Chiral Symmetry Restoration, Thermal Resonances and the $U(1)_A$ symmetry

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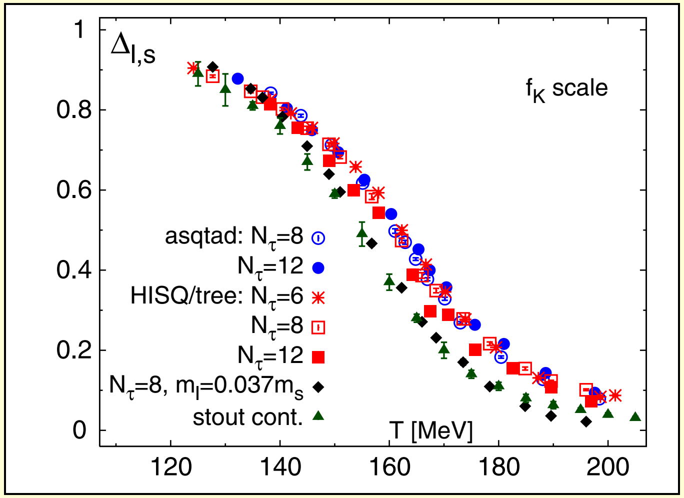
Chiral/deconfinement CROSSOVER transition @$T_c \sim$ 155 MeV
($\mu_B = 0$, $N_f = 2+1$, physical masses)

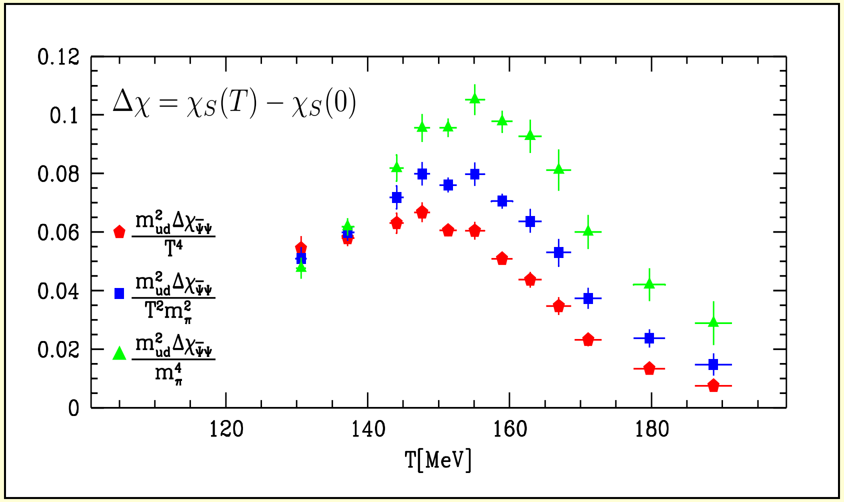
Down to $T_c^0 \sim 132$ MeV in chiral limit (true phase transition for $N_f = 2$ massless)

Well established in lattice through:

- **Inflection point of (subtracted) light quark condensate** $\langle \bar{q}q \rangle_l (T)$
- **Peak of scalar susceptibility** $\chi_S(T) = -\frac{\partial}{\partial m_l} \langle \bar{q}q \rangle_l = \int_x \left[\langle \bar{q}q(x)\bar{q}q(0) \rangle_T - \langle \bar{q}q \rangle_l^2 \right]$

Y.Aoki, S. Borsanyi et al (Budapest-Wuppertal) 2009, 2010
A.Bazavov et al (Hot QCD) 2012-2019





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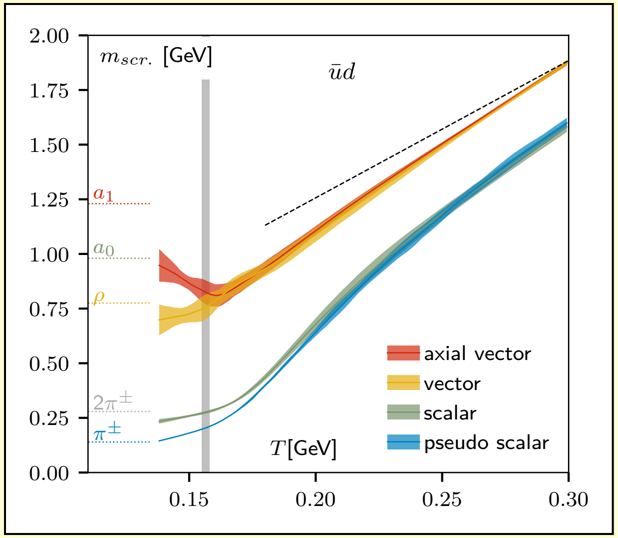
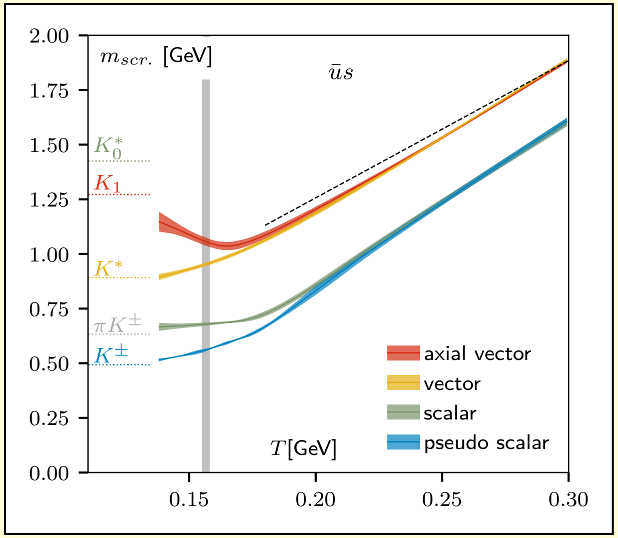
Well established in lattice through:

chiral partners (degener. of susceptibilities and screening masses):
$\rho/a_1,\ \sigma/\pi,\ K/\kappa, \ldots$

A.Bazavov et al (Hot QCD) 2019

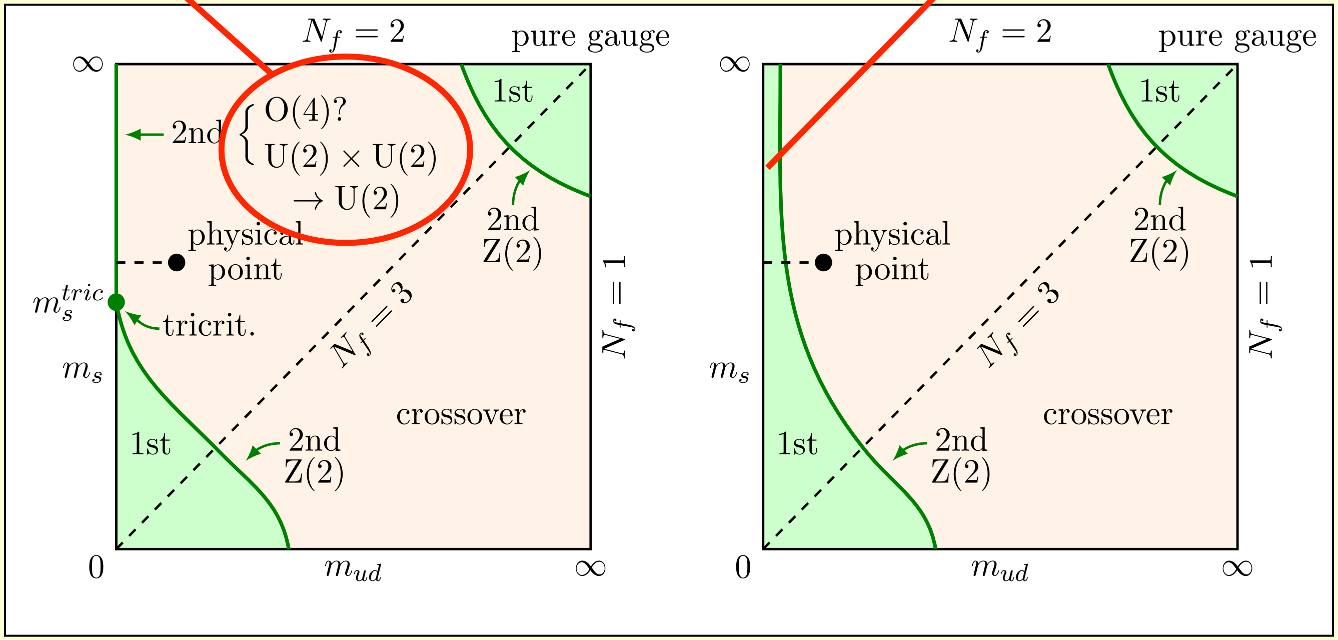




$\Rightarrow$ Nature of the chiral transition (chiral pattern) open problem with the QCD phase diagram

Universality class depends on the strenght of $U(1)_A$ breaking @T_c

Transition order can even change if $U(1)_A$ is sufficiently restored



B.B. Brandt et al 2019

Gross, Pisarski, Yaffe 1981, Shuryak 1994. Cohen 1996. Lee-Hatsuda 1996

Pisarski, Wilczek, 1984. Pelissetto, Vicari 2013. Esser, Grahl, Rischke 2015

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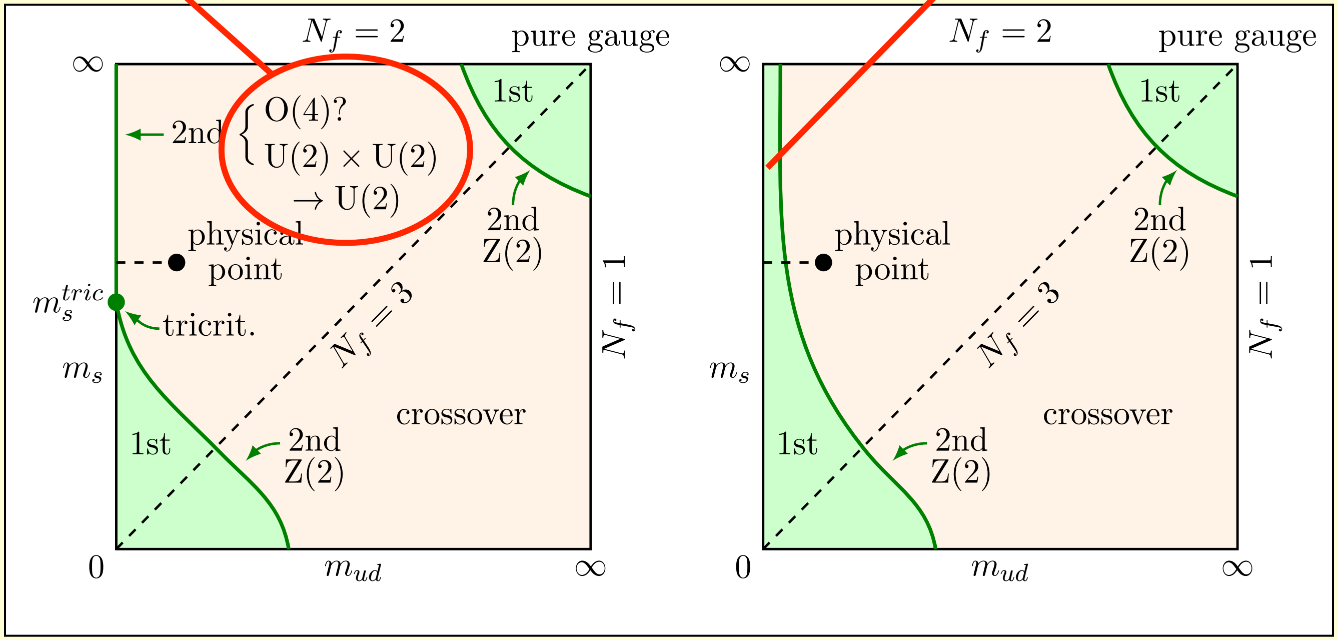

B.B. Brandt et al 2019

$\Rightarrow$ **How effective is $U(1)_A$ asymptotic restoration at T_c?**

CHIRAL PATTERNS AND PARTNERS

Some consequences of $U(1)_A$ restoration around T_c:

- $\chi_{top}(T)$ **and** $M_{\eta'}(T)$ **(visible in HIC) reduction**

  J. I. Kapusta, D. Kharzeev, L. D. McLerran, PRD53, 5028 (1996)

  T. Csorgo et al, PRL105, 182301 (2010)

  G. Grilli di Cortona, et al, JHEP 1601, 034 (2016)

  M. Ishii et al, PRD95, 114022 (2017)

  AGN, J.Ruiz de Elvira, A.Vioque-Rodríguez, JHEP11, 086 (2019)

  M. P. Lombardo, A. Trunin, IJMPA 35 (2020)

- $\eta-\eta'$ **mixing ideal** $(\eta\sim\eta_l,\ \eta'\sim\sqrt{2}\eta_s)$

  M. Ishii et al PRD93, 016002 (2016)

  AGN, J.Ruiz de Elvira, JHEP1603, 186 (2016), PRD97, 074016 (2018), PRD98, 014020 (2018)

