Color-Flavor Dependent NJL Model and QCD Phase Diagram

By

Aftab Ahmad

In collaboration with

Ali Murad

Institute of Physics, Gomal University, D.I.Khan, Pakistan

19th International Conference on Hadron Spectroscopy and Structure , July 26-31, 2021, Mexico

Outline

- Motivation and Objective
- QCD Lagrangian and NJL Model
- Schwinger-Dyson's Equations (SDE)
- Color-flavor dependence of NJL Model and QCD Phase diagram
- Results and Discussions

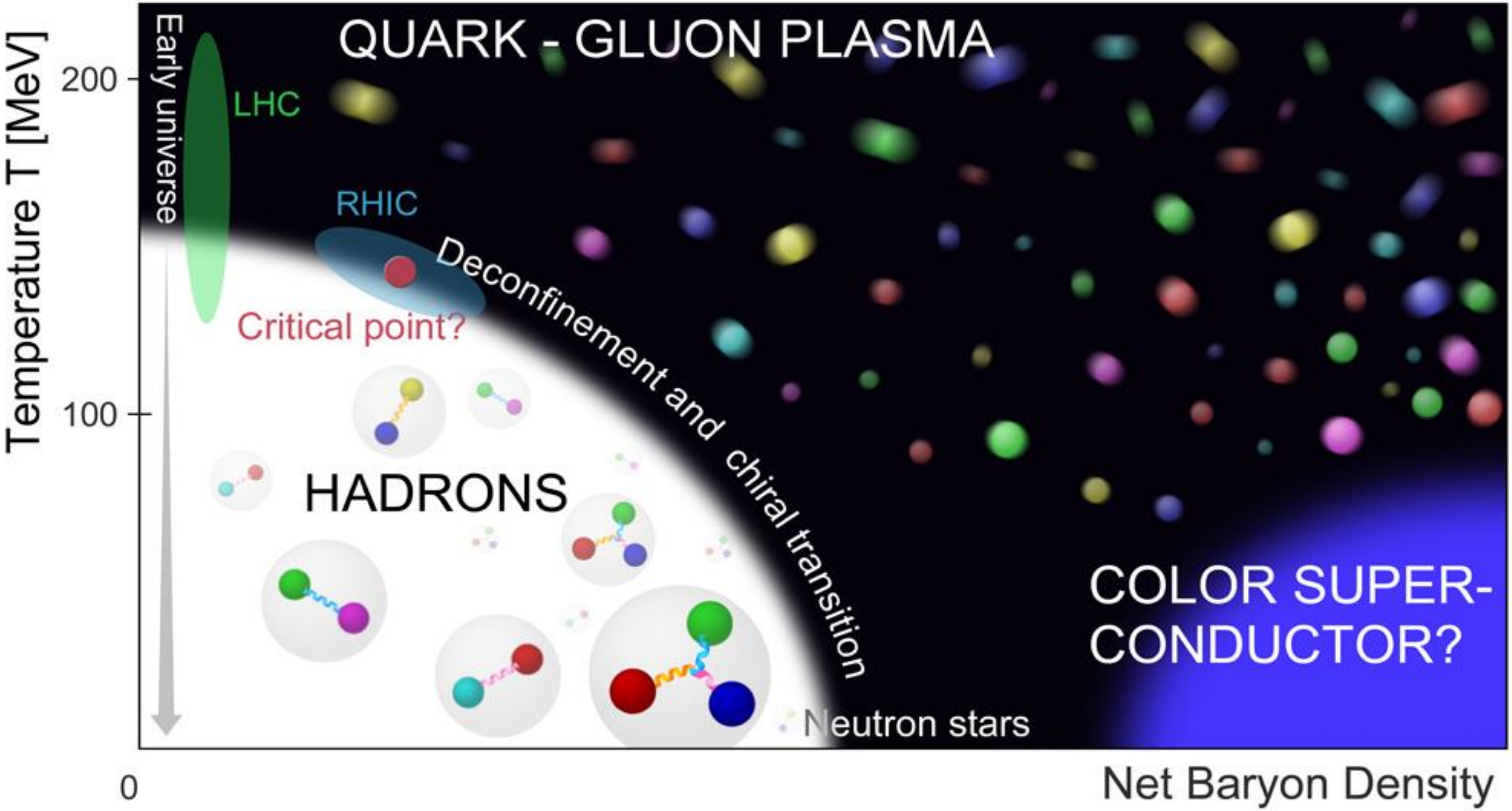
Motivation and Objective

- Dynamical chiral symmetry breaking and/or restoration in the NJL model for higher number of light quark flavors and colors.

- What are the critical number of flavors and colors in the NJL model for chiral-symmetry-breaking and its restoration at finite temperature and density?

- The impact of higher number of colors and flavors on the critical end point in the QCD Phase diagram

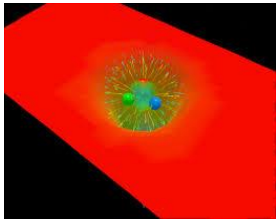
Cartoon Picture of QCD Phase Diagram



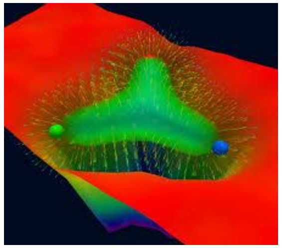
Guido Cossu, KEK, Tsukuba

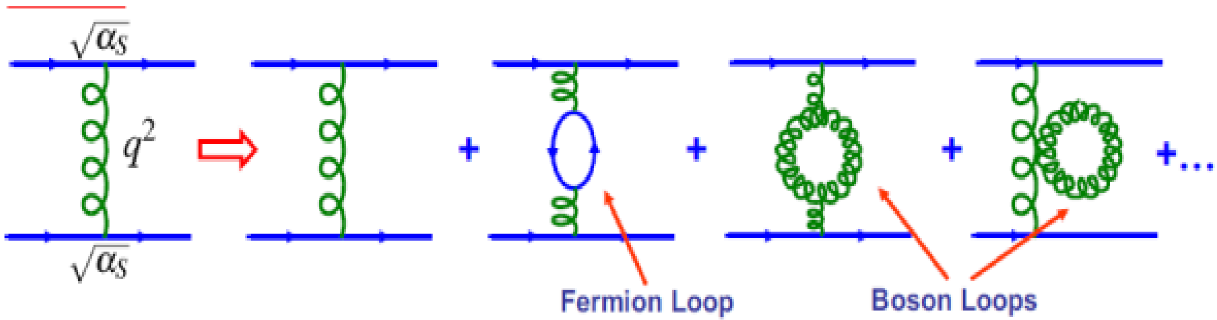
Introduction to QCD

- ***Weakly interacting* at high energies (> a few GeV)**
  - **Asymptotic freedom**
  - **Accessible by perturbation theory**



- ***Strongly interacting* at low energies**
  - **Confinement**
  - **Perturbation theory fails**
  - **Genuine non-perturbative effects (SDE, Lattice, Effective models etc.)**





[QCD] [QED]

Introduction to QCD
Lagrangian

- The QCD is defined by the Lagrangian

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma^{\mu}\mathrm{D}_{\mu} - \hat{m})q - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a} \qquad \text{(i)}$$

[Buballa,2005]

Where D_{μ} is the covariant derivative,

$$\mathrm{D}_{\mu} = \partial_{\mu} - ig\,\frac{\lambda^{a}}{2}A^{a}_{\mu} \qquad \text{(ii)}$$

$G^{a}_{\mu\nu}$ is the gluon field strength,

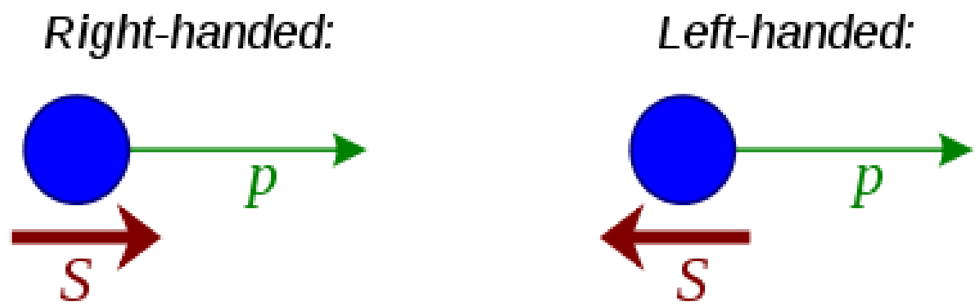
$$G^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - A^{a}_{\mu}\partial_{\nu} + gf^{abc}A^{b}_{\mu}A^{c}_{\nu} \qquad \text{(iii)}$$

