

# New developments to localize the QCD critical point

Jacquelyn Noronha-Hostler  
University of Illinois Urbana-Champaign

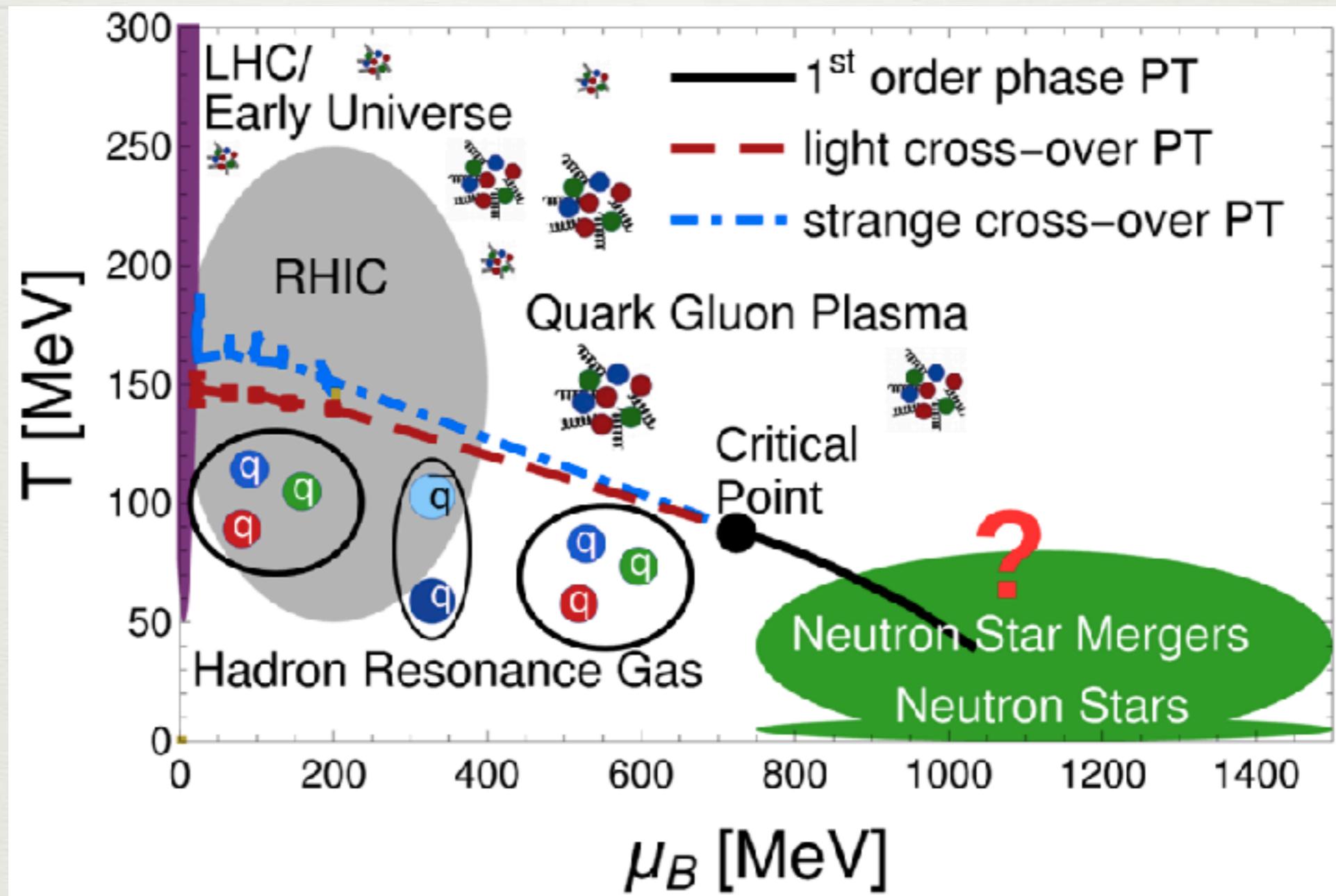
HADRON 2021, Mexico City



Illinois Center for Advanced Studies of the Universe



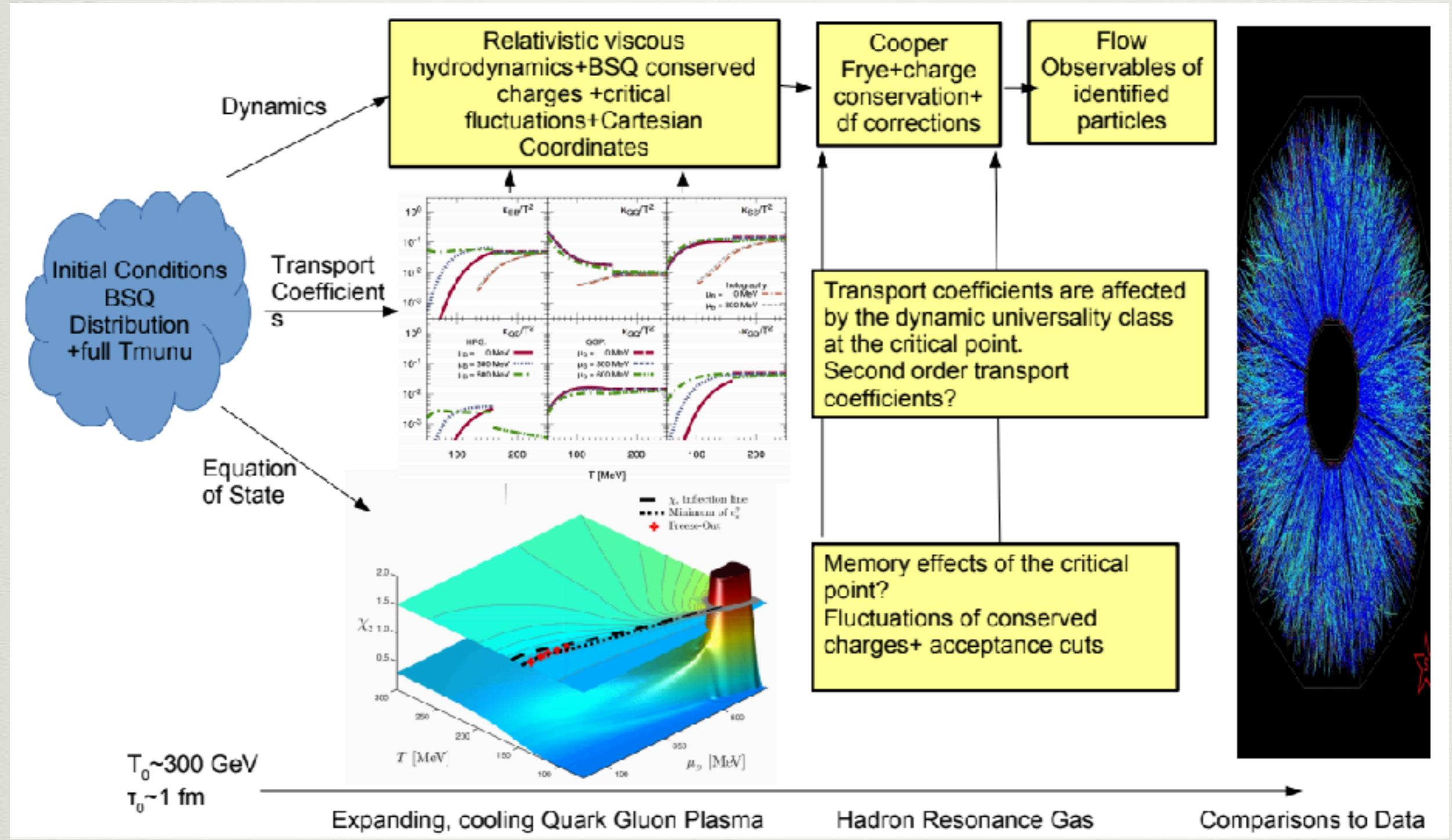
# QCD Phase Diagram



**Light transition** *Phys.Lett. B738 (2014) 305-310*; **Strange Transition** Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, *arXiv:1805.00088*;  
**Neutron Star (mergers)** V. Dexheimer *arXiv:1708.08342*; **Holography** Critelli, JNH, et al, *Phys.Rev. D96 (2017) no.9, 096026*

# Needed Theoretical development

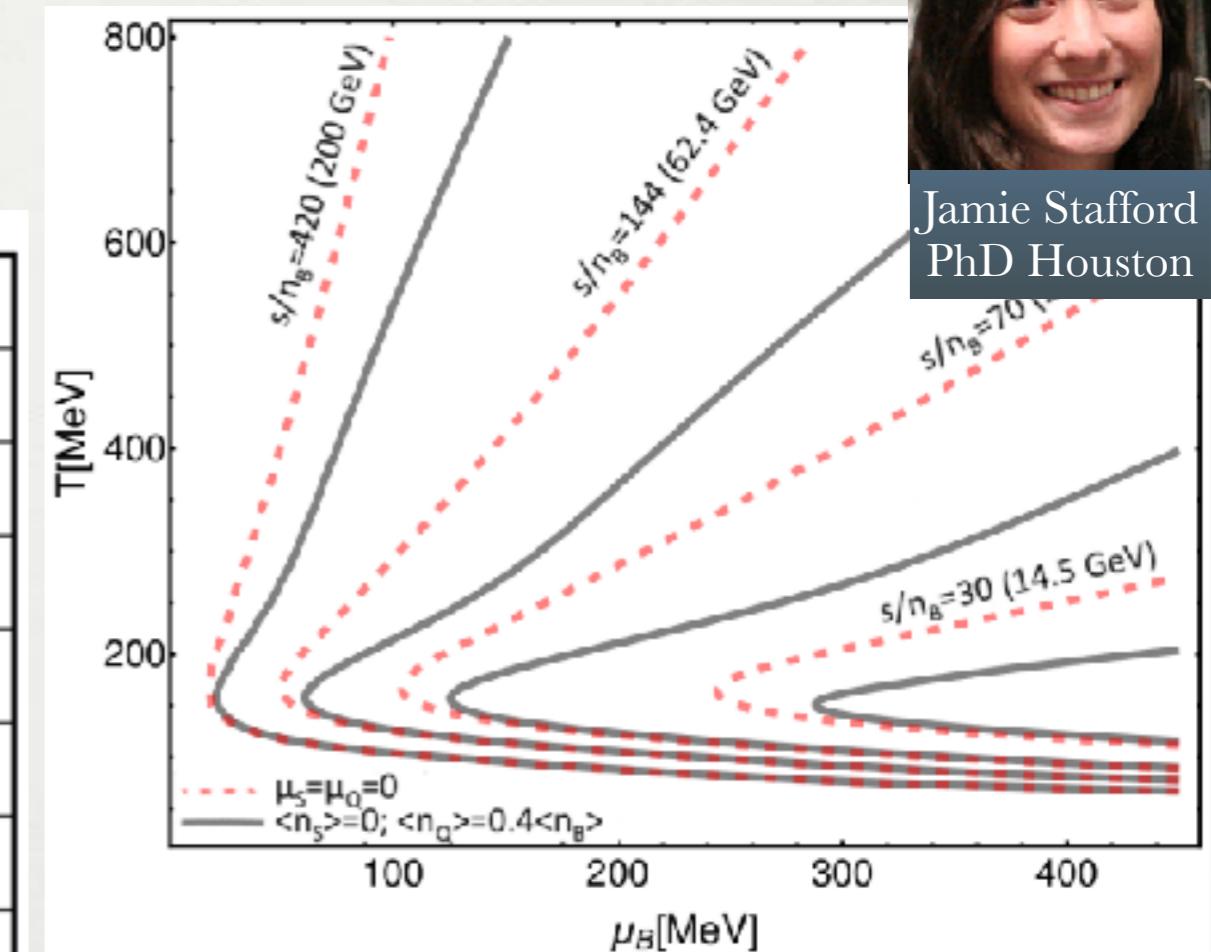
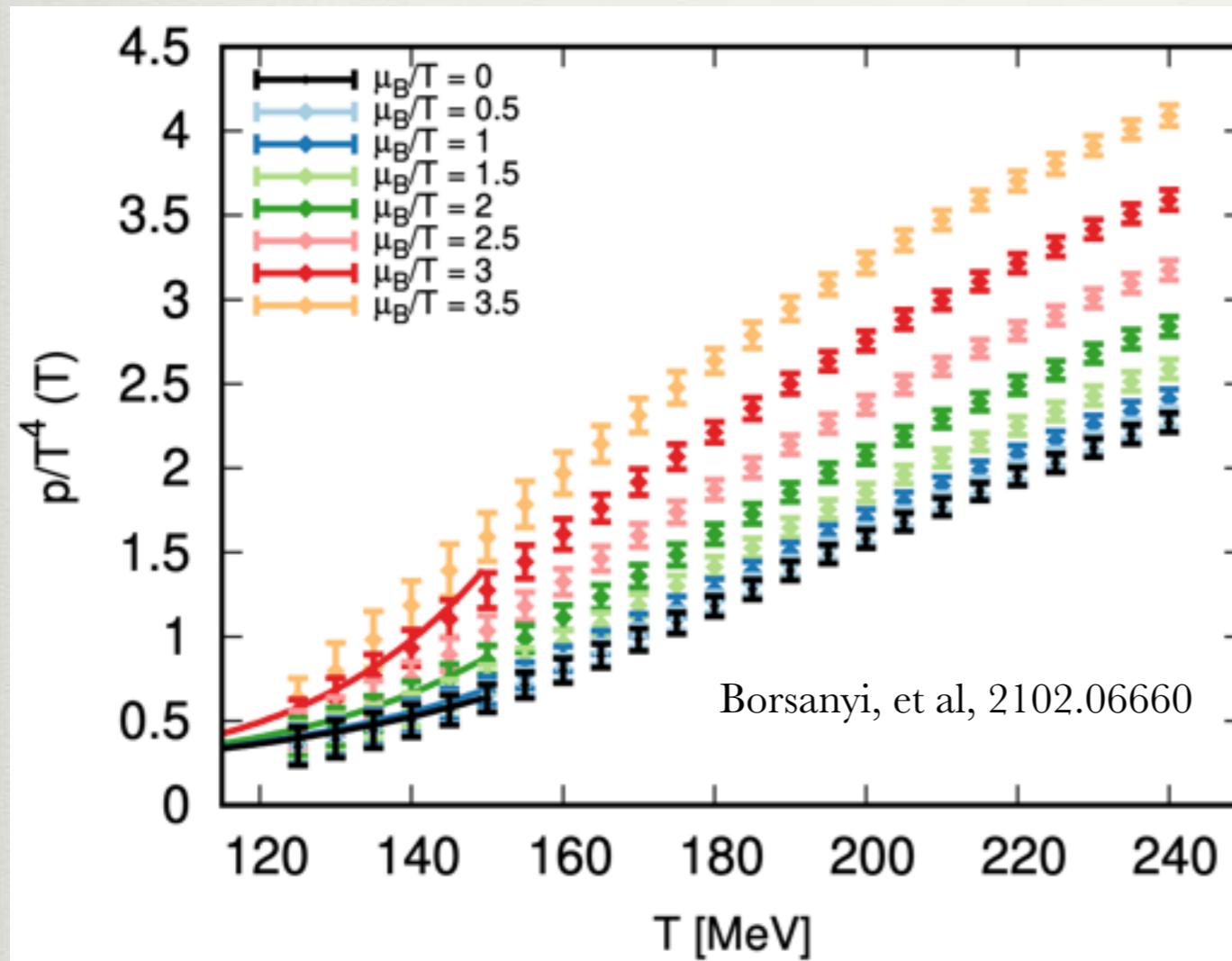
Review: Dexheimer, Noronha, JNH, Ratti, Yunes, *J.Phys.G* 48 (2021) 7, 073001



$$\mu_B^{max} \sim 525\text{MeV}$$

# EOS: Lattice QCD (only)

New resummation scheme to reach larger  $\mu_B$



2 open-source codes that reconstruct the BSQ EOS

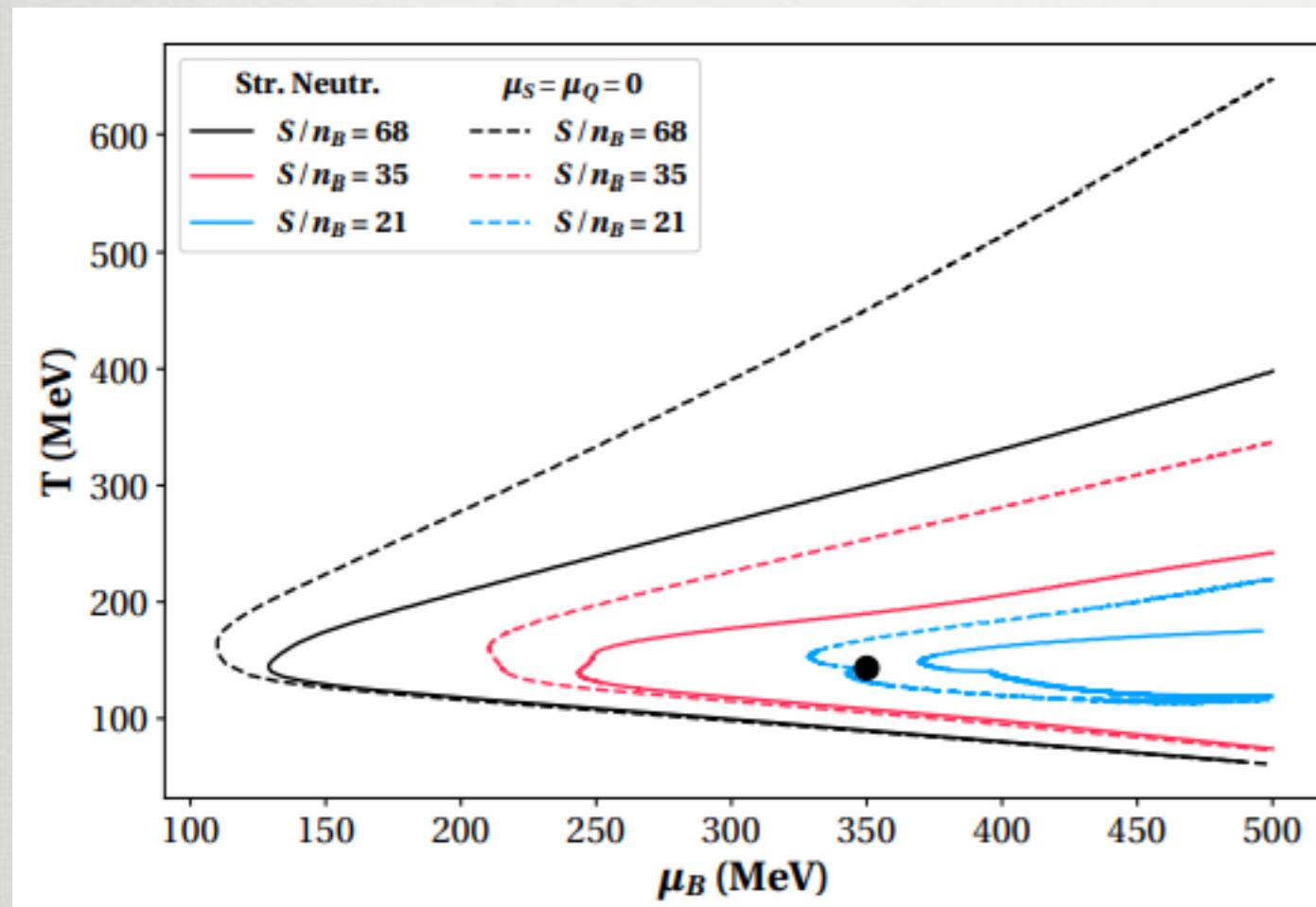
JNH, Stafford, et al, *Phys.Rev.C* 100 (2019) 6, 064910  
Monnai et al, *Phys.Rev.C* 100 (2019) 2, 024907

$$\mu_B^{max} \sim 450 - 600 \text{ MeV}$$

# EOS: Lattice + criticality

Taylor series does not capture criticality

Couple Lattice QCD to 3D Ising model



Stafford, JNH et al, arXiv:2103.08146  
 Parotto, JNH et al, *Phys.Rev.C* 101 (2020) 3, 034901

- 2 conditions:
- Baryon conservation:  $\mu_B > 0, \mu_S = \mu_Q = 0$
  - Heavy-Ion Trajectory:  $\mu_B > 0, \langle \rho_S \rangle = 0, \langle \rho_Q \rangle = 0.4 \langle \rho_B \rangle$



