Radiative Transitions of Charmonium States in Covariant Confined Quark Model

Gurjav Ganbold

Bogolubov Laboratory of Theoretical Physics, JINR, Dubna

Institute of Physics and Technology, MAS, Ulaanbaatar

in collaboration with T.Gutsche (Uni Tuebingen),
M.A.Ivanov (BLTP JINR Dubna),
V.Lyubovitskij (Uni Tuebingen)

Outline

- Motivation
- Approach
  - Compositeness condition
  - Infrared confinement (cutoff)
- Charmonium states: ground states and orbital excitations
  - Dominant (one-photon) radiative decays of states:
    - Vector 1^{--} : J/Ψ
    - Scalar 0^{++}: χ_{c0}
    - Axial 1^{++} : χ_{c1}
    - Axial 1^{+-} : h_c
    - Tensor 2^{++} : χ_{c2}
- Numerical results:
  - Model parameters
  - Renormalized couplings
  - Decay widths
  - Deconfinement limit
- Summary and outlook

Charmonium States:

♦ Charmonium states are intensively searched (LHCb, BES-III, BELLE, ...).

♦ Charmonium states are unusual:
- the quark masses are much larger than the confinement scale
- have low-lying excited states (L=1, $J^{PC} = 0^{++}$, 1^{++}, 1^{+-}, 2^{++}).

♦ These $c\bar{c}$ mesons have narrow widths, one-photon decay modes are dominant.

♦ Small binding energy -> an ideal testing ground to validate model assumptions.

♦ Discrepancies still exist between the theoretical predictions and world data.

Covariant Confined Quark Model:

- Lagrangian-based formulation ⇒ full Lorentz invariance is kept.
- Direct inclusion of many-quark states (baryons, tetraquarks,...) is available.
- Wide application to and convincing results obtained in:
  - Strong decays
  - Electroweak transitions
  - Heavy meson and boson physics
  - Beyond the Standard Model … [papers in PRD, PLB, NPB, ...]

CCQM approach for Meson-Quark Interaction

- Hadrons *H(x)* interact by *quark exchanges,* with hadron-quark coupling g_H.

$$L_{\rm int} = g_H H(x) J_H(x)$$

T. Branz et al., **PRD81**, 034010 (2010)

- Interpolating quark current (for meson):

$$J_H(x) = \int dx_1 \int dx_2 \, F_H(x; x_1, x_2)\, \bar{q}(x_2) \Gamma_H q(x_1)$$

$$\Gamma_P = i\gamma^5 ; \quad \Gamma_V = \gamma^\mu$$

- Vertex function (trans. inv.)

$$\omega_j = m_j / (m_1 + m_2)$$

$$F_H(x; x_1, x_2) = \delta(x - \omega_1 x_1 - \omega_2 x_2) \cdot \Phi_H(|x_1 - x_2|^2)$$

$$\Phi_H(-p^2) = \exp\left(\frac{p^2}{\Lambda_H^2}\right)$$

Λ_H ~ hadron "size"

- Quark propagator **(**in the Schwinger representation**):**

$$S_m(\hat{p}) = \frac{m + \hat{p}}{m^2 - p^2} = (m + \hat{p}) \cdot \int_0^\infty d\alpha_1 \exp\left[-\alpha_1 \left(m^2 - p^2\right)\right]$$

- The **compositeness condition** eliminates the bare fields from consideration.

$$Z_H = \left\langle H_{bare} \,|\, H_{phys} \right\rangle^2 = 1 - g_{ren}^2 \, \Pi'_H(M_H^2) = 0$$

- Any hadronic matrix element containing loops can be finally written in the form

$$\Pi^0 = N_c \int_0^1 d^n\alpha \; f(\alpha_1, \alpha_2, \ldots, \alpha_n)$$

- Convert the set of Fock-Schwinger parameters into a simplex by adding

$$1 = \int_0^\infty dt\, \delta\left(1 - \sum_{i=1}^n \alpha_i\right)$$

$$\Pi^0 = N_c \int_0^\infty dt\, t^{n-1} \int_0^1 d^n\alpha \; \delta\left(1 - \sum_{i=1}^n \alpha_i\right) f(t\alpha_1, t\alpha_2, \ldots, t\alpha_n)$$

- The integral diverges for $t \to \infty$, if the kinematic variables allow for the appearance of branch points corresponding to the creation of free quarks.

$$\Pi^0 = N_c \int_0^{1/\lambda^2} dt\, t^{n-1} \int_0^1 d^n\alpha \; \delta\left(1 - \sum_{i=1}^n \alpha_i\right) f(t\alpha_1, t\alpha_2, \ldots, t\alpha_n)$$

- Threshold singularities disappear by introducing λ – *the* infrared cutoff parameter

- **Infrared confinement** is introduced to guarantee the absence of all possible *thresholds* corresponding to quark production.

Model parameters

A meson in the model is characterized by:

- the global infrared confinement parameter **λ** (universal)
- the constituent quark masses **m_1** & **m_2**
- the meson size parameter **Λ_H** (free)

- totally **1+4+N** parameters for **N** hadrons → **1+5/N ≈ 1** per hadron

+ The model parameters are determined by minimizing *χ2* in a fit to the latest data and some lattice results. The errors of the fitted parameters are of order ~10%

- Global parameters:

$$\lambda = 0.181\,\text{GeV},$$
$$m_{ud} = 0.241\,\text{GeV}, \qquad m_s = 0.428\,\text{GeV},$$
$$m_c = 1.67\,\text{GeV}, \qquad m_b = 5.07\,\text{GeV}$$

G.Ganbold et al., J.Phys. G 42, 075002 (2015).

