

HADRON2021, July 28<sup>th</sup>, 2021

# Unveiling the proton structure via TMD gluon distributions

**Francesco Giovanni Celiberto**

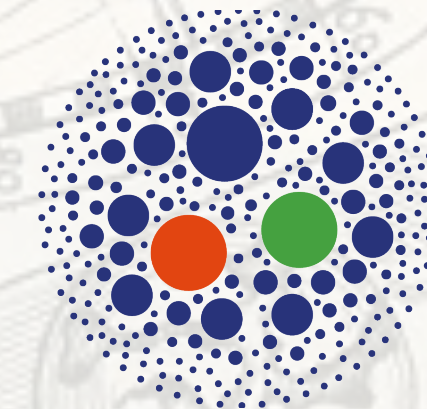
ECT\*/FBK Trento & INFN-TIFPA

**ECT\***

EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS



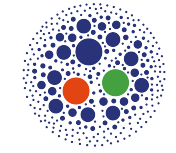
Trento Institute for  
Fundamental Physics  
and Applications



**HAS QCD**

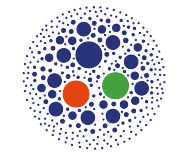
HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS

# Gluon TMDs: a largely unexplored territory



**Theory:** different **gauge-link** structures...

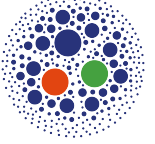
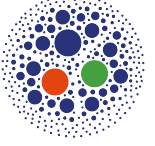
...more diversified kind of **modified universality!**

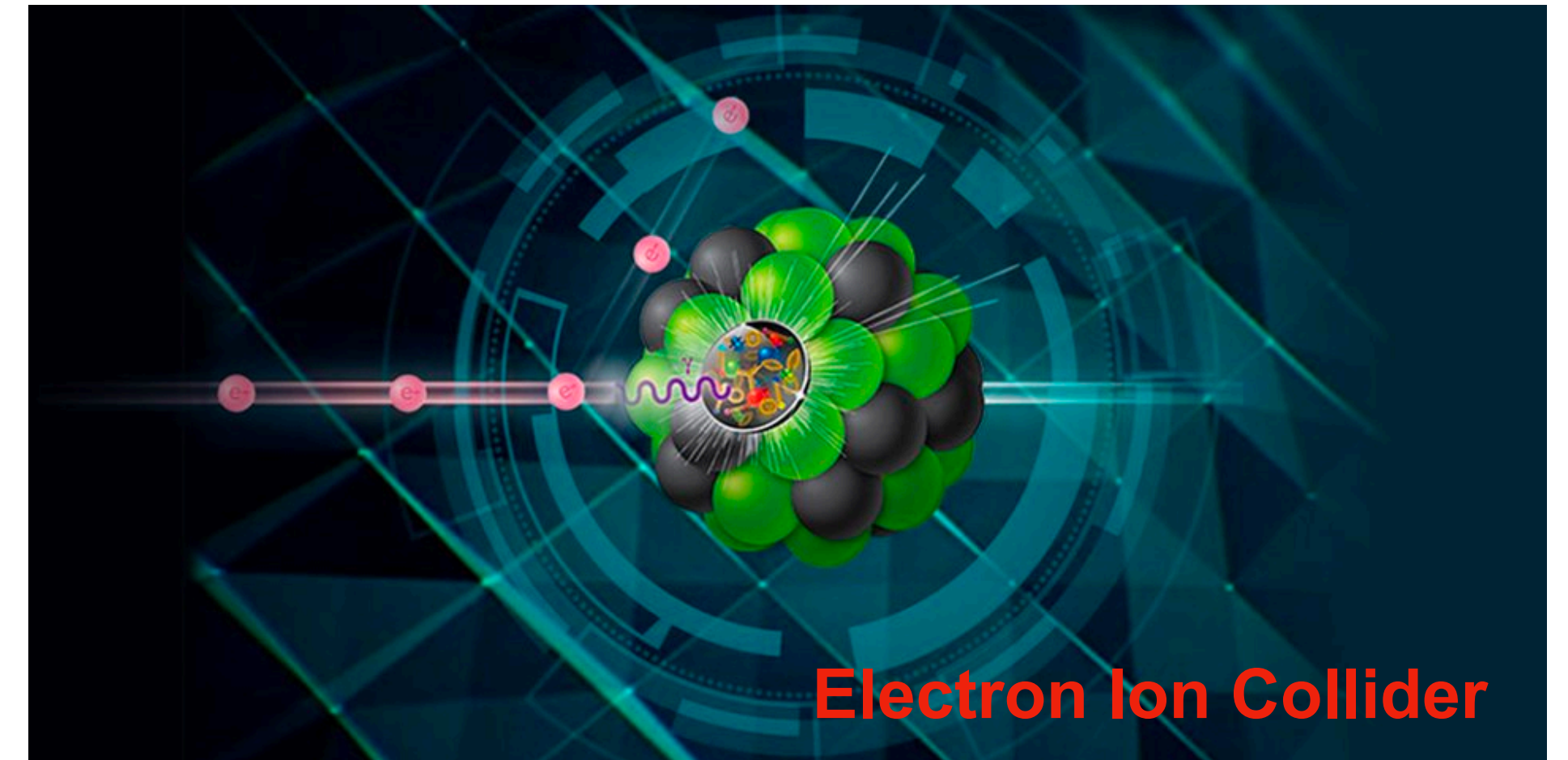


**Pheno:** golden channels for extraction

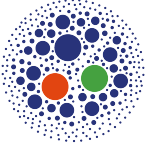
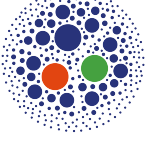
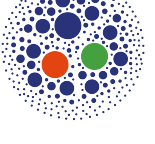
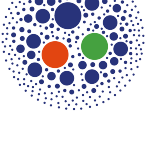
of quark TMDs are subleading for gluon TMDs

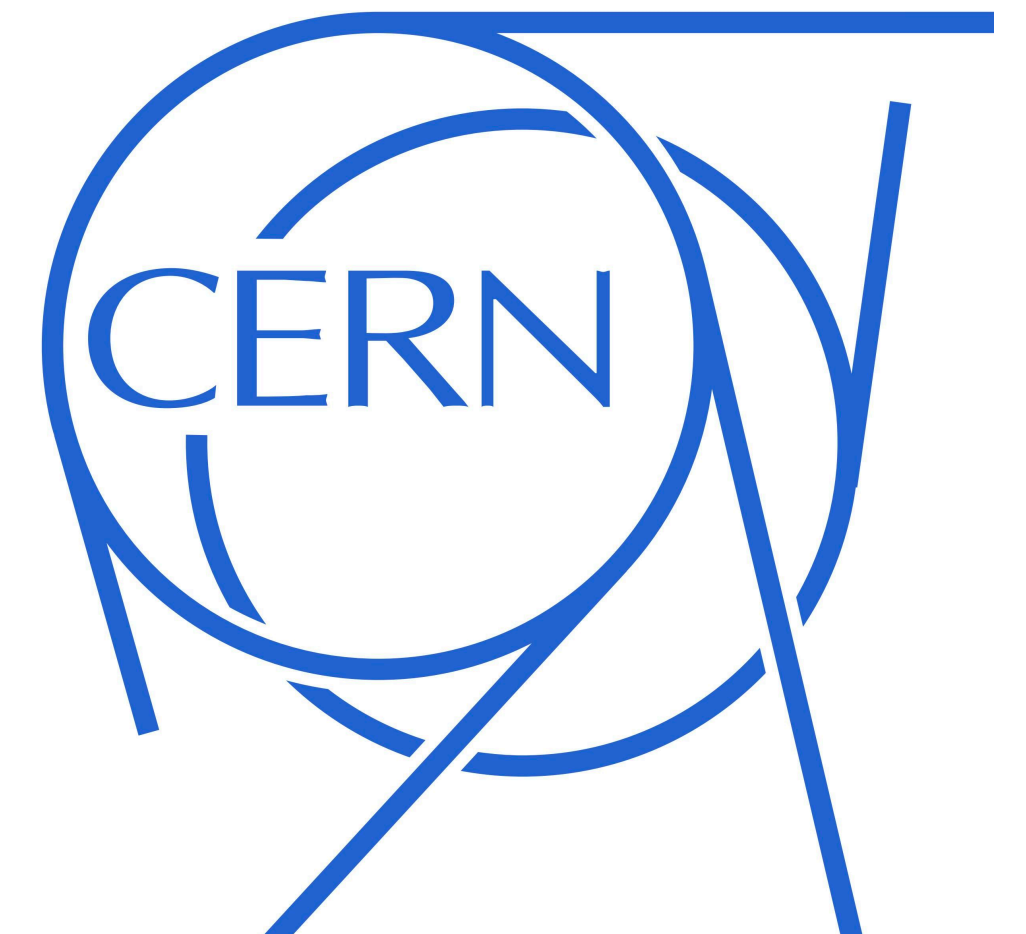
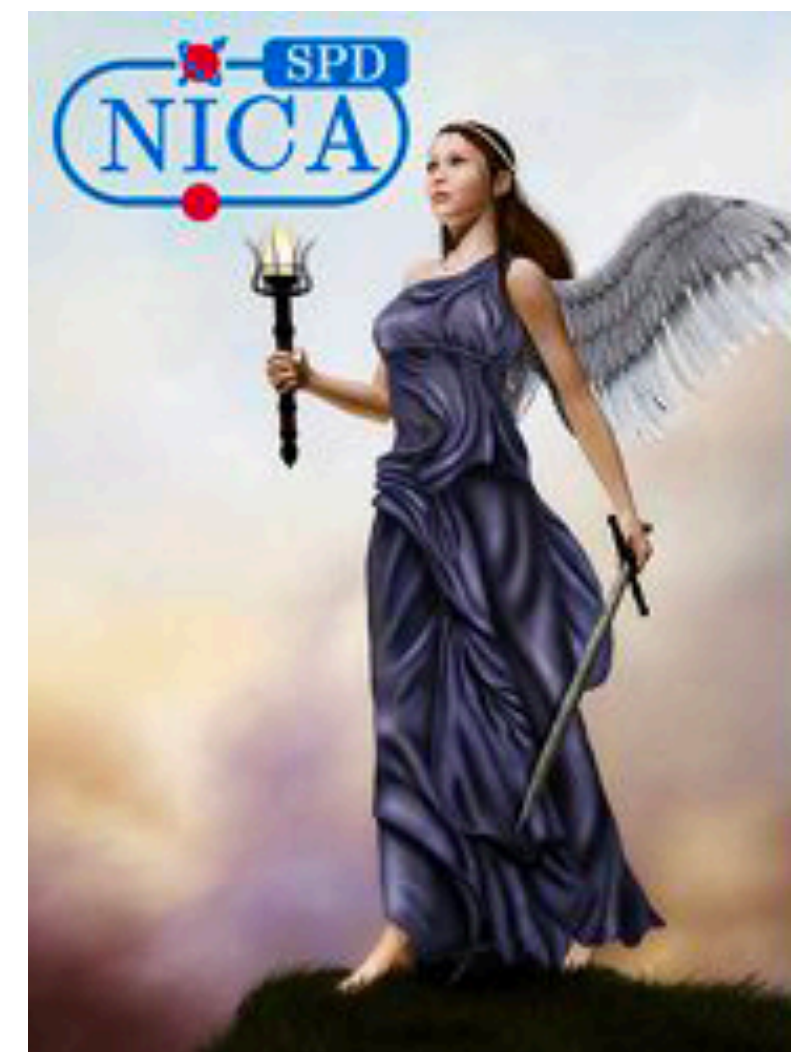
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...more diversified kind of **modified universality!**
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## Motivation

-  Gluon-TMD PDFs: *core* sector of **EIC** studies
-  Need for a *flexible* model, suited to *pheno*
-  **Unpolarized** and **polarized gluon TMDs**
-  *Consistent* framework for quark TMDs

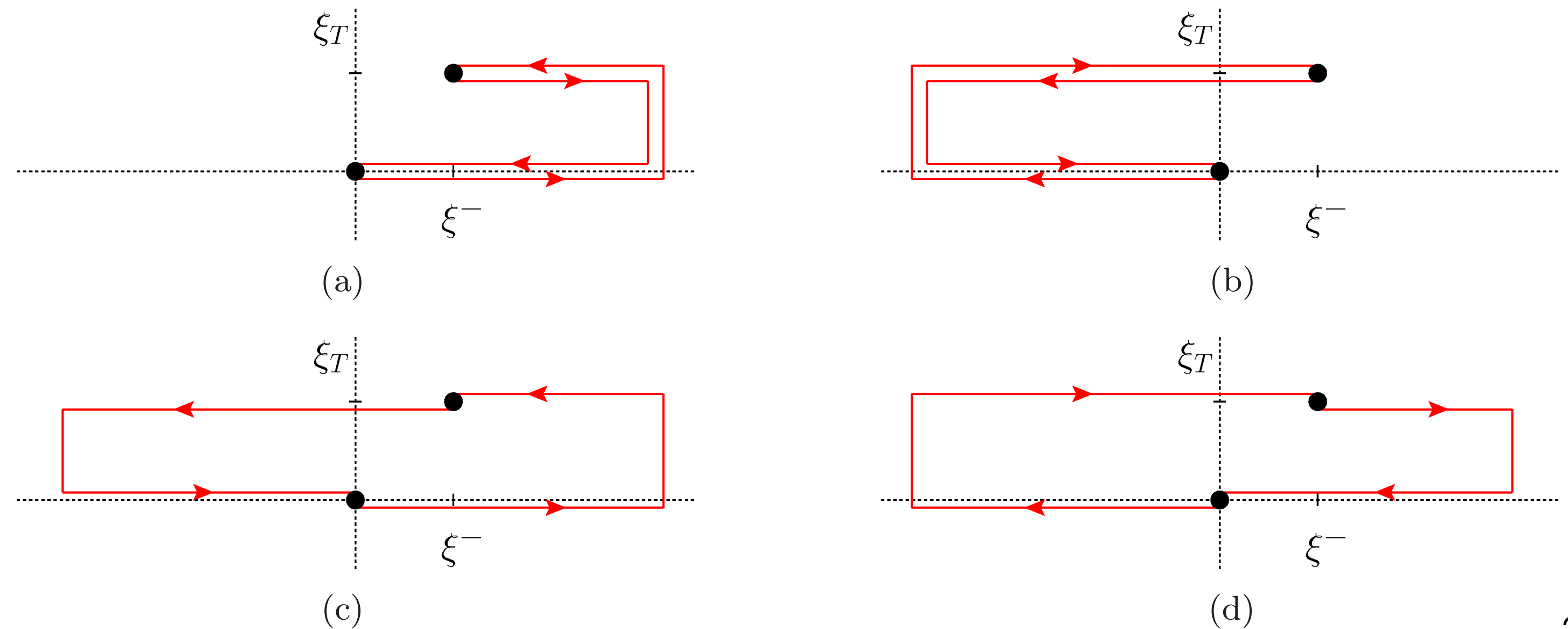


# Gluon TMDs: gauge links and modified universality

- \* **Single-spin asymmetries** → process dependence of TMDs via **gauge links**
- \* **Color flow** → integration paths of gauge links calculable
- \* Gluon TMDs → more complicated structure with respect to quark **staple links**
- \* **Factorization-preserving** processes → two main kinds of **modified universality**
- \* Different classes of processes → distinct gluon TMDs, **not related** to each other

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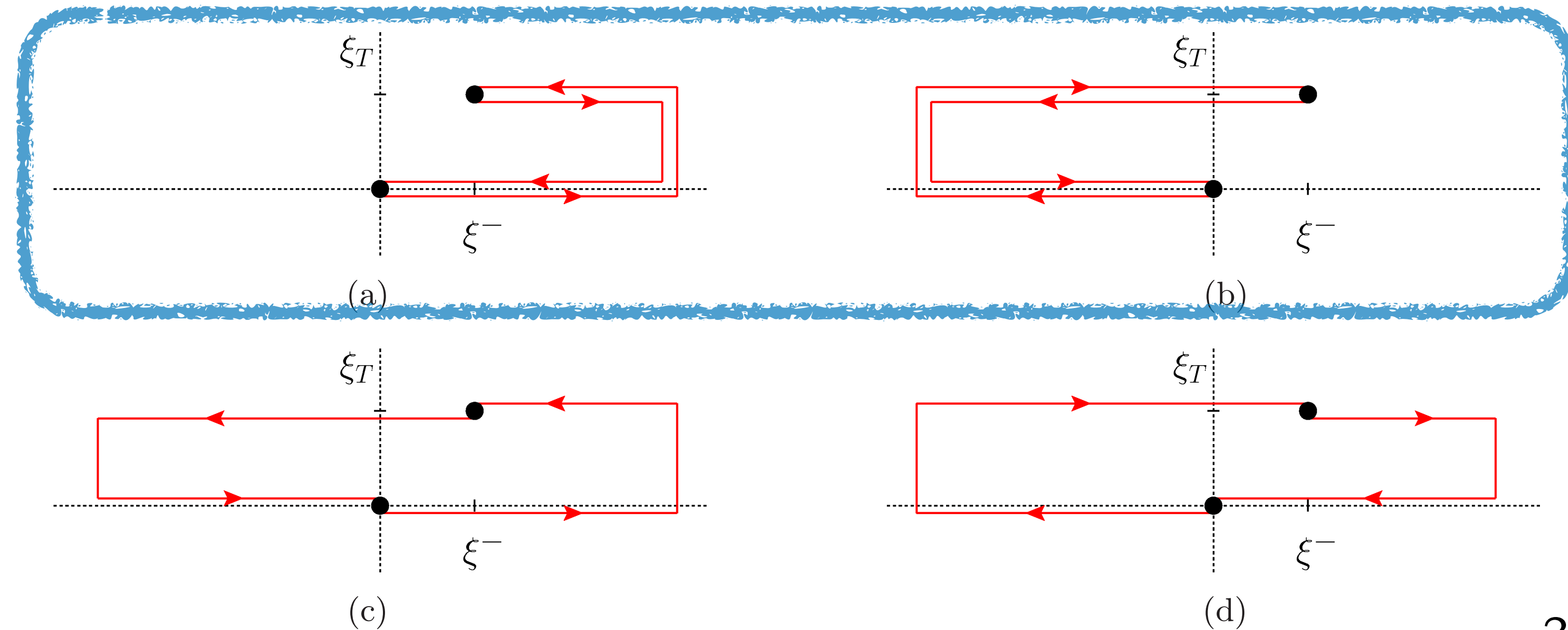


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## Weizsäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]

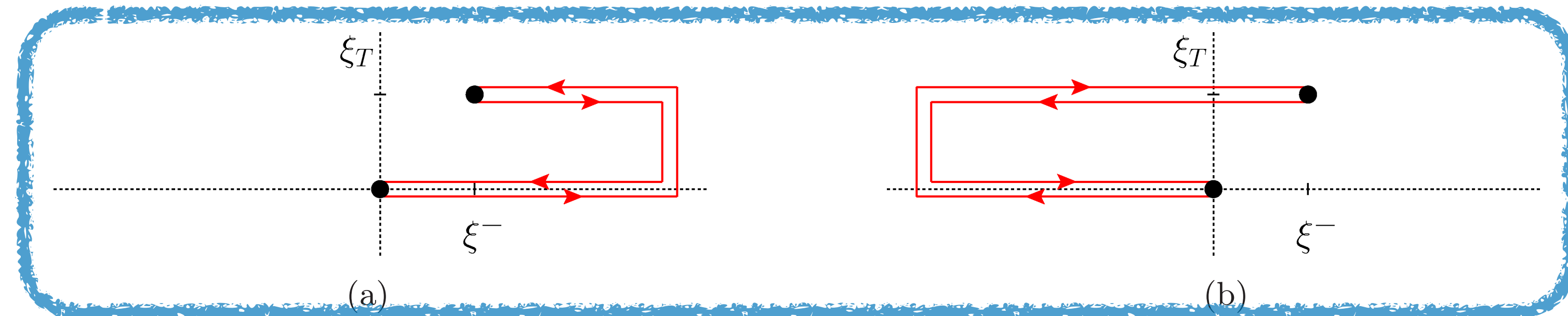


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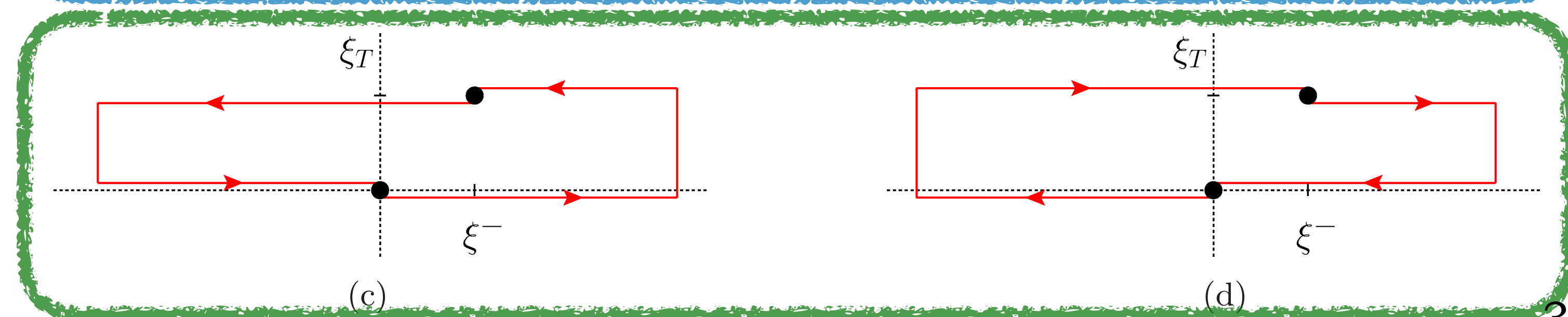
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## Dipole (DP)

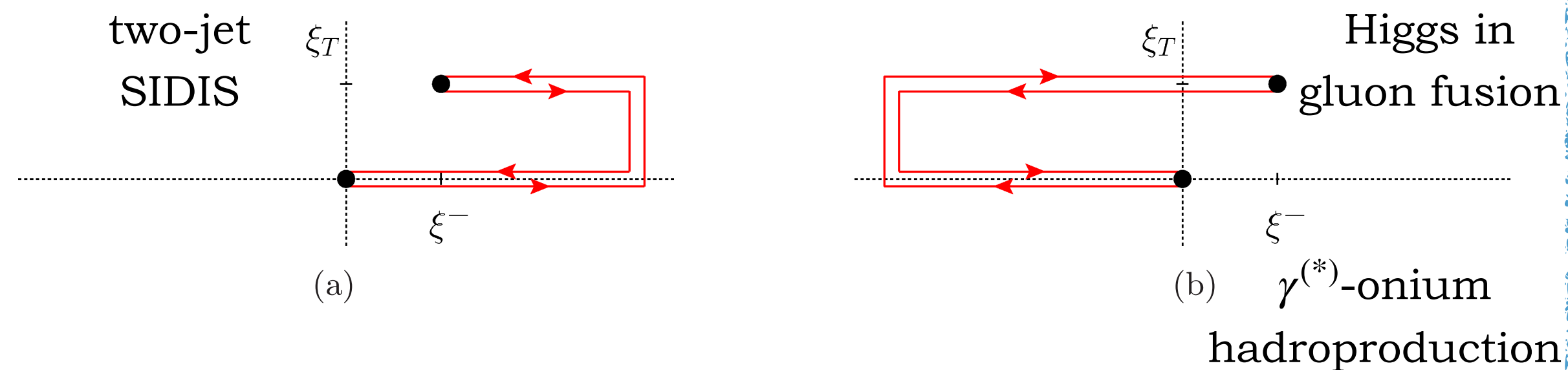
(c) [ + , - ] or (d) [ - , + ]



# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]



\* Color flow annihilated within final/initial state

\*  $f$ -type gluon TMDs  $\rightarrow f^{abc}$  color structure

\* Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$

$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

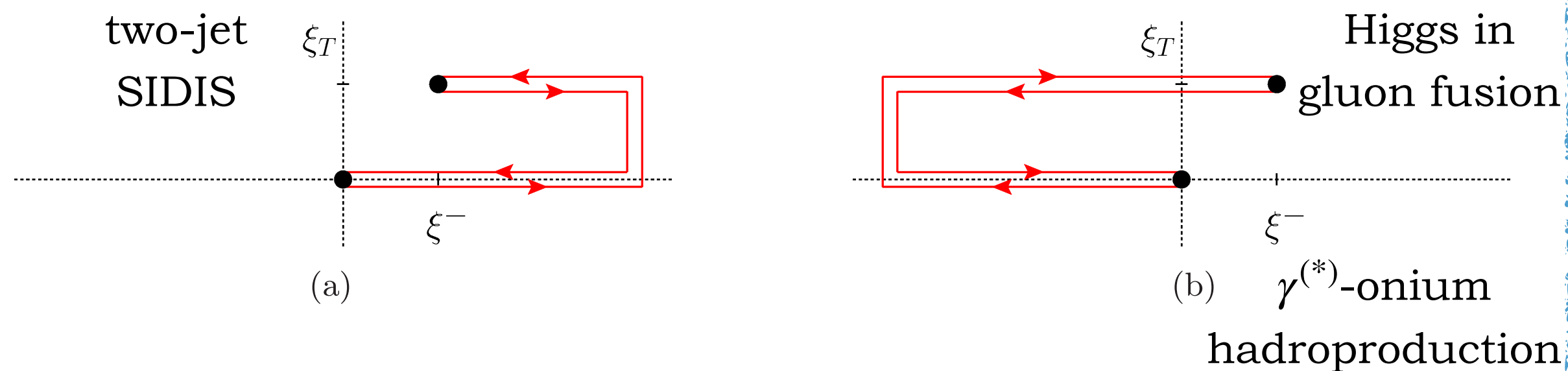
\* Phenomenology: Higgs, quarkonia or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS



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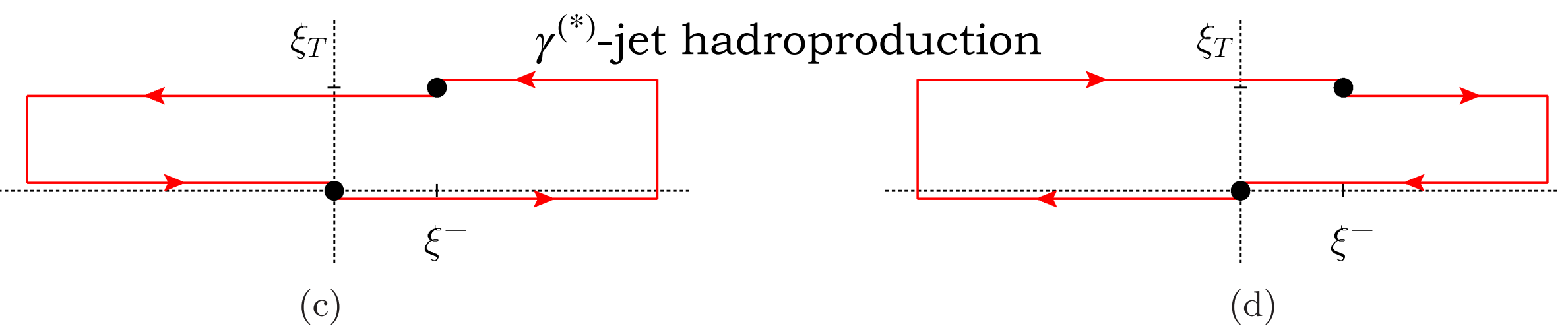
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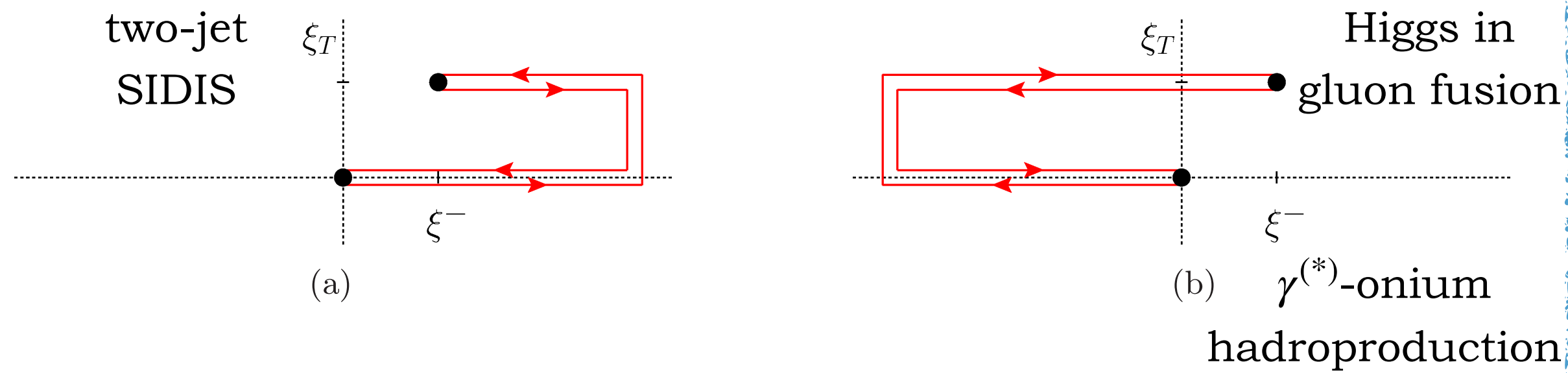
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- \* Phenomenology: single hadron or  $\gamma^{(*)}$ -jet hadroproduction, SIDIS or Drell-Yan (subleading)