- A^{a}_{μ} : gluon field with (a: color indices) and (μ: four vector indices)
- λ^{a} and f^{abc}: SU(3) Gell-Mann matrices and structure constant
- g : is the QCD coupling strength
- γ^{μ}: is the Dirac gamma matrix
- $\hat{m}$: diag(m_u, m_d,….) is mass matrix in flavor space
- q: is the quark field with six flavor (u, d, s, c, b, t)

Chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$

Introducing the left-and right-quarks fields through the projectors

$$\Psi_R = \frac{(1+\gamma^5)}{2}\Psi, \qquad \Psi_L = \frac{(1-\gamma^5)}{2}\Psi,$$



The Dirac Part of the QCD Lagrangian can be decomposed as

$$\mathcal{L}_{QCD} = \bar{\Psi}_L(i\gamma\mu D_\mu)\Psi_L + (\bar{\Psi}_R\, i\gamma\mu D_\mu)\Psi_R - \bar{q}_{L(m)}q_R - \bar{q}_{R(m)}q_L$$

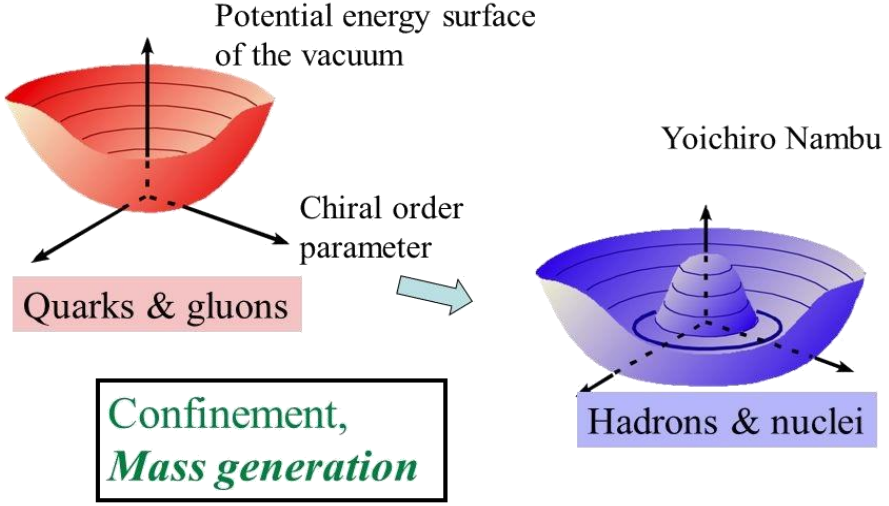
For massless quark the right and left handed components decouple and so the Lagrangian has a chiral symmetry.

Introduction to QCD

Spontaneous chiral symmetry breaking

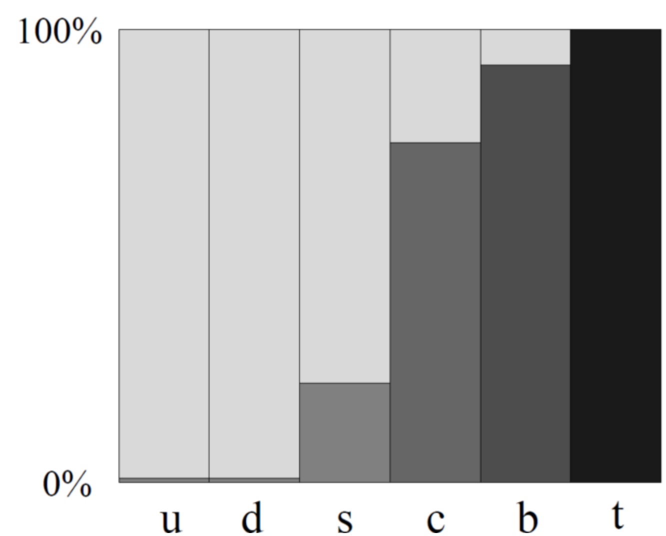
- If the QCD Lagrangian has chiral symmetry, but the vacuum does not satisfy this symmetry, then one says that the symmetry has been broken spontaneously.
- Spontaneous symmetry breaking is related to existence of non-vanishing quark condensate $\langle \bar{q}q \rangle$, which acts as an order parameter for spontaneous symmetry breaking.

Spontaneous breaking of ***chiral (χ) symmetry***



[web2.ph.utexas.ed

Chiral Symmetry Vs Higgs



[P. Braun-Munzinger, J. Wambach, arXiv:0801.4256]

Dynamical mass generation in the NJL Model

We Start with QCD in the effective manner through Nambu-Jona-Lasinio (NJL) type interaction from the Lagrangian

Yoichiro Nambu,
Nobel prize 2008

$$\mathcal{L} = \bar{\psi}(i \not\partial - m)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$

Four-Fermion interaction → { Scalar interaction, Axil-vector interaction }

- Schwinger's-Dyson's-equations and QCD gap equation

$$S^{-1}(p) = S_0^{-1}(p) - \Sigma(p)$$

Bare quark propagator: $$iS_0(p) = i\frac{\not{p} + mI}{p^2 - m^2 + i\epsilon}$$

Dressed quark Propagator: $$iS(p) = i\frac{\not{p} + MI}{p^2 - M^2 + i\epsilon}$$

Self-Energy: $$\Sigma(p) = \int \frac{d^4k}{(2\pi)^4} g^2 \Delta_{\mu\nu}(p-k)\frac{\lambda^a}{2}\gamma_\mu S_f(k)\frac{\lambda^a}{2}\Gamma_\nu(p,k)$$

In NJL-Model: $$g^2\Delta_{\mu\nu} = G\delta_{\mu\nu}$$

In $SU(N_c)$ representatior

$$\sum_{a=1}^{8}\frac{\lambda^a}{2}\frac{\lambda^a}{2} = \frac{1}{2}\left(N_c - \frac{1}{N_c}\right)$$

NJL- Model Gap Equation

NJL- Model Gap Equation:

$$M = m + 8i\mathcal{G}^{N_c}(N_f) \int \frac{d^4k}{(2\pi)^4} \frac{M}{k^2 - M^2 + i\epsilon}$$

Quark-antiquark Condensate:

$$\langle \bar{q}q \rangle = -i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\, S(p) \quad \text{or} \quad -\langle \bar{q}q \rangle = \frac{M - m}{2\mathcal{G}^{N_c}(N_f)}$$

Finite Temperature and Chemical Potential:

$$\int \frac{d^4k}{i(2\pi)^4} f(k_0, \mathbf{k}) \to T \sum_n \int \frac{d^3k}{(2\pi)^3} f(i\omega_n + \mu, \mathbf{k}) \;, \; \omega_n = (2n+1)\pi T$$

$$M = m + 4\mathcal{G}^{N_c}(N_f) M \int_0^\Lambda \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{E_k} (1 - n_F(T,\mu) - \bar{n}_F(T,\mu))$$

$$n_F(T,\mu) = \frac{1}{e^{(E_k - \mu)/T} + 1} \;, \quad \bar{n}_F(T,\mu) = \frac{1}{e^{(E_k + \mu)/T} + 1}$$

Modelling of the effective NJL Coupling

- For color

$$\mathcal{G}^{N_c}(N_f) = [\frac{1}{2}\left(N_c - \frac{1}{N_c}\right)]G(N_f)$$

- For flavor

$$G(N_f) \longrightarrow \frac{9}{2}G\sqrt{1 - \frac{(N_f - 2)}{N_f^c}}$$

Mass should have this kind of relation with flavor.

$$M \backsim \sqrt{N_f^c - N_f}$$

[Bashir et al. , PRD 88, 054003 (2013)], [Ahmad et al , JPG 48 (2021) 7, 075002]

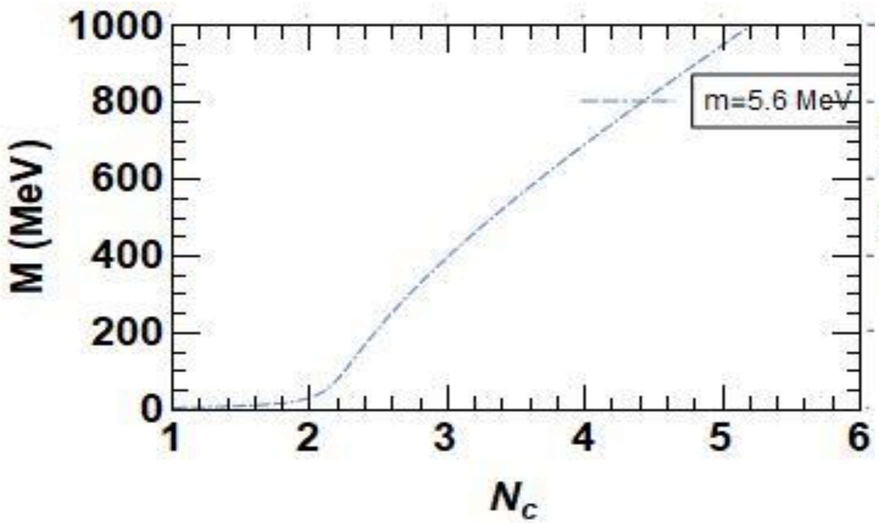
For fixed $Nc = 3$, $N_f = 2$, $G = 2.44/\Lambda^2$, $\Lambda = 587.9$ and m = 5.6 MeV , we get

