Transverse-momentum-dependent Parton Distribution Function Within Basis Light-front Quantization Framework

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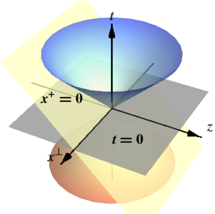
Table of Contents

Light-front quantization and Hamiltonian formalism (Brodsky, Pauli, and Pinsky 1998)

- $v = (v^+, v^-, v^\perp) = (v^0 + v^3, v^0 - v^3, v^1, v^2)$



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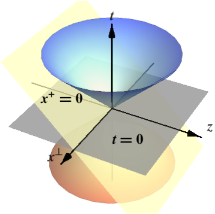

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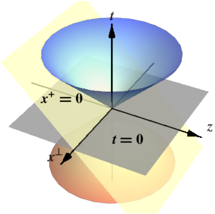

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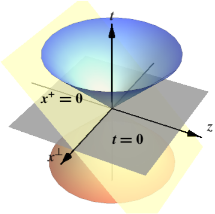
- $\hat{H} |P, \Lambda\rangle = M^2 |P, \Lambda\rangle$
- $$|P, \Lambda\rangle = \sum_n \sum_{\lambda_1 \ldots \lambda_n} \int \prod_{i=1}^n \left[\frac{dx_i d^2 k_i^\perp}{\sqrt{x_i} 16\pi^3} \right] 16\pi^3 \delta \left(1 - \sum_{i=1}^n x_i \right) \delta^{(2)} \left(\sum_{i=1}^n k_i^\perp \right) \psi^\Lambda_{\lambda_1 \ldots \lambda_n}(\{x_i, k_i^\perp\}) |n, x_i P^+, x_i P^\perp + k_i^\perp, \lambda_i\rangle\rangle$$

Light-front quantization and Hamiltonian formalism (Brodsky, Pauli, and Pinsky 1998)



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- Effect of light-front boost (Soper 1971)
  - Longitudinal boost serves merely to rescale the other Poincáre generators
  - Transverse boost reduce to be Galilean.

Light-front quantization and Hamiltonian formalism (Brodsky, Pauli, and Pinsky 1998)

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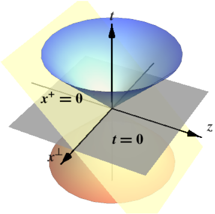


LFWF $\psi^{\Lambda}_{\lambda_1\ldots\lambda_n}(\{x_i, k_i^\perp\})$ contains all the information about the internal structure of the bound state $|P,\Lambda\rangle$ and is also invariant under boost.

Discrete basis and LFWF $\psi^{\Lambda}_{\lambda_1 \ldots \lambda_n}(\{x_i, k_i^{\perp}\})$

How to solve this eigen equation ?

$$\hat{H} \, |P, \Lambda\rangle = M^2 \, |P, \Lambda\rangle$$

Discrete basis and LFWF $\psi^{\Lambda}_{\lambda_1 \ldots \lambda_n}(\{x_i, k_i^{\perp}\})$

Solution of BLFQ

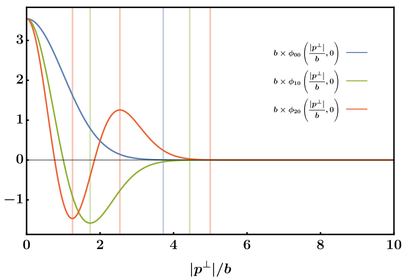
Discrete basis and their direct product

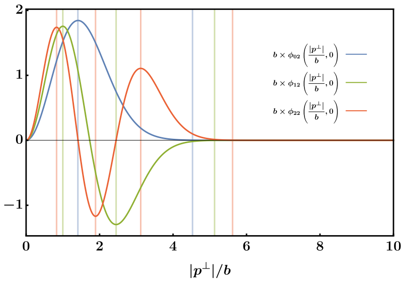
Truncation

Discrete basis and LFWF $\psi^{\Lambda}_{\lambda_1 \ldots \lambda_n}(\{x_i, k_i^{\perp}\})$

Solution of BLFQ

Discrete basis and their direct product	Truncation
2D HO $\phi_{nm}(p^{\perp})$ in the transverse plane	$\sum_i (2n_i + \lvert m_i \rvert + 1) \leq N_{\max}$