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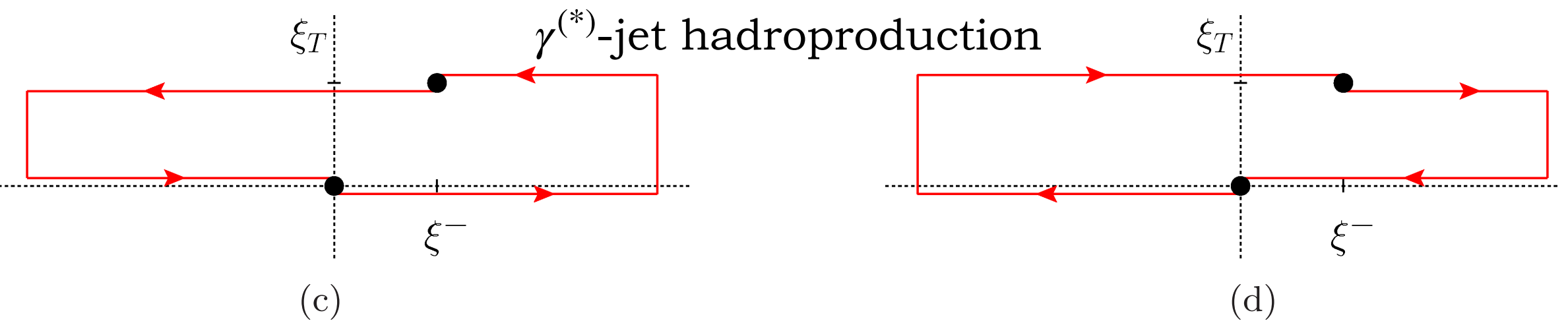
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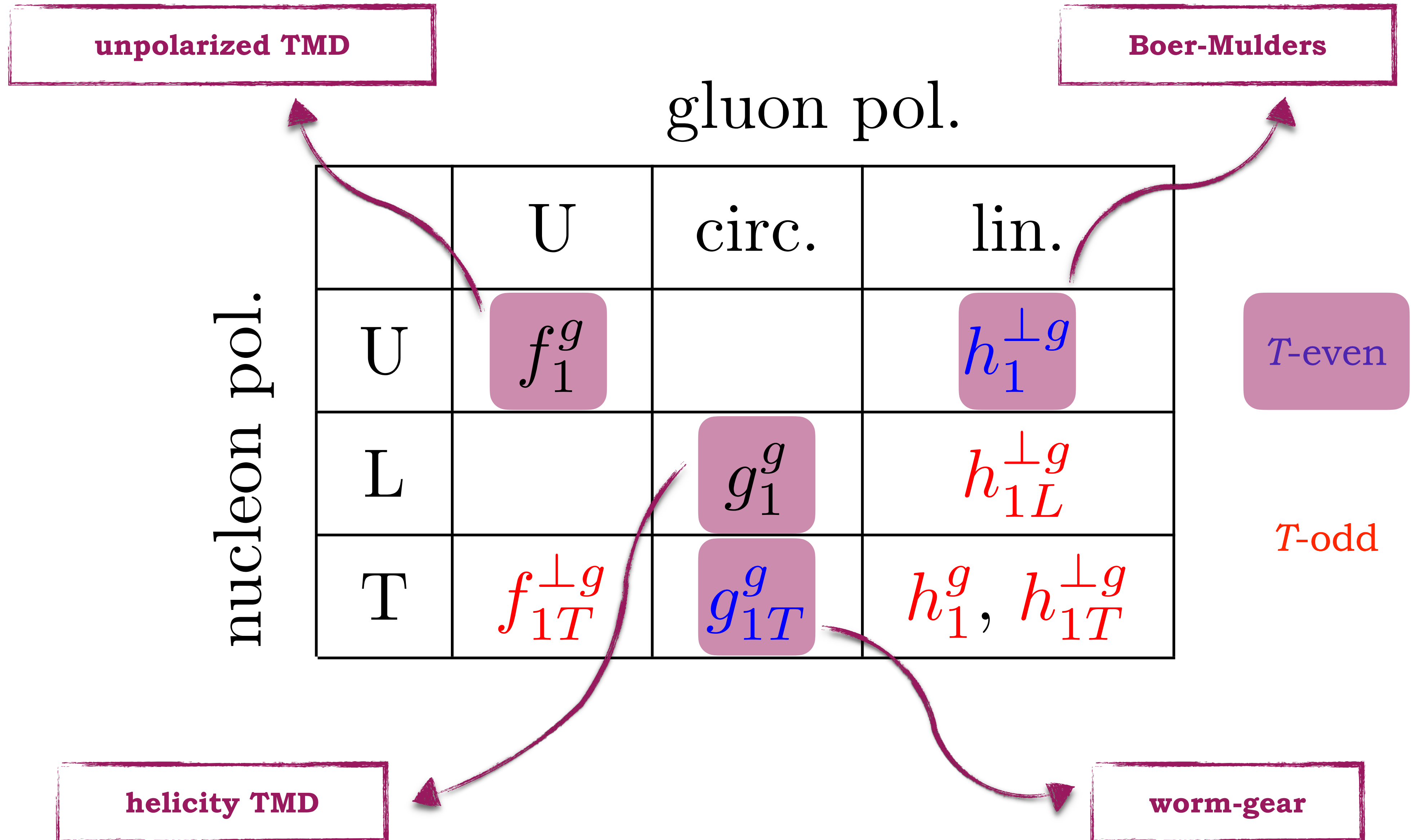
Gauge link  $\rightarrow$  two main independent sets of TMDs, **not related** to each other

# ***T*-even and *T*-odd gluon TMD PDFs at twist-2**

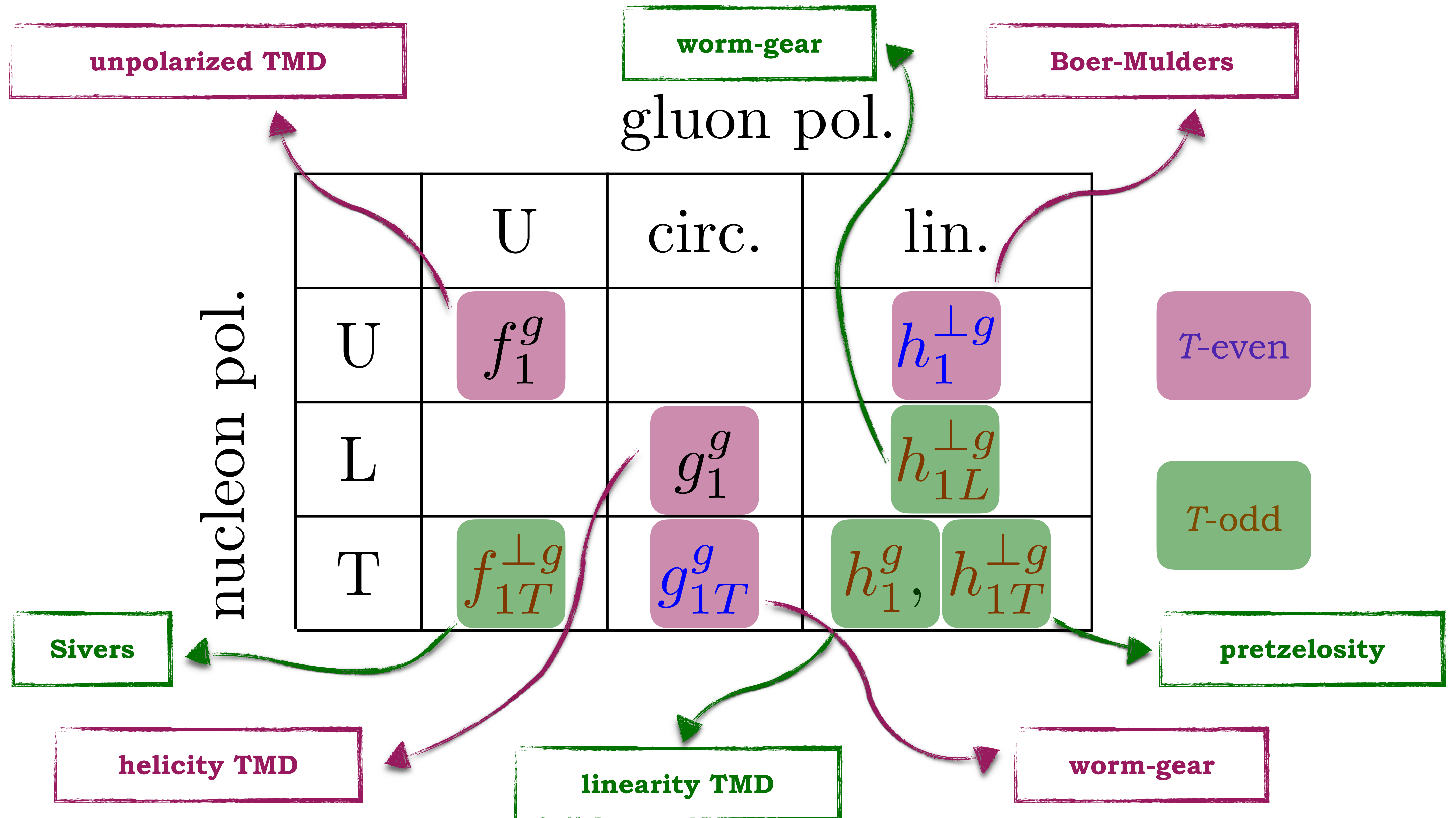
gluon pol.

nucleon pol.		U	circ.	lin.	
	U	$f_1^g$		$h_1^{\perp g}$	<i>T</i> -even
	L		$g_1^g$	$h_{1L}^{\perp g}$	<i>T</i> -odd
	T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$	

# $T$ -even and $T$ -odd gluon TMD PDFs at twist-2

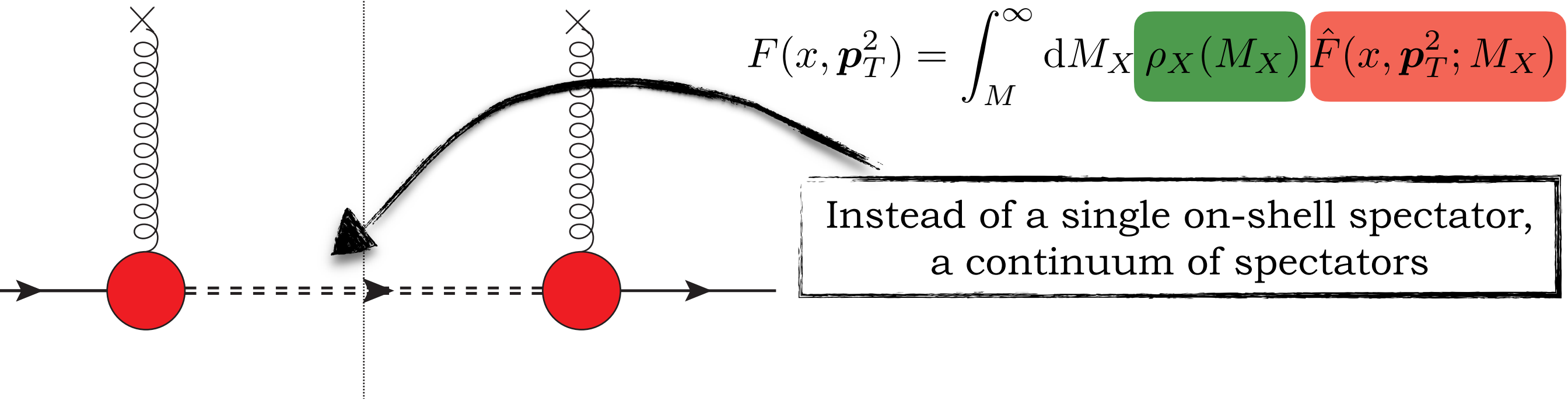


# $T$ -even and $T$ -odd gluon TMD PDFs at twist-2



# Our model at a glance

## Spectator-system spectral-mass function



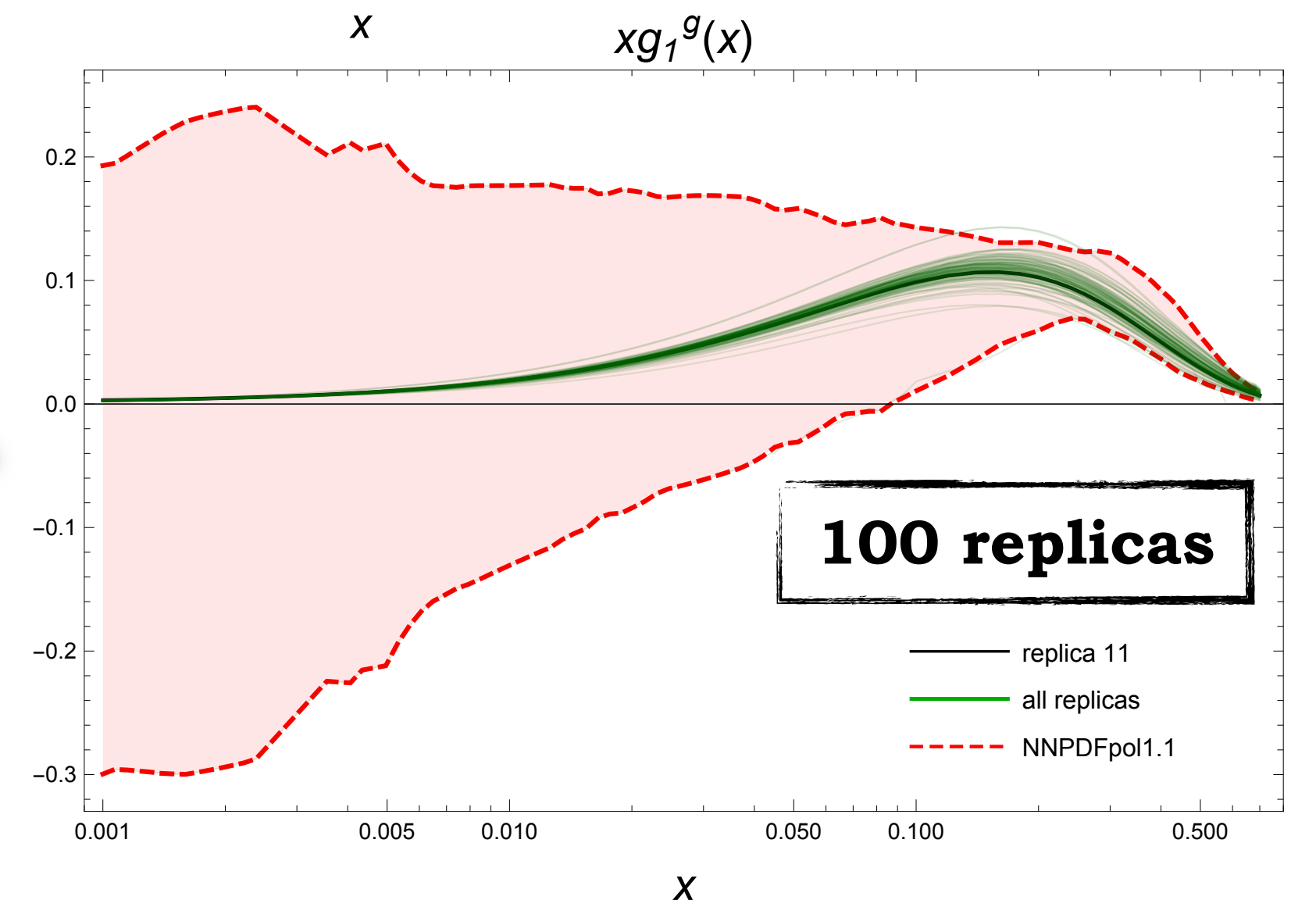
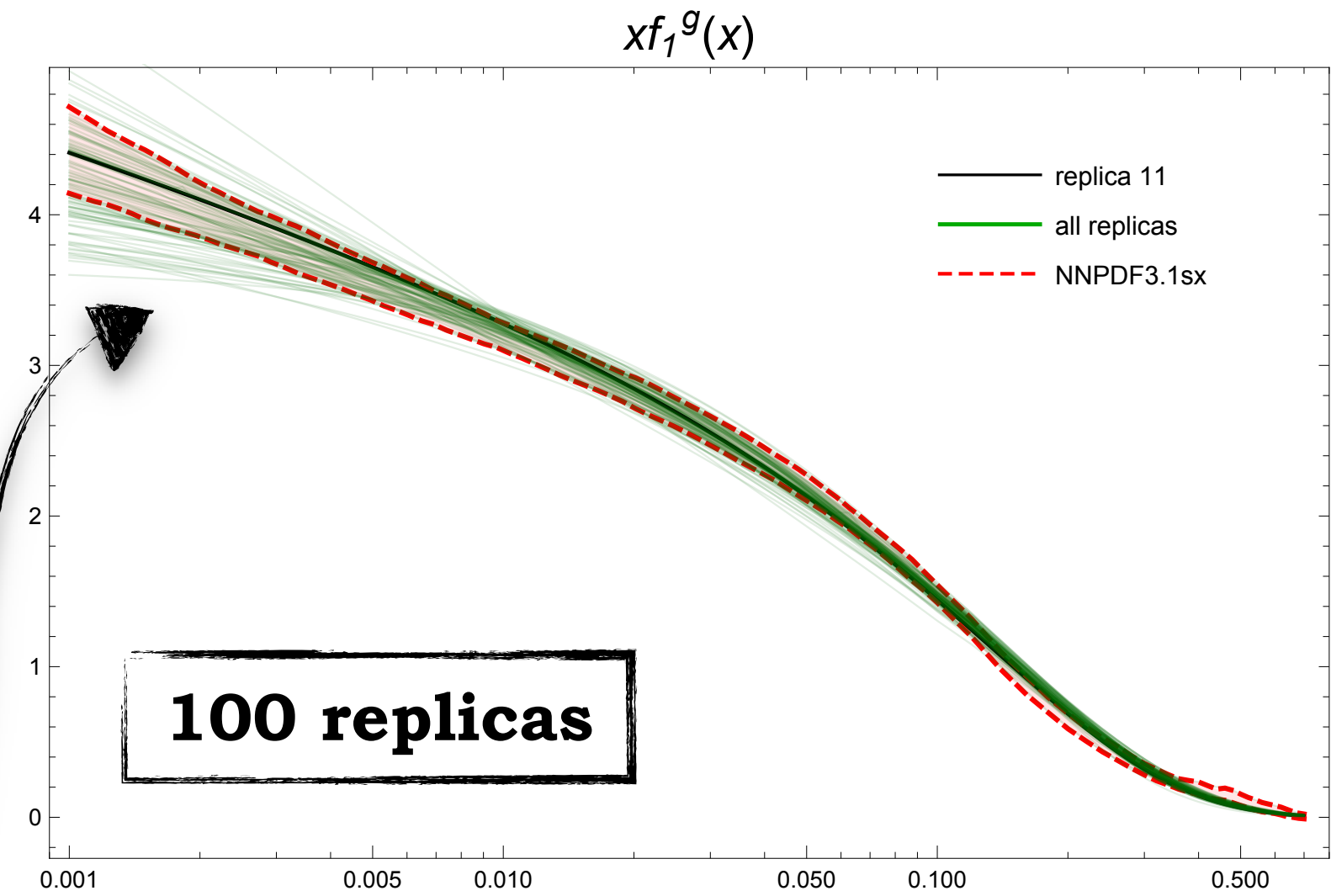
$$F(x, p_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, p_T^2; M_X)$$

Spectral function **learns** small- and moderate- $x$  info encoded in **NNPDF** collinear parametrizations (NNPDF3.1sx + NNPDFpol1.1)

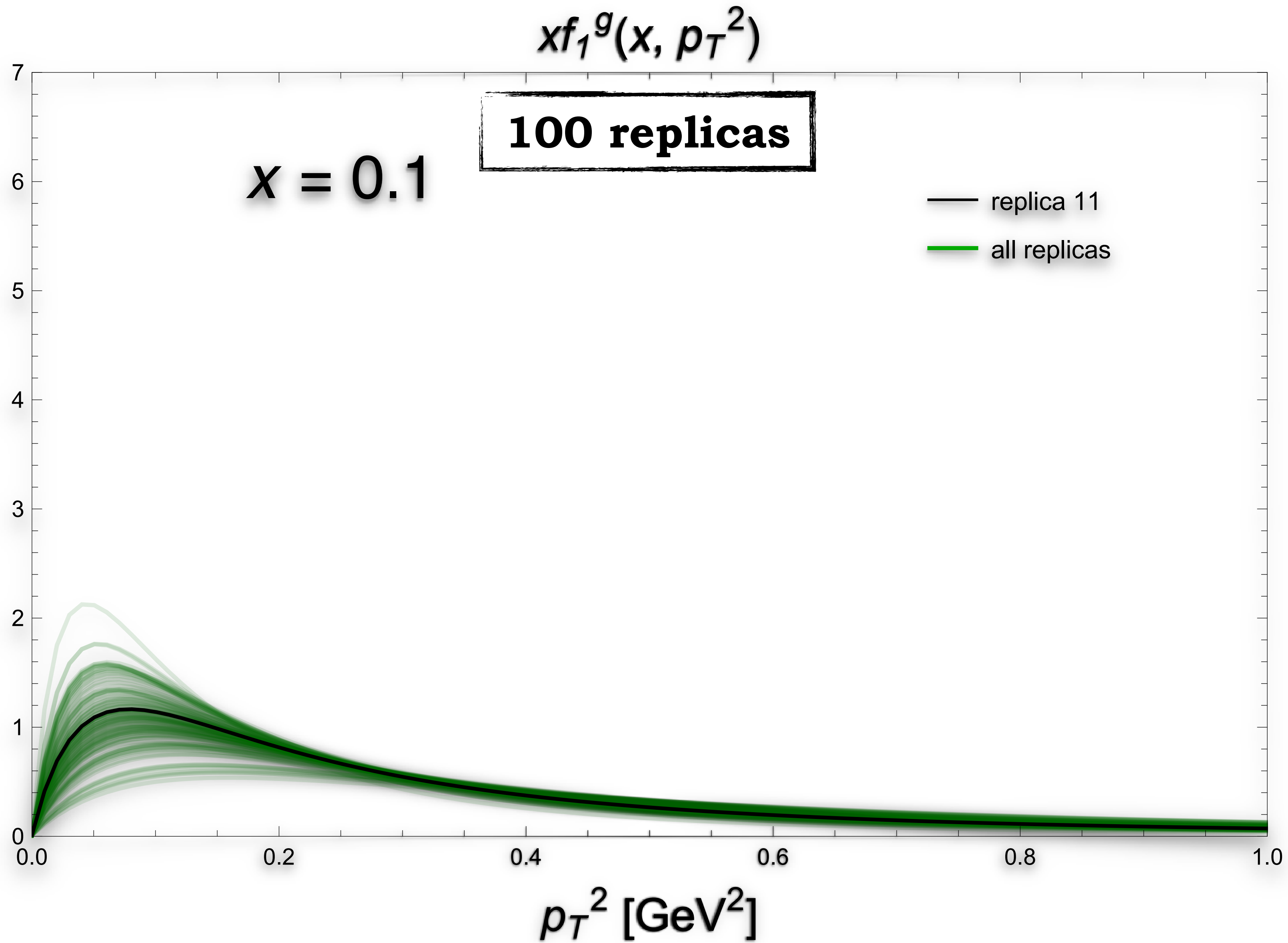
- ✓ **Simultaneous fit** of  $f_1$  and  $g_1$  PDFs
- ✓ Inclusion of small- $x$  resummation effects (**BFKL**)
- ✓ Calculation of all twist-2  $T$ -even gluon TMDs

## Link with collinear factorization

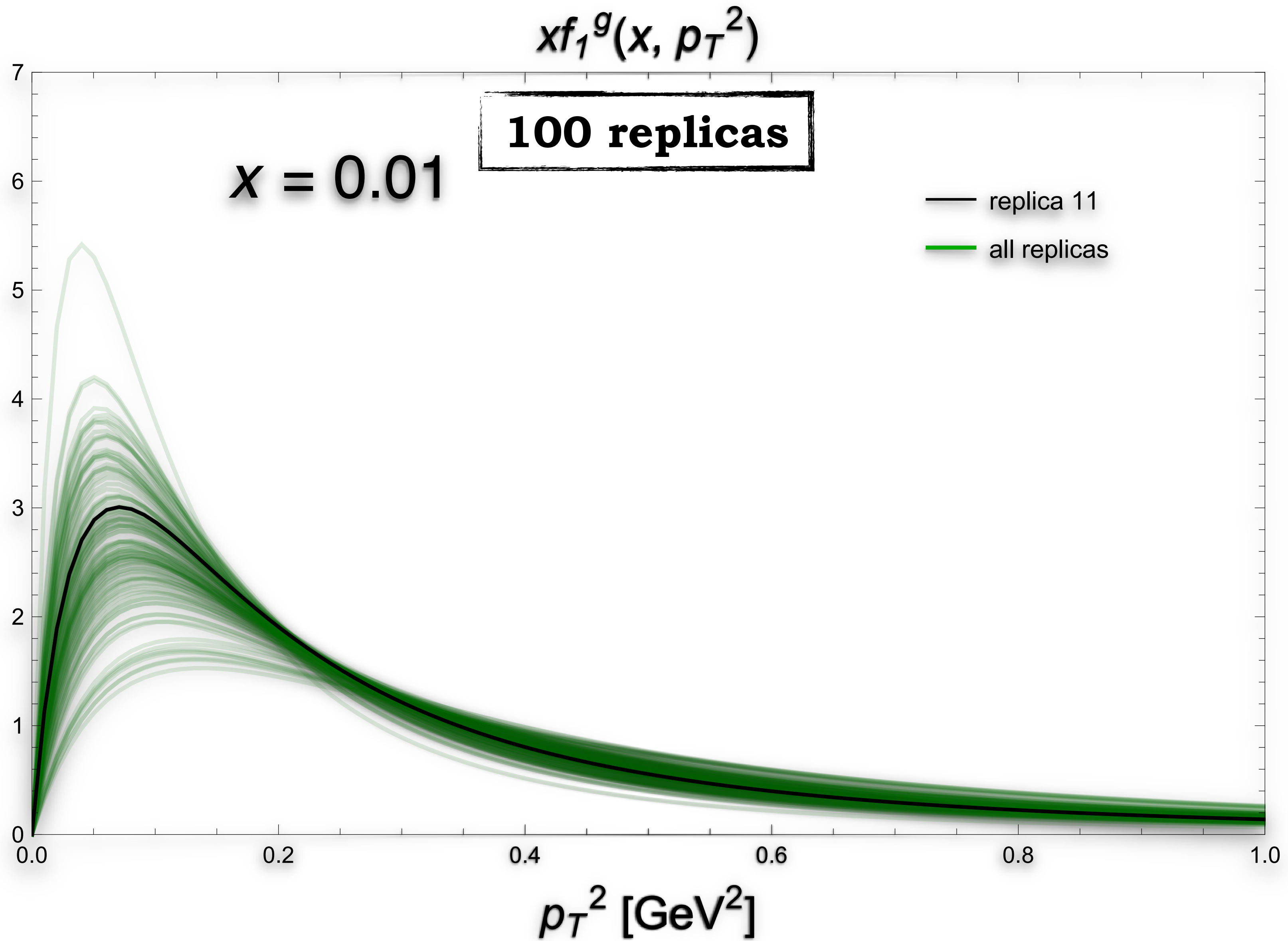
$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution



# Unpolarized gluon TMD

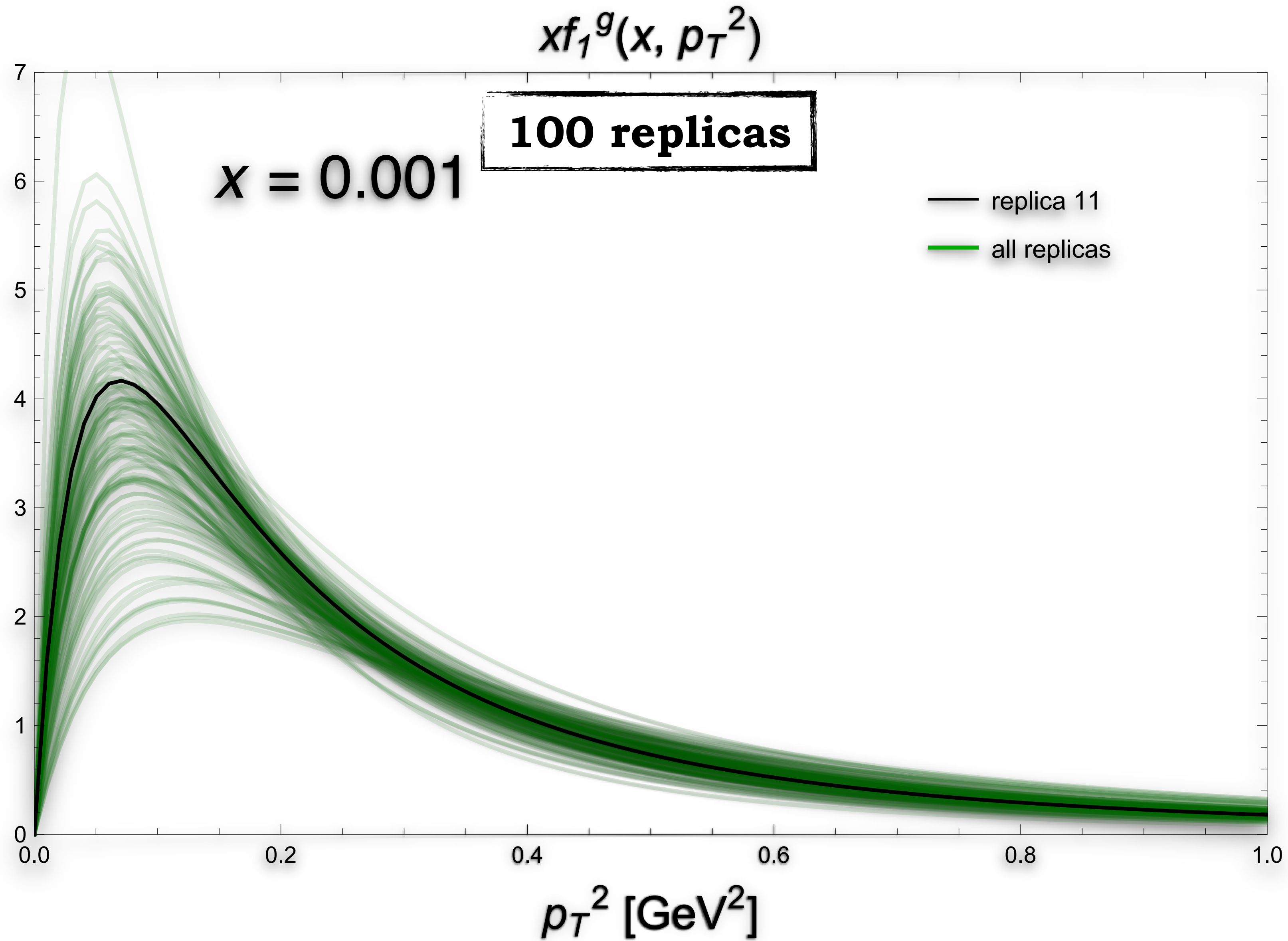


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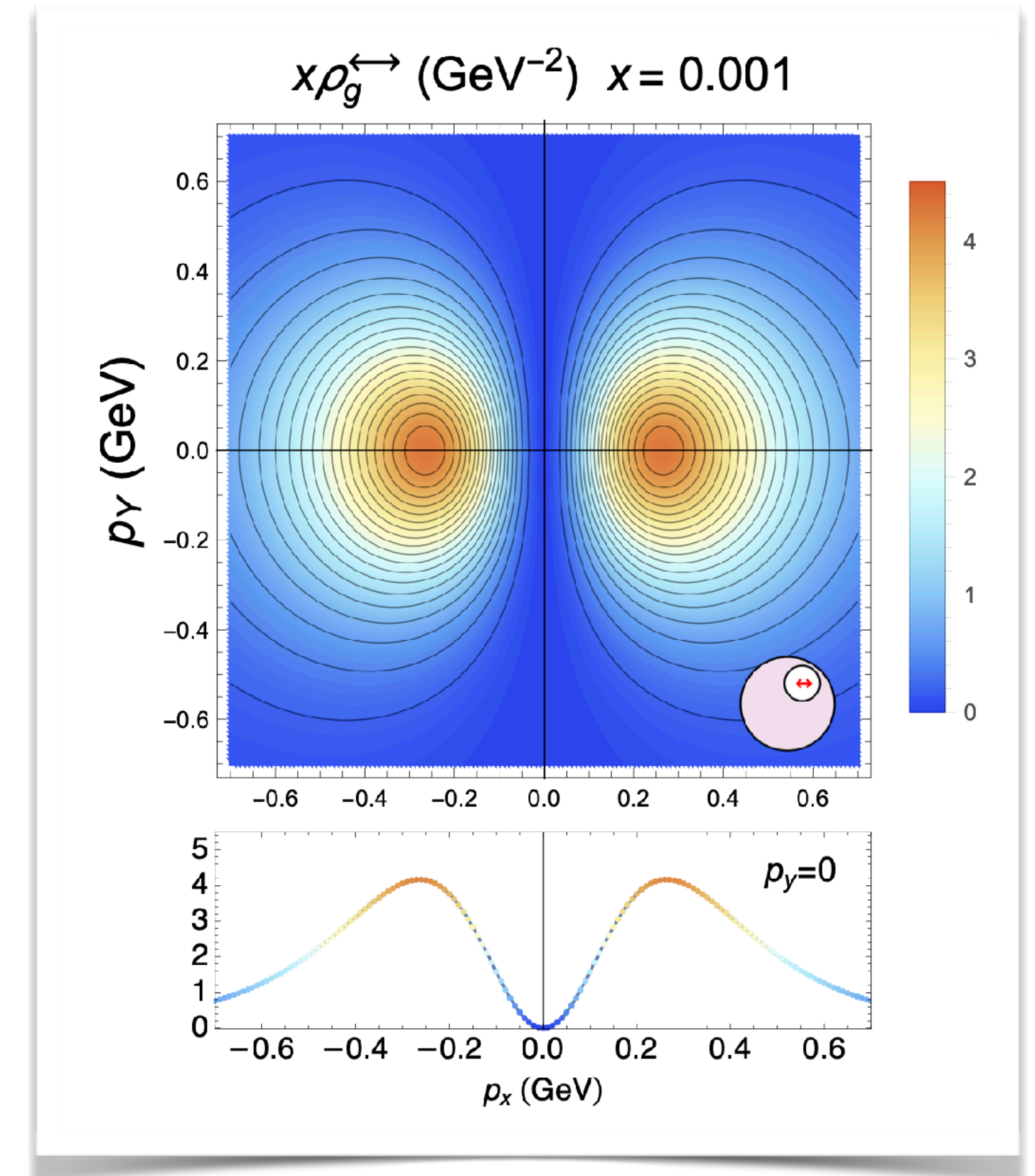
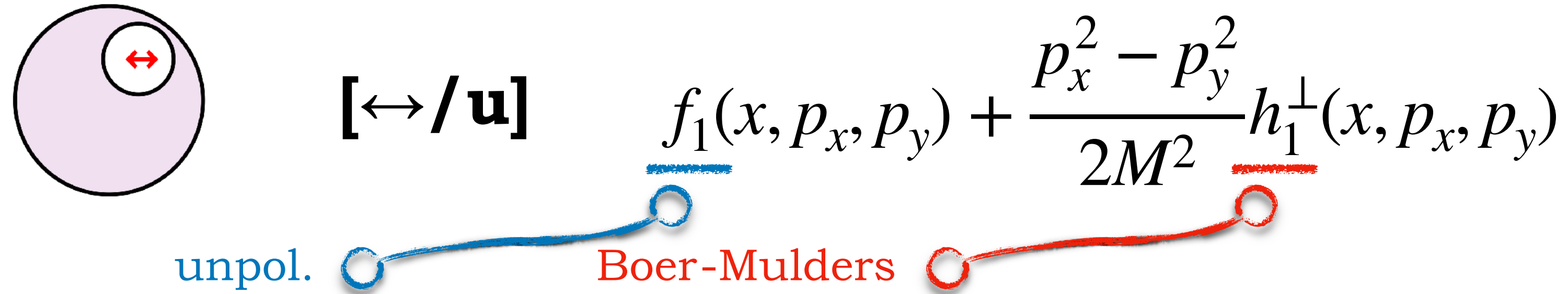




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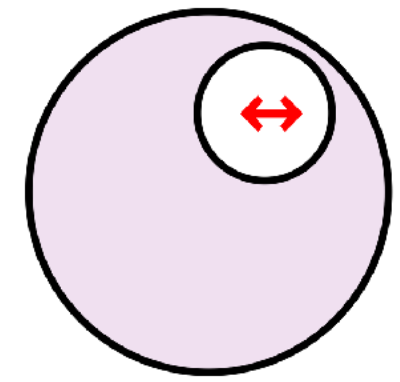


# Boer-Mulders effect in unpolarized $pp$ collisions



[\[A. Bacchetta, F.G.C., M. Radici, P. Taelis \(2020\)\]](#)

# Boer-Mulders effect in unpolarized $pp$ collisions

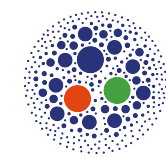


$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

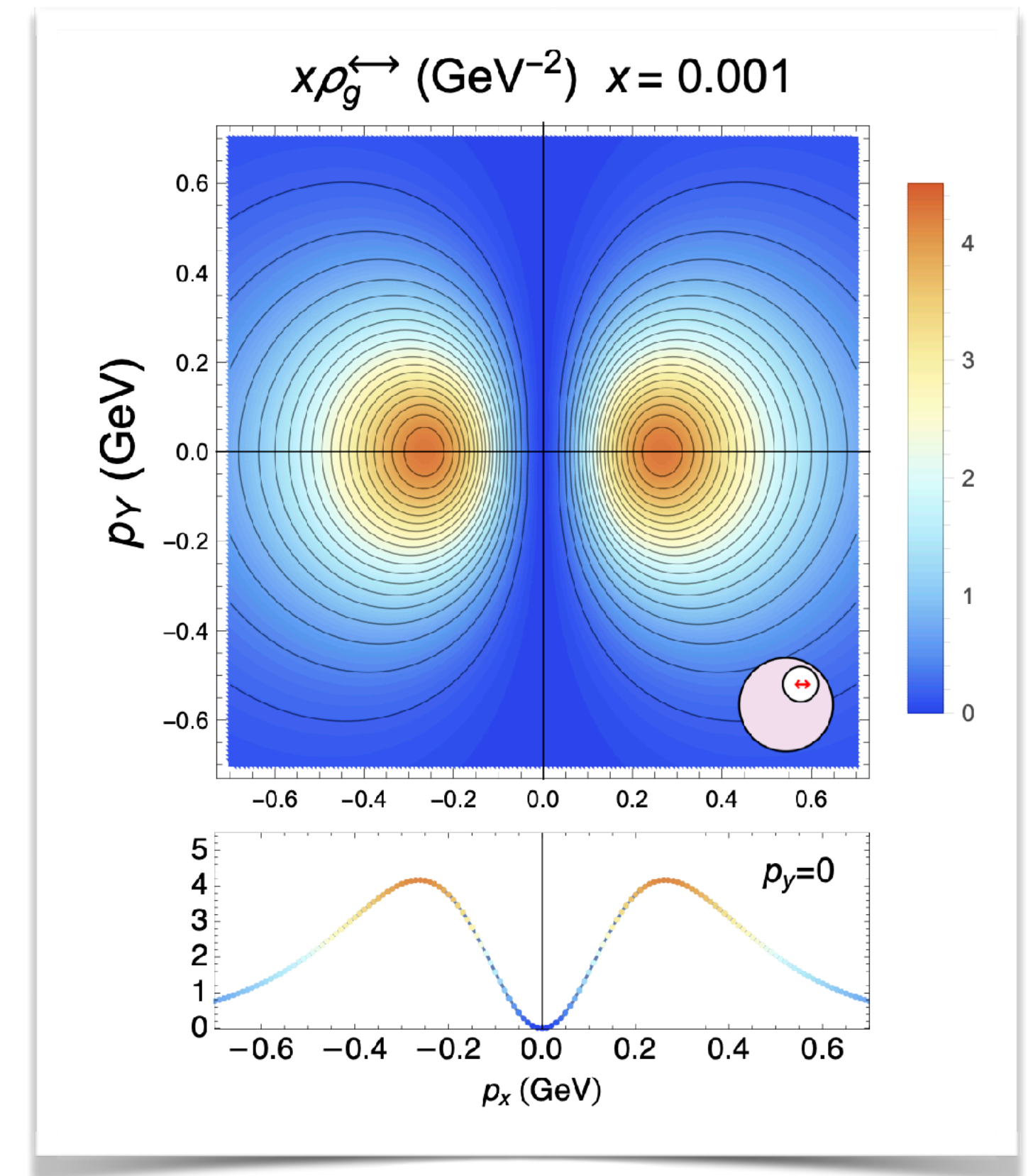
Boer-Mulders



(Pseudo)scalar Higgs  $p_T$ -distribution

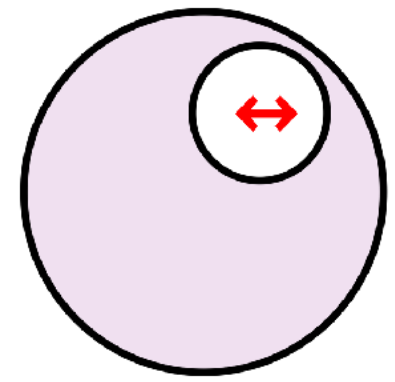
$$\frac{E d\sigma^{H(A)}}{d^3\vec{q}} \Big|_{q_T \ll m_H} = \frac{\pi\sqrt{2}G_F}{128m_H^2 S} \left(\frac{\alpha_s}{4\pi}\right)^2 |\mathcal{A}_{H(A)}(\tau)|^2 \times \left( C [f_1^g f_1^g] \pm C [w_H h_1^{\perp g} h_1^{\perp g}] \right) + \mathcal{O}\left(\frac{q_T}{m_H}\right)$$

[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]  
(Higgs+jet angular distributions) [D. Boer, C. Pisano (2015)]



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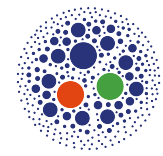
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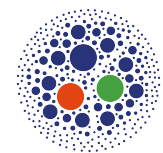
Boer-Mulders



## (Pseudo)scalar Higgs $p_T$ -distribution

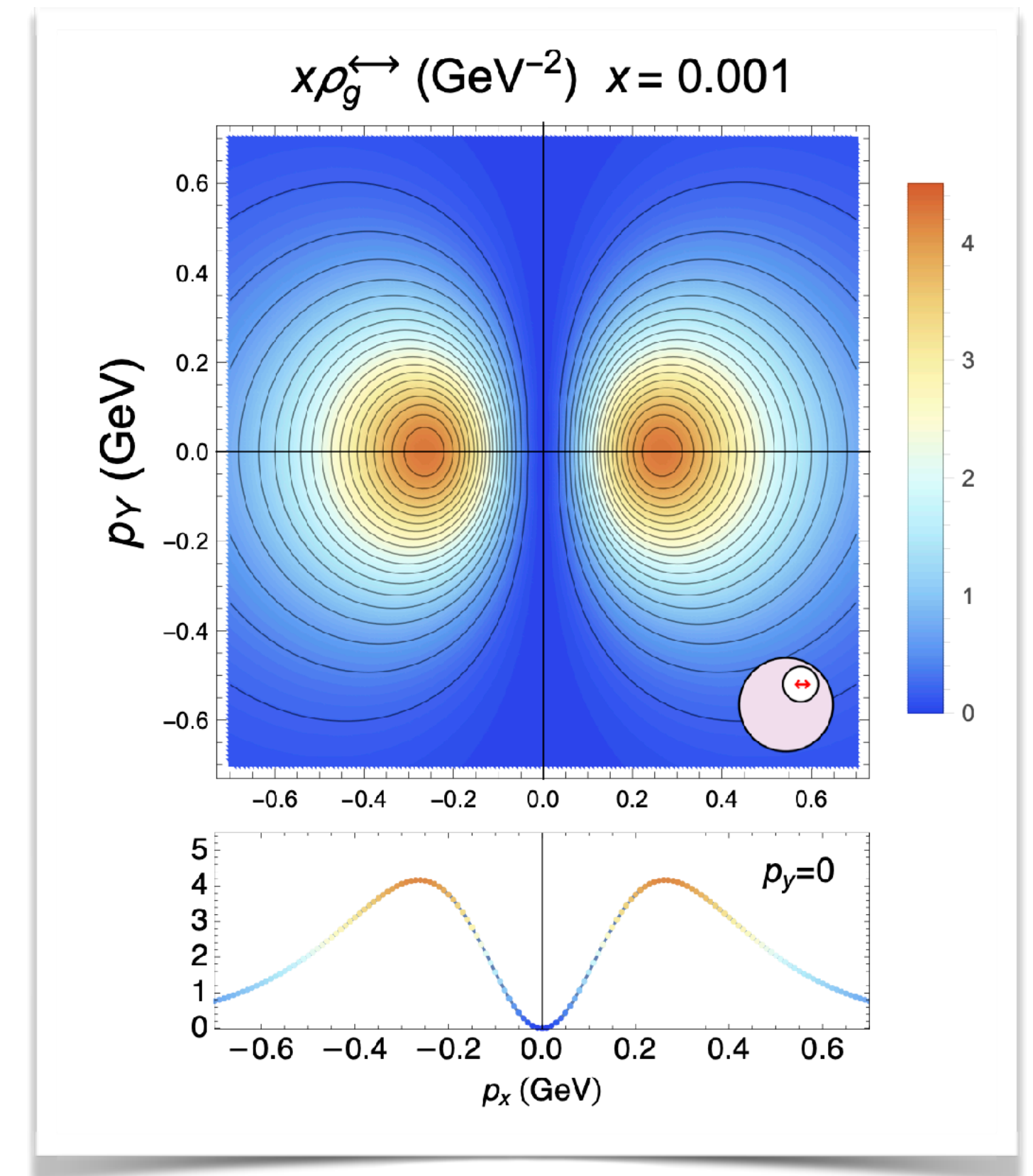
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## Model prediction at low- $x$



$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$



[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

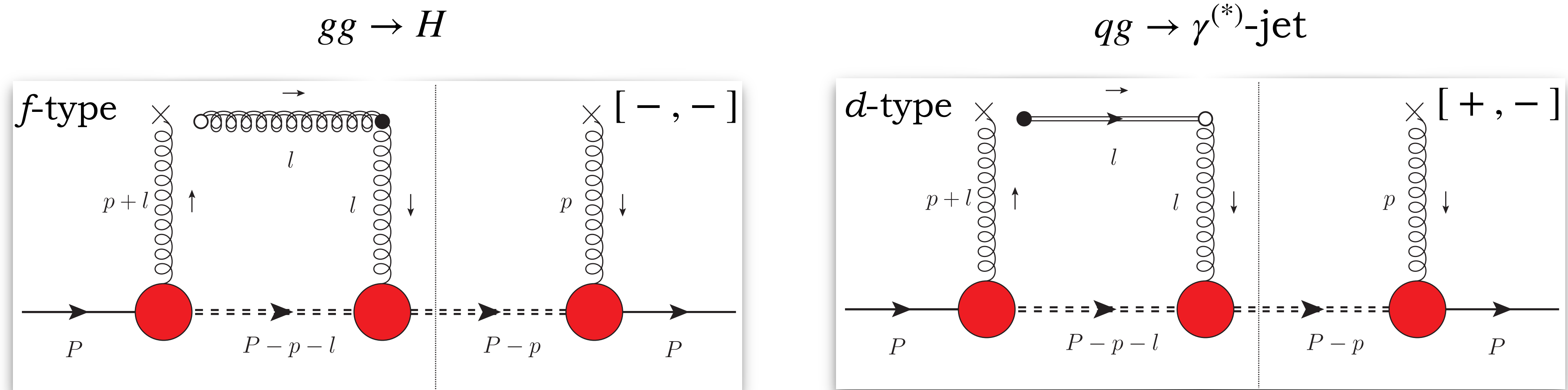
**...towards twist-2  
T-odd gluon TMDs**

# ***T*-odd gluon TMDs in a spectator model**

-  No residual gluon-spectator interaction at tree level
-  *Interference* with one-gluon exchange (*eikonal*)

# $T$ -odd gluon TMDs in a spectator model

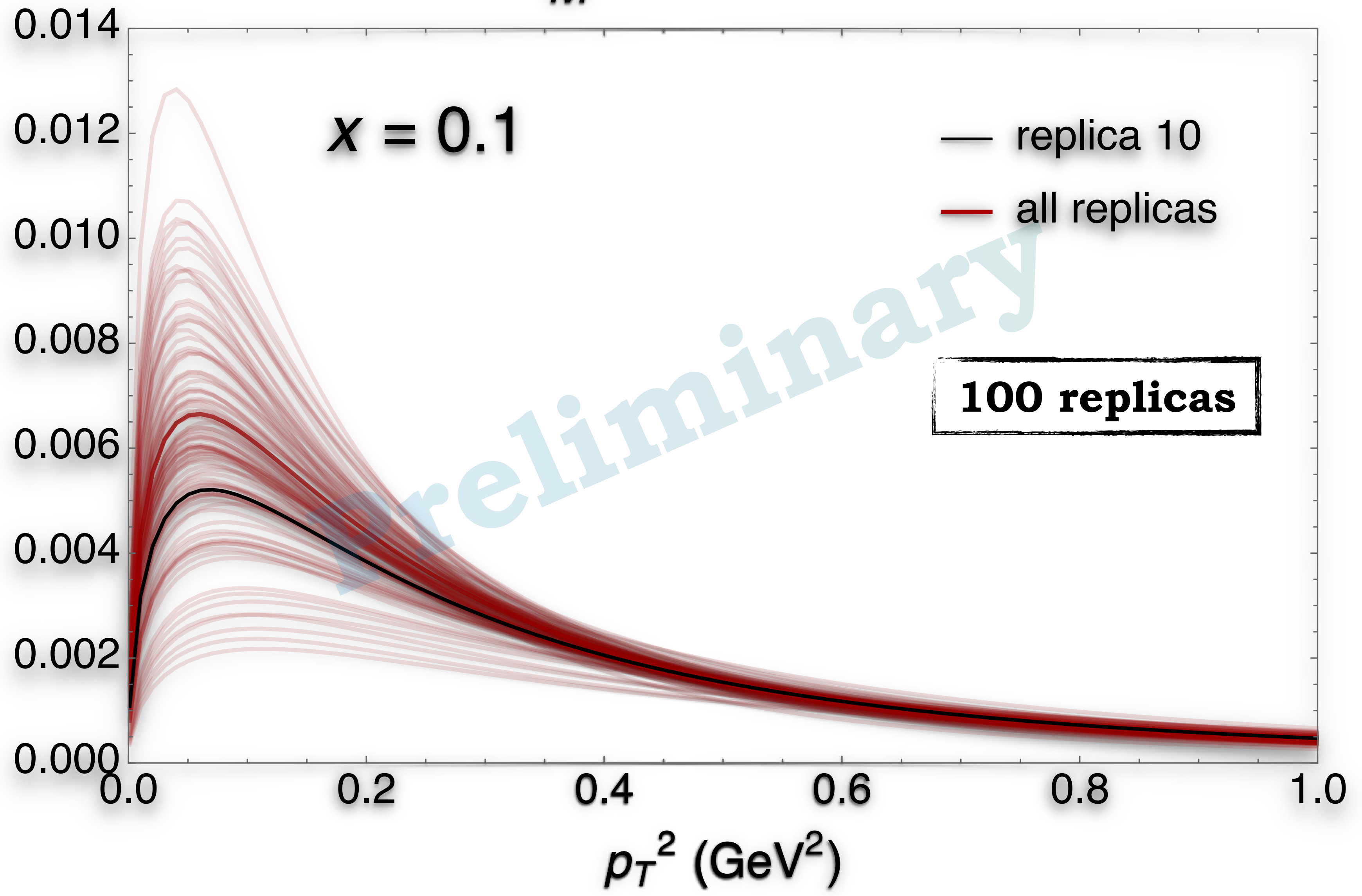
- No residual gluon-spectator interaction at tree level
- *Interference* with one-gluon exchange (*eikonal*)



- Leading-twist one-gluon-exchange of the gauge-link operator
- Sensitivity to WW/DP structures
- Calculation of **Sivers** function *underway!*

# Sivers gluon TMD

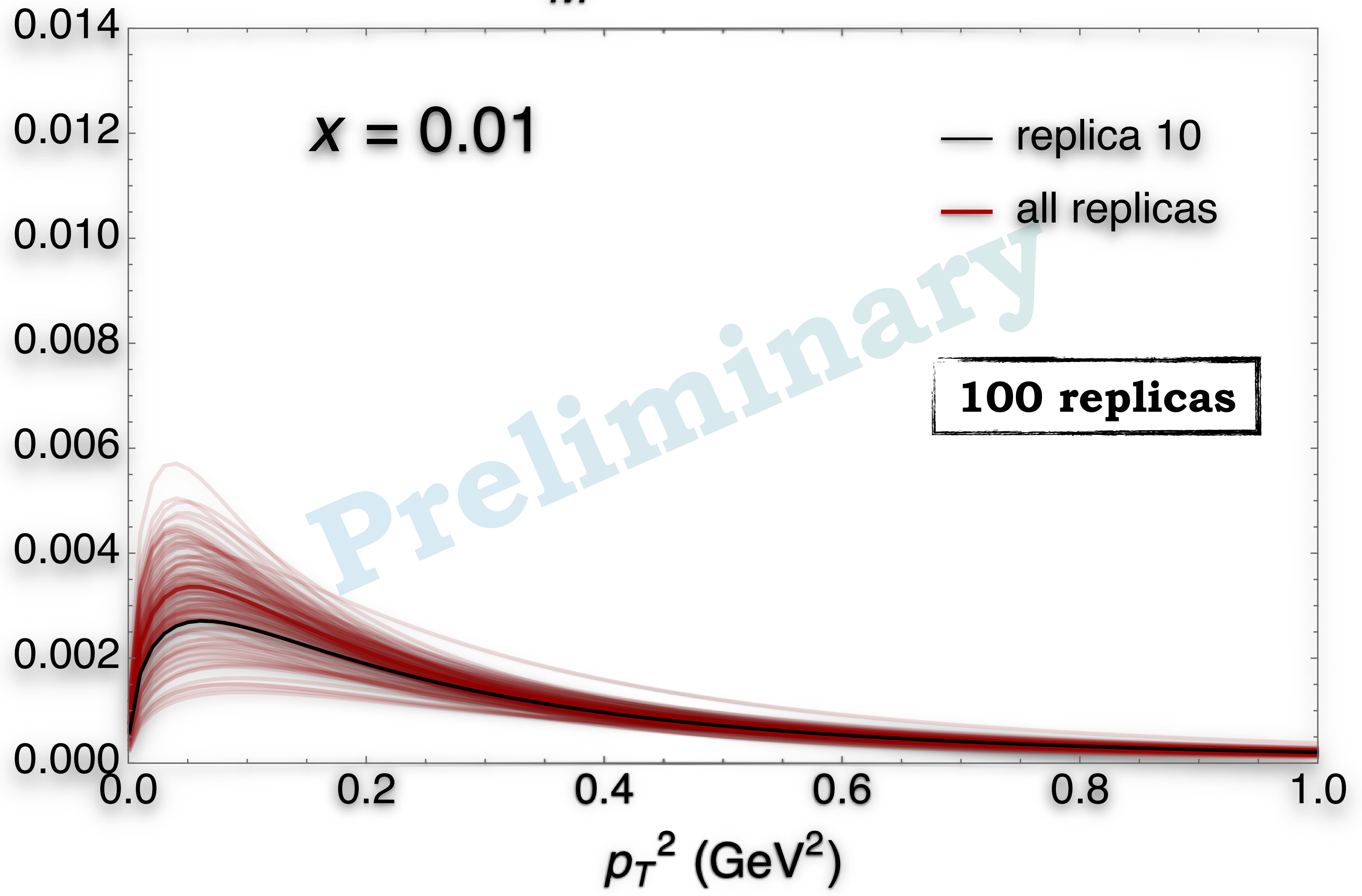
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$





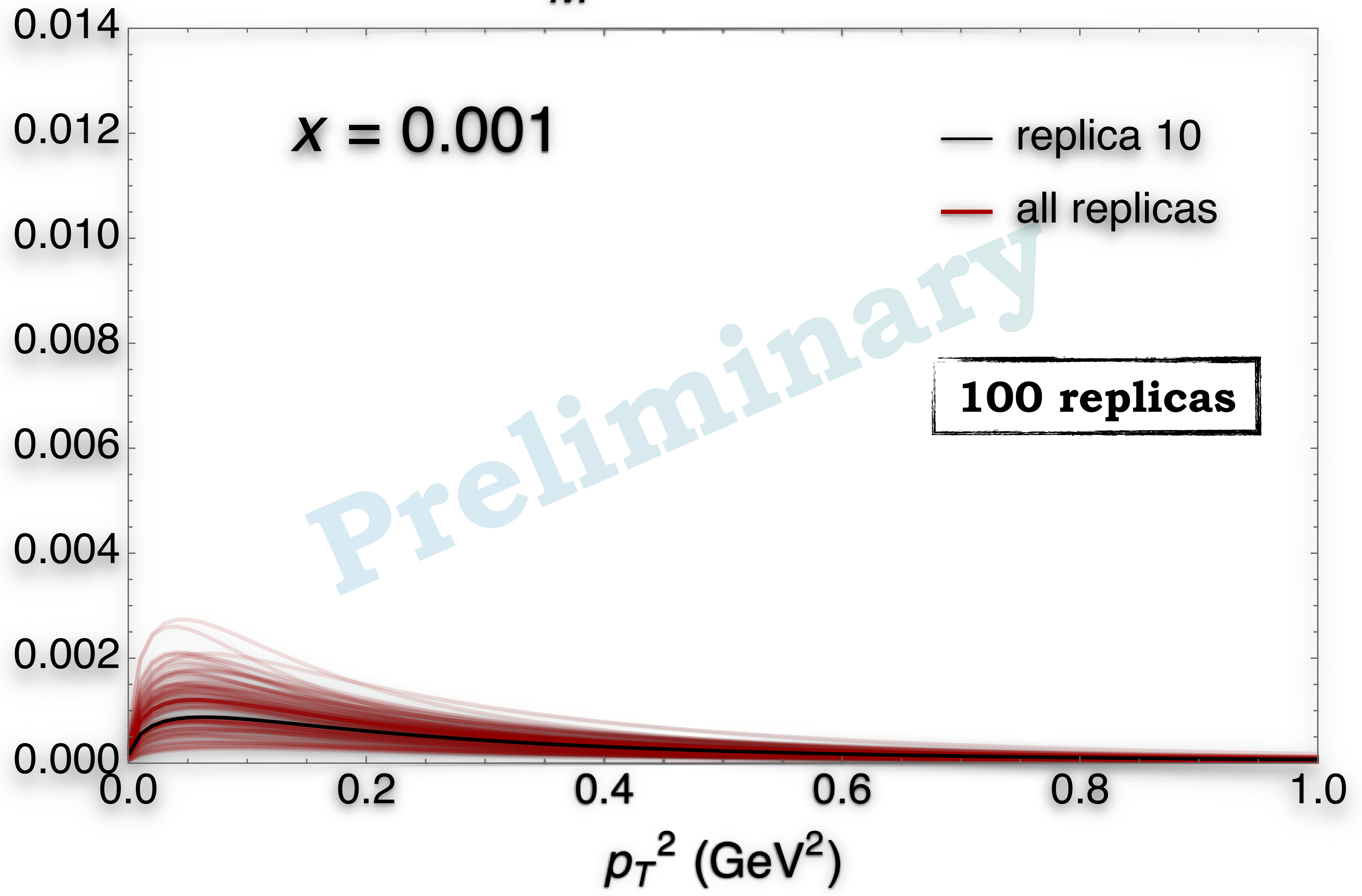
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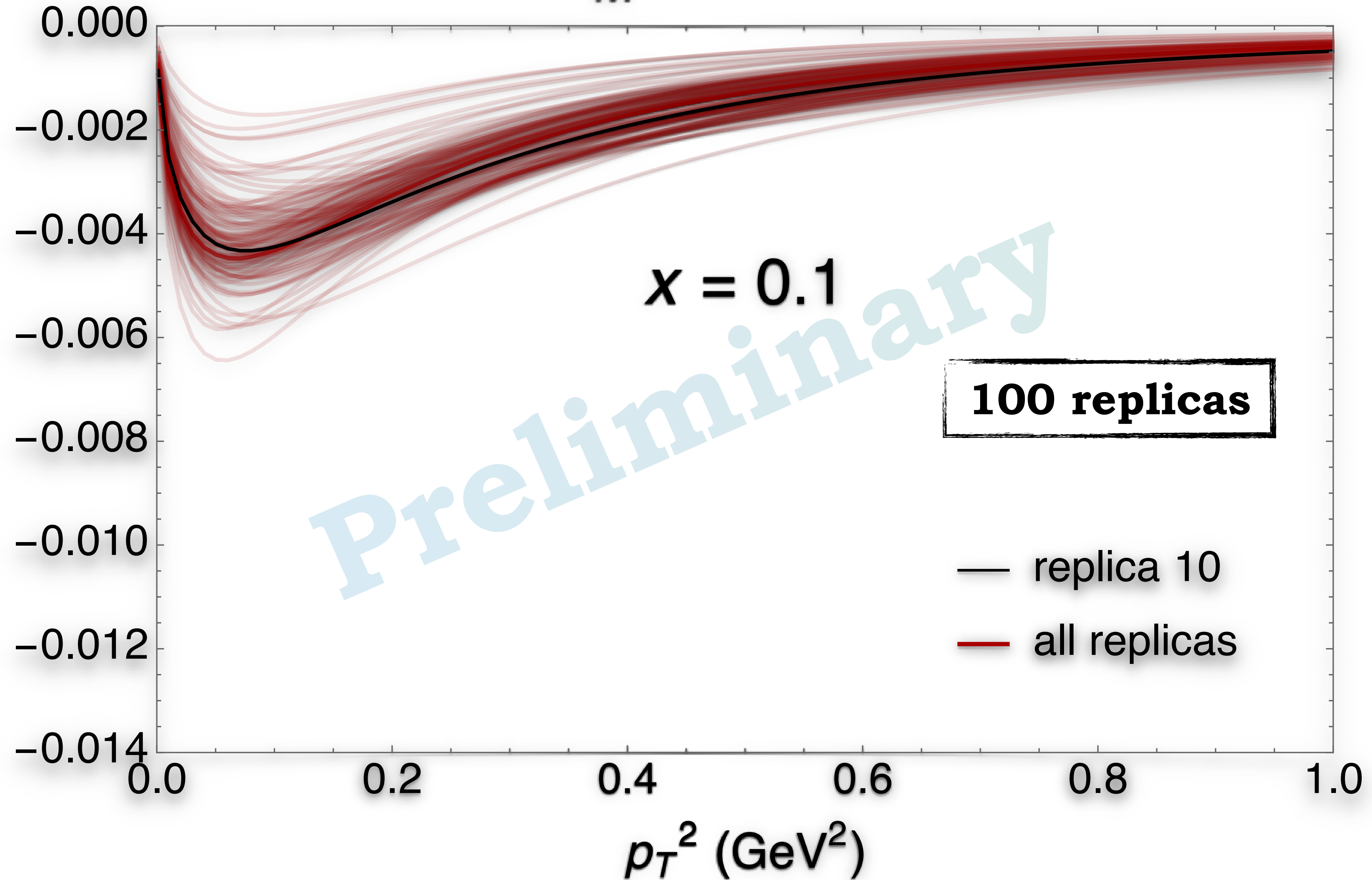
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Preliminary