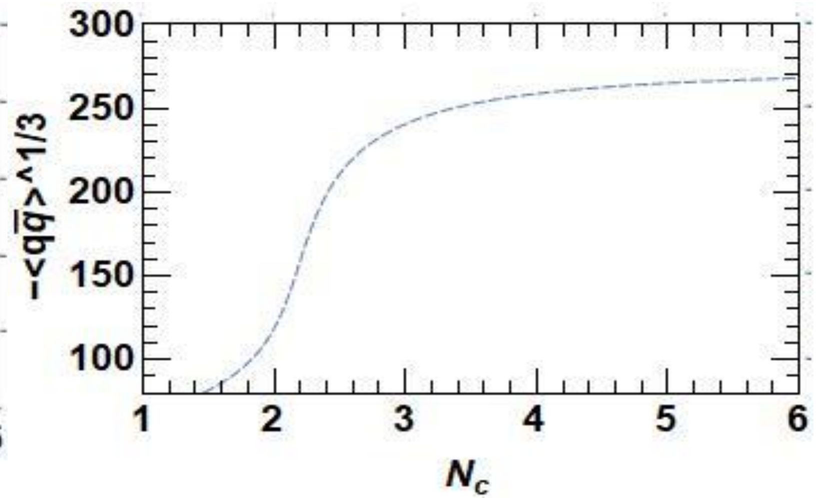
M = 399.44 MeV , $\langle\bar{q}q\rangle$ = $-(240.8MeV)^3$ and the pion decay constant $f_\pi = 92.4\text{MeV}$

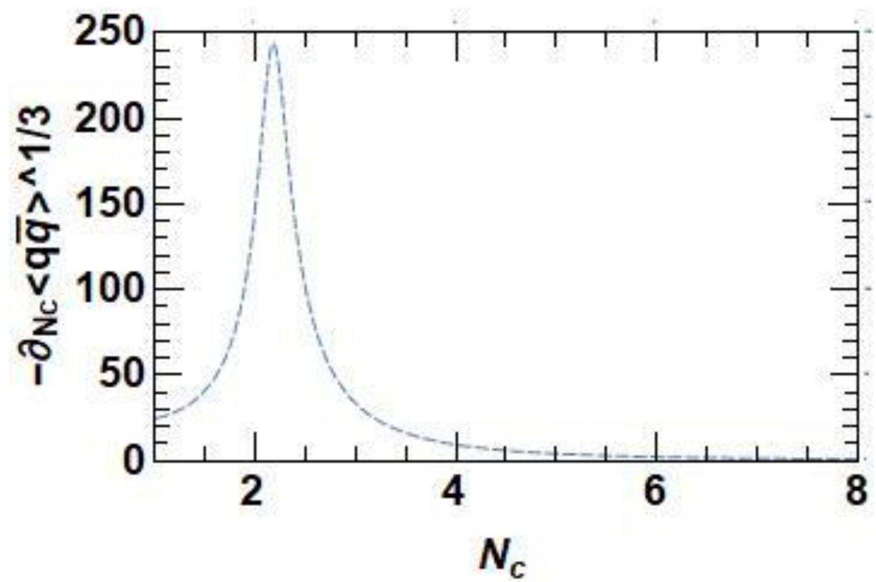
[Buballa, *Phys.Rept.* 407 (2005) 205-376]

Numerical Results

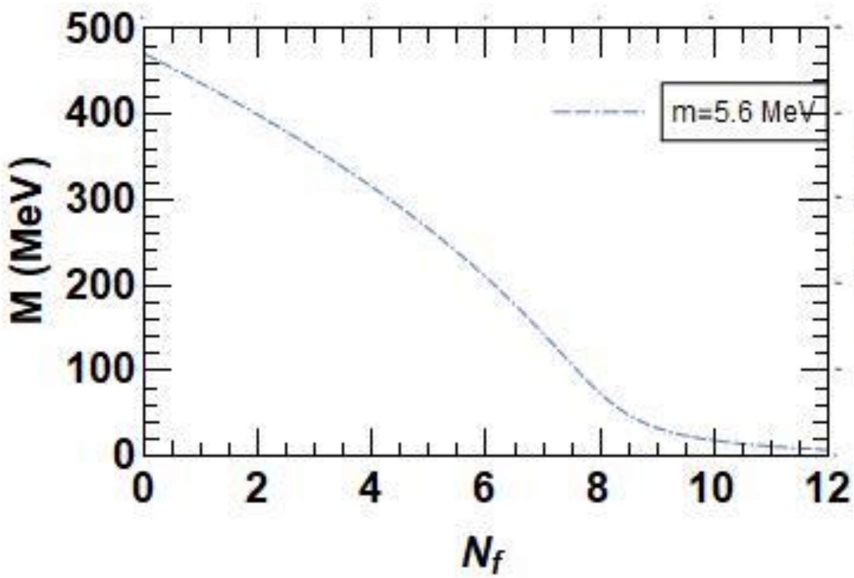
Chiral Symmetry Breaking for Higher Number of Color and for Nf=2

