Good and bad diquark properties and spatial correlations in lattice QCD

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based on arXiv:2106.09080

Diquarks - an attractive concept

"The concept of diquarks is almost as old as the quark model, and actually predates QCD [1]"

$\rightsquigarrow$ Snowmass '20, [1] PR 155, 1601 (1967)

- Successful for low-lying baryons and exotic hadrons.
  - But, experimental evidence has been elusive.
- Well founded in QCD with many predictions. Light quarks:
  - special "good" $(\bar{3}_F, \bar{3}_c, J^P = 0^+)$ configuration
  - "good" diquarks experience attraction effect
  - large mass splitting in good, bad and not-even-bad
  - non-vanishing size or compact?
- For heavy quarks, with HQSS, diquarks can act as single antiquarks $[QQ] \leftrightarrow \bar{Q}$.

$\rightsquigarrow$ opportunities for exotic hadrons, see B. Colquhoun on Tue.

good, bad and not-even

Diquark operator:

$$\boxed{D_\Gamma = q^c C \Gamma q'}$$

$\rightsquigarrow$ c, C =charge conjugation

$\rightsquigarrow$ Γ acts on Dirac space

J^P	C	F	Op: Γ
0^+	$\bar{3}$	$\bar{3}$	γ_5, $\gamma_0\gamma_5$
1^+	$\bar{3}$	6	γ_i, σ_{i0}
0^-	$\bar{3}$	6	$\mathbb{1}$, γ_0
1^-	$\bar{3}$	$\bar{3}$	$\gamma_i\gamma_5$, σ_{ij}

towards a clearer understanding and footing in QCD

Goal: Measure diquark properties in QCD non-perturbatively

- **spectrum:** *[diquark] mass differences are fundamental characteristics of QCD* (Jaffe '05, arXiv:hep-ph/0409065)
- **spatial correlations:** *study attraction and special status of the "good" diquark*
- **structure:** *estimate size and shape of the "good" diquark*

A gauge invariant probe - static quark as spectator

- A problem for the lattice is that diquarks are colored, i.e. not-gauge invariant.
  - Could fix a gauge, but then properties are gauge-dependent (masses, sizes,...)

$\rightsquigarrow$ `lattice and Dyson-Schwinger, see e.g. [15-20] in 2106.09080`

- **Alternative:** Static spectator quark Q ($m_Q \to \infty$) cancels in mass differences.
  - Diquark properties exposed in a gauge-invariant way.

$\rightsquigarrow$ `hep-lat/0510082, hep-lat/0509113, hep-lat/0609004, arxiv:1012.2353`

$$\boxed{C_\Gamma(t) \sim \exp\left[-t\left(m_{D_\Gamma} + m_Q + \mathcal{O}(m_Q^{-1})\right)\right]}$$

$\rightsquigarrow$ $t \to$ large, $m_Q \to$ large

- **Lattice correlator:** Diquark embedded in a static-light-light baryon

$$C_\Gamma(t) = \sum_{\vec{x}} \left\langle [D_\Gamma Q](\vec{x}, t)\ [D_\Gamma Q]^\dagger(\vec{0}, 0)\right\rangle$$

$\rightsquigarrow$ static quark=Q and $D_\Gamma = q^c C\Gamma q$
$\rightsquigarrow$ flavor combinations ud, ℓs, ss'
$\rightsquigarrow$ static-light mesons $[\bar{Q}\Gamma q]$

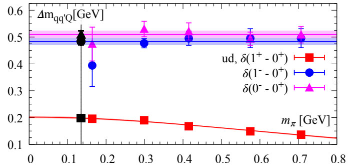
setting up on the lattice - we recycle

- $n_f = 2+1$ full QCD, $32^3 \times 64$, $a = 0.090$fm, $a^{-1} = 2.194$GeV (PACS-CS gauges)
- $\boxed{m_\pi = 164, 299, 415, 575, 707\,\text{MeV}}$, $m_s \simeq m_s^{\text{phys}}$, propagators re-used from before
- Quenched gauge $a \simeq 0.1$fm, $\boxed{m_\pi^{\text{valence}} = 909\,\text{MeV}}$, to match `hep-lat/0509113`