- $U(1)_A @ T_c$ **affects critical point at** $\mu_B\neq 0$

  M. Mitter, B. J. Schaefer, PRD89, 054027 (2014)

$O(4)$ and $U(1)_A$ partners for scalar/pseudoscalar nonets: $I = 0, 1$

$\psi_l^T = (u, d)$

$I = 1$ $\qquad$ $I = 0$

$$\begin{array}{ccc} \pi^a = i\bar{\psi}_l \gamma_5 \tau^a \psi_l & \xleftrightarrow{SU(2)_A} & \sigma_l = \bar{\psi}_l \psi_l \\ \updownarrow U(1)_A & & \updownarrow U(1)_A \\ \delta^a = \bar{\psi}_l \tau^a \psi_l & \xleftrightarrow{SU(2)_A} & \eta_l = i\bar{\psi}_l \gamma_5 \psi_l \end{array}$$

Lowest meson states:

$\pi^a \to$ **pion**, $\quad \delta^a \to a_0(980)$

$\sigma_l, \ \sigma_s = \bar{s}s \to f_0(500), f_0(980)$ **(mixed)**

$\eta_l, \ \eta_s = i\bar{s}\gamma_5 s \to \eta, \eta'$ **(mixed)**

Chiral SSB: $SU(2)_V \times SU(2)_A \, [\approx O(4)] \to SU(2)_V \, [\approx O(3)]$

Not fully settled in the lattice:

$$\begin{array}{ccc} \pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi_l & \xleftrightarrow{SU(2)_A} & \sigma_l = \bar{\psi}_l\psi_l \\ \updownarrow_{U(1)_A} & & \updownarrow_{U(1)_A} \\ \delta^a = \bar{\psi}_l\tau^a\psi_l & \xleftrightarrow{SU(2)_A} & \eta_l = i\bar{\psi}_l\gamma_5\psi_l \end{array}$$

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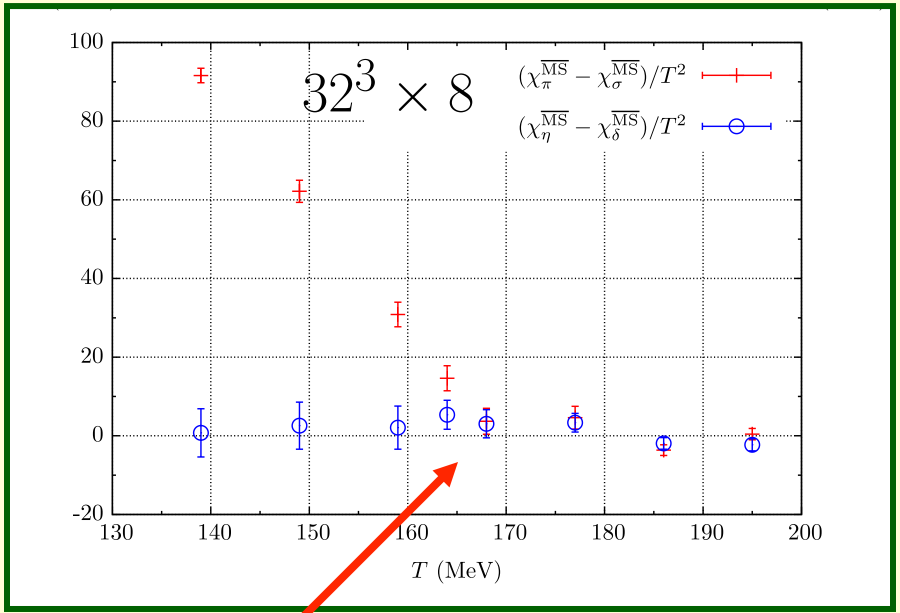
$N_f = 2 + 1$ **susceptibilities**

(for physical m_{ud}, m_s masses)

Buchoff et al (LLNL/RBC coll) PRD89 (2014)



$O(4)$ **OK (with large uncertainties in $\chi_\eta - \chi_\delta$)**

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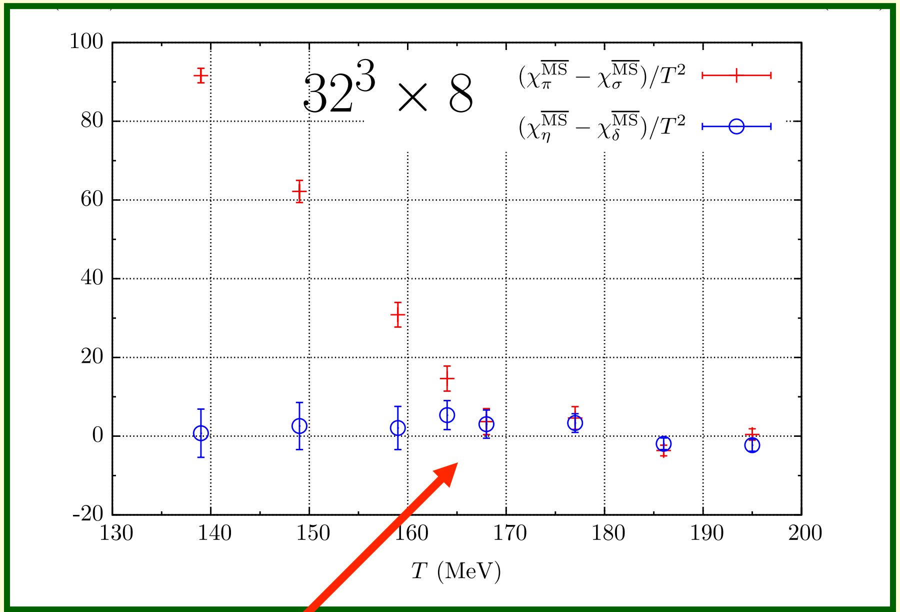
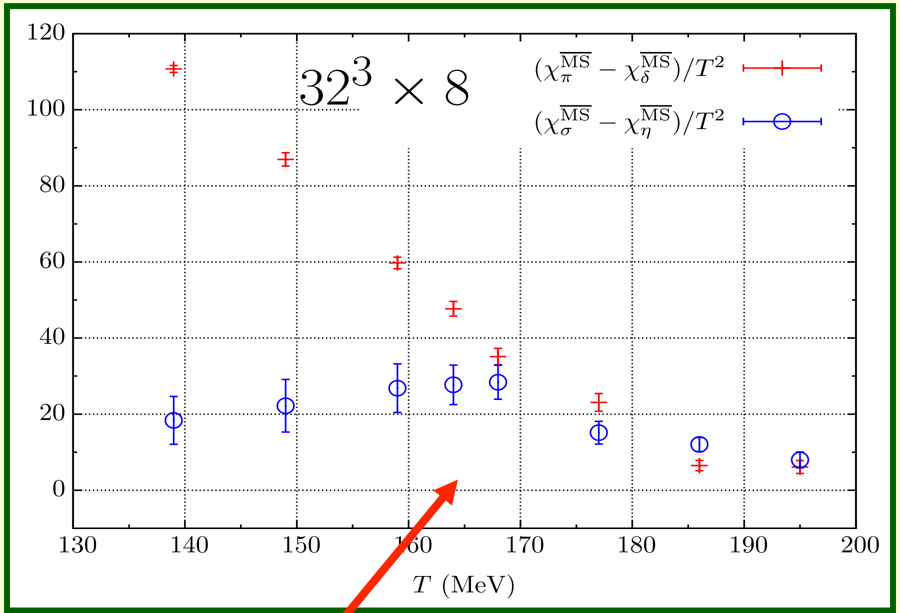




$O(4)$ **OK** (**with large uncertainties in** $\chi_\eta - \chi_\delta$)

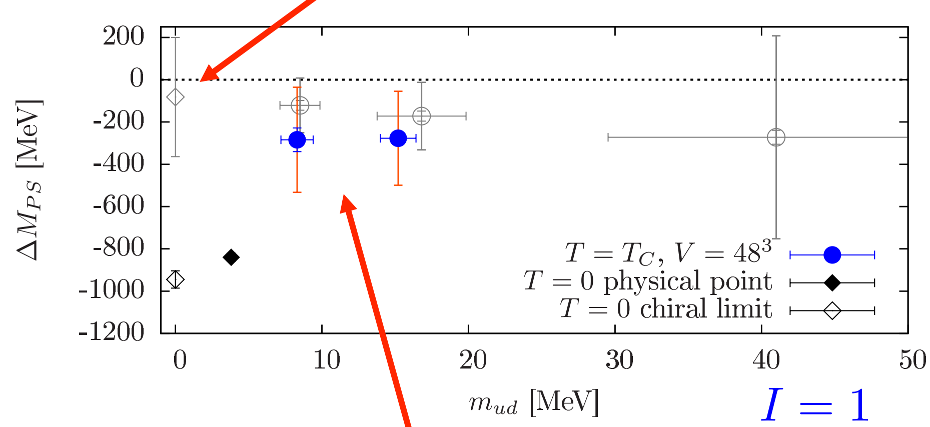
significant $U(1)_A$ **breaking @** T_c **for physical masses**

Aoki et al, 2012, Cossu et al, 2013 ($N_f = 2$, $\hat{m} \to 0$)

Brandt et al 2016, ($N_f = 2$, $\hat{m} \neq 0$)
2019 ($N_f = 2$, **incl.** $\hat{m} \to 0$ **screening masses**)

$$\begin{array}{ccc} \pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi_l & \overset{SU(2)_A}{\longleftrightarrow} & \sigma_l = \bar{\psi}_l\psi_l \\ \updownarrow_{U(1)_A} & & \updownarrow_{U(1)_A} \\ \delta^a = \bar{\psi}_l\tau^a\psi_l & \overset{SU(2)_A}{\longleftrightarrow} & \eta_l = i\bar{\psi}_l\gamma_5\psi_l \end{array}$$

Compatible with $U(1)_A$ restoration @T_c in chiral limit



Small $U(1)_A$ breaking for $m \neq 0$, increasing with larger volumes

WARD IDENTITIES

AGN, J.Ruiz de Elvira, JHEP 2016, PRD 2018

Connect in particular $\chi_{S,P}$ and $\langle \bar{q}_i q_i \rangle$ from QCD generat.funct.

⇩

$$\boxed{\chi_P^{ls}(T) = -2\frac{m_l}{m_s}\chi_{5,disc}(T) = -\frac{2}{m_l m_s}\chi_{top}(T)}$$

$$\chi_P^{ls} = \int_T dx \langle \mathcal{T} \eta_l(x) \eta_s(0) \rangle$$

$\chi_{5,disc} = \frac{1}{4}\left(\chi_P^{\pi} - \chi_P^{\eta_l}\right)$ **measures** $O(4) \times U(1)_A$ **restoration**

$\chi_{top} = -\frac{1}{36}\int_T dx \langle \mathcal{T} A(x) A(0) \rangle$ **topological susceptibility**, $A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

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- $SU(2)_A$ **transforms (**η_s **invariant)** $P_{ls} \xrightarrow{(*)} \langle \delta \eta_s \rangle = 0$ **by parity**

$$\boxed{\eta_l \overset{O(4)}{\sim} \delta \Rightarrow \chi_P^{ls} \overset{O(4)}{\sim} 0 \Rightarrow \chi_{5,disc} \overset{O(4)}{\sim} 0, \ \chi_{top} \overset{O(4)}{\sim} 0}$$

$(*)\ \eta_l \to i\bar{\psi}_l \gamma_5 e^{i\frac{\pi}{2}\gamma_5 \tau^b}\psi_l = -\delta^b$

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Also Azcoiti 2016 with similar WI

$\Longrightarrow O(4) \times U(1)_A$ **pattern for *exact* chiral restoration**

(hence consistent with Cossu, Aoki, Brandt et al $N_f = 2$, $m_l \to 0$**)**

$I = 1/2$ SECTOR (K/κ): Role of strangeness

AGN, J.Ruiz de Elvira, A.Vioque-Rodríguez, D.Alvarez EPJC 2021

- $K^a = i\bar{\psi}\gamma_5\lambda^a\psi \leftrightarrow \kappa^a = \bar{\psi}\lambda^a\psi$ $\quad O(4)$ **and** $U(1)_A$ **degenerated**

  (Kaon and $K_0^*(700)$ or κ)

$\psi^T = (u, d, s), a = 4, \ldots 7$

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- **WIs for this sector:**

$$\boxed{\chi_P^K(T) = -\frac{\langle \bar{q}q \rangle_l (T) + 2 \langle \bar{s}s \rangle (T)}{m_l + m_s}}$$

$$\boxed{\chi_S^\kappa(T) = \frac{\langle \bar{q}q \rangle_l (T) - 2 \langle \bar{s}s \rangle (T)}{m_s - m_l}}$$

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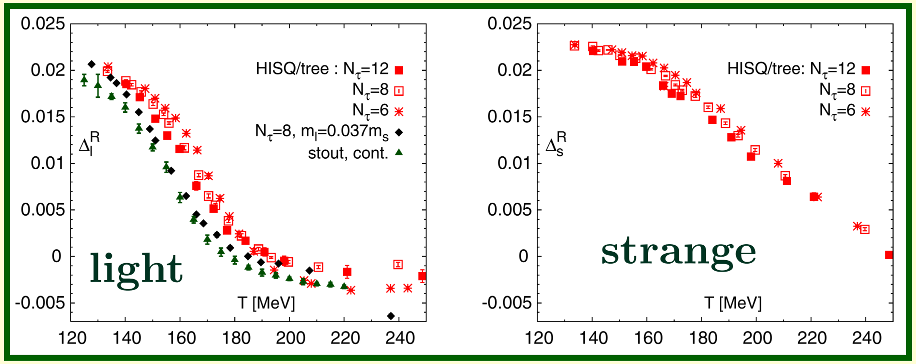