- Central values of the **size parameters** Λ_H (in GeV)

π	K	D	D_s	B	B_s	B_c	η_c	η_b	
0.87	1.02	1.71	1.81	1.90	1.94	2.50	2.06	2.95	
ρ	ω	Φ	J/ψ	K*	D*	D^*_s	B*	B^*_s	ϒ
0.61	0.50	0.91	1.93	0.75	1.51	1.71	1.76	1.71	2.96

Charmonium Radiative Decays

Charmonium states $^{2S+1}L_J$ ($L \leq 1$): dominant radiative decay modes PDG-2021

State	J^{PC}	Current	Mass (MeV)	Full width (Γ)	Mode	Fraction (Γ_ι / Γ)
$\eta_c(^1S_0)$	0^{-+}	$i\bar{q}\gamma_5 q$	2983.9 ± 0.5	32.0 ± 0.7 MeV	$\gamma + \gamma$	$(1.58 \pm 0.11) \times 10^{-4}$
$J/\Psi(^3S_1)$	1^{--}	$\bar{q}\gamma_\mu q$	3096.9 ± 0.0006	92.9 ± 2.8 keV	$\gamma + \eta_c$	$(1.7 \pm 0.4) \times 10^{-4}$
$\chi_{c0}(^3P_0)$	0^{++}	$\bar{q} I q$	3414.71 ± 0.30	10.8 ± 0.6 MeV	$\gamma + J/\Psi$	$(1.40 \pm 0.05) \times 10^{-4}$
$\chi_{c1}(^3P_1)$	1^{++}	$\bar{q}\gamma_\mu \gamma_5 q$	3510.67 ± 0.05	0.84 ± 0.04 MeV	$\gamma + J/\Psi$	$(34.3 \pm 1.0) \times 10^{-4}$
$h_c(^1P_1)$	1^{+-}	$i\bar{q}\overleftrightarrow{\partial}_\mu \gamma_5 q$	3525.38 ± 0.11	0.7 ± 0.4 MeV	$\gamma + \eta_c$	$(51 \pm 6) \times 10^{-4}$
$\chi_{c2}(^3P_2)$	2^{++}	$\frac{i}{2}\bar{q}\left(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu\right) q$	3556.17 ± 0.07	1.97 ± 0.09 MeV	$\gamma + J/\Psi$	$(19.0 \pm 0.5) \times 10^{-4}$

- Nonrelativistic **potential model** [12] and in the **Coulomb gauge approach** [34] result in large widths $\Gamma(J/\psi \to \gamma\eta_c(1S)) \simeq 2.9$ keV, about a factor of 2 larger than the world data.
- **Quark models** fail to reproduce the measured branching width $\Gamma(J/\psi \to \gamma\eta_c)$ and, instead, obtain a significantly larger value [10, 12, 35].
- **Constituent quark models** describe the radiative transitions of J/ψ, $\psi(2S)$, χ_{cJ}, h_c and $\psi(3770)$ [17], but the numerical results differ from the worldwide data.
- **Lattice QCD** [18, 20] carried out on the radiative transition properties of χ_{c0}, χ_{c1}, however, good descriptions are still not obtained due to technical restrictions.

+ First observation of decays @LHCb in ***p-p*** collision at energy (c.m.) ***8 TeV***
Measured ratios of branching fractions [*R.Aaji et al., PRL 119, 062001 (2017)*]

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c1} p K^-)}{\mathcal{B}(\Lambda_b^0 \to J/\psi p K^-)} = 0.242 \pm 0.014 \pm 0.013 \pm 0.009,$$

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \to J/\psi p K^-)} = 0.248 \pm 0.020 \pm 0.014 \pm 0.009,$$

$$\frac{\mathcal{B}(\Lambda_b^0 \to \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \to \chi_{c1} p K^-)} = 1.02 \pm 0.10 \pm 0.02 \pm 0.05,$$

Comparing with meson section:

$$\frac{B(B \to \chi_{c2} K)}{B(B \to \chi_{c1} K)}$$

Belle (2008), **BaBar** (2009), **LHCb** (2013): ***SUPPRESSED***

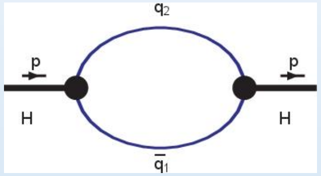
M.Beneke NPB811 (2009) Factorization approach: ***SUPPRESSED***

Belle Collaboration (2016): ***SUPPRESSION LESSENED***
if additional particles are present in the final state

Renormalized couplings

- The renormalization coupling g_H is defined from the compositeness condition

$$Z_H = 1 - g_H^2 \tilde{\Pi}'_H(M_H^2) = 0\,, \qquad \tilde{\Pi}'_H(p^2) = \frac{d}{dp^2}\,\tilde{\Pi}_H^{(1)}(p^2)$$



- The requirement $Z_H = 0$ implies that the physical state does not contain the bare state and is appropriately described as a bound state. It effectively excludes the constituent degrees of freedom from the physical state space.
- The interaction leads to a dressed physical particle, i.e. its mass and wave function have to be renormalized.

- For a meson the mass operator corresponding to the self-energy diagram

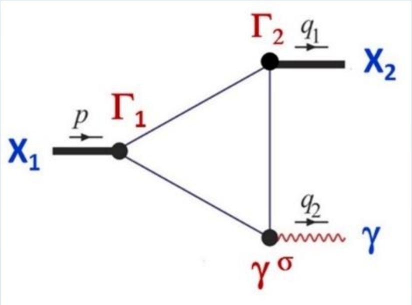
$$\tilde{\Pi}_H(p) \;=\; N_c \int \frac{dk}{(2\pi)^4 i} \widetilde{\Phi}_H^2\left(-k^2\right) \mathrm{tr}\left[\Gamma_H \widetilde{S}_1(\hat{k}+w_1\hat{p})\Gamma_H \widetilde{S}_2(\hat{k}-w_2\hat{p})\right]$$

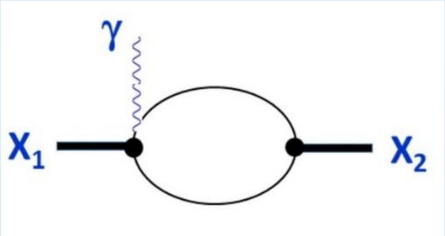
Matrix elements

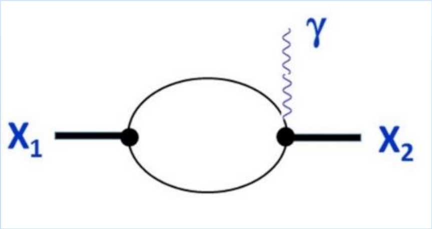
The invariant matrix element for the one-photon radiative transition $X_1 \to \gamma X_2$

$$\mathfrak{M}_{X_1\to\gamma X_2} = i(2\pi)^4\delta^4(p - q_1 - q_2)\,\varepsilon_{X_1}\varepsilon_{X_2}\varepsilon_\gamma\, T_{X_1\to\gamma X_2}(q_1, q_2)$$

In **LO**, transition amplitude $T_{X1\to\gamma X2}(q_1, q_2)$ is described by **'triangle'+ 'bubble' diagrams**







The contributions given by the bubble-type diagrams are small and do not exceed the common errors (±10%) of our calculations.