Discrete basis and LFWF $\psi^{\Lambda}_{\lambda_1\ldots\lambda_n}(\{x_i, k_i^{\perp}\})$

Solution of BLFQ

Discrete basis and their direct product	Truncation
2D HO $\phi_{nm}(p^{\perp})$ in the transverse plane	$\sum_i (2n_i + \|m_i\| + 1) \le N_{\max}$
Plane-wave in the longitudinal direction	$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$

Solution of BLFQ

Discrete basis and their direct product	Truncation
2D HO $\phi_{nm}(p^{\perp})$ in the transverse plane	$\sum_i (2n_i + \lvert m_i \rvert + 1) \leq N_{\max}$
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Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$

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Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$	Fock sector truncation
$\lvert\alpha\rangle = \otimes_i \lvert\alpha_i\rangle$	

Discrete basis and LFWF $\psi^{\Lambda}_{\lambda_1 \ldots \lambda_n}(\{x_i, k_i^{\perp}\})$

We finally get the LFWF we want

$$\hat{H}\,|P,\Lambda\rangle = M^2\,|P,\Lambda\rangle$$

$$\Downarrow$$

$$\sum_{|\beta\rangle} \langle\alpha|H|\beta\rangle\,\langle\beta|P,\Lambda\rangle = M^2\,\langle\alpha|P,\Lambda\rangle$$

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Eigen problem of a **VERY BIG** matrix

Discrete basis and LFWF $\psi^{\Lambda}_{\lambda_1 \dots \lambda_n}(\{x_i, k_i^{\perp}\})$

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Eigen problem of a **VERY BIG** matrix

A bunch of mature algorithms exist

Need to use the supercomputer

Discrete basis and LFWF $\psi^{\Lambda}_{\lambda_1\ldots\lambda_n}(\{x_i, k_i^{\perp}\})$

We finally get the LFWF we want

$$\hat{H}\,|P,\Lambda\rangle = M^2\,|P,\Lambda\rangle$$

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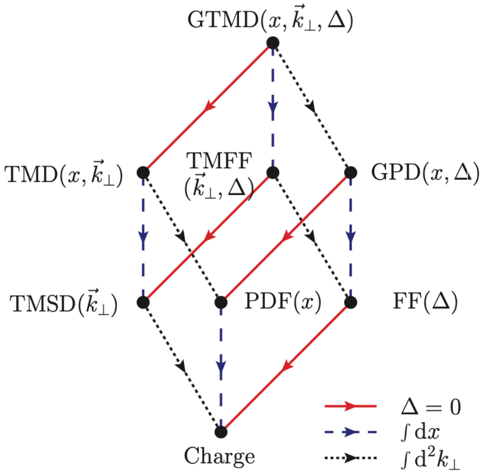
$$\sum_{|\beta\rangle} \langle\alpha|H|\beta\rangle\,\langle\beta|P,\Lambda\rangle = M^2\,\langle\alpha|P,\Lambda\rangle$$

$$\Downarrow\ \langle\alpha|P,\Lambda\rangle = \psi(\alpha)$$

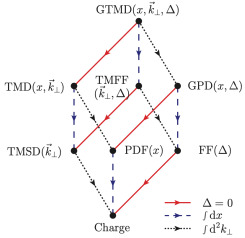
$$\psi^{\Lambda}_{\lambda_1\ldots\lambda_n}(\{x_i, k_i^{\perp}\}) = \sum_{\{n_i,m_i\}}\left[\psi(\alpha)\times\prod_i \phi^{m_i}_{n_i}(p_i^{\perp})\right]$$



$$W^{[\Gamma]}_{\lambda\lambda'}(P, k, \Delta, N; \eta) = \frac{1}{2}\int \frac{d^4 z}{2(2\pi)^4} e^{ik\cdot z} \langle p', \lambda' |\, \bar{\psi}(-\tfrac{1}{2}z)\, \Gamma\, \mathcal{W}(-\tfrac{1}{2}z, \tfrac{1}{2}z \,|\, n)\, \psi(\tfrac{1}{2}z)\, | p, \lambda\rangle$$