- **WIs for this sector:**

$$\chi_P^K(T) = -\frac{\langle \bar{q}q \rangle_l (T) + 2\langle \bar{s}s \rangle (T)}{m_l + m_s}$$

$\Rightarrow$ **decreases, faster just below T_c (from $\langle \bar{q}q \rangle_l$)**

$$\chi_S^\kappa(T) = \frac{\langle \bar{q}q \rangle_l (T) - 2\langle \bar{s}s \rangle (T)}{m_s - m_l}$$

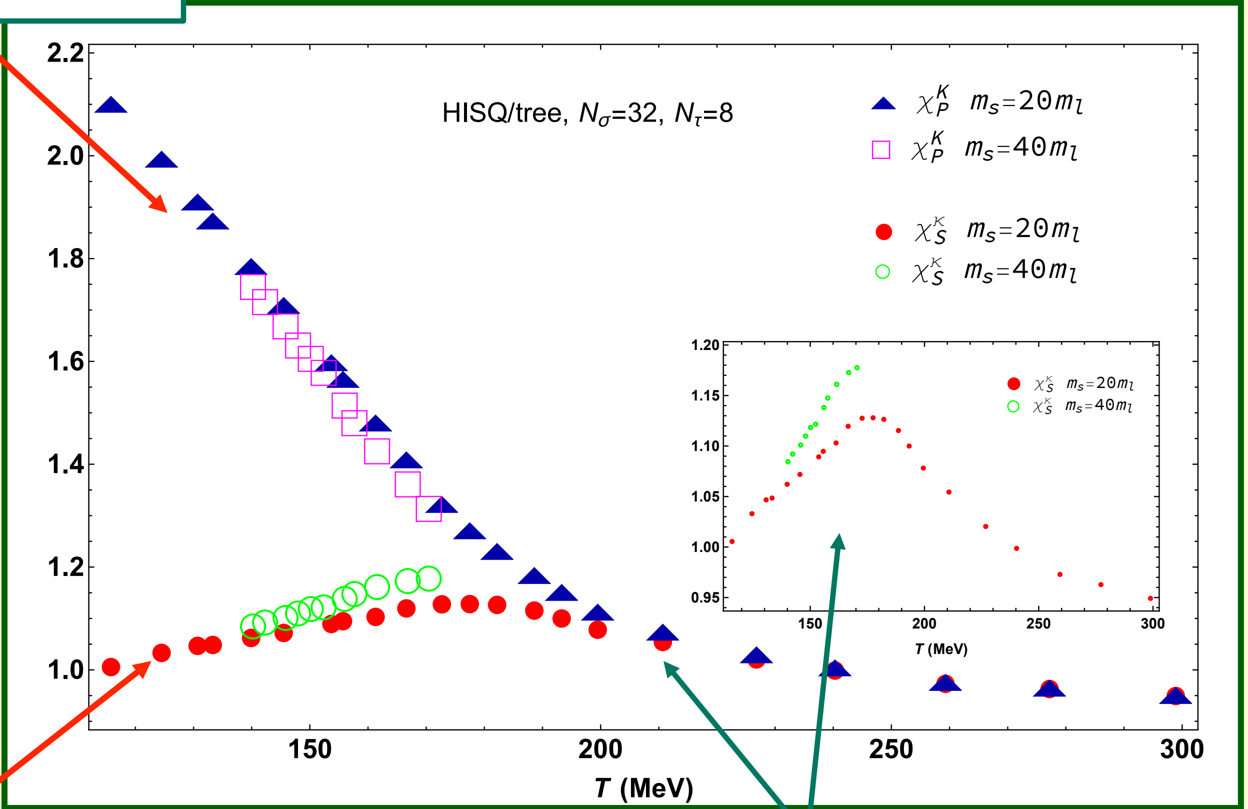
$\Rightarrow$ **Increases driven by $\langle \bar{q}q \rangle_l$, decreases as $\langle \bar{s}s \rangle$ takes over (above T_c) and $\chi_S^\kappa \to \chi_P^K$**

$\Rightarrow \chi_S^\kappa$ PEAK

Reconstructed susceptibilities from WI and lattice condensate data (no direct results yet)

Lattice points from
A.Bazavov et al (Hot QCD) 2012-14 $(N_f = 2+1)$

$$\chi_P^K(T) = -\frac{\langle \bar{q}q \rangle_l (T) + 2\langle \bar{s}s \rangle (T)}{m_l + m_s}$$



$$\chi_S^\kappa(T) = \frac{\langle \bar{q}q \rangle_l (T) - 2\langle \bar{s}s \rangle (T)}{m_s - m_l}$$

- $\chi_S^\kappa \to \chi_P^K$ in $O(4) \times U(1)_A$ region, above χ_S^κ peak
- Peak behaviour driven by m_l/m_s

In addition ...

$$\boxed{\chi_S^{\kappa}(T) - \chi_P^{K}(T) = \frac{2m_s}{m_s^2 - m_l^2}\left[\langle \bar{q}q\rangle_l(T) - 2\frac{m_l}{m_s}\langle \bar{s}s\rangle(T)\right]}$$

In addition ...

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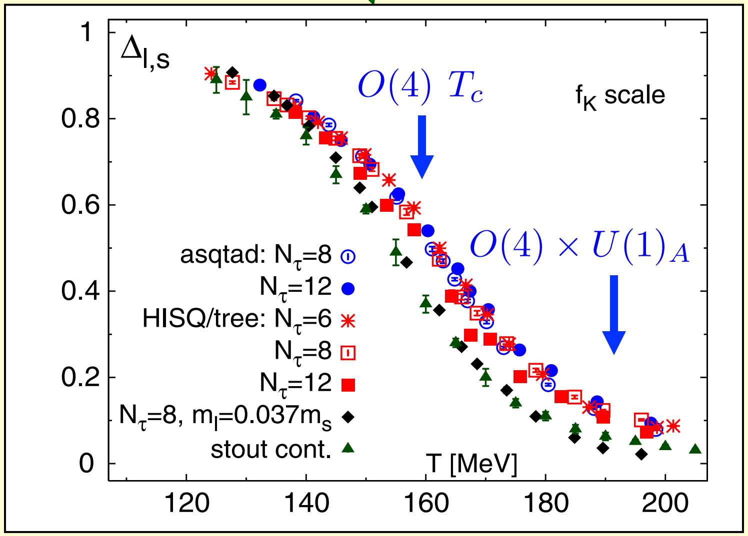
subtracted condensate $\Delta_{l,s}$
(removes lattice finite-size divergences)

$\Rightarrow$ $N_f = 2$, **consistent with** $O(4)\times U(1)_A$ **restoration for** $m_l, \langle\bar{q}q\rangle_l \to 0^+$



$\Rightarrow$ **In physical limit provides well-determined** $O(4)\times U(1)_A$ **breaking above** T_c **via** $\Delta_{l,s}$ **tail modulated by** $\langle\bar{s}s\rangle$

EFFECTIVE THEORIES

- Provide systematic analysis below the transition in terms of physical hadrons

- SU(3) and U(3) ChPT model-independent framework for light mesons (π, K, η, η') within $1/N_c \sim m_q \sim T^2 \sim p^2$ counting [(1)]

- Light meson scattering dominant interactions in the thermal bath. Unitarized scattering generates (thermal) resonances [(2)]

- HRG approach includes heavier states and describes most observables below T_c [(3)]

- Still, relevant observables for chiral and $U(1)_A$ restoration can be understood within (U)ChPT $>>>$

(1) Gasser, Leutwyler, Gerber, Kaiser, Herrera-Siklody et al, AGN, Ruiz de Elvira, ...

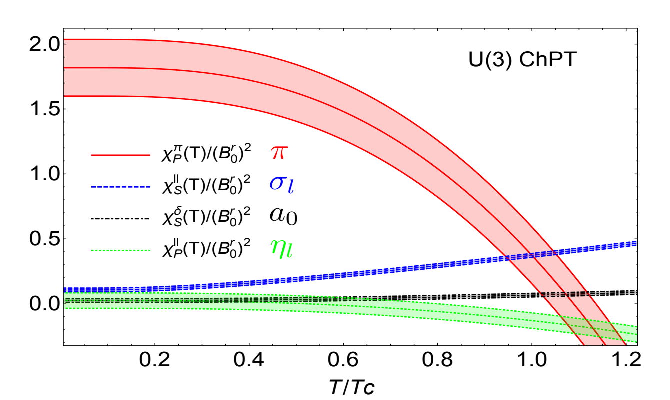
(2) Cabrera, Dobado, Fernández-Fraile, AGN, Llanes-Estrada, Peláez, Ruiz de Elvira, Torres-Andrés, Vioque, $\cdots$

(3) Karsch, Tawfik, Redlich, Tawfik-Toublan, Huovinen, Petreczcky, Jankowski, Blaschke, Spalinski, ...

Partners in $U(3)$ ChPT

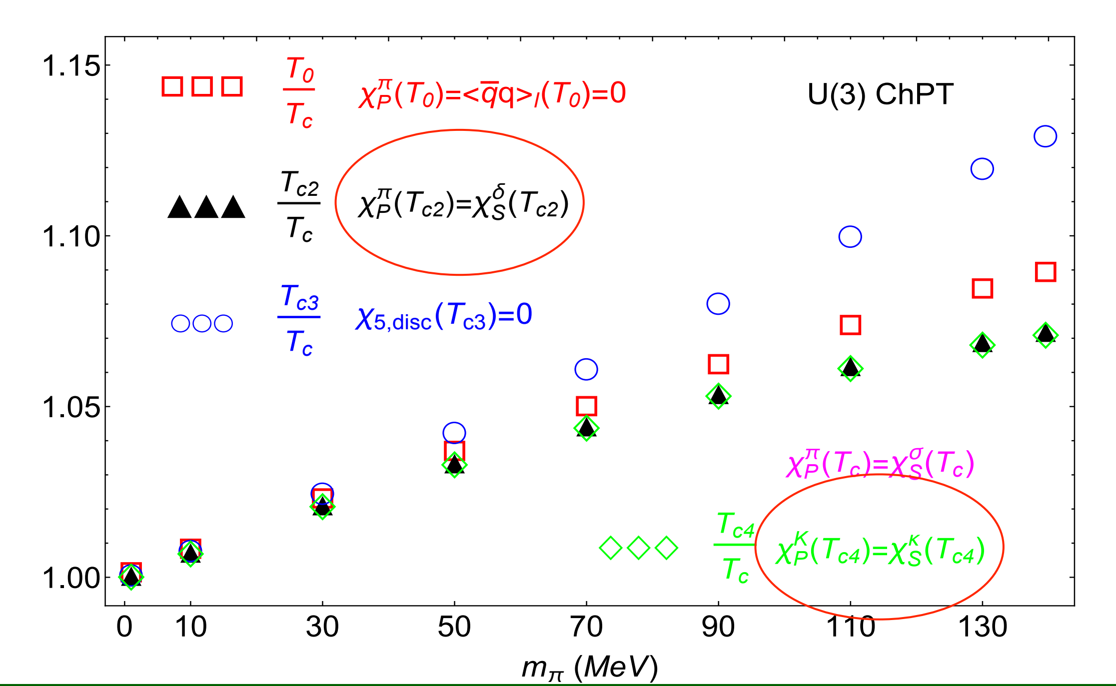
AGN, J.Ruiz de Elvira, PRD 2018

$1/N_c \sim m_q \sim T^2 \sim p^2$ counting



$T_{U(1)_A} \sim 1.1 T_{chiral}$
(within ChPT uncertainties)

LECs from
X. K. Guo et al 2015



$\rightarrow O(4) \times U_A(1)$ **in chiral limit**

(note $K-\kappa$ and $\pi-\delta$ coincidence)

$\Rightarrow$ Compatible with WIs, lattice and models (NJL Ishii et al PRD 2017)

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, PRD 2018

$\Rightarrow \chi_S$ **saturated by lightest** $p=0$ **state** $f_0(500)$ **(**σ**)**

$$\boxed{\chi_S(T) \simeq \chi_S(0)\frac{M_S^2(0)}{M_S^2(T)}}$$

$$M_S^2(T) = \mathbf{Re}(s_{pole}(T)) \sim \mathbf{Re}\Sigma_{f_0}(T)$$

Pole from UChPT $\pi\pi$ **scattering at finite** T
(AGN et al 2002)

