Gluon transversity and TMDs for spin-1 hadrons

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19th International Conference on
Hadron Spectroscopy and Structure (Haron2021)
Mexico City, Mexico, July 26-31, 2021
https://indico.nucleares.unam.mx/event/1541/

Refs. SK and Qin-Tao Song, PRD 101 (2020) 054011 & 094013;
PRD 103 (2021) 014025; arXiv:2106.15849;
A. Arbuzov *et al*., Prog. Nucl. Part. Phys. 119 (2021) 103858.

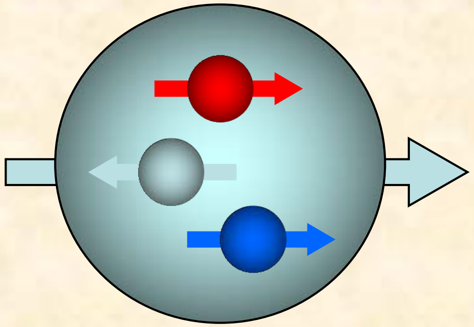
July 28, 2021

Contents

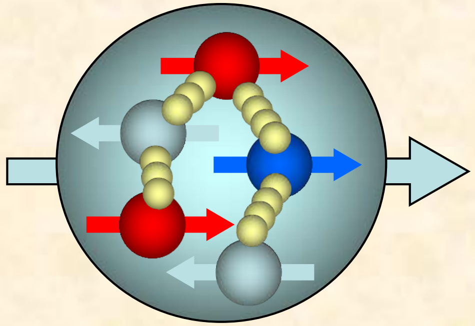
Note on our notations
Gluon transversity: $\Delta_T g$
Tensor-polarized gluon distribution: $\delta_T g$

Nucleon spin

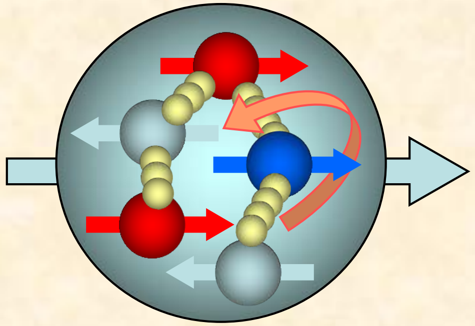
Almost none of nucleon spin is carried by quarks! → **Nucleon spin crisis!?**



Naïve Quark Model



Sea-quarks and gluons?

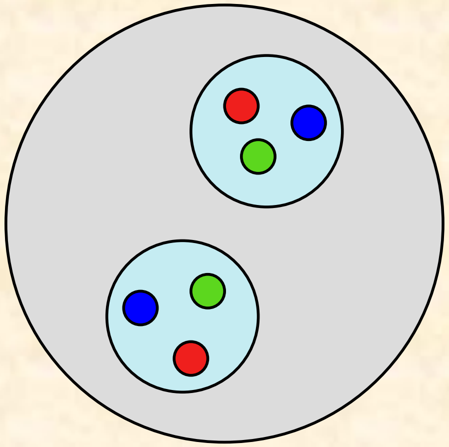


Orbital angular momenta ?

"old" standard model

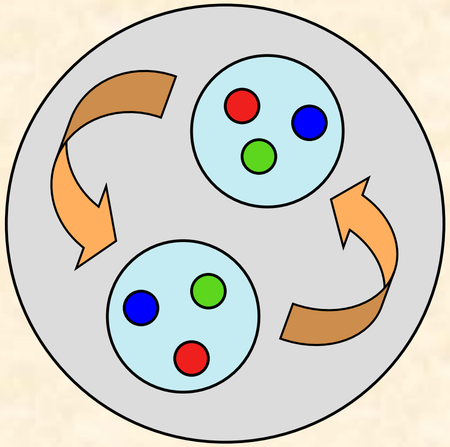
Tensor structure b_1 (*e.g.* deuteron)

Tensor-structure crisis!?



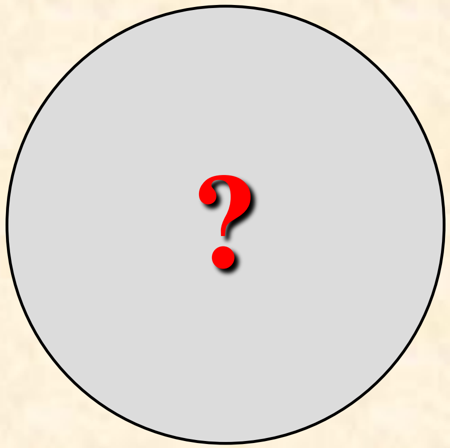
only S wave

$b_1 = 0$



S + D waves

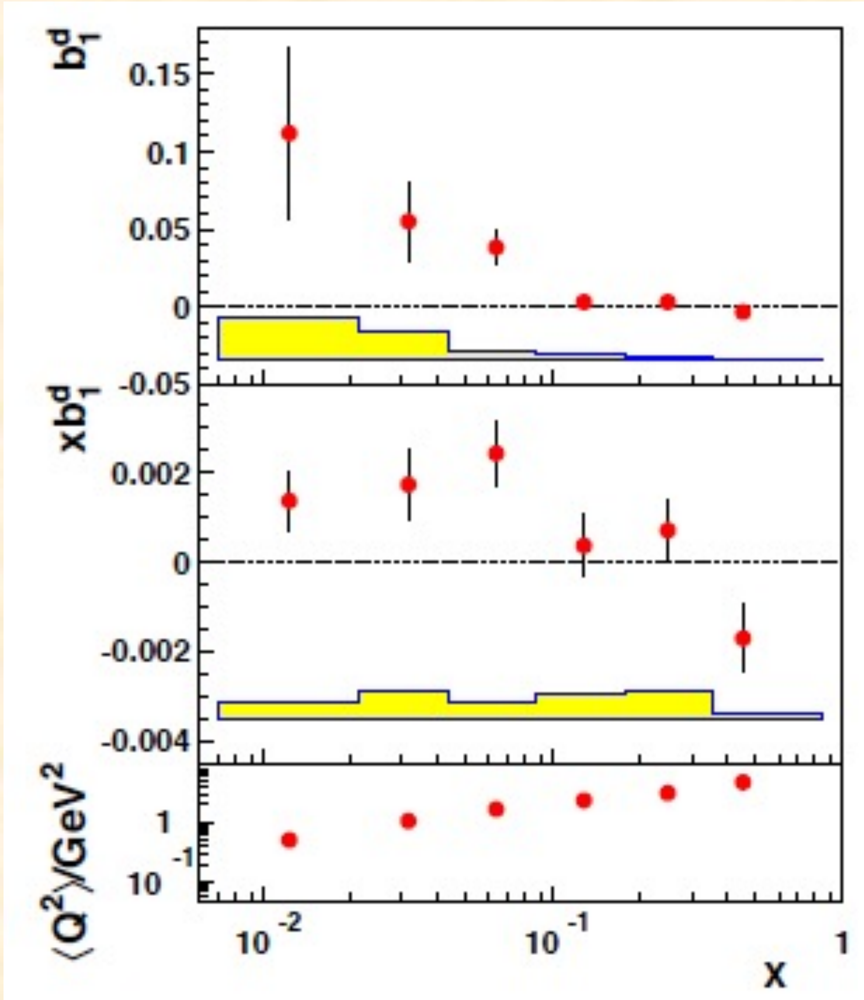
standard model $b_1 \neq 0$



$b_1^{\text{experiment}} \neq b_1^{\text{"standard model"}}$

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.



27.6 GeV/c $\rightleftarrows$, 0

$\longrightarrow \bullet$

positron **deuteron**

b_1 **measurement in the kinematical region**

$0.01 < x < 0.45, \;\; 0.5\ \text{GeV}^2 < Q^2 < 5\ \text{GeV}^2$

b_1 **sum in the restricted** Q^2 **range** $Q^2 > 1\ \text{GeV}^2$

$$\int_{0.02}^{0.85} dx\, b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5\ \text{GeV}^2$

$$\int dx\, b_1^D(x) = \lim_{t \to 0} -\frac{5}{12}\frac{t}{M^2} F_Q(t) + \underline{\sum_i e_i^2 \int dx \delta_T \bar{q}_i(x)} = 0 \ ?$$

b_1 sum rule: F. E. Close and SK, PRD 42 (1990) 2377.

$$\int \frac{dx}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3}\int dx\left[u_v - d_v\right] + \underline{\frac{2}{3}\int dx\left[\bar{u} - \bar{d}\right]} \neq 1/3$$

Drell-Yan experiments probe these antiquark distributions.

Standard model prediction for b_1 of deuteron

Convolution model: $A_{hH,hH}(x,Q^2)=\varepsilon^{*\mu}_{h'}W^{H'H}_{\mu\nu}\varepsilon^{\nu}_{h}=\int\frac{dy}{y}\sum_s f^H_s(y)\hat{A}_{hs,hs}(x/y,\,Q^2)$

$b_1=A_{+0,+0}-\frac{A_{++,++}+A_{+-,+-}}{2},\quad \hat{A}_{+\uparrow,+\uparrow}=F_1-g_1,\quad \hat{A}_{+\downarrow,+\downarrow}=F_1+g_1$

Nucleon momentum distribution: $f^H(y)\equiv f^H_\uparrow(y)+f^H_\downarrow(y)=\int d^3p\,y\,|\phi^H(\vec{p})|^2\delta\left(y-\frac{E-p_z}{M_N}\right)$

D-state admixture: $\phi^H(\vec{p})=\phi^H_{\ell=0}(\vec{p})+\phi^H_{\ell=2}(\vec{p})$

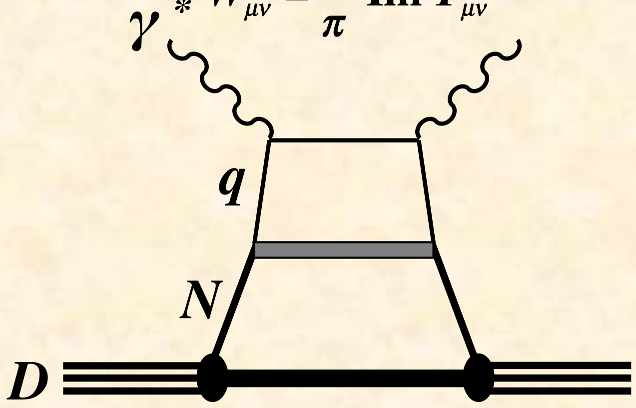
$$b_1(x)=\int\frac{dy}{y}\delta_T f(y)F_1^N(x/y,\,Q^2),\qquad y=\frac{Mp\cdot q}{M_N P\cdot q}\simeq\frac{2p^-}{P^-}$$

$$\delta_T f(y)=f^0(y)-\frac{f^+(y)+f^-(y)}{2}$$

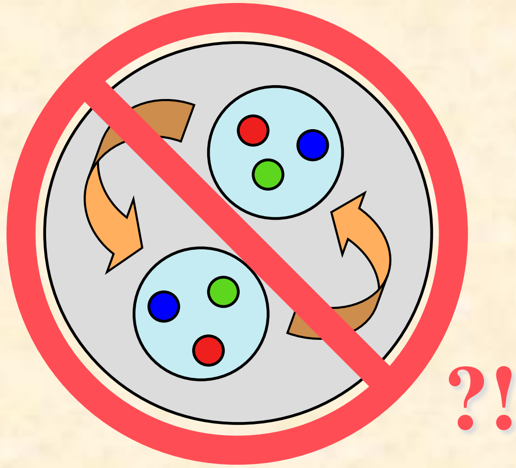
$$=\int d^3p\,y\left[-\frac{3}{4\sqrt{2}\pi}\phi_0(p)\phi_2(p)+\frac{3}{16\pi}|\phi_2(p)|^2\right](3\cos^2\theta-1)\delta\left(y-\frac{p\cdot q}{M_N\nu}\right)$$