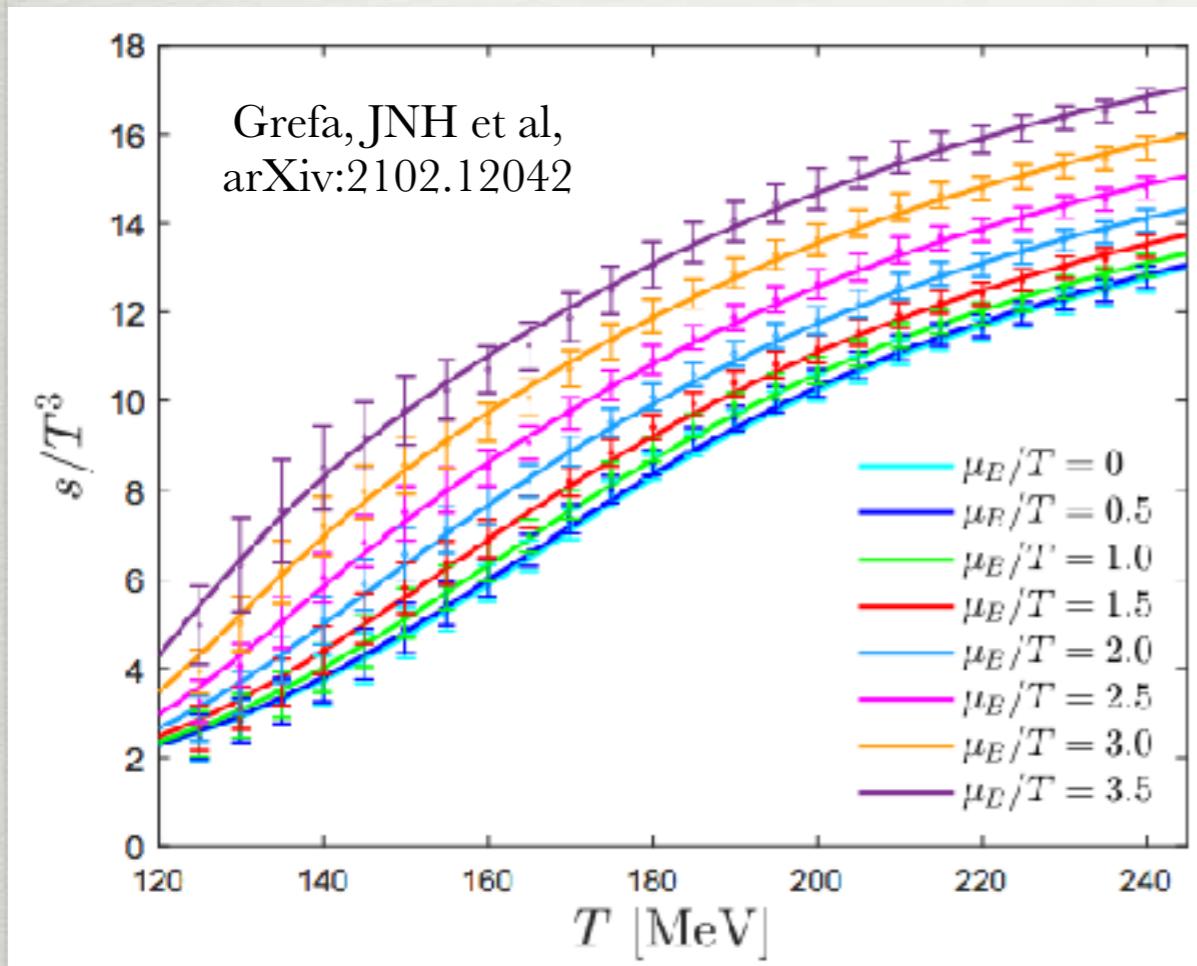
Debora Mroczeck  
UIUC PhD Student



Paolo Parotto  
PD Wuppertal

$$\mu_B^{max} \sim 1100 \text{ MeV}$$

# EOS: holography

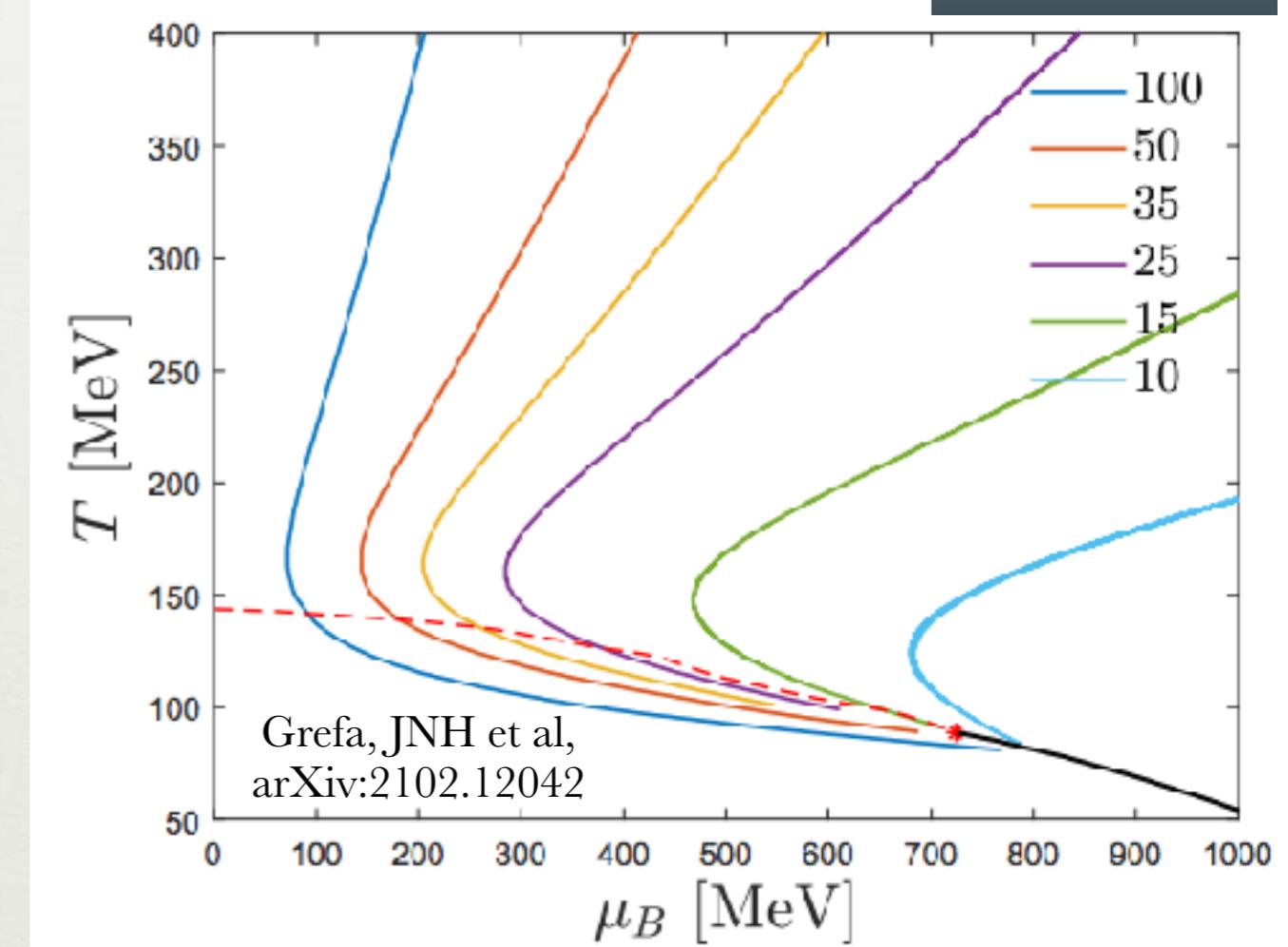


Fitting to just Lattice  
QCD at  $\mu_B = 0$  captures  
 $\mu_B > 0$  EOS

Predicts CP at  
 $\{T_c = 89, \mu_B = 724\}$  MeV

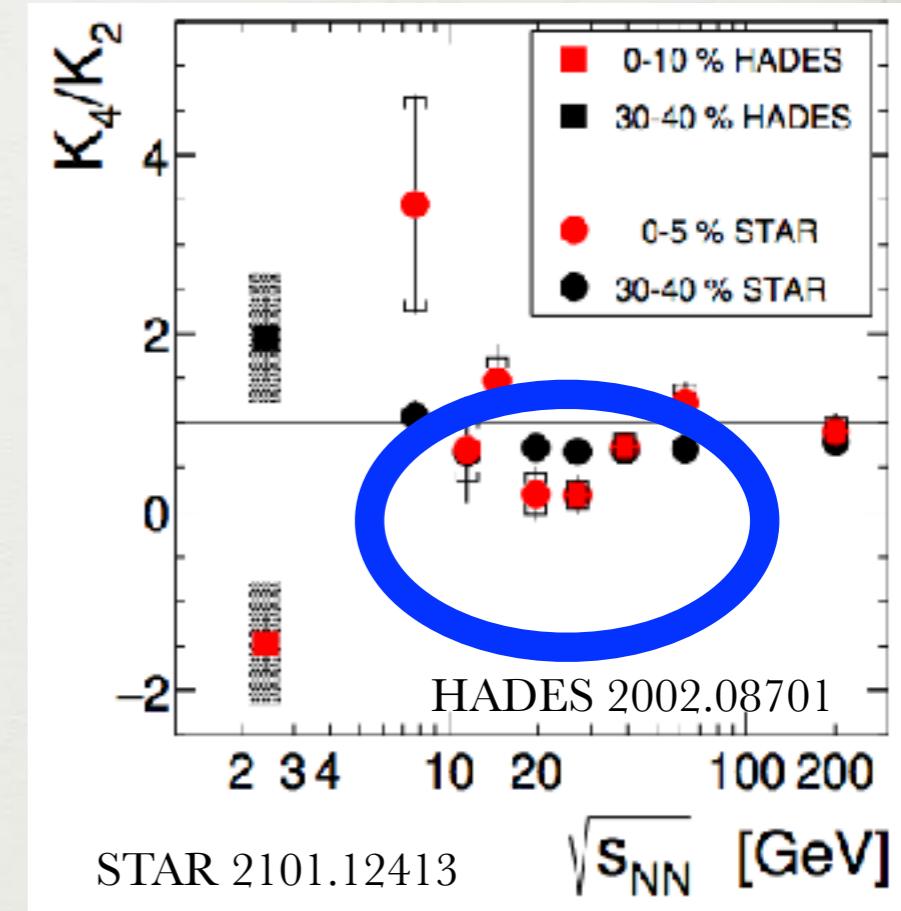
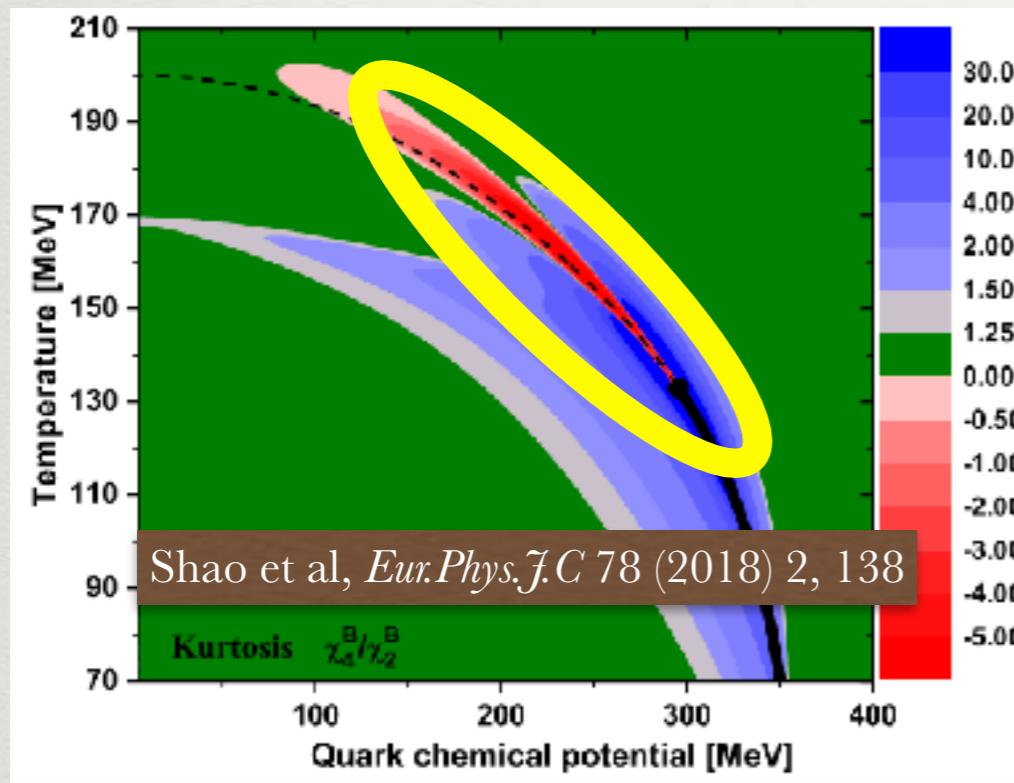


Joaquin Grefa  
UH PhD Student



# Signature of the critical point: Kurtosis

3D Ising (leading order terms only) Stephanov. PRL 107 (2011)



Non-monotonic approach to CP, thought to be signature

Still signature in 3D Ising model with all sub-leading terms?  
Generic signature in all models?

# Dip NOT a clear CP signature

Models without criticality  
already see a dip

- Baryon conservation  
Braun-Munzinger et al, Nucl. Phys. A960, 114 (2017);  
arXiv:2007.02463, see also Bzdak et al *Phys.Rev.C* 87 (2013) 1, 014901

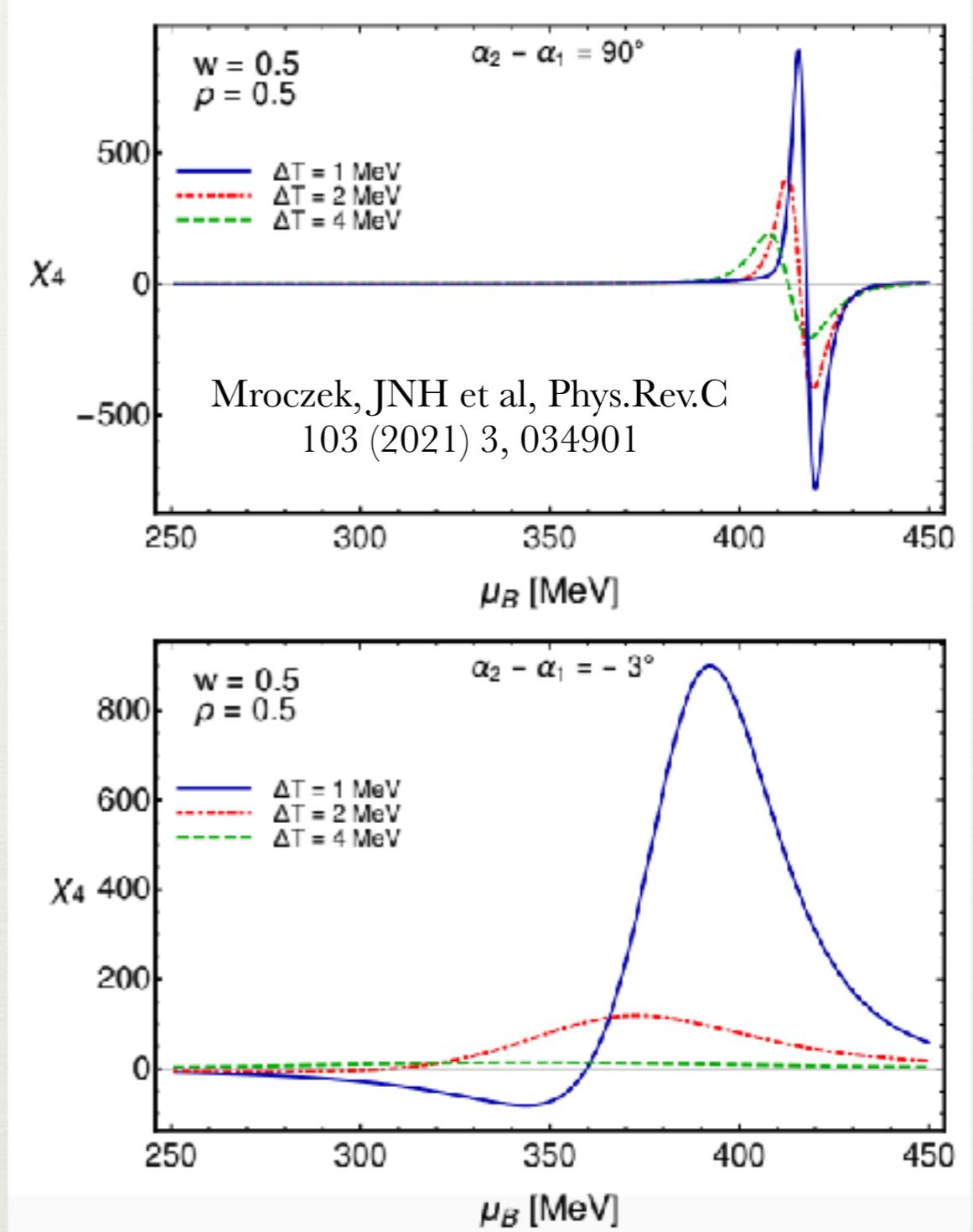
- Lattice QCD Taylor series

Braun-Munzinger et al, Nucl. Phys. A960, 114 (2017);  
arXiv:2007.02463, see also Bzdak et al *Phys.Rev.C* 87 (2013) 1, 014901

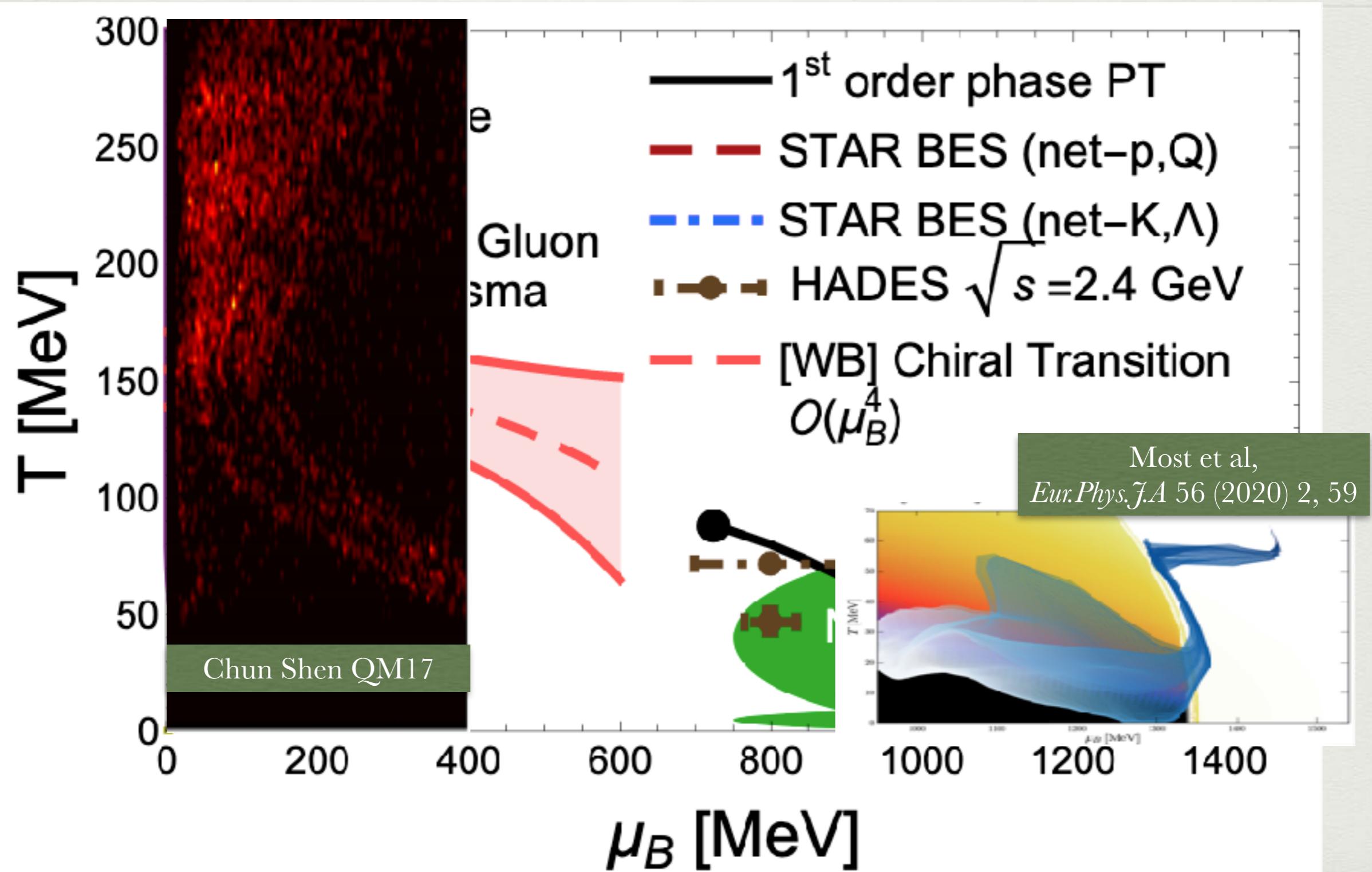
- UrQMD

Braun-Munzinger et al, Nucl. Phys. A960, 114 (2017);  
arXiv:2007.02463, see also Bzdak et al *Phys.Rev.C* 87 (2013) 1, 014901

3D Ising+sub-leading terms,  
only peak clear signature!