Taking into account the uncertainty of the experimental data, we drop the bubble-type diagrams without loss in accuracy of our estimates.

Transition amplitude

G.Ganbold et al.,
arXiv:2107.0877v1 [hep-ph]

$$T_{X_1\to\gamma X_2}(q_1,q_2)=g_{X_1}g_{X_2}e_c e N_c \iiint\limits_0^1 d\alpha_1\, d\alpha_2\, d\alpha_3$$
$$\cdot\int\frac{d^4k}{(2\pi)^4 i}\exp\left\{k^2(\alpha_1+\alpha_2+\alpha_3+s_1+s_2)+2k^\nu R^\nu+R_0\right\}$$
$$\cdot\,\mathrm{tr}\left[\Gamma_2(m_c+\hat{k}+\tfrac{1}{2}\hat{p})\,\Gamma_1(m_c+\hat{k}-\tfrac{1}{2}\hat{p})\gamma^\sigma(m_c+\hat{k}-\tfrac{1}{2}\hat{p}+\hat{q}_2)\right]$$
$$=T^{inv}_{X_1,X_2,\gamma}(q_1,q_2)+T^{res}_{X_1,X_2,\gamma}(q_1,q_2).$$

$$q_2^\sigma\cdot T^{(inv)}_{X_1\to\gamma X_2}(q_1,q_2)=0.$$

$$\Gamma_1=\{\gamma^\mu, I, \gamma^\mu\gamma_5, \overleftrightarrow{\partial}_\nu\gamma^5, i(\gamma^\mu\overleftrightarrow{\partial}_\nu+\gamma^\nu\overleftrightarrow{\partial}_\mu)/2\}$$

$$T^{(inv)}_{X_1\to\gamma X_2}(q_1,q_2)=\frac{g_{X_1}\,g_{X_2}\,e_c\,e\,N_c}{(2\pi)^2}\int\limits_0^{1/\lambda^2}dt\,\frac{t^2}{(s+t)^2}\iiint\limits_0^1 d\alpha_1\,d\alpha_2\,d\alpha_3\,\delta(1-\alpha_1-\alpha_2-\alpha_3)$$
$$\cdot f_{\Gamma_1,\Gamma_2}(p,q_1,q_2,m_c,s,t,\alpha_1,\alpha_2,\alpha_3)\cdot\exp\left(-t\,z_0+\frac{t\,s}{s+t}z_1+\frac{s^2}{s+t}z_2\right), (1$$

Transition $J/\Psi(^3S_1) \to \gamma + \eta_c(^1S_0)$

A typical electromagnetic M1 transition between ground states, from the vector J/ψ ($\Gamma_1 = \gamma_\rho$) to the pseudoscalar η_c ($\Gamma_2 = i\gamma_5$) by radiating a photon ($\Gamma_\gamma = \gamma_\sigma$)

Trace factor:

$$f_{\Gamma_1,\Gamma_2} = m_c\,\epsilon^{q_1 q_2 \rho\sigma}\,, \qquad \epsilon^{q_1 q_2 \rho\sigma} \equiv \epsilon^{\mu\nu\rho\sigma} q_1^\mu q_2^\nu\,;$$

Transition amplitude:

$$T^{(inv)\rho\sigma}_{J/\psi\to\gamma\eta_c} = g_{J/\psi}\, g_{\eta_c}\, C(p^2, q_1^2, q_2^2)\,\epsilon^{q_1 q_2 \rho\sigma}\,;$$

Decay width:

$$\Gamma(J/\psi \to \gamma\eta_c) = \frac{\alpha}{24}\, g^2_{J/\psi}\, g^2_{\eta_c}\, M^3_{J/\psi} \left(1 - \frac{M^2_{\eta_c}}{M^2_{J/\psi}}\right)^3 \cdot \left[C(M^2_{J/\psi}, M^2_{\eta_c}, 0)\right]^2\,;$$

Transition $\chi_{c0}\,({}^3P_0) \rightarrow \gamma + J/\Psi\,({}^3S_1)$

The one-photon radiative transition amplitude of the orbitally excited (scalar) charmonium into the vector ground-state reads

$$T^{(inv)\rho\sigma}_{\chi_{c0}\to\gamma J/\psi}(q_1,q_2) = g_{\chi_{c0}}\, g_{J/\psi}\, d(p^2,q_1^2,q_2^2)\cdot\left(q_1^\sigma q_2^\rho - g_{\rho\sigma}(q_1\cdot q_2)\right)$$

Trace factor:

$$d = \frac{e_c e N_c m_c}{(2\pi)^2}\int\limits_0^{1/\lambda^2} dt \frac{t^2}{(s+t)^2}\iint\limits_0^1 d\alpha_1 d\alpha_2 \left(\frac{1}{2}+\frac{t\alpha_3 + s_2/2}{s+t}\right) e^{-t\,z_0 + \frac{t\,s}{s+t}\,z_1}$$

Decay width:

$$\Gamma(\chi_{c0}\to\gamma J/\psi) = \frac{\alpha}{24} g^2_{\chi_{c0}}\, g^2_{J/\psi}\, M^3_{\chi_{c0}} \left(1-\frac{M^2_{J/\psi}}{M^2_{\chi_{c0}}}\right)^3 \cdot \left[d(g_{\chi_{c0}}, g_{J/\psi}, M^2_{\chi_{c0}}, M^2_{J/\psi}, 0)\right]^2 .$$