S-D term D-D term

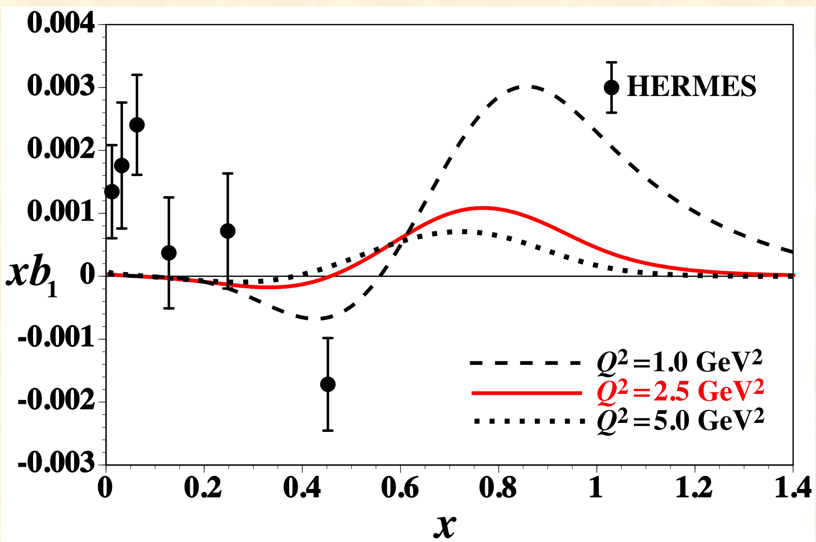
$W_{\mu\nu}=\frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}$



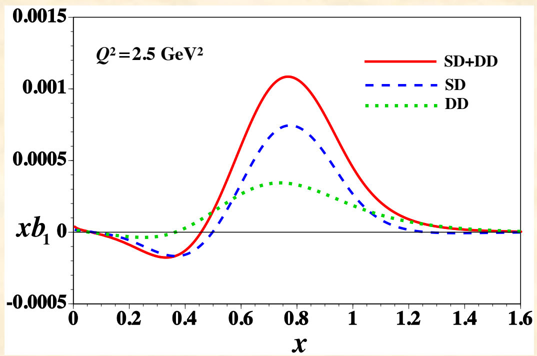
Standard model of the deuteron



S + D waves



W. Cosyn, Yu-Bing Dong, S. Kumano, M. Sargsian, Phys. Rev. D 95 (2017) 074036.



$|b_1(\text{theory})|\ll|b_1(\text{HERMES})|$ at $x<0.5$

Standard convolution model does not work for the deuteron tensor structure!?

Possible studies on gluon transversity

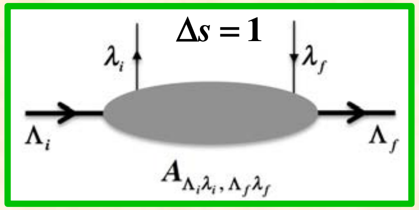
S. Kumano and Qin-Tao Song, Phys. Rev. D 101 (2020) 054011 & 094013;
A. Arbuzov *et al*., Prog. Nucl. Part. Phys. 119 (2021) 103858.

Gluon transversity $\Delta_T g$

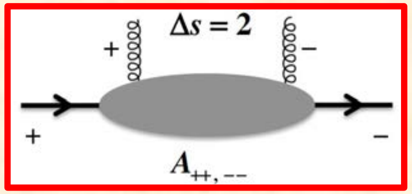
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, **conservation** $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

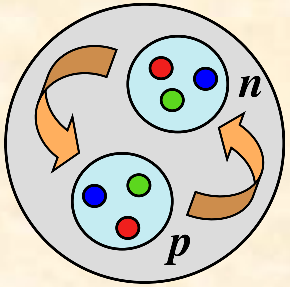
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \to \lambda_f = -\frac{1}{2}$ **quark spin flip** $(\Delta s = 1)$



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, $\cancel{A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)}$ **not possible for nucleon**



Note: Gluon transversity does not exist for spin-1/2 nucleons.

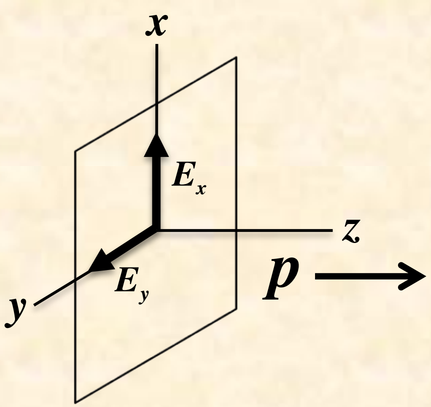


S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \ \Leftrightarrow$ **still** $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$**?**

Gluon transversity distribution in deuteron



Linear-polarization difference: $d\sigma(E_x - E_y) \propto \Delta_T g$

$$\Delta_T g(x) = \int \frac{d\xi^-}{2\pi} x p^+ \, e^{ixp^+\xi^-} \left\langle pE_x \left| A^x(0)A^x(\xi) - A^y(0)A^y(\xi) \right| pE_x \right\rangle_{\xi^+ = \vec{\xi}_T = 0}$$

$$= g_{\hat{x}/\hat{x}} - g_{\hat{y}/\hat{x}}$$

$g_{\hat{y}/\hat{x}}$ = gluon distribution with the gluon linear polarization ε_y in the deuteron linear polarization E_x

Polarization vectors $\vec{E}_x = \vec{\varepsilon}_x = (1,\ 0,\ 0)$, $\vec{E}_y = \vec{\varepsilon}_y = (0,\ 1,\ 0)$

Spin and tensor of the deuteron

$$S^\mu = \frac{1}{M}\varepsilon^{\mu\nu\alpha\beta} p_\nu \,\mathrm{Im}(E^*_\alpha E_\beta), \quad T^{\mu\nu} = -\frac{1}{3}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - \mathrm{Re}(E^{\mu *} E^\nu)$$

$$E^\mu = (0,\ \vec{E}), \quad \vec{E}_\pm = \frac{1}{\sqrt{2}}(\mp 1,\ -i,\ 0), \quad \vec{E}_0 = (0,\ 0,\ 1)$$

- $\vec{E}_+, \vec{E}_0, \vec{E}_-$: Spin states with z-components of spin $s_z = +1,\ 0,\ -1$
- $\vec{E}_x = (1,\ 0,\ 0)$, $\vec{E}_y = (0,\ 1,\ 0)$: Linear polarizations

  $\rightarrow$ to measure gluon transversity

(1) Prepare $s_x = 0$ [$\vec{E}_x = (1,\ 0,\ 0)$] by taking the quantization axis x and $s_y = 0$ [$\vec{E}_y = (0,\ 1,\ 0)$] by taking the quantization axis y.

(2) Combination of transverse polarizations.

$$\boldsymbol{S} = (S_T^x,\ S_T^y,\ S_L),$$

$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}$$

$$S_{TT}^{xy} = S_{LT}^x = S_{LT}^y = 0$$

Polarizations	$\vec{E}$	S_T^x	S_T^y	S_L	S_{LL}	S_{TT}^{xx}
Longitudinal $+z$	$\frac{1}{\sqrt{2}}(-1,\ -i,\ 0)$	0	0	+1	$+\frac{1}{2}$	0
Longitudinal $-z$	$\frac{1}{\sqrt{2}}(+1,\ -i,\ 0)$	0	0	−1	$+\frac{1}{2}$	0
Transverse $+x$	$\frac{1}{\sqrt{2}}(0,\ -1,\ -i)$	+1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $-x$	$\frac{1}{\sqrt{2}}(0,\ +1,\ -i)$	−1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $+y$	$\frac{1}{\sqrt{2}}(-i,\ 0,\ -1)$	0	+1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Transverse $-y$	$\frac{1}{\sqrt{2}}(-i,\ 0,\ +1)$	0	−1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Linear x	$(1,\ 0,\ 0)$	0	0	0	$+\frac{1}{2}$	−1
Linear y	$(0,\ 1,\ 0)$	0	0	0	$+\frac{1}{2}$	+1

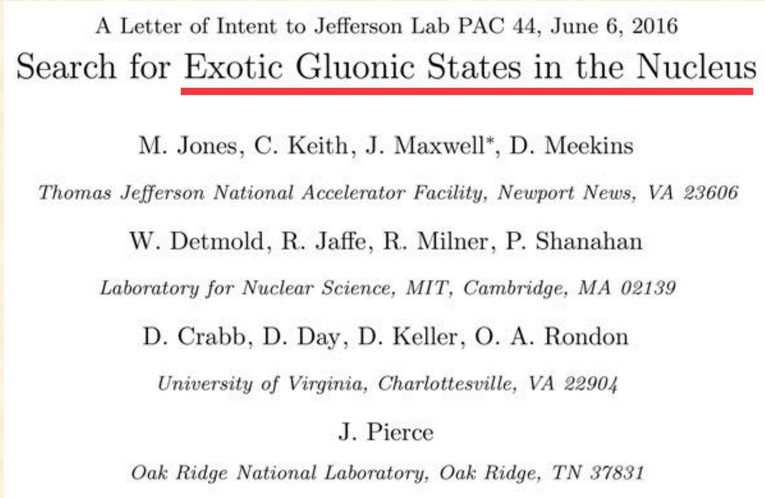
Transverse polarization

Linear polarization

Letter of Intent at Jefferson Lab (middle 2020’s)

Jefferson Lab,
Electron accelerator ~12 GeV

LoI, arXiv:1803.11206



A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

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Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

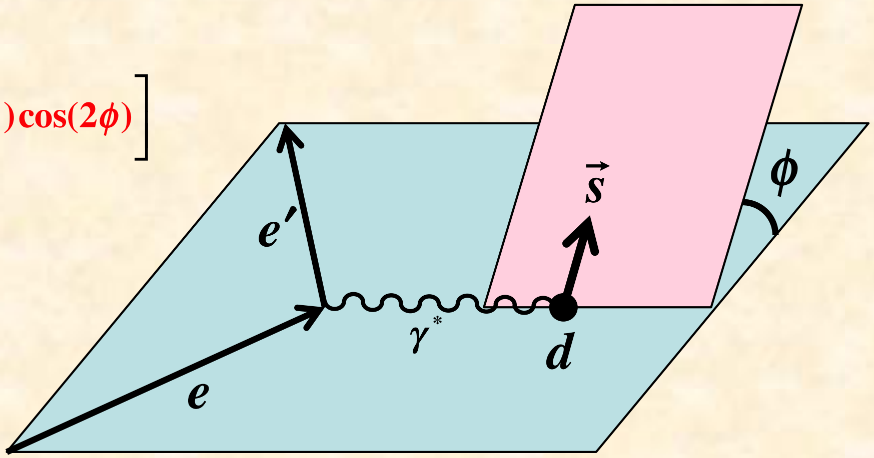
For development of polarized deuteron target, see D. Keller, D. Crabb, D. Day Nucl. Inst. Meth. Phys. Res. A981 (2020) 164504.