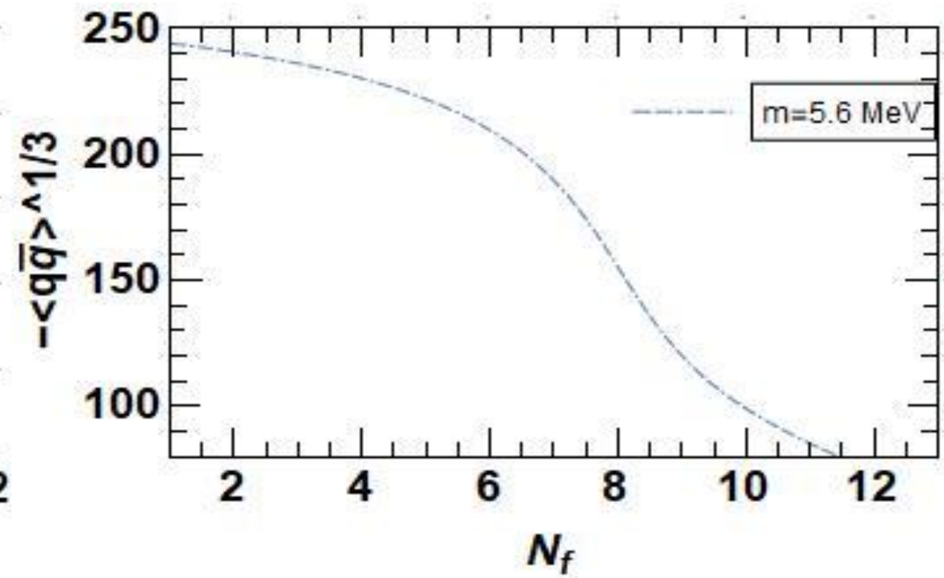


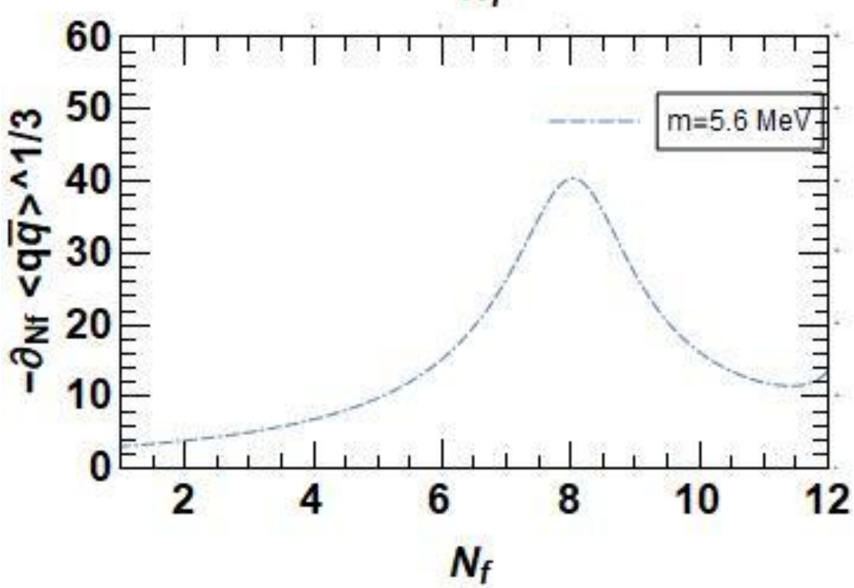




Chiral Symmetry Breaking for Higher Number of flavors with fixed Nc=3



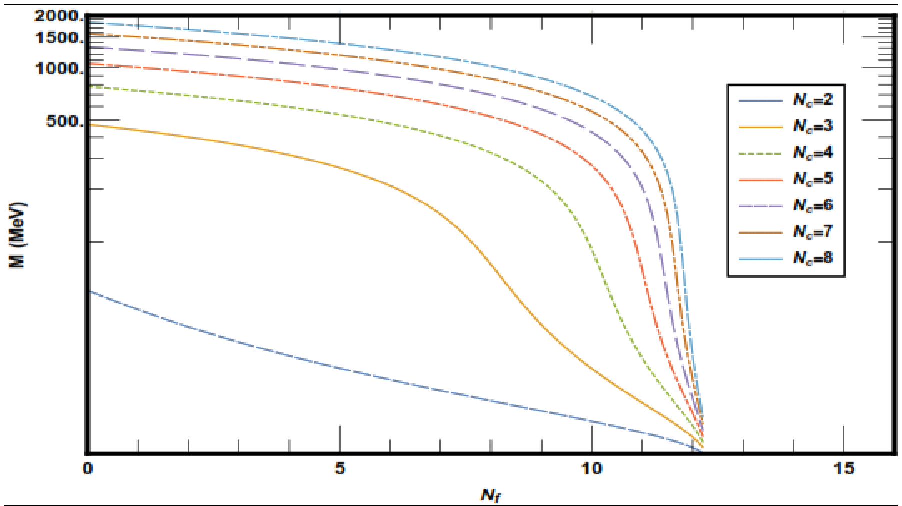


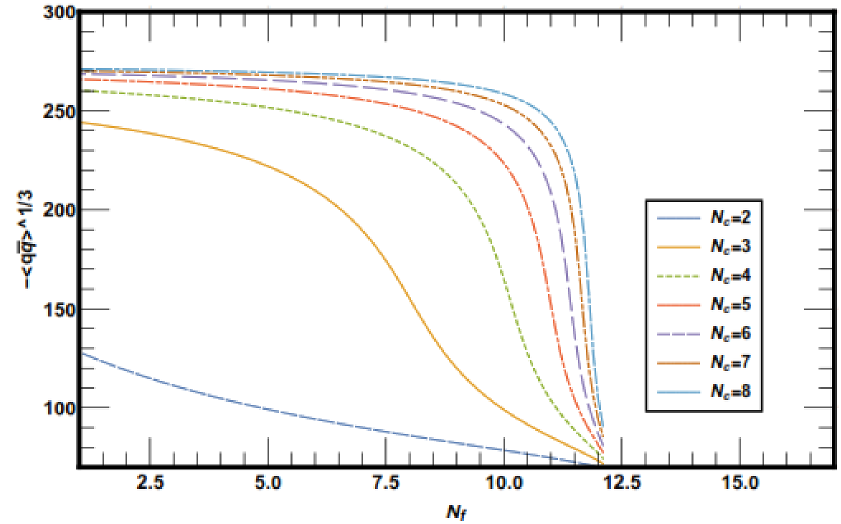


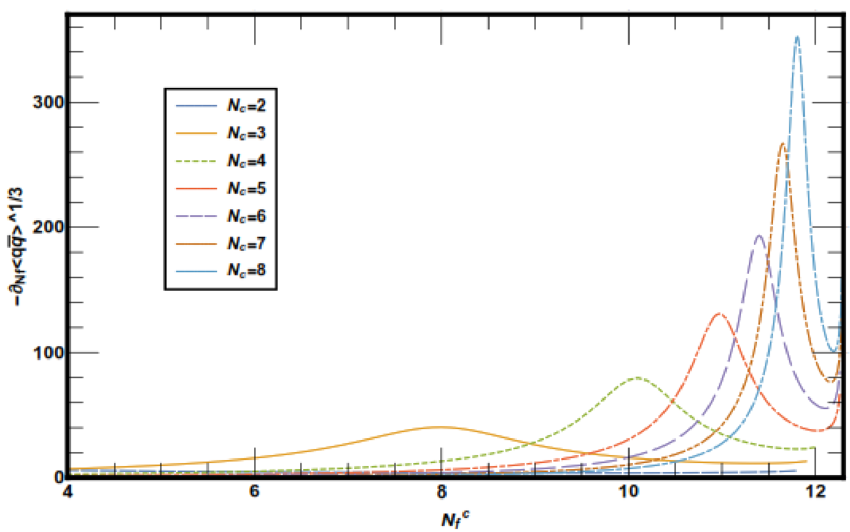
N_c	N_f	**Method**	**Reference**
3	8 ± 1	SDE	Bashir et al.,PRD88,054003(2013)
3	≤ 8	Lattice QCD	Appelquist et al.,PRL100:171607(2008)
3	10 ± 0.29	RGF Eqn's	Gies et al., EPJC46,433(2006)
3	8	CI-model	Ahmad et al.,JPG48,075002(2021)
3	8	NJL-model	Our results