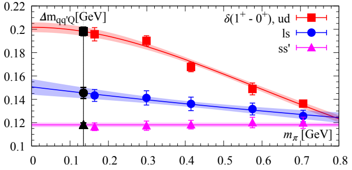
I. Diquark spectroscopy

Lattice spectroscopy - diquark-diquark differences

ud 0^+ versus 1^+, 0^- and 1^-



$(1^+ - 0^+)_{qq'}$ splitting



We consider differences of $qq'Q$ baryons:

$$C_\Gamma^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)$$

$\rightsquigarrow$ Q drops out

$\rightsquigarrow$ measures diquark-diquark mass difference

Bad-good diquark splitting:

- Special status of good diquark observed
- Good 0^+ ud diquark lies lowest in the spectrum
- Bad 1^+ ud diquark 100-200 MeV above
- 0^- and 1^- ud diquarks ~ 0.5 GeV above
- Pattern repeated in ℓs and ss'

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral limit: $\sim$ const
- Heavy-quark limit: decreases $\sim 1/(m_{q_1} m_{q_2})$, with $m_\pi \sim (m_{q_1} + m_{q_2})$

$$\delta(1^+ - 0^+)_{q_1 q_2} = A / \left[1 + (m_\pi / B)^{n \in 0,1,2}\right]$$

Lattice spectroscopy - diquark-quark differences

We consider differences of a $qq'Q$ baryon and a light-static meson:

$$C^{qq'Q}_{\Gamma=\gamma_5}(t) - C^{q'\bar{Q}}_{\gamma_5}(t)$$

$\rightsquigarrow$ Q drops out

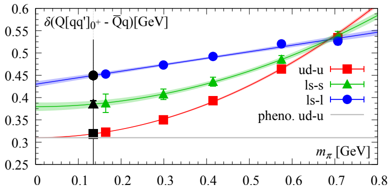
$\rightsquigarrow$ diquark-quark mass difference

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral vs. heavy-quark limiting behaviours, as before

$$\delta(Q[q_1q_2]_{0^+} - \bar{Q}q_2) = C\;\left[1 + (m_\pi/D)^{n\in 0,1,2}\right]$$

$Qqq' - \bar{Q}q'$ splittings



Diquark-quark splitting:

- Established relative masses between a good diquark and an [anti]quark
- May prove useful in identifying favourable tetra-, pentaquark channels
- Omits possible distortions through additional light quarks, Pauli-blocking, spin-spin interactions ...

Diquark spectroscopy - phenomenological estimates

We want to compare our results with phenomenology

- Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
- For pheno estimates use charm and bottom hadron masses where leading $\mathcal{O}(1/m_Q)$ $(Q = c, b)$ can be cancelled

Four estimates considered:

- $\delta(1^+ - 0^+)_{ud}$: $\boxed{\frac{1}{3}\left(2M(\Sigma_Q^*) + M(\Sigma_Q)\right) - M(\Lambda_Q)}$
- $\delta(1^+ - 0^+)_{us}$: $\boxed{\frac{2}{3}\left(M(\Xi_Q^*) + M(\Sigma_Q) + M(\Omega_Q)\right) - M(\Xi_Q) - M(\Xi_Q')}$
- $\delta(Q[ud]_{0^+} - \bar{Q}u)$: $\boxed{M(\Lambda_Q) - \frac{1}{4}\left(M(P_{Qu}) + 3M(V_{Qu})\right)}$

  $\rightsquigarrow$ P_{Qu}, V_{Qu} are the ground-state, heavy-light mesons
- $\delta(Q[us]_{0^+} - \bar{Q}s)$:

$$\boxed{M(\Xi_Q) + M(\Xi_Q') - \frac{1}{2}(M(\Sigma_Q) + M(\Omega_Q)) - \frac{1}{4}(M(P_{Qs}) + 3M(V_{Qs}))}$$