Transition $\chi_{c1}\,({}^3P_1) \to \gamma + J/\Psi\,({}^3S_1)$

The transition amplitude with four seemingly independent Lorentz structures reads

$$
\begin{aligned}
T^{(inv)\mu\rho\sigma}_{\chi_{c1}\to\gamma J/\psi}(q_1,q_2) \;=\; & g_{\chi_{c1}}\, g_{J/\psi}\, \big[\epsilon^{q_2\mu\sigma\rho}(q_1\cdot q_2)\, W_1 + \epsilon^{q_1 q_2\sigma\rho} q_1^\mu\, W_2 \\
& +\; \epsilon^{q_1 q_2 \mu\rho} q_2^\sigma\, W_3 + \epsilon^{q_1 q_2\mu\sigma} q_1^\rho\, W_4 - \epsilon^{q_1\mu\sigma\rho}(q_1\cdot q_2)\, W_4\big]
\end{aligned}
$$

Helicity amplitudes:

$$
\begin{aligned}
H_L \;&=\; i g_{\chi_{c1}}\, g_{J/\psi}\, \frac{M^2_{\chi_{c1}}}{M_{J/\psi}} |\vec{q}_2|^2 \Big[W_1 + W_3 - \frac{M^2_{J/\psi}}{M_{\chi_{c1}}|\vec{q}_2|} W_4\Big]\,, \qquad |\vec{q}_2| = \frac{M^2_{\chi_{c1}} - M^2_{J/\psi}}{2M_{\chi_{c1}}}\,, \\
H_T \;&=\; -i g_{\chi_{c1}}\, g_{J/\psi}\, M_{\chi_{c1}} |\vec{q}_2|^2 \Big[W_1 + W_2 - \Big(1 + \frac{M^2_{J/\psi}}{M_{\chi_{c1}}|\vec{q}_2|}\Big) W_4\Big]\,.
\end{aligned}
$$

Decay width:

$$
\Gamma(\chi_{c1} \to \gamma J/\Psi) = \frac{\alpha\, g^2_{\chi_{c1}}\, g^2_{J/\psi}}{12\pi} \frac{|\vec{q}_2|}{M^2_{\chi_{c1}}} \Big(|H_L|^2 + |H_T|^2\Big)
$$

Transition $h_c\,(^1P_1) \to \gamma + \eta_c\,(^1S_0)$

The gauge invariant transition amplitude reads

$$T^{(inv)\rho\sigma}_{h_c\to\gamma\eta_c}(q_1,q_2) = g_{h_c}\, g_{\eta_c}\, h(p^2,q_1^2,q_2^2)\cdot(q_2^\rho q_1^\sigma - g_{\rho\sigma}(q_1\cdot q_2))$$

Square of Matrix element:

$$|\mathcal{M}_{h_c\to\gamma\eta_c}|^2 \sim |\varepsilon^\rho_{h_c}\,\varepsilon^\sigma_\gamma\,T^{(inv)\rho\sigma}_{h_c\to\gamma\eta_c}(q_1,q_2)|^2 = \frac{1}{2} g^2_{h_c}\, g^2_{\eta_c}\, M^4_{h_c}\left(1 - M^2_{\eta_c}/M^2_{h_c}\right)^2 \cdot |h(p^2,q_1^2,q_2^2)|^2$$

Decay width:

$$\Gamma(h_c\to\gamma\eta_c) = \frac{\alpha\, g^2_{h_c}\, g^2_{\eta_c}}{24\,(1+2S)} M^3_{h_c}\left(1 - \frac{M^2_{\eta_c}}{M^2_{h_c}}\right)^3 \cdot |h(M^2_{h_c}, M^2_{\eta_c}, 0)|^2$$

With spin **S=1**

Transition $\chi_{c2}\,({}^3P_2) \to \gamma + J/\Psi\,({}^3S_1)$

The one-photon radiative transition amplitude of the orbitally excited tensor state into the vector ground-state charmonium reads

$$T^{(inv)\mu\nu\rho\sigma}_{\chi_{c2}\to\gamma J/\psi}(q_1,q_2) = g_{\chi_{c2}}\, g_{J/\psi}\, \Big\{ A\cdot\Big(g^{\mu\rho}\Big[g^{\sigma\nu}(q_1\cdot q_2) - q_1^\sigma\, q_2^\nu\Big] + g^{\nu\rho}\Big[g^{\sigma\mu}(q_1\cdot q_2) - q_1^\sigma\, q_2^\mu\Big]\Big) + B\cdot\Big(g^{\sigma\rho}\Big[q_1^\mu\, q_2^\nu + q_1^\nu\, q_2^\mu\Big] - g^{\mu\sigma} q_1^\nu\, q_2^\rho - g^{\nu\sigma} q_1^\mu\, q_2^\rho\Big)\Big\}\,, \qquad (40)$$

Decay width:

$$\Gamma(\chi_{c2}\to\gamma J/\psi) = \frac{\alpha\, g^2_{\chi_{c2}}\, g^2_{J/\psi}}{4(1+2S)}\, M^3_{\chi_{c2}} \left(1 - \frac{M^2_{J/\psi}}{M^2_{\chi_{c2}}}\right)\cdot\left(C_A\cdot A^2 + C_{AB}\cdot A\cdot B + C_B\cdot B^2\right)$$

With spin **S=2**

Coefficients:

$$\begin{aligned} C_A &= \frac{1}{\xi}\left(\frac14 + \frac73\xi - \frac{31}{6}\xi^2 + \frac73\xi^3 + \frac14\xi^4\right) = 0.195717, \qquad \xi = M^2_{J/\psi}/M^2_{\chi_{c2}} = 0.758384,\\ C_{AB} &= -\frac{1}{\xi}\left(\frac12 - \frac13\xi - 2\xi^2 + 3\xi^3 - \frac76\xi^4\right) = -0.0257553, \qquad (43)\\ C_B &= \frac{1}{\xi}\left(\frac14 - \frac23\xi + \frac16\xi^2 + \xi^3 - \frac{13}{12}\xi^4 + \frac13\xi^5\right) = 0.00225945\,. \end{aligned}$$

Modified Vertex for Charmonium

CCQM: The non-local vertex function $\Phi_H(-p^2)$ characterizes the quark distribution inside the hadron. It is unique for the given hadron, each hadron has its own adjustable parameter Λ_H related to the hadron 'size'.