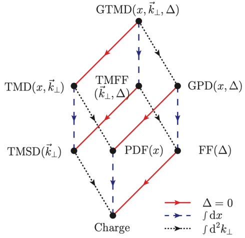
Plot from (B. Pasquini and Lorce' 2012)

- Reduction from GTMD.



$$\Phi^{[\Gamma]}(x, k^\perp; P, S) = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z^\perp}{2(2\pi)^3} e^{ik\cdot z} \times \langle P, S|\, \bar{\Psi}(0)\Gamma\mathcal{U}(0^\perp, z^\perp)\Psi(z)\,|P, S\rangle\,|_{z^+=0}$$

- Reduction from GTMD.
- Parameterization.

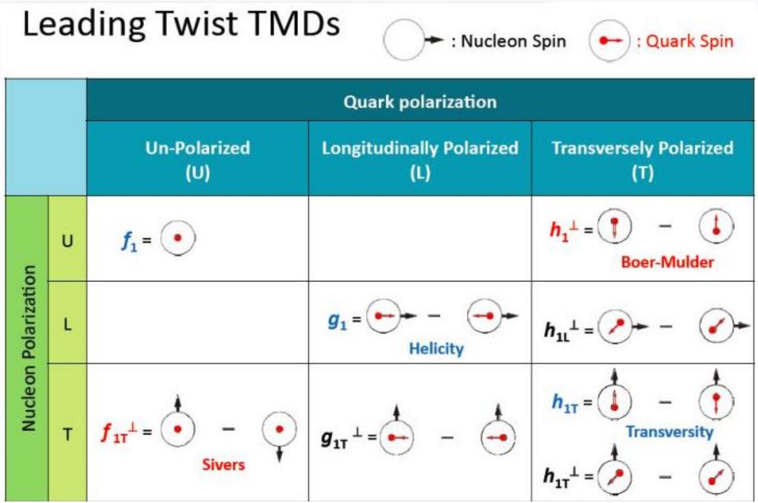


$$\Phi^{[\gamma^+]}(x, k^\perp; P, S) = f_1^e - \frac{\epsilon^{ij} k^i S^j}{M_e} f_{1T}^{\perp e}\,,$$

$$\Phi^{[\gamma^+\gamma^5]}(x, k^\perp; P, S) = S^3 g_{1L}^e + \frac{k^\perp \cdot S^\perp}{M_e} g_{1T}^e\,,$$

$$\Phi^{[i\sigma^{j+}\gamma^5]}(x, k^\perp; P, S) = S^j h_1^e + S^3 \frac{k^j}{M_e} h_{1L}^{\perp e} + S^i \frac{2k^i k^j - (k^\perp)^2 \delta^{ij}}{2M_e^2} h_{1T}^{\perp e} + \frac{\epsilon^{ij} k^i}{M_e} h_1^{\perp e}\,.$$

- Reduction from GTMD.
- Parameterization.
- Eight leading twist TMDs.



Plot taken from Prof. Yuan Feng's talk on 8/27/2014.

- Reduction from GTMD.
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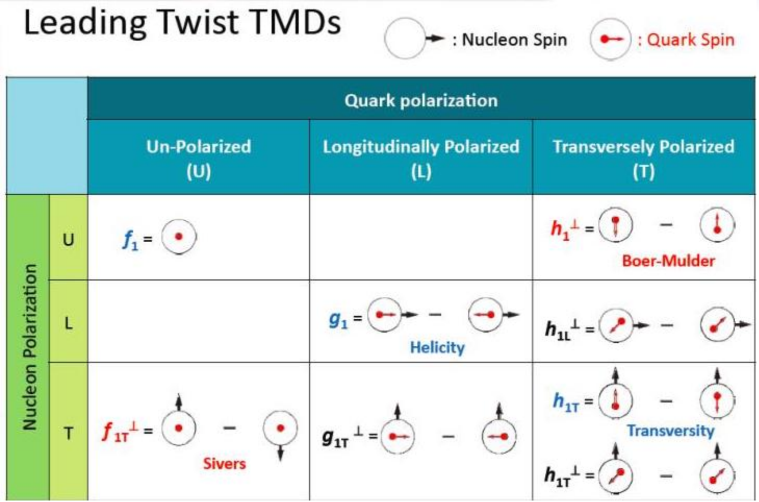
- Eight leading twist TMDs.
- Two of them (red in Prof. Yuan Feng's plot) need more care.