Electron scattering with polarized-deuteron target

$$\left.\frac{d\sigma}{dx\,dy\,d\phi}\right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4}\left[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2) - \frac{1}{2}x(1-y)\Delta(x,Q^2)\cos(2\phi) \right]$$

$$\Delta(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3}\Delta_T g(y,Q^2)$$

By looking at the deuteron-polarization angle ϕ, the quark transversty $\Delta_T g$ can be measured.



Our motivation by considering the JLab experiment

We proposed to use hadron accelerator facilities for studying the gluon transversity.

Advantages:

- **Independent experiment from JLab**
- **Different kinematical regions: larger Q^2, smaller x**
- **Hadron facilities are often useful for probing gluon distributions (namely a leading effect).**
- **Hadron cross sections are generally larger (not for Drell-Yan).**
- **The gluon transversity could be measured in a different form from the integral $\int_x^1 \frac{dy}{y^3} \Delta_T q(y, Q^2)$ in the JLab experiment.**

→ In our PRD 101 (2020) 054011 & 094013 , we proposed proton-deuteron Drell-Yan process by considering the Fermilab-E1039.

However, our formalism is valided for Drell-Yan experiments at any other facilities.

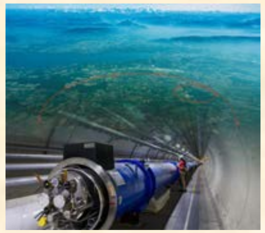
Fermilab-MI

NICA

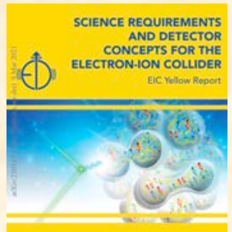
RHIC (fixed target)

GSI-FAIR

J-PARC

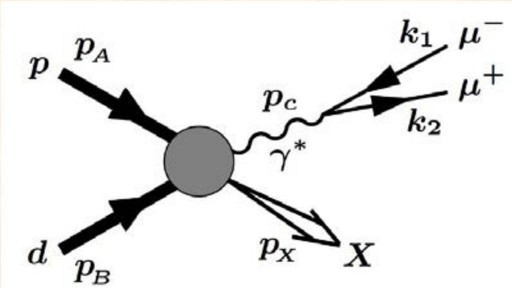


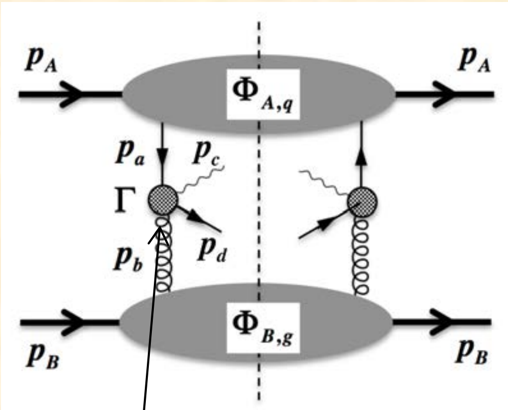
LHC (fixed target)
COMPASS/AMBER

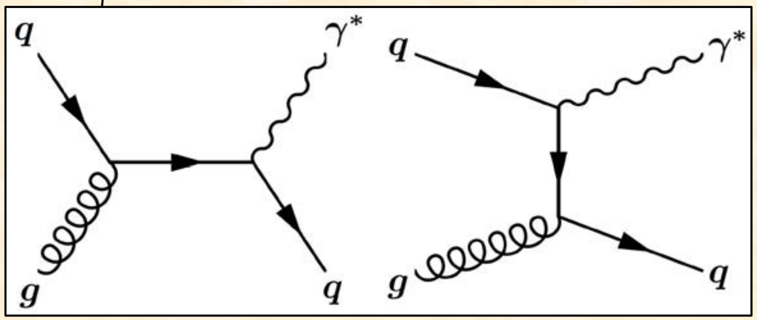


EIC,
EicC

Proton-deuteron Drell-Yan cross section







Drell-Yan cross section

$$d\sigma_{pd\to\mu^+\mu^-X} = \int_0^1 dx_a \int_0^1 dx_b f_a(x_a) f_b(x_b) d\hat{\sigma}_{ab\to\mu^+\mu^-d}, \quad M_{ab\to\mu^+\mu^-d} = eM^{\mu}_{\gamma^*\to\mu^+\mu^-}\frac{-1}{Q^2}eM_{ab\to\gamma^* d}$$

In terms of lepton tensor $L^{\mu\nu}$ and hadron tensor $W_{\mu\nu}$

$$\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy} = \frac{\alpha^2}{12\pi^2 Q^4}\left[\int d\Phi_2(q;\, k_1,\, k_2)\; 2L^{\mu\nu}\right]W_{\mu\nu}$$

dilepton phase space: $d\Phi_2(q;\, k_1,\, k_2) = \delta^4(q-k_1-k_2)\dfrac{d^3k_1}{2E_1(2\pi)^3}\dfrac{d^3k_2}{2E_2(2\pi)^3}$

$$L^{\mu\nu} = 2(k_1^{\mu}k_2^{\nu} + k_1^{\nu}k_2^{\mu} - k_1\cdot k_2 g^{\mu\nu})$$

$$W_{\mu\nu} = \overline{\sum_{\substack{\text{spin,}\\ \text{color}}}}\sum_q e_q^2 \int_{\min(x_a)}^1 dx_a \frac{\pi}{p_g^-(x_a - x_1)} \mathrm{Tr}\left[\Gamma_{\nu\beta}\left\{\Phi_{q/A}(x_a) + \Phi_{\bar{q}/A}(x_a)\right\}\hat{\Gamma}_{\mu\alpha}\Phi^{\alpha\beta}_{g/B}(x_b)\right], \quad \hat{\Gamma}_{\nu\beta} = \gamma^0\Gamma_{\nu\beta}\gamma^0$$

Collinear correlation functions

Refs. A. Bacchetta and P. J. Mulders, Phys. Rev. D 62 (2000) 114004.
D. Boer et al., JHEP 10 (2016) 013.
T. van Daal, arXiv:1812.07336 (Ph.D. Thesis)

$$\Phi_{q/A}(x_a) = \frac{1}{2}\left[\bar{\not{n}} f_{1,q/A}(x_a) + \gamma_5 \bar{\not{n}} S_{A,L} g_{1,q/A}(x_a) + \bar{\not{n}}\gamma_5 \not{S}_{A\perp} h_{1,q/A}(x_a)\right]$$

$$\Phi_{q/B}(x_b) = \frac{1}{2}\left[\not{n} f_{1,q/B}(x_b) + \gamma^5 \not{n}\, S_{B,L}\, g_{1,q/B}(x_b) + i\sigma_{\mu\nu}\gamma^5 n^{\mu} S^{\nu}_{B,T} h_{1,q/B}(x_b) + \not{n}\, S_{LL}\, f_{1LL,q/B}(x_b) + \sigma_{\mu\nu} n^{\nu} S^{\mu}_{B,LT} h_{1LT,q/B}(x_b)\right]$$

$$\Phi^{ij}_{g/B}(x_b) = \frac{1}{2}\left[-g_T^{ij} f_{1,g/B}(x_b) + i\epsilon_T^{ij} S_{B,L} g_{1L,g/B}(x_b) - g_T^{ij} S_{B,LL} f_{1LL,g/B}(x_b) + S^{ij}_{B,TT} h_{1TT,g/B}(x_b)\right]$$

Gluon transversity: $\Delta_T g = h_{1TT,g}$
(Sorry to use two differenent notations in a talk.)

Proton-deuteron Drell-Yan cross section

Drell-Yan cross section

$$\frac{d\sigma_{pd\to\mu^+\mu^-X}(E_x-E_y)}{d\tau\, dq_T^2\, d\phi\, dy} = \frac{\alpha^2\alpha_s C_F q_T^2}{6\pi s^3}\cos(2\phi)\int_{\min(x_a)}^{1} dx_a \frac{1}{(x_a x_b)^2 (x_a - x_1)(\tau - x_a x_2)^2} \sum_q e_q^2 x_a \left[q_A(x_a)+\bar{q}_A(x_a)\right] x_b \Delta_T g_B(x_b)$$

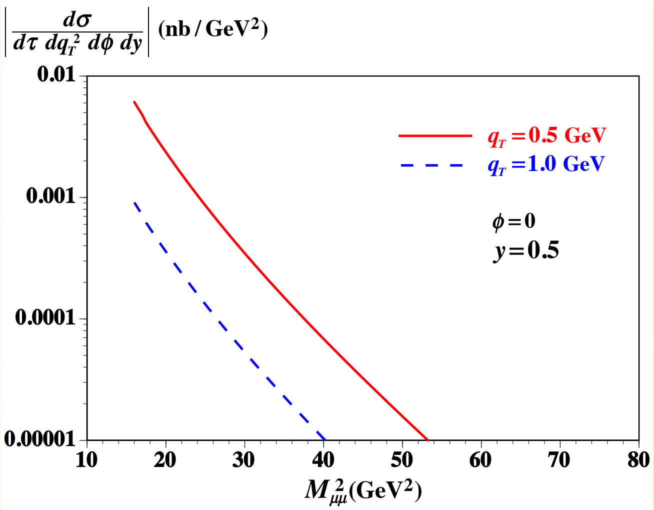
$$C_F = \frac{N_c^2-1}{2N_c}, \quad \min(x_a) = \frac{x_1-\tau}{1-x_2}, \quad x_b = \frac{x_a x_2 - \tau}{x_a - \tau}$$