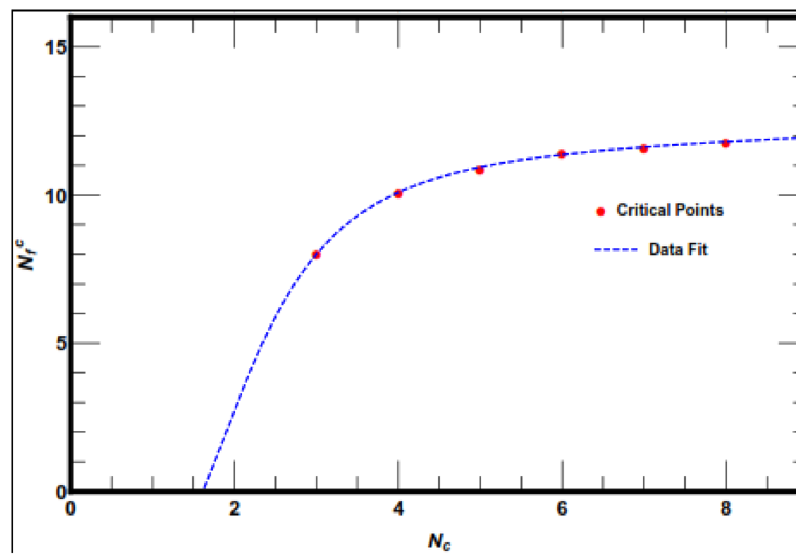
NUMERICAL RESULTS

Critical Number of Flavors for Chiral Symmetry Restoration With Various Colors





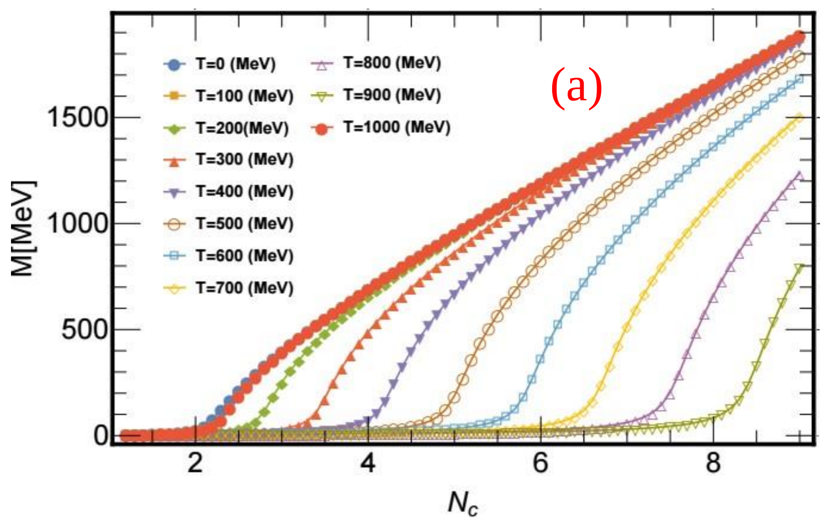


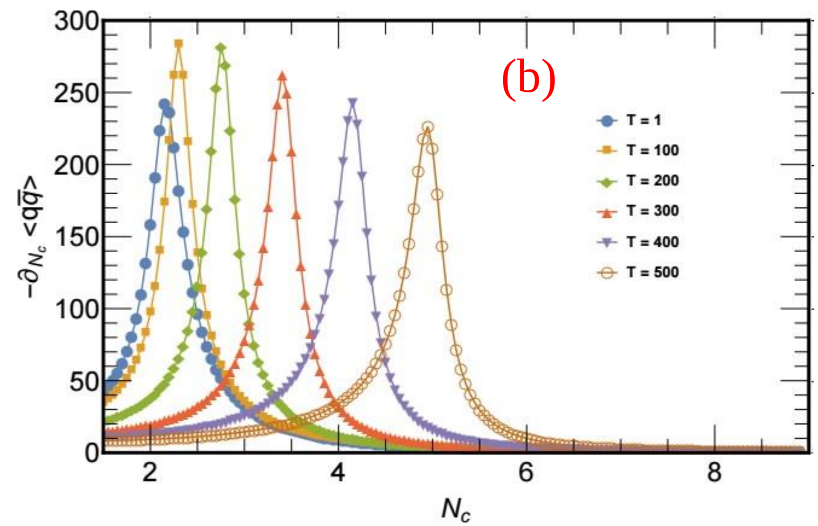


NUMERICAL RESULTS

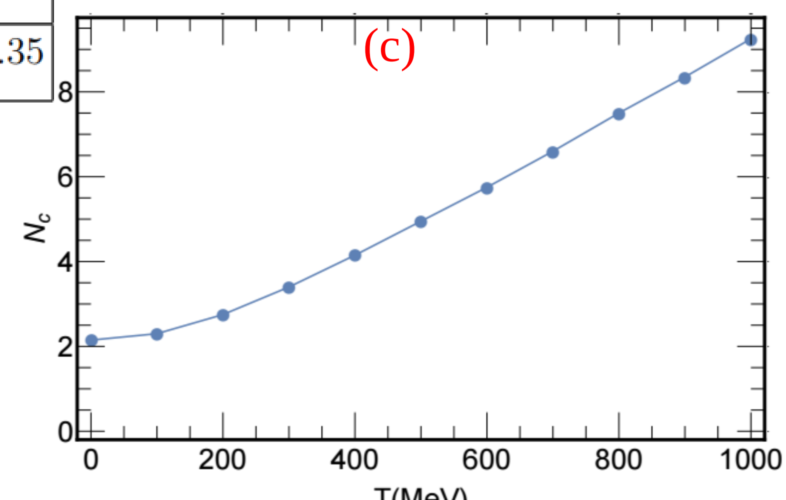
Critical Number of colors for chiral symmetry breaking at $T \neq 0$ but $\mu = 0$

- For fixed 2- flavors, with bare mass m = 5.6 MeV





T (MeV)	0	100	200	300	400	500	600	700	800	900
Critical N_c	2.15	2.3	2.75	3.4	4.15	4.95	5.75	6.6	7.5	8.35

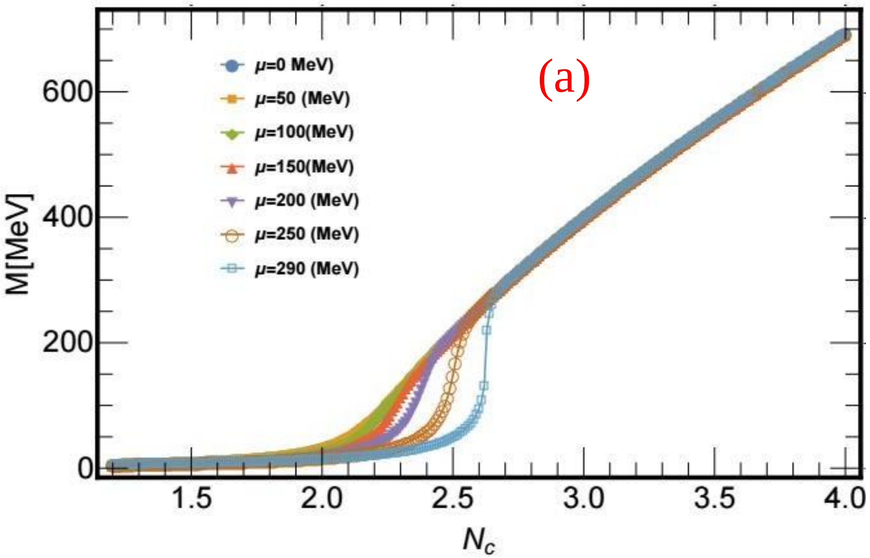


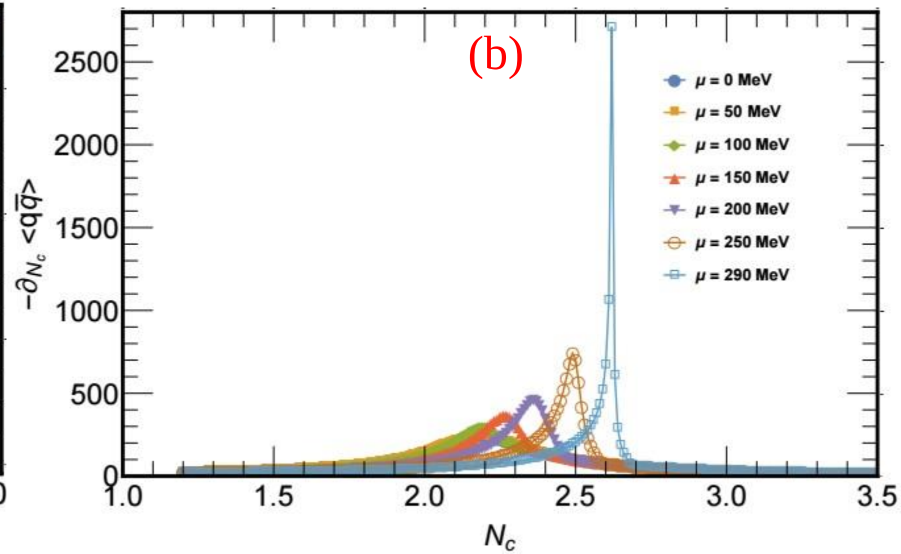
- Table. Data for Fig. (c)
- Fig. (a): Mass Vs Number of colors
- Fig. (b): The peaks of quark condensate
- Fig. (c): The critical number of colors

NUMERICAL RESULTS

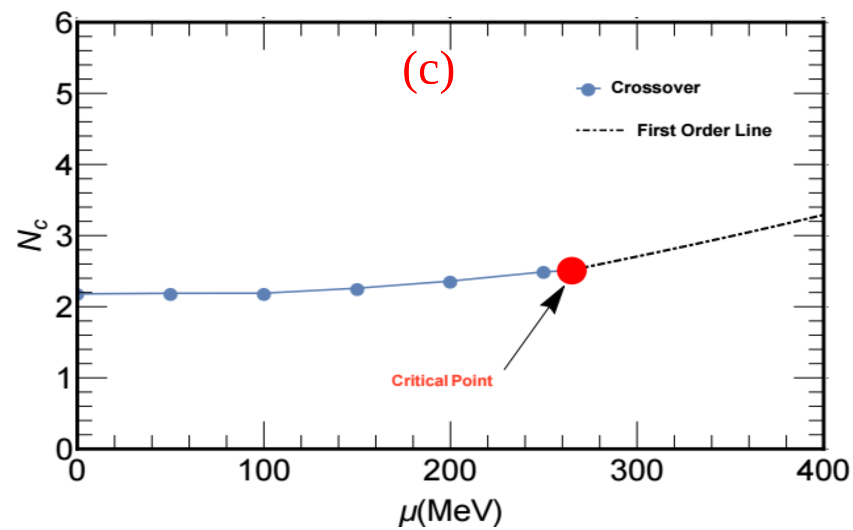
Critical Number of colors for Chiral Symmetry Breaking at T = 0 and $\mu \neq 0$

- For fixed 2- flavors, with bare mass m = 5.6 MeV





μ (MeV)	0	50	100	150	200	250	290
Critical N_c	2.18	2.189	2.19	2.26	2.36	2.49	2.51



- Table. Data for Fig. (c)
- Fig. (a): Mass Vs Number of colors
- Fig. (b): The peaks of quark condensate
- Fig. (c): The critical number of colors