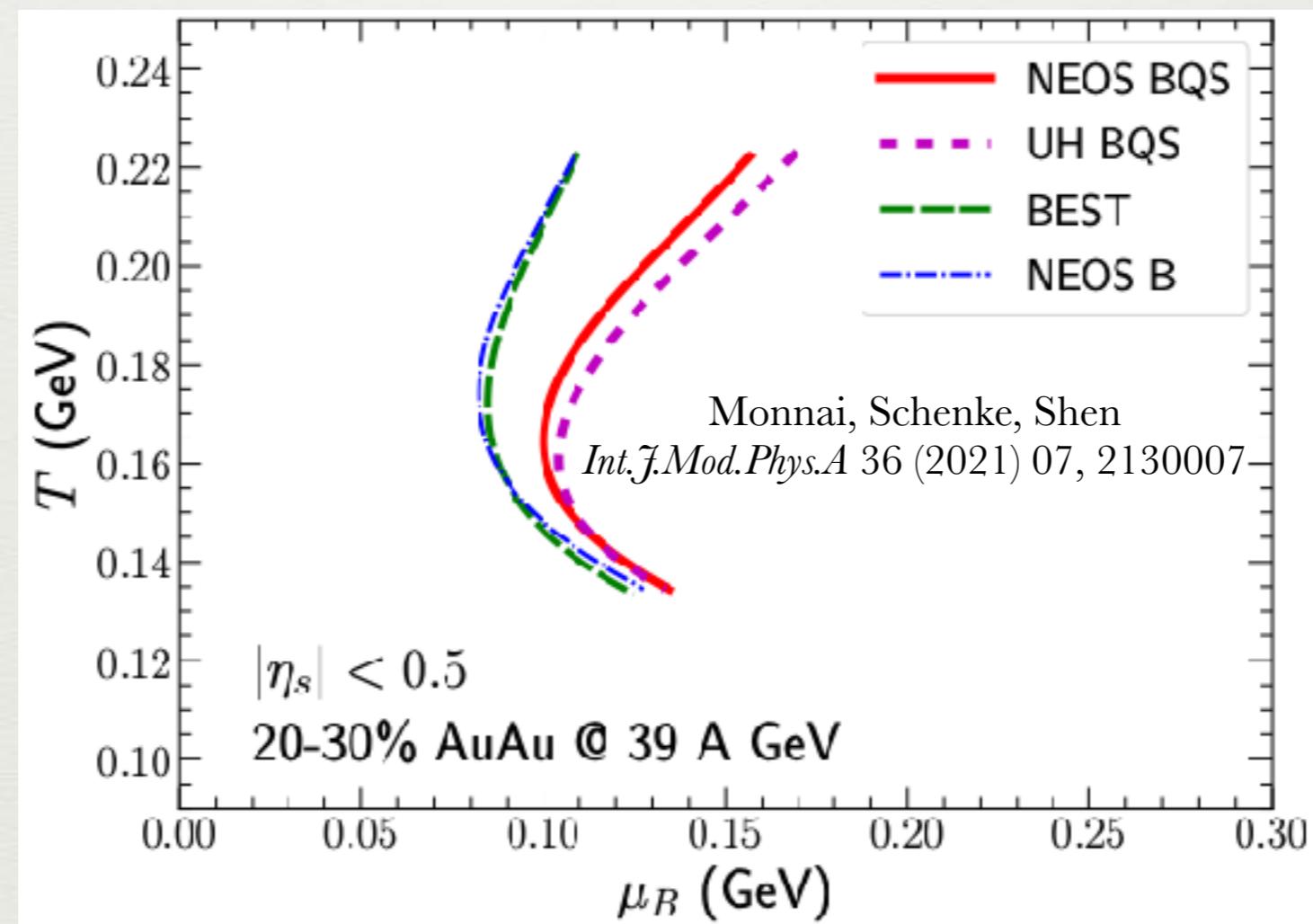


# QCD Phase Diagram Out-of-Equilibrium



# Averaged trajectories vs. EOS

Differences between only baryon conservation vs. BSQ influence the trajectories through the QCD phase diagram.  
Shown with  $\eta/s = 0.08$



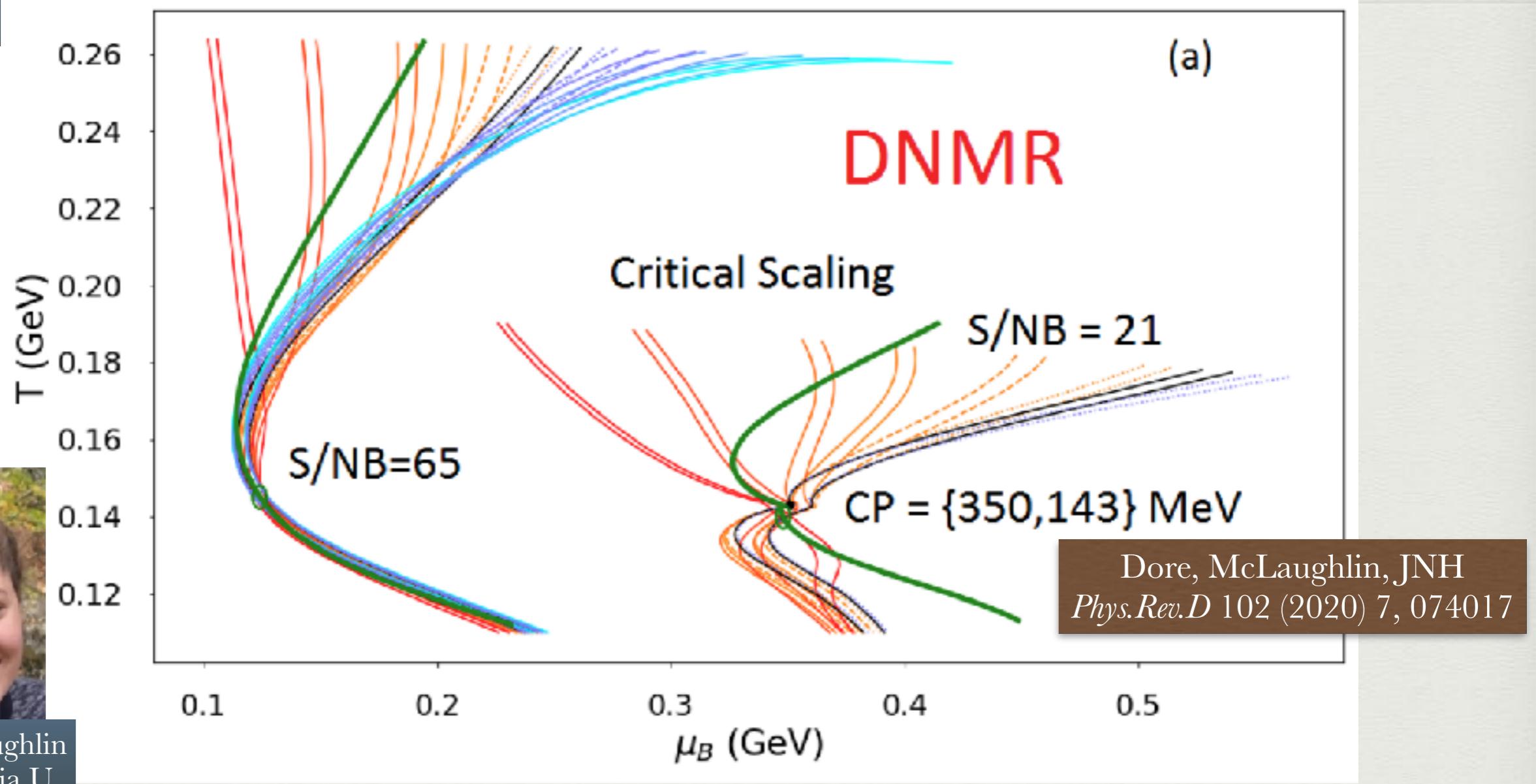


Travis Dore  
PhD UIUC

# Far from equilibrium causes further deviations from ideal

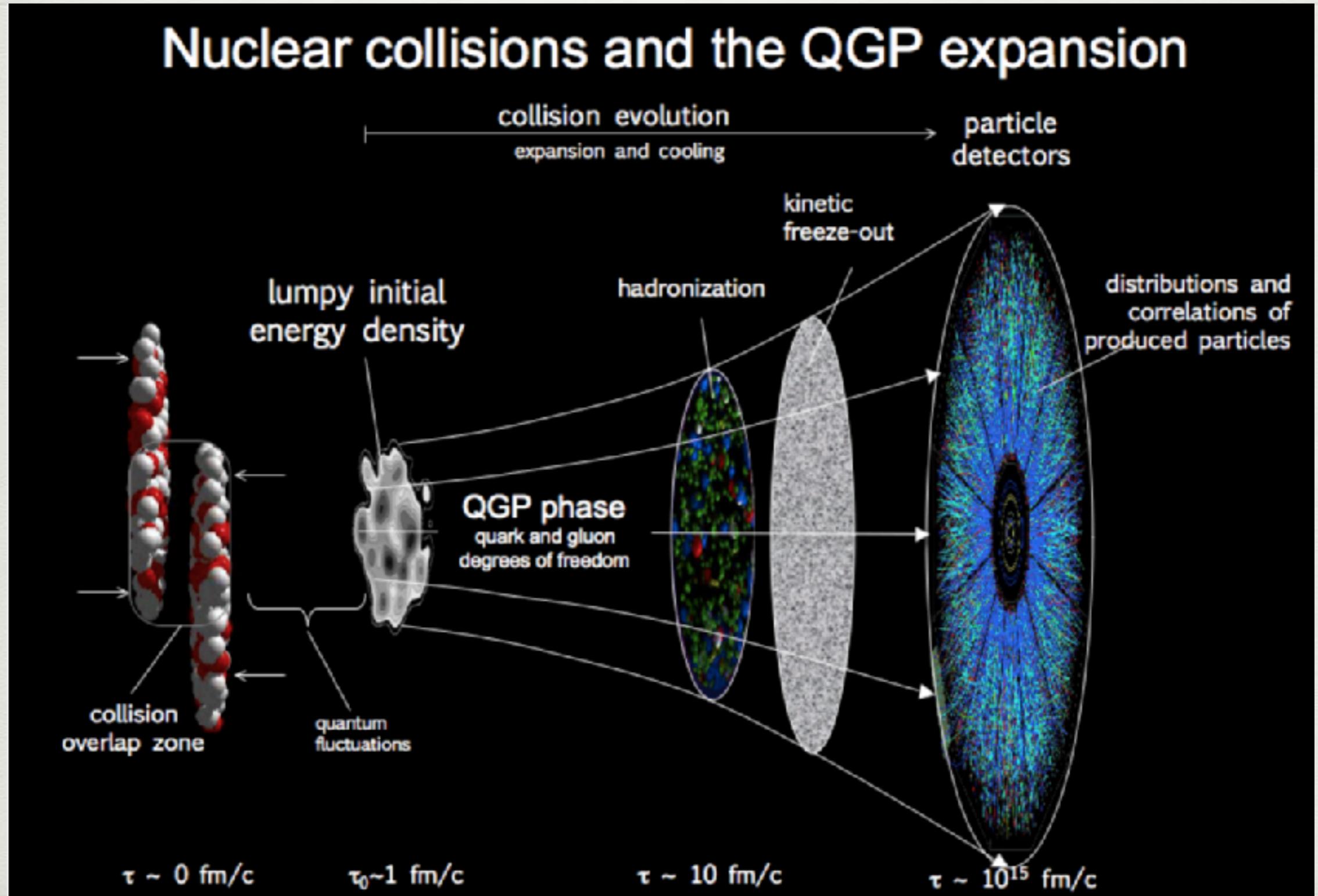


Emma McLaughlin  
PhD Columbia U.

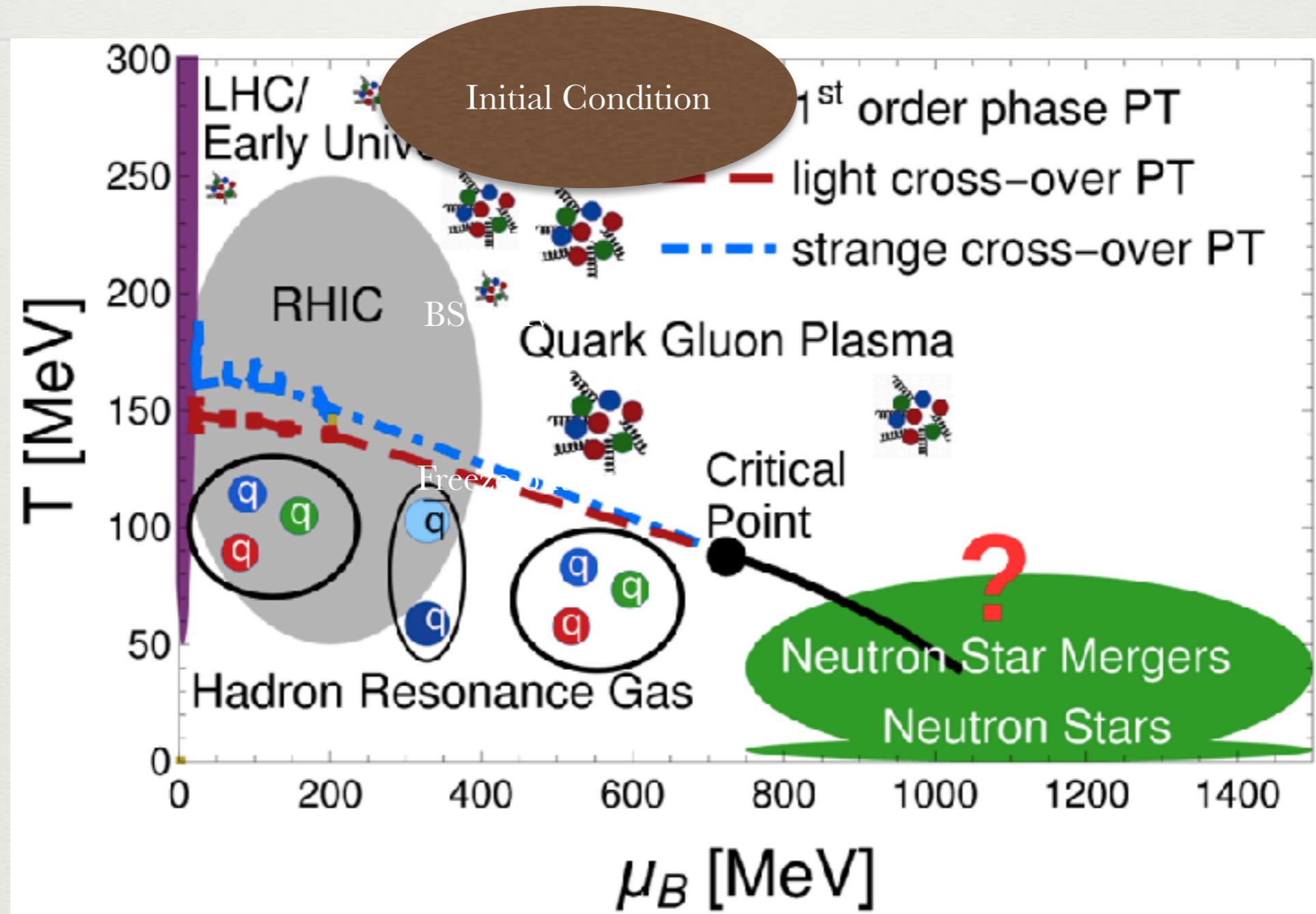


Far-from-equilibrium initial conditions in heavy-ion collisions can dramatically affect the passage through the phase diagram

# Heavy-ion collisions explore the QCD phase diagram *dynamically*



# QCD Phase Diagram



**Light transition** Phys.Lett. B738 (2014) 305-310; **Strange Transition** Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, arXiv:1805.00088 ;  
**Neutron Star (mergers)** V. Dexheimer arXiv:1708.08342; **Holography** Critelli, JNH, et al, Phys.Rev. D96 (2017) no.9, 096026

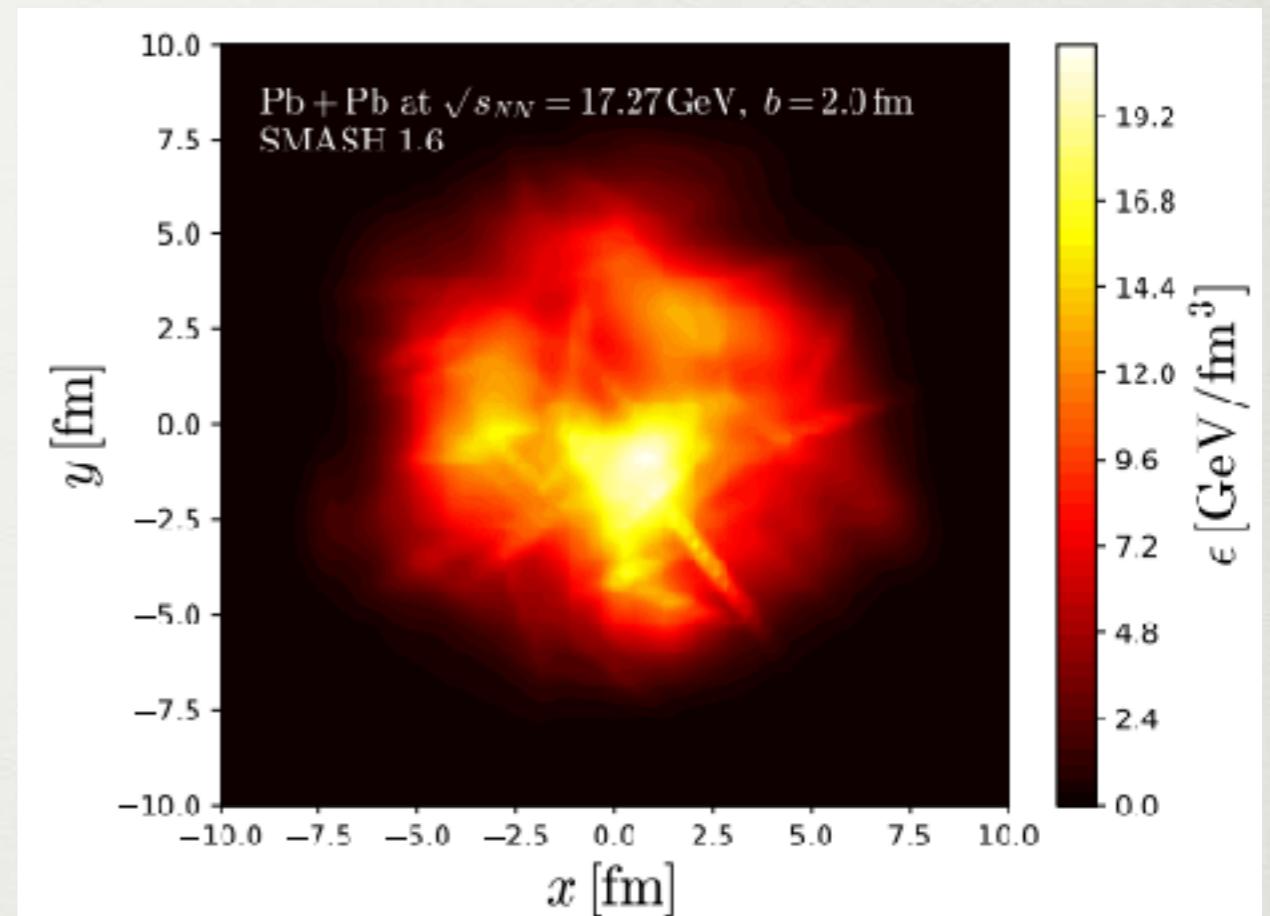
# Initializing finite baryon density with nucleons

Strings between nucleons to simulate baryon stopping

Werner, Phys.Rept. 232 (1993) 87-299 ; Jeon and Kapusta, Phys.Rev. C56 (1997) 468-480 ; Bialis, Bzdak, Bozek, Acta Phys.Polon. B49 (2018) 103; Schenke and Shen Phys.Rev. C97 (2018) no.2, 024907

Transport (UrQMD/SMASH)

Karpenko, et al;Phys.Rev. C91 (2015) no.6, 064901



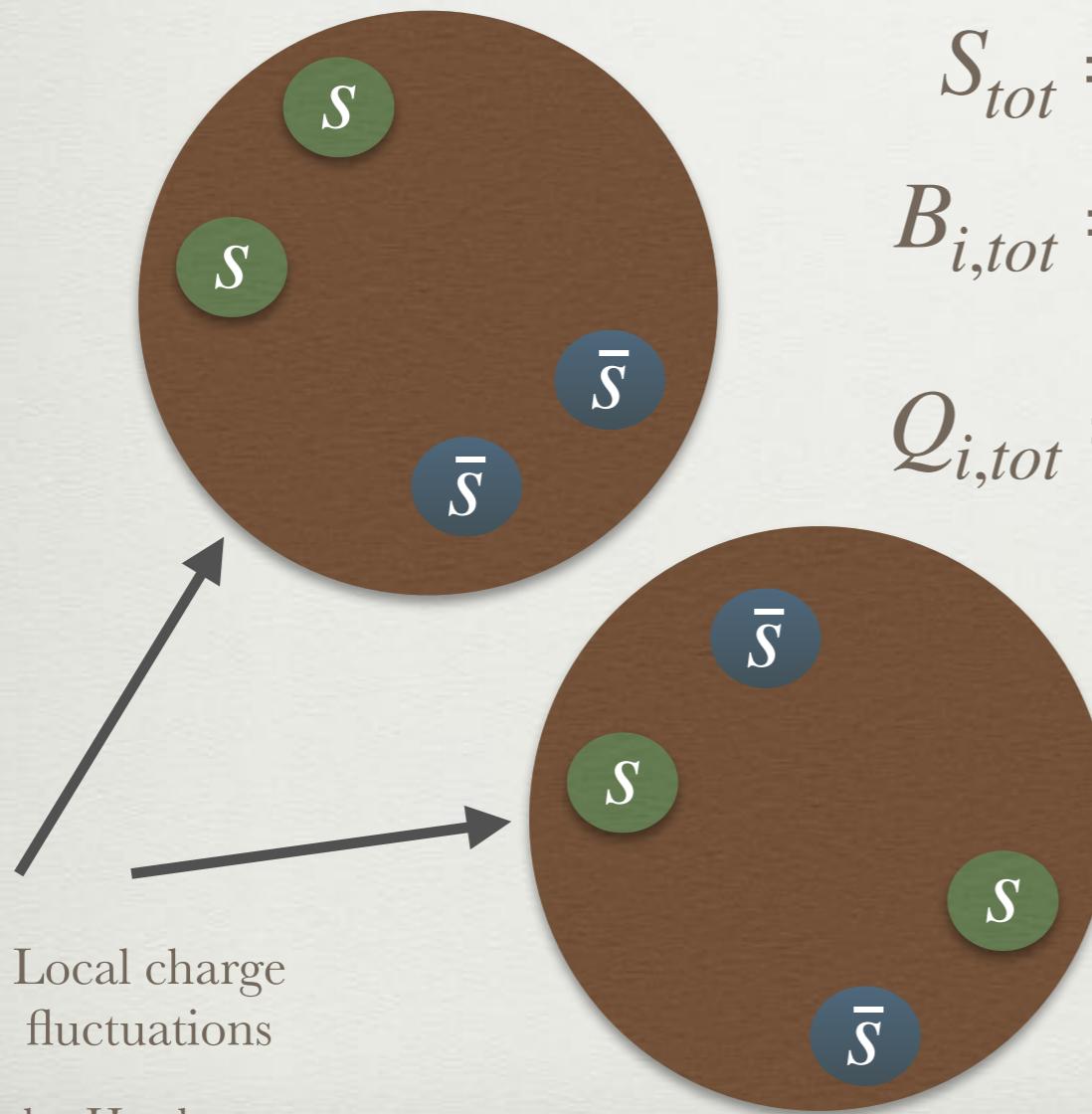
Mohs et al, J.Phys.G 47 (2020) 6, 065101

What about at the quark/gluon level?