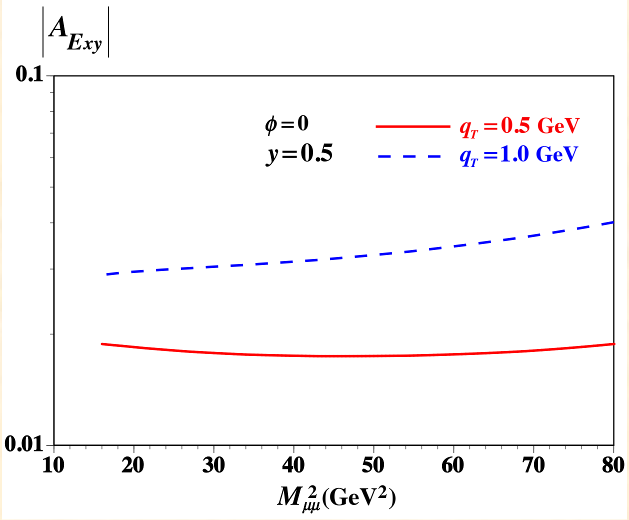
= (unpolarized PDFs of proton) * (gluon transversity distribution in the deuteron)

- Consider the Fermilab-E1039 experiment with the proton beam of $p = 120$ GeV
- No available $\Delta_T g$, so we may tentatively assume $\Delta_T g = \Delta g_p + \Delta g_n$ $\left(\text{or } \frac{\Delta g_p + \Delta g_n}{2}, \frac{\Delta g_p + \Delta g_n}{4} \right)$
- CTEQ14 for $q(x)+\bar{q}(x)$, NNPDFpol1.1 for $\Delta g(x)$

Cross section: Dimuon mass squred ($M_{\mu\mu}^2 = Q^2$) dependence

Spin asymmetry: $$A_{E_{xy}} = \frac{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x) - \frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x) + \frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}$$





New proposal at Fermilab-PAC in January, 2022 (D. Keller) !

Experimental possibility at Fermilab in 2020's

Polarized fixed-target experiments at the Main Injector, Proton beam = 120 GeV

© Fermilab

J-PARC?

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Fermilab-E1039

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

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Fermilab experimentalists are interested in the gluon transversity by replacing the E1039 proton target for the deuteron one. (Spokesperson of E1039: D. Keller) However, there was no theoretical formalism until our work.

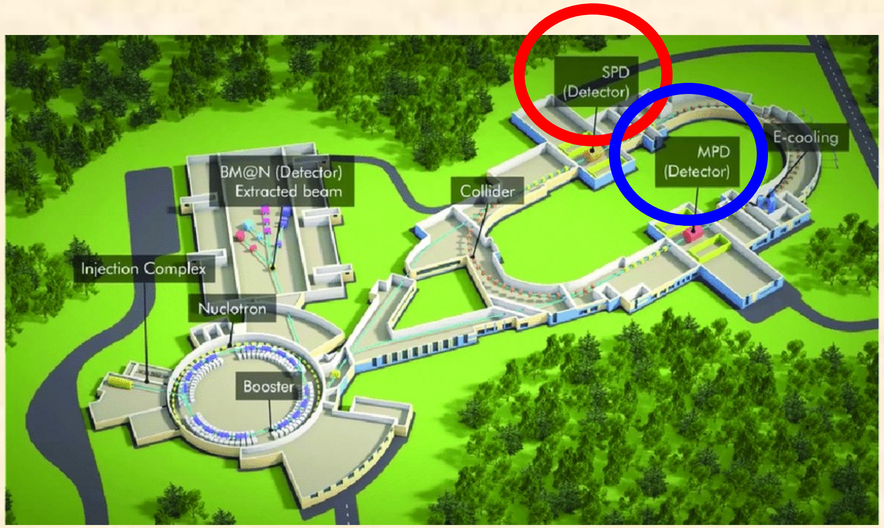
The Transverse Structure of the Deuteron with Drell-Yan

D. Keller[1]

[1] *University of Virginia, Charlottesville, VA 22904*

New proposal is being written for a Fermilab-PAC in 2022.

Nuclotron-based Ion Collider fAcility (NICA)



SPD (Spin Physics Detector for physics with polarized beams)
MPD (MultiPurpose Detector for heavy ion physics)

$\vec{p}+\vec{p}: \quad \sqrt{s_{pp}} = 12 \sim 27$ GeV

$\vec{d}+\vec{d}: \quad \sqrt{s_{NN}} = 4 \sim 14$ GeV

$\vec{p}+\vec{d}$ is also possilbe.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al*. (NICA project), Nucl. Part. Phys. 119 (2021) 103858..

Unique opportunity in high-energy spin physics, especially on the deuteron spin physics.
→ Theoretical formalisms need to be developed.

It is a timely project in 2020's in competition with JLab, Fermilab, and EIC/EicC (possibly also AMBER, J-PARC, GSI-FAIR).

TMDs for spin-1 hadrons

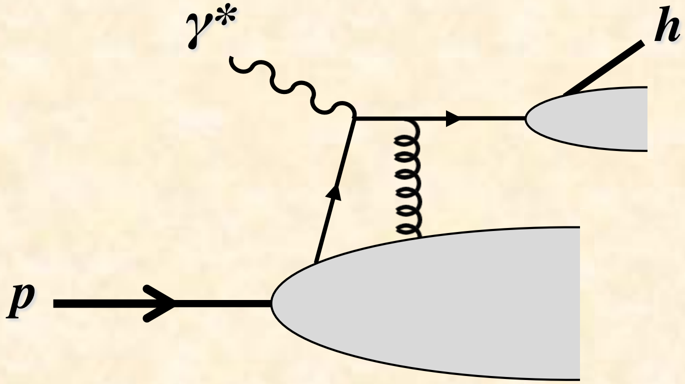
S. Kumano and Qin-Tao Song,
Phys. Rev. D 103 (2021) 014025.

Importance of color flow (gauge link) in semi-inclusive DIS and Drell-Yan processes

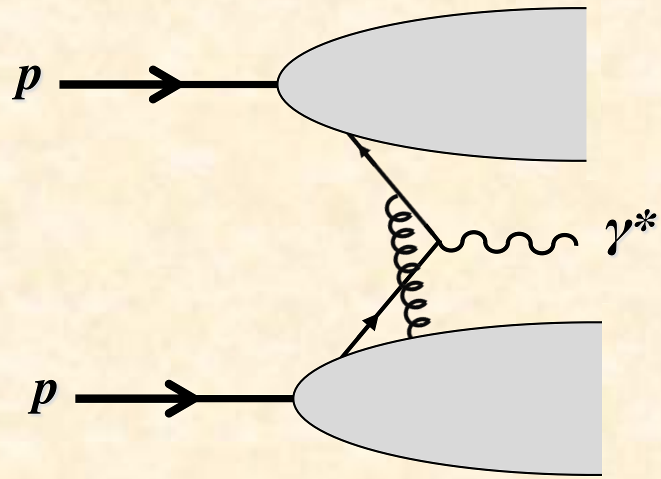
$$\boxed{q(x,k_\perp)=\int\frac{dz^- d^2z_\perp}{2(2\pi)^3}\,e^{-ixp^+z^-+i\vec{k}_\perp\cdot\vec{z}_\perp}\langle p|\bar{\psi}(z^-,\vec{z}_\perp)\,\gamma^+U(z^-,\vec{z}_\perp;0)\psi(0)|p\rangle\Big|_{z^+=0}}$$

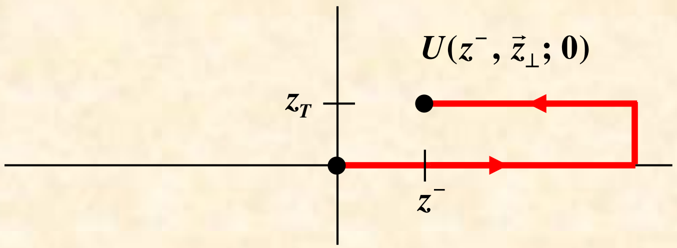
Semi-inclusive DIS (deep inelastic scattering):

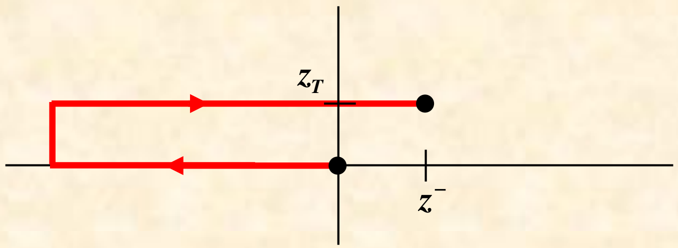
$e+p\to e'+h+X$



Drell-Yan process: $p+p\to\mu^+\mu^-+X$







Twsit-2 TMDs for spin-1/2 nucleons and spin-1 hadrons

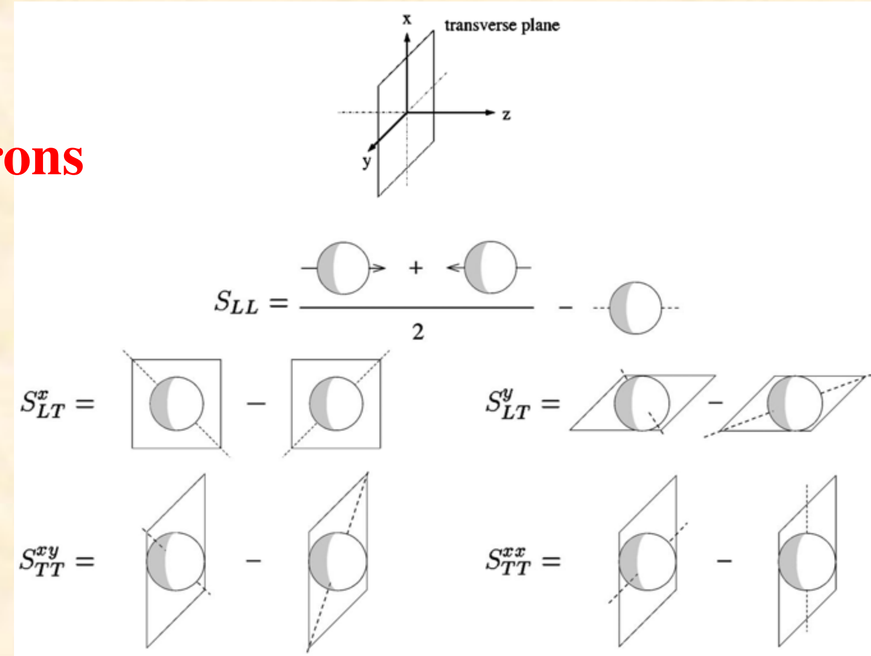
Twist-2 TMDs

Bacchetta-Mulders, PRD 62 (2000) 114004.