NUMERICAL RESULTS

Chiral Symmetry Breaking at T = 0 and $\mu \neq 0$, for various number of colors

- For fixed 2- flavors, with bare mass m = 5.6 MeV

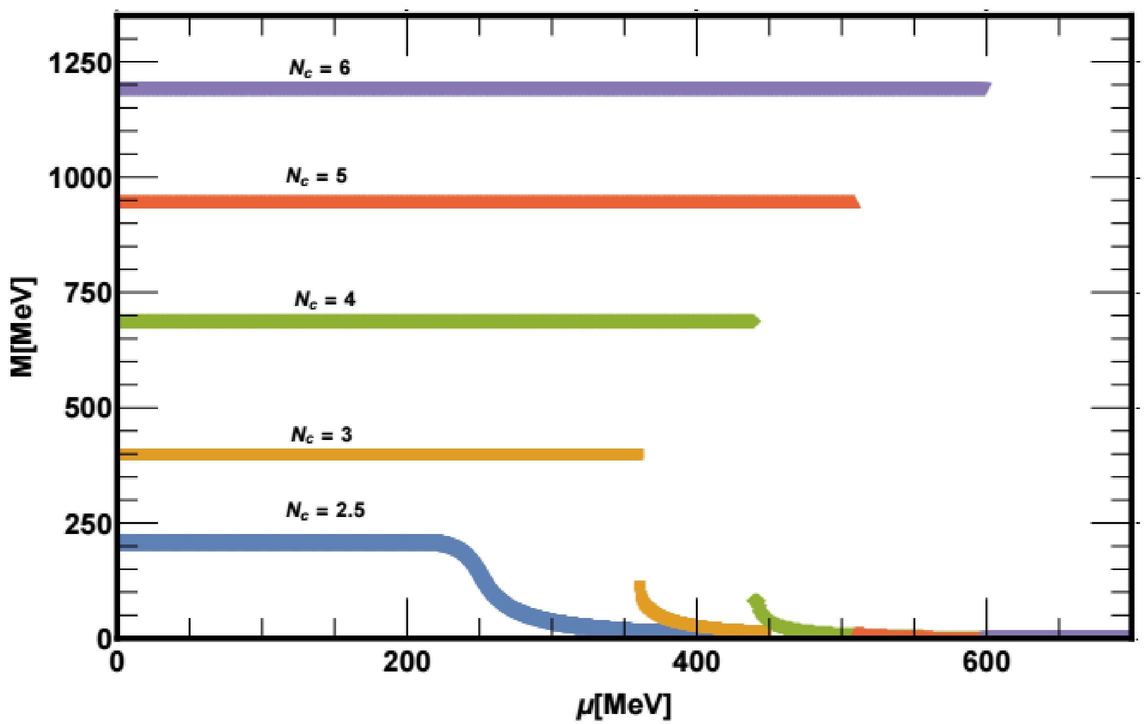


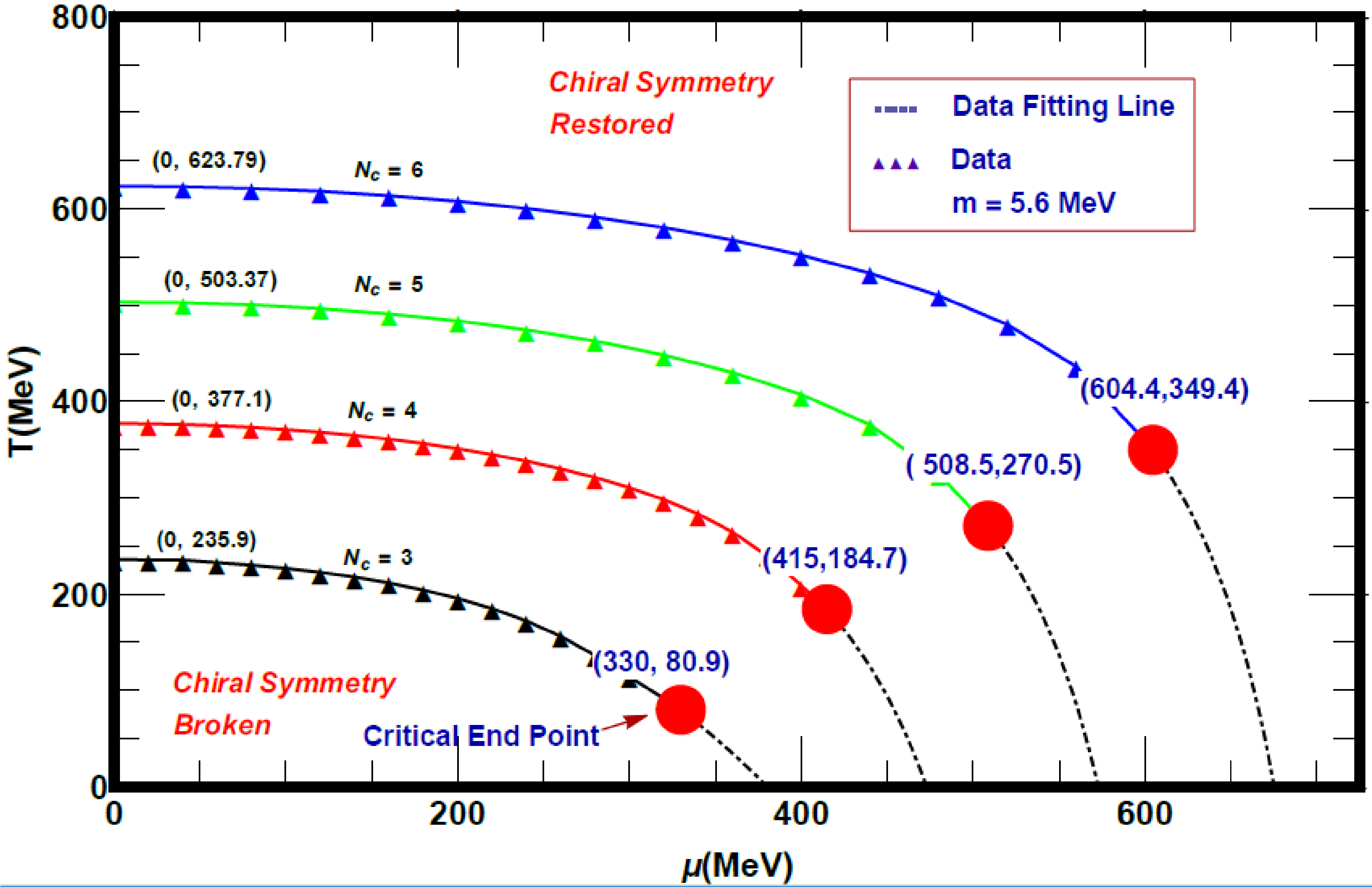
Fig. Mass Vs chemical potential

- The phase transition is crossover for $N_c \leq 2.51$, while first order for higher number of N_c .

NUMERICAL RESULTS

QCD Phase Diagrams for various Number of Colors (Combined)