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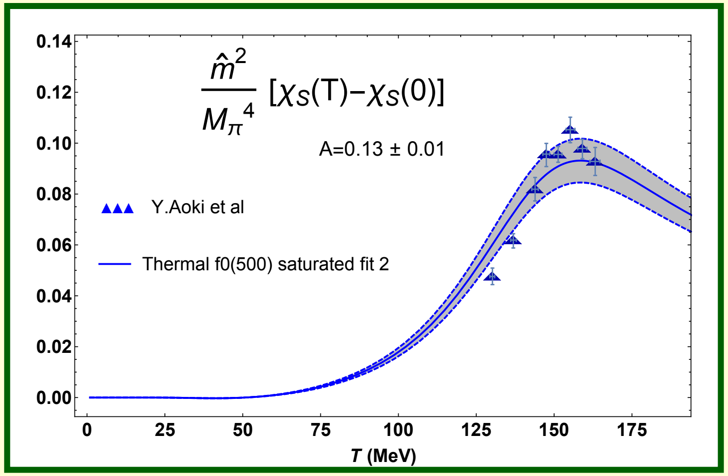
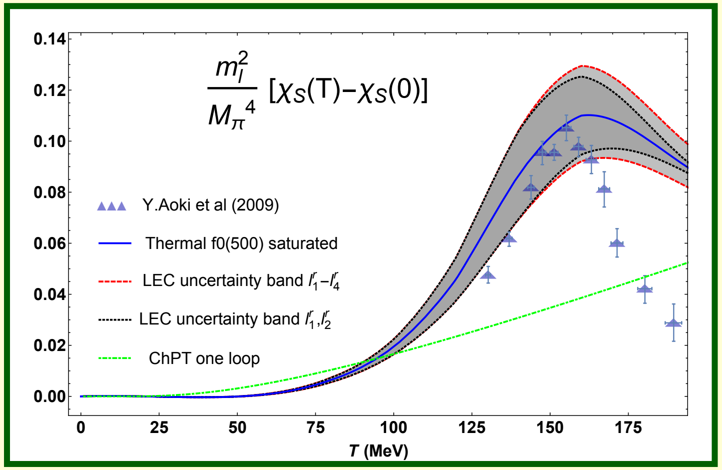


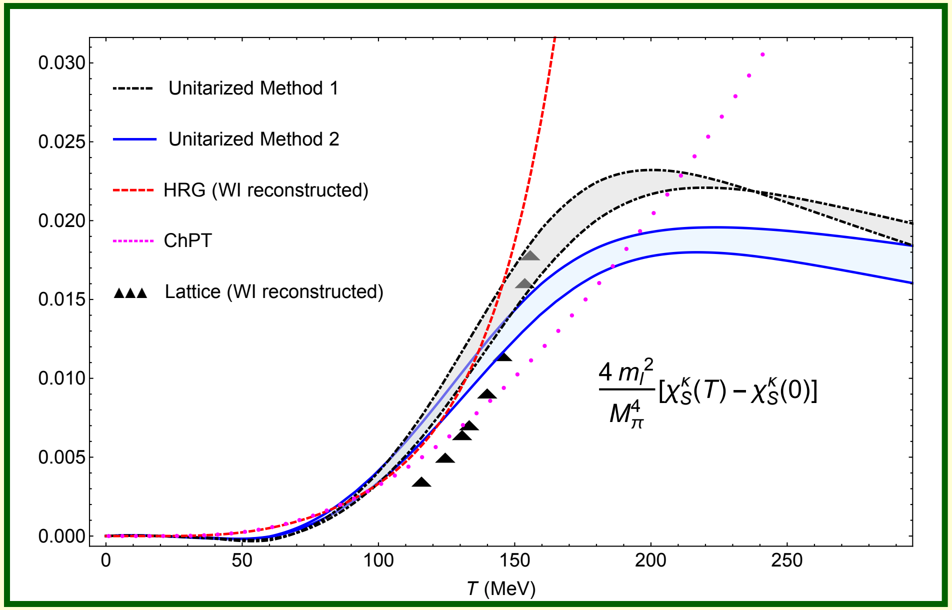


- $\Rightarrow$ **Reproduces expected peak, OK with lattice**
- $\Rightarrow$ **Thermal interactions crucial**
- $\Rightarrow$ **Uncertainties within LEC range**

Saturated κ scalar susceptibility in UChPT

AGN, J.Ruiz de Elvira, A.Vioque-Rodríguez, D.Alvarez EPJC 2021

$\Rightarrow$ χ_S^κ saturated now by $K_0^*(700)$ thermal pole in πK scattering



thermal interactions crucial
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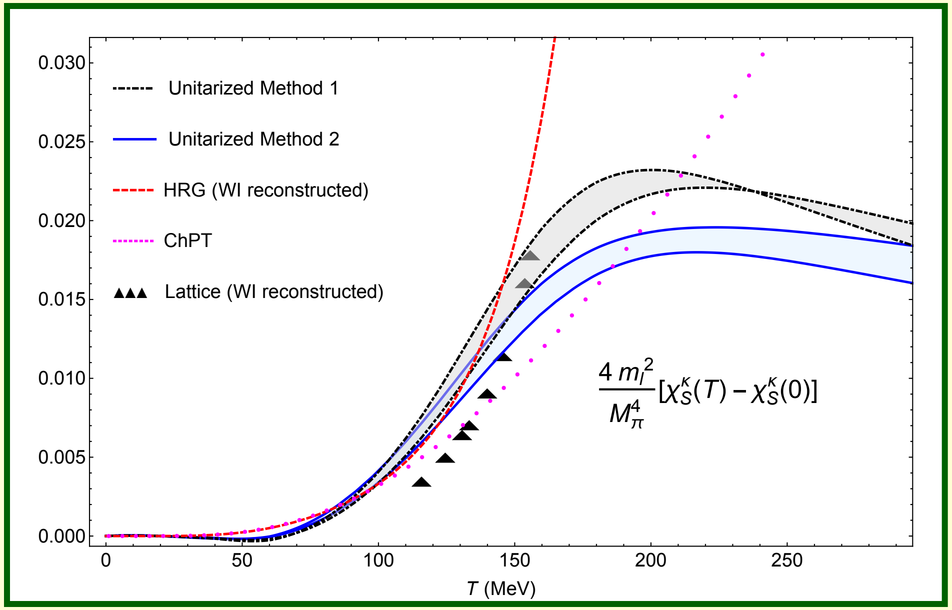
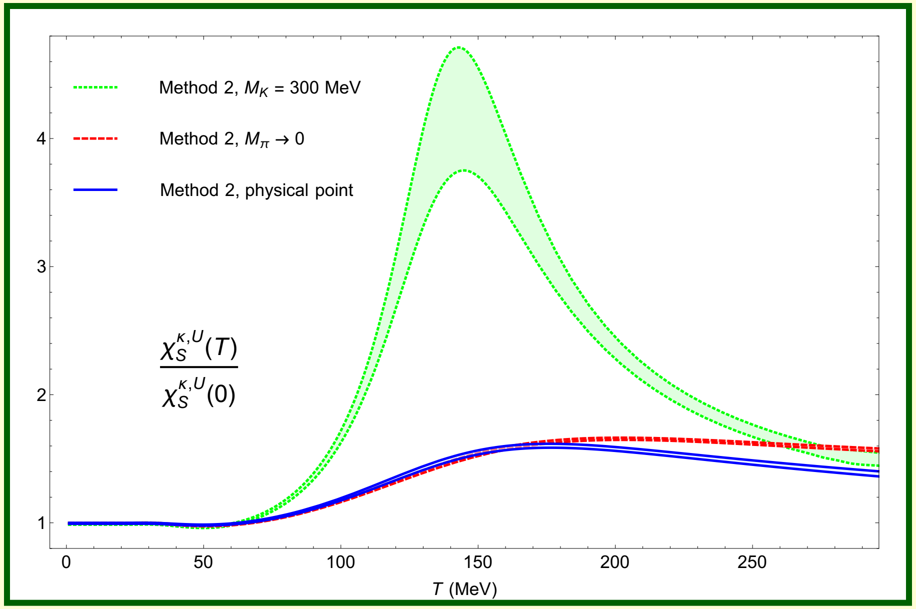




thermal interactions crucial to reproduce peak

- $m_l/m_s \to 0^+$: $\chi_S^\kappa \to \chi_P^K$ **at lower** T
- $m_l/m_s \to 1$: $\chi_S^\kappa \to \chi_S^\sigma$ $SU(3)$ **deg,**

(**Oller, Ruiz de Elvira, Meissner, ... @** $T=0$)

CONCLUSIONS

- ⋆ **Nature of chiral transition and interplay with $U(1)_A$ key for QCD phase diagram**

- ⋆ $U(1)_A$ **breaking @T_c stronger for $N_f = 2+1$ than $N_f = 2$**
  (role of strangeness and chiral limit crucial)

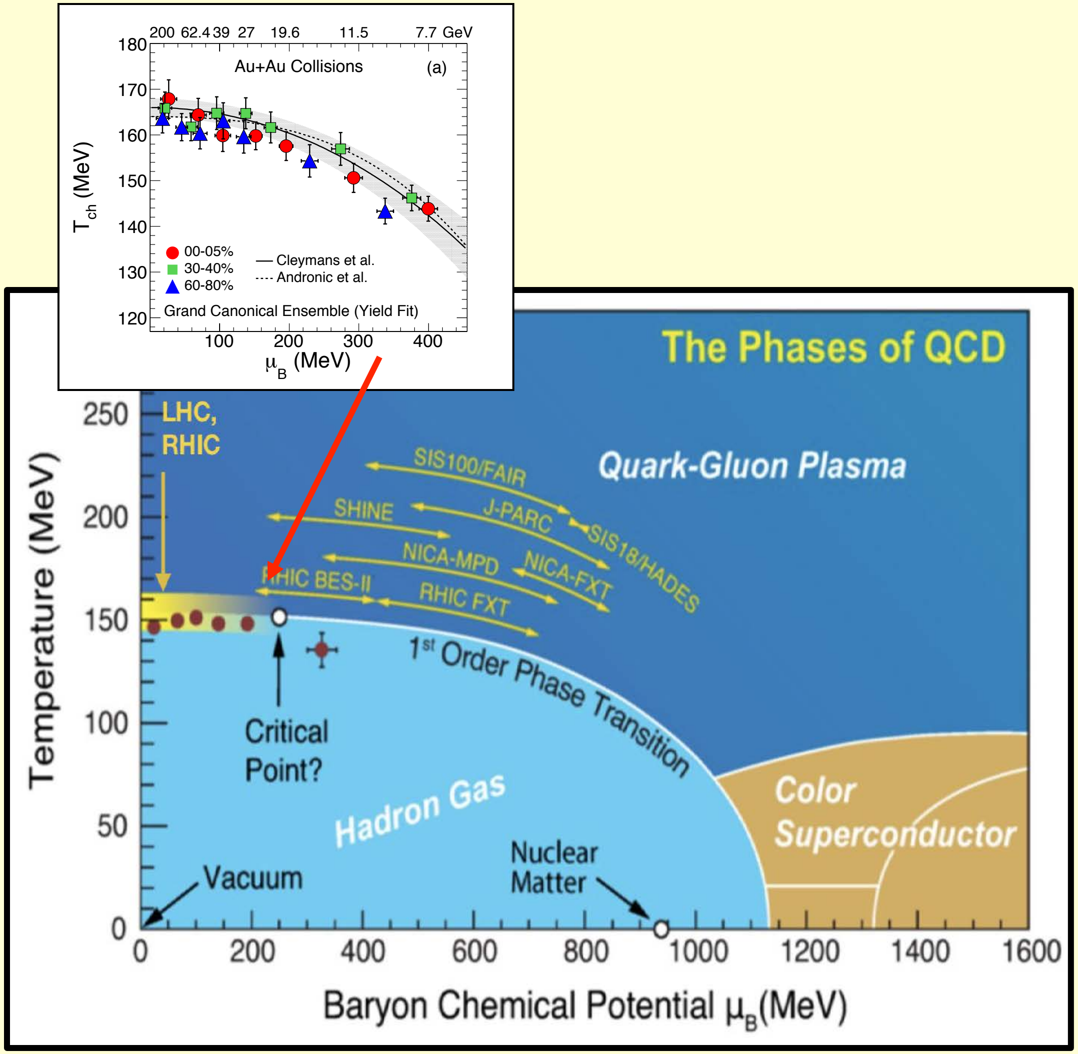
- ⋆ **WI** $\Rightarrow O(4) \times U(1)_A$ **for exact chiral restoration of S/P nonet**
  OK with $N_f = 2$ lattice

- ⋆ **WI** $\Rightarrow K/\kappa$ **suitable channel as alternative $O(4) \times U(1)_A$ sign driven by $\langle \bar{s}s \rangle$ around χ_S^κ peak**

- ⋆ **Eff.Theo.** $\Rightarrow$ **- Patterns and partners OK with WI**
  - Saturated χ_S, χ_S^κ with thermal $f_0(500)$, $K_0^*(700)$
  OK with lattice and WI for chiral & $U(1)_A$ rest.