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Spin-1/2 nucleon

Spin-1 hadrons



Twist-2 collinear PDFs

$[\cdot\cdot\cdot]$= chiral odd

Hadron \ Quark	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					
L			$g_{1L}\,(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}\,(b_1)$					
LT						*1
TT						

*1 Because of the time-reversal invariance, the collinear PDF $h_{1LT}(x)$ vanishes. However, since the time-reversal invariance cannot be imposed in the fragmentation functions, we should note that the corresponding fragmentation function $H_{1LT}(z)$ should exist as a collinear fragmentation function. (see our PRD paper for the details)

TMD correlation functions for spin-1 hadrons

$$\Phi_{ij}(k,P,T) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi} \left\langle P,T \middle| \bar\psi_j(0) W(0,\xi)\psi_i(\xi) \middle| P,T\right\rangle$$
$$W(0,\xi) = P\exp\left[-ig\int_0^\xi d\xi\cdot A(\xi)\right]$$

Spin vector: $S^\mu = S_L \frac{P^+}{M}\bar n^\mu - S_L\frac{M}{2P^+}n^\mu + S_T^\mu$

Tensor: $T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar n^\mu \bar n^\nu + \frac{P^+}{M}\bar n^{\{\mu}S_{LT}^{\nu\}} - \frac{2}{3}S_{LL}\left(\bar n^{\{\mu}n^{\nu\}} - g_T^{\mu\nu}\right) + S_{TT}^{\mu\nu} - \frac{M}{2P^+}n^{\{\mu}S_{LT}^{\nu\}} + \frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu\right]$

Tensor part (twist-2): Bacchetta, Mulders, PRD 62 (2000) 114004

$$\Phi(k,P,T) = \left(\frac{A_{13}}{M}I + \frac{A_{14}}{M^2}\not P + \frac{A_{15}}{M^2}\not k + \frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu} + \left[A_{17}\gamma_\nu + \left(\frac{A_{18}}{M}P^\rho + \frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho} + \frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}$$

Tensor part (twist-2, 3, 4): n^μ **dependent terms are added for up to twist 4.**

[For the spin-1/2 nucleon: Goeke, Metzand, Schlegel, PLB 618 (2005) ,90; Metz, Schweitzer, Teckentrup, PLB 680 (2009) 141.]

Kumano-Song-2020, for the details see KEK-TH-2258

$$\Phi(k,P,T\,|\,n) = \boxed{\left(\frac{A_{13}}{M}I + \frac{A_{14}}{M^2}\not P + \frac{A_{15}}{M^2}\not k + \frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu} + \left[A_{17}\gamma_\nu + \left(\frac{A_{18}}{M}P^\rho + \frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho} + \frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}}$$

Bacchetta -Mulders

$$+\left(\frac{B_{21}M}{P\cdot n}k_\mu + \frac{B_{22}M^3}{(P\cdot n)^2}n_\mu\right)n_\nu T^{\mu\nu} + i\gamma_5\varepsilon_{\mu\tau\rho\sigma}P^\rho\left(\frac{B_{23}}{(P\cdot n)M}k^\tau n^\sigma k_\nu + \frac{B_{24}M}{(P\cdot n)^2}k^\tau n^\sigma n_\nu\right)T^{\mu\nu}$$

$$+\left[\frac{B_{25}}{P\cdot n}\not n k_\mu k_\nu + \left(\frac{B_{26}M^2}{(P\cdot n)^2}\not n + \frac{B_{28}}{P\cdot n}\not P + \frac{B_{30}}{P\cdot n}\not k\right)k_\mu n_\nu + \left(\frac{B_{27}M^4}{(P\cdot n)^3}\not n + \frac{B_{29}M^2}{(P\cdot n)^2}\not P + \frac{B_{31}M^2}{(P\cdot n)^2}\not k\right)n_\mu n_\nu + \frac{B_{32}M^2}{P\cdot n}\gamma_\mu n_\nu\right]T^{\mu\nu}$$

$$-\left[\varepsilon_{\mu\tau\rho\sigma}\gamma^\tau P^\rho\left(\frac{B_{34}}{P\cdot n}n^\sigma k_\nu + \frac{B_{33}}{P\cdot n}k^\sigma n_\nu + \frac{B_{35}M^2}{(P\cdot n)^2}n^\sigma n_\nu\right) + \varepsilon_{\lambda\tau\rho\sigma}k^\lambda\gamma^\tau P^\rho n^\sigma\left(\frac{B_{36}}{P\cdot nM^2}k_\mu k_\nu + \frac{B_{37}}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{38}M^2}{(P\cdot n)^3}n_\mu n_\nu\right)\right]\gamma_5 T^{\mu\nu}$$

$$+\varepsilon_{\mu\tau\rho\sigma}k^\tau P^\rho n^\sigma\left(\frac{B_{39}}{(P\cdot n)^2}k_\nu + \frac{B_{40}M^2}{(P\cdot n)^3}n_\nu\right)\not n\gamma_5 T^{\mu\nu}$$

$$+\sigma_{\rho\sigma}\left[P^\rho k^\sigma\left(\frac{B_{41}}{(P\cdot n)M}k_\mu n_\nu + \frac{B_{42}M}{(P\cdot n)^2}n_\mu n_\nu\right) + P^\rho n^\sigma\left(\frac{B_{43}}{(P\cdot n)M}k_\mu k_\nu + \frac{B_{44}M}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{45}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu}$$

$$+\sigma_{\rho\sigma}\left[k^\rho n^\sigma\left(\frac{B_{46}}{(P\cdot n)M}k_\mu k_\nu + \frac{B_{47}M}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{48}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu} + \sigma_{\mu\sigma}\left[n^\sigma\left(\frac{B_{49}M}{P\cdot n}k_\nu + \frac{B_{50}M^3}{(P\cdot n)^2}n_\nu\right) + \left(\frac{B_{51}M}{P\cdot n}P^\sigma + \frac{B_{52}M}{P\cdot n}k^\sigma\right)n_\nu\right]T^{\mu\nu}$$

New terms in our paper

From this correlation function, new tensor-polarized TMDs are defined in twist-3 and 4 in addition to twist-2 ones.

Terms associated with $n = \frac{1}{\sqrt{2}}(1,\ 0,\ 0,\ -1)$

Twist-3 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2}\mathrm{Tr}\left[\Phi^{[\Gamma]}(x, k_T, T)\,\Gamma\right] = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \Phi(k, P, T\,|\,n)\,\Gamma\right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2}F^{\perp}(x, k_T^2)$$

$$\Phi^{[\gamma^i]}(x, k_T, T) = \frac{M}{P^+}\left[f_{LL}^{\perp}(x, k_T^2)\frac{S_{LL}k_T^i}{M} + f'_{LT}(x, k_T^2)S_{LT}^i - f_{LT}^{\perp}(x, k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - f'_{TT}(x, k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + f_{TT}^{\perp}(x, k_T^2)\frac{k_T^i k_T\cdot S_{TT}\cdot k_T}{M^3}\right]$$

$$\Phi^{[1]}(x, k_T, T) = \frac{M}{P^+}\left[e_{LL}(x, k_T^2)S_{LL} - e_{LT}^{\perp}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + e_{TT}^{\perp}(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right]$$

$$\Phi^{[i\gamma_5]}(x, k_T, T) = \frac{M}{P^+}\left[e_{LT}(x, k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} - e_{TT}(x, k_T^2)\frac{S_{TT\mu\rho}k_T^{\rho}\varepsilon_T^{\mu\nu}k_{T\nu}}{M^2}\right]$$

$$\Phi^{[\gamma^i\gamma_5]}(x, k_T, T) = \frac{M}{P^+}\left[-g_{LL}^{\perp}(x, k_T^2)\frac{S_{LL}\varepsilon_T^{ij}k_{Tj}}{M} - g'_{LT}(x, k_T^2)\varepsilon_T^{ij}S_{LTj} + g_{LT}^{\perp}(x, k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}S_{LT}\cdot k_T}{M^2} + g'_{TT}(x, k_T^2)\frac{\varepsilon_T^{ij}S_{TTjl}k_T^l}{M} - g_{TT}^{\perp}(x, k_T^2)\frac{\varepsilon_T^{ij}k_{Tj}k_T\cdot S_{TT}\cdot k_T}{M^3}\right]$$

$$\Phi^{[\sigma^{-+}]}(x, k_T, T) = \frac{M}{P^+}\left[h_{LL}(x, k_T^2)S_{LL} - h_{LT}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + h_{TT}(x, k_T^2)\frac{k_T\cdot S_{TT}\cdot k_T}{M^2}\right]$$

$$\Phi^{[\sigma^{ij}]}(x, k_T, T) = \frac{M}{P^+}\left[h_{LT}^{\perp}(x, k_T^2)\frac{S_{LT}^i k_T^j - S_{LT}^j k_T^i}{M} - h_{TT}^{\perp}(x, k_T^2)\frac{S_{TT}^{il}k_{Tl}k_T^j - S_{TT}^{jl}k_{Tl}k_T^i}{M^2}\right]$$

*2, *3 Because of the time-reversal invariance, the collinear PDFs $g_{LT}(x)$ and $h_{LL}(x)$ do not exist. However, the corresponding new collinear fragmentation functions $G_{LT}(z)$ and $H_{LL}(z)$ should exist. (see our PRD paper for the details)

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^{\perp}$ $[e]$			$g^{\perp}$		$[h]$
L		$f_L^{\perp}$ $[e_L]$	$g_L^{\perp}$		$[h_L]$	
T		$f_T, f_T^{\perp}$ $[e_T, e_T^{\perp}]$	$g_T, g_T^{\perp}$		$[h_T], [h_T^{\perp}]$	
LL	$f_{LL}^{\perp}$ $[e_{LL}]$			$g_{LL}^{\perp}$		$[h_{LL}]$
LT	$f_{LT}, f_{LT}^{\perp}$ $[e_{LT}, e_{LT}^{\perp}]$			$g_{LT}, g_{LT}^{\perp}$		$[h_{LT}], [h_{LT}^{\perp}]$
TT	$f_{TT}, f_{TT}^{\perp}$ $[e_{TT}, e_{TT}^{\perp}]$			$g_{TT}, g_{TT}^{\perp}$		$[h_{TT}], [h_{TT}^{\perp}]$

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					*3
LT	f_{LT}			*2		
TT						

New TMDs $[\cdots]$= chiral odd **New collinear PDFs**

Twist-4 TMDs for spin-1 hadrons

$$\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2}\mathrm{Tr}\left[\Phi^{[\Gamma]}(x, k_T, T)\,\Gamma\right] = \frac{1}{2}\mathrm{Tr}\left[\int dk^- \Phi(k, P, T \mid n)\,\Gamma\right], \quad F(x, k_T^2) \equiv F'(x, k_T^2) - \frac{k_T^2}{2M^2} F^{\perp}(x, k_T^2)$$

$$\Phi^{[\gamma^-]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[f_{3LL}(x, k_T^2) S_{LL} - f_{3LT}(x, k_T^2)\frac{S_{LT}\cdot k_T}{M} + f_{3TT}(x, k_T^2)\frac{k_T \cdot S_{TT}\cdot k_T}{M^2}\right]$$

$$\Phi^{[\gamma^-\gamma_5]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[g_{3LT}(x, k_T^2)\frac{S_{LT\mu}\varepsilon_T^{\mu\nu}k_{T\nu}}{M} + g_{3TT}(x, k_T^2)\frac{S_{TT\mu\rho}k_T^{\rho}\varepsilon_T^{\mu\nu}k_{T\nu}}{M^2}\right]$$

$$\Phi^{[\sigma^{i-}]}(x, k_T, T) = \frac{M^2}{P^{+2}}\left[h_{3LL}^{\perp}(x, k_T^2)\frac{S_{LL}k_T^i}{M} + h'_{3LT}(x, k_T^2) S_{LT}^i - h_{3LT}^{\perp}(x, k_T^2)\frac{k_T^i S_{LT}\cdot k_T}{M^2} - h'_{3TT}(x, k_T^2)\frac{S_{TT}^{ij}k_{Tj}}{M} + h_{3TT}^{\perp}(x, k_T^2)\frac{k_T^i k_T \cdot S_{TT}\cdot k_T}{M^3}\right]$$

*4 Because of the time-reversal invariance, $h_{3LT}(x)$ does not exist; however, the corresponding new collinear fragmentation function $H_{3LT}(z)$ should exist because the time-reversal invariance does not have to be imposed.