$$\Lambda_X = \{\Lambda_{\eta c}, \Lambda_{J/\psi}, \Lambda_{\chi c0}, \Lambda_{\chi c1}, \Lambda_{hc}, \Lambda_{\chi c2}\}$$

These charmonium members have the same quark content and possess physical masses in a relative narrow interval $\sim$3÷3.5 GeV.
For this specific case we use the Ansatz: the charmonium 'size' is proportional to its physical mass, i.e., $\Lambda_X = \varrho \cdot M_X$ with $\varrho > 0$ - a common adjustable parameter:

$$\varrho \equiv \Lambda_X / M_X$$

Subsequently, we further use the charmonium *vertex function* defined as

$$\widetilde{\Phi}_X\left(-p^2\right) = \exp\left(\frac{1}{\varrho^2} \cdot \frac{p^2}{M_X^2}\right)$$

Numerical results

For further numerical evaluation we keep the basic CCQM parameters:

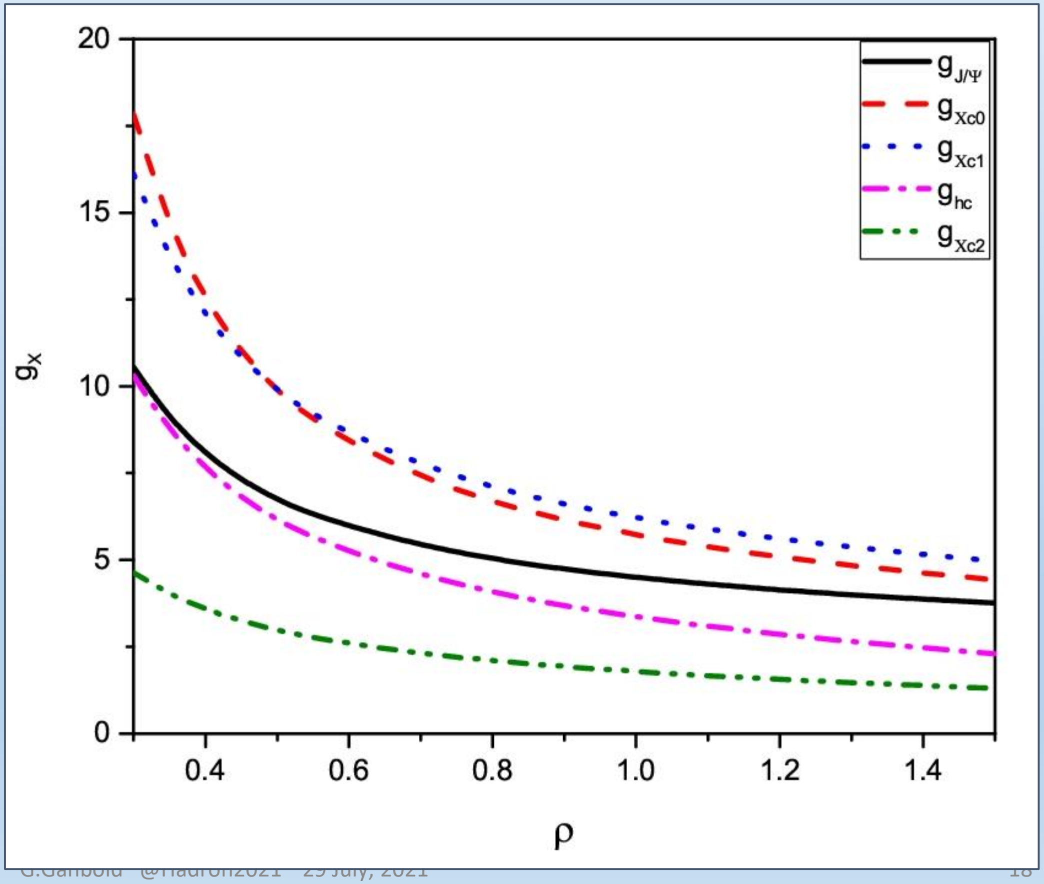
- ♦ the universal infrared cutoff parameter $\lambda = 0.181$ GeV
- ♦ the constituent charm quark mass in the range of ±10% around $m_c = 1.67$ GeV.

- ♦ We vary $\varrho > 0$ to fit the latest experimental data from PDG-2021.

