Properties of low-lying charmonia and bottomonia from lattice QCD + QED

Jonna Koponen
jkoponen@uni-mainz.de

Hadron 2021
The 19th International Conference on Hadron Spectroscopy and Structure

July 29, 2021

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Helmholtz-Institut Mainz

Collaborators

- Christine Davies (University of Glasgow)
- Ben Galloway (University of Glasgow)
- Daniel Hatton (University of Glasgow)
- Peter Lepage (Cornell University)
- Andrew Lytle (University of Illinois)

References

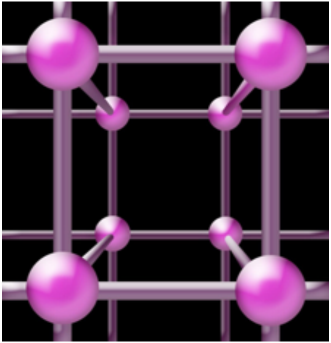
1. Charmonium: PRD **102** (2020) 054511, arXiv 2005.01845
2. J/ψ tensor decay constant: PRD **102** (2020) 094509, arXiv 2008.02024
3. Bottomonium: PRD **103** (2021) 054512, arXiv 2101.08103
4. b quark mass: PRD **103** (2021) 114508, arXiv 2102.09609

Overview

Motivation: precise and accurate SM predictions

Lattice QCD calculations for many quantities have reached, or surpassed, statistical precision of 1%

- masses and decay constants of ground-state pseudoscalar mesons are good examples
- meson masses used for tuning, and determining quark masses



The precision of modern lattice QCD results means that sources of small SYSTEMATIC UNCERTAINTY that could appear at the percent level need to be understood. Here we focus on QED effects.

Lattice ensembles

We use lattice configurations generated by the MILC collaboration

- 17 gluon field ensembles
- six different lattice spacings from very coarse ($a \approx 0.15$ fm) to exafine ($a \approx 0.03$ fm)
- $n_f = 2+1+1$: light, s and c quarks in the sea, $m_l = (m_u + m_d)/2$
- one ensemble with $n_f = 1+1+1+1$ (physical m_u and m_d)
- various light sea quark masses, including (close to) physical masses, to control chiral extrapolation
- Highly Improved Staggered Quark (HISQ) action for both sea and valence quarks
  - removes tree-level a^2 discretisation errors
  - For heavy quarks 'Naik' term is adjusted to remove $(am)^4$ errors at tree-level
- heavy-HISQ: use several heavy valence quark masses $m_h > m_c$ to extract quantities at physical b mass

QED on the lattice

Lattice QCD with quenched QED

- quenched QED = include effects from the valence quarks having electric charge (the largest QED effect) but neglect effects from the electric charge of the sea quarks
- generate a random momentum space photon field $A_\mu(k)$ for each QCD gluon field configuration and set zero modes to zero using the QED$_L$ formulation (QED in finite box)
- Fourier transform A_μ into position space
- $\exp(ieQA_\mu)$ gives the desired $U(1)$ field (Q=quark electric charge in units of the proton charge e)
- this approach is known as the stochastic approach to quenched QED, in contrast to the perturbative approach
- c and b lattice quark masses tuned separately in pure QCD and QCD+QED so that J/ψ and Υ masses match experiment

Meson correlators

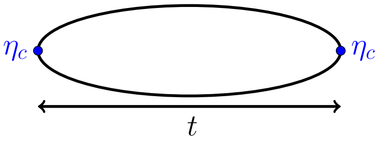
We calculate the quark-line connected correlation functions of pseudoscalar and vector mesons on each ensemble and use a multi-exponential fit to extract amplitudes and energies:

$$C_{\text{2-point}}(t) = \sum_i A_i \left(e^{-E_i t} + e^{-E_i (L_t - t)} \right)$$

The decay constants are related to the ground state amplitude and meson mass:

$$f_P = 2m_q \sqrt{\frac{2A_0^P}{(M_o^P)^3}}, \quad f_V = Z_V \sqrt{\frac{2A_0^V}{M_0^V}}.$$

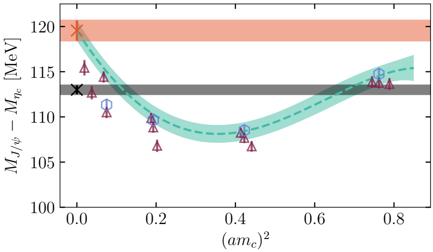
Z_V is a renormalisation factor for the lattice vector current.