# 3 conserved charges: BSQ

QCD has not only baryon number but strangeness and electric charge conservation

Quark Gluon Plasma



$$S_{tot} = 0$$

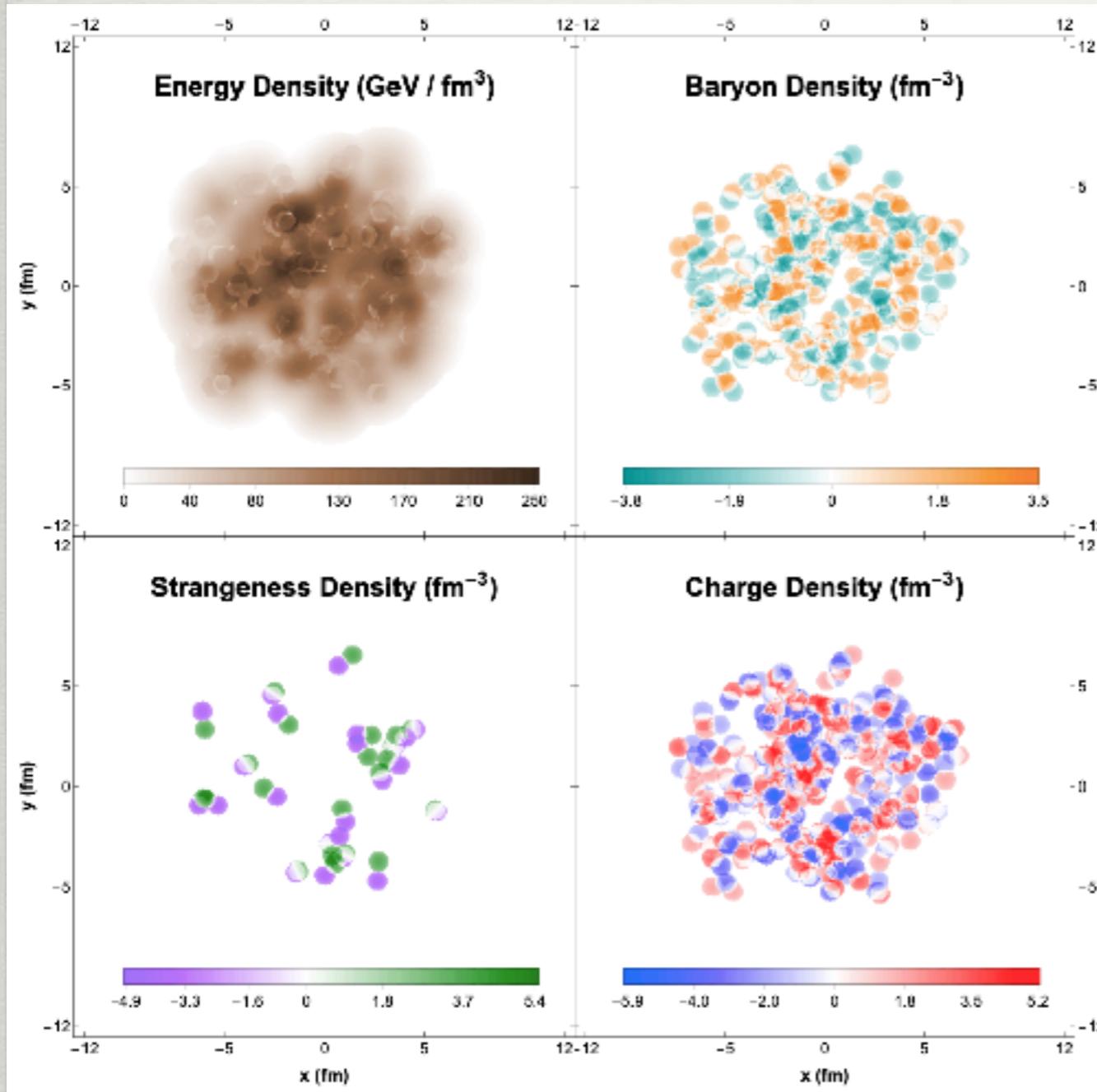
$$B_{i,tot} = B_{f,tot}$$

$$Q_{i,tot} = Q_{f,tot}$$

Each grid point in an initial condition needs to initialize

$$\{T, \rho_B, \rho_S, \rho_Q\}$$

# ICCING- Initializing Conserved Charges in Nuclear Geometries

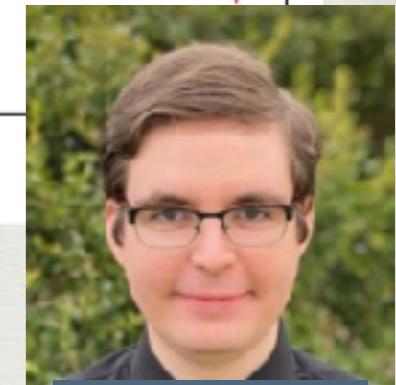
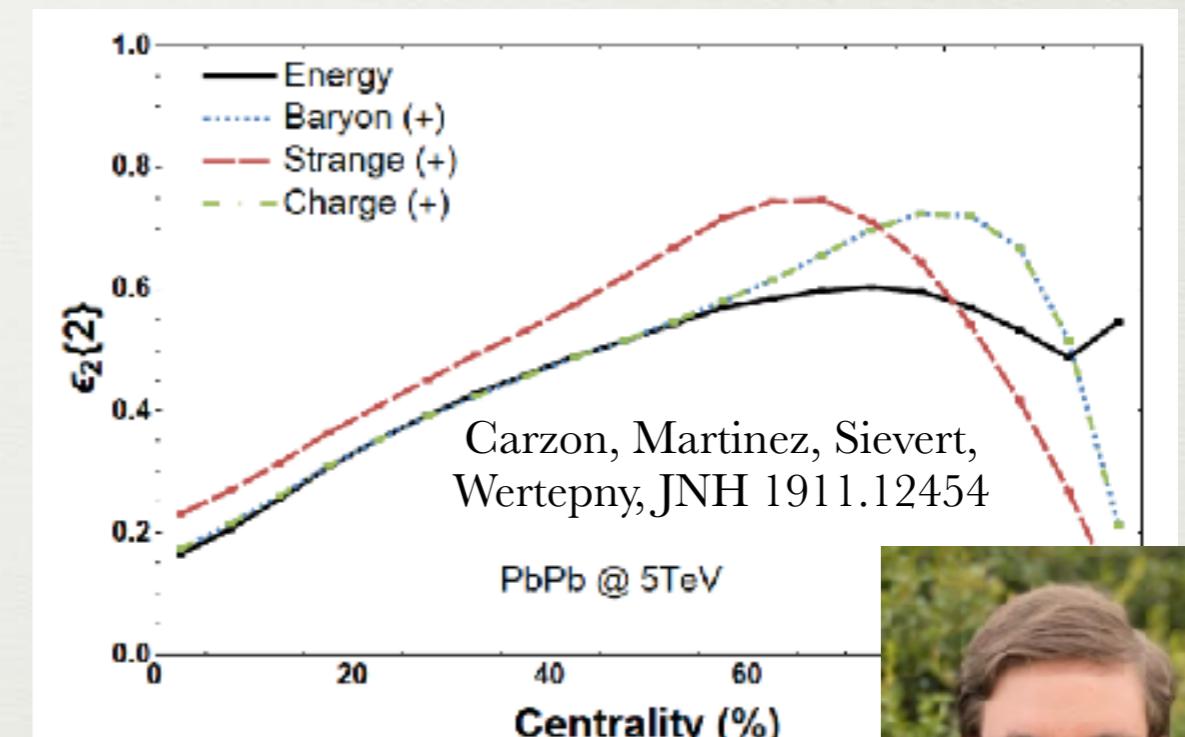


Theoretical development

**Martinez, Sievert, Wertepny**

JHEP 02 (2019) 024; JHEP 1807 (2018) 003

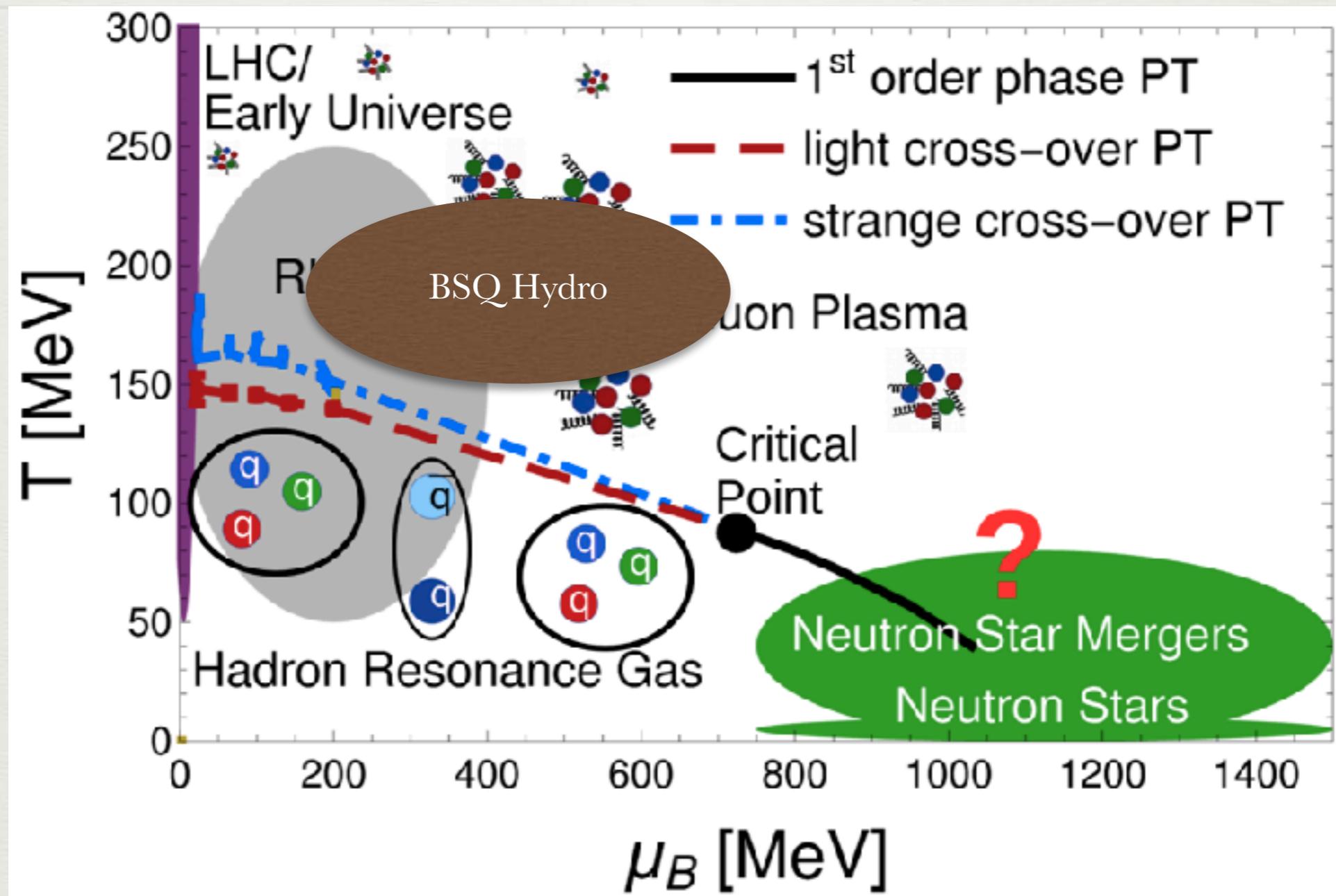
$$g \rightarrow q\bar{q}$$



Patrick Carzon  
PhD UIUC

ICCING: Carzon, Martinez, Sievert, Wertepny, JNH  
[1911.10272](https://arxiv.org/abs/1911.10272); [1911.12454](https://arxiv.org/abs/1911.12454)

# QCD Phase Diagram



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**Neutron Star (mergers)** V. Dexheimer *arXiv:1708.08342*; **Holography** Critelli, JNH, et al, *Phys.Rev. D96 (2017) no.9, 096026*

# Upgrades to Hydrodynamics

**Equation of State**  
Baryon, Strangeness,  
Electric Charge  
EOS+CP

## Transport Coefficients

Shear and bulk viscosity

$$\eta T/w(T, \mu_B, \mu_S, \mu_Q)$$

BSQ Diffusion

$$\kappa_{BB}, \kappa_{BS}, \dots$$

## BSQ Hydro

Ideal BSQ

$$\partial_\mu B^\mu = 0, B^\mu = \rho_B u^\mu + n_B^\mu$$

Diffusion

$$\kappa_{BB} \Delta^{\mu\nu} \partial_\nu (\mu_B/T)$$

BSQ cross terms

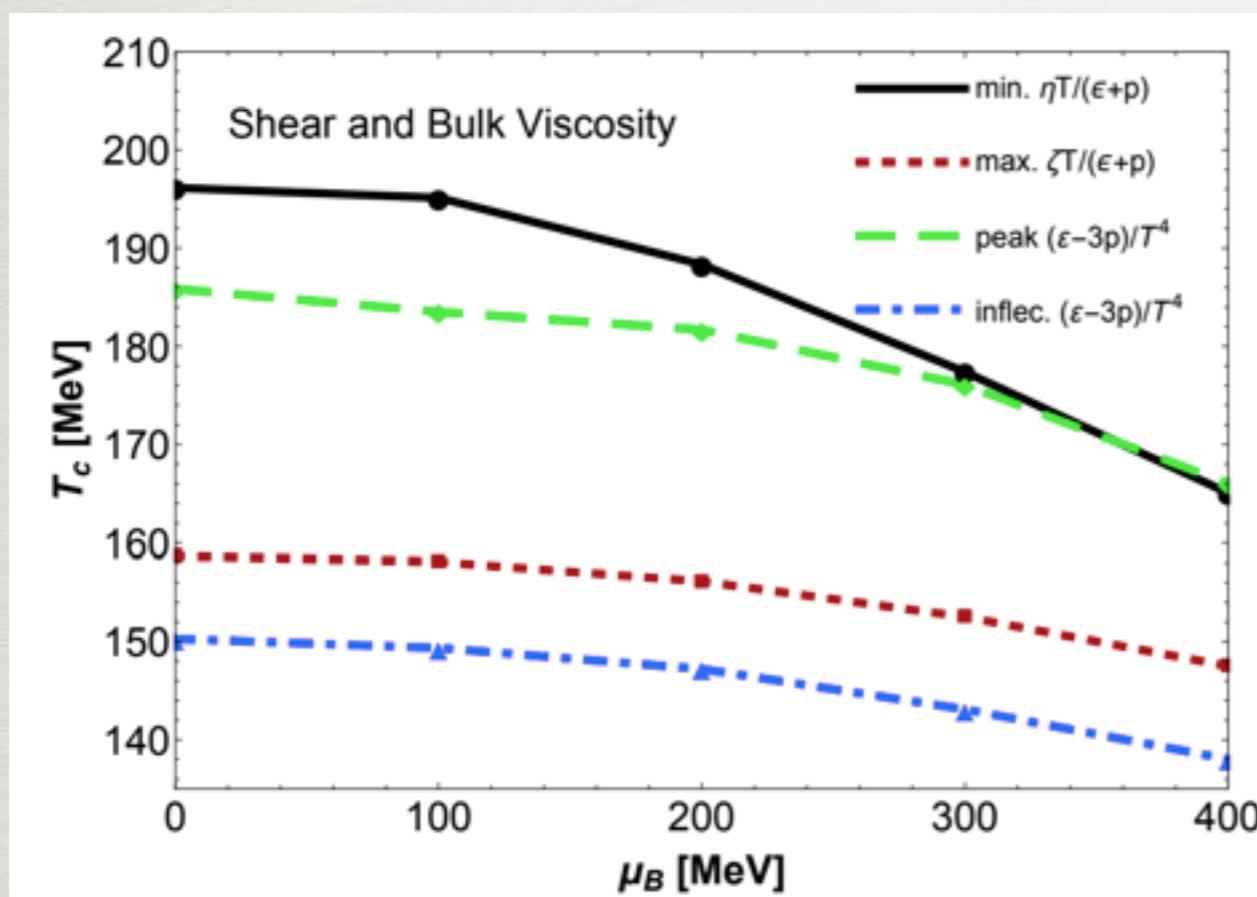
$$\kappa_{BS} \Delta^{\mu\nu} \partial_\nu (\mu_B/T)$$

# Viscosity at finite densities

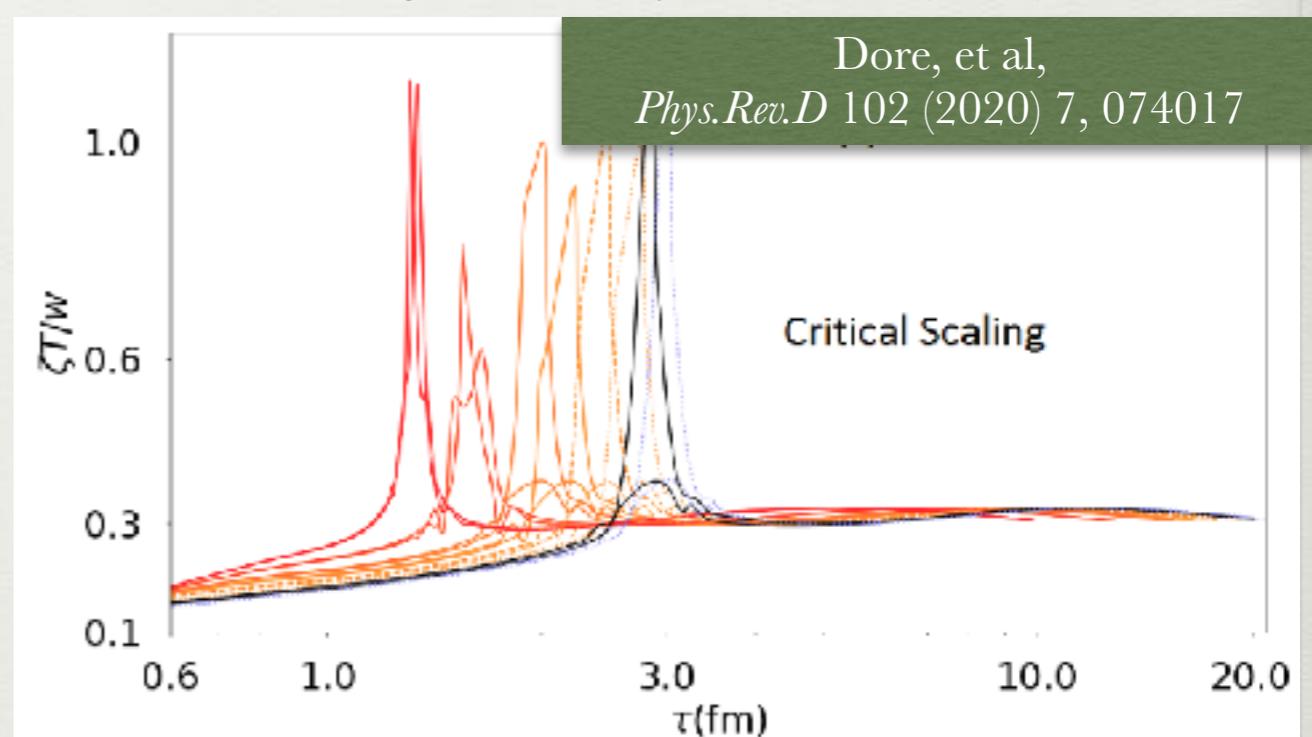
Temperature of minimum of shear and maximum of bulk viscosity

Critical Scaling:

$$\left(\frac{\zeta T}{w}\right)_{CS} = \frac{\zeta T}{w} \left[ 1 + \left(\frac{\xi}{\xi_0}\right)^3 \right]$$



Monnai, Mukherjee, Yin; Phys.Rev.C 95 (2017) 3, 034902

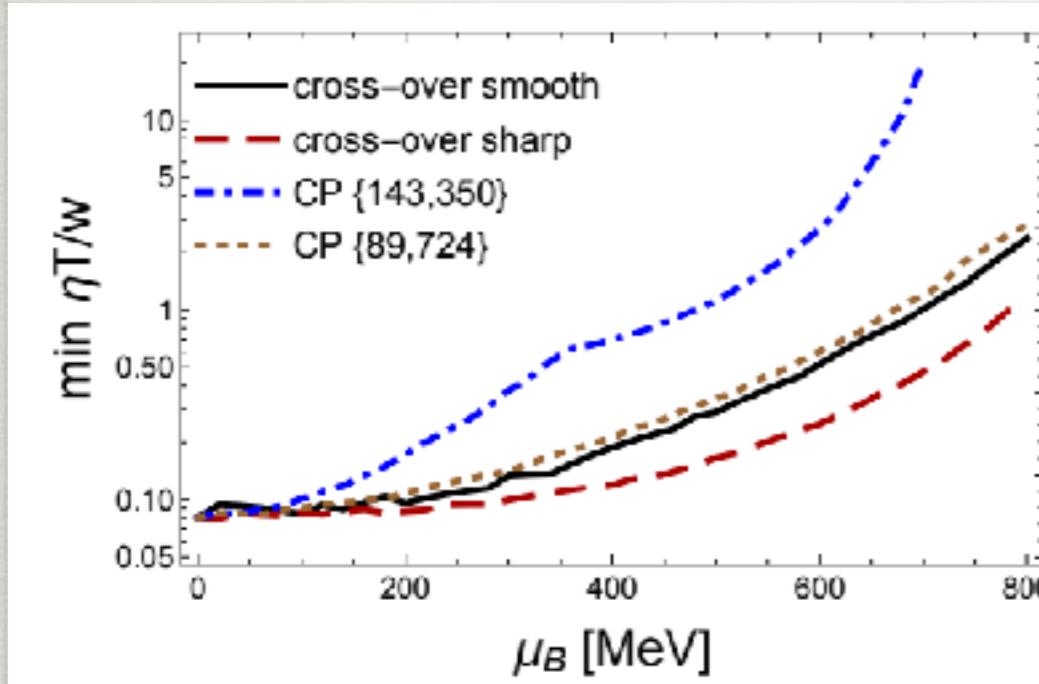


Rougemont, JN Het al, Phys.Rev.  
D96 (2017) no.1, 014032

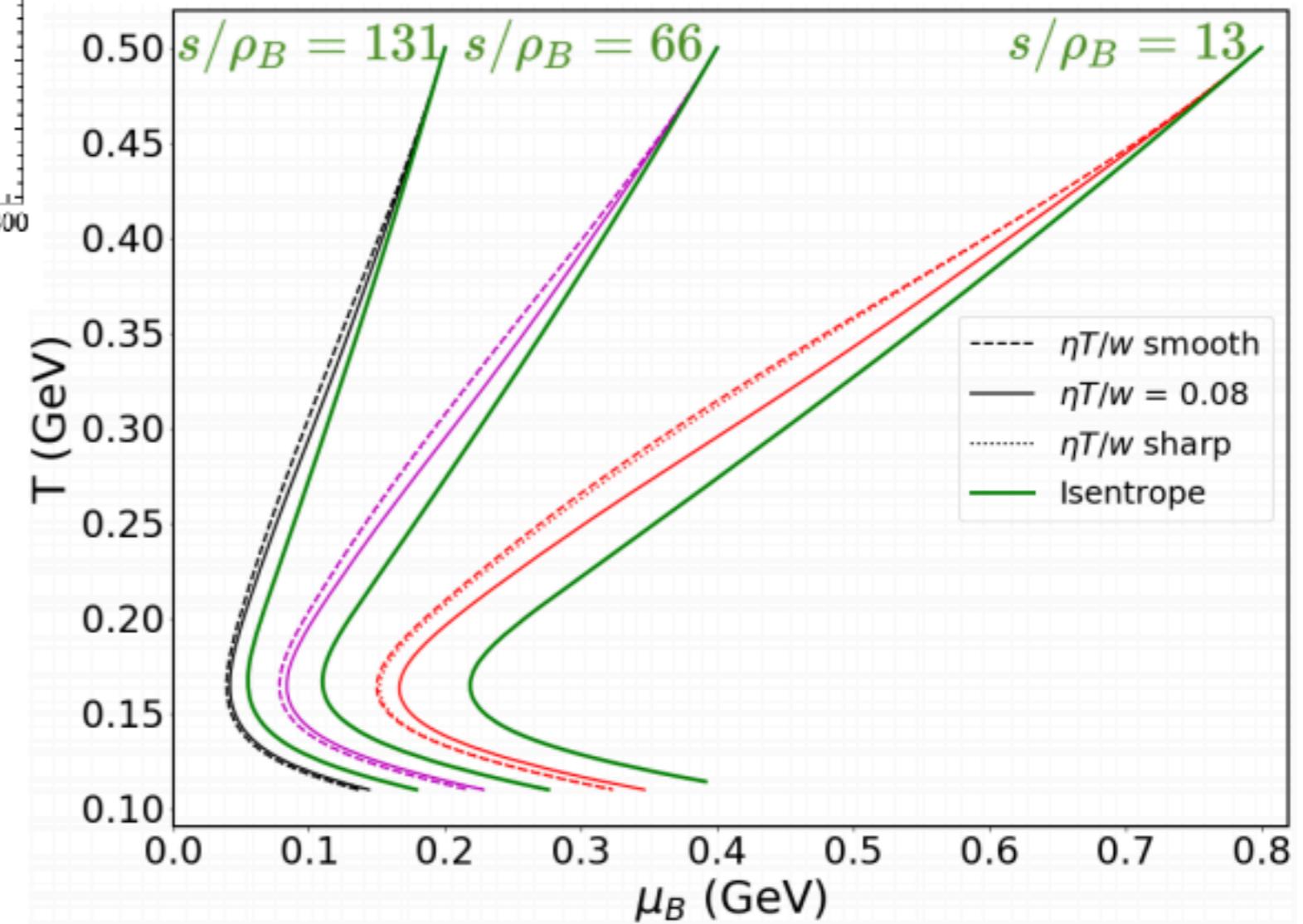
Note  $\zeta T/w \propto 1/3 - c_s^2$

# Consequences of $\eta T/w$ across $\mu_B$

McLaughlin, JNH, et al, [2103.02090](#)