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^{\perp}]$
L			g_{3L}		$[h_{3L}^{\perp}]$	
T		$f_{3T}^{\perp}$	g_{3T}		$[h_{3T}], [h_{3T}^{\perp}]$	
LL	f_{3LL}					$[h_{3LL}^{\perp}]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}], [h_{3LT}^{\perp}]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}], [h_{3TT}^{\perp}]$

New TMDs

$[\cdots]$= chiral odd

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						*4
TT						

New collinear PDFs

Sum rules for TMDs of spin-1 hadrons

Twist-2 TMDs

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Time-reversal invariance in colliear corrlation functions (PDFs)

$$\int d^2k_T \Phi_{\text{T-odd}}(x, k_T^2) = 0$$

Sum rules for the TMDs of spin-1 hadrons

$$\int d^2k_T\, h_{1LT}(x,k_T^2) = 0, \qquad \int d^2k_T\, g_{LT}(x,k_T^2) = 0,$$

$$\int d^2k_T\, h_{LL}(x,k_T^2) = 0, \qquad \int d^2k_T\, h_{3LT}(x,k_T^2) = 0$$

For example, in the twist-4

$$\int d^2k_T\, h_{3LT}(x,k_T^2) \equiv \int d^2k_T \left[h'_{3LT}(x,k_T^2) - \frac{k_T^2}{2M^2} h_{3LT}(x,k_T^2) \right] = 0$$

$$\Phi^{[\sigma^{i-}]} = \frac{M^2}{P^{+2}} \Big[h_{3LL}^\perp(x,k_T^2) S_{LL} \frac{k_T^i}{M} + h'_{3LT}(x,k_T^2) S_{LT}^i - h_{3LT}^\perp(x,k_T^2) \frac{k_T^i S_{LT}\cdot k_T}{M^2} - h'_{3TT}(x,k_T^2) \frac{S_{TT}^{ij} k_{T\,j}}{M} + h_{3TT}^\perp(x,k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{k_T^i}{M} \Big]$$

Twist-3 TMDs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		$f_T, f_T^\perp$ $[e_T, e_T^\perp]$	$g_T, g_T^\perp$		$[h_T], [h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	$f_{LT}, f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			$g_{LT}, g_{LT}^\perp$		$[h_{LT}], [h_{LT}^\perp]$
TT	$f_{TT}, f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			$g_{TT}, g_{TT}^\perp$		$[h_{TT}], [h_{TT}^\perp]$

Twist-4 TMDs

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}], [h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}], [h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}], [h_{3TT}^\perp]$

New fragmentations for spin-1 hadrons

Corresponding fragmentation functions exist for the spin-1 haddrons

simply by changing function names and kinematical variables.

TMD distribution functions: $f,\ g,\ h,\ e\ ;\quad x,\ k_T,\ S,\ \ T,\ \ M,\ \ n,\ \gamma^+,\ \sigma^{i+}$

$$\Downarrow$$

TMD fragmentation functions: $D,\ G,\ H,\ E\ ;\ \ z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

Fragmentation functions:
X. Ji, Phys. Rev. D 49, 114 (1994).

Summary on our recent spin-1 TMD studies

TMDs of spin-1 hadrons

- **TMDs: interdisciplinary field of physics**
  (Color Aharonov-Bohm effect, Color entanglement)
- **We proposed new 30 TMDs and 3 PDFs in twist 3 and 4.**
- **New sum rules for TMDs.**
- **New TMD fragmentation functions.**

Twist-3 TMD: $f_{LL}^{\perp}, e_{LL}, f_{LT}, f_{LT}^{\perp}, e_{1T}, e_{1T}^{\perp}, f_{TT}, f_{TT}^{\perp}, e_{TT}, e_{TT}^{\perp},$
$g_{LL}^{\perp}, g_{LT}, g_{LT}^{\perp}, g_{TT}, g_{TT}^{\perp}, h_{1L}, h_{LT}, h_{LT}^{\perp}, h_{TT}, h_{TT}^{\perp}$

Twist-4 TMD: $f_{3LL}, f_{3LT}, f_{3TT}, g_{3LT}, f_{3TT}, h_{3LL}^{\perp}, h_{3LT}, h_{3LT}^{\perp}, h_{3TT}, h_{3TT}^{\perp}$

Twist-3 PDF: e_{LL}, f_{LT}

Twist-4 PDF: f_{3LL}

Sum rules: $\int d^2k_T\, g_{LT}(x, k_T^2) = \int d^2k_T\, h_{LL}(x, k_T^2) = \int d^2k_T\, h_{3LL}(x, k_T^2) = 0$

TMD distribution functions: $f,\ g,\ h,\ e;\quad x,\ k_T,\ S,\ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

$$\Downarrow$$

TMD fragmentation functions: $D,\ G,\ H,\ E;\quad z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

Twist-2 relation and sum rule for PDFs of spin-1 hadrons

(analogous to Wandzura-Wilczek relation and the Burkhardt-Cottingham sum rule)

S. Kumano and Qin-Tao Song,
arXiv:2106.15849, submitted for publication.

Wandzura-Wilczek and Burkhardt-Cottingham relations for g_1 and g_2

Structure functions: $\int \frac{d(P^+\xi^-)}{2\pi} e^{ixP^+\xi^-} \left\langle P,S \right| \bar{\psi}(0)\gamma^\mu \gamma_5 \psi(\xi) \left| P,S \right\rangle_{\xi^+=\vec{\xi}_T=0} = 2M_N \left[g_{1L}(x)\bar{n}^\mu S\cdot n + g_T(x) S_T^\mu + g_{3L}(x)\frac{M_N^2}{(P^+)^2} n^\mu S\cdot n \right]$

$$S^\mu = S_L \frac{P^+}{M}\bar{n}^\mu - S_L \frac{M_N}{2P^+} n^\mu + S_T^\mu, \quad P^\mu = P^+ \bar{n}^\mu + \frac{M_N^2}{2P^+} n^\mu, \quad S\cdot n = S_L \frac{P^+}{M_N}$$

$$g_1(x) = \frac{1}{2}\left[g_{1L}(x) + g_{1L}(-x) \right], \quad g_1(x) + g_2(x) = \frac{1}{2}\left[g_T(x) + g_T(-x) \right]$$

Operators: $R^{\sigma\{\mu_1\cdots\mu_{n-1}\}} = i^{n-1}\bar{\psi}\gamma^\sigma \gamma_5 D^{\{\mu_1}\cdots D^{\mu_{n-1}\}}\psi = R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} + R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}} = \text{twist 2} + \text{twist 3}$

$$R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} = \frac{1}{n}\left[S^\sigma P^{\{\mu_1} P^{\mu_2}\cdots P^{\mu_{n-1}\}} + S^{\mu_1} P^{\{\sigma} P^{\mu_2}\cdots P^{\mu_{n-1}\}} + S^{\mu_2} P^{\{\mu_1} P^{\sigma}\cdots P^{\mu_{n-1}\}} + \cdots \right]$$

$$R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}} = \frac{1}{n}\left[(n-1) S^\sigma P^{\{\mu_1} P^{\mu_2}\cdots P^{\mu_{n-1}\}} - S^{\mu_1} P^{\{\sigma} P^{\mu_2}\cdots P^{\mu_{n-1}\}} - S^{\mu_2} P^{\{\mu_1} P^{\sigma}\cdots P^{\mu_{n-1}\}} - \cdots \right]$$

$$\left\langle P,S \right| R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} \left| P,S \right\rangle = \frac{2}{n} a_n M_N \left[S^\sigma P^{\mu_1}\cdots P^{\mu_{n-1}} + P^{\mu_1} S^\sigma \cdots P^{\mu_{n-1}} + \cdots \right]$$

$$\left\langle P,S \right| R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}} \left| P,S \right\rangle = \frac{2}{n} d_n M_N \left[(S^\sigma P^{\mu_1} - P^\sigma S^{\mu_1}) P^{\mu_2}\cdots P^{\mu_{n-1}} + (S^\sigma P^{\mu_2} - P^\sigma S^{\mu_2}) P^{\mu_1}\cdots P^{\mu_{n-1}} + \cdots \right]$$

$$\frac{1}{2M_N (P^+)^{n-1}} n_{\mu_1}\cdots n_{\mu_{n-1}} \left\langle P,S \right| R^{\sigma\{\mu_1\cdots\mu_{n-1}\}} \left| P,S \right\rangle = \bar{n}^\sigma (S\cdot n)\int_{-1}^1 dx x^{n-1} g_{1L}(x) + S_T^\sigma \int_{-1}^1 dx x^{n-1} g_T(x)$$

$$= \frac{1}{2M_N (P^+)^{n-1}} n_{\mu_1}\cdots n_{\mu_{n-1}} \left\langle P,S \right| R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} \left| P,S \right\rangle + \frac{1}{2M_N (P^+)^{n-1}} n_{\mu_1}\cdots n_{\mu_{n-1}} \left\langle P,S \right| R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}} \left| P,S \right\rangle$$

$$\to \int_{-1}^1 dx x^{n-1} g_{1L}(x) = a_n, \quad \int_{-1}^1 dx x^{n-1} g_T(x) = \frac{1}{n} a_n + \frac{n-1}{n} d_n$$

$$\to \int_0^1 dx x^{n-1} g_1(x) = \int_{-1}^1 dx x^{n-1} \frac{1}{2} g_{1L}(x) = \frac{1}{2} a_n, \quad \int_0^1 dx x^{n-1}[g_1(x) + g_2(x)] = \int_{-1}^1 dx x^{n-1}\frac{1}{2} g_T(x) = \frac{1}{2n} a_n + \frac{n-1}{2n} d_n$$

$$\to \int_0^1 dx x^{n-1} g_2(x) = \int_0^1 dx x^{n-1}\left[-g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) \right] + \frac{n-1}{2n} d_n$$

If we write $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) + \bar{g}_2(x)$

$\to$ $g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$ (Wandzura-Wilczek relation), $\quad \int_0^1 dx x^{n-1}\bar{g}_2(x) = \frac{n-1}{2n} d_n$

$\to$ $\int_0^1 dx\, g_2(x) = 0$ (Burkhardt-Cottingham sum rule)

Note: Twist-3 operators $R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$ are obtained by the Tayler expansion of $\xi_\mu \bar{\psi}(0)(\partial^\mu \gamma^\sigma - \partial^\sigma \gamma^\mu)\psi(\xi)$, which needs to be investigated in details for finding the details of twist-3 terms.