BACKUP SLIDES

QCD PHASE DIAGRAM



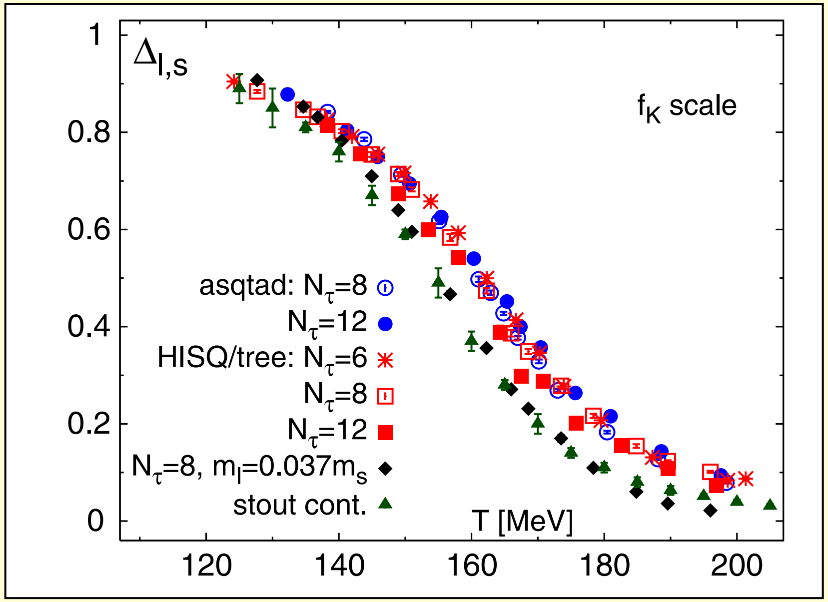
A.Bazavov et al, USQCD whitepaper 2019

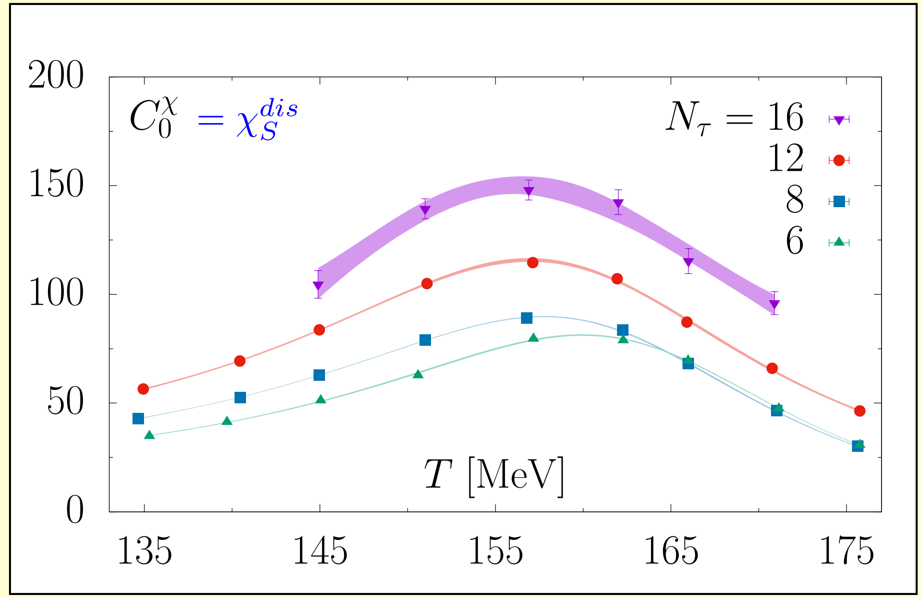
MAIN CHALLENGES:

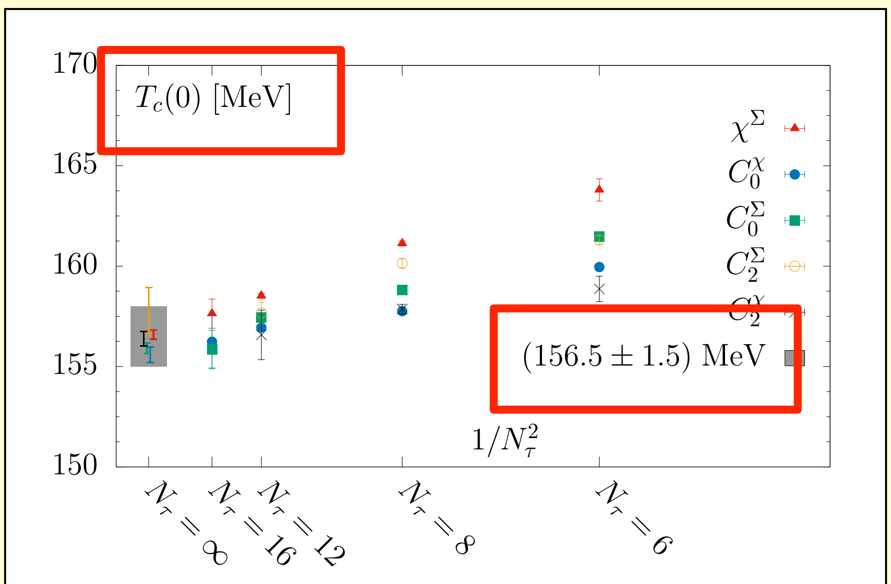
- Higher density beyond BES
- Existence and nature of the Critical point
- Nature of the Chiral Transition and interplay with $U(1)_A$

CONDENSATE AND SCALAR SUSCEPTIBILITY

A.Bazavov et al (Hot QCD) 2012-2019



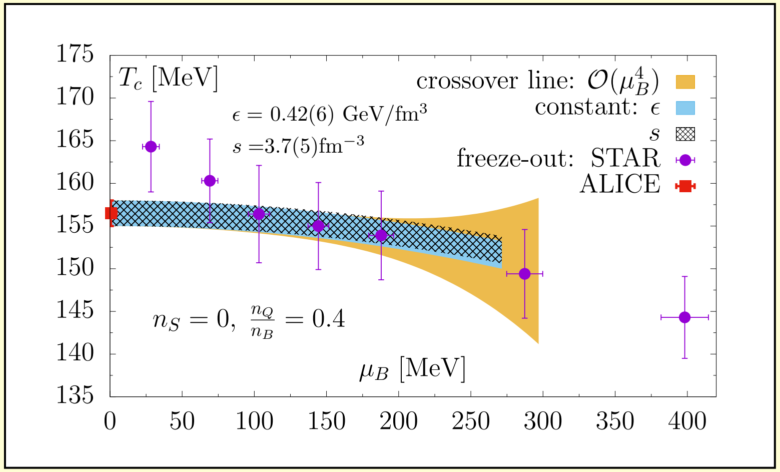




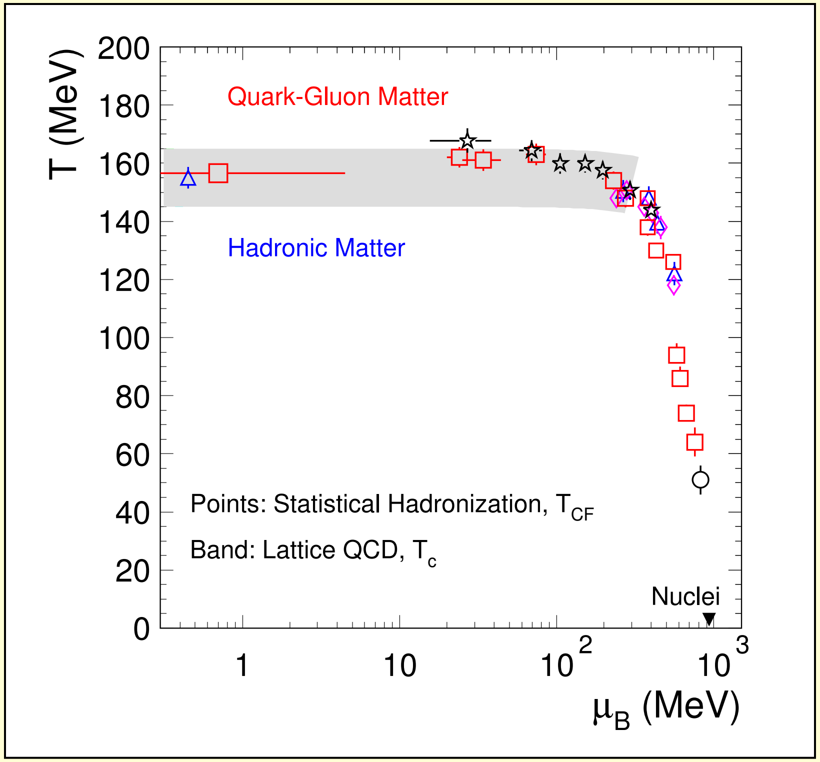
$$\Delta_{l,s} = \frac{\langle \bar{q}q \rangle_T - (2m_q/m_s)\,\langle \bar{s}s \rangle_T}{\langle \bar{q}q \rangle_0 - (2m_q/m_s)\,\langle \bar{s}s \rangle_0}$$

Chiral limit $T_c^0 = 132^{+3}_{-6}$ MeV
with reasonable $O(4)$ scaling
(Ding et al 2019)

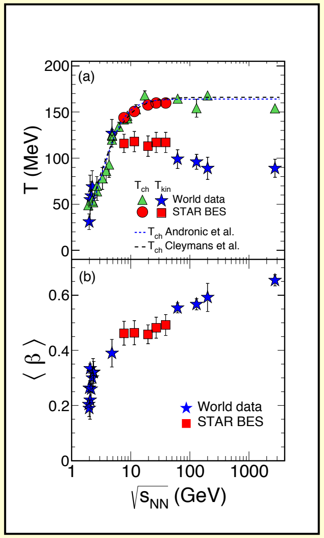
QCD phase diagram explored in HIC → chemical freeze-out close to phase boundary

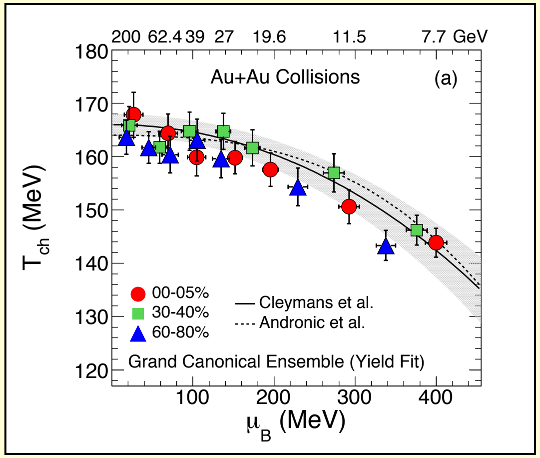


A.Bazavov et al (Hot QCD) 2018
(μ_B through Taylor expansion)



Andronic et al 2018 (ALICE)



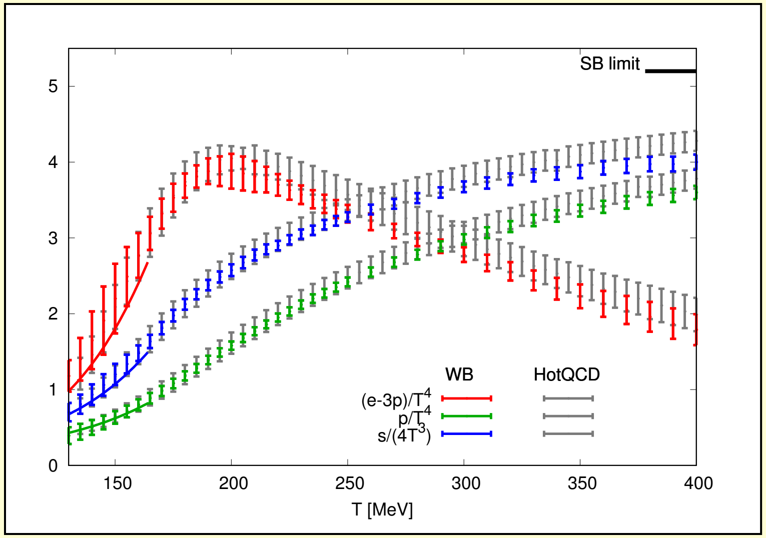


BES (STAR Adamczyk et al 2017)

Chemical FO from Hadron Statistical Model fit to hadron yields (central ALICE data)

OTHER HIGHLIGHTS OF QCD TRANSITION

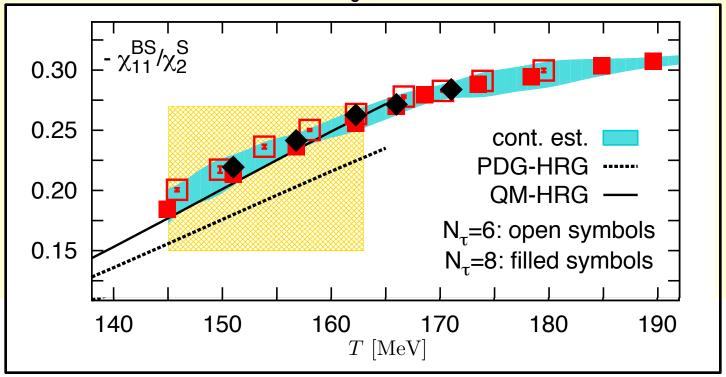
Pressure, entropy, trace anomaly



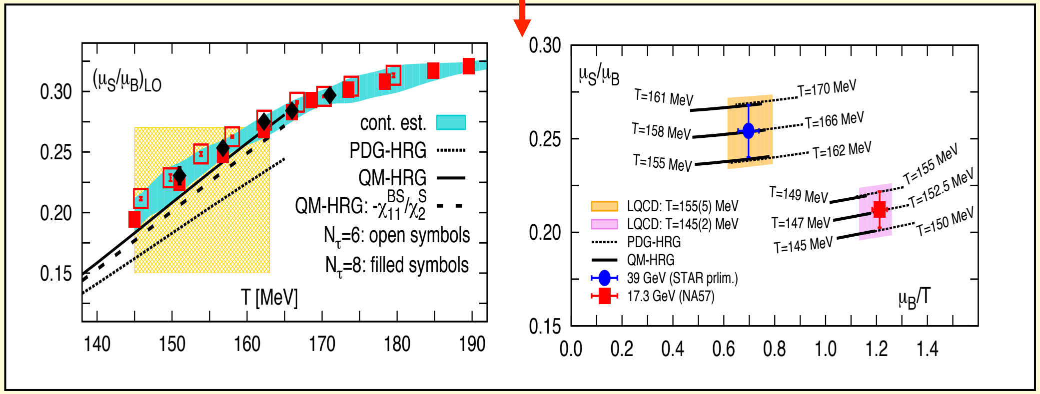
From C.Ratti 2018 (2014 WB, HotQCD data)