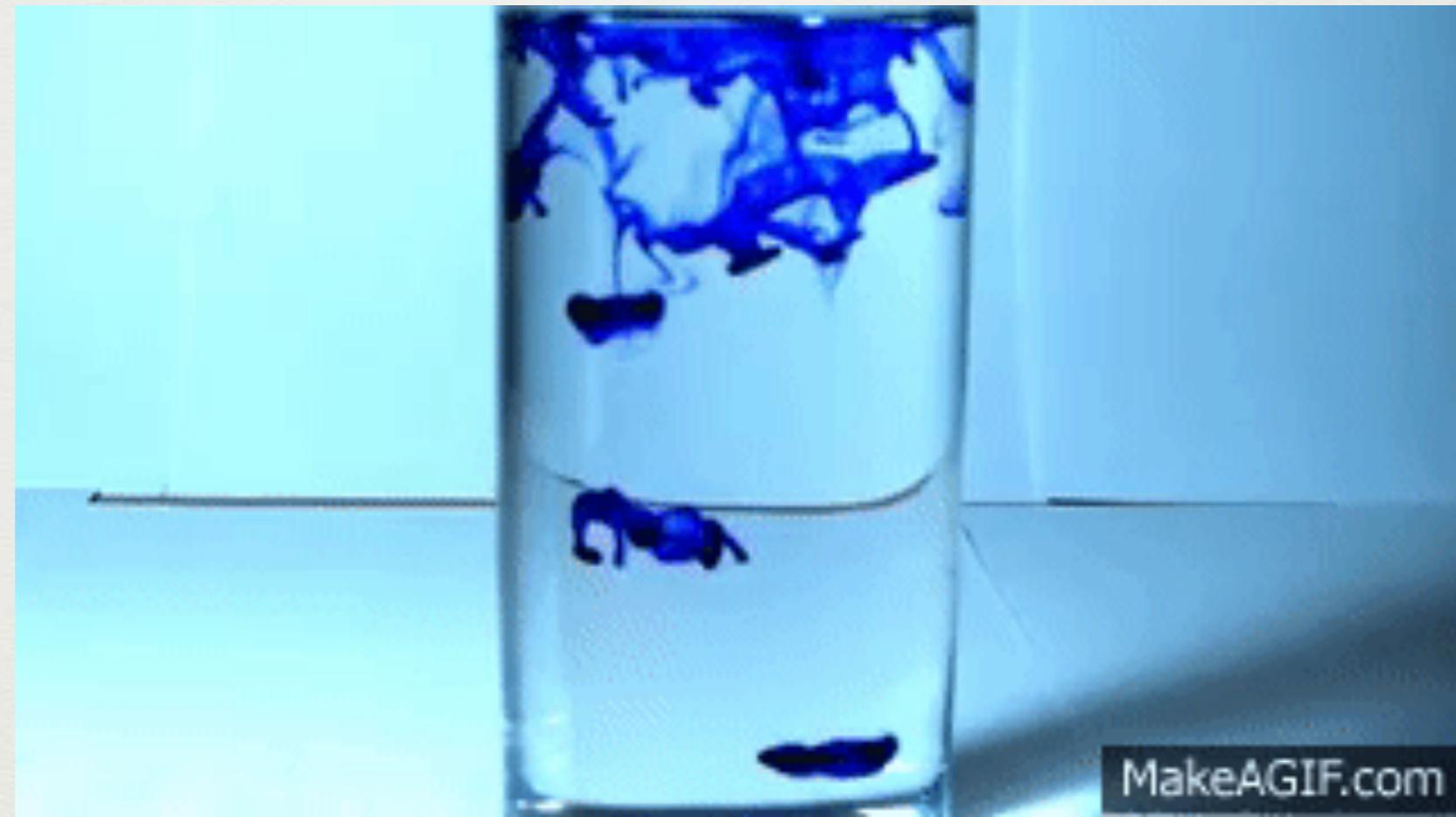


Transition line affects  
magnitude of  $\eta T/w$



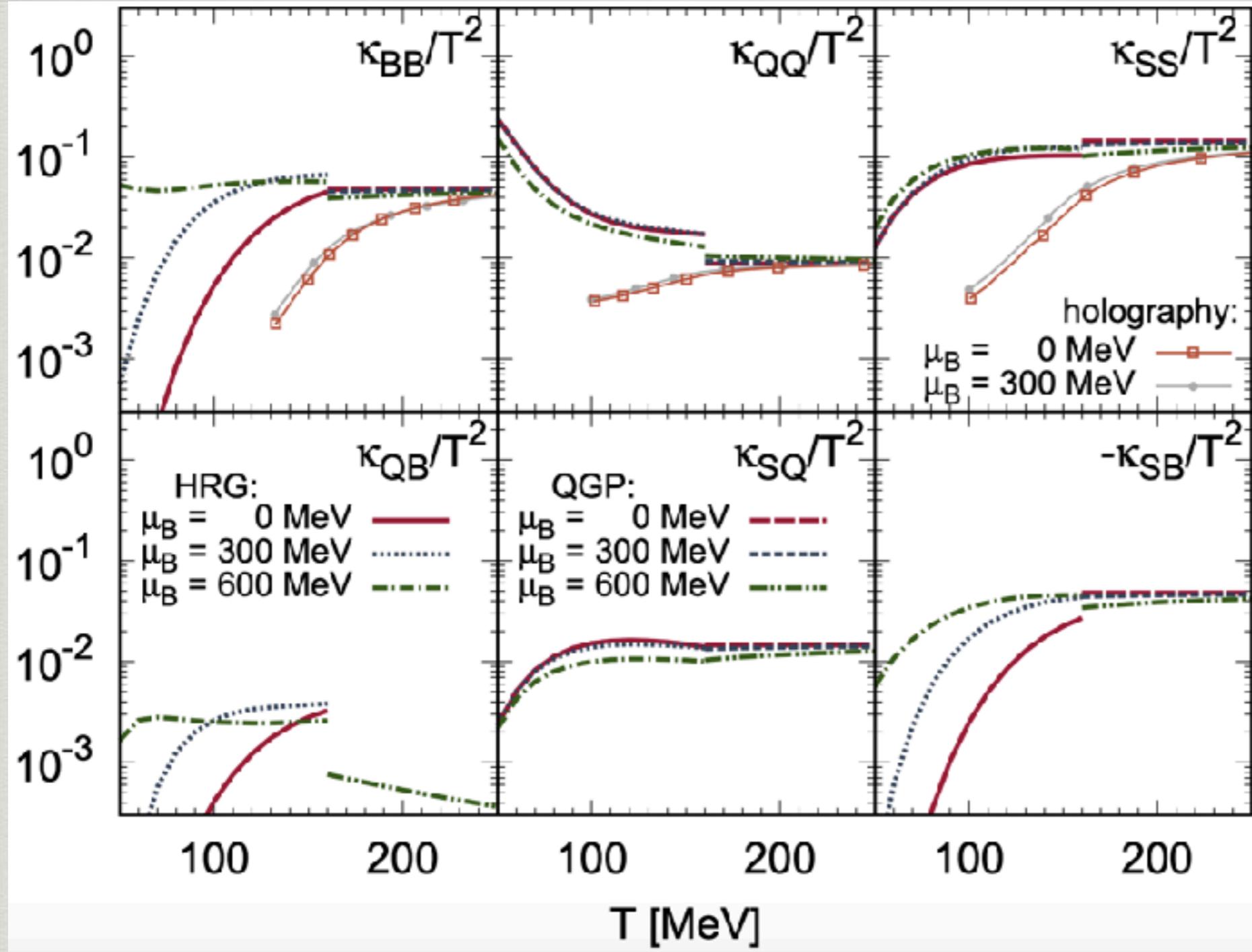
Viscous effects more  
prevalent at low beam  
energies

# BSQ diffusion



How do quarks spread out over time? How quickly do u, d, s quarks diffuse through out the QGP since they carry multiple conserved charges?

# BSQ Diffusion Matrix



Kinetic Theory

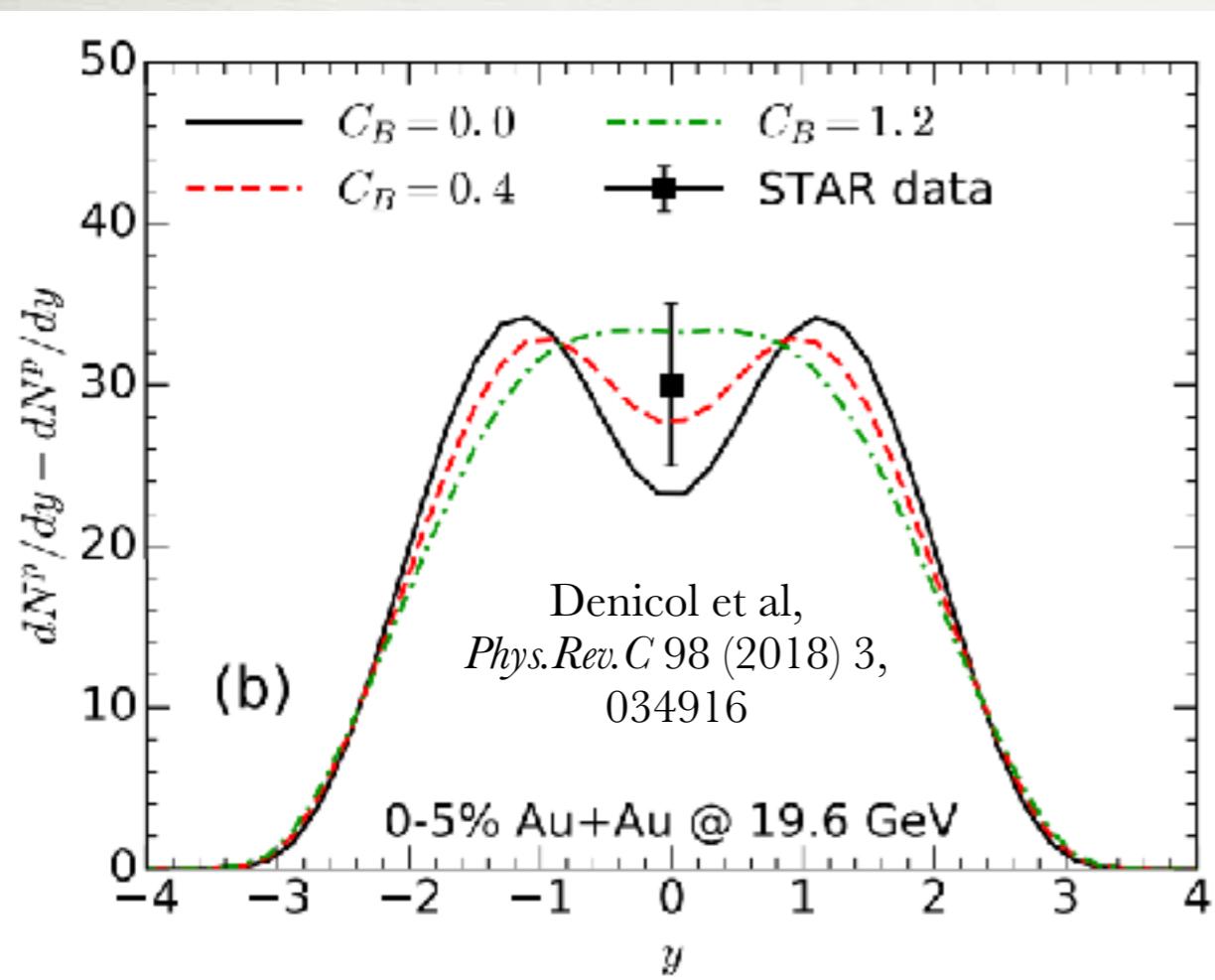
Greif, Fotakis, Denicol, Greiner  
*Phys.Rev.Lett.* 120 (2018) no.24,  
 242301

Holography (black  
hole engineering)

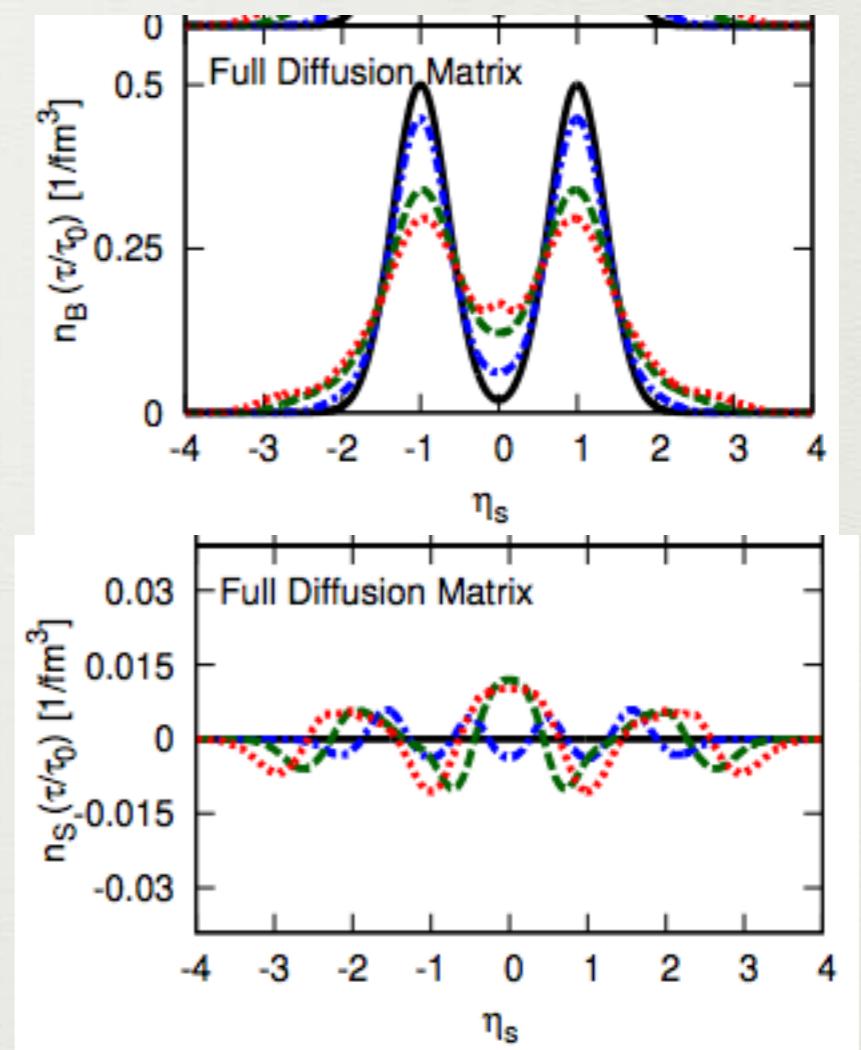
Rougemont, Critelli, JNH, Noronha,  
 Ratti *Phys.Rev. D96* (2017) no.1,  
 014032

# Initial hydro results

## 3+1 B hydro



## 1+1 BS hydro



Fotakis et al *Phys.Rev.D* 101 (2020) 7, 076007

See also, Du, Heinz Comput.Phys.Commun. 251 (2020) 107090

# BSQ equations of motion



Lydia Spychalla  
REU UIUC

- Additional



Christopher Plumberg  
UIUC postdoc

shear+bulk viscosity gives 10 coupled  
2+1 BSQ relativistic viscous  
hydrodynamics+ICCING

Coming Soon!

Almaalol, Carzon, Cruz Camacho,  
Dore, Mroczek, Plumberg, Spychalla,  
Sievert, JNH

thermodynamic derivatives

- Criticality + relativistic viscous hydro still open question!

Stephanov, Nahrgang, Basar, Yin, Schaeffer, Yee, An, Martinez, Teaney, Akamatsu, Yan,  
Rajagopal, Weller, Ridgway, Du, Heinz, Song

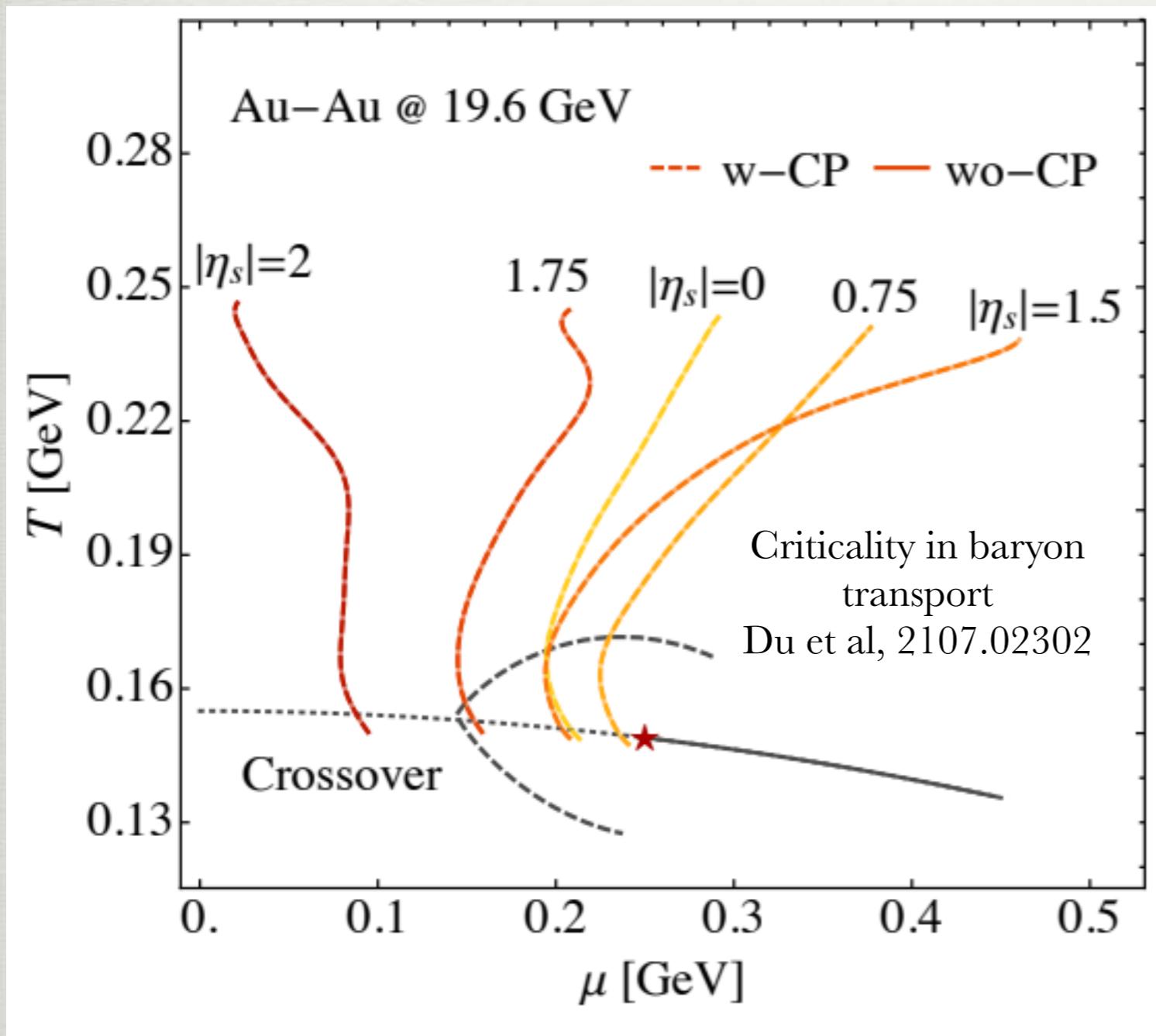


Dekra Almaalol  
PhD Kent  
Soon UIUC postdoc



Nikolas Cruz Camacho  
PhD UIUC

# Critical fluctuations in 1+1D hydro

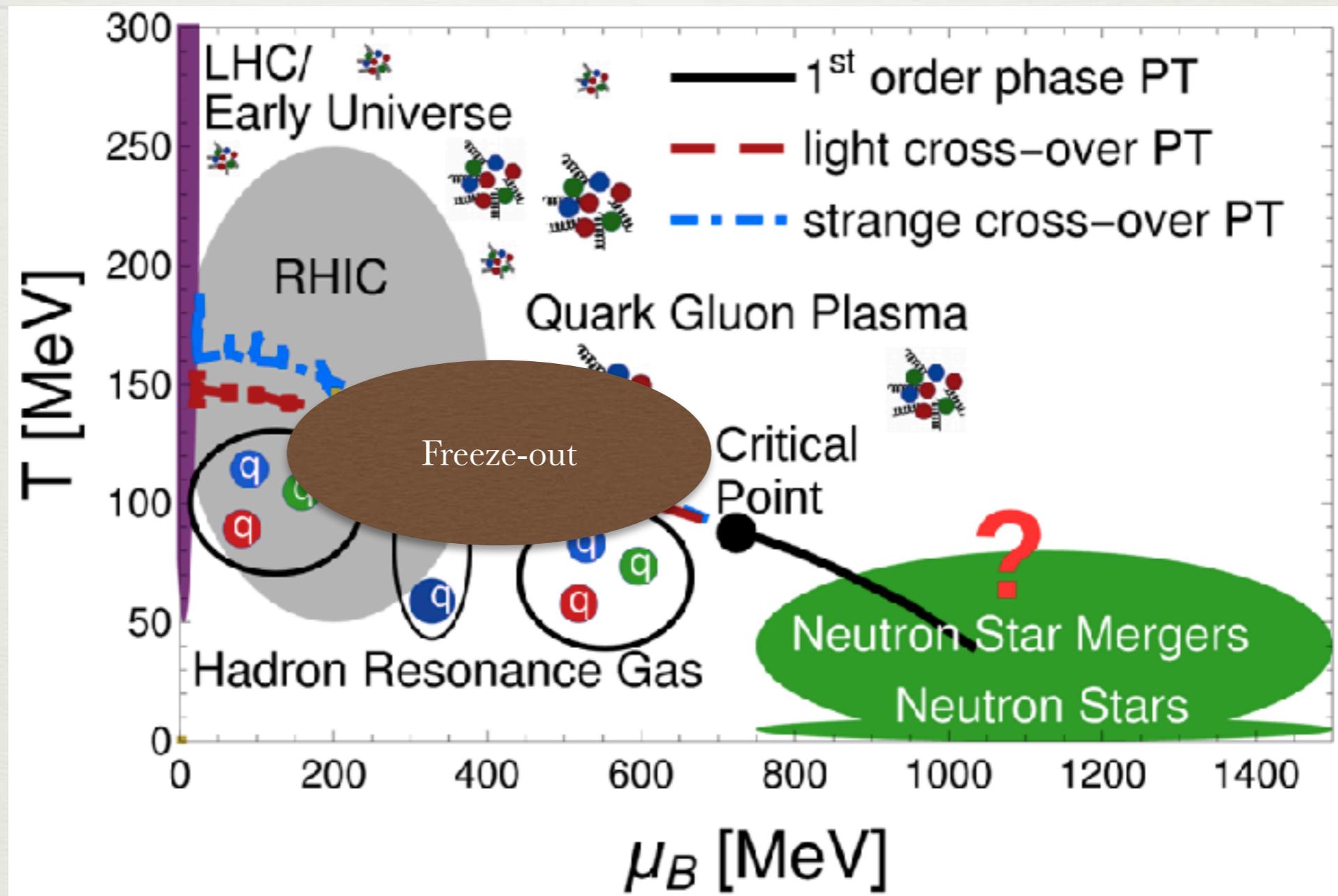


Critical slow down (from baryon transport), no noticeable affect.