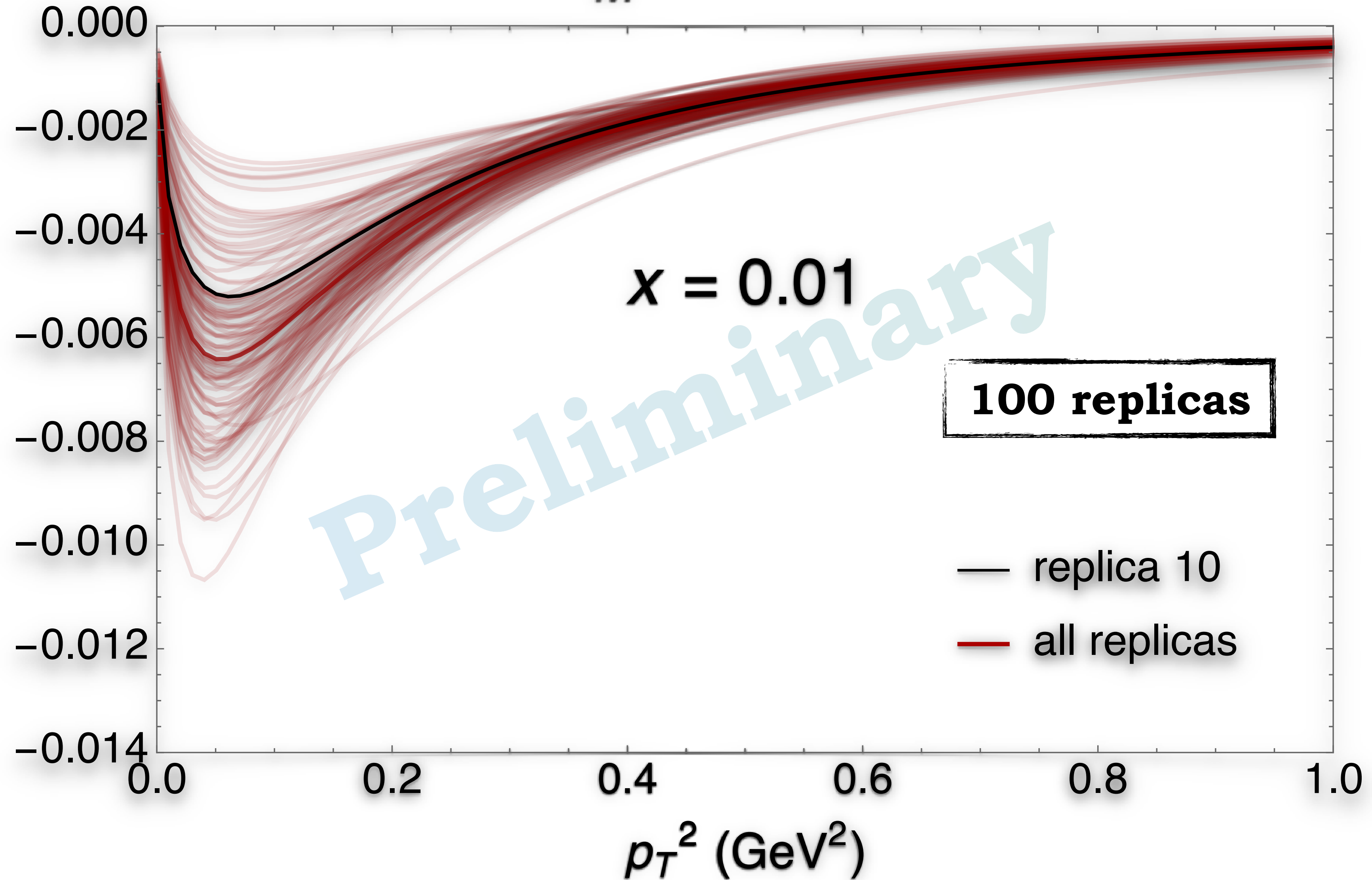
# Linearity gluon TMD

$$x \frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)$$



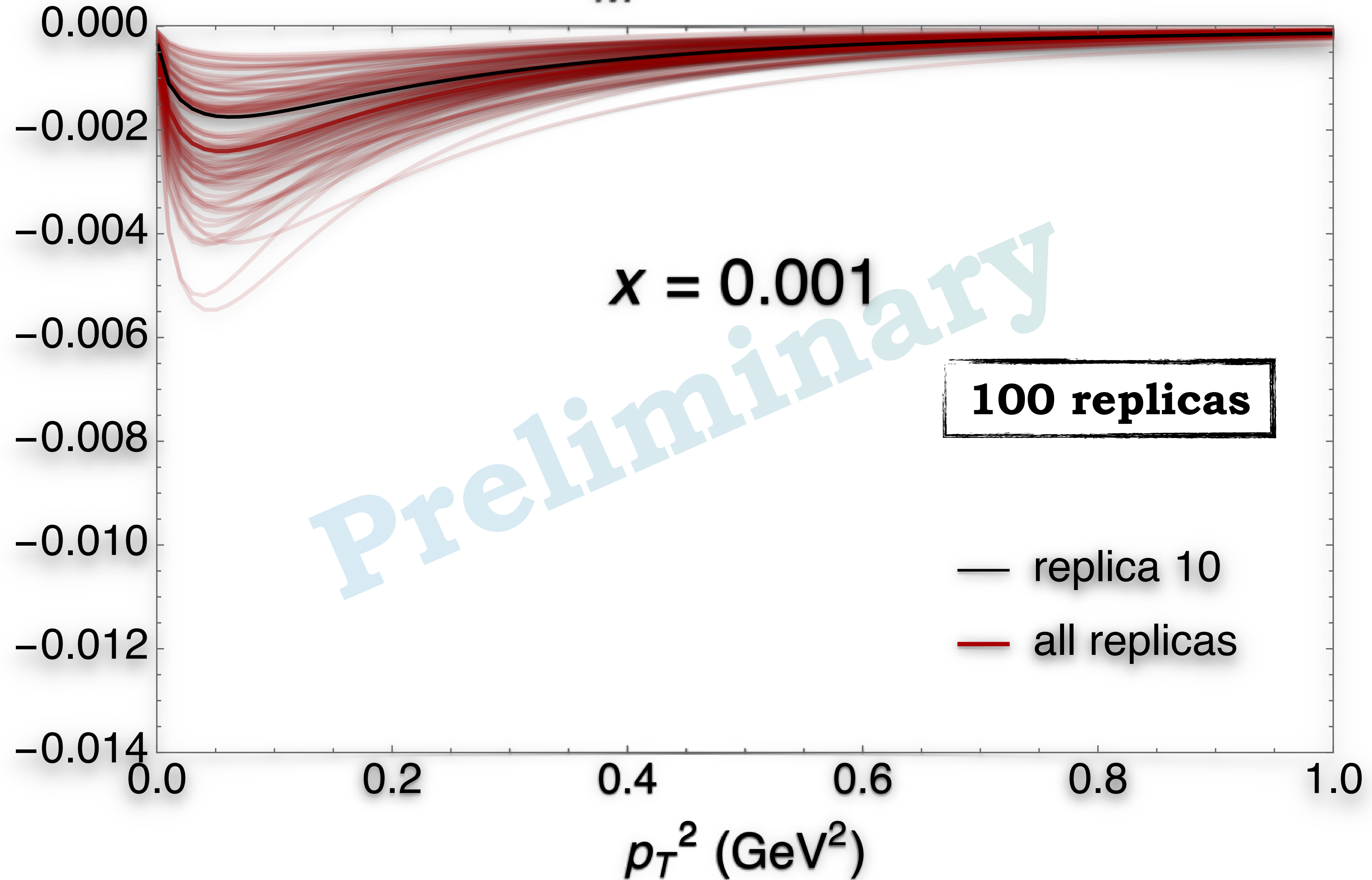
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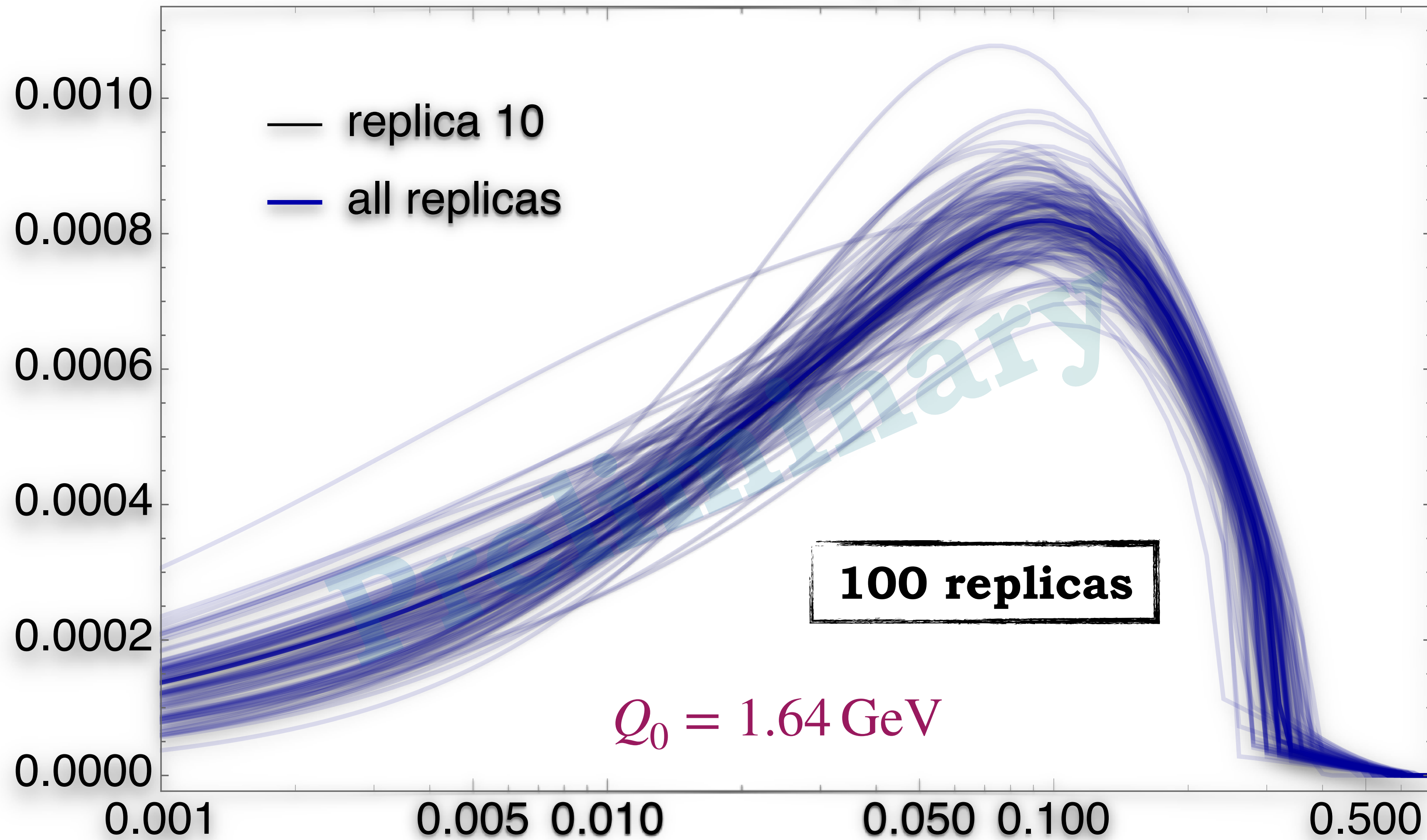


# Qiu-Sterman twist-3 gluon PDF

$$f_{1T}^{\perp(f)}(x) = \int d^2p_T \frac{p_T^2}{2M^2} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

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# Checkpoints and further steps

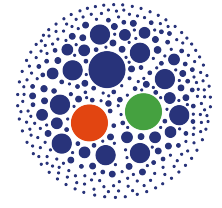
- ☑ Systematic calculation of all twist-2  $T$ -even gluon TMDs
- ☑ Spectral mass to catch small- and large- $x$  effects
- ☑ **Simultaneous fit** of  $f_1$  and  $g_1$  PDFs via **replica method**



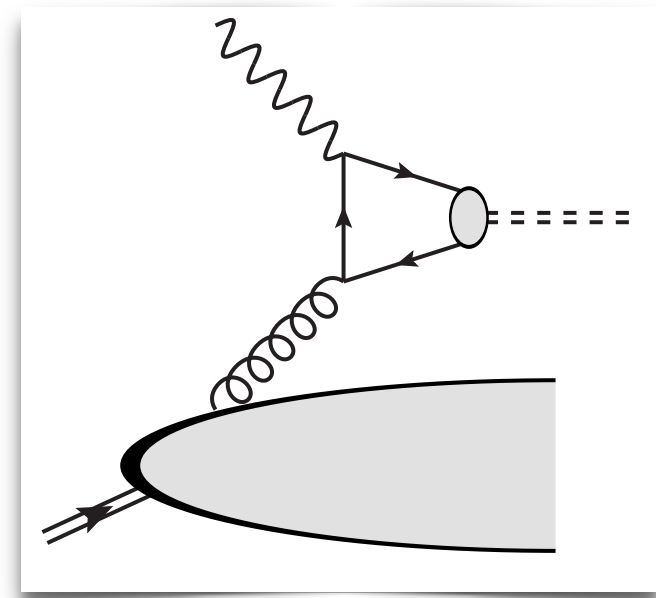
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- Twist-2  $T$ -odd gluon TMDs (**Sivers**, etc.) in progress!
- Pheno: **spin asymmetries**, **pseudodata** and **impact studies**
- Evolution: extension to quark TMDs in the same framework
- Explorative studies on gauge-link sensitivity and factorization
- Studies on GPD and small- $x$  UGD sectors

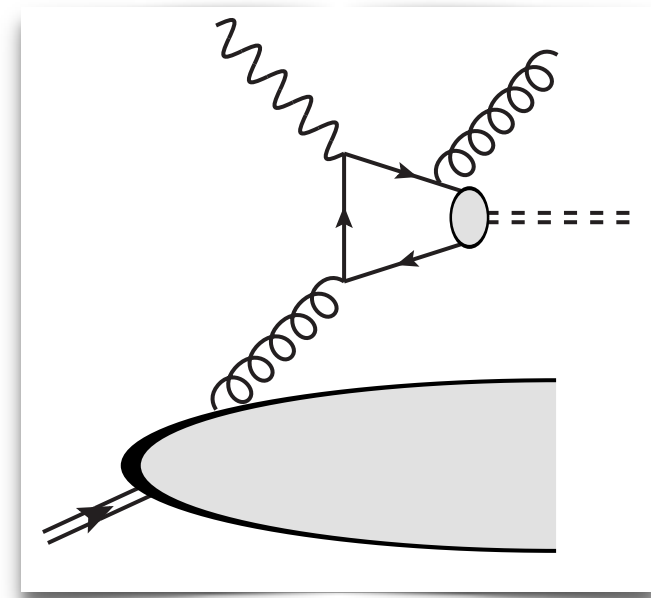
# Hadronic structure and quarkonia



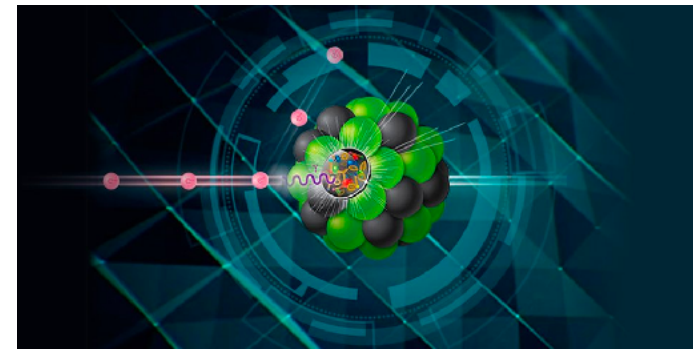
## SIDIS



CO



CO + CS



EIC Yellow Report Document  
[\(EICUG website\)](#)

[\[EICUG \[arXiv:2103.05419\]\]](#)

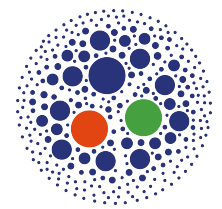
On the physics potential to study the gluon content of proton and deuteron at NICA SPD

[\[NICA Collaboration \[arXiv:2011.15005\]\]](#)

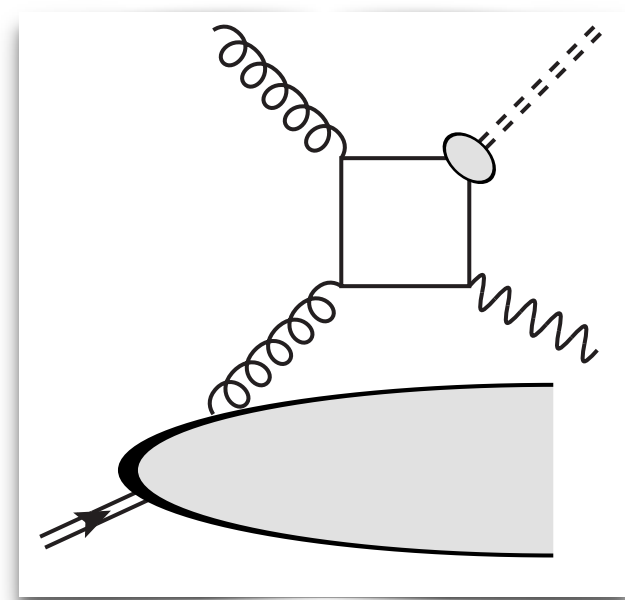
Perspectives for quarkonium studies at the high-luminosity LHC

[\(QAT 2021 Workshop\)](#)

[\[Quarkonia As Tools Collaboration \[arXiv:2012.14161\]\]](#)