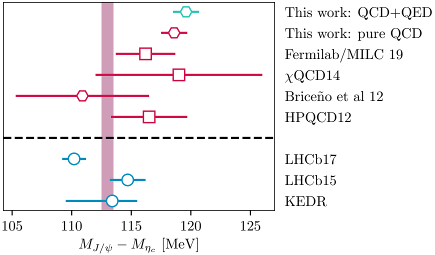


We then take the results at different lattice spacings and extrapolate to continuum, taking into account $(am)^{2n}$ and $(a\Lambda)^{2n}$ discretisation effects. Terms that take into account mistuned sea quark masses are also included. For bottomonium, we map out the dependence in quark mass to extract the result at physical m_b.

Charmonium hyperfine splitting

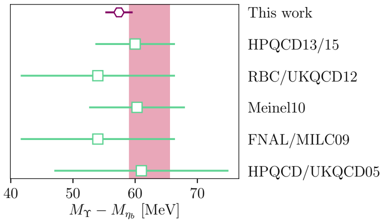
Our QCD+QED result for the charmonium hyperfine splitting is $M_{J/\psi} - M_{\eta_c} = 120.3(1.1)$ MeV.





Quark-line disconnected correlation functions are not included in the lattice calculation. The difference in the results is taken to be the effect of the η_c decay to two gluons (prohibited in the lattice calculation):
$\Delta M_{\eta_c}^{\text{annihln}} = +7.3(1.2)$ MeV

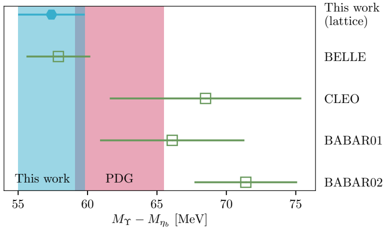
Bottomonium hyperfine splitting

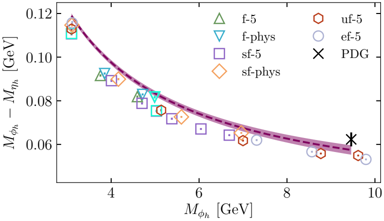


Map out the dependence in m_h to extract the result at physical m_b.

Our QCD+QED result is
$M_\Upsilon - M_{\eta_b} = 57.5(2.3)(1.0)$ MeV.

The missing quark-line disconnected contributions (the second uncertainty) are expected to be smaller for bottomonium than charmonium. We find good agreement with experiment.





Decay constants

The decay constant of a pseudoscalar meson P (e.g. η_c or η_b) is defined in terms of the axial current as

$$\langle 0|A_\alpha|P\rangle = p_\alpha f_P.$$

Using PCAC relation this can be written as

$$\langle 0|\bar{\Psi}_q \gamma_5 \Psi_q|P\rangle = \frac{(M_0^P)^2}{2m_q} f_P.$$

For a vector meson like J/ψ the vector decay constant is defined through the vector current

$$\langle 0|\bar{\Psi}_q \gamma_\alpha \Psi_q|V\rangle = f_V M_V \epsilon_\alpha,$$

where ϵ is the polarisation vector of the meson.