Fluctuations of conserved charges (Q,B,S)

Bazavov et al 2014



related to strangeness freeze-out conditions



$$n_S = 0,\ n_Q/n_B = 0.4$$

Ward Identities

Formally from QCD through A/V transformations:

$$\left\langle \frac{\delta \mathcal{O}_P(y)}{\delta \alpha_A^a(x)} \right\rangle = -\left\langle \mathcal{O}_P(y)\bar{\psi}(x)\left\{\frac{\lambda^a}{2},\mathcal{M}\right\}\gamma_5\psi(x)\right\rangle + i\frac{\delta_{a0}}{\sqrt{6}}\langle \mathcal{O}_P(y)A(x)\rangle$$

$$\left\langle \frac{\delta \mathcal{O}_S(y)}{\delta \alpha_V^a(x)} \right\rangle = \left\langle \mathcal{O}_S(y)\bar{\psi}(x)\left[\frac{\lambda^a}{2},\mathcal{M}\right]\psi(x)\right\rangle$$

$\lambda^0 = \sqrt{2/3}\,\mathbb{1}$, $A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

$\mathcal{O}_P^b = i\bar{\psi}\gamma_5\lambda^b\psi \equiv P^b \rightarrow$ **1p vs 2p fns** $\rightarrow \langle\bar{q}q\rangle$ **vs** χ_P

$\mathcal{O}_P^{bc} = P^b S^c \rightarrow$ **2p vs 3p** $\rightarrow$ **ch.partners vs meson vertices**
(e.g. $\chi^\sigma - \chi^\pi \sim \sigma\pi\pi, \ldots$ **)**

$\mathcal{O}_S^b = \bar{\psi}\lambda^b\psi \equiv S^b \rightarrow \langle\bar{q}q\rangle$ **vs** χ_S **for** κ **sector** $b = 4, \ldots, 7$

WARD IDENTITIES obtained from the QCD generating functional may shed light on chiral patterns and partners

AGN, J.Ruiz de Elvira, 2016, 2018

- π **SECTOR** $\rightarrow \langle \bar{q}q \rangle_l (T) = -\hat{m}\chi_P^\pi(T)$
- K **SECTOR** $\rightarrow \langle \bar{q}q \rangle_l (T) + 2\langle \bar{s}s \rangle (T) = -(\hat{m} + m_s)\chi_P^K(T)$
- η, A **SECTOR** $\rightarrow \eta/\eta'$ **mixing &** $U_A(1)$ **anomaly enter:**

$$\begin{aligned}
\chi_P^{\eta_l}(T) &= -\frac{\langle \bar{q}q \rangle_l (T)}{\hat{m}} - \frac{4}{\hat{m}^2}\chi_{top}(T) \\
\chi_P^{\eta_s}(T) &= -\frac{\langle \bar{s}s \rangle (T)}{m_s} - \frac{1}{m_s^2}\chi_{top}(T) \\
\chi_P^{ls} &= -\frac{\hat{m}}{2m_s}\left[\chi_P^\pi(T) - \chi_P^{\eta_l}(T)\right] = -\frac{2}{\hat{m}m_s}\chi_{top}(T)
\end{aligned}$$

$\chi_P^{ab} \equiv \int_T dx \left\langle P^a(x)P^b(0)\right\rangle, \ \langle \bar{q}q \rangle_l = \left\langle \bar{u}u + \bar{d}d \right\rangle, \ \hat{m} = m_u = m_d$ $\qquad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

$\chi_{top} \equiv -\frac{1}{36}\int_T dx \langle \mathcal{T} A(x)A(0)\rangle$ **TOPOLOGICAL SUSCEPTIBILITY**

WI AND LATTICE SCREENING MASSES

Assuming soft T behavior for residues and M_{sc}/M_{pole} of correlators $K_{P,S}$:

$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2} \rightarrow$ **measured in lattice**

$$\frac{M_\pi^{sc}(T)}{M_\pi^{sc}(0)} \sim \left[\frac{\chi_P^\pi(0)}{\chi_P^\pi(T)}\right]^{1/2} = \left[\frac{\langle \bar{q}q\rangle_l(0)}{\langle \bar{q}q\rangle_l(T)}\right]^{1/2}$$

$$\frac{M_K^{sc}(T)}{M_K^{sc}(0)} \sim \left[\frac{\chi_P^K(0)}{\chi_P^K(T)}\right]^{1/2} = \left[\frac{\langle \bar{q}q\rangle_l(0) + 2\langle \bar{s}s\rangle(0)}{\langle \bar{q}q\rangle_l(T) + 2\langle \bar{s}s\rangle(T)}\right]^{1/2}$$

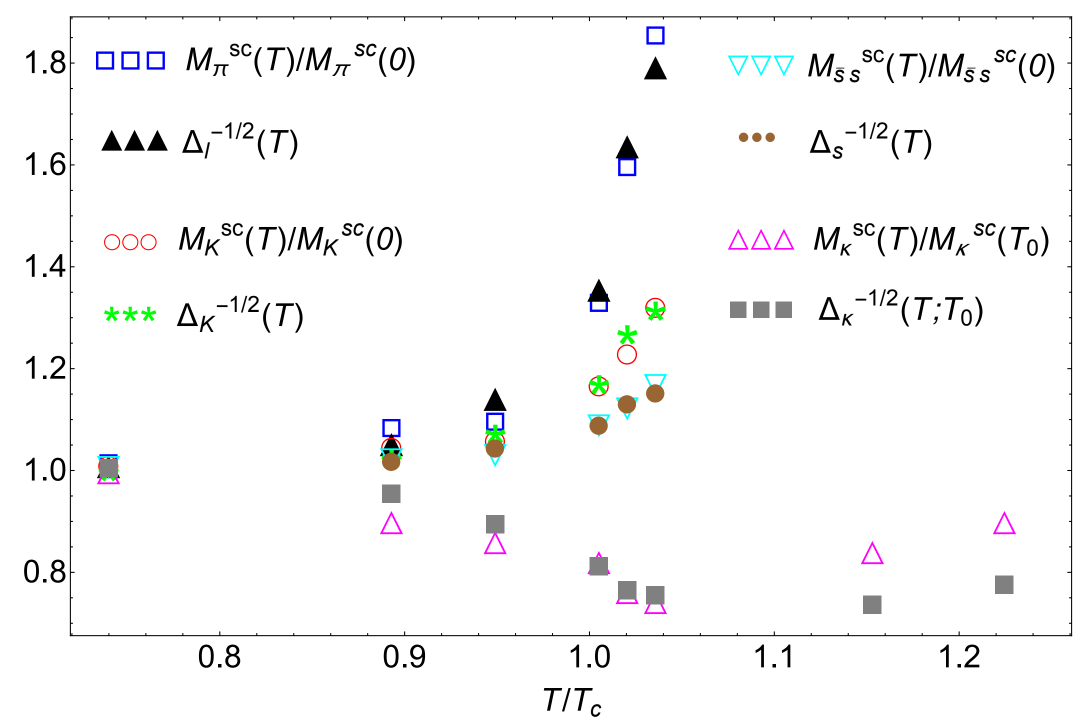
$$\frac{M_{\bar{s}s}^{sc}(T)}{M_{\bar{s}s}^{sc}(0)} \sim \left[\frac{\chi_P^{\bar{s}s}(0)}{\chi_P^{\bar{s}s}(T)}\right]^{1/2} \sim \left[\frac{\langle \bar{s}s\rangle(0)}{\langle \bar{s}s\rangle(T)}\right]^{1/2}$$

$$\frac{M_\kappa^{sc}(T)}{M_\kappa^{sc}(0)} \sim \left[\frac{\chi_S^\kappa(0)}{\chi_S^\kappa(T)}\right]^{1/2} = \left[\frac{\langle \bar{q}q\rangle_l(0) - 2\langle \bar{s}s\rangle(0)}{\langle \bar{q}q\rangle_l(T) - 2\langle \bar{s}s\rangle(T)}\right]^{1/2}$$

Anomalous contrib. $\frac{\hat{m}}{m_s}$ **suppressed**

WI AND LATTICE SCREENING MASSES

Same lattice setup for masses (Cheng et al EPJC'11) and condensates (PRD'08)



- $<5\%$ **deviations** below T_c from predicted WI scaling
- Δ_i **subtracted condensates** with two fit parameters
- **Rapid T_c increase** in $M_\pi^{sc} \sim \langle \bar{q}q \rangle_l^{-1/2}$.

  Softer $M_K^{sc} \sim \left(\langle \bar{q}q \rangle_l + 2\langle \bar{s}s \rangle\right)^{-1/2}$. **Even softer** $M_{\bar{s}s}^{sc} \sim \langle \bar{s}s \rangle^{-1/2}$
- κ **minimum** from condensate diff. (last two points not fitted)

WI and Screening Masses

Subtracted Condensates have the right critical behavior in lattice, avoiding $T=0$ finite-size divergences $\langle \bar{q}_i q_i \rangle \sim m_i/a^2 + \ldots$:

$$\Delta_l(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + \langle \bar{q}q \rangle_l^{ref}}{\langle \bar{q}q \rangle_l^{ref}}$$

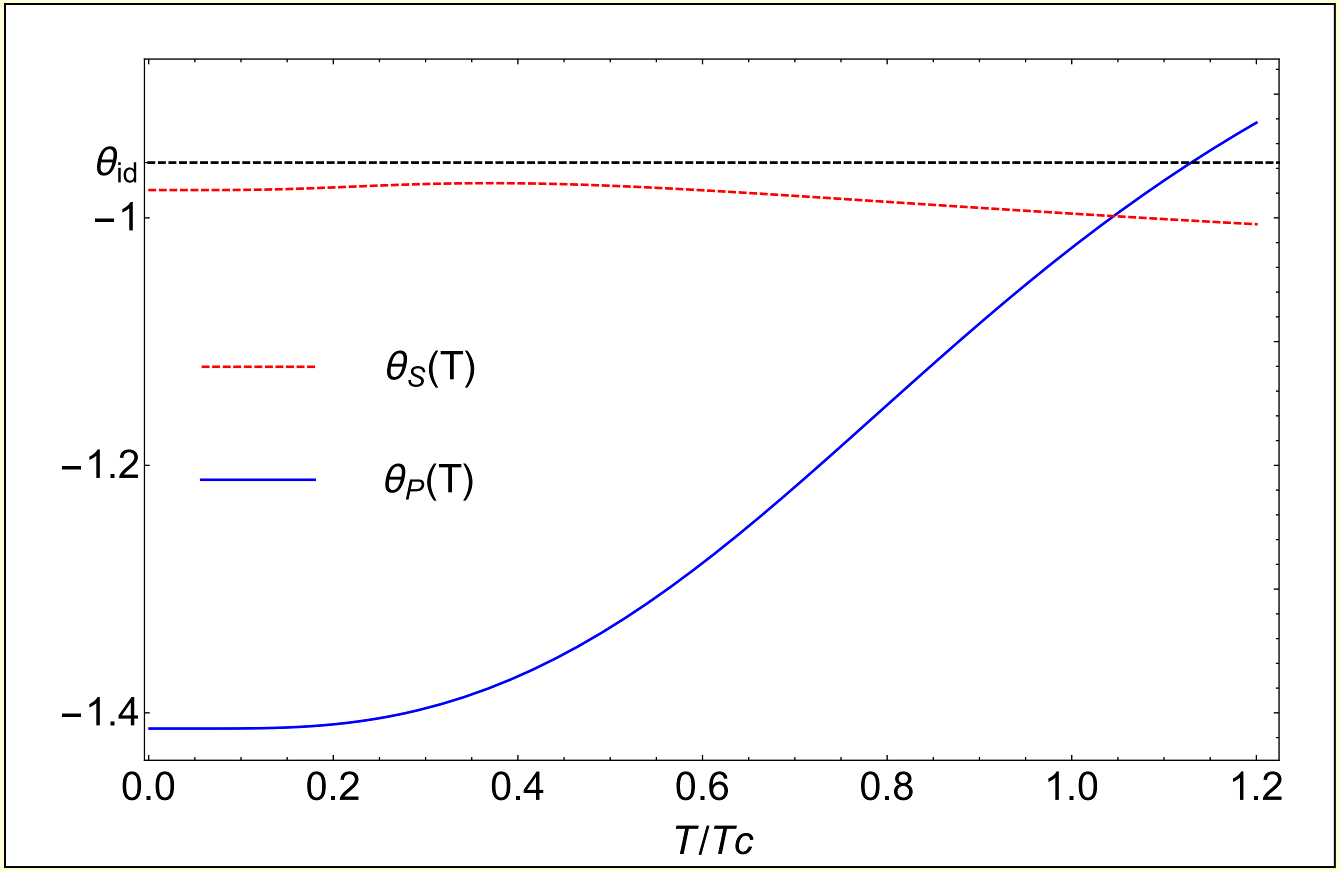
$$\Delta_K(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}$$

$$\Delta_s(T) = \frac{2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{s}s \rangle^{ref}}{\langle \bar{s}s \rangle^{ref}}$$

$$\Delta_\kappa(T;T_0) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l (T_0) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T_0) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}$$

$$\begin{aligned} r_1^3 \langle \bar{q}q \rangle_l^{ref} &= 0.750 \\ r_1^3 \langle \bar{s}s \rangle^{ref} &= 1.061 \\ r_1 &\simeq 0.31 \text{ fm} \end{aligned}$$