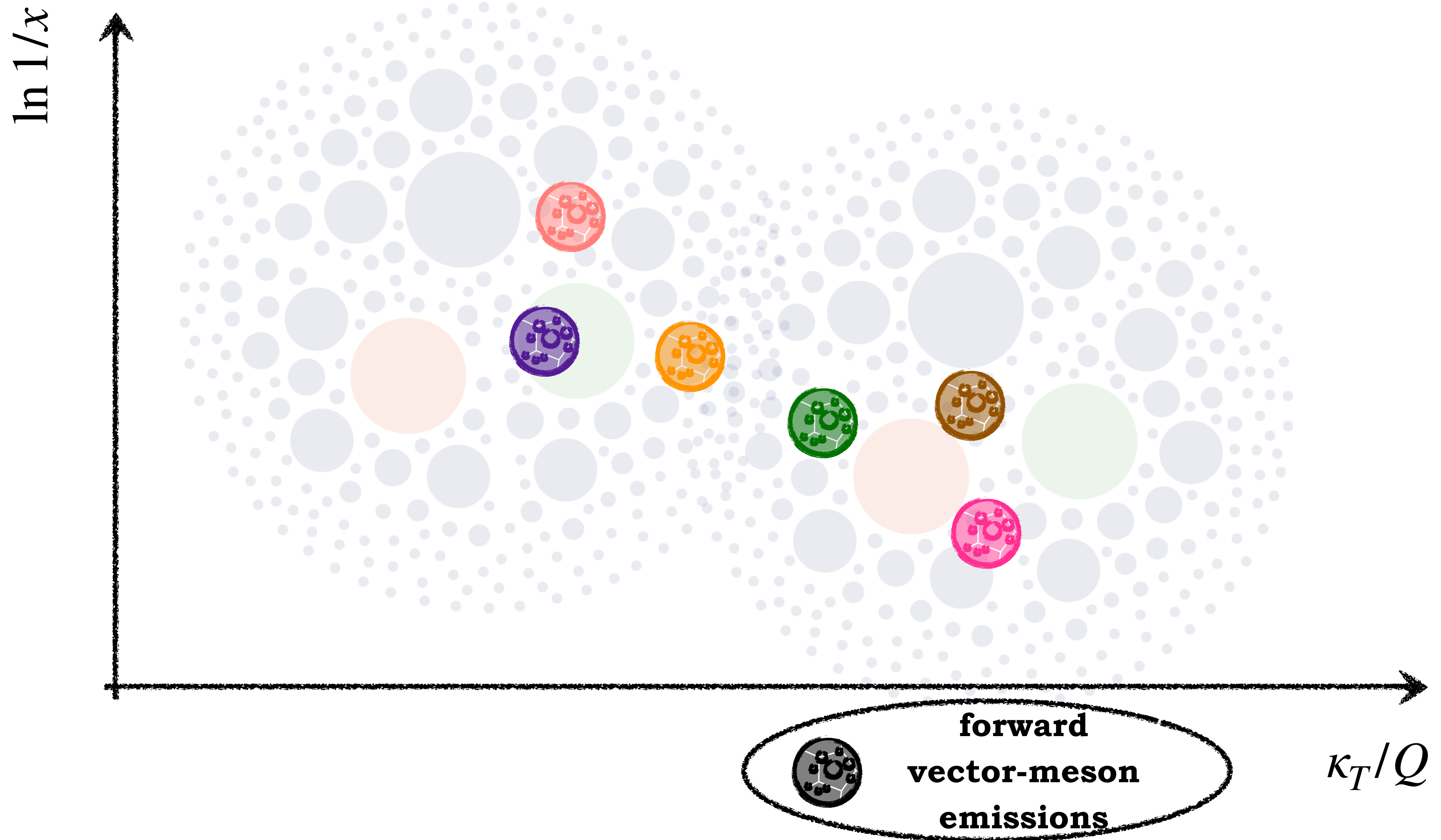
## Hadroproduction



CO + CS

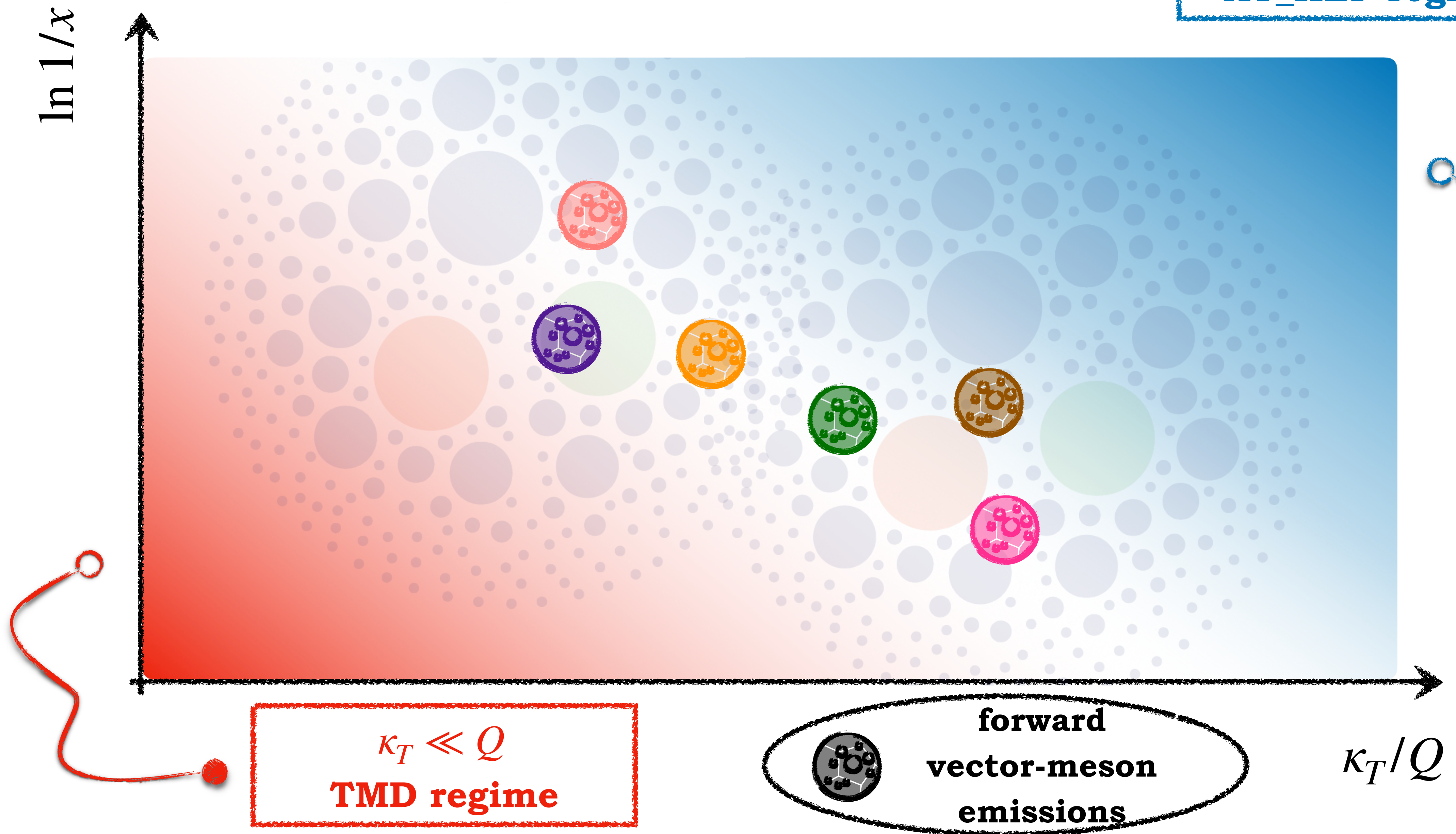
# Hadronic structure and vector mesons

# Hadronic structure and vector mesons



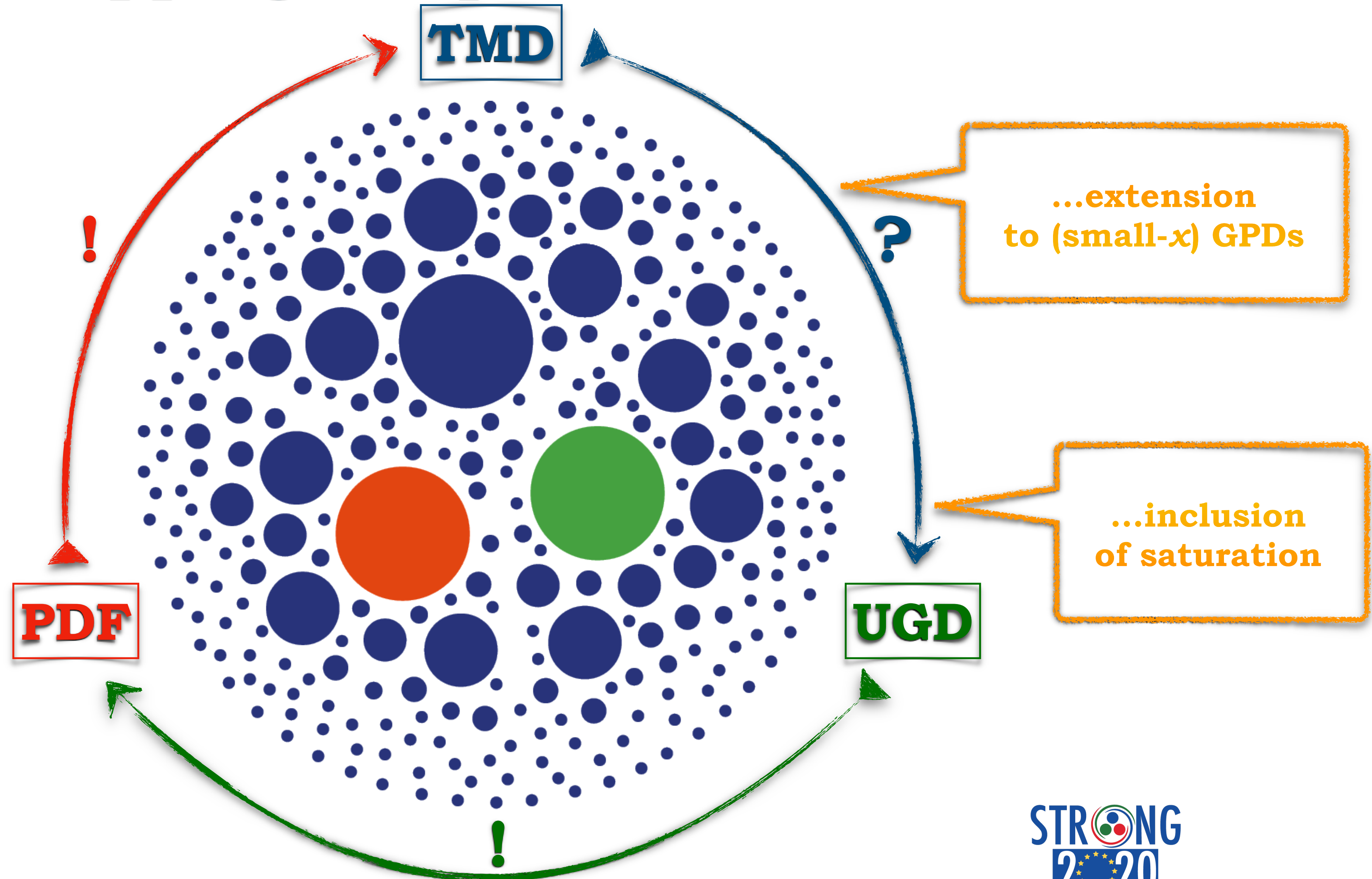
# Hadronic structure and vector mesons

$x \ll 1; \kappa_T \approx Q \gg \Lambda_{\text{QCD}}$   
**AT\_HEF regime**



**...towards a  
*multi-lateral*  
formalism**

# Mapping the proton content



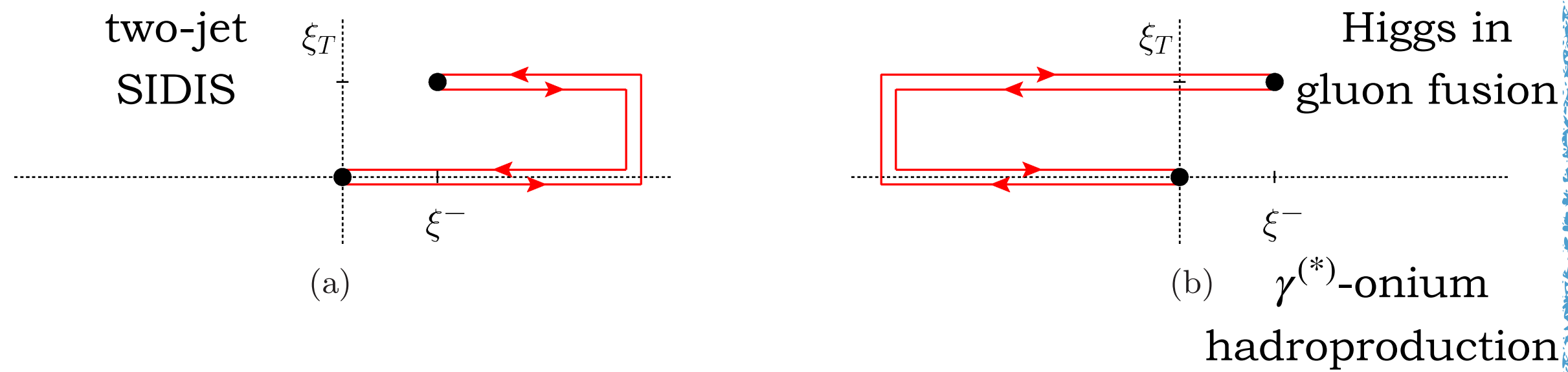
**Backup  
slides**



# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

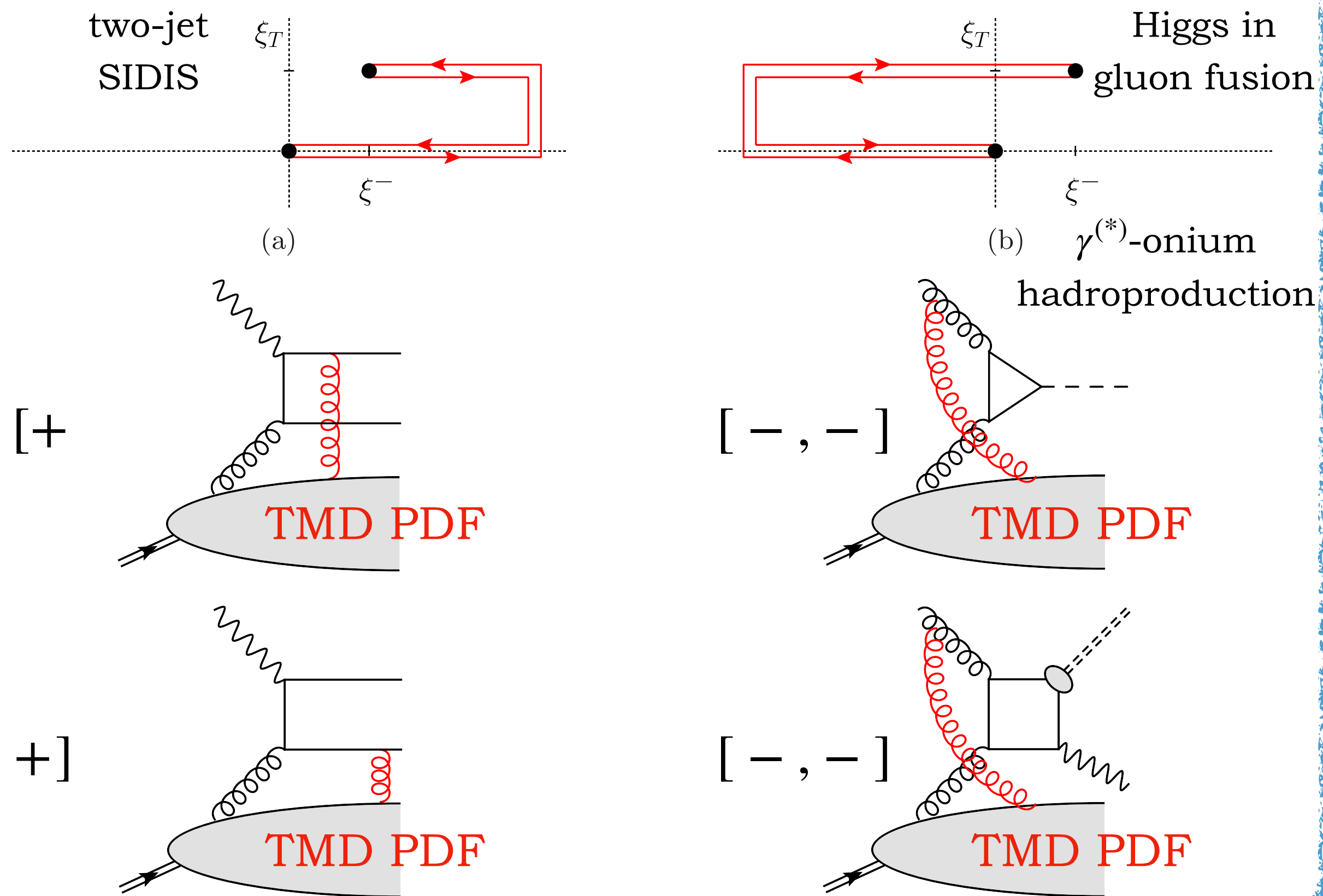
(a)  $[+, +]$  or (b)  $[-, -]$



# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

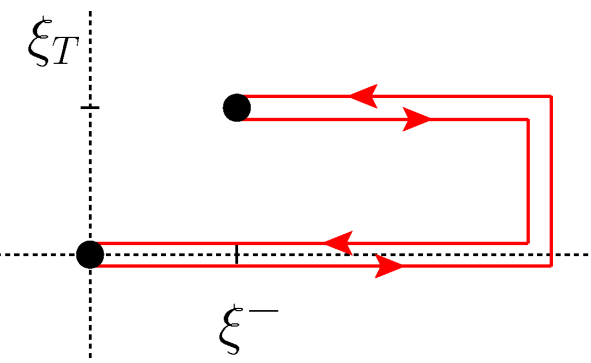


# Accessing WW and DP gluon TMDs

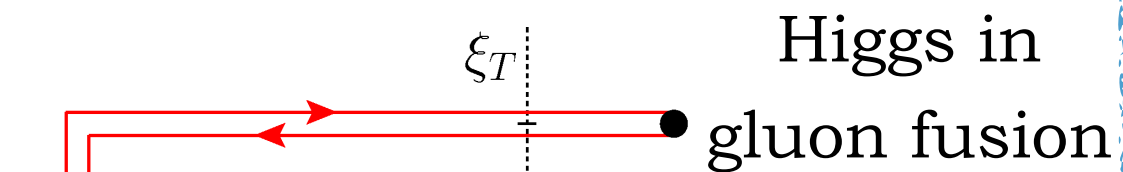
## Weizsäcker-Williams (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

two-jet  
SIDIS



(a)

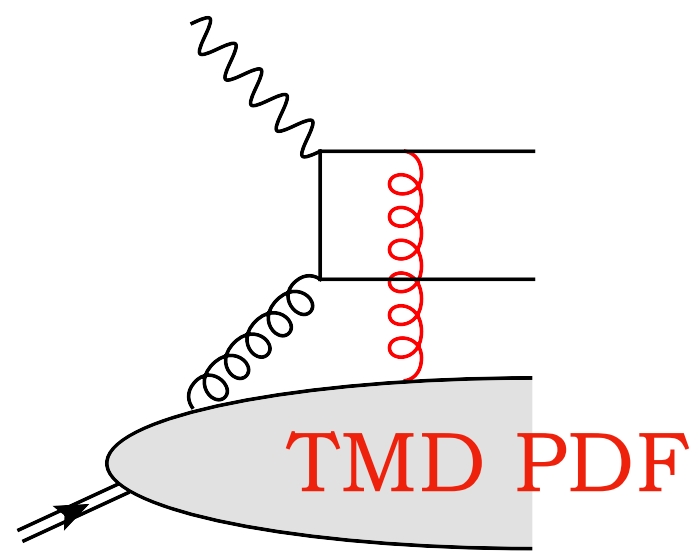


(b)

Higgs in  
gluon fusion

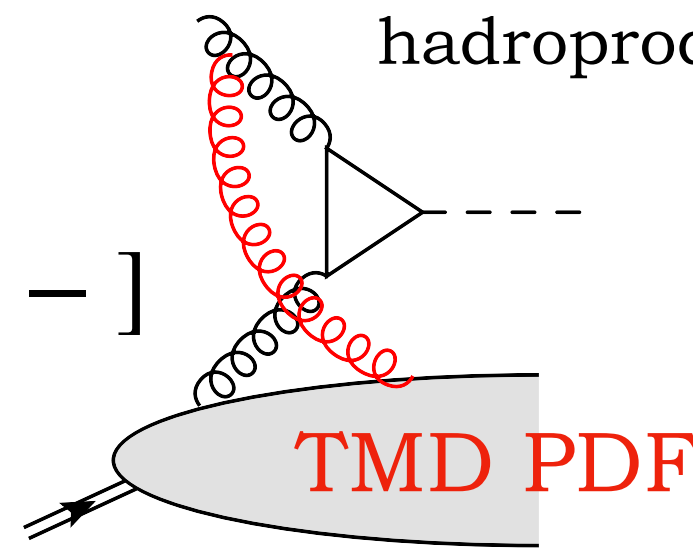
$\gamma^{(*)}$ -onium  
hadroproduction

[+]



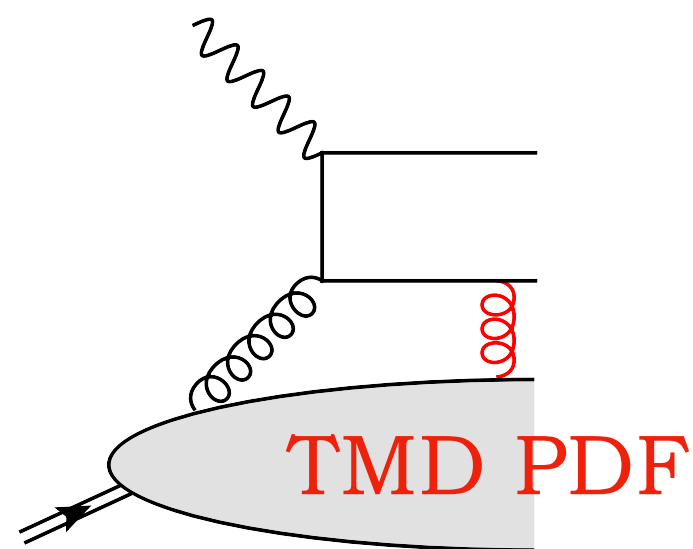
TMD PDF

[-, -]



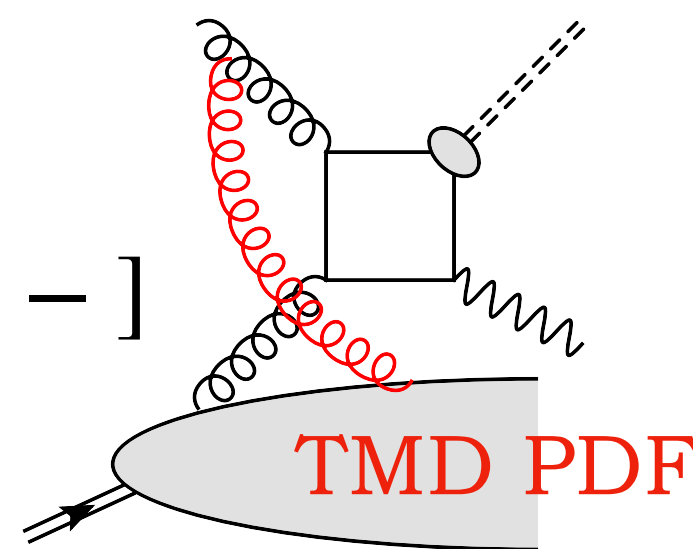
TMD PDF

+] ]



TMD PDF

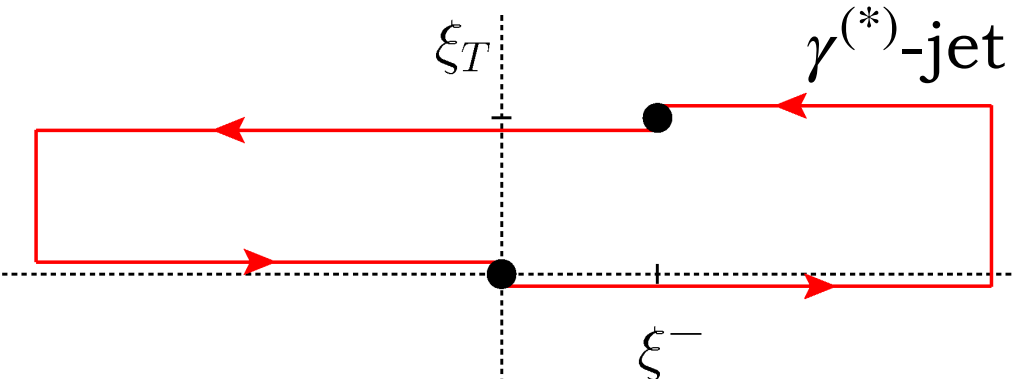
[-, -]



TMD PDF

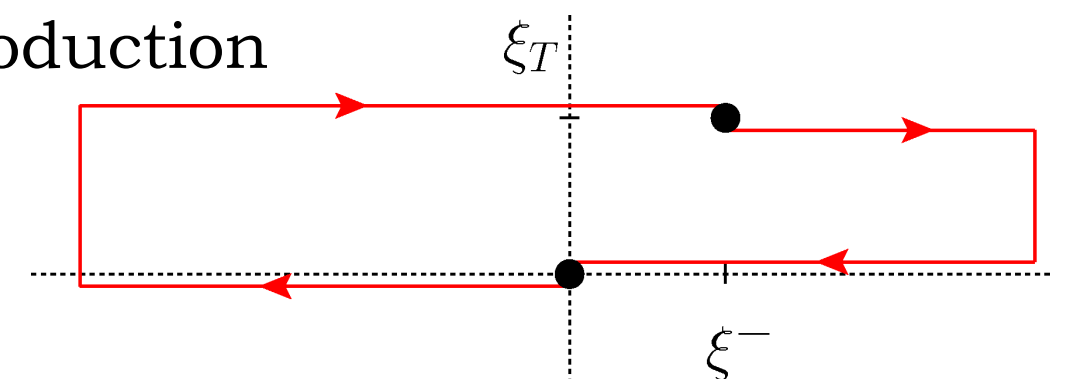
## Dipole (DP)

(c)  $[+, -]$  or (d)  $[-, +]$



(c)

$\gamma^{(*)}$ -jet hadroproduction



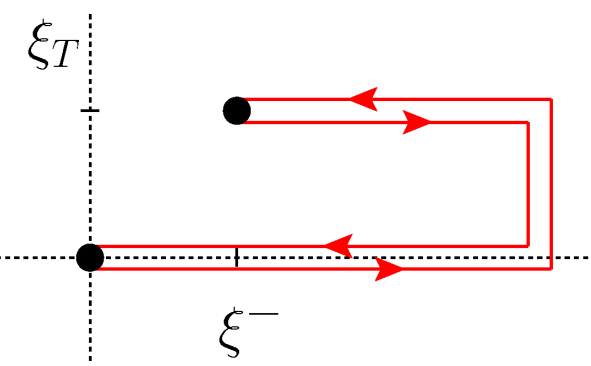
(d)

# Accessing WW and DP gluon TMDs

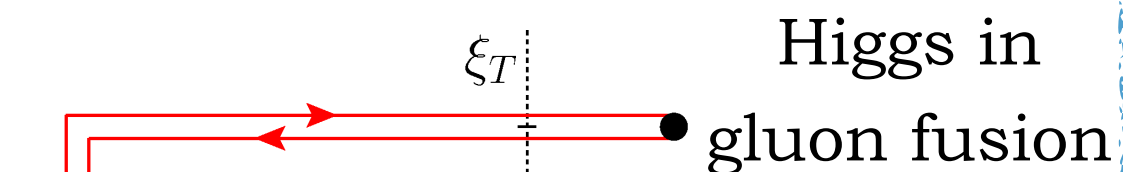
## Weizsäcker-Williams (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

two-jet  
SIDIS



(a)

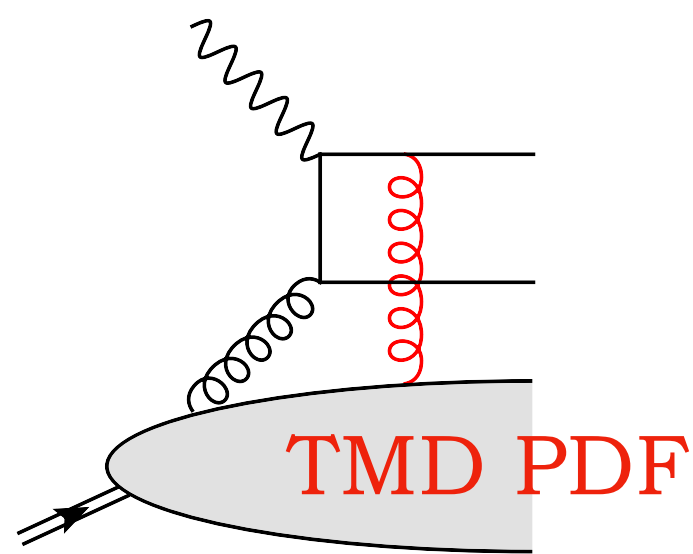


(b)

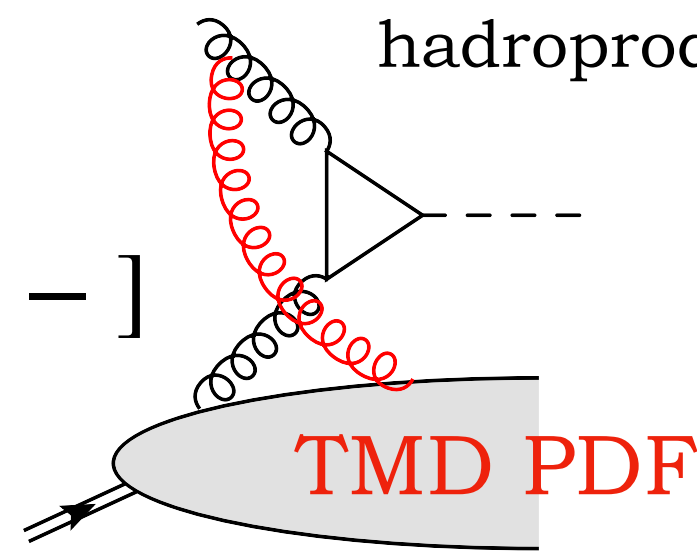
Higgs in  
gluon fusion

$\gamma^{(*)}$ -onium  
hadroproduction

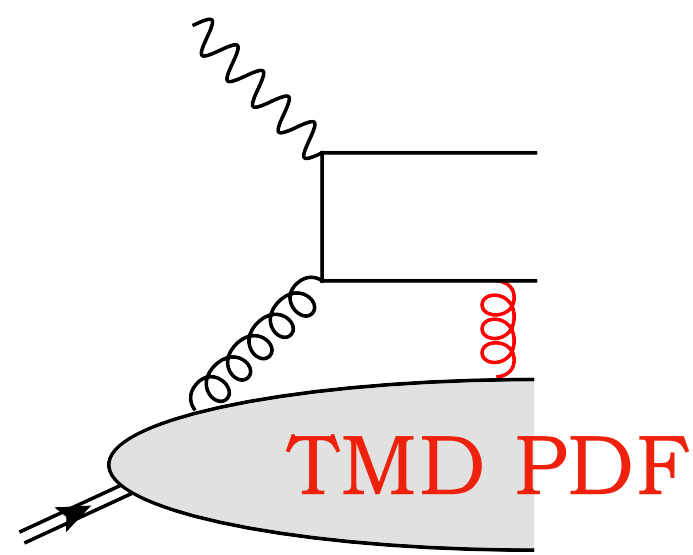
[+]



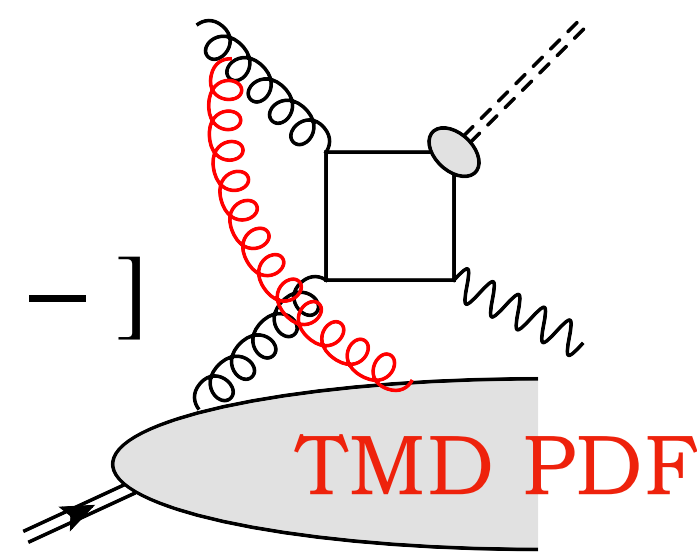
[-, -]



[+]

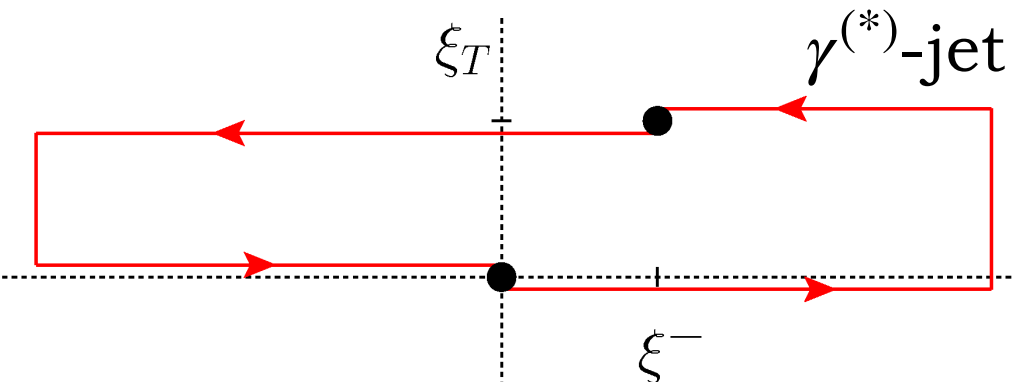


[-, -]

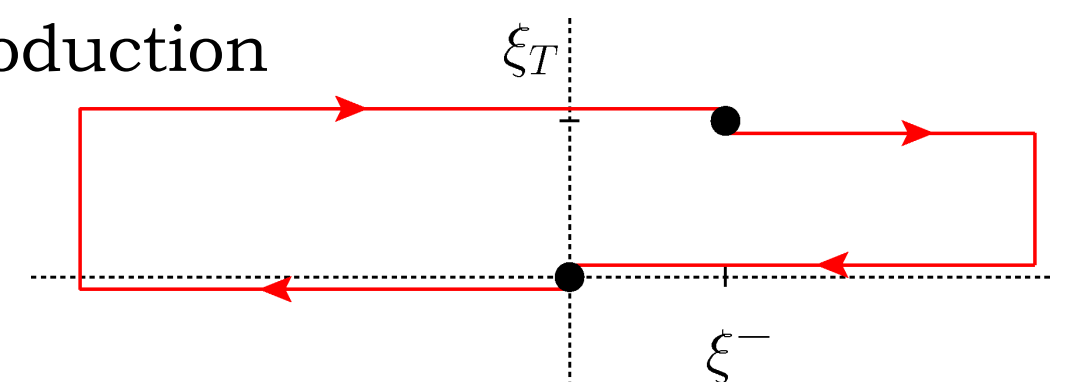


## Dipole (DP)

(c)  $[+, -]$  or (d)  $[-, +]$



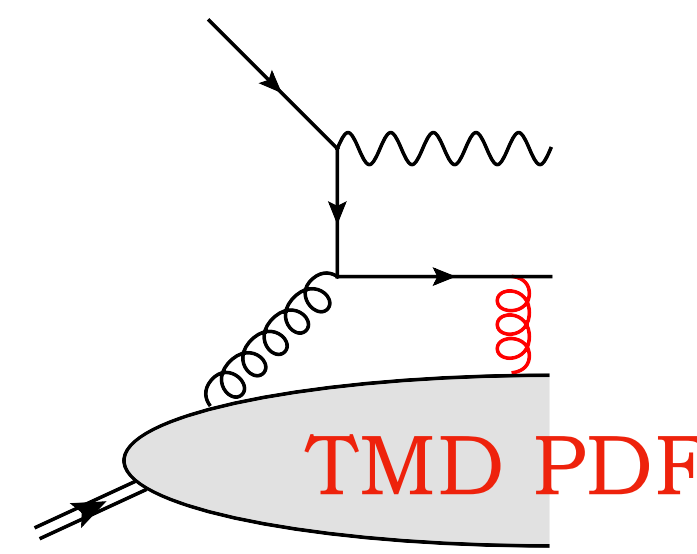
(c)



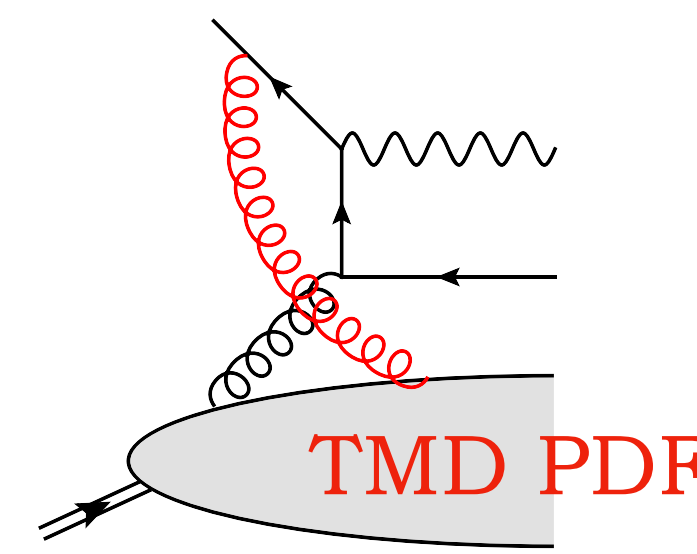
(d)

$\gamma^{(*)}$ -jet hadroproduction

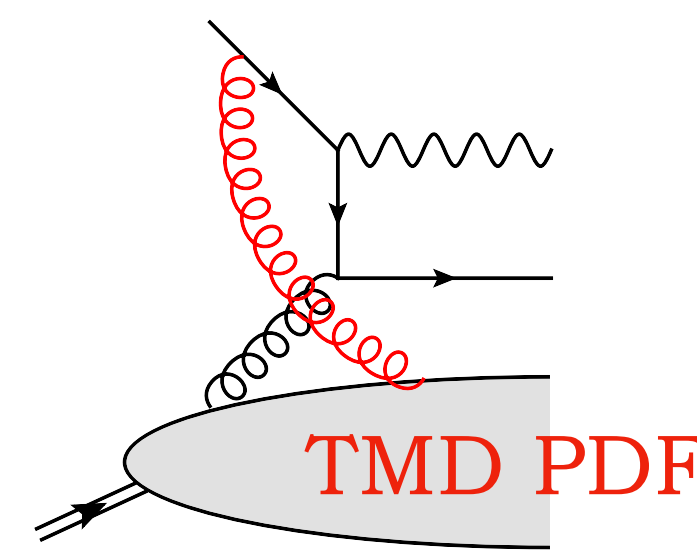
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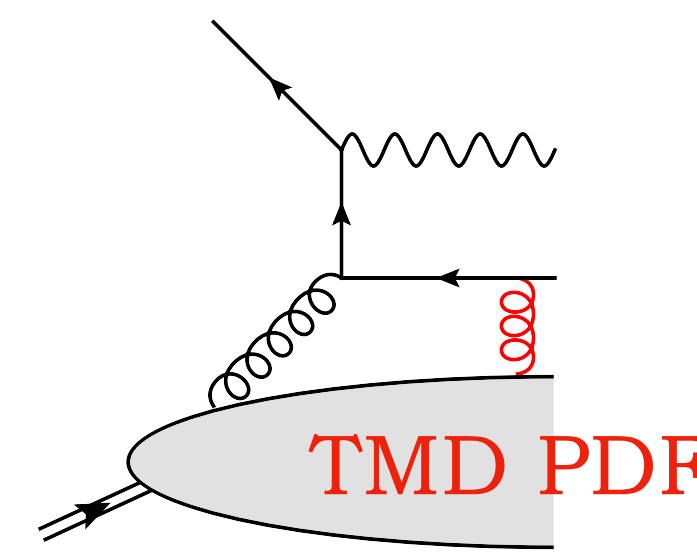
[-]



-]



[+]



# Dihadron hadroproduction and factorization breaking

\* Proof of factorization violation  [T. J. Rogers, P. J. Mulders (2010)]

\* Assumed factorization in SCET and CGC

\* Significance of low- $x$  studies

\* Size of factorization-breaking effects small?