The tensor decay constant of the vector meson is

$$\langle 0|\bar{\Psi}_q \sigma_{\alpha\beta} \Psi_q|V\rangle = i f_V^T(\mu)(\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha).$$

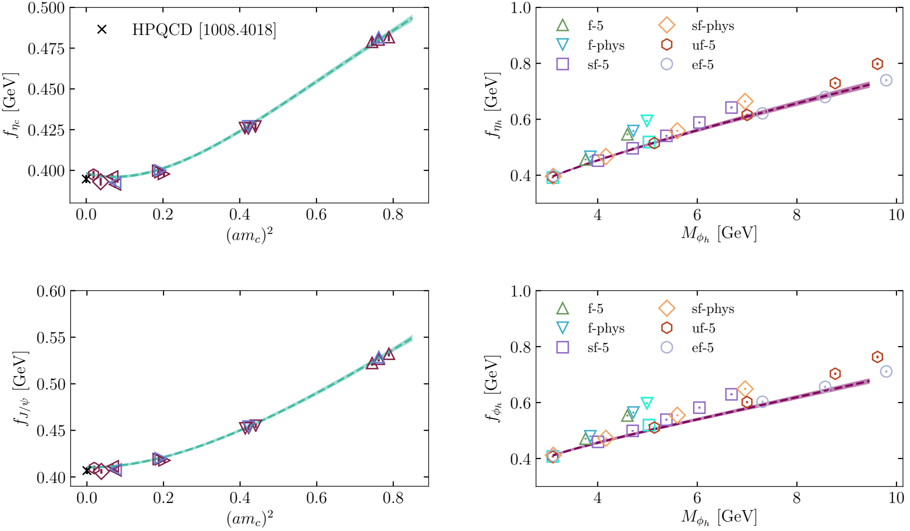
NOTE: Tensor decay constant is scale- and scheme-dependent, unlike the vector decay constant f_V.

Recall that the decay constants can be written in terms of meson masses and amplitudes as

$$f_P = 2m_q \sqrt{\frac{2A_0^P}{(M_o^P)^3}},$$

$$f_V = Z_V \sqrt{\frac{2A_0^V}{M_0^V}}, \quad f_T = Z_T \sqrt{\frac{2A_0^T}{M_0^V}}.$$

Decay constants f_{η_c}, $f_{J/\psi}$, f_{η_b} and f_{Υ}

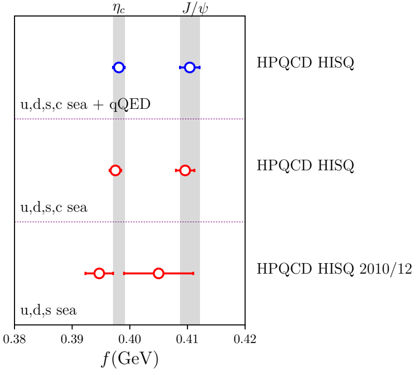


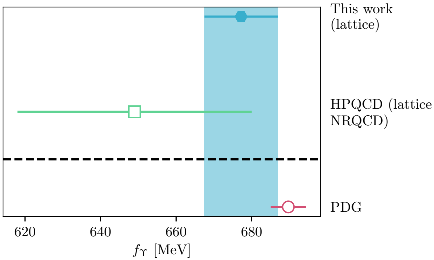
Comparison of decay constant results

Vector and pseudoscalar meson decay constants and their ratios:

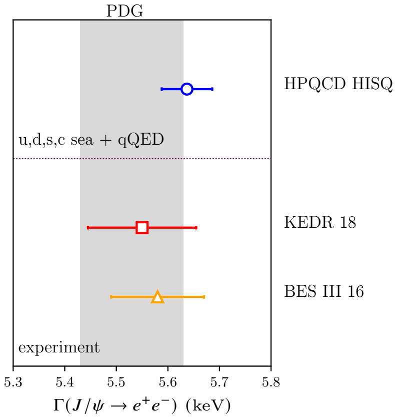
$$f_{J/\psi} = 410.4(1.7)\ \text{MeV}$$
$$f_{\eta_c} = 398.1(1.0)\ \text{MeV}$$

$$\frac{f_{J/\psi}}{f_{\eta_c}} = 1.0284(19)$$
$$\frac{f_\Upsilon}{f_{\eta_b}} = 0.9454(99)$$
$$f_\Upsilon = 677.2(9.7)\ \text{MeV}$$
$$f_{\eta_b} = 724(12)\ \text{MeV}$$





Leptonic widths



The partial decay width of a vector meson to a lepton pair is directly related to the decay constant:

$$\Gamma(\phi_h \to l^+l^-) = \frac{4\pi}{3}\alpha^2_{\mathrm{QED}} Q^2 \frac{f^2_{\phi_h}}{M_{\phi_h}},$$

where Q is the electric charge of the quark.

