Quantum baryon number fluctuations in subsystems of a hot relativistic fermionic gas

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Based on: arXiv:2105.02125

26-31 July 2021
HADRON2021
Virtual

NARODOWE CENTRUM NAUKI

Supported by IFJPAN and the NCN Grants No. 2016/23/B/ST2/00717 and No. 2018/30/E/ST2/00432

Motivation:

The Joint Institute for Computational Fundamental Science

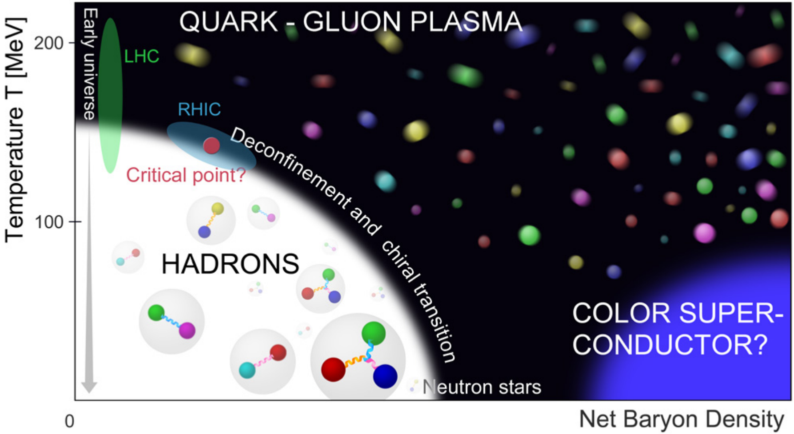


Figure: Theoretical diagram based on Lattice QCD simulations depicting expected quark-gluon phase transition.

Motivation:

Fluctuations of various physical quantities play a very important role in all fields of physics, as they reveal the information about

- possible phase transitions
- formation of structures in the Early Universe
- dissipative phenomena

Most common fluctuations we deal with are those arising from quantum uncertainty relation or those present in thermodynamic systems.

Motivation:

- We discuss fluctuations of the baryon number density in a hot and dense relativistic gas of fermions.
- Our analysis is relevant for relativistic heavy-ion physics, in particular, in the context of the beam energy scan (BES).
- Hunt for the conjectured critical endpoint in the QCD phase diagram has triggered vast theoretical and experimental studies of many fluctuation observables.
- Study of fluctuations of baryon number might provide an excellent opportunity to study the critical phenomena.

Basic concepts and definitions:

- We consider the fluctuation of the baryon number in the subsystem S_a of the thermodynamic system S_V described by the grand canonical ensemble characterized by the temperature (T) and the baryon chemical potential (μ).

- The volume V of the larger system S_V is larger than the characteristic volume of the subsystem S_a.

- We derive a compact formula that defines quantum fluctuations of the baryon number operator in subsystems of a hot and dense relativistic gas.

- Then we apply this formula to get physical insights into situations expected in relativistic heavy-ion collisions.

Basic concepts and definitions:

A quantum field operator for spin-$\frac{1}{2}$ particle has the standard form:

$$\psi(t,\boldsymbol{x}) = \sum_r \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\boldsymbol{k}}}} \Big(U_r(\boldsymbol{k}) a_r(\boldsymbol{k}) e^{-ik\cdot x} + V_r(\boldsymbol{k}) b_r^\dagger(\boldsymbol{k}) e^{ik\cdot x} \Big),$$

where $a_r(\boldsymbol{k})$ and $b_r^\dagger(\boldsymbol{k})$ are annihilation and creation operators for particles and antiparticles, respectively, satisfying the canonical commutation relations $\{a_r(\boldsymbol{k}), a_s^\dagger(\boldsymbol{k}')\} = (2\pi)^3\delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$ and $\{b_r(\boldsymbol{k}), b_s^\dagger(\boldsymbol{k}')\} = (2\pi)^3\delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$, whereas $\omega_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2+m^2}$ is the energy of a particle.