Renormalization couplings

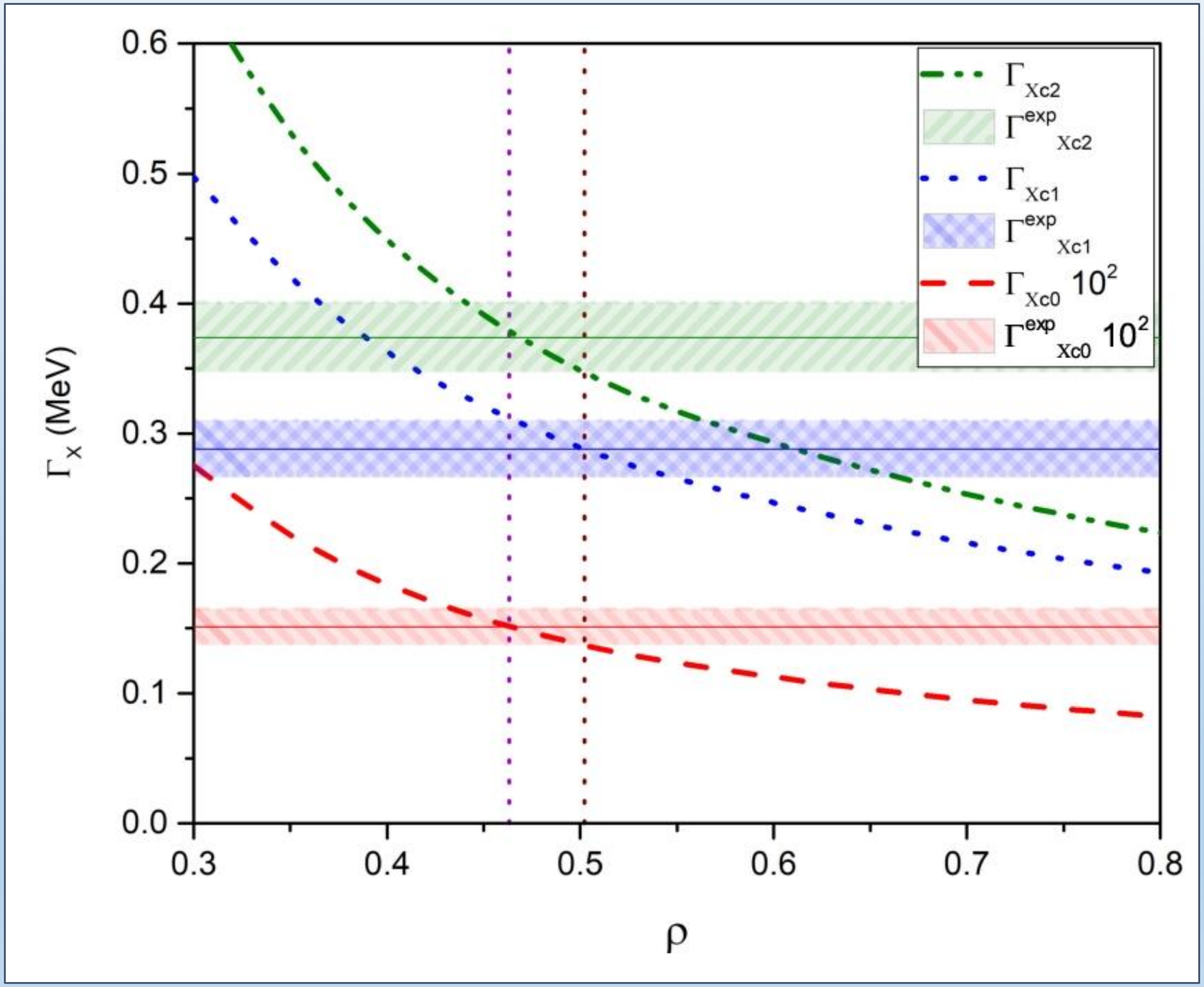
First we calculate g_H . They are strictly fixed by the compositeness requirements and do not constitute further free parameters, although keep indirect dependencies on basic model parameters.

G.Ganbold et al., arXiv:2107.0877v1 [hep-ph]

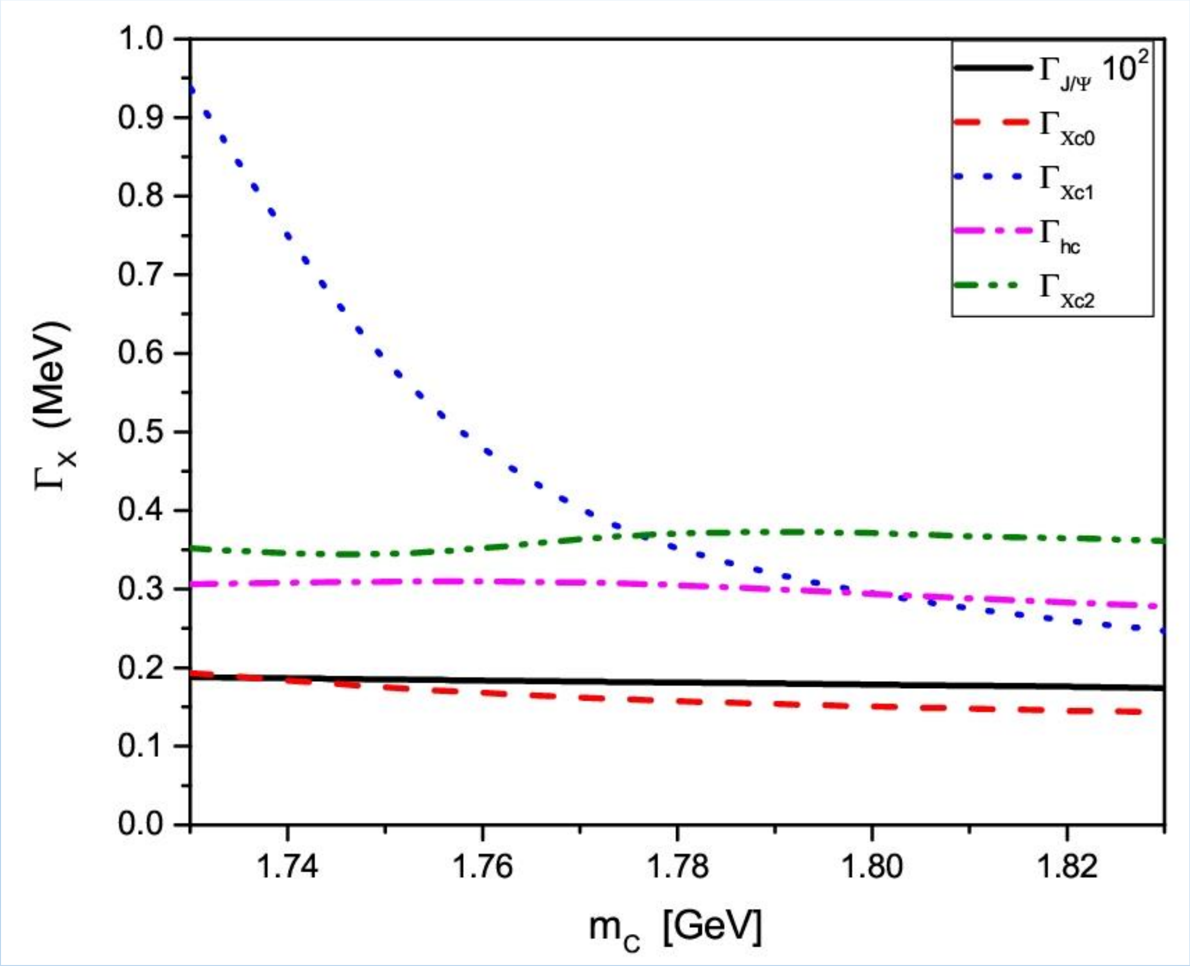


Having calculated g_x we are able to estimate the partial widths of the one-photon radiative decays of the excited (*L=1*) charmonium states χ_{c0} , χ_{c1} and χ_{c2} to find the optimal values of the 'slope' parameter $\varrho > 0$ at different

$m_c \in [1.78 \div 1.82]$ GeV



Having calculated $\boldsymbol{g_x}$ we are able to estimate the partial widths of the one-photon radiative decays of the excited (***L=1***) charmonium states $\boldsymbol{\chi_{c0}}$, $\boldsymbol{\chi_{c1}}$ and $\boldsymbol{\chi_{c2}}$ to find the optimal values of the c-quark mass $\mathbf{m_c}$ at different ’slope’ parameter $\boldsymbol{\varrho > 0}$.