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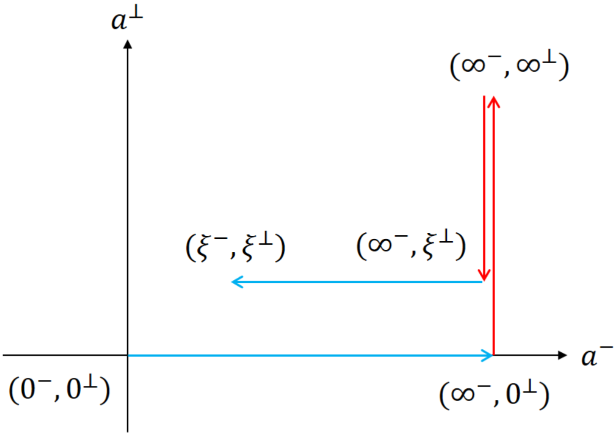


Schematic plot of integration contour of gauge link, taken SIDIS as example.

Basics of the electron calculation (Hu et al. 2021)

Hamiltonian

$$\hat{H} = \hat{H}_{\text{QED}} + \hat{H}'$$

$$\hat{H}_{\text{QED}} = P^+ \hat{P}^-_{\text{QED}} - (\hat{P}^\perp)^2$$

$$P^-_{\text{QED}} = \int \mathrm{d}^2 x^\perp \mathrm{d}x^- \left[\frac{1}{2}\bar{\Psi}\gamma^+ \frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2}A^j (i\partial^\perp)^2 A^j + e j^\mu A_\mu\right]$$

$$\hat{H}' = \lambda_L(\hat{H}_{CM} - 2b^2 \hat{I})$$

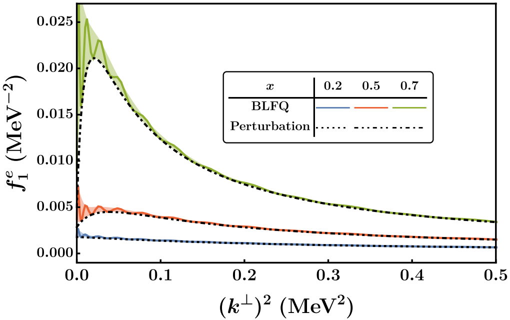
Truncations and Parameter

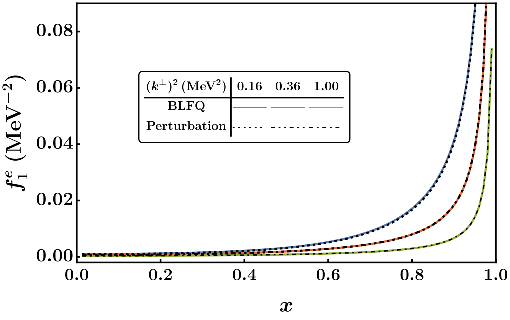
$$N_{\max} = 100, \quad K = 100, \quad |e_{\text{phy}}\rangle = |e\rangle + |e\gamma\rangle$$

we use physical coupling α

$b = M_e$ and bare electron mass $m_e = M_e + \Delta m_e$, where Δm_e is the mass counter-term

Good agreement with perturbative calculation (Bacchetta, Mantovani, and Barbara Pasquini 2016)





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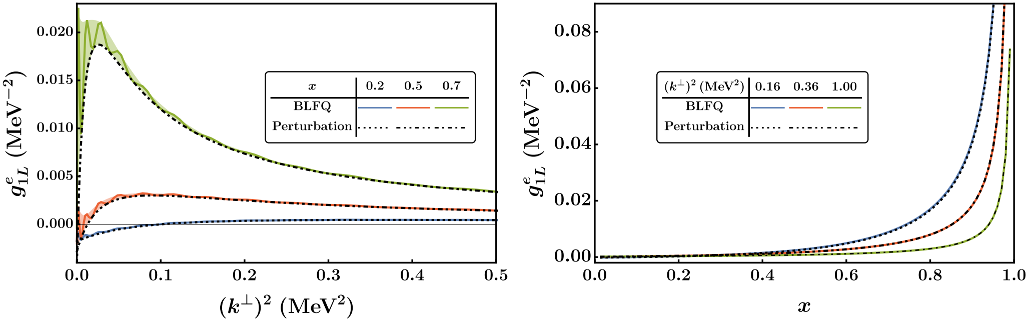