Bulk viscous effects still relevant  
 $\eta \sim \xi^{-19}$ ,  $D_B \sim \xi^{-1}$ ,  $\zeta \sim \xi^3$

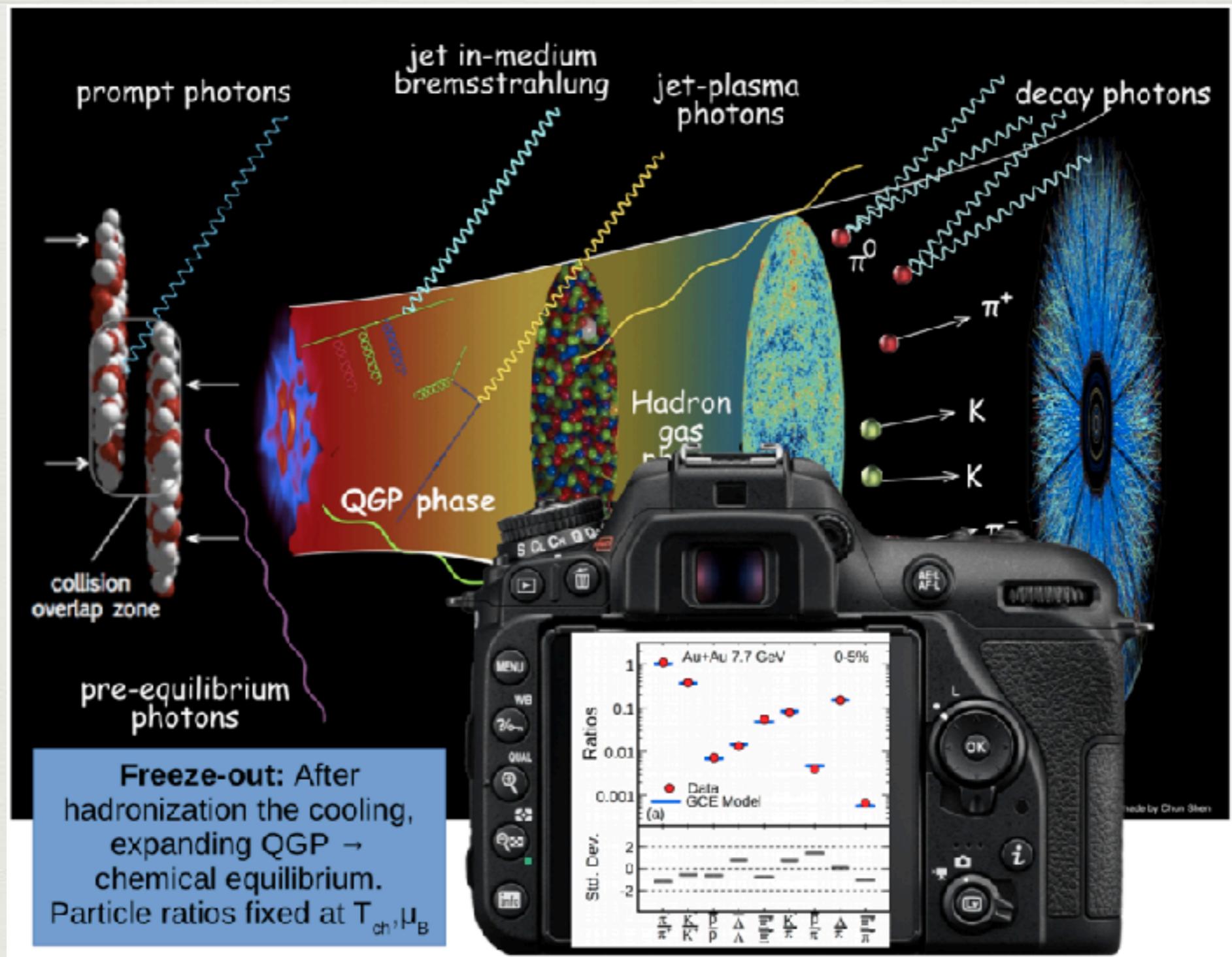
Monnai et al, *Phys.Rev.C* 95 (2017) 3, 034902;  
Dore, McLaughlin, JNH  
*Phys.Rev.D* 102 (2020) 7, 074017; Du et al,  
2107.02302

# QCD Phase Diagram



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**Neutron Star (mergers)** V. Dexheimer *arXiv:1708.08342*; **Holography** Critelli, JNH, et al, *Phys.Rev. D96 (2017) no.9, 096026*

# Freeze-out: finding the cross-over temperature

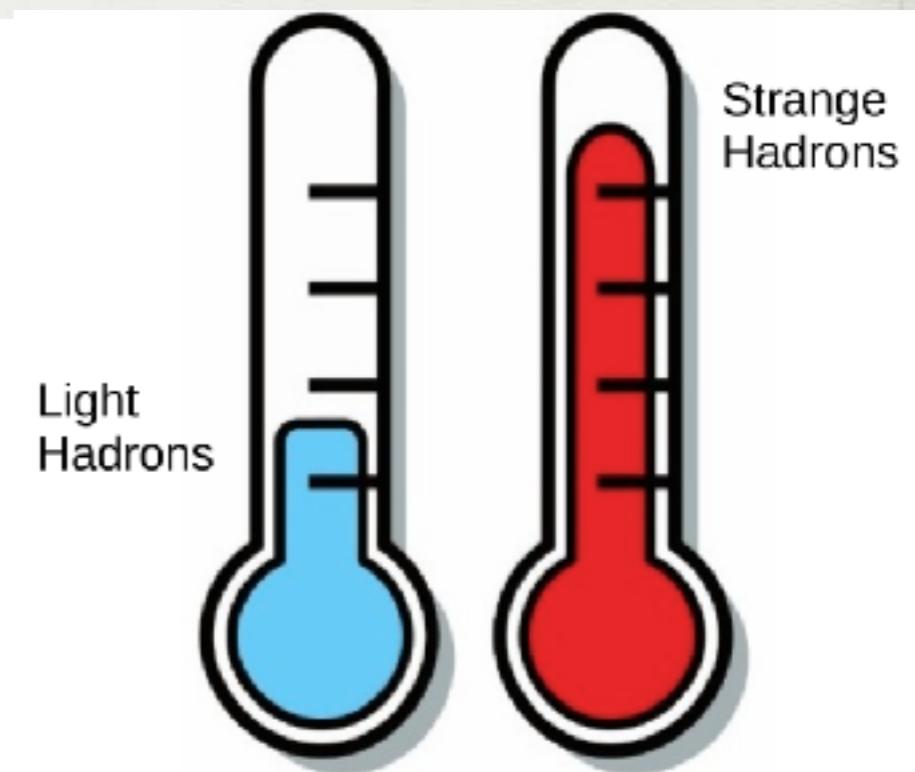
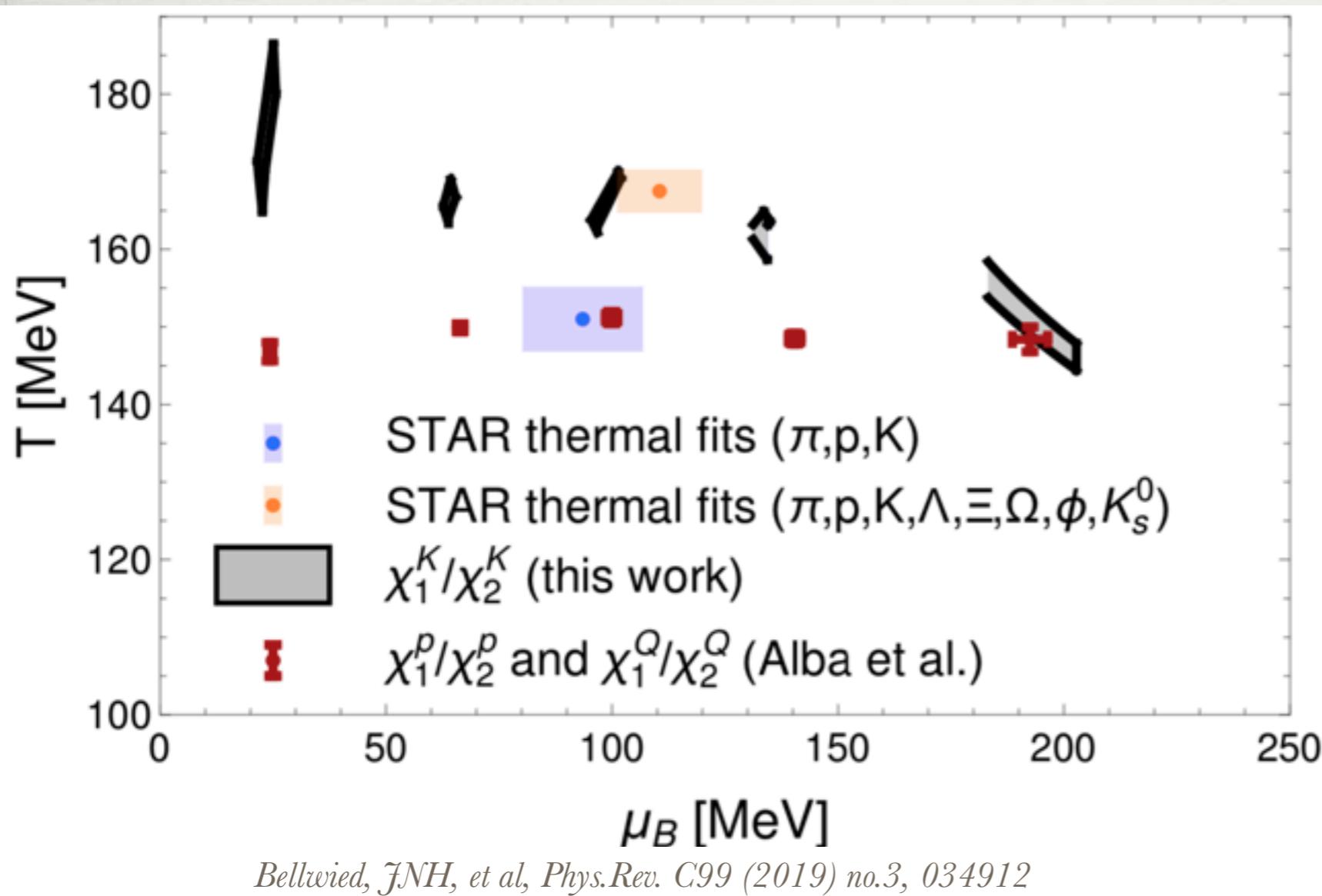


# Hadron resonance gas at large $\rho_B$



- Conservation of  $B,S,Q$  in MC sampling  
D. Oliinychenko *et al*, *Phys.Rev.C* 102 (2020) 3, 034904
- Cooper Fry diffusion corrections  
$$f(T, \mu_B, \mu_s, \mu_Q) + \delta f_\eta + \delta f_\zeta + \delta f_{D_{B,S,Q}}$$
- N-body interactions/potentials  
SMASH A. Sorensen, Koch 2011.06635

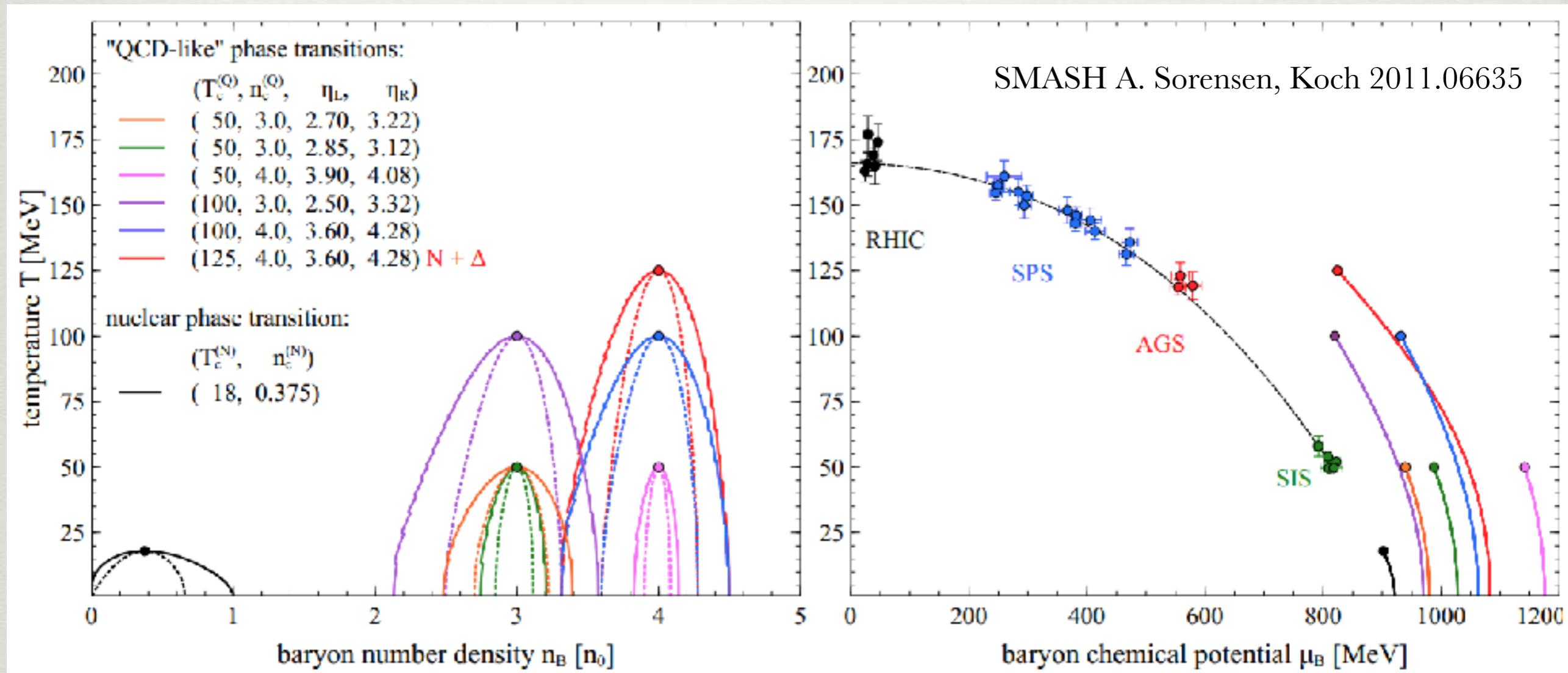
# Flavor hierarchy in freeze-out



Thermal fits also see  
flavor hierarchy

[STAR] *Phys. Rev. C* 96 (2017) 44904; Alba et al, *Phys.Rev.C* 101 (2020) 5, 054905, Flor et al, *Phys.Lett.B* 814 (2021) 136098

# Phase transitions in hadronic transport



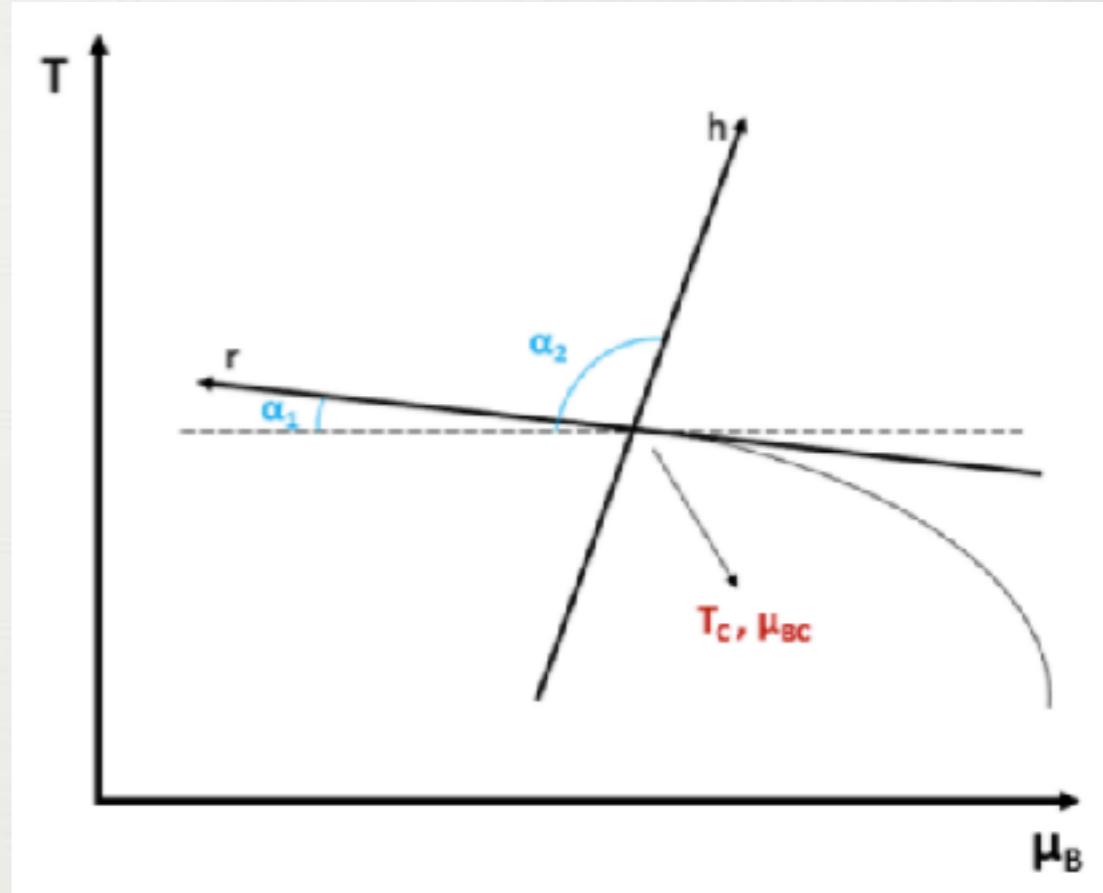
Such an approach allows for better systematic checks

# Conclusions and Outlook

- EOS from Lattice QCD reaching larger  $\mu_B$ , models hint at a CP
- BSQ charges likely will affect final flow harmonics
- Studies of BSQ viscous hydrodynamics underway
- Unanswered questions about transport coefficients at finite  $\mu_B$
- Possible signs of a flavor hierarchy at freeze-out