$\varrho \in [0.47 \div 0.53]$

Finally by fitting the latest experimental data PDG-2021 on the partial widths of the dominant one-photon radiative decay of the orbitally-excited charmonium states χ_{c0}, χ_{c1} and χ_{c2} we fix the optimal values of model parameters.

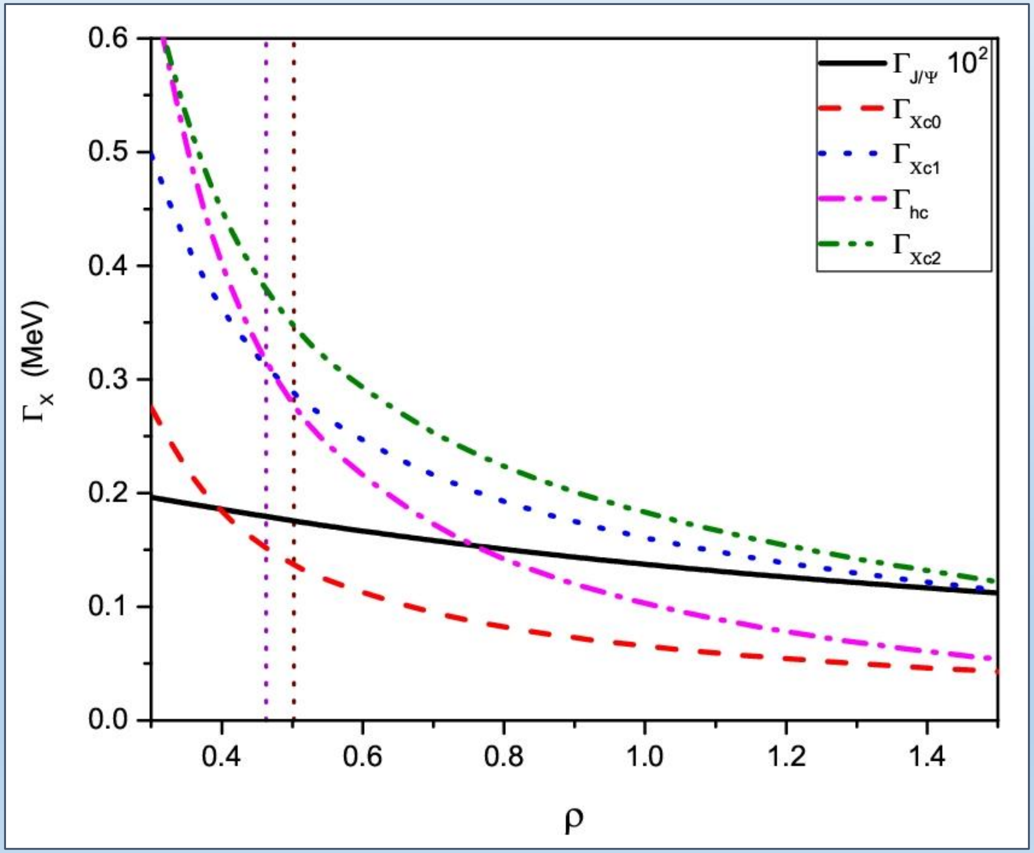
Having fixed the model parameters we calculate the partial widths of the dominant one-photon radiative decays of the ground ($J/\psi \rightarrow \gamma+\eta_c$) and orbital-excited ($h_c \rightarrow \gamma+J/\psi$) states in dependence on ϱ, together with the curves for χ_{cJ}, J={0,1,2}

λ = 0.181 MeV

m_c = 1.80 GeV

ϱ = 0.485

G.Ganbold et al., arXiv:2107.0877v1 [hep-ph]



Some theoretical predictions of the partial widths (in units of keV) of the dominant radiative decay of the charmonium states below the $D\bar{D}$ threshold in comparison with recent data.

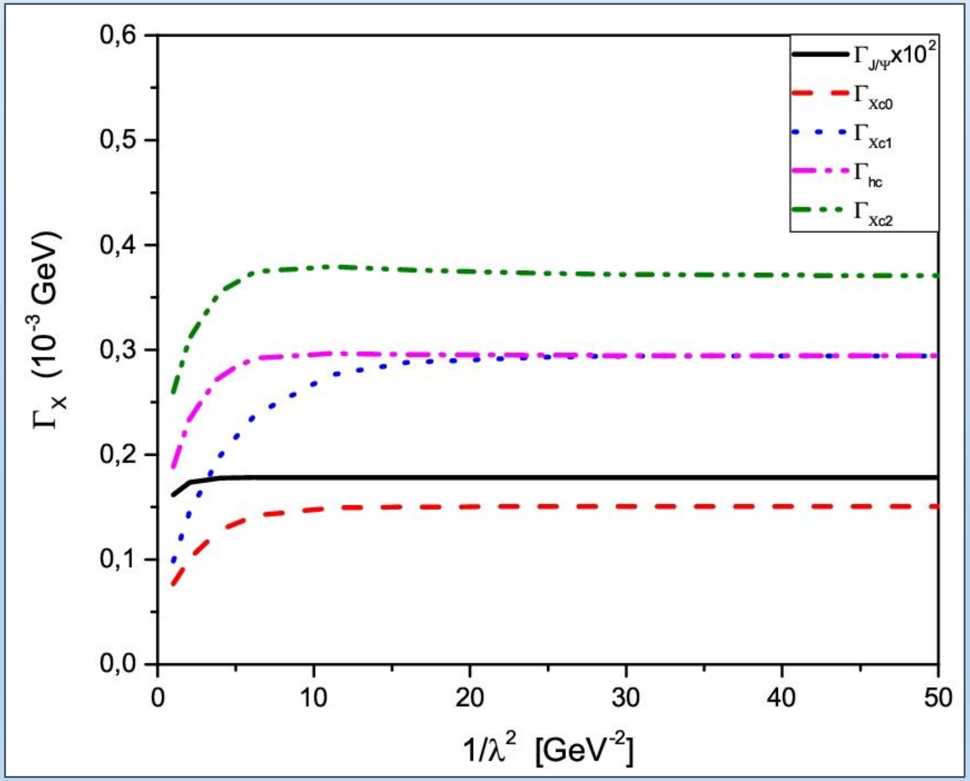
G.Ganbold et al., arXiv:2107.0877v1 [hep-ph]

