	87.8 ± 17.1	59.4 ± 11.6	56.4 ± 11.0	63.4 ± 12.4

Table 11: The widths of some decays of $\eta_2(1645)$ and $\eta_2(1870)$. The choice of angles is discussed in the text.

Iso-singlet mixing

"Physical states are admixtures of flavor states."

Iso-singlet mixing due to $U(1)_A$ breaking (mixing between singlet and octet states or $|\bar{n}n\rangle$ and $|\bar{s}s\rangle$ states).

- Large mixing angle ($\theta \sim -40^\circ$) among the pseudoscalars $\Rightarrow$ mass difference between η and η'.
- Small mixing angle ($\theta \sim -3^\circ$) among the vectors $\Rightarrow$ ω and ϕ are nearly the same as flavor states.
- Similar mixing observed among other iso-singlets:
  $f_1(1285) - f_1'(1420)(\theta \sim +24^\circ$ (LHCb, 2013)),
  $h_1(1170) - h_1(1415)(\theta \sim \pm 3^\circ *$ (BESIII, 2017)), etc.

Kaon mixing

"Physical states are admixtures of flavor states."

"Kaons no C"!

- Strange states with same J^P can mix.
- Axial kaons ($K_1(1270)$, $K_1(1400)$) are mixtures of the $K_{1,A}$ and $K_{1,B}$ states ($\theta \in [-35°, +72°]$ $*$ (Tayduganov, 2012)).
- Kaon mixing in $J = 1^+$ sector decides the mixing angles of the respective iso-singlets, through the Gell-Mann-Okubo mass relations (Cheng, 2021; source of uncertainty: large width of $a_1(1260)$).
- 2^- kaons ($K_2(1770)$, $K_2(1820)$) are mixtures of ${}^1D_2$ (2^{-+}) and ${}^3D_2$ (2^{--}) states ($\theta \sim -39°$ $*$ (Barnes, 2003)).