RUHR-UNIVERSITÄT BOCHUM
INSTITUT FÜR THEORETISCHE PHYSIK II

RUB

Nucleon distribution amplitudes in the chiral quark-soliton model

June-Young Kim
Hadron2021, 28.07.2021

with M.V. Polyakov
H.-Ch. Kim.

Light-cone wave function(LCWF)

- ✓ In the light cone formalism or infinite momentum frame (IMF), nucleon state can be written as /V. L. Chernyak et al (1977), S.Brodsky et al. (1979), A.V.Efremov et al. (1980), G.Lepage et al. (1980)/

$$|P\rangle = \psi_{3q}|qqq\rangle + \psi_{3q+1g}|qqqg\rangle + \psi_{3q+q\bar{q}}|qqqq\bar{q}\rangle + \ldots$$

- ✓ According to the factorization theorem, the hard exclusive amplitude can be expressed as a convolution of the hard kernel with a non-perturbative contribution. In the case of the form factor, the non-perturbative object is defined as distribution amplitudes (DAs) of hadron involved in the process./V. L. Chernyak et al (1977), V. L. Chernyak et al ZPC42 (1989) /

$$G_M \propto \varphi_N \otimes T_H \otimes \varphi_N$$

- ✓ The DAs describe the distribution of quark inside the baryon in the longitudinal momentum fraction. The nucleon distribution amplitudes probe the 3Q light-cone wave function./V. Braun et al. NPB 589 (2000)/

$$\Phi_{3,4,5,6}, \ \Psi_{4,5}, \ \Xi_{4,5}$$

Nucleon Distribution amplitudes

✓ The formal definition of the nucleon DAs /V. L. Chernyak et al (1977)/

$$\langle 0|\epsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) d^k_\gamma(a_3 z)|N(p,\lambda)\rangle$$
$$= \Big[(\not p C)_{\alpha\beta}(\gamma_5 N)_\gamma V(a_i z\cdot p) + (\not p\gamma_5 C)_{\alpha\beta} N_\gamma A(a_i z\cdot p) + (i\sigma_{\mu\nu}p^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5 N)_\gamma T(a_i z\cdot p)\Big]$$

- Lorentz invariant function $V, A, T(a_i p\cdot z)$
- origin of configuration space

$$V(0) = T(0) = f_N$$

- permutation symmetry of the first two quarks and isospin symmetry of the nucleon

$$|P_\uparrow\rangle = \frac{1}{4\sqrt{6}}\int\left[\frac{dx}{\sqrt{x}}\right]\varphi_N(x_i)\Big[|u^\uparrow u^\downarrow d^\uparrow\rangle - |u^\uparrow d^\downarrow u^\uparrow\rangle\Big] \qquad \varphi_N(x_i) = V(x_i) - A(x_i)$$

DAs in various models

✓ Two limits of DAs: non-relativistic (NR) limit and asymptotic (AS) limit /V. L. Chernyak et al. NPB246 (1984)/

$$\varphi_N^{NR}(x_i) \propto \delta\left(x_1 - \frac{1}{3}\right)\delta\left(x_2 - \frac{1}{3}\right)\delta\left(x_3 - \frac{1}{3}\right)$$