$\rightsquigarrow$ P_{Qs}, V_{Qs} are the ground-state, heavy-strange mesons

Diquark spectroscopy - comparing results

- We summarise the main spectroscopy results as:

All in [MeV]	$\delta E_{\text{lat}}(m_\pi^{\text{phys}})$	δE_{pheno}	$\delta E^{\text{bottom}}_{\text{pheno}}$	$\delta E^{\text{charm}}_{\text{pheno}}$
$\delta(1^+ - 0^+)_{ud}$	198(4)	206(4)	206	210
$\delta(1^+ - 0^+)_{\ell s}$	145(5)	145(3)	145	148
$\delta(1^+ - 0^+)_{ss'}$	118(2)			
$\delta(Q[ud]_{0^+} - \bar{Q}u)$	319(1)	306(7)	306	313
$\delta(Q[\ell s]_{0^+} - \bar{Q}s)$	385(9)	397(1)	397	398
$\delta(Q[\ell s]_{0^+} - \bar{Q}\ell)$	450(6)			

$\rightsquigarrow$ updated pheno using PDG '21
$\rightsquigarrow$ use the bottom estimate for static
$\rightsquigarrow$ use charm-bottom difference as estimate for deviation from static
$\Rightarrow \lesssim \mathcal{O}(7)$MeV deviation

- Overall, very good agreement observed.

II. Diquark structure

Diquarks - spatial correlations

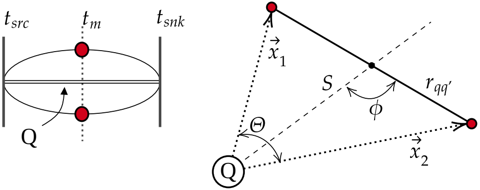
We access (good) diquark structure information through density-density correlations:

$$\boxed{C_\Gamma^{dd}(\vec{x}_1,\vec{x}_2,t) = \left\langle \mathcal{O}_\Gamma(\vec{0},2t)\ \rho(\vec{x}_1,t)\rho(\vec{x}_2,t)\ \mathcal{O}_\Gamma^\dagger(\vec{0},0)\right\rangle}$$

$\rightsquigarrow$ $\mathcal{O}_\Gamma = q^c C\Gamma q$ and $\rho(\vec{x},t) = \bar{q}(\vec{x},t)\gamma_0 q(\vec{x},t)$

$\rightsquigarrow$ $t_m = (t_{snk}+t_{src})/2$ to minimize excited states

Main tool: Correlations between two light quarks' relative positions to the static quark



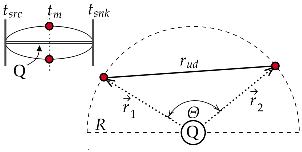
$\rightsquigarrow$ $\vec{r}_{ud} = \vec{x}_2 - \vec{x}_1$ and $\vec{S} = (\vec{x}_1 - \vec{x}_2)/2$

$$\boxed{\rho_2(r_{ud},S,\phi;\Gamma) = C_\Gamma^{dd}(\vec{x}_1,\vec{x}_2,t_m)}$$

Note, when S and r_{ud} fixed, distance between static quark Q and light quarks q, q' is

- Minimized for $\phi=\pi$, possible disruption due to Q is largest
- Maximized for $\phi=\pi/2$, possible disruption due to Q is smallest

Good diquark attraction



Setting $\phi = \pi/2$:

- $|\vec{x}_1| = |\vec{x}_2| = R$, use R, Θ:

$$\rho_2^\perp(R,\Theta) = \rho_2(r_{ud}, S, \pi/2)$$

- Attraction visible through increase in $\rho_2^\perp$ for small Θ at any fixed R

Two limiting cases for the two quarks:

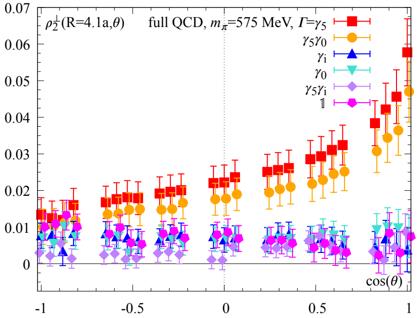
- ◦ $\cos(\Theta) = 1$ on top of each other
- ◦ $\cos(\Theta) = -1$ opposite each other

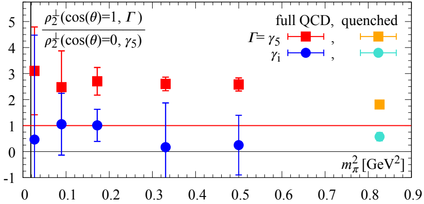
"Lift" as qualitative criterion:

$$\frac{\rho_2^\perp(R,\Theta=0,\Gamma)}{\rho_2^\perp(R,\Theta=\pi/2,\gamma_5)}$$

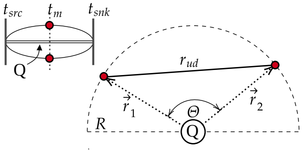
Increase observed in good diquark only

Spatial correlation over Θ





Good diquark size



- Distance between quarks:

  $r_{ud} = R\sqrt{2\,(1-\cos(\Theta))}$

  $\rightsquigarrow$ different visualisation

- $\rho_2^\perp(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$

  $\rightsquigarrow$ "characteristic size" r_0

- Need to control:
  - interference from Q

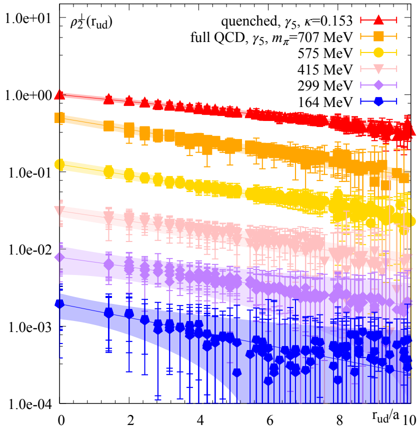
    $\rightsquigarrow$ we limit analysis to $r_{ud} < R$
  - periodicity effects

    $\rightsquigarrow$ in practice we find $L = 5r_0$

- Further checks:

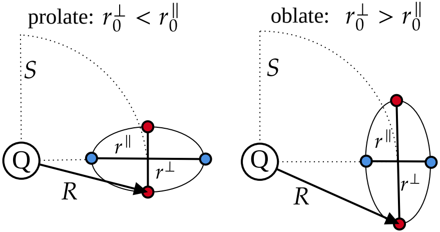
  $A(R, r_{ud} = 0) \sim \exp(-R/R_0)$

Spatial correlation over r_{ud}



- $r_{ud} = 0$ normalised, offset for each m_π
- all R shown simultaneously
- combined fits over $\forall R$ with shared r_0

Shape of good diquarks - studying oblateness



Tangential and radial spatial correlation decay

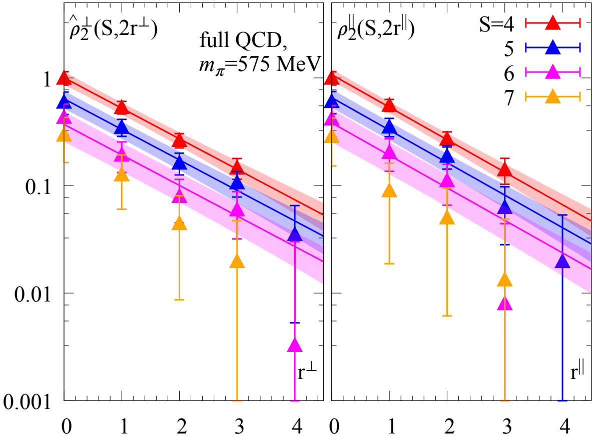
As opposed to before $R \neq$fixed:

- ◦ $\phi = \pi$: radial correlation, size $\rightsquigarrow r_0^\parallel$
- ◦ $\phi = \pi/2$: tangential correlation, size $\rightsquigarrow r_0^\perp$

◦ $r_0^\perp / r_0^\parallel$ gives information on shape:
- $= 1$, spherical
- $\neq 1$, prolate/oblate

- Probe $J = 0$ nature of good diquark
- Diquark polarisation through static quark?

Oblateness - results



Goal:

- $r_0^{\perp}$, $r_0^{||}$ at fixed S

Technical issue:

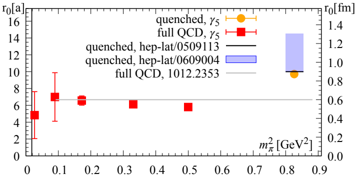
- (||) as before: $R = S$
- ($\perp$) different: $R = \sqrt{(r^{\perp})^2 + S^2}$

Solution:

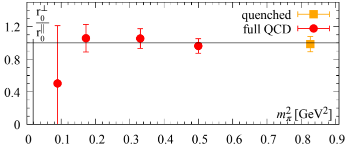
- Introduce "nuisance" paremeter R_0
- Adjusted in figure
- Parallel lines $\rightsquigarrow r_0^{\perp} = r_0^{||}$

Diquark structure - overview

Size dependence $r_0(m_\pi)$



Shape dependence $r_0^\perp / r_0^\parallel(m_\pi)$



Good diquark size:

- Agreement w/ prev. quenched and dynamical
- Refinement through our results
- $r_0 \simeq \mathcal{O}(0.6)$fm weak m_π dependence

$\rightsquigarrow \sim r_{\text{meson, baryon}}$, arXiv:1604.02891

$r_0(m_\pi)$ dependence:

- $m_{q,q'} \uparrow$ should produce more compact object
- But, diquark attraction$\downarrow$ works opposite
- Former effect dominates at large m_π?
- But, in quenched diquarks definitely larger...

$r_0^\perp / r_0^\parallel(m_\pi)$ dependence:

- Ratio $\simeq 1$ for all m_π
- Consistent w/ scalar, $J = 0$, shape
- No diquark polarisation through Q observed

Summary

Gauge invariant approach to diquarks in $n_f = 2 + 1$ lattice QCD

- Lattice setup with short chiral extrapolations, continuum limit still required

Diquark spectroscopy

- Special status of "good" diquark confirmed, attraction of 198(4)MeV over "bad"
- Chiral and flavor dependence modelled through simple Ansatz
- Very good agreement with phenomenological estimates

Diquark structure

- $q - q$ attraction in good diquark induces compact spatial correlation
- Good diquark size $r_0 \simeq \mathcal{O}(0.6)$fm $\sim r_{\text{meson, baryon}}$, weakly m_π dependent
- Good diquark shape appears nearly spherical

Outlook

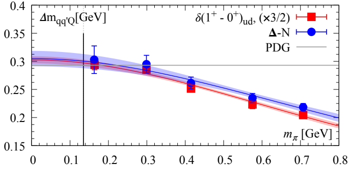
- Results provide clear, quantitative support for the good diquark picture
- Hope to refine diquark model parameters
- Insights for studies of exotic tetraquarks (esp. doubly heavy), heavy-baryons, etc.
- In future, extend towards tetraquarks, finite temperature ...

Thank you for your attention.

Further material

Δ-Nucleon mass difference

$[\Delta - N](m_\pi)$



Measured the mass difference of $\Delta - N$

- Prediction: $\delta(\Delta - N) = 3/2 \times \delta(1^+ - 0^+)_{ud}$
- Same Ansatz as before
- Prediction holds well, even at fairly large m_π