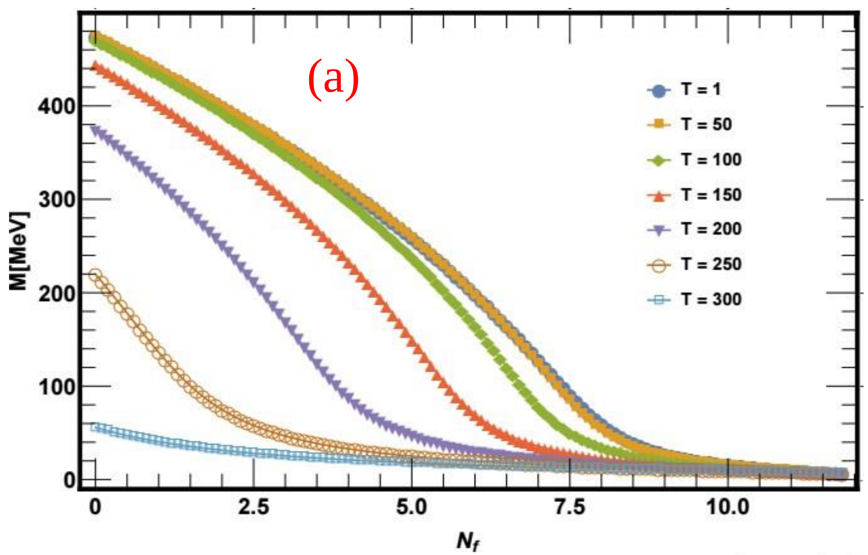
- For fixed 2- flavors

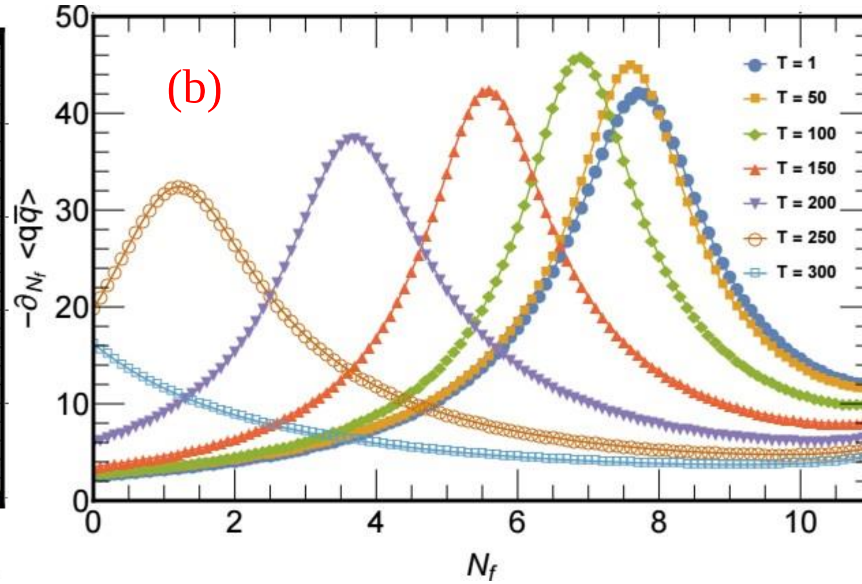


NUMERICAL RESULTS

Critical Number of Flavors for Chiral Symmetry Restoration at $T \neq 0$ but $\mu = 0$

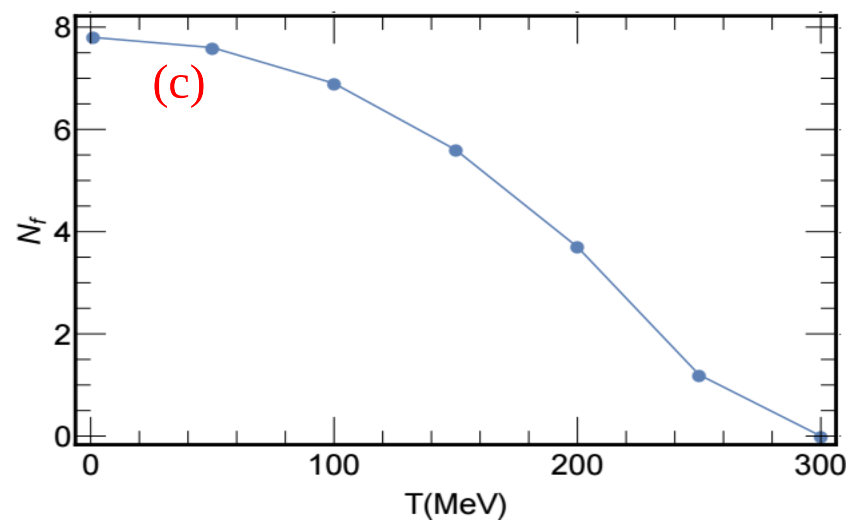
- For fixed 3-colors with bare mass m = 5.6 MeV





T (MeV)	0	50	100	150	200	250	300
Critical N_f	8	7.8	6.9	5.6	3.7	1.2	0.1

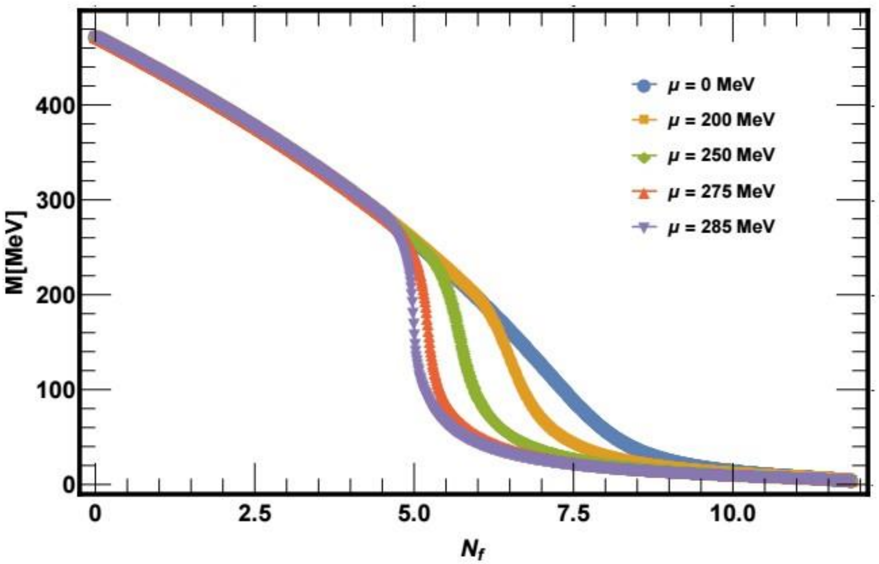
- Table. Data for Fig. (c)
- Fig. (a): Mass Vs Number of flavors
- Fig. (b): The peaks of quark condensate
- Fig. (c): The critical number of flavors

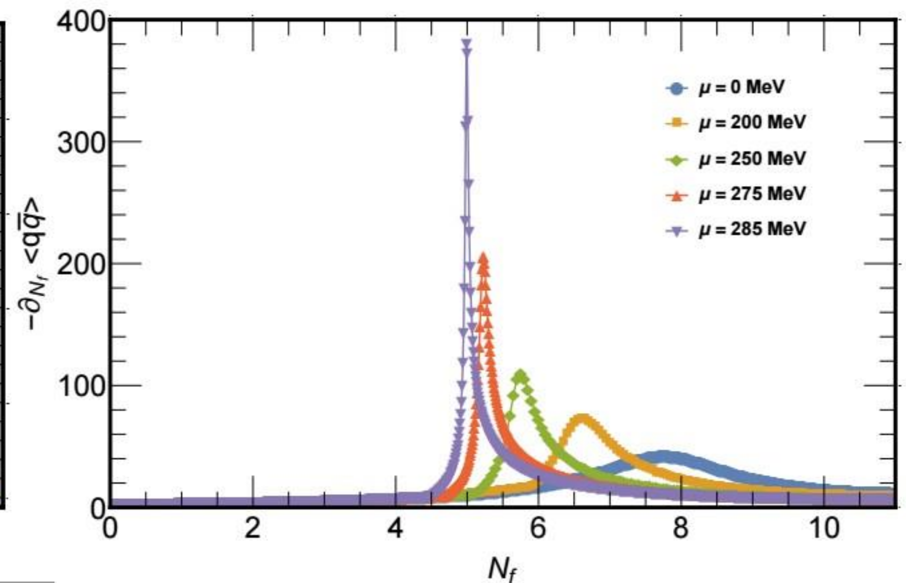


NUMERICAL RESULTS

Critical Number of Flavors for Chiral Symmetry Restoration at T = 0 and $\mu \neq 0$

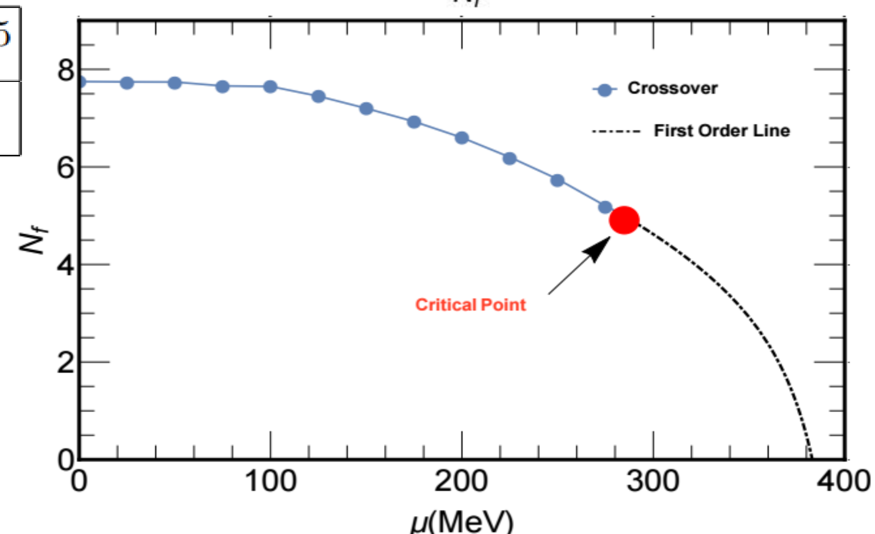
- For fixed 3-colors, with bare mass m = 5.6 MeV





μ (MeV)	0	50	100	150	175	200	225	250	275	285
Critical N_f	8	7.8	7.65	7.2	6.93	6.6	6.2	5.75	5.2	5