$$\downarrow$$

$$G_M^n > 0,\ G_M^p < 0\ (\mathrm{NR})$$

$$\varphi_N^{\mathrm{AS}}(x_i) = 120 x_1 x_2 x_3$$

$$\downarrow$$

$$G_M^p / G_M^n \to 0\ (\mathrm{AS})$$

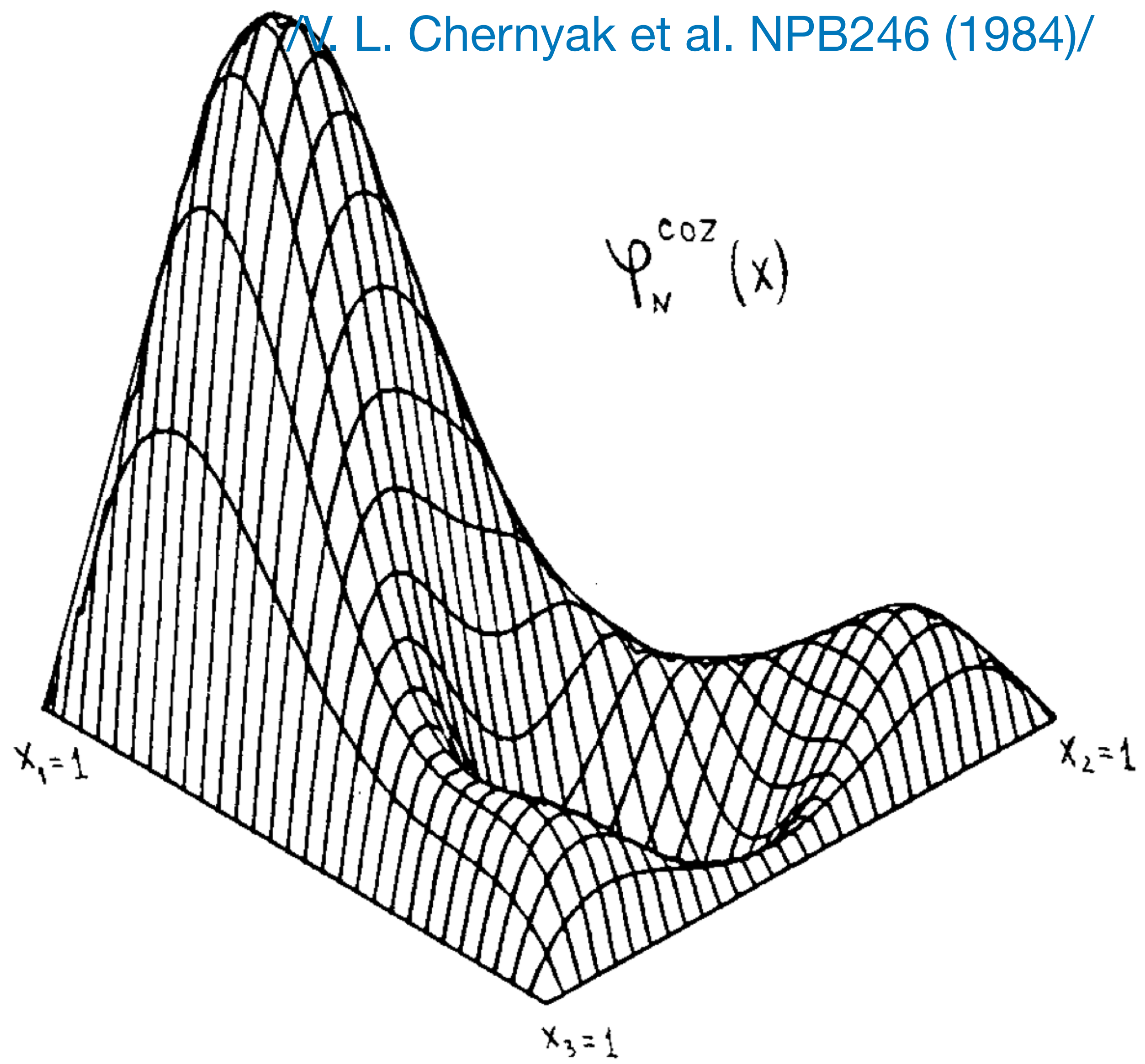
- DA is different from both limits.

✓ QCD sum rules /V. L. Chernyak et al. NPB246 (1984)/

- strong asymmetry ($x_1 \leftrightarrow x_2$)
- normalization constant f_N

$$f_N(\mu = 1\ \mathrm{GeV}) = (5.0 \pm 0.3) \times 10^{-3}\ \mathrm{GeV}^2$$

/V. L. Chernyak et al. NPB246 (1984)/



DAs in various models

✓ Lattice QCD /G. S. Bali et al. EPJA (2019)/

- weak asymmetry($x_1 \leftrightarrow x_2$) and almost symmetric
- normalization constant f_N

$$f_N^{N_f=2+1}(\mu = 2\ \mathrm{GeV}) = 3.54^{+6}_{-4} \times 10^{-3}\mathrm{GeV}^2$$