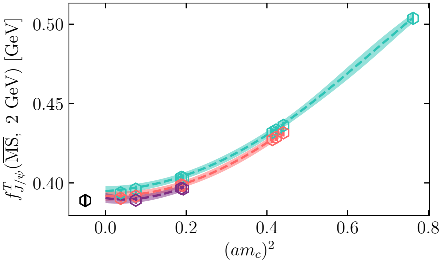
Our results are:

$$\Gamma(J/\psi \to e^+e^-) = 5.637(47)(13)\ \mathrm{keV}$$
$$\Gamma(\Upsilon \to e^+e^-) = 1.292(37)(3)\ \mathrm{keV}$$

There is no experimental decay rate that can be directly compared with the pseudoscalar decay constant.

J/ψ tensor decay constant $f^T_{J/\psi}$

1. Extract $\sqrt{2A_0^T/M_0^T}$ from tensor-tensor correlators.
2. Calculate the renormalisation factor Z_T^{SMOM}. Convert f^T to the $\overline{MS}$ scheme at multiple scales μ using RI-SMOM scheme as an intermediate scheme on each ensemble.
3. Run all the $\overline{MS}$ tensor decay constants at a range of scales μ to a reference scale of $2\ \mathrm{GeV}$ using a three-loop calculation of the tensor current anomalous dimension. Here $\mu = 2, 3, 4$ GeV.



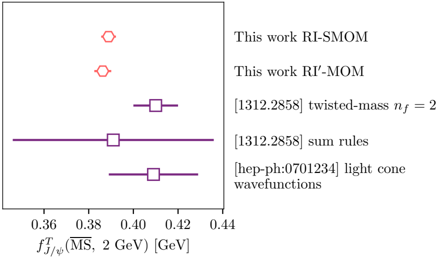
4. Fit all of the results for the $\overline{MS}$ decay constant at $2\ \mathrm{GeV}$ to a function that allows for discretisation effects and condensate contamination coming from Z_T^{SMOM}.

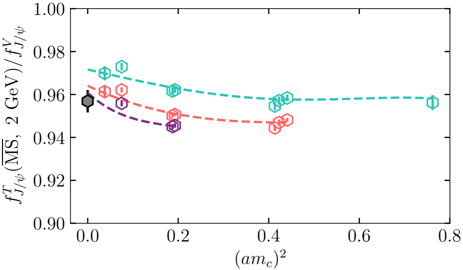
J/ψ tensor decay constant $f^T_{J/\psi}$ and ratio $f^T_{J/\psi}/f^V_{J/\psi}$

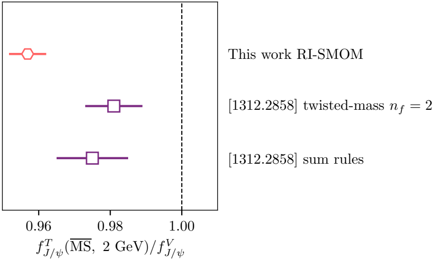
Our (pure QCD) results are:

$$f^T_{J/\psi}(\overline{MS}, 2\,\mathrm{GeV}) = 0.3927(27)\ \mathrm{GeV}$$

$$\frac{f^T_{J/\psi}(\overline{MS}, 2\,\mathrm{GeV})}{f^V_{J/\psi}} = 0.9569(52)$$



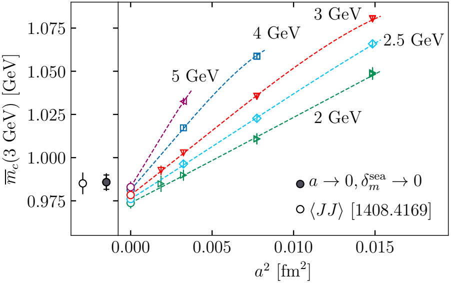




Potentially useful for tests of BSM physics.