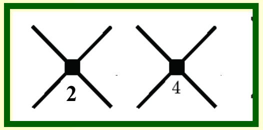
Mixing in $U(3)$ ChPT



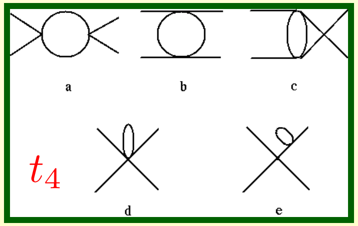
$$\frac{1}{2}\left[\chi^{88}_{P,S}(T)-\chi^{00}_{P,S}(T)\right]\sin\left[2\theta_{P,S}(T)\right]+\chi^{08}_{P,S}\cos\left[2\theta_{P,S}(T)\right]=0,$$

($\eta\eta'$ correlator vanishing, leading order)

Unitarized ChPT and resonances at finite T



t_2 t_4



ChPT partial waves $\rightarrow$

$$t^{IJ} = t_2^{IJ} + t_4^{IJ} + \ldots$$

THERMAL UNITARITY $\rightarrow$

$$\textbf{Im}\ t(s;T) = \sigma(s;T)|t(s;T)|^2$$

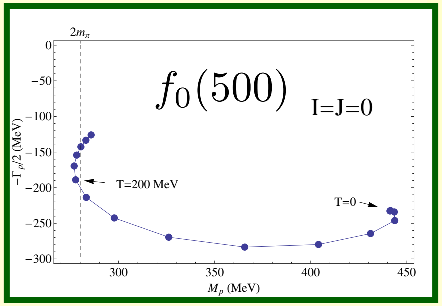
$$(s \geq (M_1+M_2)^2)$$

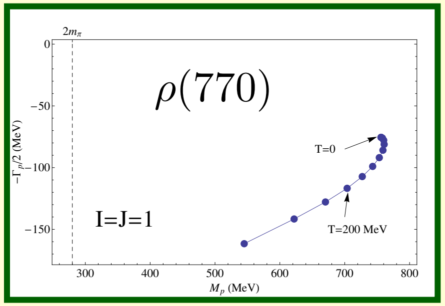
Thermal Resonances (Riemann 2nd sheet poles)

$$s_p(T) = \left[M_p(T) - i\frac{\Gamma_p(T)}{2}\right]^2$$

IAM $T \neq 0$

$$t^U(s;T) = \frac{\left[t_2(s)\right]^2}{t_2(s) - t_4(s;T)}$$





$$t_4(s) \rightarrow t_4(s;T)$$

$$\sigma(s;T) = \sigma(s)[1 + n_B(E_1) + n_B(E_2)]$$

Thermal phase space

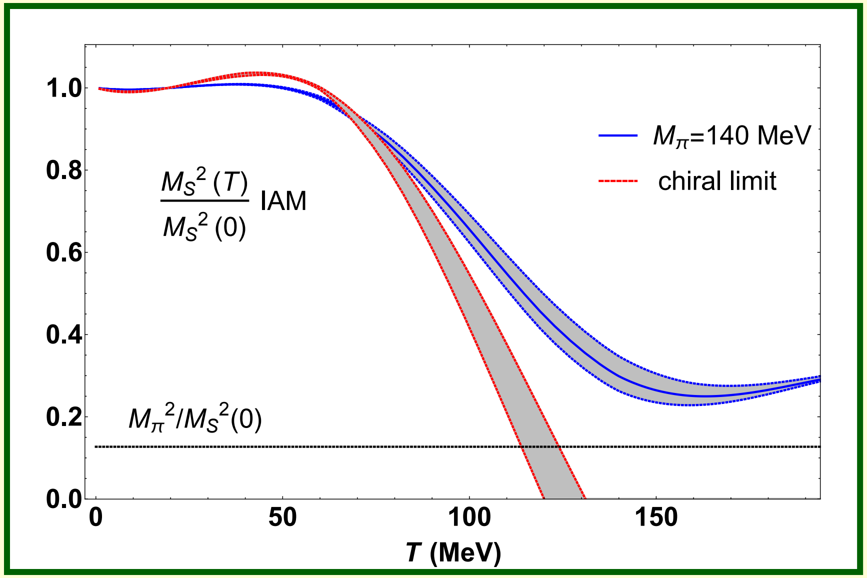
Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, PRD 2018

$\Rightarrow$ χ_S **saturated by lightest** $p=0$ **state** $f_0(500)$ (σ)

$$\boxed{\chi_S(T) \simeq \chi_S(0)\frac{M_S^2(0)}{M_S^2(T)}}$$

$$M_S^2(T) = \mathbf{Re}(s_{pole}(T)) \sim \mathbf{Re}\Sigma_{f_0}(T)$$

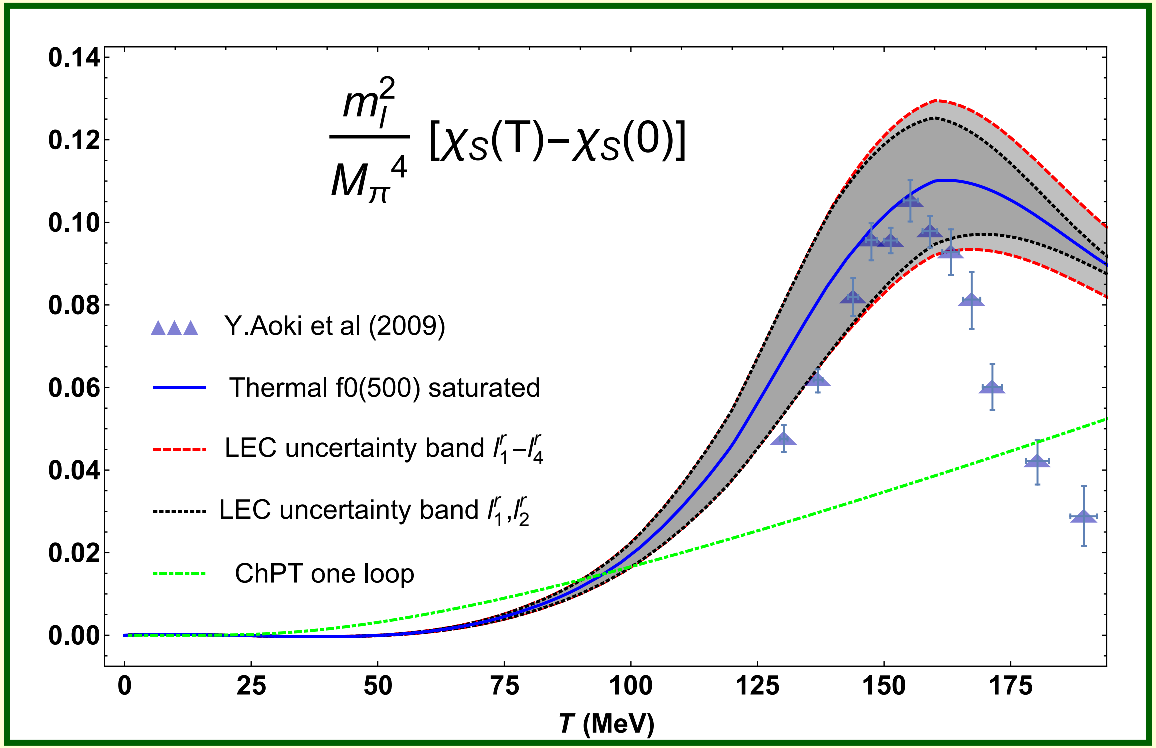


Pole from UChPT $\pi\pi$ scattering at finite T
(AGN et al 2002)

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, 2018

$$\chi_S^U(T) = \frac{\chi_S^{ChPT}(0) M_S^2(0)}{M_S^2(T)}$$



- Consistent with lattice transition peak.

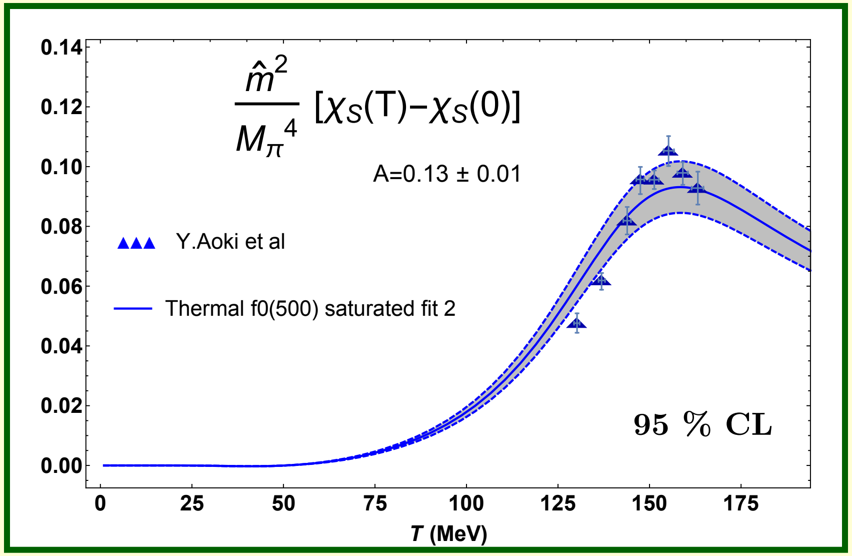
- LECs and uncertaintities from unitarized $T=0$ fit in Hanhart, Peláez, Ríos PRL100 (2008)

$$s_p = 446.5 - i220.4 \ \mathrm{MeV}$$

- Consistent T_c reduction and χ_S growth near chiral limit

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

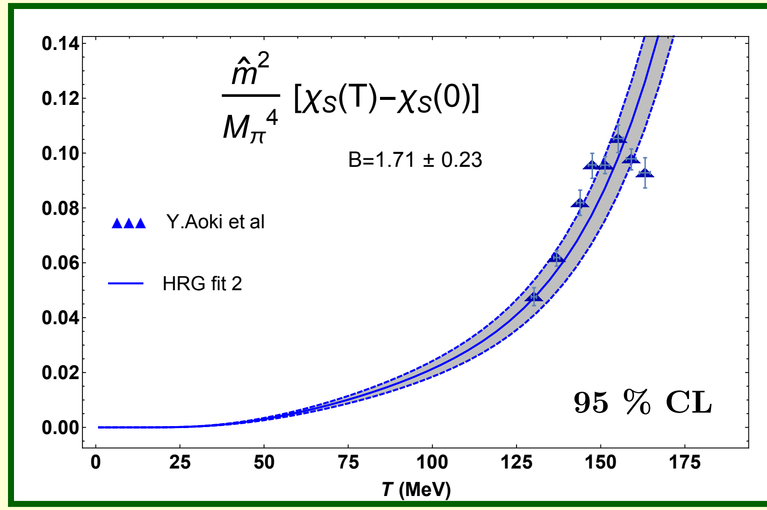
S.Ferreres, AGN, A.Vioque, PRD 2018



$$\chi_S(T) = A\frac{m_\pi^4}{4m_q^2}\frac{M_S^2(0)}{M_S^2(T)} \qquad (A_{ChPT} \simeq 0.14)$$

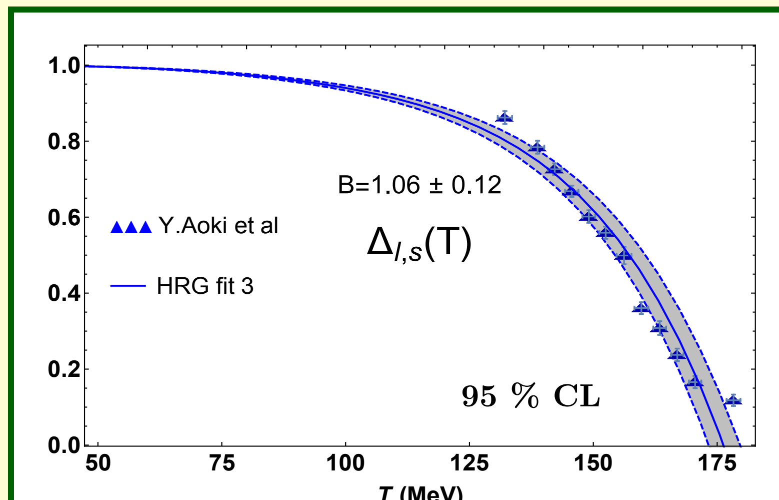
Fit	A	B	χ^2/dof	T_{max}(MeV)
Thermal f_0 fit 1	0.13± 0.02	—	6.25	155
Thermal f_0 fit 2	0.13± 0.01	—	4.93	165
HRG fit 1	—	1.90± 0.02	1.33	155
HRG fit 2	—	1.71± 0.23	10.30	165
HRG fit 3	—	1.06± 0.12	3.77	155

- Thermal f_0 reproduces peak around T_c
- Thermal interactions crucial
- HRG monotonic with conflicting $\Delta_{l,s}, \chi_S$ fits



HRG Jankowski et al 2013

free energy density normalization B fitted.