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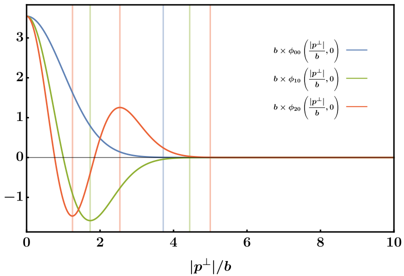
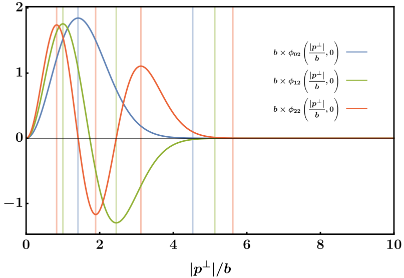
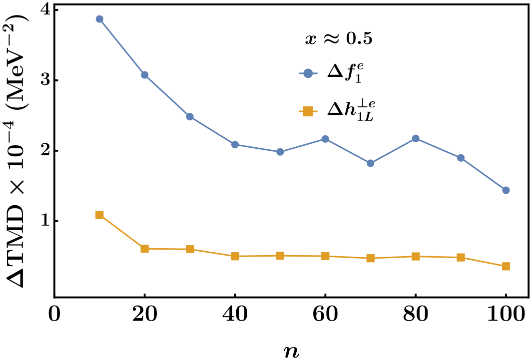
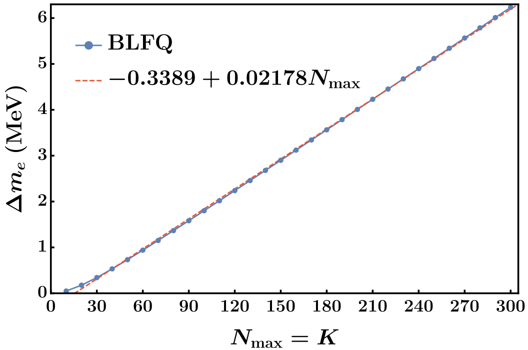




Convergence of the electron results

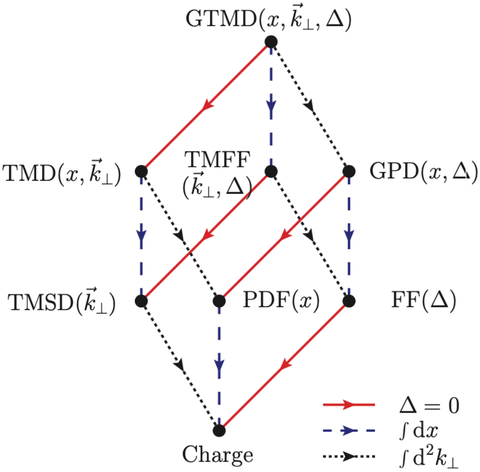


Decrease of the oscillation in the transverse plane of BLFQ results with increasing N_{max}



Increase of mass counter-term with increasing N_{max}

Self-consistency check



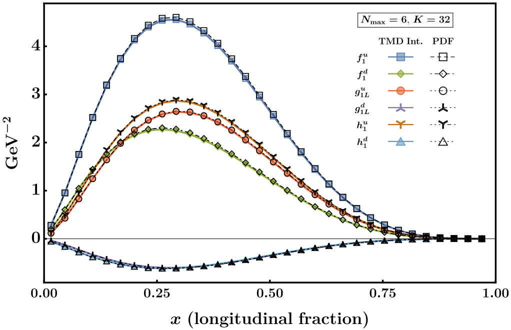
We use the same parameter set as (Mondal et al. 2020).