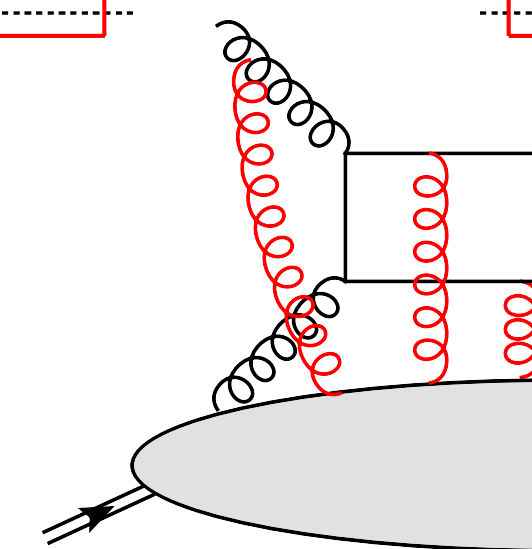
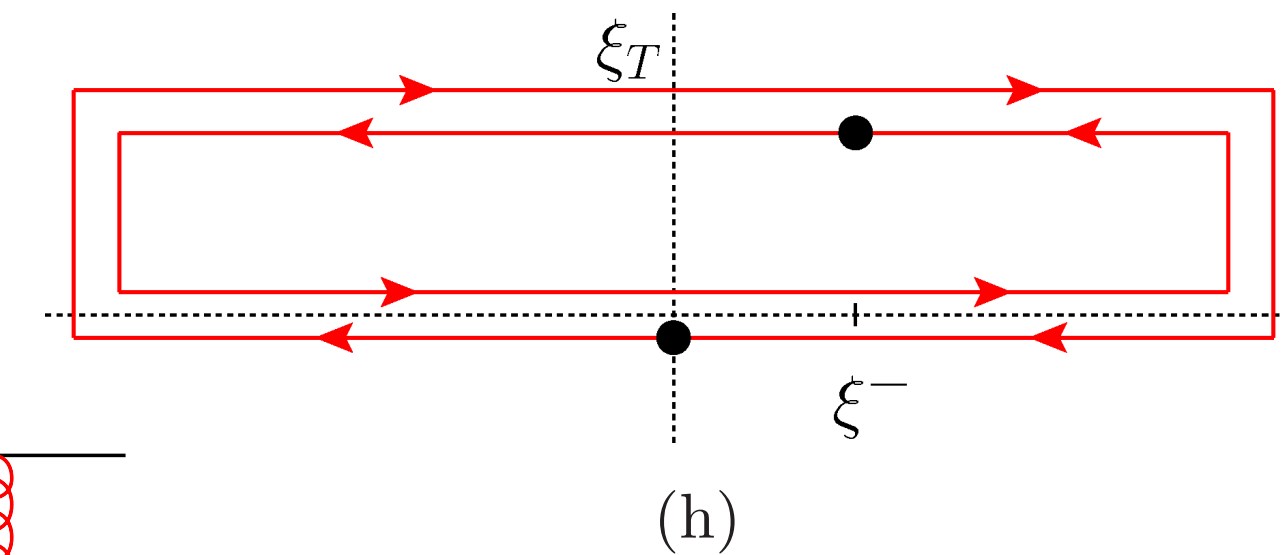
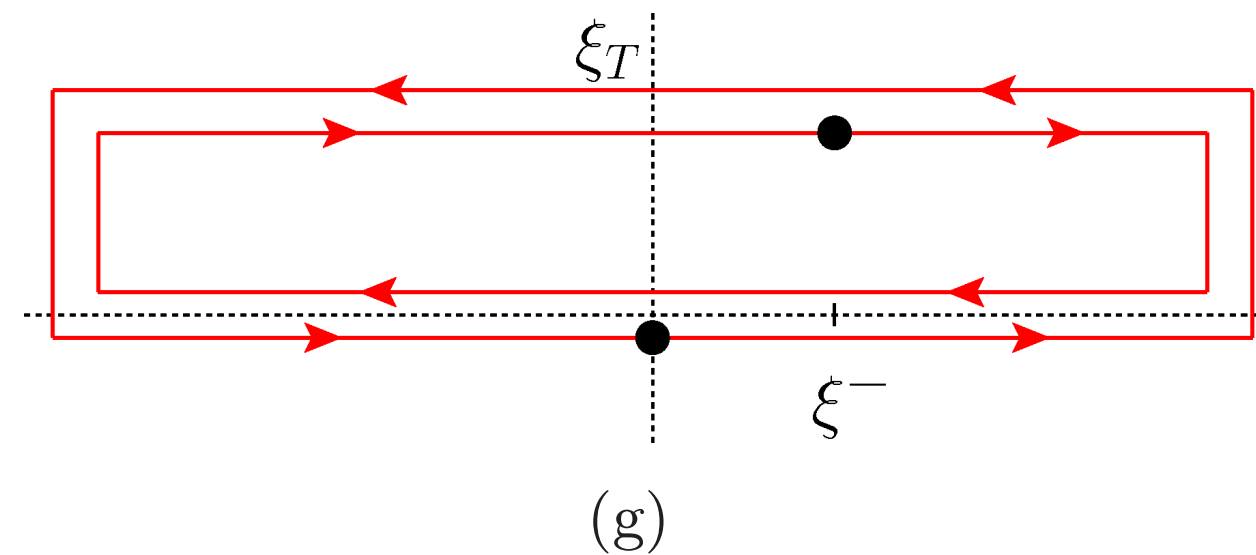
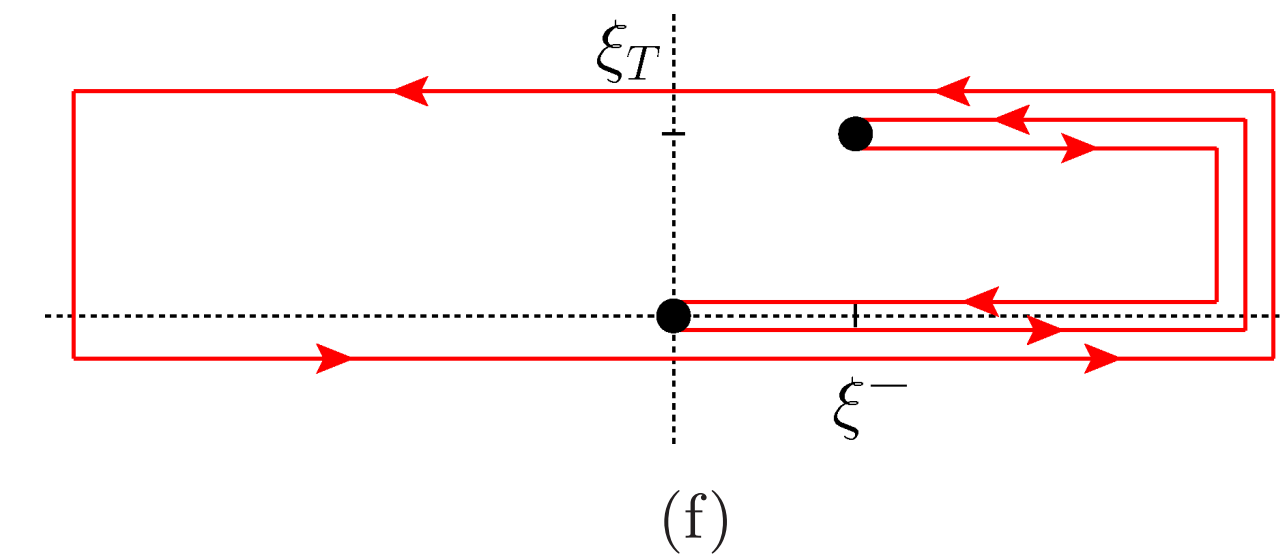
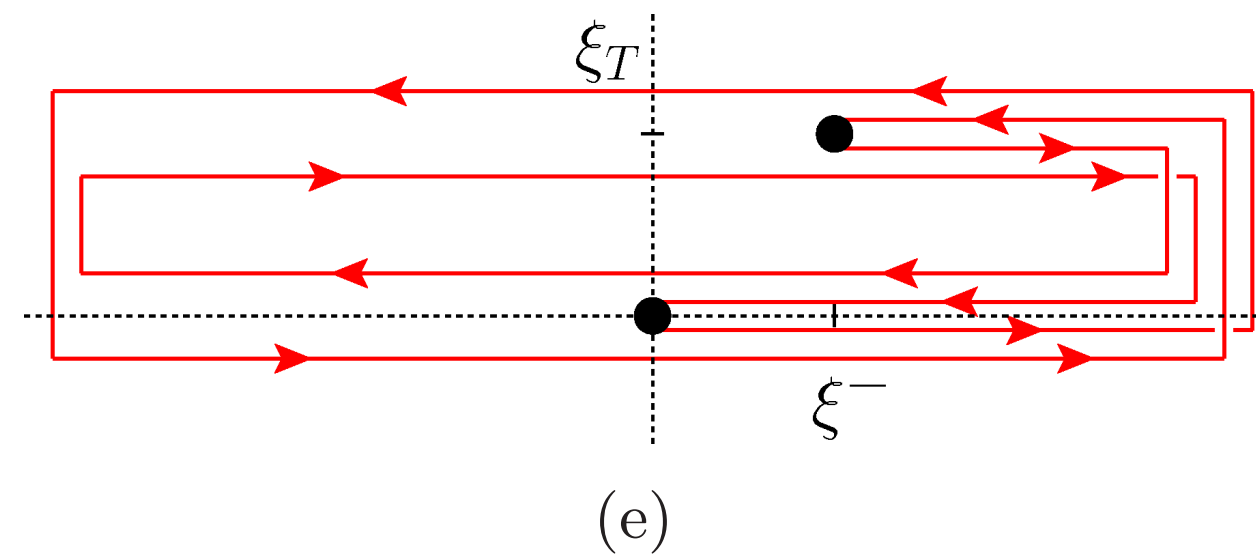
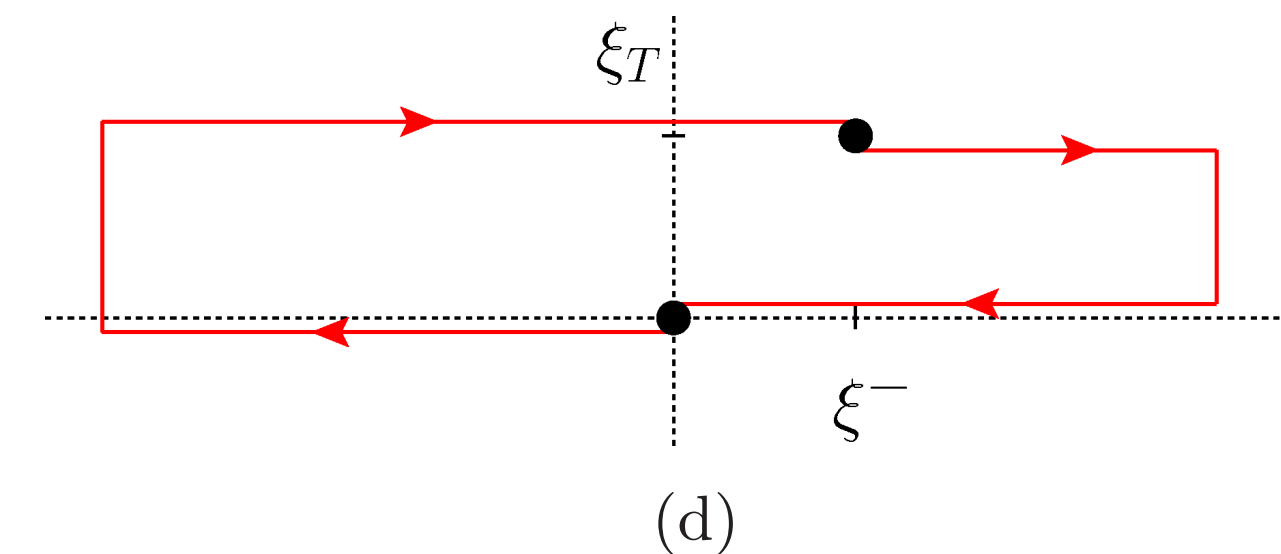
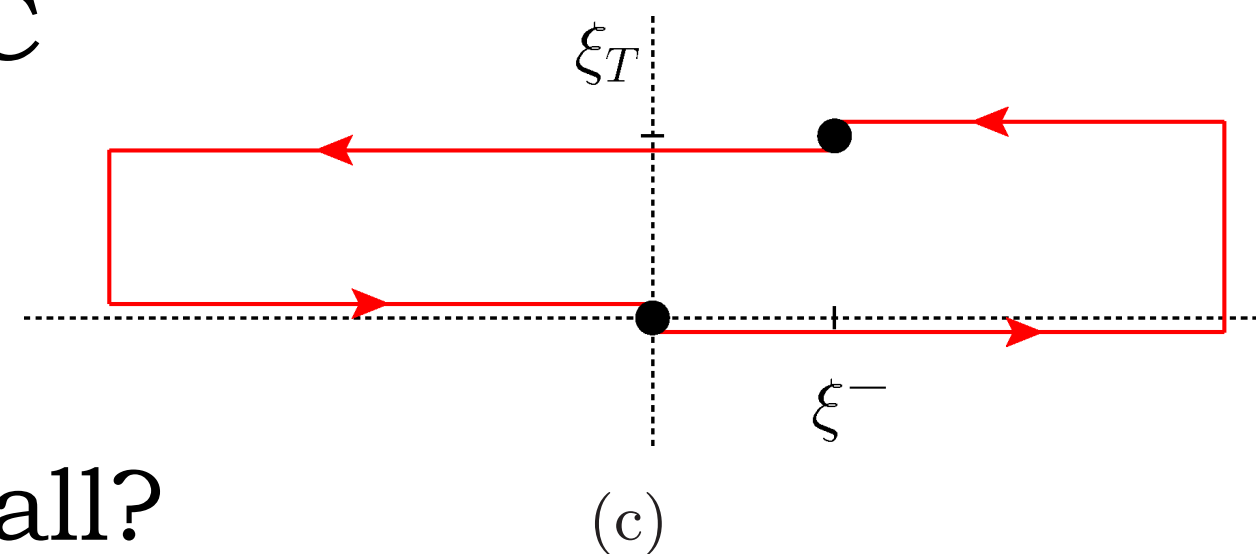
\* DP TMDs:

(c)  $[+, -]$  and (d)  $[-, +]$

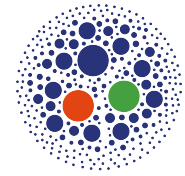
\* Appearance of new gauge **loop links**:

(e)  $[+\square, +\square]$ , (f)  $[+, +\square]$ ,

(g)  $[\square, \square]$ , and (h)  $[\square, \square]$

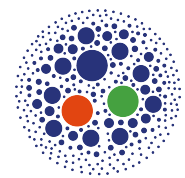
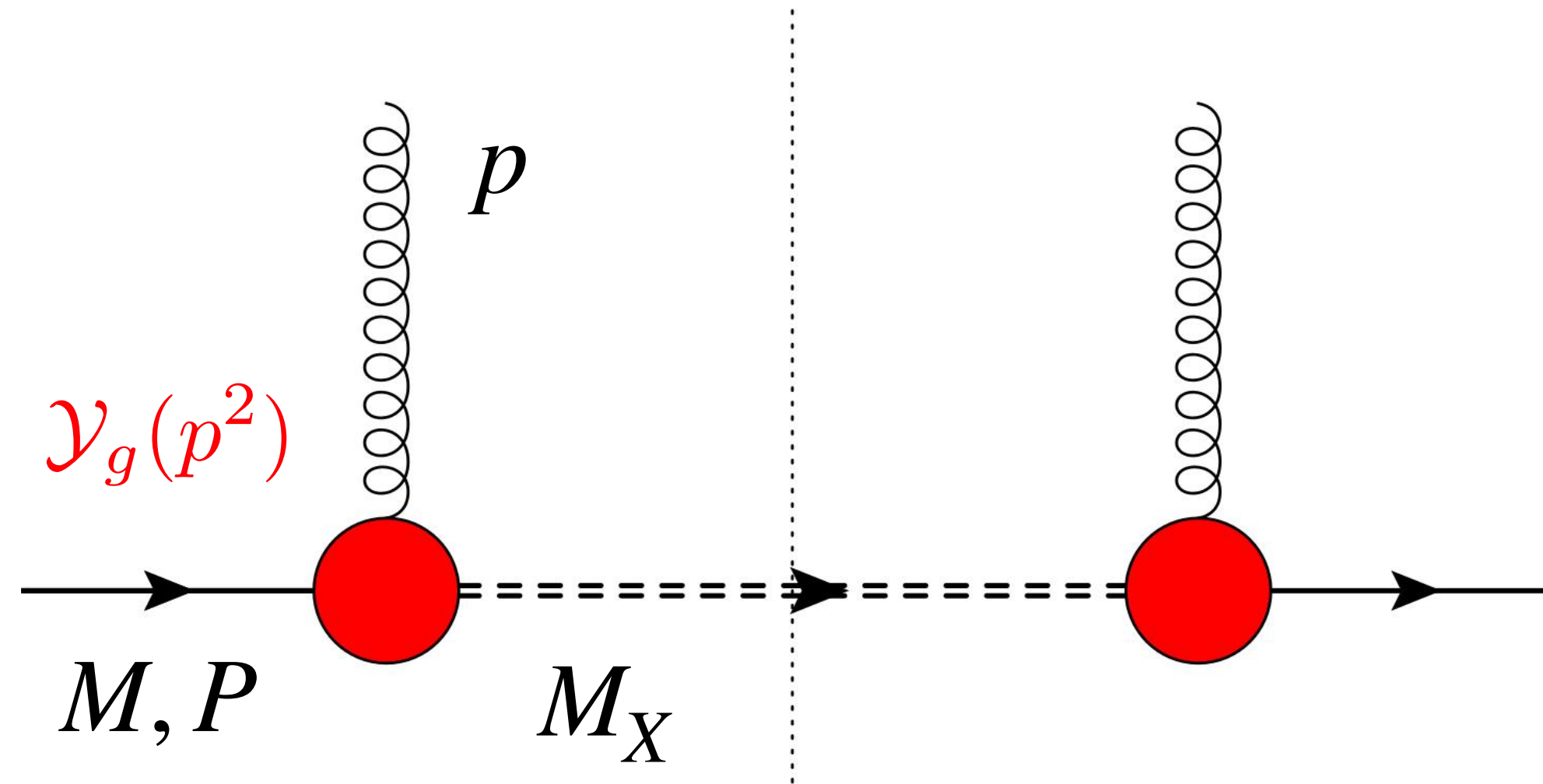


# Assumptions of the model



## Effective vertex

Lowest Fock state:  
**tri-quark** spectator  
 on-shell and  
 with mass  $M_X$



## Spin-1/2 spectator (gluon)

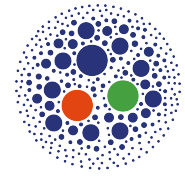
$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} \text{Tr} \left[ (\not{P} + M) \frac{1 + \gamma^5 \not{\not{p}}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M) \right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$



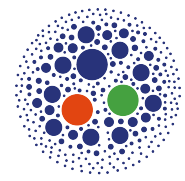
mimics proton form factors  
 (conserved EM current  
 of a free nucleon)

# Assumptions of the model



## Link with collinear factorization

$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution

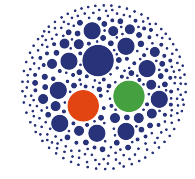


## Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high  $p_T$
3. Compensates log divergences arising from  $p_T$ -integration
4. Adds three more parameters:  $\kappa_{1,2}$  and  $\Lambda_X$

# Assumptions of the model



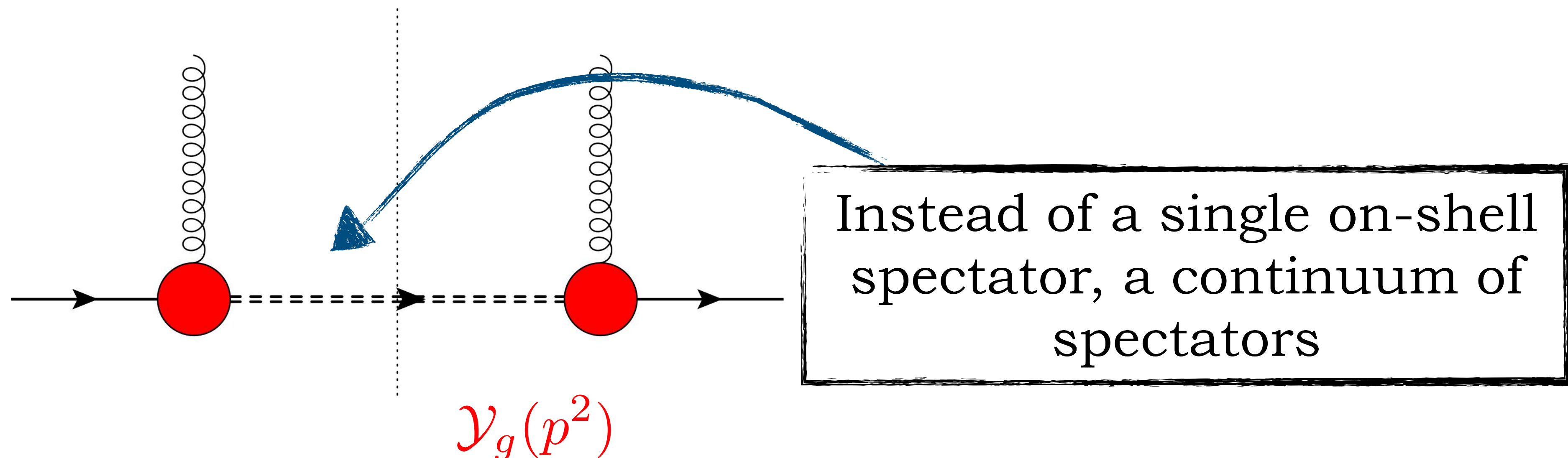
## Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]





# Assumptions of the model



## Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X \left( M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\} \right) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

low- $x$  (high- $\mu^2$ ) tail  $\propto (a - b)$

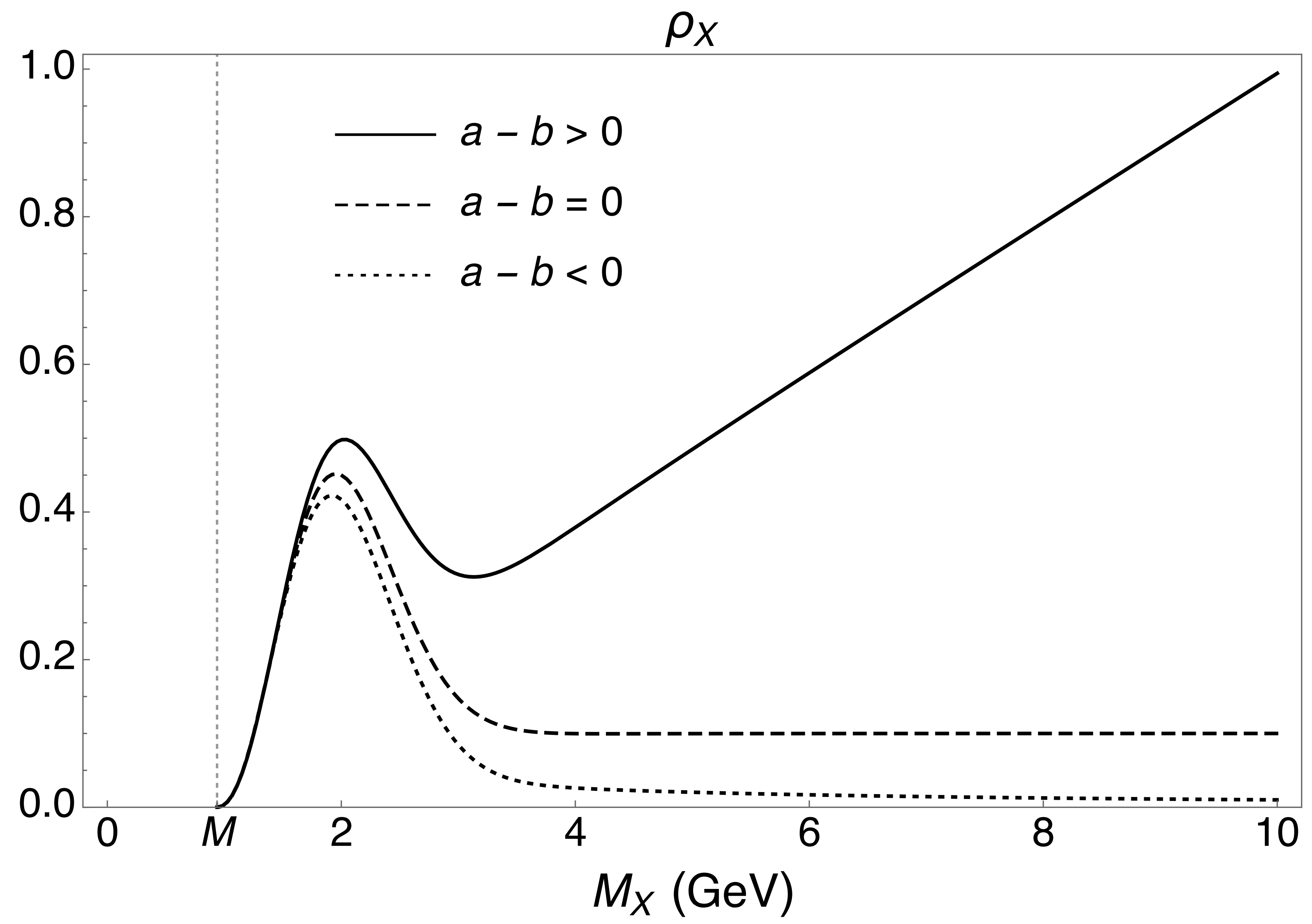
$q\bar{q}$  contributions energetically available at large  $M_X$