- Table. Data for Fig. (c)
- Fig. (a): Mass Vs Number of flavors
- Fig. (b): The peaks of quark condensate
- Fig. (c): The critical number of flavors



NUMERICAL RESULTS

Critical Number of Flavors for Chiral Symmetry Restoration at $T = 0$ MeV and $\mu \neq 0$

- For fixed 3- colors, with bare mass m = 5.6 MeV

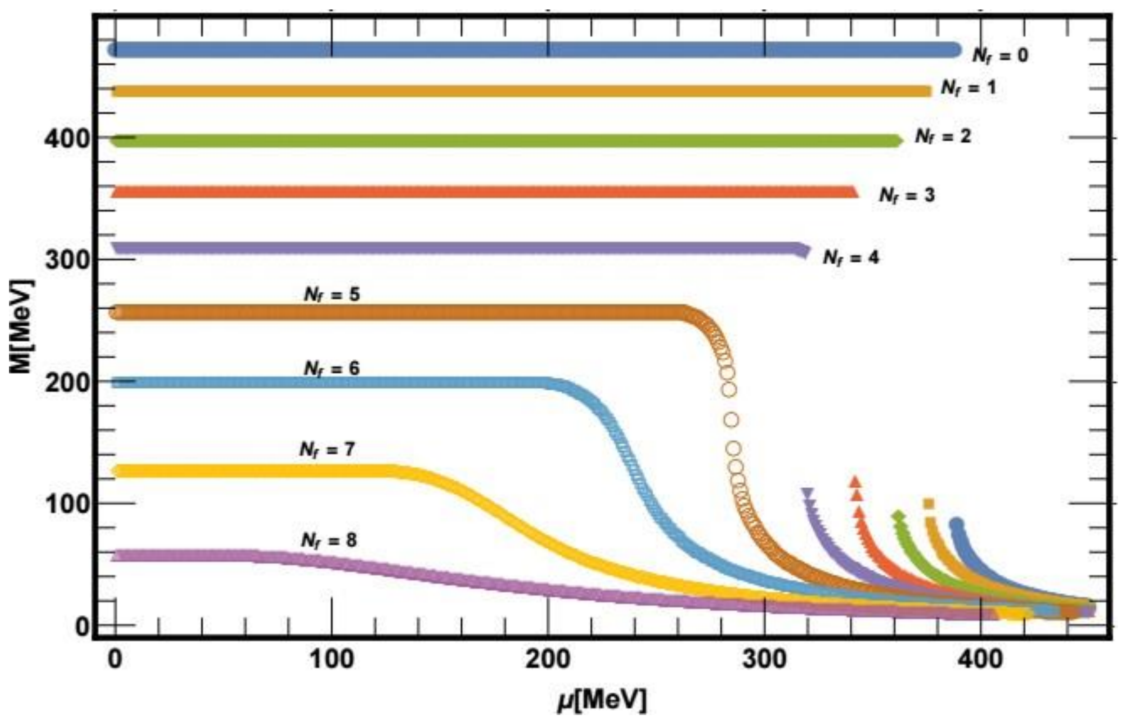


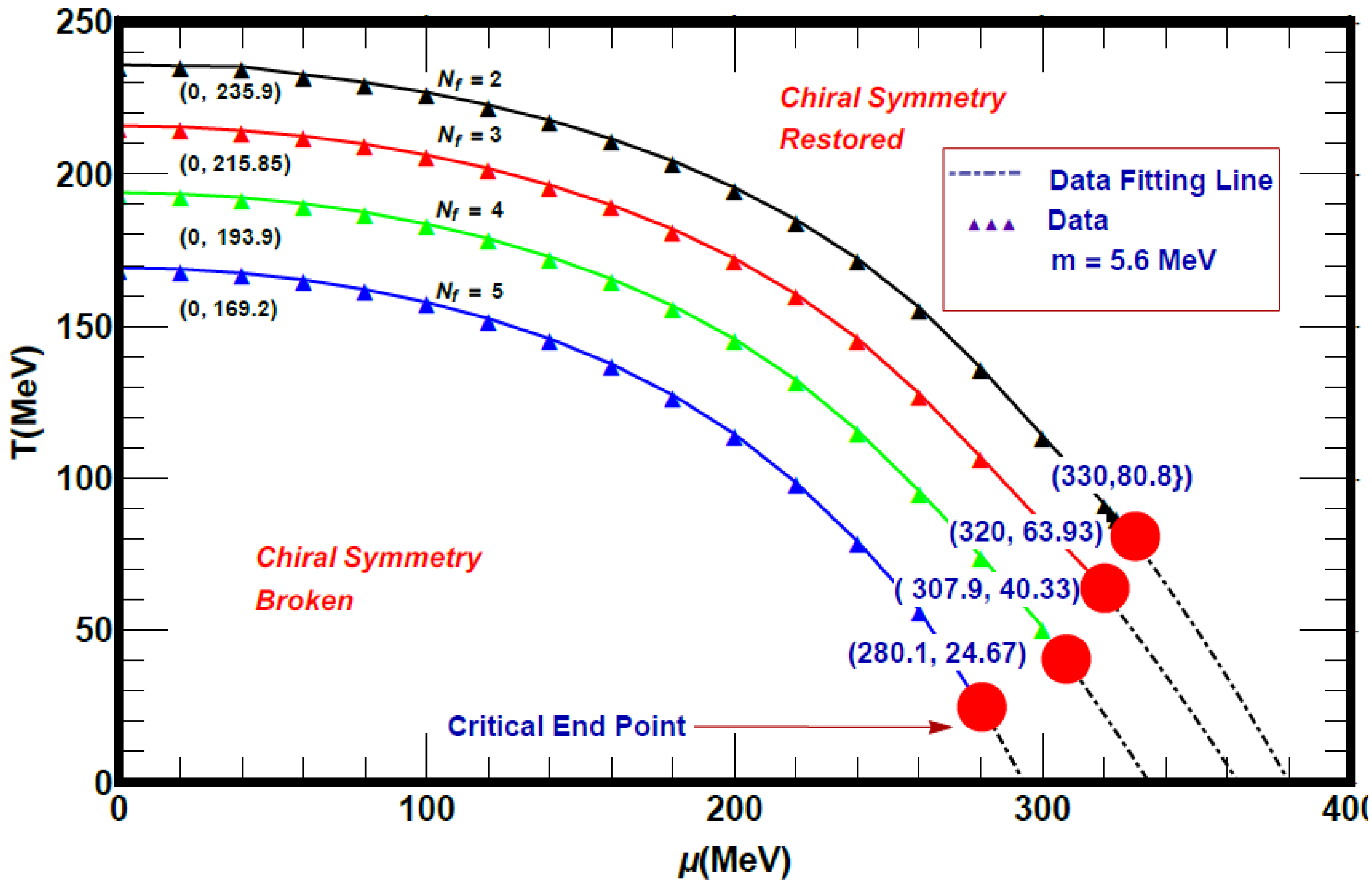
Fig. Mass Vs chemical potential at T = 0 MeV

- The phase transition is crossover for $N_f > 5$, while first order for $N_c \leq 5$.

NUMERICAL RESULTS

QCD Phase Diagrams for Higher Number of Flavors (Combined)

- For fixed 3- colors



Summery and Conclusions

- For Fixed N_f =2: the critical number of colors for the dynamical chiral symmetry breaking is N_C^c= 2.18.
- In the presence of quark chemical potential or temperature or both, the critical number of colors for chiral phase transition increases.
- For fixed N_C= 3: the critical number of flavors for chiral phase transition is observed to be equal to N_f^c = 8.
- We noted that , if we increase the chemical potential or temperature or both, the critical
number of flavors for chiral phase transition decreases.
- For higher values of flavors, i.e. N_f> 5 the chiral phase transition is totally crossover and we cannot find first order phase transition in the QCD phase diagrams.

Summery and Conclusion:

Table-A: For fixed $N_C = 3$:

S.No	N_f	m(MeV)	T_c(in MeV)	CEP($\mu_{Ep}, T_{Ep})$ in MeV
01	2	5.6	235	(330, 81)
02	3	5.6	216	(320, 64)
03	4	5.6	194	(308, 40.33)
04	5	5.6	169	(280, 25)

[Buballa, *Phys.Rept.* 407 (2005) 205-376]

Table-B: For fixed $N_f = 2$:

S.No	N_c	m(MeV)	T_c(MeV)	(μ_{Ep}, T_{Ep})(MeV)
01	3	5.6	235	(330, 81)
02	4	5.6	377	(415, 184.7)
03	5	5.6	503	(508.5, 270.5)
04	6	5.6	623	(604.4, 349.4)

Thank you for your attention

and

Thanks for the Organizers of Hadron-2021