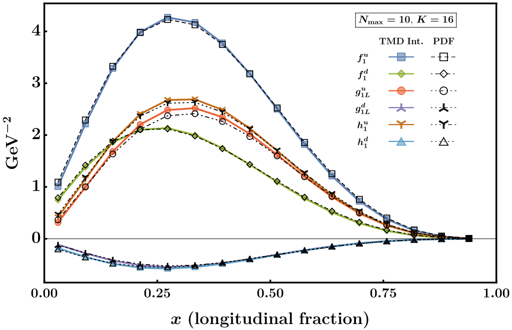
$$\int d^2k^\perp \, \mathrm{TMD}(x, k^\perp)$$

$$\| ?$$

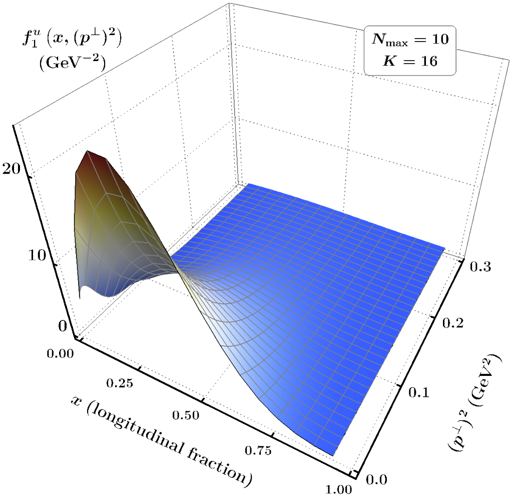
$$\mathrm{GPD}(x, \Delta = 0)$$

Self-consistency check

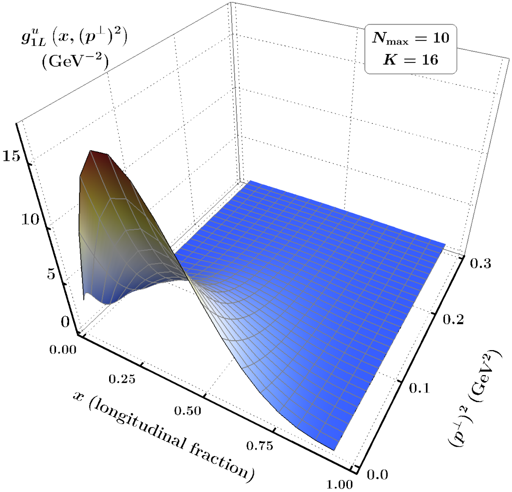




TMD results



$f_1^u\,(x,(p^\perp)^2)$
(GeV^{-2})
$N_{\max}=10$
$K=16$
20
10
0
0.00
0.25
0.50
0.75
1.00
x (longitudinal fraction)
0.0
0.1
0.2
0.3
$(p^\perp)^2$ (GeV^2)



$g_{1L}^u\,(x,(p^\perp)^2)$
(GeV^{-2})
$N_{\max}=10$
$K=16$
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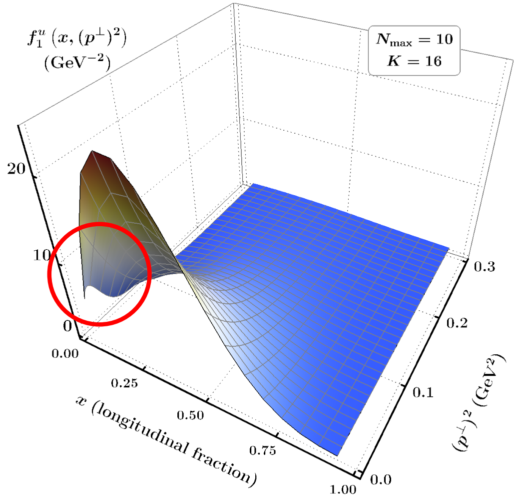
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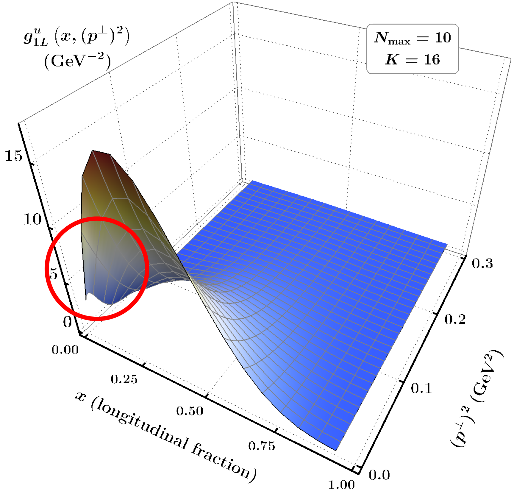
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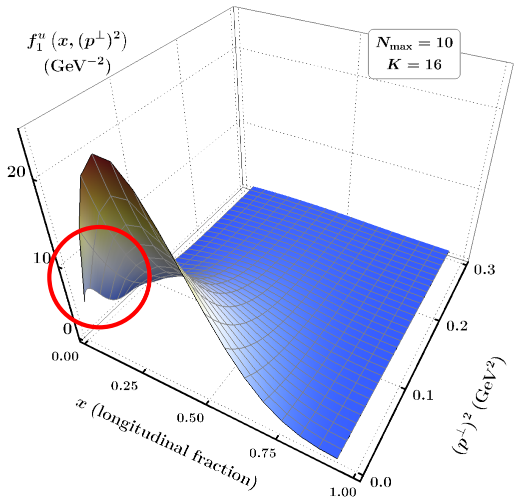
TMD results