Quark masses: Recipe

1. Tune the lattice charm mass so that the J/ψ mass takes its experimental value on each ensemble.
2. Calculate the mass renormalisation factor Z_m^{SMOM}. Convert the mass to the $\overline{MS}$ scheme at multiple scales μ using RI-SMOM scheme as an intermediate scheme on each ensemble.
3. Run all the $\overline{MS}$ quark masses at a range of scales μ to a reference scale of 3 GeV using the $\overline{MS}$ β function.



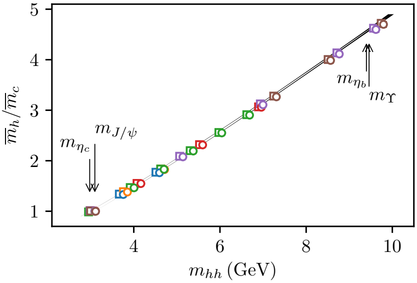
4. Fit all of the results for the $\overline{MS}$ mass at 3 GeV to a function that allows for discretisation effects and condensate contamination from Z_m^{SMOM}.
5. Repeat all steps for QCD+QED.

Ratio of quark masses

Ratio of quark masses: in pure QCD

$$\frac{\overline{m}_b(\mu)}{\overline{m}_c(\mu)} = \frac{m_b^{\text{tuned}}}{m_c^{\text{tuned}}} + \mathcal{O}(\alpha_s(\pi/a)a^2)$$

We have very few data points at physical b mass.



Instead we define functions g

$$m_{hh}^P = g_{hh}^P(\overline{m}_h/\overline{m}_c),$$
$$m_{hh}^V = g_{hh}^V(\overline{m}_h/\overline{m}_c),$$

for pseudoscalar and vector meson masses, where

$$g_{hh}^P(1) = M_{\eta_c}^{\text{cont}}, \quad g_{hh}^V(1) = M_{J/\psi}^{\text{cont}}.$$

The ratio can be obtained by solving

$$M_{\eta_b}^{\text{cont}} = g_{hh}^P(\overline{m}_b/\overline{m}_c) \text{ or}$$
$$M_{\Upsilon}^{\text{cont}} = g_{hh}^V(\overline{m}_b/\overline{m}_c).$$

We used a cubic spline fit to do this.

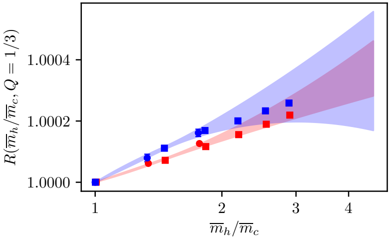
Ratio of quark masses: adding QED

Adding QED can be done in steps:

$$\left.\frac{\overline{m}_b(\mu)}{\overline{m}_c(\mu)}\right|_{**} = \frac{R(\overline{m}_b/\overline{m}_c, Q_{c,b}=0 \to \frac{1}{3})}{R(\overline{m}_c(\mu), Q_c=\frac{1}{3} \to \frac{2}{3})} \times \left.\frac{\overline{m}_b}{\overline{m}_c}\right|_{*}$$

$*$ is pure QCD and $**$ QCD+QED.

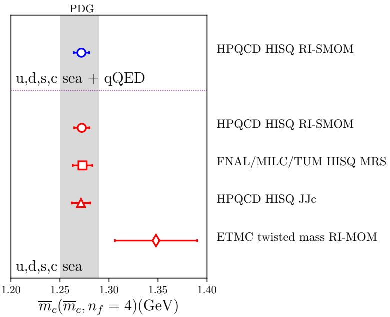
- Consider the mass ratio in theory where c and b have the same Q. $R(\overline{m}_b/\overline{m}_c, Q_{c,b}=0 \to \frac{1}{3})$ is the factor needed to change that charge from 0 (pure QCD) to $\frac{1}{3}$.
- $R(\overline{m}_c(\mu), Q_c=\frac{1}{3} \to \frac{2}{3})$ is the factor of changing charm quark electric charge Q from $\frac{1}{3}$ to $\frac{2}{3}$. Get from charm mass calculation.