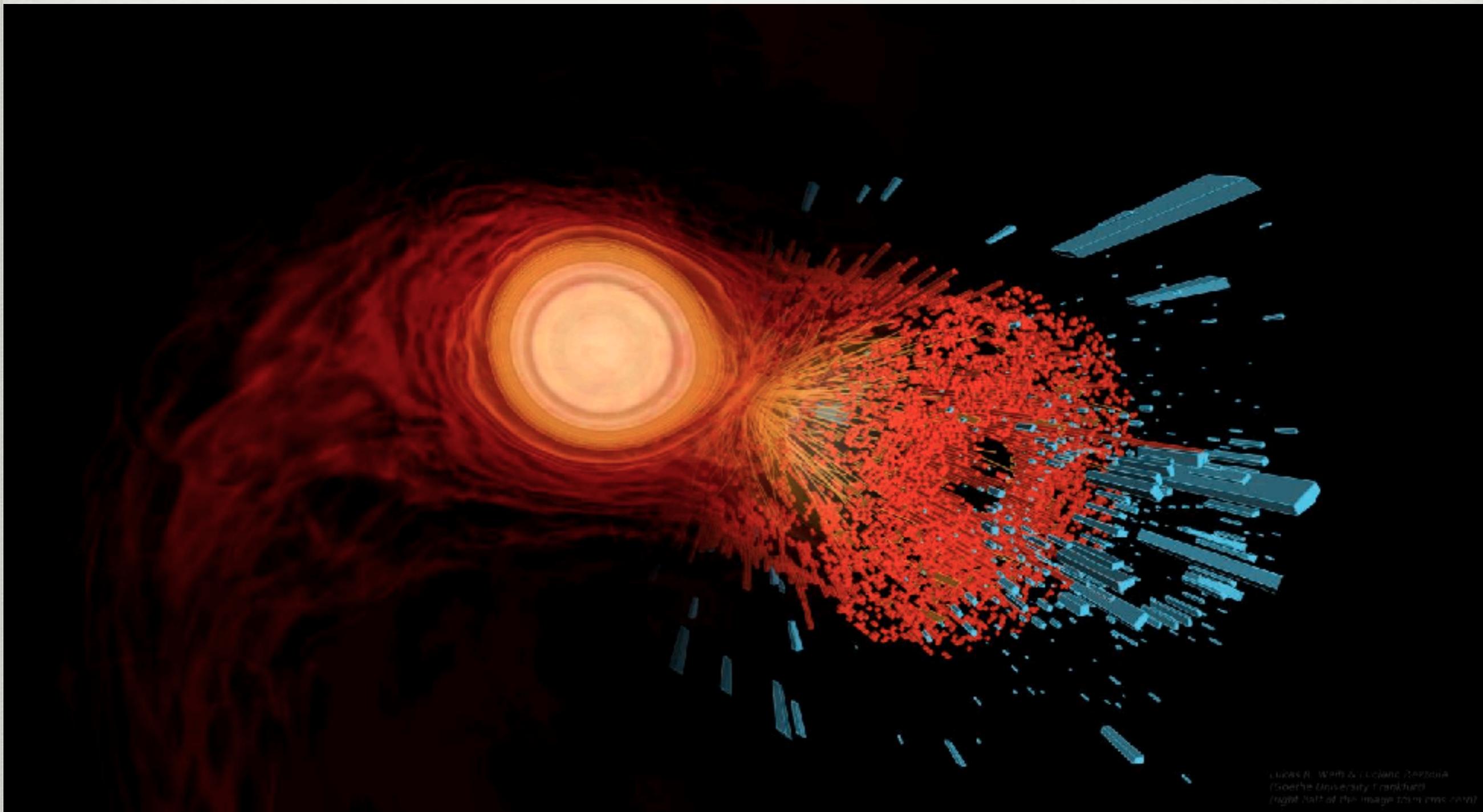
# BES-EOS: 6 unknown parameters

- 2 location:  $\{T_c, \mu_B^c\}$
- Angles:
  - $\alpha_1$  between transition line & r
  - $\alpha_2$  between r & h
- Strength of critical region:  $\rho, \omega$

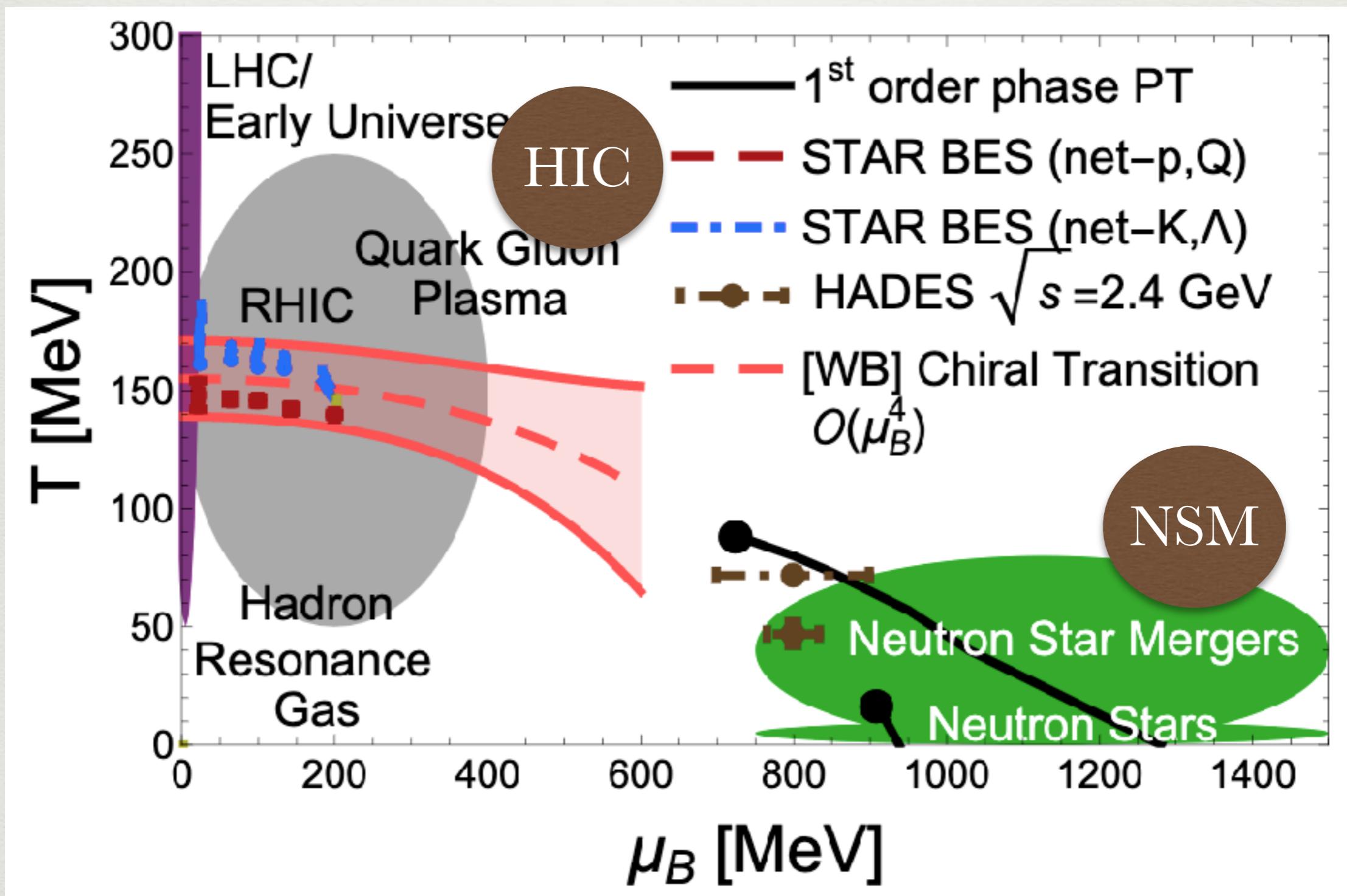


Eliminate 2: chiral transition line  
Machine Learning in progress!  
Debora Mrocze et al

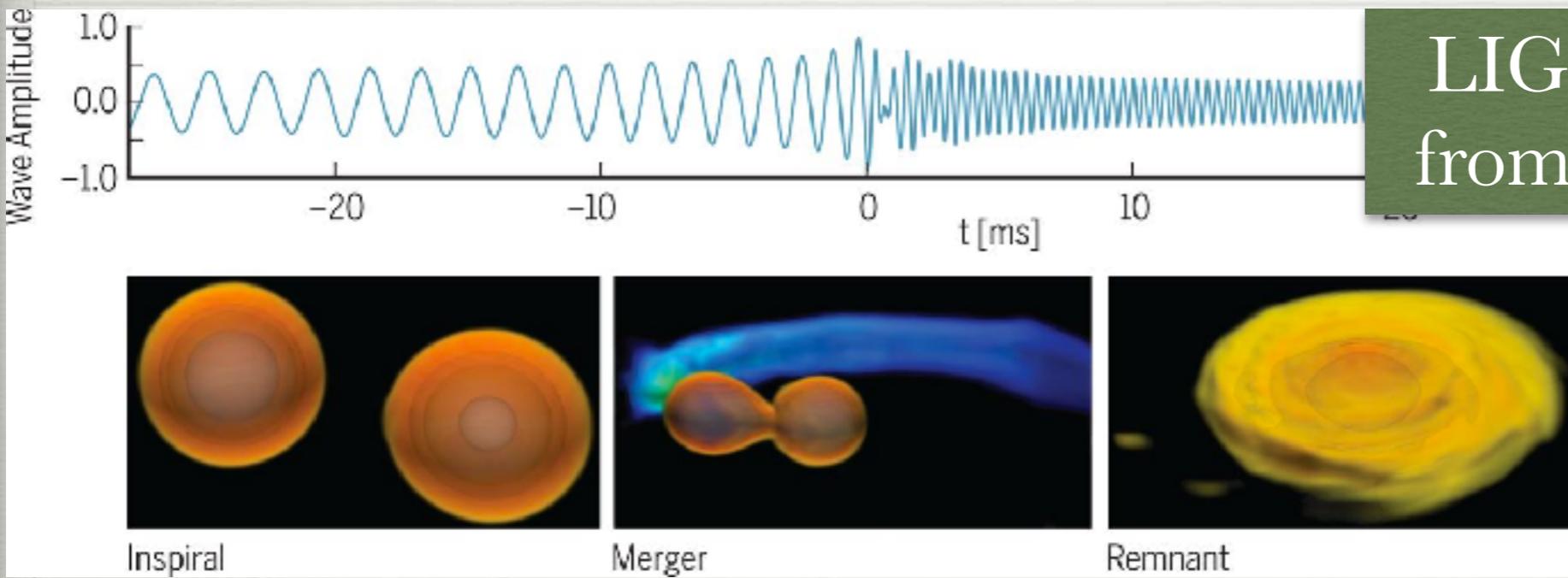
# Nucleus-Nucleus collisions = mini-neutron star mergers?



# Can heavy-ions be used to constrain the NS EOS?



# Constraints at low T and large $\rho_B$



LIGO/VIRGO: Inspiral from Gravitational Waves

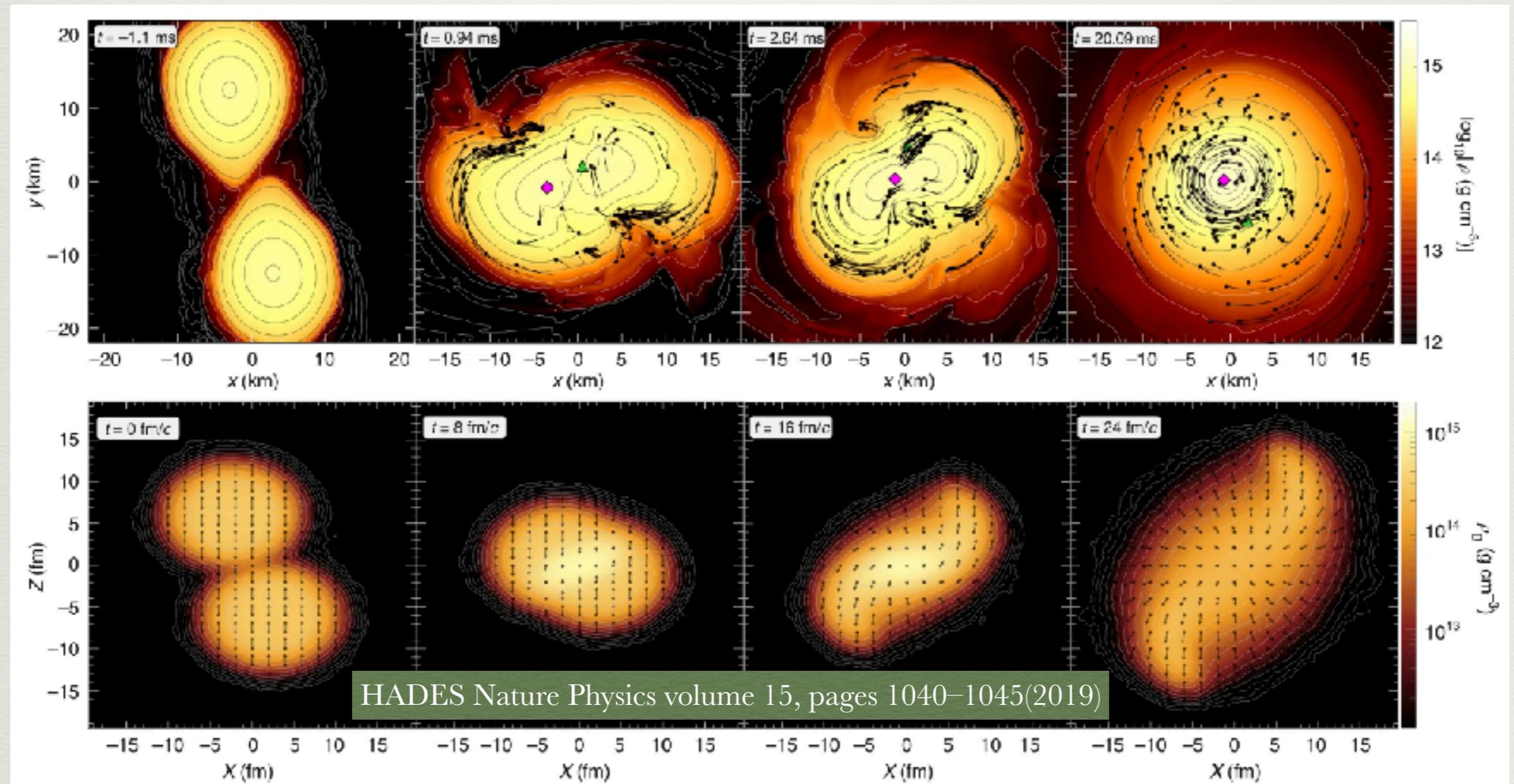
NICER: Observations of isolated Neutron Star



Late-stage Inspiral, Merger, Post-merger, ...

# HADES: Heavy-Ion Collisions vs. Neutron Stars

Comparable  $T, \rho_B$  between low-E heavy-ions (HADES) and top T's in neutron star ***mergers***.



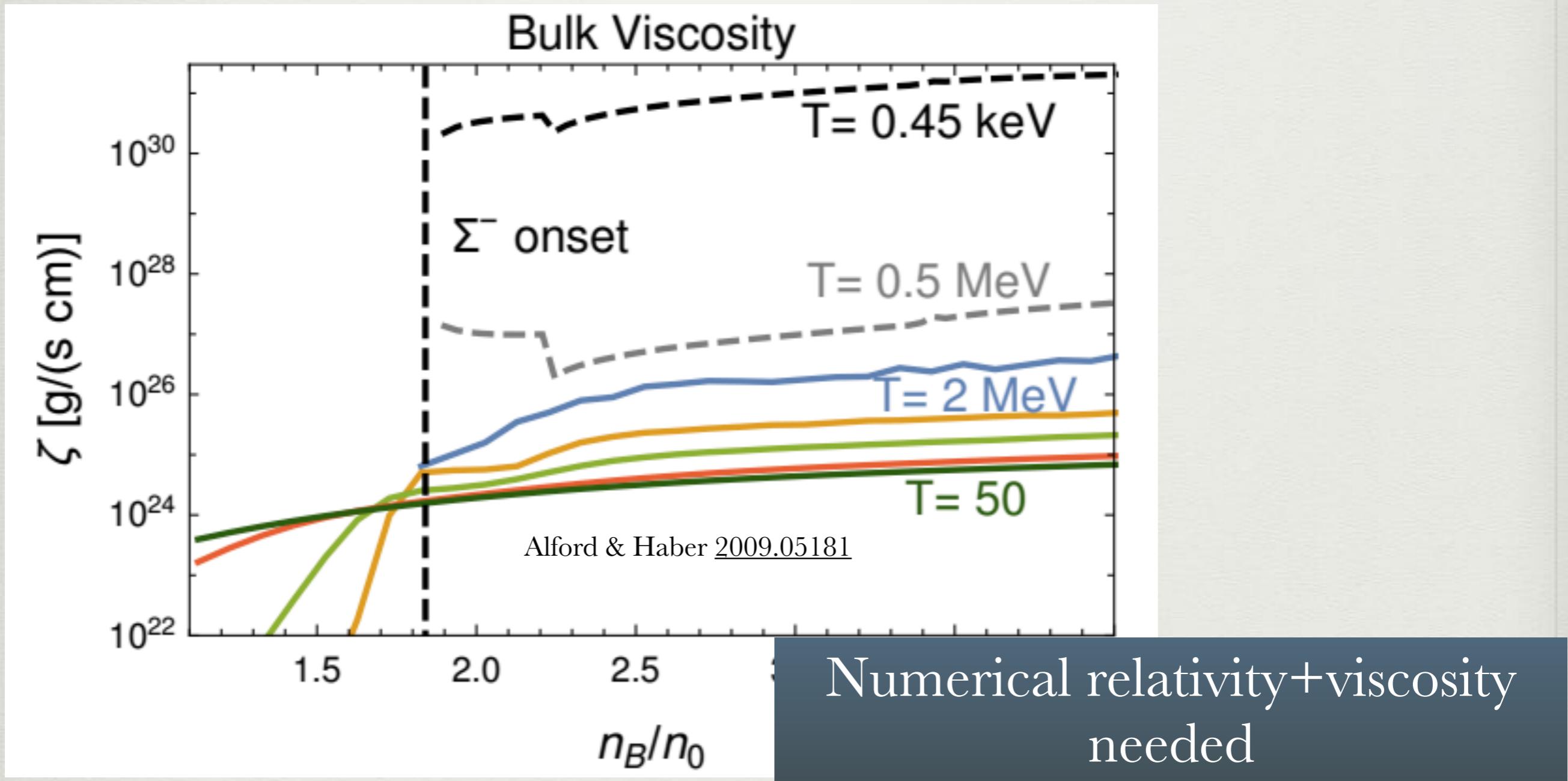
# Viscosity revolution



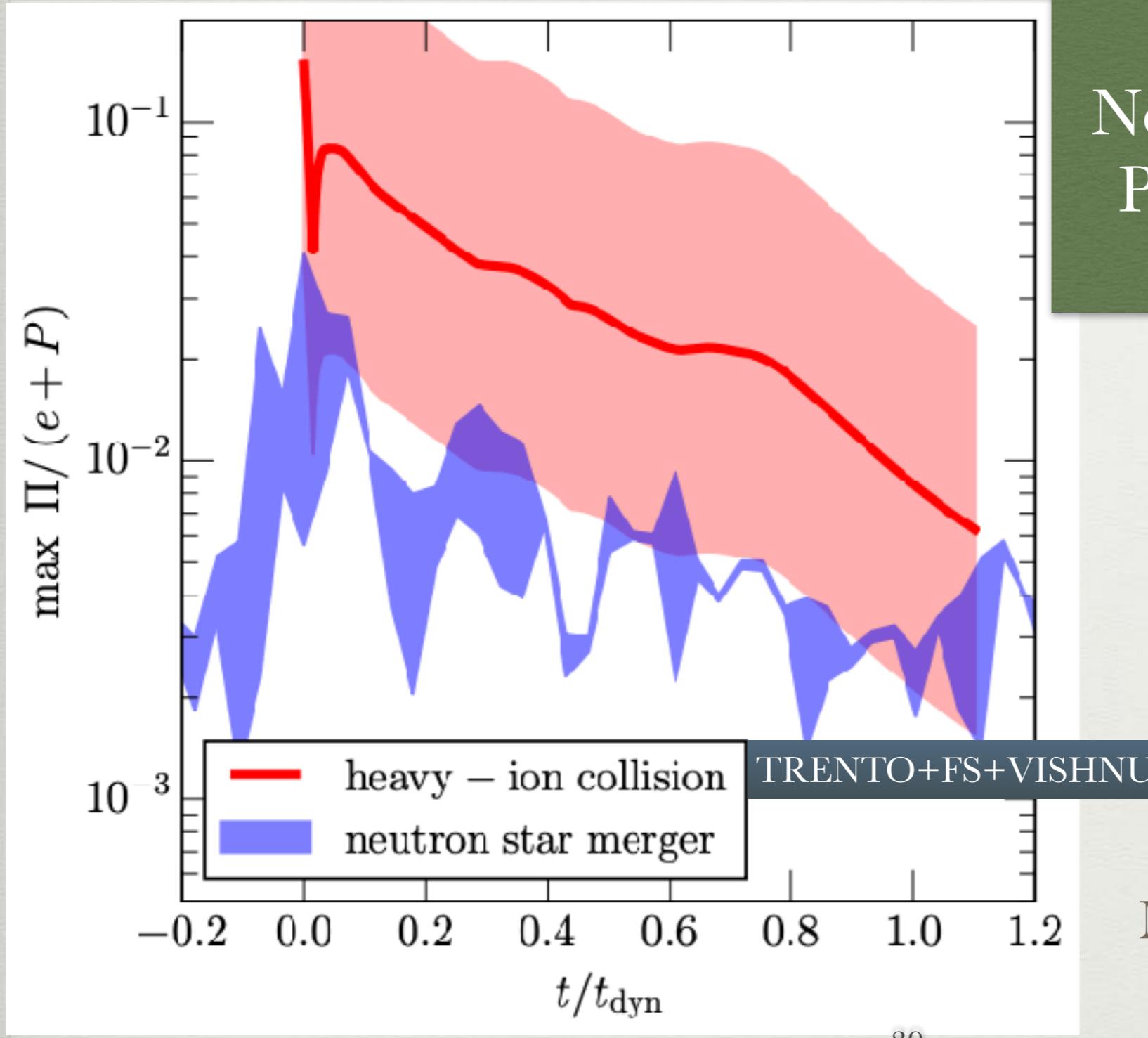
Viscosity was a game changer for heavy-ion collision - what about neutron star mergers?