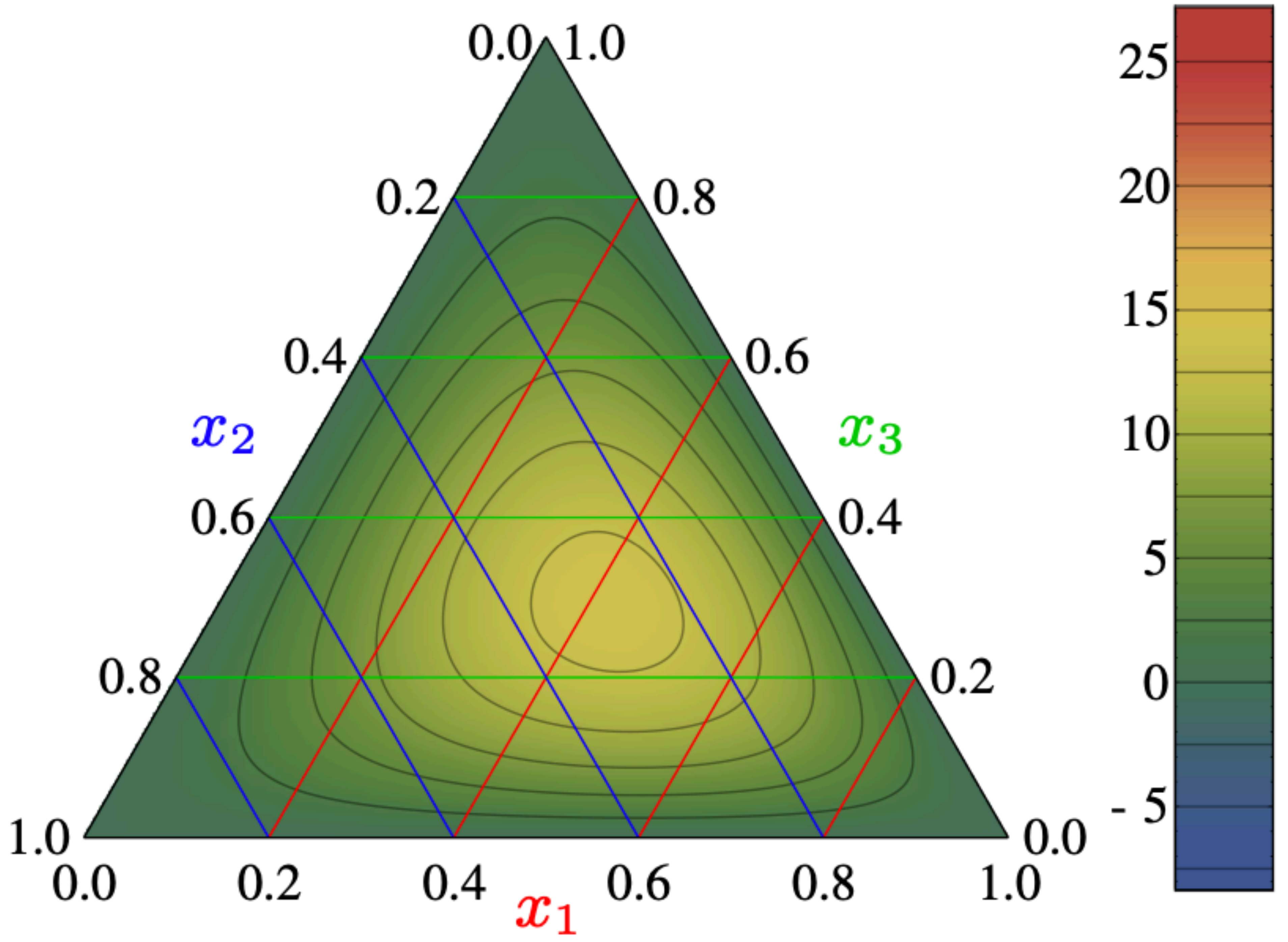
/G. S. Bali et al. EPJA (2019)/

$$f_N^{N_f=2}(\mu = 2\ \mathrm{GeV}) = 2.84^{+33}_{-33} \times 10^{-3}\mathrm{GeV}^2$$

/V.M. Braun. Et al. PRD89 (2014)/

- discrepancy between predictions from lattice QCD and QCD sum rules.

/V.M. Braun. Et al. PRD89(2014)/



✓ Models based on a dynamical Ansatz

- ✓ Z.Dziembowski PRD37 (1988),
- ✓ Z.Dziembowski, et al. PRD42 (1990),
- ✓ J.Bolz, et al. Z.Phys.A356 (1996),
- ✓ B.Pasquini, et al. PRD80 (2009).