$$\mu^2 = M_X^2 - M^2$$

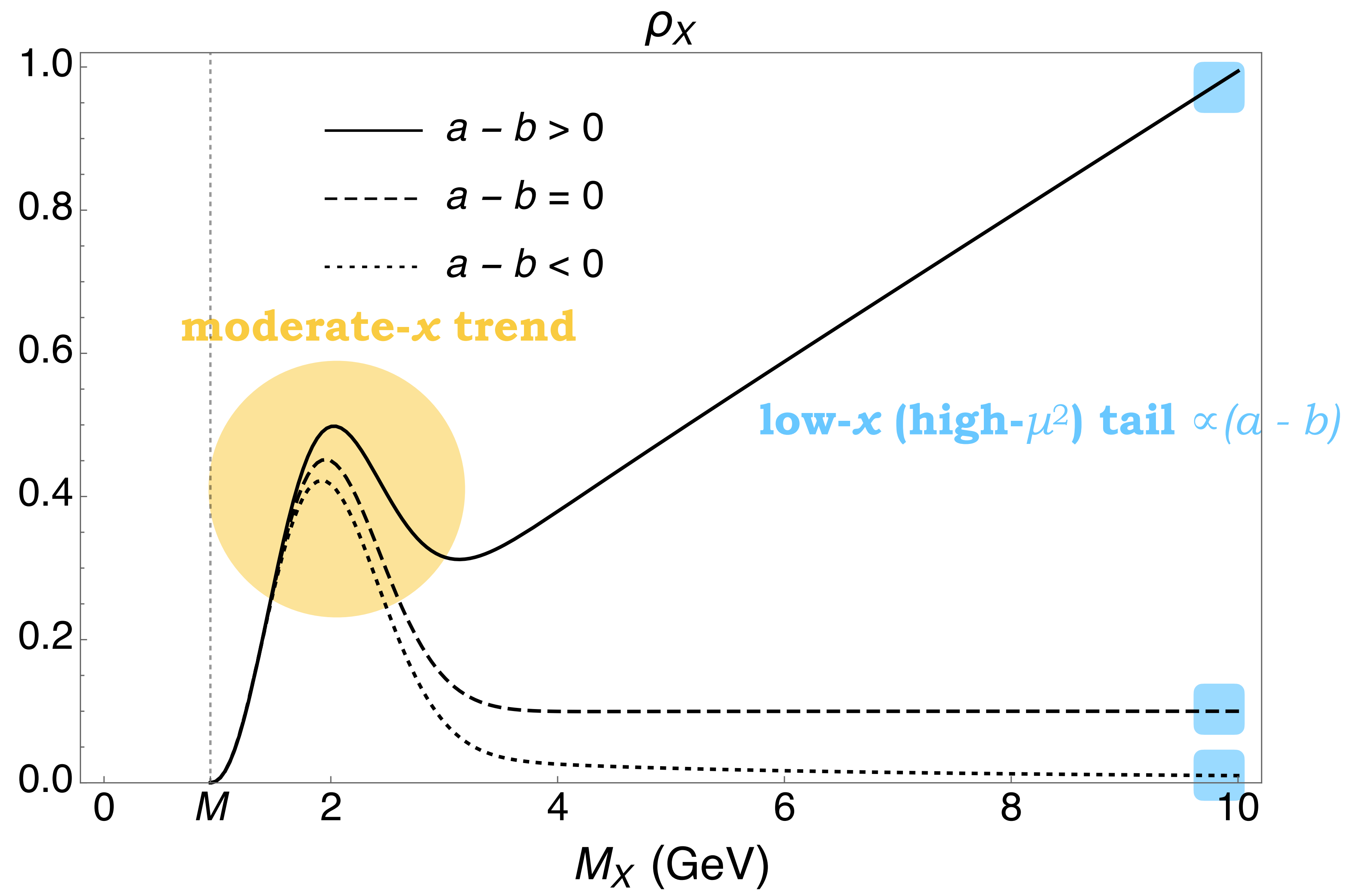
moderate- $x$  trend

pure tri-quark contribution at low  $M_X$

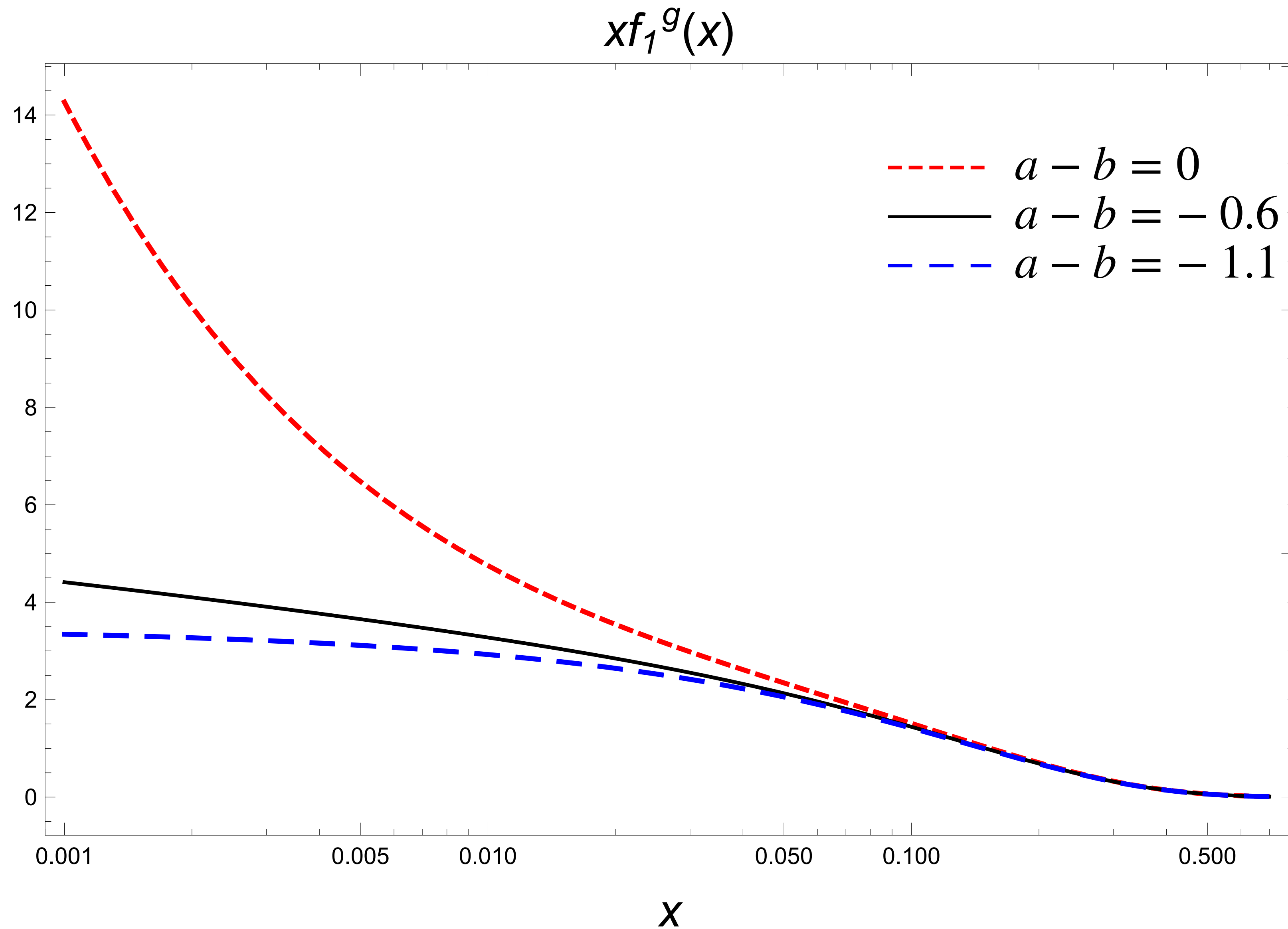
# Spectral function vs $(a - b)$



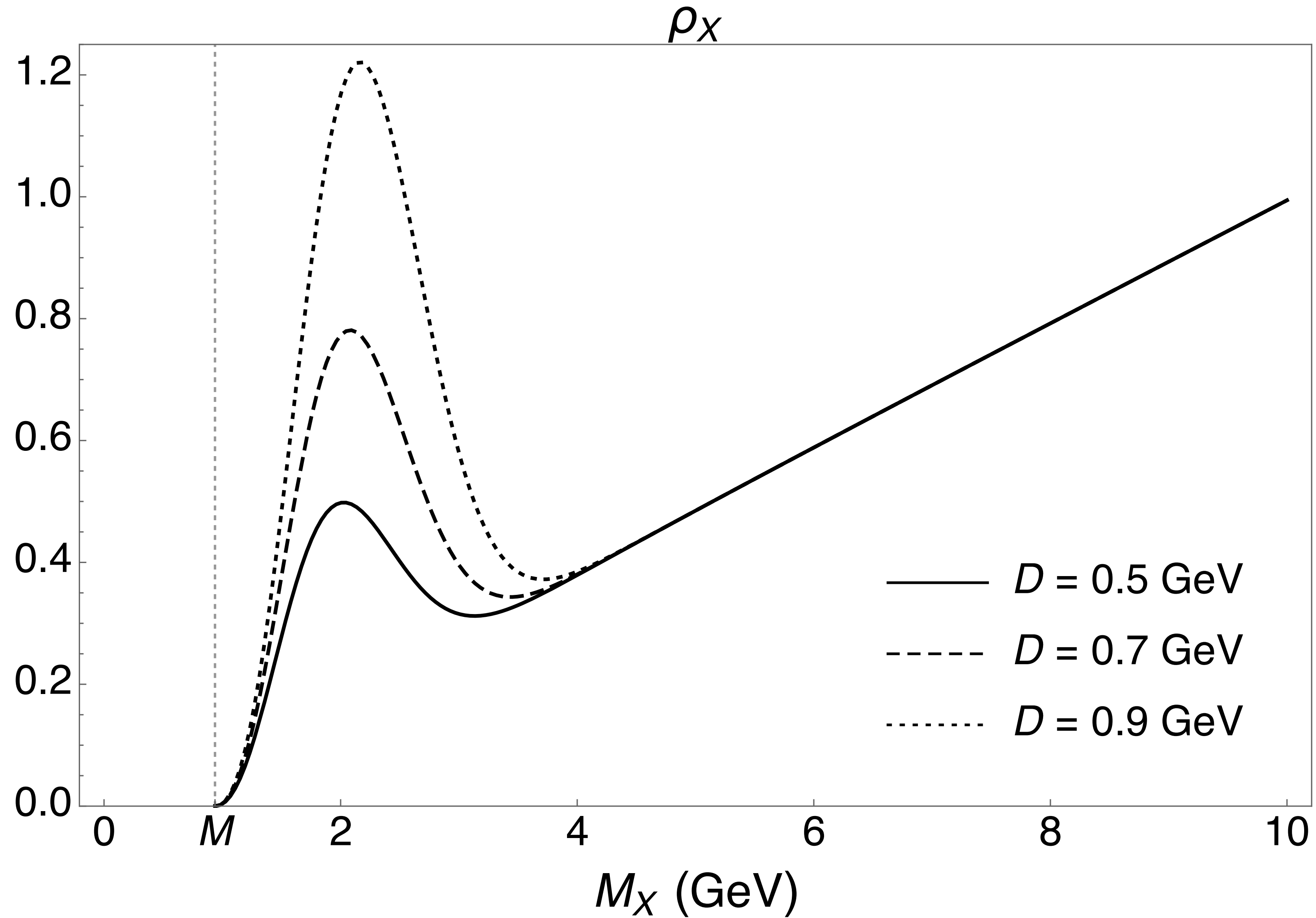
# Spectral function vs $(a - b)$



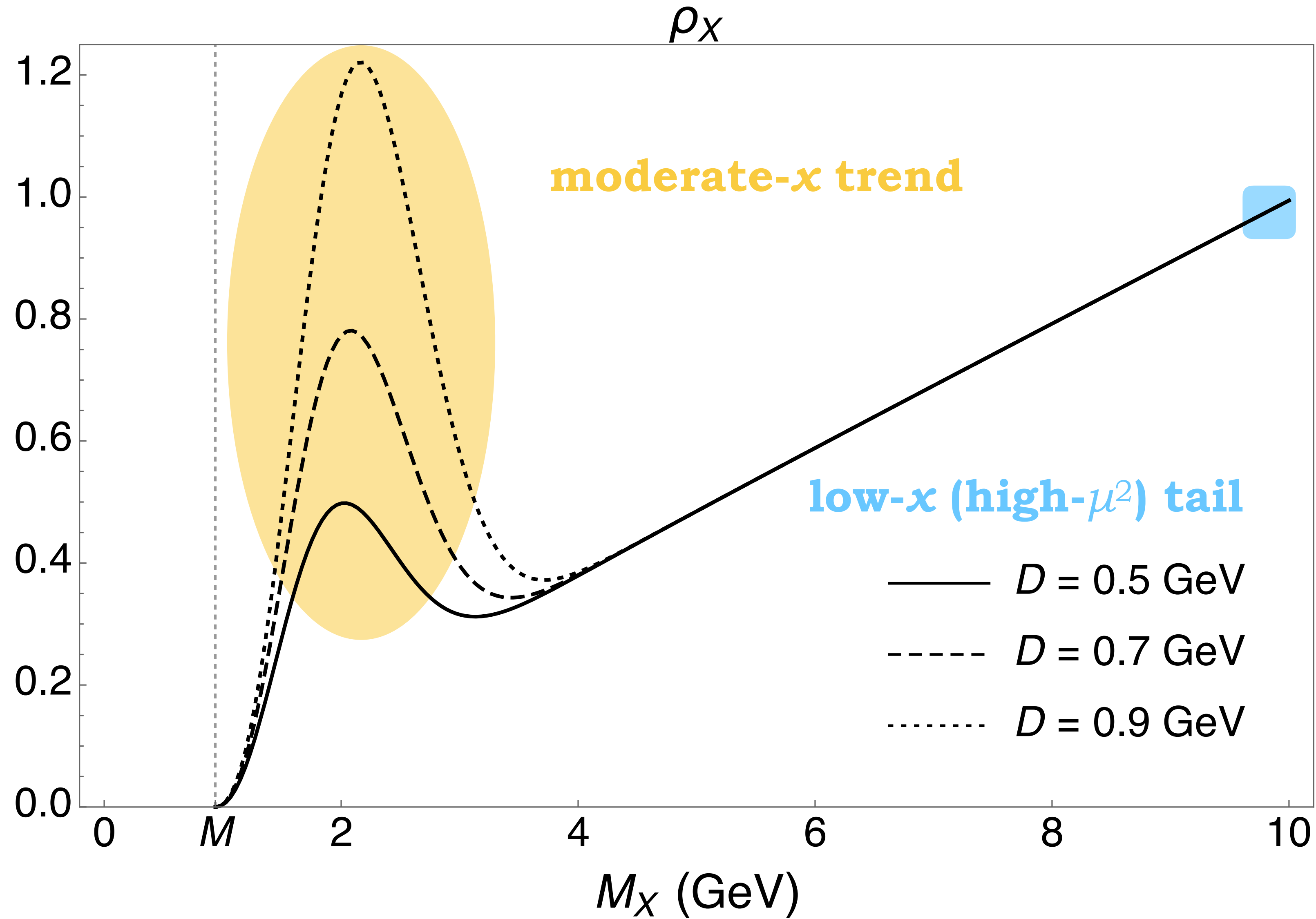
# $xf_1$ collinear PDF vs $(a - b)$



# Spectral function vs $D$

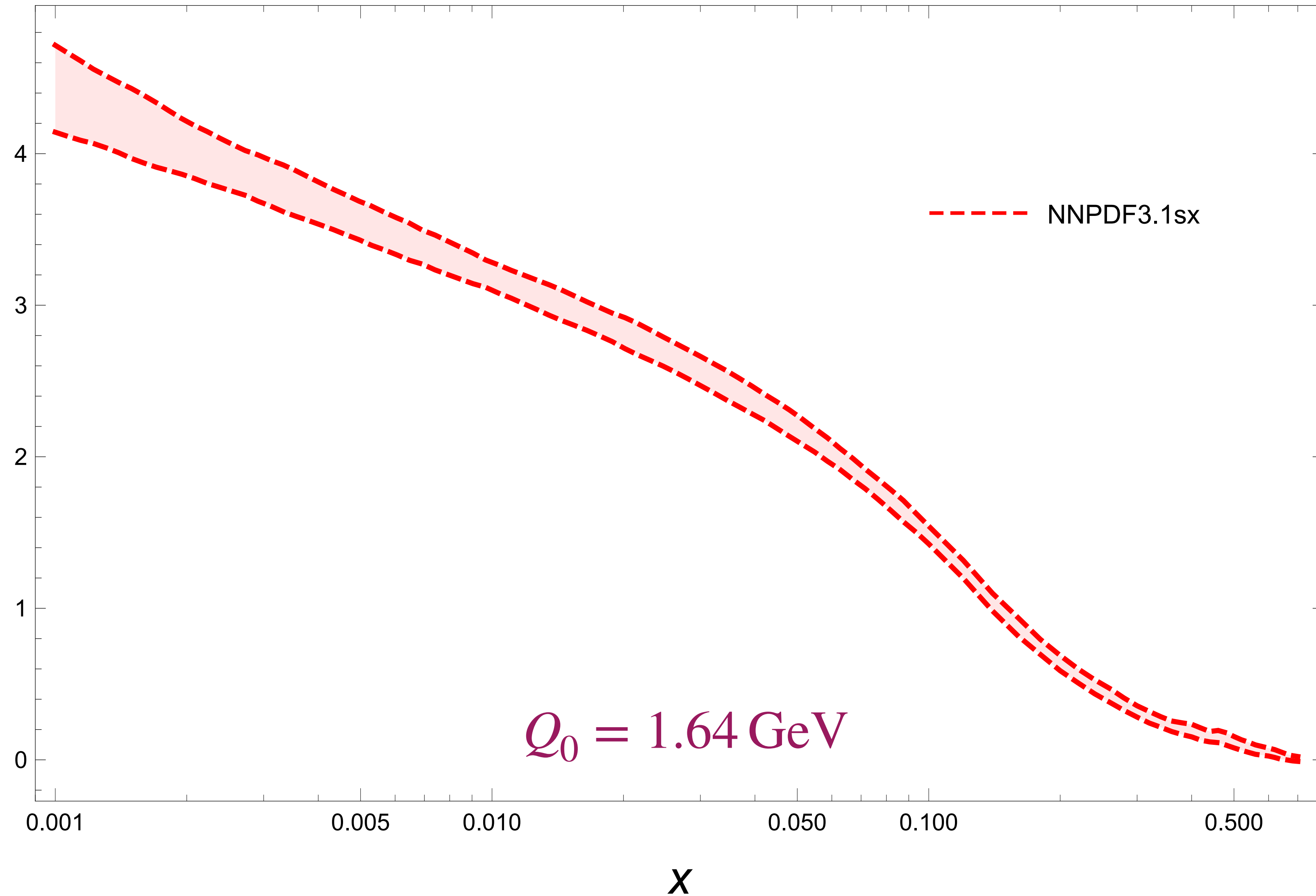


# Spectral function vs $D$



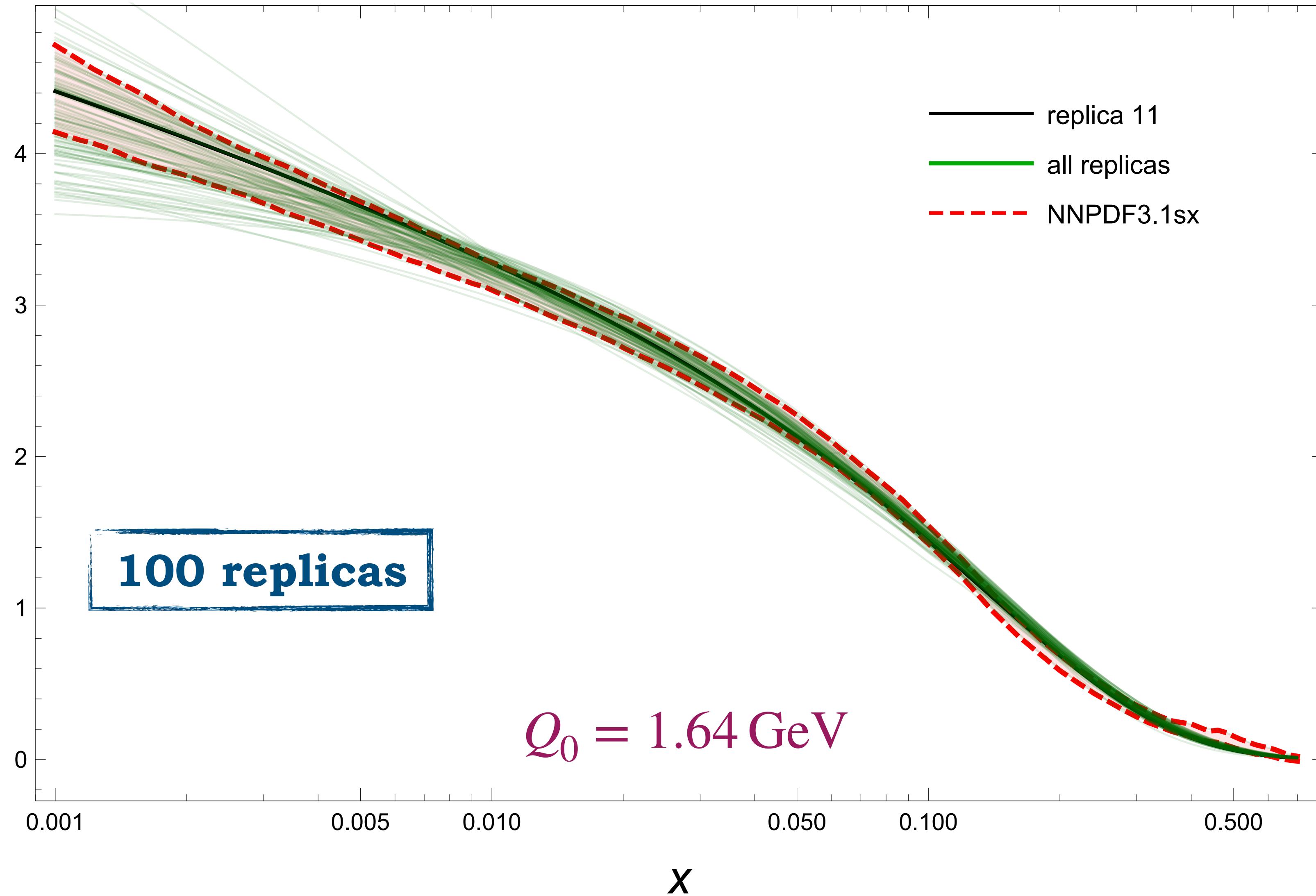
# Unpolarized gluon PDF

$$xf_1^g(x)$$



# Unpolarized gluon PDF

$$xf_1^g(x)$$



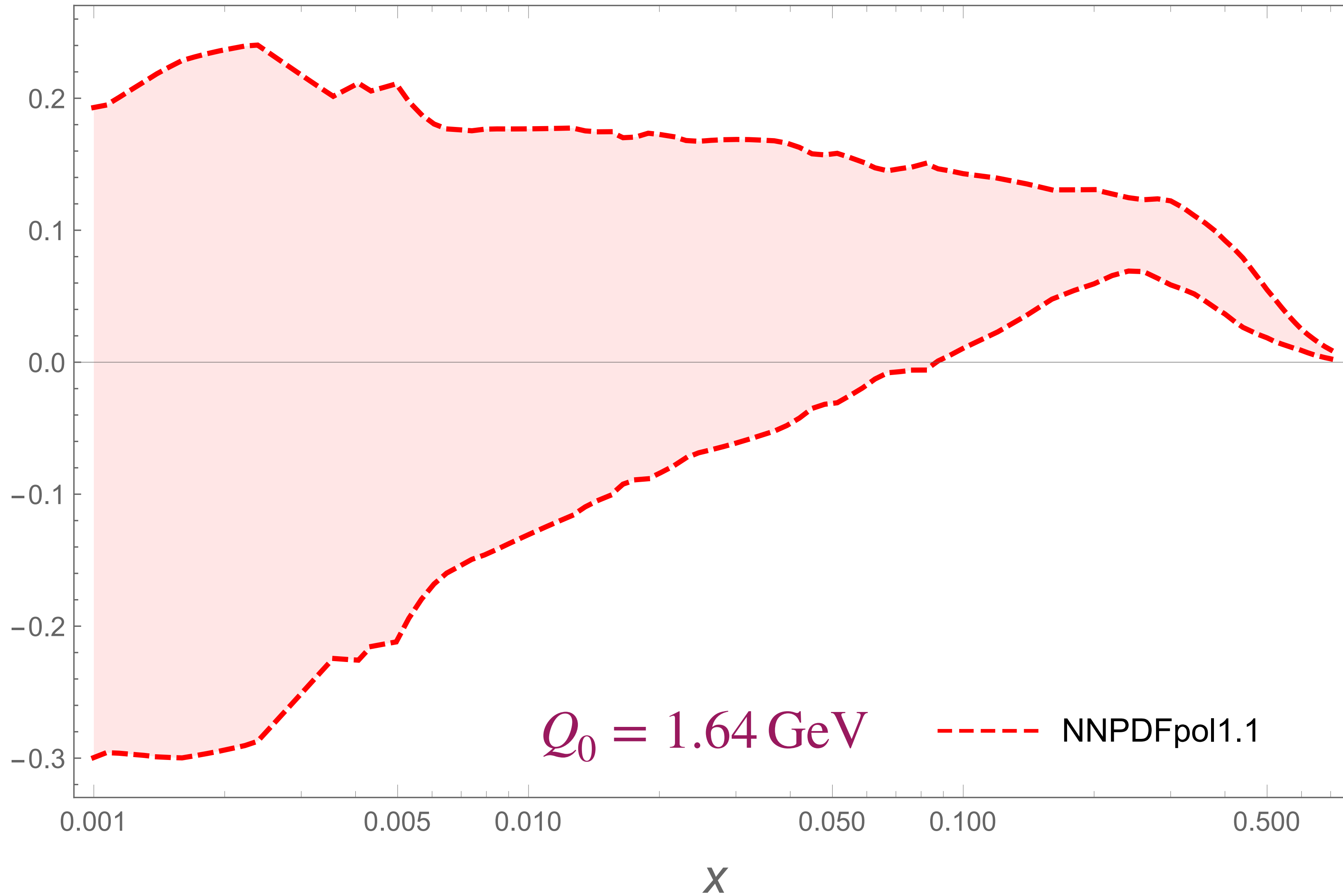
100 replicas

$Q_0 = 1.64 \text{ GeV}$



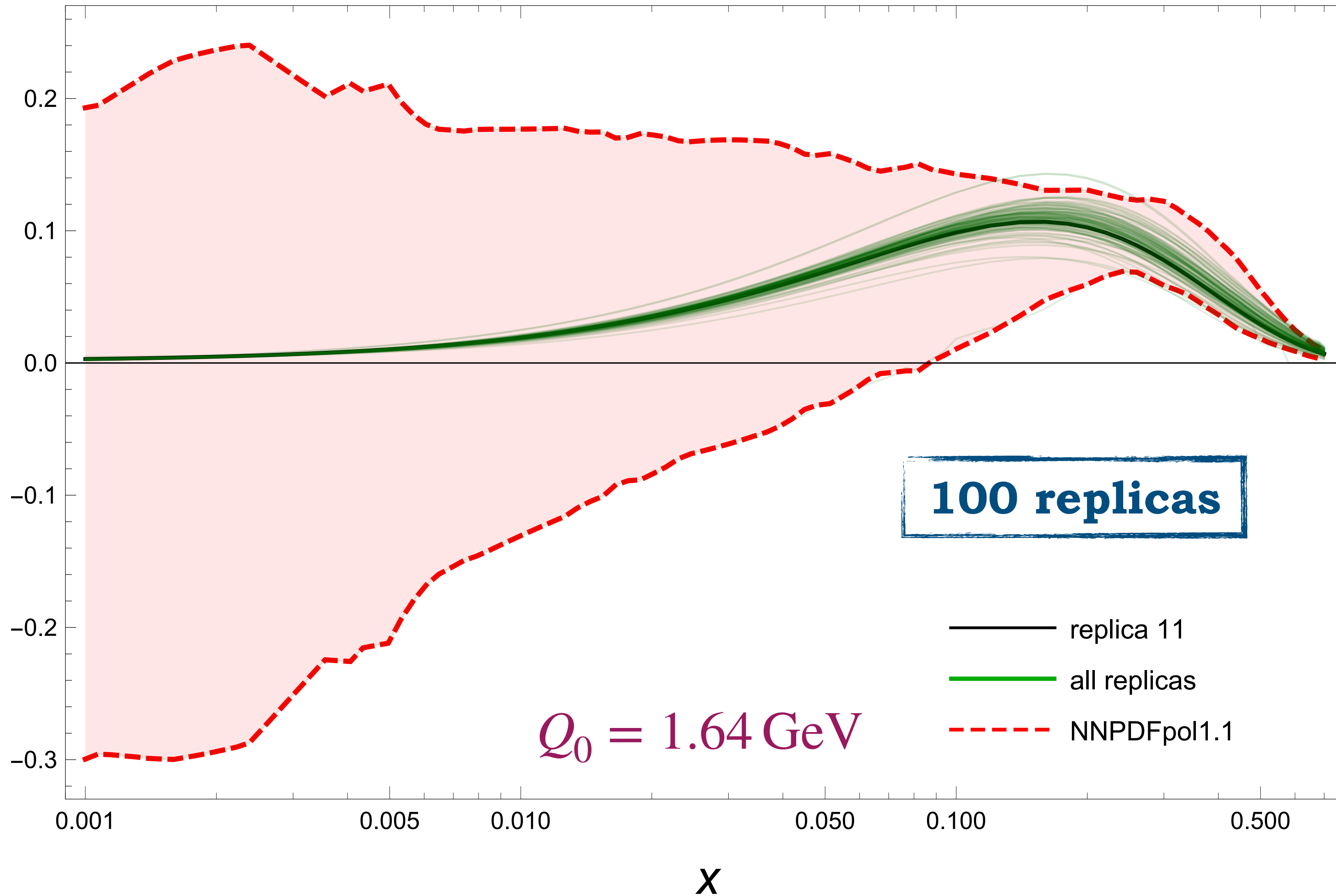
# Helicity gluon PDF

$$xg_1^g(x)$$



# Helicity gluon PDF

$$xg_1^g(x)$$



**100 replicas**

- replica 11
- all replicas
- - - NNPDFpol1.1

$Q_0 = 1.64 \text{ GeV}$

# Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

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$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

$$\langle x \rangle_g = \int_0^1 dx x f_1^g(x, Q_0) \qquad S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

# Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

$$\langle x \rangle_g = \int_0^1 dx x f_1^g(x, Q_0)$$

$$S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

Our model @  $Q_0 = 1.64$  GeV

Lattice @  $Q_0 = 2$  GeV

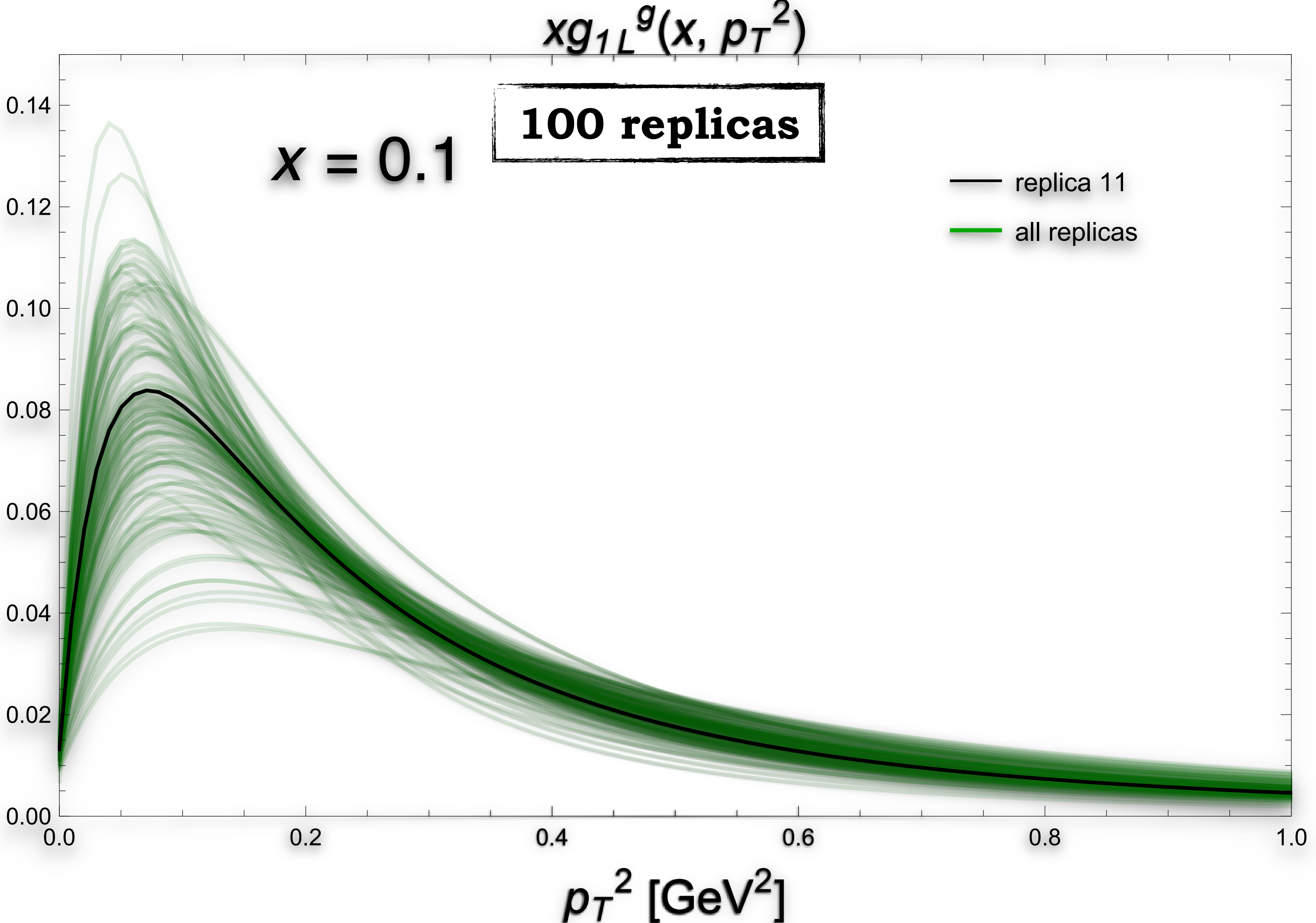
$$\langle x \rangle_g = 0.424(9)$$

$$\langle S \rangle_g = 0.159(11)$$

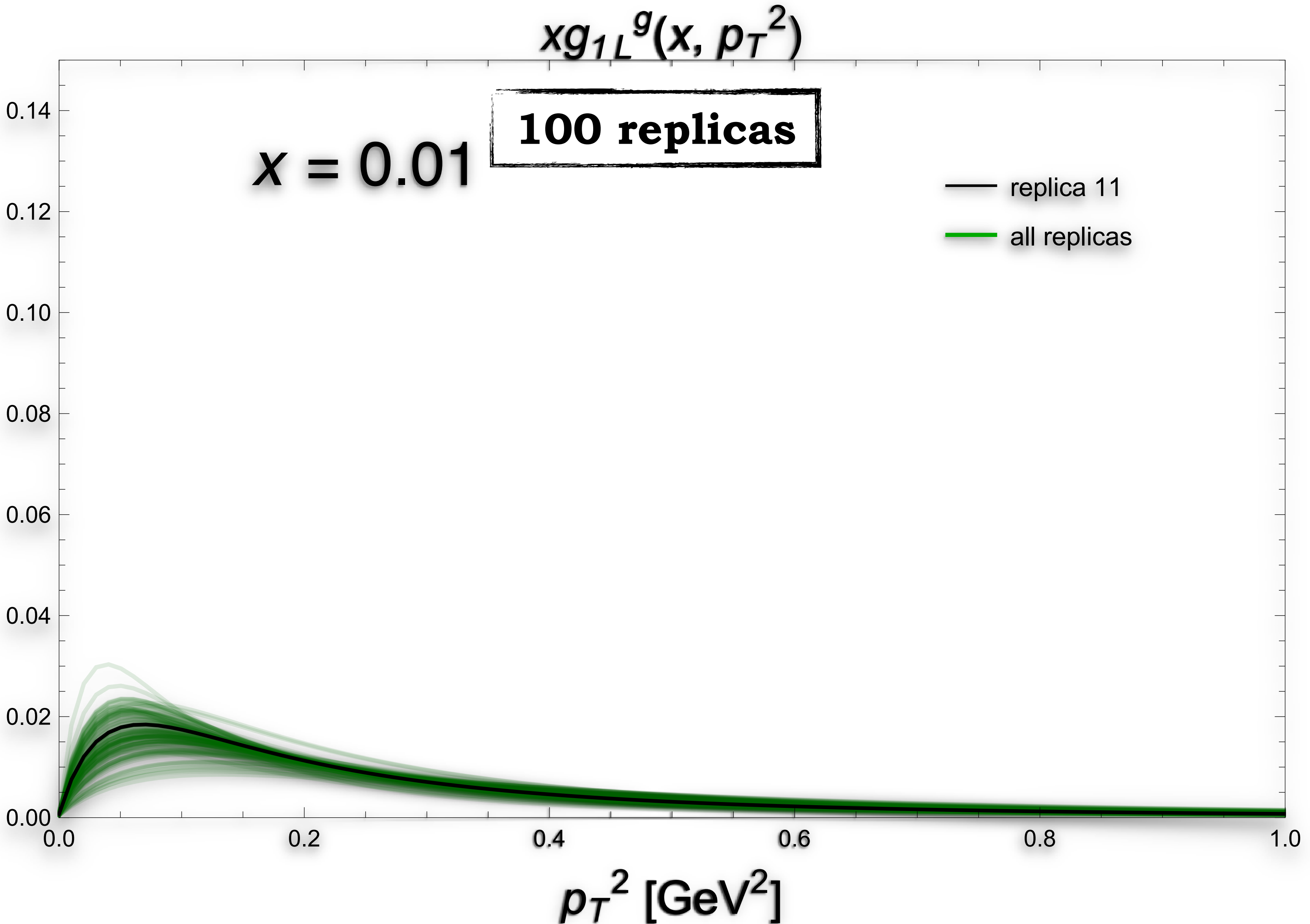
$$\langle x \rangle_g = 0.427(92)$$

$$\langle J \rangle_g = 0.187(46)$$

# Helicity gluon TMD

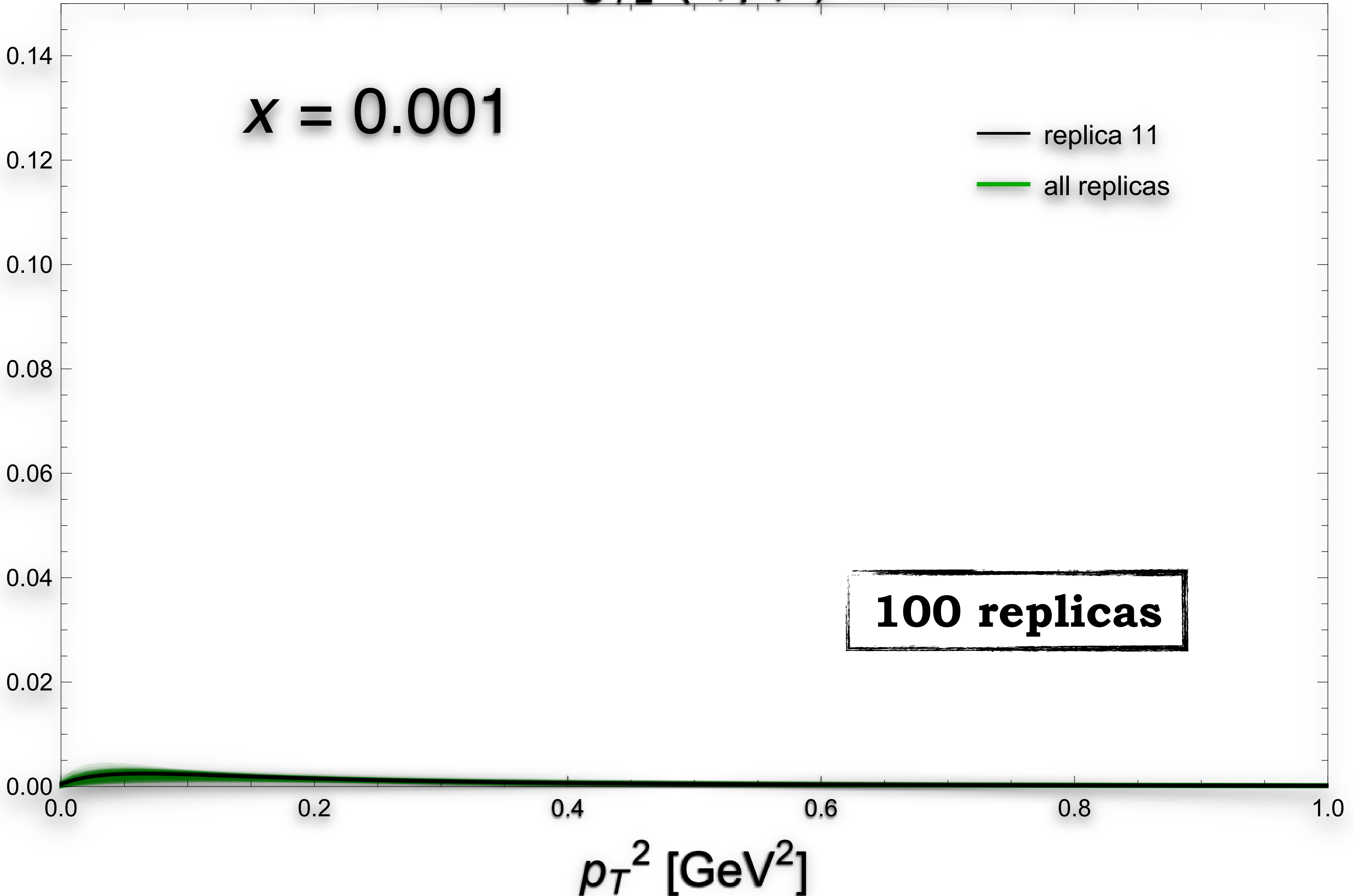


# Helicity gluon TMD



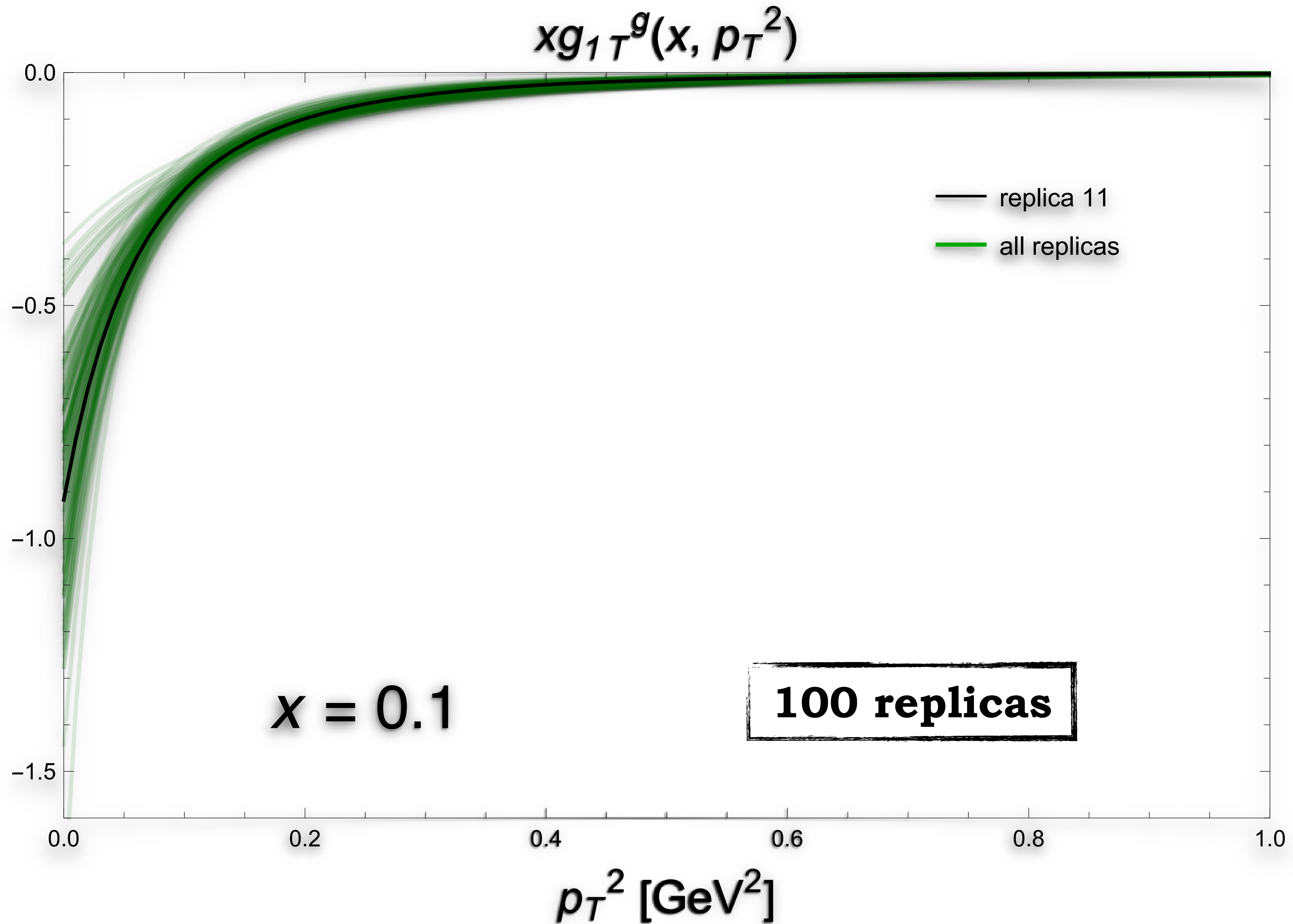
# Helicity gluon TMD

$$xg_{1L}^g(x, p_T^2)$$

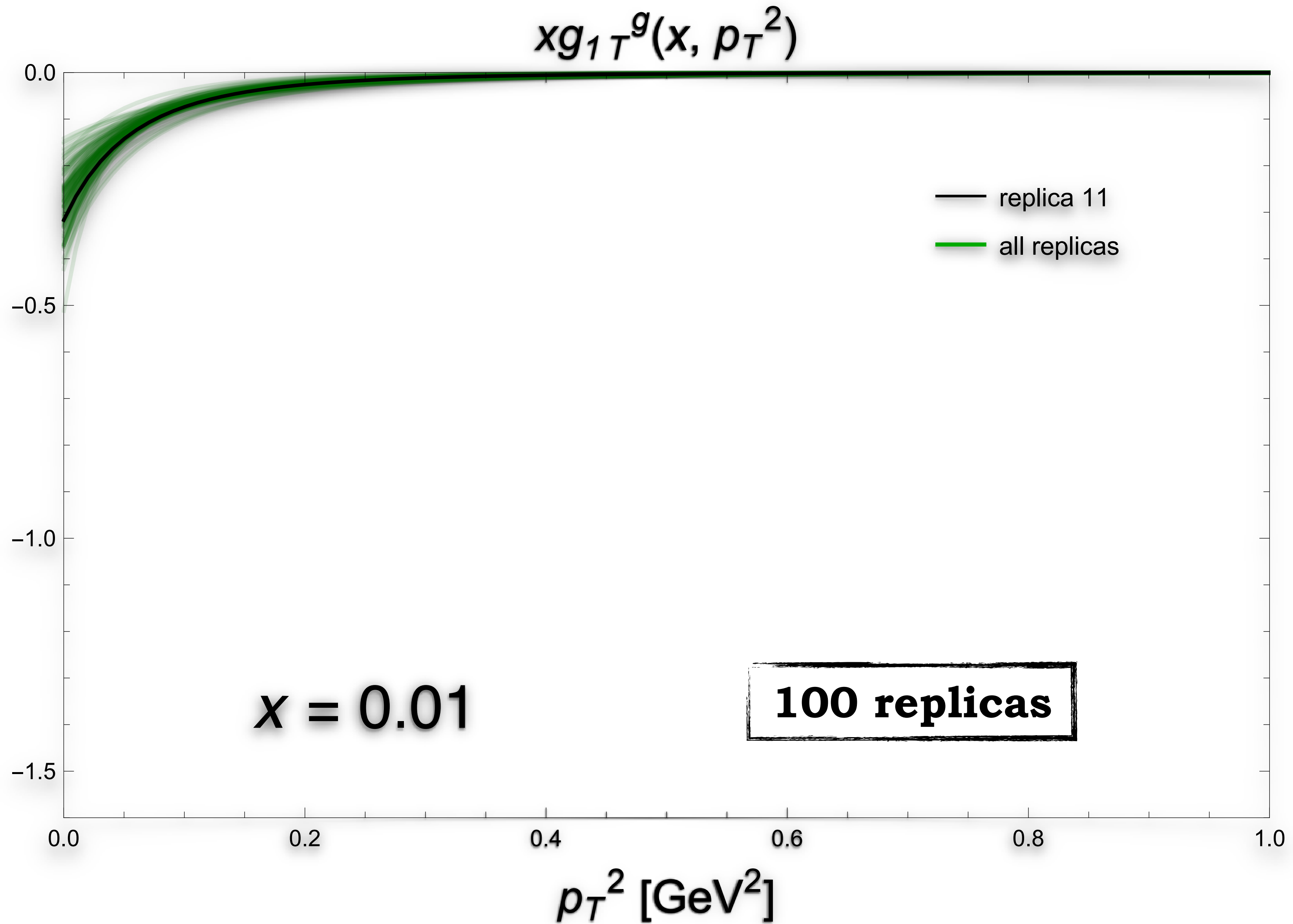




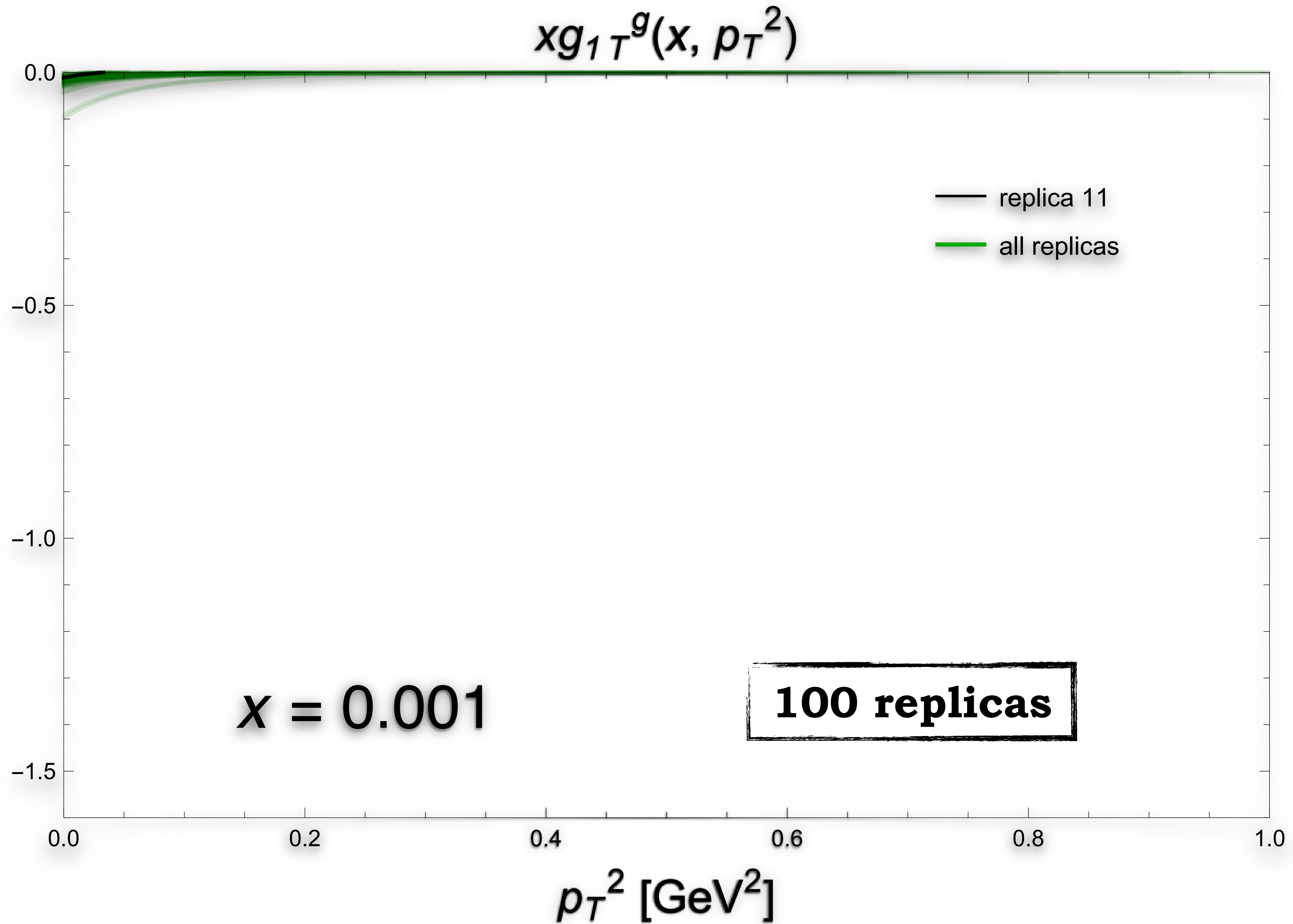
# Worm-gear gluon TMD



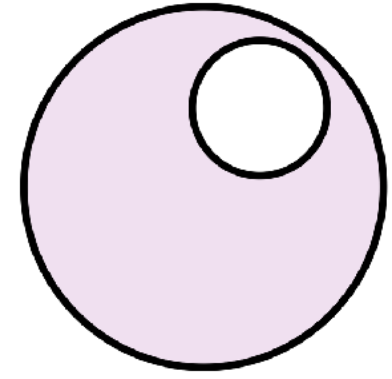
# Worm-gear gluon TMD



# Worm-gear gluon TMD



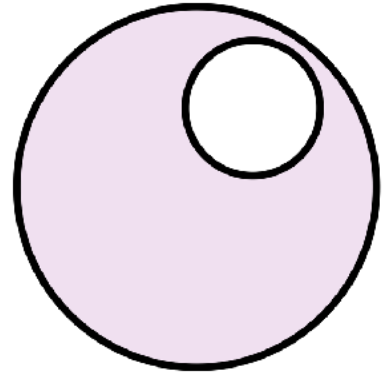
# $\rho$ -densities



**Unpolarized [u/u]**

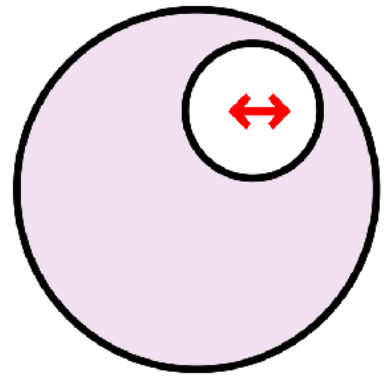
$$f_1(x, p_x, p_y)$$

# $\rho$ -densities



**Unpolarized** [**u/u**]

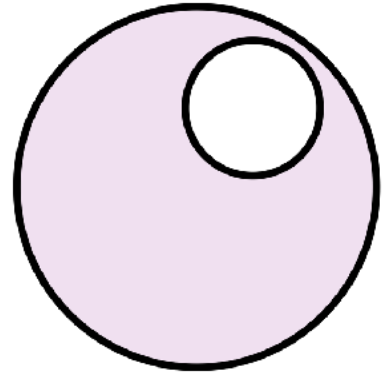
$$f_1(x, p_x, p_y)$$



**Boer-Mulders** [**↔/u**]

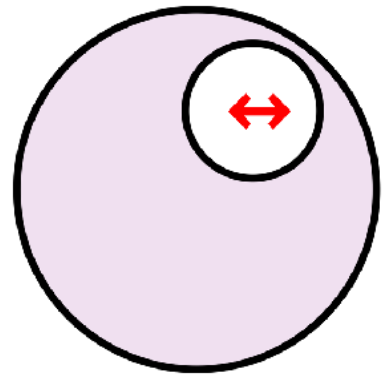
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

# $\rho$ -densities



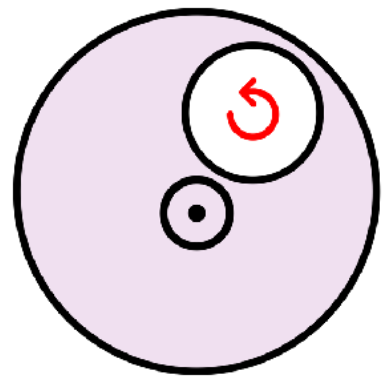
**Unpolarized** [ $\mathbf{u}/\mathbf{u}$ ]

$$f_1(x, p_x, p_y)$$



**Boer-Mulders** [ $\leftrightarrow/\mathbf{u}$ ]

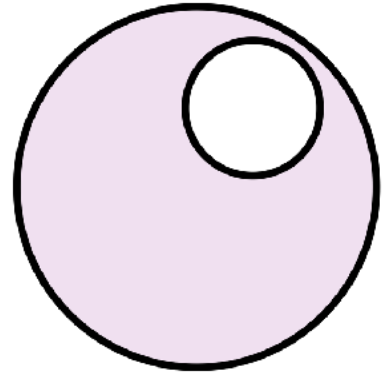
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

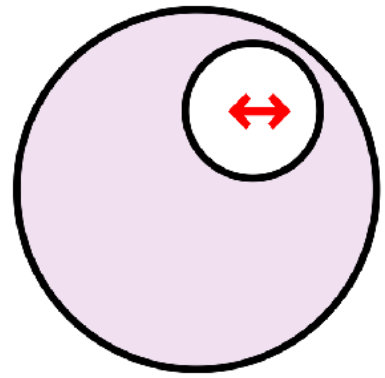
$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

# $\rho$ -densities



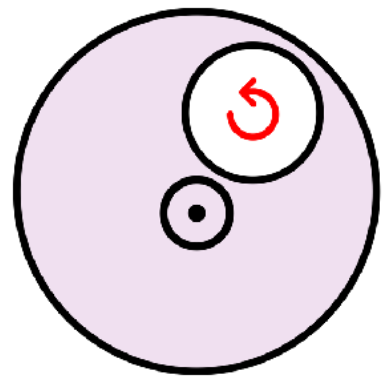
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



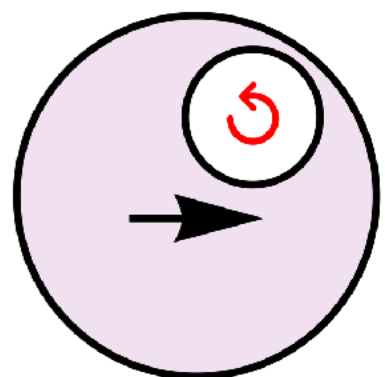
**Boer-Mulders** [ $\leftrightarrow$ /u]

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup$ /+]

$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

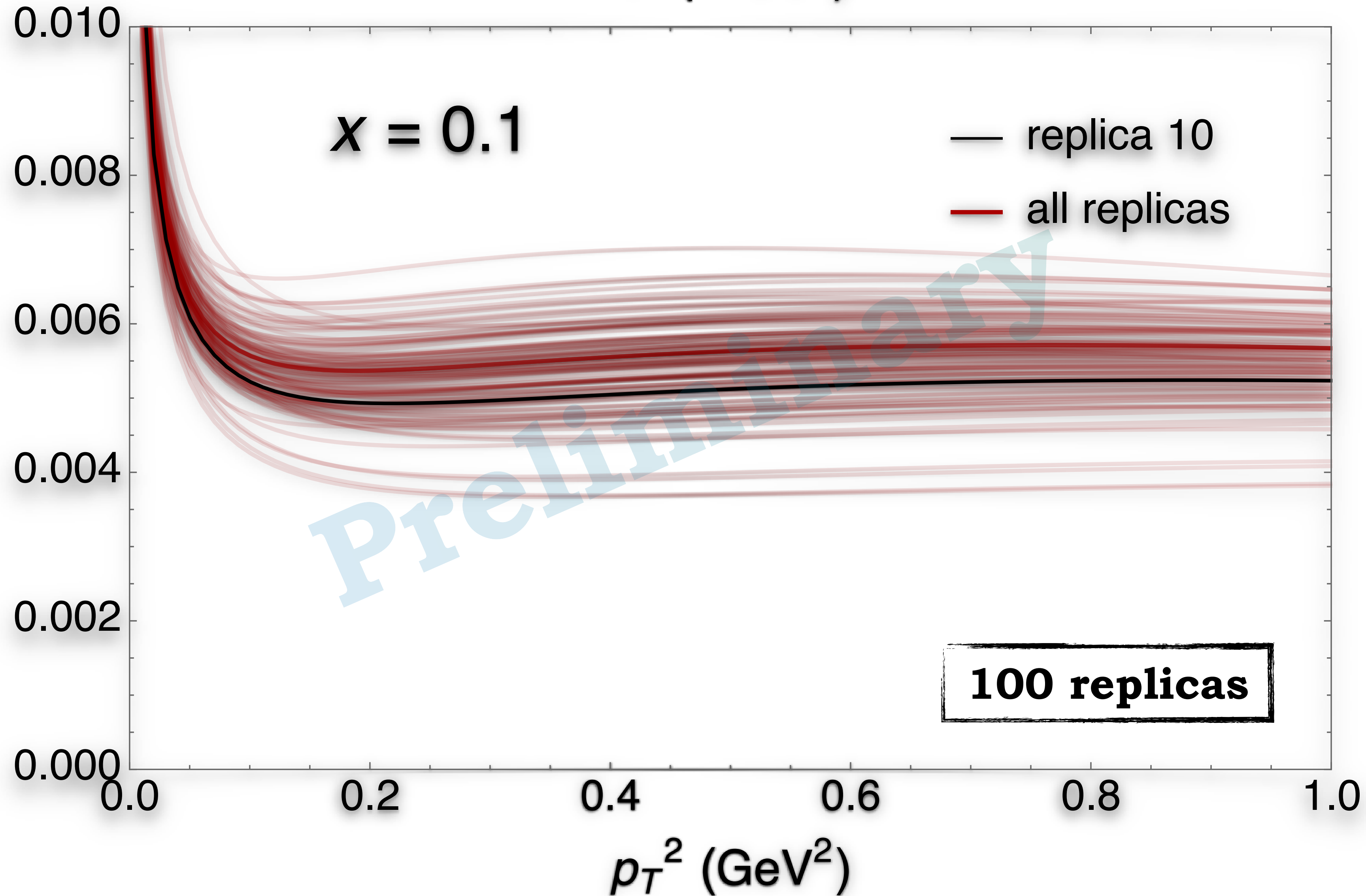


**Worm-gear** [ $\cup$ / $\rightarrow$ ]

$$f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$$

# Sivers / unpol.

$$\frac{\frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$





# Linearity / unpol.

$$\frac{\frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$