Collinear PDFs for spin-1 hadrons

may skip

Tensor polarization: $T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar{n}^\mu\bar{n}^\nu - \frac{2}{3}S_{LL}\left(\bar{n}^\mu n^\nu + \bar{n}^\nu n^\mu - g_T^{\mu\nu}\right) + \frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu + \frac{P^+}{M}(\bar{n}^\mu S_{LT}^\nu + \bar{n}^\nu S_{LT}^\mu) - \frac{M}{2P^+}(n^\mu S_{LT}^\nu + n^\nu S_{LT}^\mu) + S_{TT}^{\mu\nu}\right]$

Collinear correlation function: $\Phi(x,P,T) = \frac{1}{2}\left[f_{1LL}(x)S_{LL}\bar{\not n} + \frac{M}{P^+}e_{LL}(x)S_{LL} + \frac{M}{P^+}f_{LT}(x)S_{LT}\right]$, **up to twist-3**

Matrix element of vector operator: $\left\langle P,T\left|\,\bar\psi(0)\gamma^\mu\psi(\xi^-)\right|P,T\right\rangle = \int_{-1}^{1}dx\,e^{-ixP^+\xi^-}P^+\mathrm{Tr}\left[\Phi_{ij}(x,P,T)(\gamma^\mu)_{ji}\right] = \int_{-}^{1}dx\,e^{-ixP^+\xi^-}\,2P^+\left[f_{1LL}(x)S_{LL}\bar{n}^\mu + \frac{M}{P^+}S_{LT}^\mu f_{LT}(x)\right]$

$\alpha = 1,2$ = transverse: $\left\langle P,T\left|\,\bar\psi(0)(\partial^\mu\gamma^\alpha - \partial^\alpha\gamma^\mu)\psi(\xi)\right|P,T\right\rangle = 2MS_{LT}^\alpha\int_{-}^{1}dx\,e^{-ixP^+\xi^-}\left[-\frac{3}{2}f_{1LL}(x) + f_{LT}(x) - \frac{d}{dx}\{xf_{LT}(x)\}\right]$

$\bar\psi(0)(\partial^\mu\gamma^\alpha - \partial^\alpha\gamma^\mu)\psi(\xi) = \bar\psi(0)(D^\mu\gamma^\alpha - D^\alpha\gamma^\mu)\psi(\xi) - \bar\psi(0)\gamma^\mu\psi(\xi)ig\int_0^1 dt\,t\,\xi_\rho G^{\rho\alpha}(t\xi)$

In the Fock-Schwinger gauge: $\xi_\mu A^\mu(\xi) = 0$, we have $A^\nu(\xi) = \int_0^1 dt\,t\xi_\mu G^{\mu\nu}(t\xi)$, $G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$

$\bar\psi(0)(D^\mu\gamma^\alpha - D^\alpha\gamma^\mu)\psi(\xi) = -\frac{i}{2}\bar\psi(0)\sigma^{\alpha\mu}\vec{\not D}\psi(\xi) - \frac{i}{2}\bar\psi(0)\overleftarrow{\not D}\sigma^{\alpha\mu}\psi(\xi) - \frac{1}{2}g\int_0^1 dt\,\xi_\nu G^{\rho\nu}(t\xi)\bar\psi(0)\gamma_\rho\sigma^{\alpha\mu}\psi(\xi) + \frac{i}{2}\bar\partial_\rho\{\bar q(0)W(0,\,\xi)\gamma^\rho\sigma^{\alpha\mu}q(\xi)\}$

$-\frac{1}{2}\xi_\mu g\int_0^1 dt\,\xi_\nu G^{\rho\nu}(t\xi)\bar\psi(0)\gamma_\rho\sigma^{\alpha\mu}\psi(\xi) = \frac{1}{2}g\int_0^1 dt\,\bar\psi(0)\left\{-i\xi_\mu G^{\alpha\mu}(t\xi)\not\xi + \xi_\mu\widetilde G^{\alpha\mu}(tx)\not\xi\gamma_5 - \left[\xi^2\widetilde G^{\alpha\sigma}(t\xi) - \xi_\mu x^\alpha\widetilde G^{\mu\sigma}(tx)\right]\gamma_\sigma\gamma_5\right\}\psi(x)$

$\bar\partial_\rho\{\bar q(0)W(0,\,\xi)\gamma^\rho\sigma^{\alpha\mu}q(\xi)\} = \bar\psi(0)\overleftarrow{\not D}\sigma^{\alpha\mu}\psi(\xi) + \bar\psi(0)\vec{\not D}\sigma^{\alpha\mu}\psi(\xi) - ig\int_0^1 dt\,\xi^\nu G_{\rho\nu}(t\xi)\bar\psi(0)\gamma^\rho\sigma^{\alpha\mu}\psi(\xi)$

$\xi_\mu\left[\bar\psi(0)(\partial^\mu\gamma^\alpha - \partial^\alpha\gamma^\mu)\psi(\xi)\right] = g\int_0^1 dt\,\bar\psi(0)\left\{i\left(t - \frac{1}{2}\right)G^{\alpha\mu}(t\xi) - \frac{1}{2}\widetilde G^{\alpha\mu}(t\xi)\gamma_5\right\}\xi_\mu\not\xi\psi(x) + \frac{1}{2}g\int_0^1 dt\,\bar\psi(0)\left[\xi_\mu\xi^\alpha\widetilde G^{\mu\sigma}(t\xi) - \xi^2\widetilde G^{\alpha\sigma}(t\xi)\right]\gamma_\sigma\gamma_5\psi(\xi)$

$-\frac{i}{2}\xi_\mu\bar\psi(0)\sigma^{\alpha\mu}(\vec{\not D} - m)\psi(\xi) - \frac{i}{2}\xi_\mu\bar\psi(0)(\overleftarrow{\not D} + m)\sigma^{\alpha\mu}\psi(\xi) + \frac{i}{2}\xi_\mu\bar\partial_\rho\{\bar\psi(0)W(0,\,\xi)\gamma^\rho\sigma^{\alpha\mu}\psi(\xi)\}$

$\xi_\mu\left[\bar\psi(0)(\partial^\mu\gamma^\alpha - \partial^\alpha\gamma^\mu)\psi(\xi)\right] \simeq g\int_0^1 dt\,\bar\psi(0)\left\{i\left(t - \frac{1}{2}\right)G^{\alpha\mu}(t\xi) - \frac{1}{2}\widetilde G^{\alpha\mu}(t\xi)\gamma_5\right\}\xi_\mu\not\xi\psi(x)$

Note: Twist-3 operators $R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$ are obtained by the Tayler expansion of $\xi_\mu\bar\psi(0)(\partial^\mu\gamma^\sigma - \partial^\sigma\gamma^\mu)\psi(\xi)$, which needs to be investigated in details for finding the details of twist-3 terms.

Multiparton distribution functions: $(\Phi_G^\nu)_{ij}(x_1,x_2) = \int\frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}e^{ix_1P^+\xi_1^-}e^{i(x_2-x_1)P^+\xi_2^-}\left\langle P,T\left|\,\bar\psi_j(0)\,gG^{+\nu}(\xi_2^-)\psi_i(\xi_1^-)\right|P,T\right\rangle$

Express Φ_G^ν in terms of possible Lorentz vectors and multiparton distribution functions with the conditions Hermiticity, parity invariance, and time-reversal invariance

$\Phi_G^\nu(x_1,x_2) = \frac{M}{2}\left[iS_{LT}^\nu F_{G,LT}(x_1,x_2) - \varepsilon_\perp^{\alpha\mu}S_{LT\mu}\gamma_5 G_{G,LT}(x_1,x_2) + iS_{LL}\gamma^\alpha H_{G,LL}^\perp(x_1,x_2) + iS_{TT}^{\alpha\mu}\gamma_\mu H_{G,TT}(x_1,x_2)\right]\bar{\not n}$

$(\Phi_G^\nu)_{ij}(\not n)_{ji}$: $\quad S_{LT}^\nu F_{G,LT}(x_1,x_2) = -\frac{i}{2M}g\int\frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}e^{ix_1P^+\xi_1^-}e^{i(x_2-x_1)P^+\xi_2^-}\left\langle P,T\left|\,\bar\psi(0)\,\not n n_\mu G^{\mu\nu}(\xi_2^-)\psi(\xi_1^-)\right|P,T\right\rangle$

$(\Phi_G^\nu)_{ij}(i\gamma_5\not n)_{ji}$: $\quad S_{LT}^\nu G_{G,LT}(x_1,x_2) = \frac{i}{2M}g\int\frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi}e^{ix_1P^+\xi_1^-}e^{i(x_2-x_1)P^+\xi_2^-}\left\langle P,T\left|\,\bar\psi(0)\,i\gamma_5\not n n_\mu\tilde G^{\mu\nu}(\xi_2^-)\psi(\xi_1^-)\right|P,T\right\rangle$

$\int\frac{d(P\cdot\xi)}{2\pi}e^{ix_1P\cdot\xi}\langle P,T|\,g\int_0^1 dt\,\bar\psi(0)\left\{i\left(t - \frac{1}{2}\right)G^{\mu\nu}(t\xi) - \frac{1}{2}\gamma_5\tilde G^{\mu\nu}(t\xi)\right\}\xi_\mu\not\xi\psi(\xi)|P,T\rangle_{\xi^+=\vec\xi_T=0} = -2MS_{LT}^\nu\mathcal{P}\int_0^1 dx_2\frac{1}{x_1 - x_2}\left[\frac{\partial}{\partial x_1}\{F_{G,LT}(x_1,x_2) + G_{G,LT}(x_1,x_2)\} + \frac{\partial}{\partial x_2}\{F_{G,LT}(x_2,x_1) + G_{G,LT}(x_2,x_1)\}\right]$