J^{PC}	Radiative Decay	CCQM λ=0.181	CCQM λ=0	PDG-2021	Cornwell potential [31]	Cornwell potential LWL[31]	Lattice QCD [21]	Constit.Q.M [17]
1^{--}	$\Gamma(J/\Psi \to \gamma\,\eta_c)$	1.771	1.771	1.58 ± 0.43			2.64(11)	1.25
0^{++}	$\Gamma(\chi_{c0} \to \gamma\, J/\Psi)$	142.0	142.0	151 ± 14	118	128		128
1^{++}	$\Gamma(\chi_{c0} \to \gamma\, J/\Psi)$	296.7	297.0	288 ± 22	315	266		275
1^{+-}	$\Gamma(h_c \to \gamma\,\eta_c)$	290.8	290.7	357 ± 270			720(50)(20)	587
2^{++}	$\Gamma(\chi_{c0} \to \gamma\, J/\Psi)$	358.1	356.7	374 ± 27	419	353		467

- + $\Gamma(\chi_{cJ} \to \gamma + J/\psi)$ results are close to the recent LHCb data.
- + $\Gamma(J/\psi \to \gamma + \eta_c)$ = 1.77 keV slightly (~12 %) exceeds the recent average data.
- + $\Gamma(h_c \to \gamma + \eta_c)$ = 0.291 MeV leads to *'theoretical full decay width'* $\Gamma^{theor}(h_c) \simeq (0.57 \pm 0.12)$ MeV.

Deconfinement limit

The infrared cutoff parameter in CCQM plays an important role by removing all possible threshold singularities corresponding to the creation of free quarks, and is taken to be universal (λ = 0.181 GeV) for all physical processes. However, in some specific cases these singularities do not appear. Particularly, for m_c = 1.80 GeV we obtain $M_X < 2m_c$ for all charmonium states under consideration and the corresponding integrals converge. Then, we can use even the full integration range $t \in [0, \infty)$, i.e., with $\lambda \to 0$.



The partial decay widths do not change for $1/\lambda^2 > 20$ GeV^{-2} while λ = 0.181 GeV corresponds to $1/\lambda^2$ = 30.52 GeV^{-2}.

Our theoretical estimates on the charmonium states remain unchanged in the deconfinement limit $\lambda \to 0$.

Discussion (short)

♦ Our calculation within the CCQM

$\Gamma(J/\psi \to \gamma+\eta_c) = 1.77$ keV

slightly (about 12%) exceeds the average value of the recent data [PDG-2021].

♦ Our calculations for the central values of the partial decay widths

$\Gamma(\chi_{c0} \to \gamma+J/\psi) = 142.0$ keV ,

$\Gamma(\chi_{c1} \to \gamma+J/\psi) = 296.7$ keV ,

$\Gamma(\chi_{c2} \to \gamma+J/\psi) = 358.1$ keV

are close to the recent LHCb data.

♦ Our calculation within the CCQM

$\Gamma(h_c \to \gamma+\eta_c) = 0.291$ MeV is in agreement with the recent data.

♦ The present world data for the full decay width of $h_c(^1P_1)(3525)$ cannot be used to test the various predictions due to their large uncertainties. On the other hand, the fractional width for the one-photon radiative decay of $h_c(^1P_1)(3525)$ is detected more accurately. By combining the latest value for the fractional width of [PDG-2021] with our estimate we may calculate the *'theoretical full decay width'* for h_c as follows:

$$\Gamma^{theor}(h_c) \simeq (0.57 \pm 0.12)\ \text{MeV}. \qquad (**)$$

Hereby, we admitted a relevant ~ 10% uncertainty for $\Gamma(h_c \to \gamma+\eta_c)$.

Compared with data $\Gamma^{exp}(h_c) \simeq (0.7 \pm 0.4)$ MeV [PDG-2021], the 'prediction' (**) is located in a more narrow interval.

Summary and Outlook

- The dominant radiative transitions of the charmonium states $\eta_c(^1S_0)$, $J/\psi(^3S_1)$, $\chi_{c0}(^3P_0)$, $\chi_{c1}(^3P_1)$, $h_c(^1P_1)$ and $\chi_{c2}(^3P_2)$ have been studied within the CCQM.
  The gauge-invariant LO transition amplitudes are expressed by using either the Lorentz structures, or the helicity amplitudes.
- The renormalization couplings of the charmonium states have been strictly fixed to exclude the constituent degrees of freedom from the of physical states.
- We keep the basic model parameters m_c = 1.80 GeV, λ = 0.181 GeV and additionally introduce only one common adjustable parameter $\varrho > 0$.
  The optimal value $\varrho = 0.485$ is fixed by fitting the data for the triplet $\chi_{cJ}(^3P_J)$.
- We calculated the fractional widths for states $J/\psi(^3S_1)$ and $h_c(^1P_1)$ in good agreement with the latest data. By using the fraction data and our estimated partial decay width we recalculate the 'theoretical full width'
  $\Gamma^{theor}(h_c) \simeq (0.57 \pm 0.12)$ MeV compared with latest data $\Gamma^{exp}(h_c) \simeq (0.7 \pm 0.4)$ MeV.
- We also repeated our calculations by gradually decreasing the global cutoff parameter and revealed that the results do not change for any $\lambda < 0.181$ GeV up to the 'deconfinement' limit.
- This approach may be extended to other sections of hadron physics, particularly to investigate
  - light mesons (scalar, isoscalar, …)
  - radial excitations (charmonia and bottomia)
  - exotics (glueballs, tetraquark, X-Y-Z mesonlike objects, …)
  - heavy meson and baryon decays