Chiral quark-soliton model (ChiQSM)

✓ Effective chiral Lagrangian

$$\mathcal{L} = \overline{\psi}(x)(i\partial\!\!\!/ - MU^{\gamma^5})\psi(x)$$

✓ Saddle point equation of motion

$$\frac{\delta M_N[U]}{\delta U} = 0$$

✓ Relativistic invariance

- stationary mean field - nucleon at rest
- time-dependent mean field - moving nucleon

$$U(t,\boldsymbol{x}) = \bar{U}\left(\frac{\boldsymbol{x} - \boldsymbol{v}t}{\sqrt{1-v^2}}\right)$$

✓ E.Witten NPB160 (1979)

✓ E.Witten NPB223 (1984)

✓ D.I.Diakonov, V.Y.Petrov, P.V.Pobylitsa. NPB306 (1988)

✓ C.V. Christov et al, PPNP37 (1996)

✓ D.I. Diakonov *et al. Nucl.Phys.B* 480 (1996)

✓ D.I. Diakonov et al. PRD56 (1997)

Chiral quark-soliton model

✓ Baryon wave function

$$|B\rangle = \prod_{\text{color}}^{N_c} \int \frac{d^3k}{(2\pi)^3} F(\boldsymbol{k}) a^\dagger(\boldsymbol{k}) |\Omega\rangle$$

✓ Vacuum wave function

- quark-antiquark pair wave function W(p, p')
- Generate higher Fock states 5Q, 7Q, 9Q, …

$$|\Omega\rangle = \exp\left(\int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^3} a^\dagger(\boldsymbol{p}) W(\boldsymbol{p}, \boldsymbol{p'}) b^\dagger(\boldsymbol{p'}) \right) |0\rangle$$

✓ 3Q wave function in IMF

- translational and rotational zero modes
- normalization constant c_0

$$|B_\lambda\rangle^{3q} = c_0 \int [dk_\perp] \int \left[\frac{dx}{\sqrt{x}} \right] \int dR B^*_\lambda(R) \epsilon^{\alpha_1\alpha_2\alpha_3} \prod_{n=1}^{N} R^{f_n}_{j_n} F^{j_n\sigma_n}(x_n, \boldsymbol{k}^n_\perp) a^\dagger_{\alpha_n f_n \sigma_n}(x_n, \boldsymbol{k}^n_\perp) |0\rangle.$$

✓ V.Y.Petrov, M.V. Polyakov [hep-ph/0307077] (2002),

✓ D.I.Diakonov, V.Y.Petrov Annalen Phys 13 (2004),

✓ D.I.Diakonov, V.Y.Petrov PRD72 (2005)