Twist-2 relation and sum rule

- **Twist-3 matrix element in terms of tensor-polarized PDFs**

$$\left\langle P,T \right| \bar{\psi}(0)(\partial^\mu \gamma^\alpha - \partial^\alpha \gamma^\mu)\psi(\xi) \left| P,T \right\rangle = 2MS_{LT}^{\alpha} \int_{-}^{1} dx\, e^{-ixP^+\xi^-} \left[-\frac{3}{2} f_{1LL}(x) + f_{LT}(x) - \frac{d}{dx}\{x f_{LT}(x)\} \right]$$

- **Twist-3 operator in terms of gluon field tensor**

$$\xi_\mu \left[\bar{q}(0)(\gamma^\alpha \partial^\mu - \gamma^\mu \partial^\alpha) q(\xi) \right] = g \int_0^1 dt\, \bar{q}(0) \left\{ i\left(t - \frac{1}{2}\right) G^{\alpha\mu}(t\xi) - \frac{1}{2}\gamma_5 \tilde{G}^{\alpha\mu}(t\xi) \right\} \xi_\mu \not{\xi}\, q(\xi)$$

- **Matrix element of field tensor in terms of twist-3 multiparton distribution functions**

$$\int \frac{d(P\cdot\xi)}{2\pi} e^{ix_1 P\cdot\xi} \langle P,T | g \int_0^1 dt\, \bar{\psi}(0) \left\{ i\left(t - \frac{1}{2}\right) G^{\mu\nu}(t\xi) - \frac{1}{2}\gamma_5 \tilde{G}^{\mu\nu}(t\xi) \right\} \xi_\mu \not{\xi} \psi(\xi) | P,T \rangle_{\xi^+ = \vec{\xi}_T = 0}$$

$$= -2MS_{LT}^{\nu} \mathcal{P} \int_0^1 dx_2 \frac{1}{x_1 - x_2} \left[\frac{\partial}{\partial x_1} \{ F_{G,LT}(x_1,x_2) + G_{G,LT}(x_1,x_2) \} + \frac{\partial}{\partial x_2} \{ F_{G,LT}(x_2,x_1) + G_{G,LT}(x_2,x_1) \} \right]$$

$$x \frac{df_{LT}(x)}{dx} = -\frac{3}{2} f_{1LL}(x) - f_{LT}^{(HT)}(x),$$ **Higher-twist:** $$f_{LT}^{(HT)}(x) = -\mathcal{P}\int_0^1 dy \frac{1}{x-y} \left[\frac{\partial}{\partial x} \{ F_{G,LT}(x,y) + G_{G,LT}(x,y) \} + \frac{\partial}{\partial y} \{ F_{G,LT}(y,x) + G_{G,LT}(y,x) \} \right]$$

$$\to f_{LT}(x) = \frac{3}{2}\int_x^{\varepsilon(x)} \frac{dy}{y} f_{1LL}(y) + \int_x^{\varepsilon(x)} \frac{dy}{y} f_{LT}^{(HT)}(y), \quad \varepsilon(x) = \frac{i}{\pi} P \int_{-\infty}^{\infty} dy \frac{1}{y} e^{-ixy} = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$$

Define $f^+(x) = f(x) + \bar{f}(x) = f(x) - f(-x), \quad f = f_{1LL},\ f_{LT},\ f_{LT}^{(HT)}, \quad x > 0$

$$\to f_{LT}^+(x) = \frac{3}{2}\int_x^1 \frac{dy}{y} f_{1LL}^+(y) + \int_x^1 \frac{dy}{y} f_{LT}^{(HT)+}(y)$$ $\to$ **Twist-2 relation:** $$f_{LT}^+(x) = \frac{3}{2}\int_x^1 \frac{dy}{y} f_{1LL}^+(y)$$

If we define $f_{2LT}(x) = \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$,

$$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y) + \frac{2}{3}\int_x^1 \frac{dy}{y} f_{LT}^{(HT)+}(y)$$ $\to$ **Twist-2 relation:** $$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y),$$ **Wandzura-Wilczek like**

$\to$ **Sum rule:** $\int_0^1 dx\, f_{2LT}^+(x) = 0,$ **Burkhardt-Cottingham like**

If the parton-model sum rule without the tensor-polarized antiquark distributions $\int_0^1 dx\, f_{1LL}^+(x) = \frac{2}{3}\int_0^1 dx\, b_1^+(x) = 0$ **is valid,** $\to$ **Sum rule:** $\int_0^1 dx\, f_{LT}^+(x) = 0$

Summary on the twist-2 relation and sum rule

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$$ **(Wandzura-Wilczek relation),** $\int_0^1 dx\, g_2(x) = 0$ **(Burkhardt-Cottingham sum rule)**

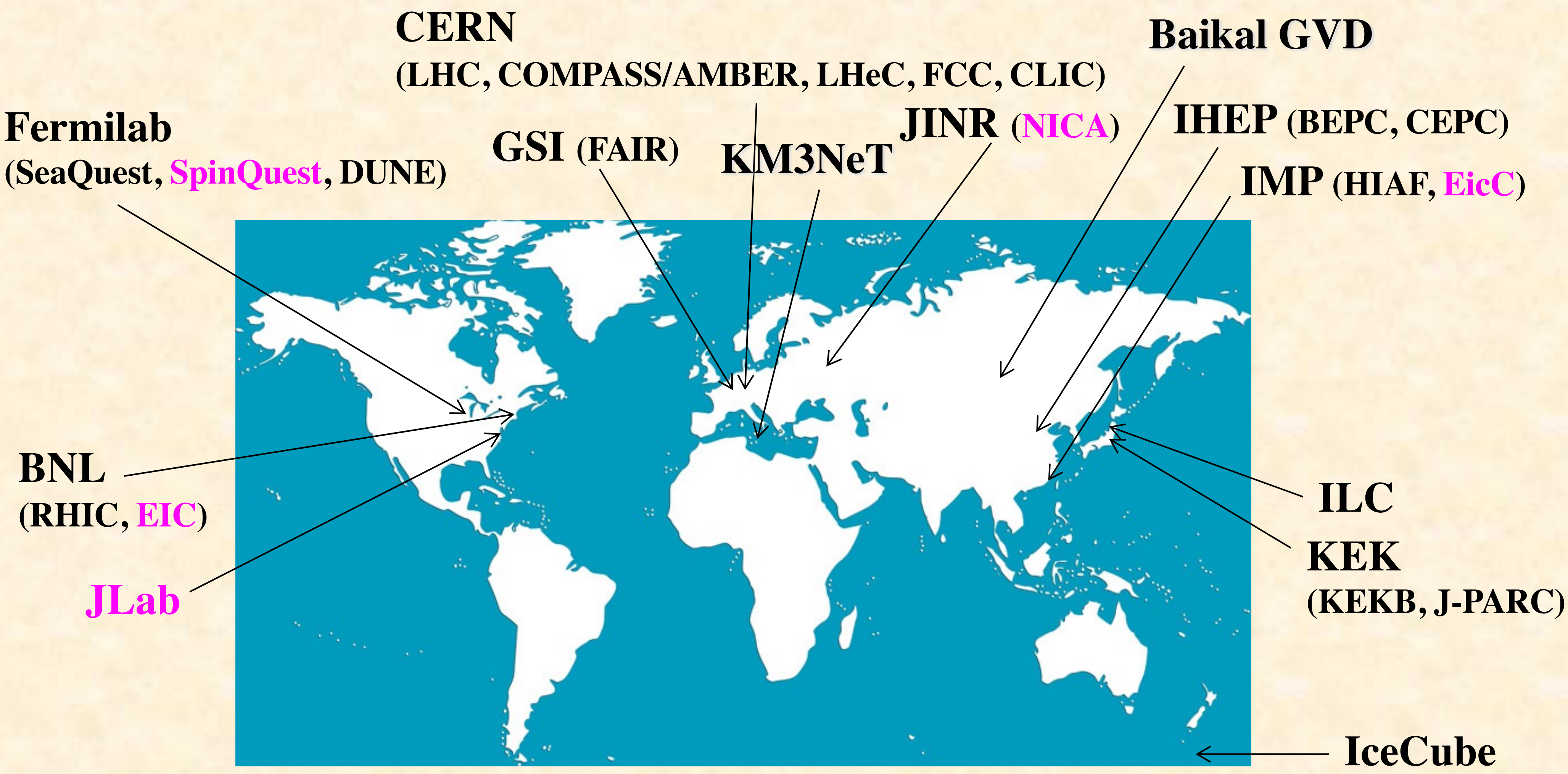
For tensor-polarized spin-1 hadrons, we obtained

$$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y),$$ $\int_0^1 dx\, f_{2LT}^+(x) = 0,$ $f_{2LT}(x) \equiv \frac{2}{3} f_{LT}(x) - f_{1LL}(x)$

$\int_0^1 dx\, f_{LT}^+(x) = 0$ **if** $\int_0^1 dx\, f_{1LL}^+(x) = \frac{2}{3}\int_0^1 dx\, b_1^+(x) = 0$

Existence of multiparton distribution functions: $F_{G,LT}(x_1,x_2),\ G_{G,LT}(x_1,x_2),\ H_{G,LL}^{\perp}(x_1,x_2),\ H_{G,TT}(x_1,x_2)$

High-energy hadron physics experiments



Facilities on spin-1 hadron structure functions including future possibilities.

Summary

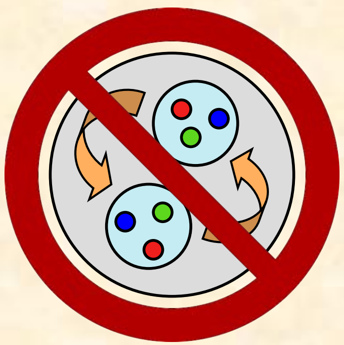
Spin-1 structure functions of the deuteron (new spin structure)

- **tensor structure in quark-gluon degrees of freedom**
- **gluon transversity: We showed that it is possible to investigate the gluon transversity at hadron accelerator facilities, for example, by the Drell-Yan process.**
- **new signature beyond "standard" hadron physics?**
- **experiments: JLab (approved), Fermilab (to be proposed), ... , NICA (in progress), AMBER?, EIC, EicC, ...**
- **TMDs: interdisciplinary field of physics to probe color degrees of freedom:**
  ***e.g.* Color Aharonov-Bohm effect, Color entanglement**

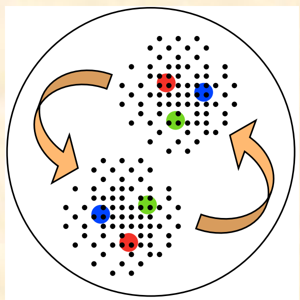
We proposed new TMDs and PDFs in twist 3 and 4 and their sum rules.

We also derive twist-2 relation and sum rule analogous to Wandzura-Wilczek relation and Burkardt-Cottingham sum rule.

We showed the existence of tensor-polarized multiparton distribution functions.



standard model



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The End

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