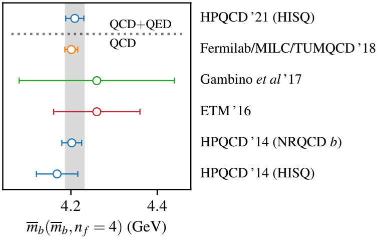


The key QED effect is $Q_c \to \frac{2}{3}$.

Ratio of quark masses in $\overline{MS}$ scheme in QCD+QED:

$$\frac{\overline{m}_b(3\text{ GeV}, n_f = 4)}{\overline{m}_c(3\text{ GeV}, n_f = 4)} = 4.586(12)$$





Charm quark mass in $\overline{MS}$ scheme in QCD+QED:

$$\overline{m}_c(3\ \text{GeV}, n_f = 4) = 0.9841(51)\ \text{GeV}$$
$$\overline{m}_c(\overline{m}_c, n_f = 4) = 1.2719(78)\ \text{GeV}$$

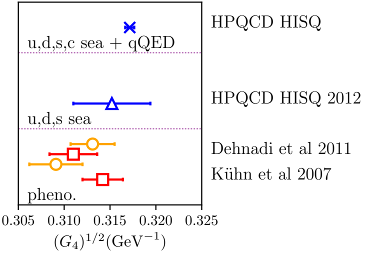
Bottom quark mass in $\overline{MS}$ scheme in QCD+QED:

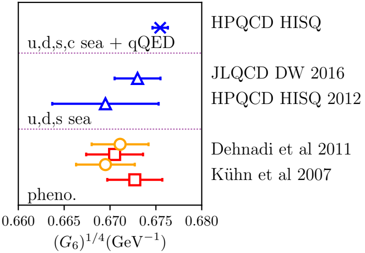
$$\overline{m}_b(3\ \text{GeV}, n_f = 4) = 4.513(26)\ \text{GeV}$$
$$\overline{m}_b(\overline{m}_b, n_f = 4) = 4.209(21)\ \text{GeV}$$
$$\overline{m}_b(\overline{m}_b, n_f = 5) = 4.202(21)\ \text{GeV}$$

Thank you!

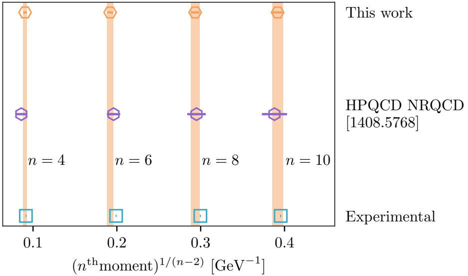
Backup slides

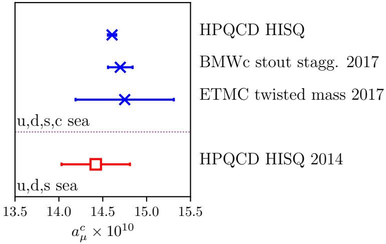
Time moments of vector current-current correlators

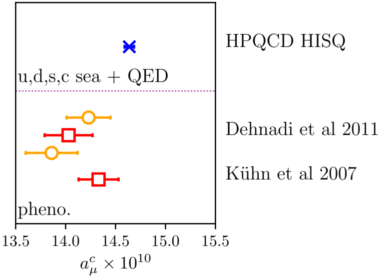




The time moments of the quarkonium vector current-current correlators can be compared with phenomenological results derived from $R(e^+e^- \to \text{ hadrons})$.
As the results are very accurate, this provides a stringent test.