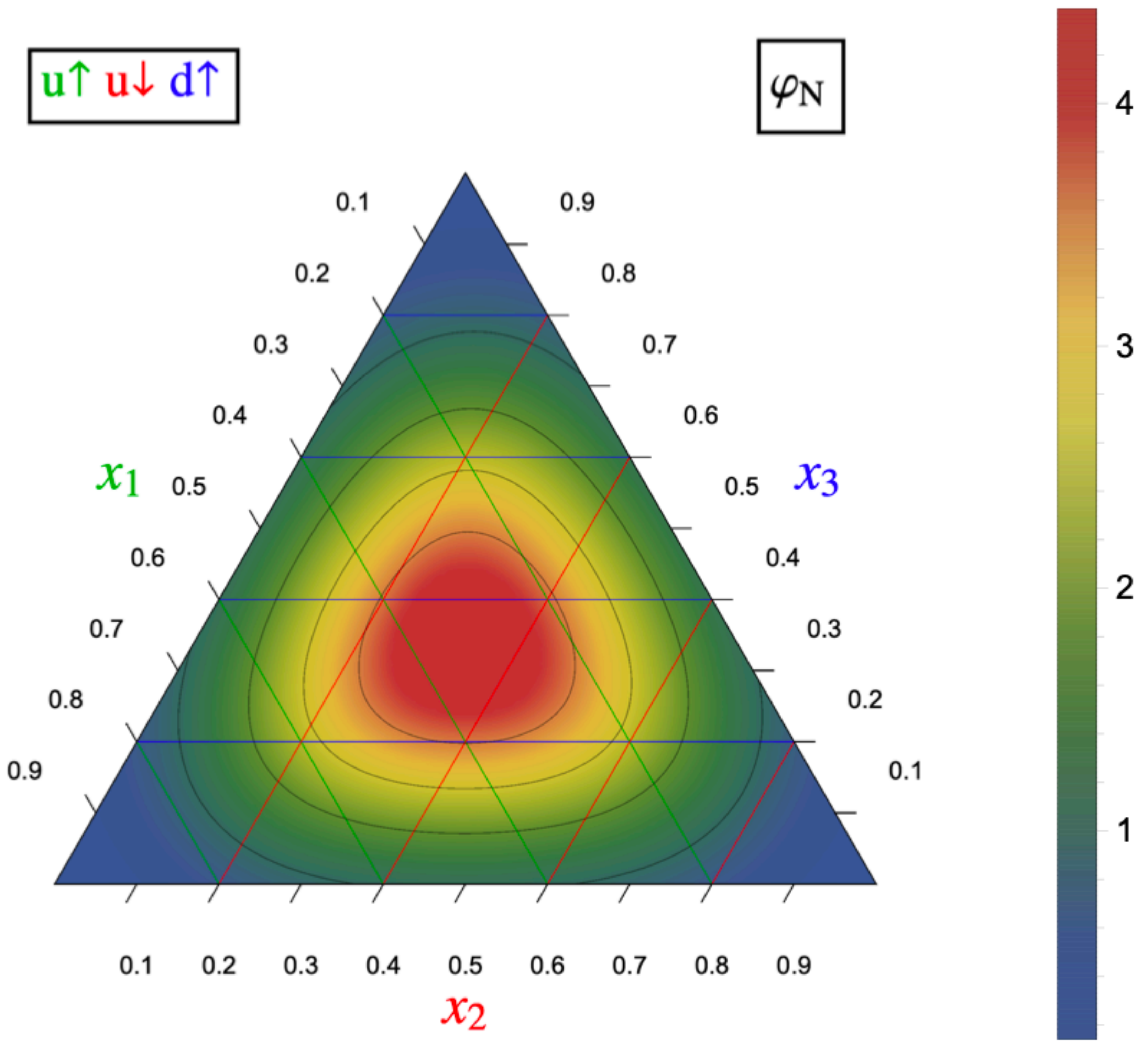
Results

✓ Leading twist-3 nucleon DA

$$\langle 0|\epsilon^{ijk}\left(u_i^{\uparrow}(a_1 z) C \not{z} u_j^{\downarrow}(a_2 z)\right) \not{z} d_k^{\uparrow}(a_3 z)|P\rangle = -\frac{1}{2}(p\cdot z)\not{z} N^{\uparrow}\int [dx] e^{-ip\cdot z\sum_i x_i a_i}\varphi_N(x_1, x_2, x_3),$$

- almost symmetric
- a rather small asymmetry
- non-zero values at the end-points
- normalization constant f_N

$$f_N(\mu \approx 0.6\ \text{GeV}) = 4.7\times 10^{-3}\ \text{GeV}^2$$



What we are missing?

✓ Normalization c_0 of the LCWF

- Valence-quark dominance?

$$\langle P|P'\rangle \overset{?}{\approx}{}^{3q} \langle P|P'\rangle^{3q}$$

- 5Q component is substantial

$$\langle P|P'\rangle =^{3q} \langle P|P'\rangle^{3q} +^{5q} \langle P|P'\rangle^{5q}...$$

$$g_A^{3q} = 1.67, \quad g_A^{5q} = 1.36, \quad g_A^{\text{ex}} = 1.27$$

- Its impact on f_N is also important.

$$f_N^{7q}(\mu \approx 2 \text{ GeV}) \sim 3.6 \times 10^{-3}\text{GeV}^2$$

✓ D.I.Diakonov, V.Y.Petrov Annalen Phys 13 (2004)

✓ D.I.Diakonov, V.Y.Petrov PRD72 (2005)

✓ C.Lorce Phys.Rev.D 79 (2009)

Lattice QCD

$$f_N^{N_f=2+1}(\mu = 2 \text{ GeV}) = 3.54^{+6}_{-4} \times 10^{-3}\text{GeV}^2$$

/G. S. Bali et al. EPJA

$$f_N^{N_f=2}(\mu = 2 \text{ GeV}) = 2.84^{+33}_{-33} \times 10^{-3}\text{GeV}^2$$

/V.M. Braun. Et al. PRD89

✓ Definite current quark mass

✓ Effect of the explicit flavor SU(3) symmetry breaking

Conclusions

- We obtained the nucleon distribution amplitudes in the chiral quark-soliton model motivated by the QCD instanton vacuum.

- The light cone wave function of the nucleon is expressed in terms of the wave function of the discrete level in the presence of the pion mean field and the quark-antiquark pair wave functions.

- We found that the result of the nucleon DA is almost symmetric, and it has a rather small asymmetry.

- The normalization constant f_N is comparable to that from the lattice QCD.

Outlook

- Distribution amplitudes of the baryon decuplet are also an interesting subject. It would be interesting to compare the normalization constants $f_\Delta^{1/2}$, $f_\Delta^{3/2}$ for the Δ baryon with those from lattice QCD.

- Higher twist-4,5,6 distribution amplitudes are under investigation.

Thank you very much!