$f_1^u(x,(p^\perp)^2)$ (GeV^{-2})
$N_{\max} = 10$
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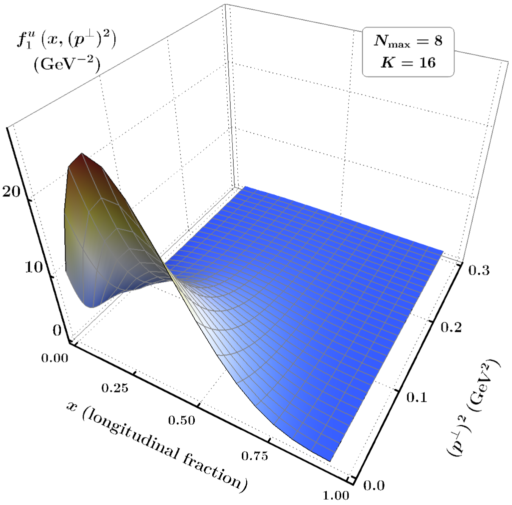
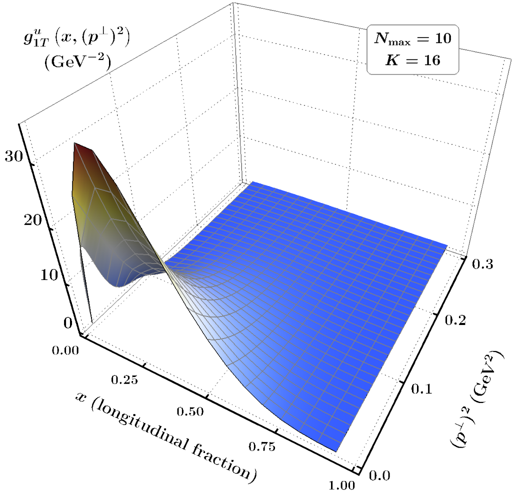

$f_1^u(x,(p^\perp)^2)$ (GeV^{-2})
$N_{\max} = 8$
$K = 16$
20
10
0
0.00
0.25
0.50
0.75
1.00
x (longitudinal fraction)
0.0
0.1
0.2
0.3
$(p^\perp)^2$ (GeV^2)

TMD results


$g_{1T}^{u}\,(x, (p^{\perp})^2)$
(GeV^{-2})
$N_{\max} = 10$
$K = 16$
30
20
10
0
0.00
0.25
0.50
0.75
1.00
x (longitudinal fraction)
0.0
0.1
0.2
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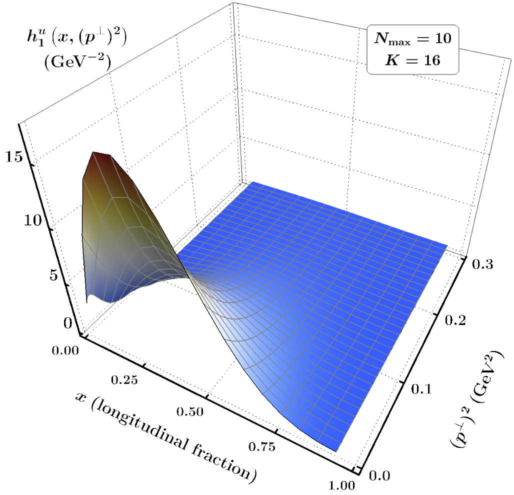
$(p^{\perp})^2$ (GeV^2)