The time moments of the quarkonium vector current-current correlators can also be used to calculate the quark-line connected HVP contribution to muon anomalous magnetic moment, a_μ. This is of particular interest, as new results are expected soon from the Muon $g-2$ experiment at Fermilab.

Our QCD+QED results are:

$$a_\mu^c = 14.638(47) \times 10^{-10}$$
$$a_\mu^b = 0.300(15) \times 10^{-10}$$

Error budgets: Charmonium

Error budget for our final result for the charmonium hyperfine splitting and decay constants including quenched QED corrections. The uncertainties shown are given as a percentage of the final result.

	ΔM_{hyp}	$f_{J/\psi}$	f_{η_c}
$a^2 \to 0$	0.13	0.09	0.03
Z_V	-	0.05	-
Pure QCD statistics	0.24	0.12	0.05
QCD+QED statistics	0.08	0.05	0.02
w_0/a	0.24	0.11	0.08
w_0	0.87	0.34	0.24
Valence mistuning	0.02	0.05	0.01
Sea mistuning	0.06	0.01	0.00
Total (%)	0.96	0.40	0.26

Error budgets: Bottomonium

	$M_\Upsilon - M_{\eta_b}$	f_Υ	f_{η_b}
statistics	2.40	0.77	0.38
SVD cut	1.48	0.44	0.67
w_0	0.55	0.61	0.59
w_0/a	0.66	0.23	0.18
Z_V	-	0.29	-
M_{ϕ_h} dependence	0.03	0.01	0.00
$1/M_{\phi_h}$ dependence	0.05	0.02	0.01
$(am_h)^{2k}$ discretisation effects	1.14	0.17	0.18
$(a\Lambda)^{2k}$ discretisation effects	0.48	0.24	0.31
$(am_h)^2(a\Lambda)^2$ discretisation effects	0.42	0.28	0.45
light and strange sea quark mistuning	1.45	0.73	0.98
charm sea quark mistuning	1.08	0.29	0.27
QED M_{ϕ_h} dependence	0.29	0.07	0.08
QED $1/M_{\phi_h}$ dependence	0.19	0.01	0.00
Total (%)	3.99	1.43	1.59

Error budgets: Charm quark mass

	$\overline{m}_c$(3 GeV)
$a^2 \to 0$	0.23
Missing α_s^4 term	0.10
Condensate	0.21
m_{sea} effects	0.00
$Z_m^{\overline{\mathrm{MS}}/\mathrm{SMOM}}$ and r	0.07
Z_m^{SMOM}	0.12
Uncorrelated m^{tuned}	0.15
Correlated m^{tuned}	0.30
Gauge fixing	0.09
μ error from w_0	0.12
QED effects	0.02
Total (%)	0.52

Error budgets: c and b quark mass ratio

	$\overline{m}_b/\overline{m}_c[m^P_{hh}]$	$\overline{m}_b/\overline{m}_c[m^V_{hh}]$	$\overline{m}_b/\overline{m}_c[\mathrm{avg}]$
$(am_h)^2 \to 0$	0.20	0.21	0.20
w_0, w_0/a	0.10	0.18	0.12
σ_u	0.12	0.12	0.09
g_m, ζ	0.05	0.05	0.05
m_{cc}	0.06	0.01	0.04
m_{bb}	0.03	0.00	0.02
$(am_h)^2\delta m^{\mathrm{sea}}_{uds} \to 0$	0.06	0.07	0.06
$\delta m^{\mathrm{sea}}_c \to 0$	0.03	0.03	0.03
$d\tilde{m}_c/dm_{cc}$	0.03	0.02	0.02
Total (%)	0.27	0.32	0.27

Error budgets: Charm quark HVP contribution to a_μ^c

	a_μ^c
$a^2 \to 0$	0.15
Z_V	0.07
Pure QCD Statistics	0.08
QCD+QED Statistics	0.01
w_0/a	0.16
w_0	0.18
Sea mistunings	0.09
Valence mistunings	0.03
$M_{J/\psi}^{\mathrm{exp.}}$	0.05
Total (%)	0.32