Basic concepts and definitions:

To perform thermal averaging, it is sufficient to know the expectation values of the products of two and four creation and/or annihilation operators

$$\langle a_r^\dagger(\boldsymbol{k}) a_s(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') f(\omega_{\boldsymbol{k}}),$$

$$\langle a_r^\dagger(\boldsymbol{k}) a_s^\dagger(\boldsymbol{k}') a_{r'}(\boldsymbol{p}) a_{s'}(\boldsymbol{p}')\rangle = (2\pi)^6 \Big(\delta_{rs'}\delta_{r's}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}')\ \delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p})$$
$$-\delta_{rr'}\delta_{ss'}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p})\ \delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}')\Big) f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'}).$$

Here $f(\omega_{\boldsymbol{k}}) = 1/(\exp(\beta(\omega_{\boldsymbol{k}}-\mu))+1)$ is the Fermi–Dirac distribution function for particles. For antiparticles, the Fermi–Dirac distribution function differs by the sign of the baryon chemical potential μ, i.e. $\bar{f}(\omega_{\boldsymbol{k}}) = 1/(\exp(\beta(\omega_{\boldsymbol{k}}+\mu))+1)$.

Basic concepts and definitions:

We define the baryon number density operator $\hat{J}^0_a$, associated with the conserved baryon current in a subsystem S_a using a smooth Gaussian profile placed at the origin of the coordinate system

$$\hat{J}^0_a = \frac{1}{(a\sqrt{\pi})^3} \int d^3\boldsymbol{x}\; \hat{J}^0(x)\; \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right)$$

where $\hat{J}^0 = \psi^\dagger \psi$.

To determine the baryon number fluctuation of the subsystem S_a, we consider the variance

$$\boxed{\sigma^2(a, m, T, \mu) = \langle : \hat{J}^0_a :: \hat{J}^0_a : \rangle - \langle : \hat{J}^0_a : \rangle^2}$$

and the normalized standard deviation as

$$\boxed{\sigma_n(a, m, T, \mu) = \frac{\left(\langle : \hat{J}^0_a :: \hat{J}^0_a : \rangle - \langle : \hat{J}^0_a : \rangle^2\right)^{1/2}}{\langle : \hat{J}^0_a : \rangle}}$$

Mean value for baryon number density operator:

Using the thermal averaging of two creation and/or annihilation operators, the thermal expectation value of $:\hat{J}^0_a:$ has the form

$$\langle :\hat{J}^0_a: \rangle = 2 \int dK \left[f(\omega_{\boldsymbol{k}}) - \bar{f}(\omega_{\boldsymbol{k}}) \right]$$

This expression agrees with the standard kinetic-theory definition, with the factor of 2 accounting for the spin degeneracy.

Quantum fluctuation expression:

$$
\begin{aligned}
\sigma^2(a, m, T, \mu) &= \langle : \hat{J}^0_a :: \hat{J}^0_a : \rangle - \langle : \hat{J}^0_a : \rangle^2 \\
&= \int \frac{dK}{\omega_{\boldsymbol{k}}} \frac{dK'}{\omega_{\boldsymbol{k}'}} (\omega_{\boldsymbol{k}} \omega_{\boldsymbol{k}'} + \boldsymbol{k} \cdot \boldsymbol{k}' + m^2) e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} \times \\
&\qquad \left[f(\omega_{\boldsymbol{k}}) \left(1 - f(\omega_{\boldsymbol{k}'})\right) + \bar{f}(\omega_{\boldsymbol{k}})(1 - \bar{f}(\omega_{\boldsymbol{k}'})) \right] \\
&- \int \frac{dK}{\omega_{\boldsymbol{k}}} \frac{dK'}{\omega_{\boldsymbol{k}'}} (\omega_{\boldsymbol{k}} \omega_{\boldsymbol{k}'} + \boldsymbol{k} \cdot \boldsymbol{k}' - m^2) e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2} \times \\
&\qquad \left[f(\omega_{\boldsymbol{k}})(1 - \bar{f}(\omega_{\boldsymbol{k}'})) + \bar{f}(\omega_{\boldsymbol{k}})(1 - f(\omega_{\boldsymbol{k}'})) \right]
\end{aligned}
$$