# Bulk viscosity in neutron star mergers

Bulk viscosity also plays a significant role in neutron star mergers



# Bulk Viscosity: Heavy-ions vs. Neutron Star Mergers



Alford, Harris, Most,  
Noronha, JNH, Pretorius,  
Plumberg, Witek, Yunes  
2107.05094

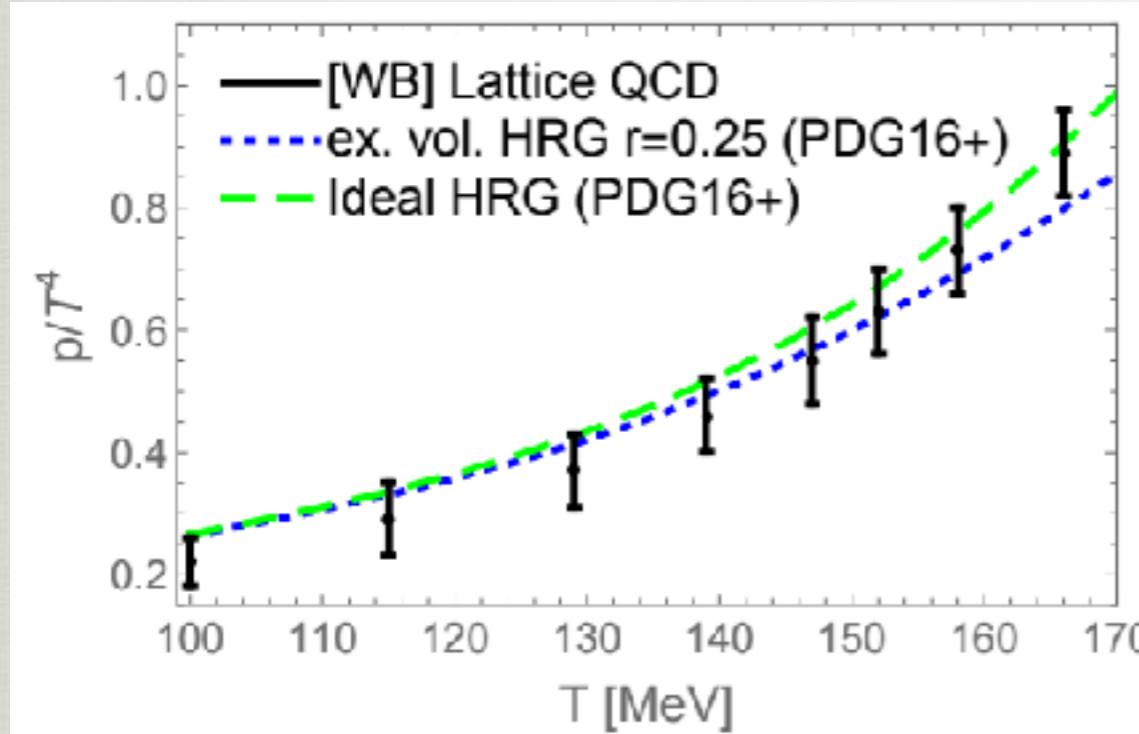
See also Pandya & Pretorius 2104.00804

Most influential post-merger

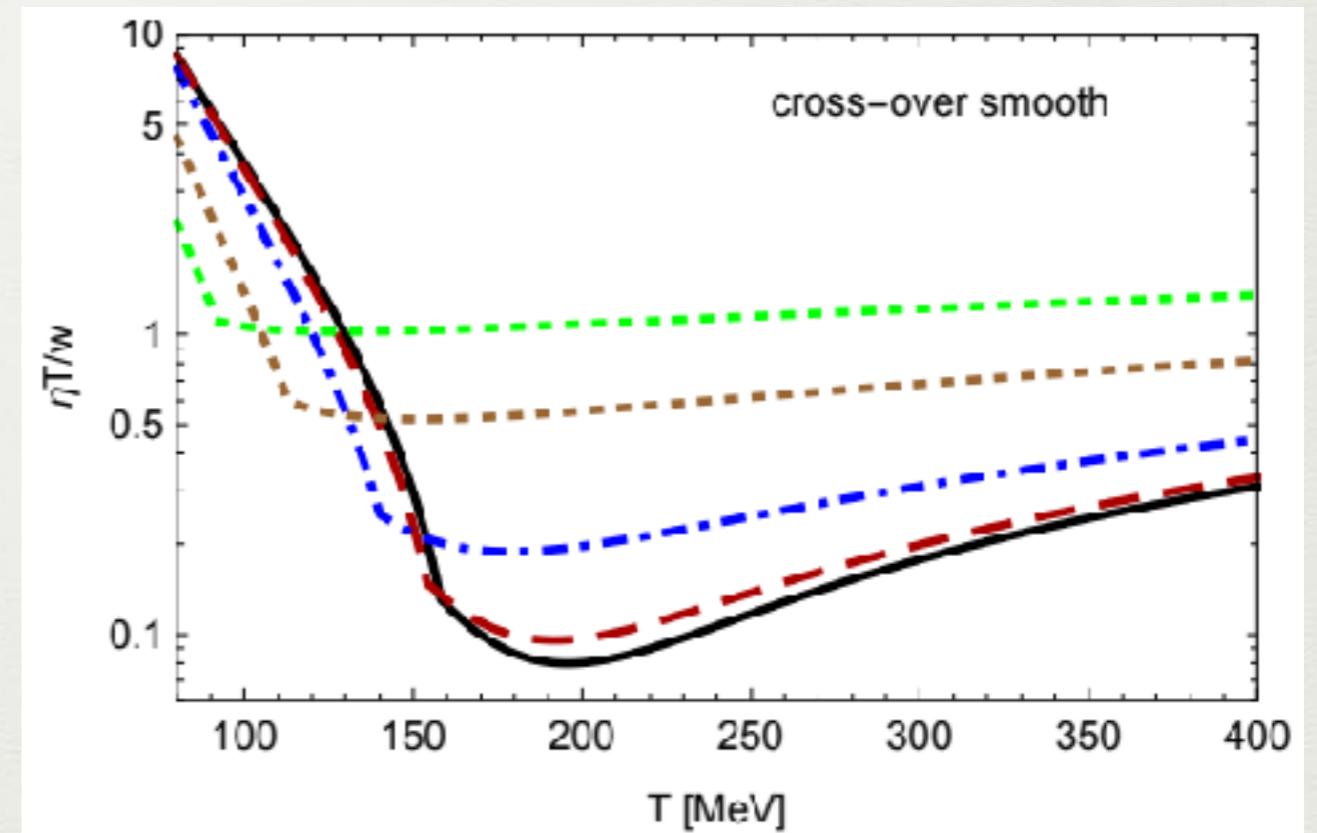
# Testable shear viscosity $\eta T/w(T, \mu_B)$

McLaughlin, JNH, et al, [2103.02090](#)

Constrain the Excluded volume from Lattice QCD



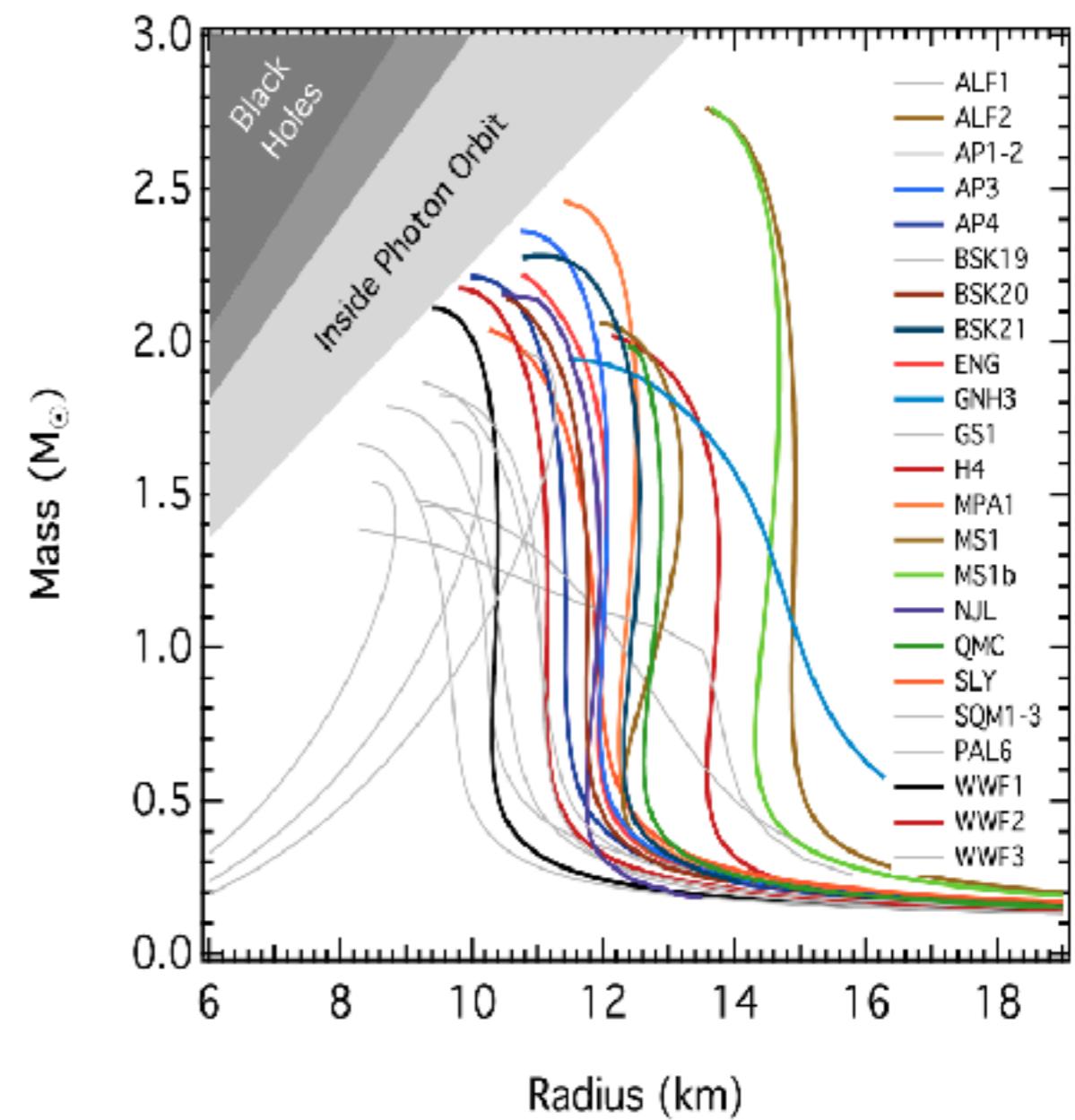
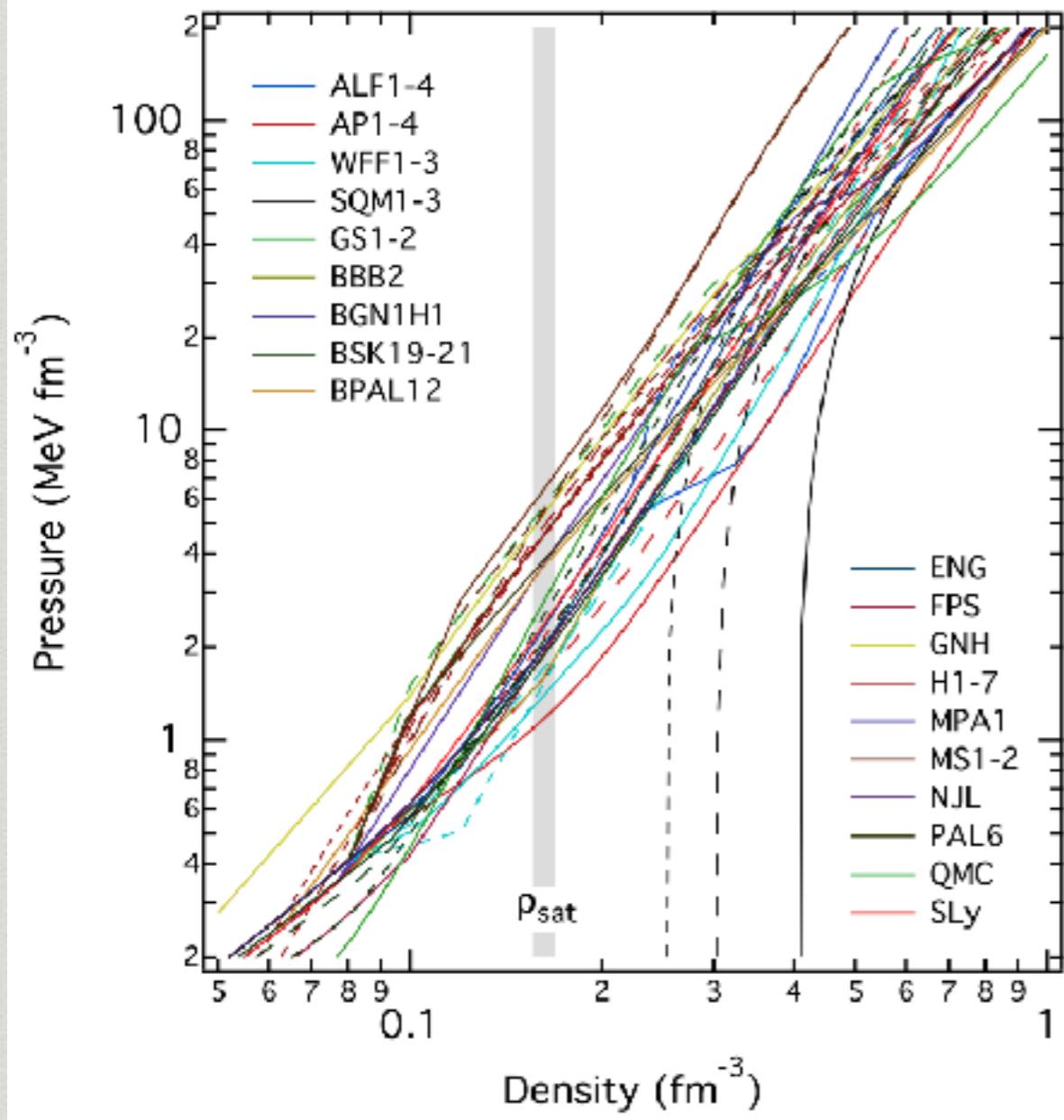
Matched to parameterized QGP shear using a variety of methods



Include critical point (different locations) and  
sharp vs. smooth cross-over

# EOS at T=0 connected to Mass vs. Radius

Plug EOS into Tolman–Oppenheimer–Volkoff (TOV) equation





Nico Yunes  
ICASU director

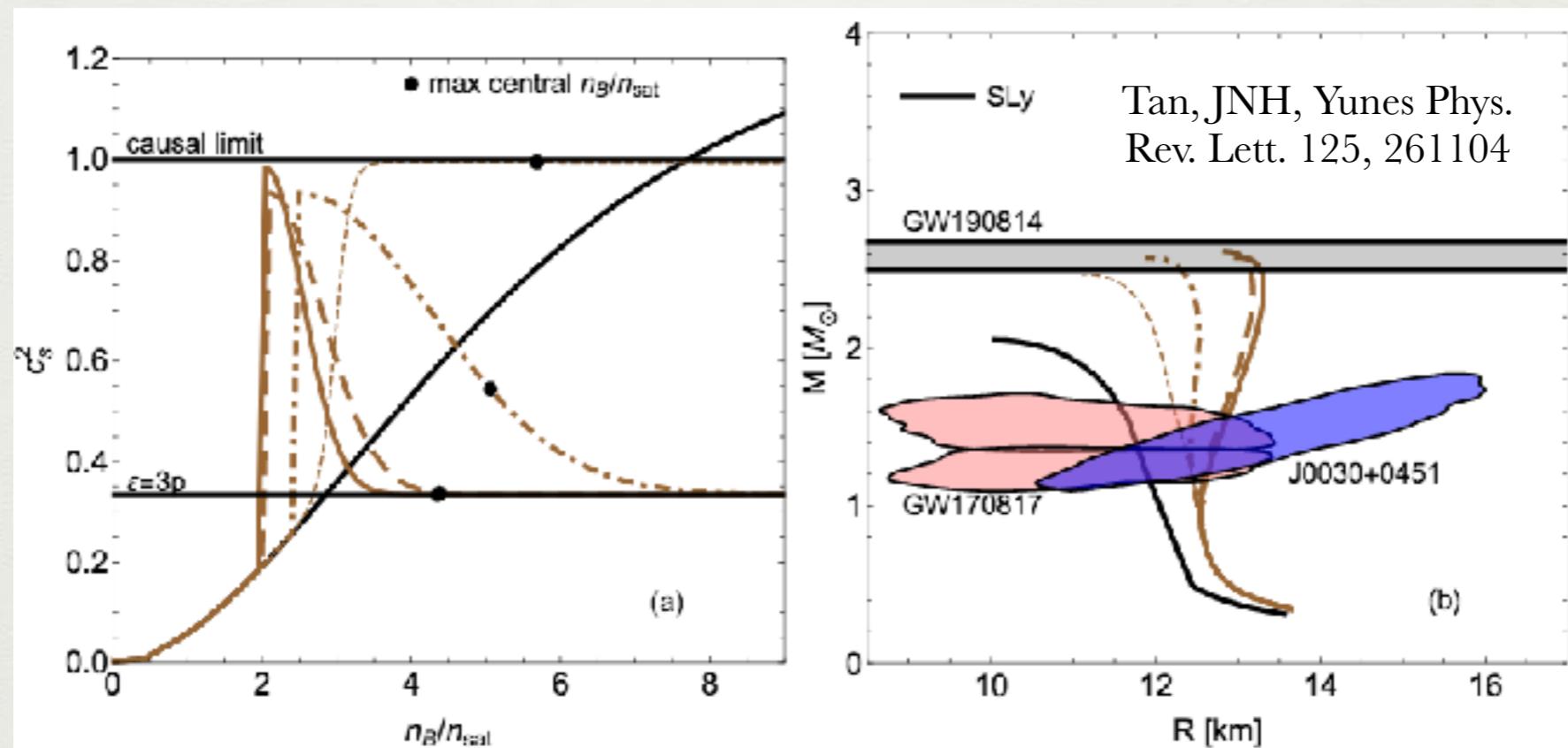
Hung Tan  
UIUC PhD Student

+Dore and Dexheimer (to appear soon)

# Heavy neutron stars

GW190814 [LIGO, Virgo], *Astrophys. J.* 896, L44 (2020)

Compact object of  $M \geq 2.5M_{\odot}$  measured, is it a neutron star?



Bump in  $c_s^2 > 1/3$ ?

Bedaque & Steiner, *Phys. Rev. Lett.* 114 (2015) 3, 031103; Alford et al, *Phys. Rev. D* 92, 083002 (2015); Ranea-Sandoval, et al, *Phys. Rev. C* 93, 045812; I. Tews, et al, *Phys. Rev. C* 98, 045804 (2018); McLerran & Reddy, *Phys. Rev. Lett.* 122, 122701 (2019); Dexheimer & Schramm, *Phys. Rev. C* 81, 045201 (2010); Jakobus, et al, arXiv:2004.07026 [nucl-th];

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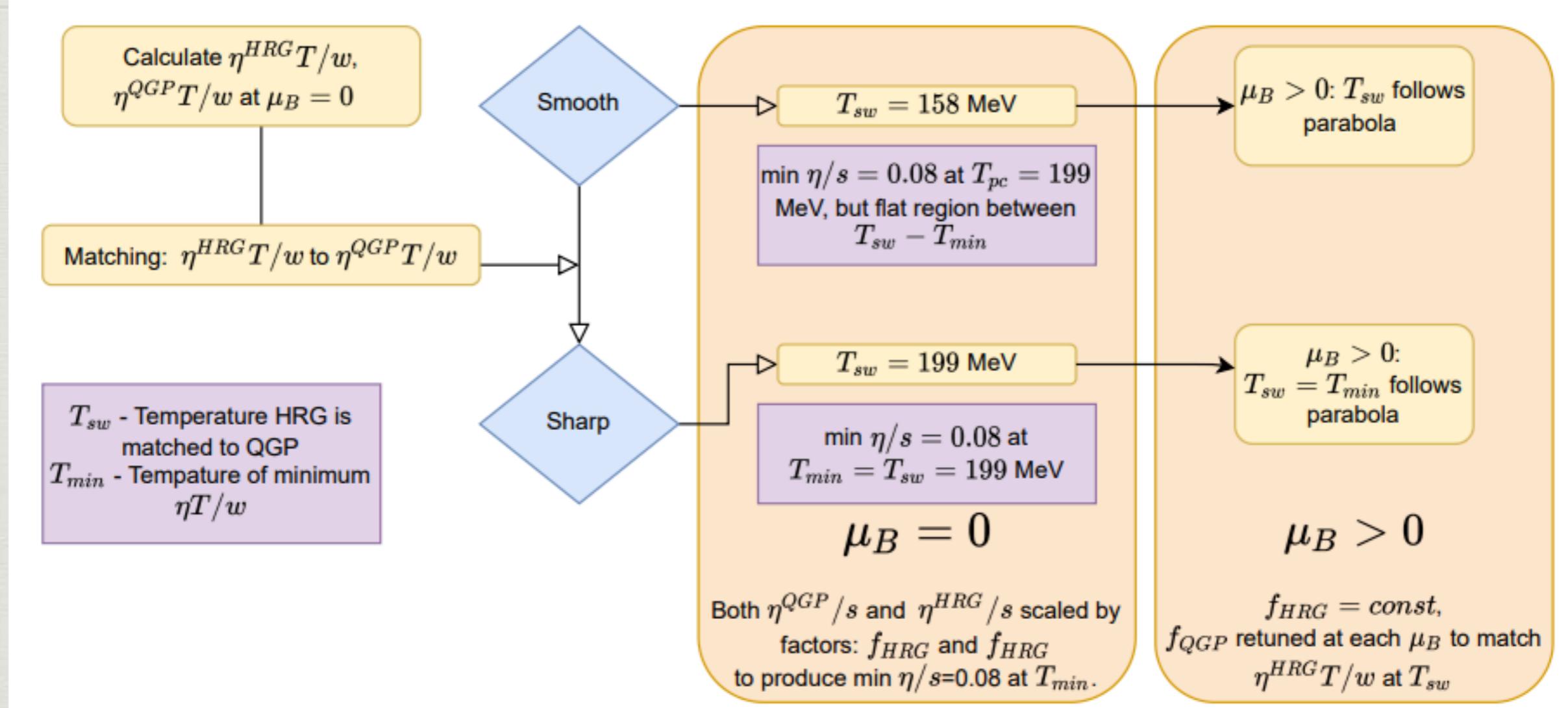
J. Noronha-Ho, Baym et al, *Astro-phys. J.* 885, 42 (2019)

Alternatives: spin/new strange interactions

Zhang & Li *Astrophys. J.* 902 (2020) 1, 38; Dexheimer et al, *Phys. Rev. C* 103 (2021) 2, 025808; Tsokarus et al, *Astrophys. J.* 905 (2020) 1, 48; Tews et al, *Astrophys. J. Lett.* 908 (2021) 1

# Testable shear viscosity $\eta T/w(T, \mu_B)$

McLaughlin et al, 2103.02090



Excluded volume in HRG

$$\eta^{HRG} = \frac{5}{64\sqrt{8}} \frac{1}{r^2} \frac{1}{n} \sum_i n_i \frac{\int_0^\infty p^3 e^{-\sqrt{p^2+m_i^2}/T+\tilde{\mu}_i} dp}{\int_0^\infty p^2 e^{-\sqrt{p^2+m_i^2}/T+\tilde{\mu}_i} dp}$$

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Parameterized QGP phase

$$\eta^{QGP}/s = f \left[ \frac{a}{\alpha_{s,HQ}^\gamma (cT/T_{YM})} + \frac{b}{(T/T_{YM})^\delta} \right]$$

# Linear vs. Cubic response

Linear response

$$V_n^{pred} = \gamma_n \mathcal{E}_n$$

Teaney,Yan,PRC83(2011)064904;Gardim,et al,PRC85(2012)024908;PRC91(2015)3,034902

Linear+cubic response

$$V_n^{pred} = \kappa_{1,n} \mathcal{E}_n + \kappa_{2,n} |\mathcal{E}_n|^2 \mathcal{E}_n$$

JNH,Yan,Gardim,Ollitrault Phys. Rev. C 93, 014909 (2016)