$h_{1}^{u}\,(x, (p^{\perp})^2)$
(GeV^{-2})
$N_{\max} = 10$
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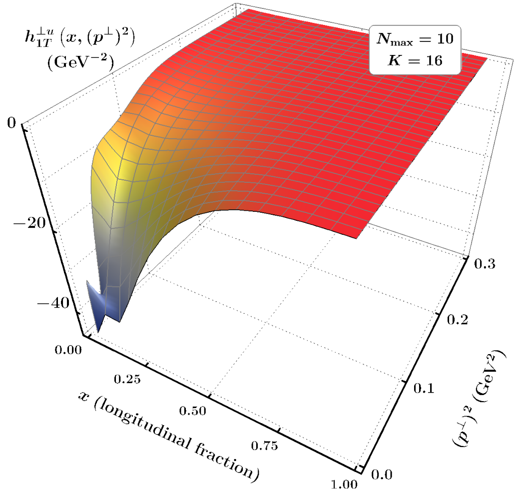
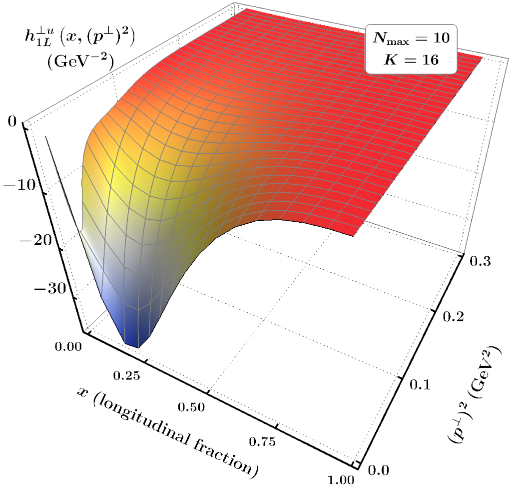
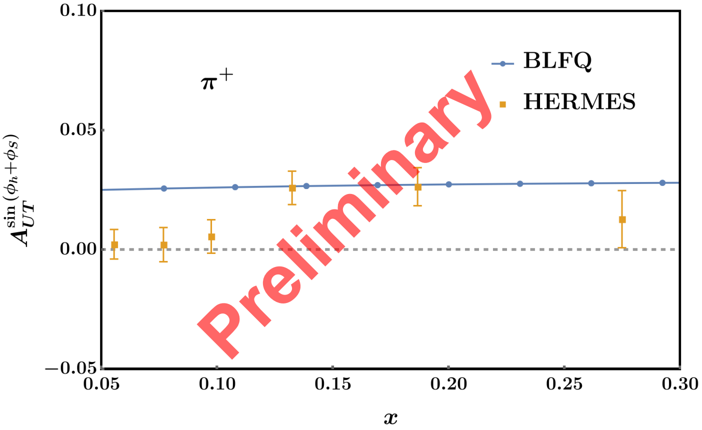

$h_{1T}^{\perp u}(x, (p^{\perp})^2)$
(GeV^{-2})
$N_{\max} = 10$
$K = 16$
0
−20
−40
0.00
0.25
0.50
0.75
1.00
x (longitudinal fraction)
0.0
0.1
0.2
0.3
$(p^{\perp})^2$ (GeV^2)


$h_{1L}^{\perp u}(x, (p^{\perp})^2)$
(GeV^{-2})
$N_{\max} = 10$
$K = 16$
0
−10
−20
−30
0.00
0.25
0.50
0.75
1.00
x (longitudinal fraction)
0.0
0.1
0.2
0.3
$(p^{\perp})^2$ (GeV^2)

Asymmetry calculations



Predictions of Collins asymmetry. Experiment data are taken from (Airapetian et al. 2010). Fragmentation functions follow that in (Maji, Chakrabarti, and Teryaev 2017).

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