Variation of normalized baryon density fluctuation:

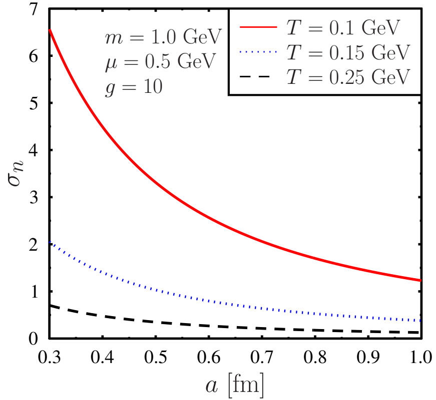


Figure: Variation of normalized fluctuation σ_n in the subsystem S_a with the scale a for different values of the temperature T and fixed particle mass $m = 1.0$ GeV and baryon chemical potential $\mu = 0.5$ GeV.

Variation of normalized baryon density fluctuation:

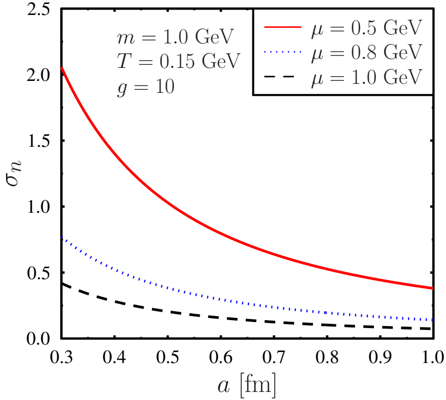


Figure: Variation of normalized fluctuation σ_n in the subsystem S_a with the scale a for different values of the baryon chemical potential μ and fixed particle mass $m = 1.0$ GeV and temperature $T = 0.15$ GeV.

Variation of normalized baryon density fluctuation:

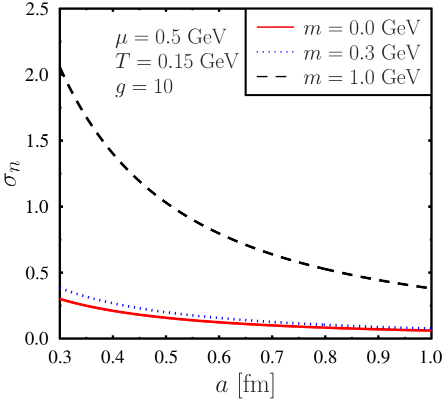


Figure: Variation of normalized fluctuation σ_n in the subsystem S_a with the scale a for different values of the particle mass and fixed temperature $T = 0.15$ GeV and baryon chemical potential $\mu = 0.5$ GeV.

Variation of normalized baryon fluctuation:

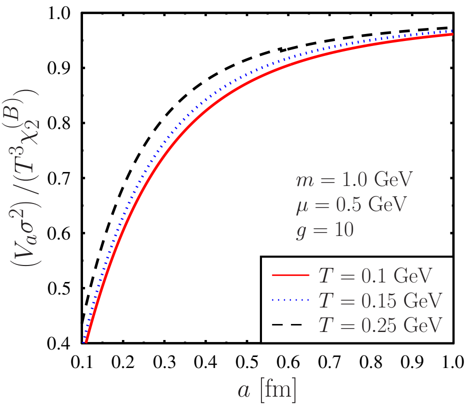


Figure: Variation of normalized fluctuation $V_a\sigma^2/(T^3\chi_2^{(B)})$ for different values of temperature (T) but with fixed baryon chemical potential (μ) and particle mass (m).