Saturated Scalar susceptibility in the LSM

S.Ferreres, AGN, A.Vioque, 2018

$$\mathcal{L}_{LSM} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \frac{\lambda}{4}\left[\Phi^T\Phi - v_0^2\right]^2 + h\sigma,$$

$$\chi_S(T) \quad = \quad \left(\frac{d^2h}{dm_q^2}\right) v(T) + \left(\frac{dh}{dm_q}\right)^2 \Delta_\sigma(k=0;T),$$

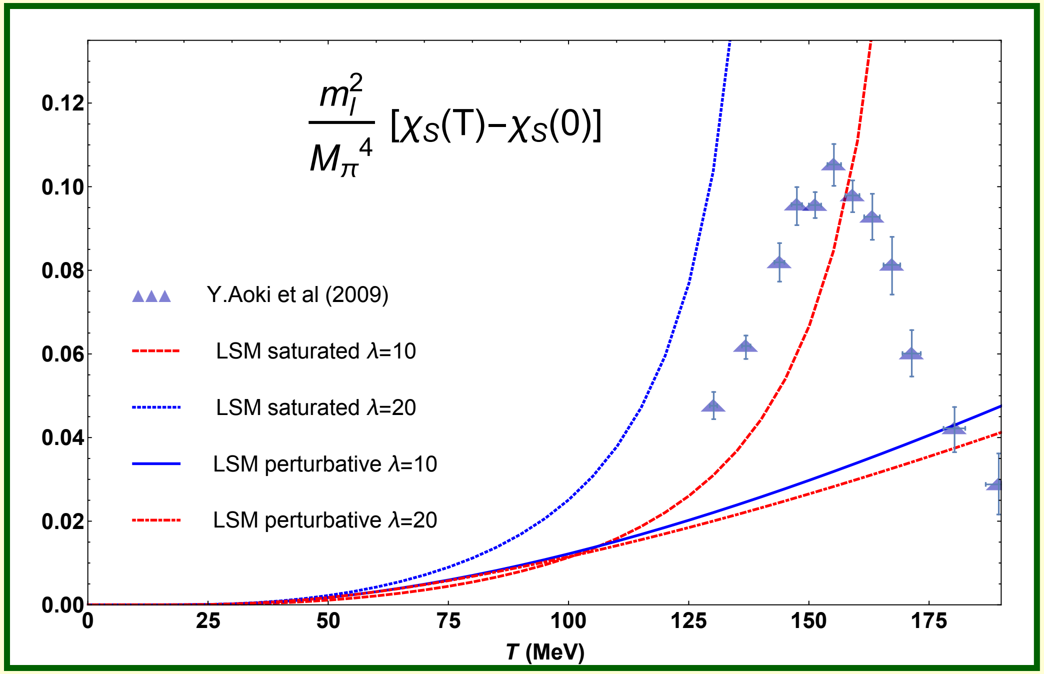
suppressed near T_c

$$M_{0\pi}^2 = \frac{h}{v} = \lambda(v^2 - v_0^2) \quad , \quad M_{0\sigma}^2 = M_{0\pi}^2 + 2\lambda v^2, \quad v = \langle\sigma\rangle$$

calculating the self-energy Σ to one loop:

M_π (MeV)	M_p (MeV)	Γ_p (MeV)	λ
0	450.0	172.5	8.4
0	775.1	550.0	20.0
140	450.0	159.2	9.6
140	750.1	550.0	21.2

$$\frac{\chi_S(T)}{\chi_S(0)} \simeq \frac{M_{0\sigma}^2 + \Sigma\left(k=0; T=0\right)}{M_{0\sigma}^2 + \Sigma\left(k=0; T\right)}$$



Saturated κ scalar susceptibility

AGN, J.Ruiz de Elvira, A.Vioque-Rodríguez, D.Alvarez 2020

$$t_U(s;T) = \frac{t_2^2(s)}{t_2(s) - \tilde{t}_4(s,T)}$$

Meth.1: $\tilde{t}_4(s;T) = 16\pi\, t_2(s)^2 \tilde{J}_{\pi K}(s;T)$

Meth.2: $\tilde{t}_4(s;T) = t_4(s;0) + 16\pi\, t_2(s)^2 \left[J_{\pi K}(s;T) - J_{\pi K}(s;0)\right]$

$$J_{\pi K}(s;T) = T \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{q^2 - M_K^2} \frac{1}{(q-Q)^2 - M_\pi^2}$$

$T = 0$ **poles (LECs Molina, Ruiz de Elvira 2020):**

$$\sqrt{s_p}^{(1)} = (731 \pm 7) - i(280 \pm 9)\ \textbf{MeV}$$
$$\sqrt{s_p}^{(2)} = (679 \pm 6) - i(289 \pm 8)\ \textbf{MeV}$$

Thermal Unitarity:

$$\textbf{Im}\, t_U(s;T) = \sigma_{\pi K}(s;T) |t_U(s;T)|^2 \quad (s \geq (M_K + M_\pi)^2)$$

$$\sigma_{\pi K}(s;T) = \frac{1}{s} \sqrt{\left(s - (M_\pi + M_K)^2\right)\left(s - (M_\pi - M_K)^2\right)}\left[1 + n(E_+) + n(E_-)\right],$$

$$E_\pm = (s \pm \Delta)/(2\sqrt{s}),\ \Delta = M_K^2 - M_\pi^2,\ n(x) = (e^{x/T} - 1)^{-1}$$

Topological Charge in $U(3)$ ChPT

AGN, J.R.Elvira, A.Vioque, 2019

$$\epsilon_{vac}(\theta)=\epsilon_{vac}(0)+\frac{1}{2}\chi_{top}\theta^2+\frac{1}{24}c_4\theta^4+\dots$$

$\sim$ **Axion mass** $\sim$ **Axion coupling**

$$\chi_{top}^{U(3),LO}=\Sigma\frac{M_0^2\bar{m}}{M_0^2+6B_0\bar{m}} \qquad c_4^{U(3),LO}=-\frac{\Sigma}{\bar{m}^{[3]}}\left(\frac{M_0^2\bar{m}}{M_0^2+6B_0\bar{m}}\right)^4$$

$$\bar{m}=\left[\frac{1}{m_u}+\frac{1}{m_d}+\frac{1}{m_s}\right]^{-1} \qquad \bar{m}^{[3]}=\left[\frac{1}{m_u^3}+\frac{1}{m_d^3}+\frac{1}{m_s^3}\right]^{-1}$$

$(-\Sigma)$ LO quark condensate $\qquad$ M_0 anomalous part of $M_{\eta'}$ $(m_{u,d,s}=0)$

- **vanish** $m_q\to 0\Rightarrow$ **expected to be well described within ChPT**
- $SU(3)$ **for** $M_0\to\infty$, $SU(2)$ **for** $M_0, m_s\to\infty$
- **Quenched** $m_q\to\infty$**:** $\chi_{top}^{LO}=F^2M_0^2/6$

  (Witten-Veneziano 1979) $\to$ **meson loops crucial**

Leutwyler,Smilga 1992: $SU(3)$ **LO**
Mao et al 2009; Bernard et al: 2012: $SU(3)$ **NLO**
Grilli et al 2016: $T\neq 0$ $SU(2)$ **NLO**

Topological Charge in $U(3)$ ChPT

AGN, J.R.Elvira, A.Vioque, 2019

$\rightarrow$ $T=0$ **results** $(m_u = m_d)$**:**

$\chi_{top}^{1/4}$ [MeV]	U(3)	SU(2)	SU(3)
LO	74(3)	75(3)	75(3)
NLO	74(3)	78(3)	83(2)
NNLO	81(2)		

$\left[\chi_{top}^{latt}\right]^{1/4} = 73(9)$ **(Bonati et al 2016)**

$b_2 = \frac{c_4}{12\chi_{top}}$	U(3)	SU(2)	SU(3)
LO	-0.01737(4)	-0.02083	-0.01960
NLO	-0.018(2)	-0.029(2)	-0.025(1)
NNLO	-0.023(2)		

$b_2^{latt} = -0.0216(15)$ **(Bonati et al 2016, gluodynamics)**

$\Rightarrow$ $SU(2)$ **dominates,** $U(3)$ η' **loops and** $\eta - \eta'$ **mixing of the same order than** $SU(3)$ K, η **loops**

$\Rightarrow$ **full** $U(3)$ **in agreement with lattice within uncertainties (LEC and lattice, larger lattice uncertainty for** b_2**)**

$\Rightarrow$ **In addition, within** $U(3)$ **ChPT, explicit expressions for the leading and subleading** $1/N_c$ **contributions can be obtained:**

$$\chi_{top} = \underbrace{\mathcal{O}(1)}_{\text{Witten-Veneziano term}} + \mathcal{O}(N_c^{-1}) + \ldots$$

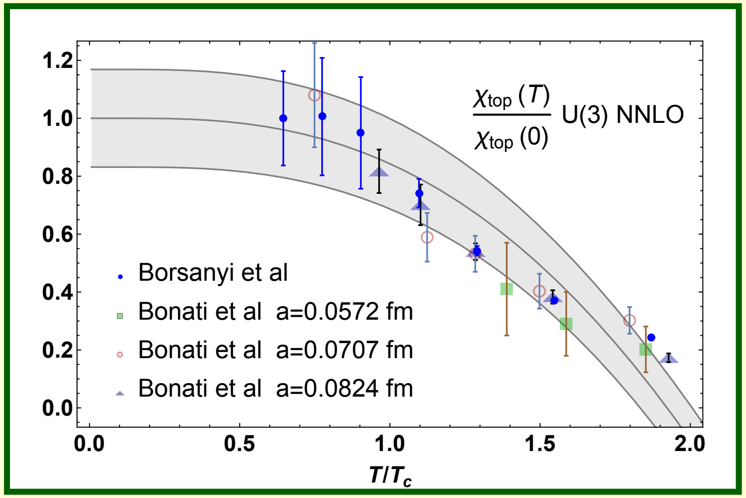
$$c_4 = \underbrace{\mathcal{O}(N_c^{-2})}_{\text{constant } \theta^4 \text{ term in } \mathcal{L}} + \mathcal{O}(N_c^{-3}) + \ldots$$

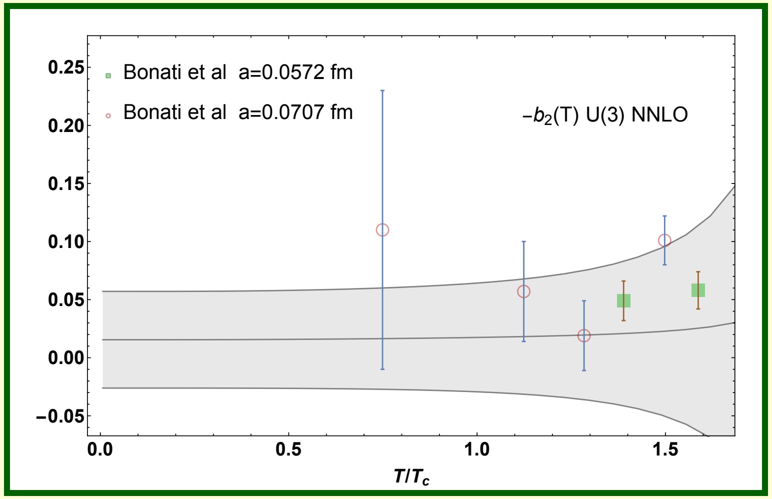
$\Rightarrow$ **OK with lattice and theo:**

Vicari, Panagopoulos 2009-2011
Bonati et al 2016
Vonk, Guo, Meissner 2019

Topological Charge in $U(3)$ ChPT

AGN, J.R.Elvira, A.Vioque, 2019





- **T-dependence captured by ChPT within uncertainties (larger for c_4) far beyond the low-T applicability regime**

- **Scales with T dominated by $\langle \bar{q}q \rangle_l^{ChPT}$ at low T**
- **However, 2nd term in WI $\chi_{top} = -\frac{1}{4}\left[m_{ud}\langle \bar{q}q\rangle_l + m_{ud}^2 \chi^{\eta_l}\right]$ relevant near T_c**

  $\rightarrow$ **finite $O(4)$-$U(1)_A$ gap in physical case**