Variation of normalized baryon fluctuation:

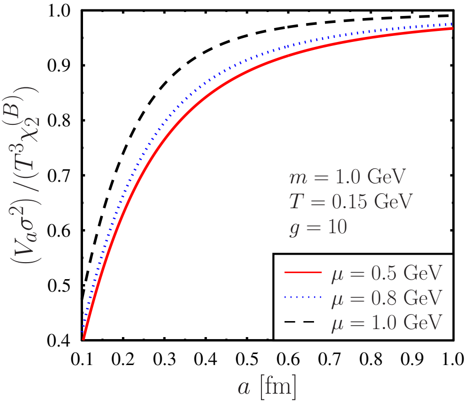


Figure: Variation of normalized fluctuation $V_a\sigma^2/(T^3\chi_2^{(B)})$ for different values of baryon chemical potential (μ) but with fixed temperature (T) and particle mass (m).

Variation of normalized baryon fluctuation:

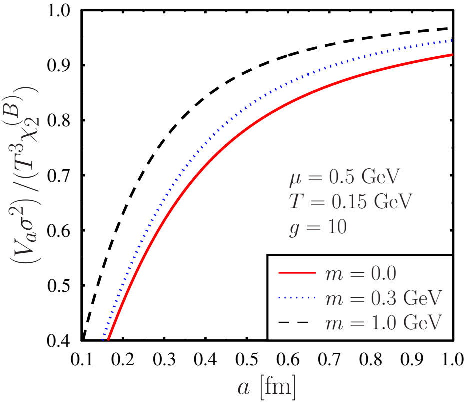
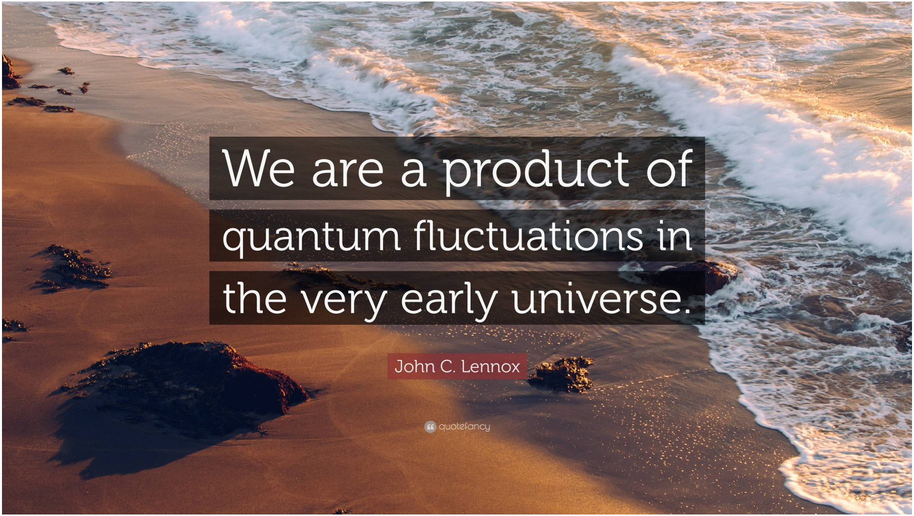


Figure: Variation of normalized fluctuation $V_a\sigma^2/(T^3\chi_2^{(B)})$ for different values of particle mass (m) but with fixed temperature (T) and baryon chemical potential (μ).

Summary:

- We have analyzed quantum baryon-number fluctuations in subsystems of a hot and dense relativistic gas of fermions
- And found that they diverge for small system sizes.
- Our results agree with the results known from statistical physics for sufficiently large system size a.
- In this way, we have delivered a useful formula that accounts for both statistical and quantum features of the fluctuations.
- The numerical results obtained here can be useful to interpret and shed new light on the heavy-ion experimental data.



Thank you for your attention!