## The 3-D Structure of the Pion From Pion-induced Drell-Yan Scattering



## Leonard Gamberg

w/ Patrick Barry, Eric Moffat, Alexei Prokudin Wally Melnitchouk, Nobuo Sato, ..... "JAM 3-D"







## The Pion as bound state QCD

- Pion plays a central/"outsized" role in hadron physics
- As a nearly massless  $\bar{q}q$  bound state Goldstone boson
  - ★ Chiral symmetry is explicitly broken by small current quark masses

Píon pole condition from Bethe-Salpeter

$$m_{\pi}^2 = rac{m}{2 \, G_{\pi} \, M \, I(m_{\pi}^2)}$$

moving at close to speed of light

Gell Mann Reiner Oaks Relation

$$f_{\pi}^2 m_{\pi}^2 = \frac{1}{2} \left( m_u + m_d \right) \left\langle \bar{u}u + \bar{d}d \right\rangle$$

 $\star$  Mass without mass" bulk of pion mass due to quantum fluctuations of  $\bar{q}q$  pairs, gluons, & energy associated with quarks





## The Pion recent progress

Also @ high energies pion reveals unique spectrum of momentum distributions,  $f_{i/\pi}(x,\mu)$ • See talk of P.C. Barry Monday: DY & LN process as predicted from Collinear Factorization

$$\frac{\mathrm{d}\sigma^{\mathrm{DY}}}{\mathrm{d}Q^{2}\mathrm{d}y} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{\mathrm{DY}}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2})$$

2) 
$$\frac{\mathrm{d}^3 \sigma^{\text{LN}}}{\mathrm{d} x_B \,\mathrm{d} Q^2 \,\mathrm{d} x_L} = \frac{4\pi\alpha^2}{x_B \,Q^4} \left(1 - y_e + \frac{y_e^2}{2}\right) F_2^{\text{LN}}(x_B \,\mathrm{d} x_B \,\mathrm{d} x_B \,\mathrm{d} x_B)$$

understanding how these contrasting manifestations of the same  $\bar{q}q$  bound state arise dynamically at different energy scales from first principles remains a major challenge in QCD

 $f_{b/B}(x_b, \mu^2)$ 









## The Pion recent progress

Also @ high energies pion reveals unique spectrum of momentum distributions,  $f_{i/\pi}(x,\mu)$ ulletSee talk of P.C. Barry Monday: DY & LN process as predicted from Collinear Factorization

Jefferson Lab Angular Momentum (JAM) Barry, Sato, Melnitchouk, C.R. Ji, PRL 2018



FIG. 2. Pion valence (green), sea quark (blue) and gluon (red, scaled by 1/10) PDFs versus  $x_{\pi}$  at  $Q^2 = 10 \text{ GeV}^2$ , for the full DY + LN (dark bands) and DY only (light bands) fits. The bands represent  $1\sigma$  uncertainties, as defined in the standard Monte Carlo determination of the uncertainties [42] from the experimental errors. The model dependence of the fit is represented by the outer yellow bands.









Barry et al *Phys.Rev.D* 103 (2021), also included transverse momentum dependent Drell-Yan data in a global QCD analysis for the first time. These data were able to be described well by our O( $\alpha_s$ ) framework, and new pion PDFs were extracted. The inclusion of the pT-dependent data helped in reduce the uncertainties of the gluon distribution at large x





Jefferson Lab Angular Momentum (JAM) Cao Barry, Sato, Melnitchouk, arXiv:2103.02159

1) 
$$\frac{d\sigma^{DY}}{dQ^{2}dy} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$\frac{d\sigma^{DY}}{dQ^{2}dy} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$\frac{\pi}{q} + \frac{\pi}{q} +$$



## Extended fit to include "large" p<sub>T</sub> Drell Yan data

### **Collinear Factorization CSS NPB 1985**

$$(x_a, x_b, y, p_{
m T}, Q^2, \mu^2) \, f_{a/A}(x_a, \mu^2) \, f_{b/B}(x_b, \mu^2)$$

 $p_T \lesssim Q$ 



Jefferson Lab Angular Momentum (JAM) Cao Barry, Sato, Melnitchouk, arXiv:2103.02159

New Monte Carlo QCD analysis of pion parton distribution functions, including, for the first time, large transverse momentum dependent pion-nucleus Drell-Yan cross sections together with  $P_T$ -integrated Drell-Yan & leading neutron electroproduction data from HERA.



#### JAM20 Pion PDFs

3) 
$$\frac{d\sigma^{\mathrm{DY}}}{dQ^2dydp_T} = \sum_{a,b} \int \mathrm{d}x_a \,\mathrm{d}x_b \,H_{a,b}^{\mathrm{FO}}(x_a, x_b, y, p_{\mathrm{T}}, Q_{\mathrm{T}})$$

## Extended fit to include "large" p<sub>T</sub> Drell Yan data



Pion valence quark, sea quark, and gluon (scaled by a factor of 1/10) PDFs extracted from the JAM Monte Carlo analysis of DY (pT-integrated and pT-dependent) and leading neutron data at a scale of  $\mu^2 = 10 \text{ GeV}^2$ 





2)









#### Jefferson Lab Angular Momentum (JAM) Cao Barry, Sato, Melnitchouk, Phys. Rev. D 103 (2021)

New Monte Carlo QCD analysis of pion parton distribution functions, including, for the first time, large transverse momentum dependent pion-nucleus Drell-Yan cross sections



3) 
$$\frac{d\sigma^{\mathrm{DY}}}{dQ^2 dy dp_T} = \sum_{a,b} \int \mathrm{d}x_a \,\mathrm{d}x_b \,H_{a,b}^{\mathrm{FO}}(x_a, x_b, y, p_{\mathrm{T}}, Q_{\mathrm{T}})$$

Extended fit to include "large" p<sub>T</sub> Drell Yan data

To describe the transverse momentum "region"  $p_T \sim k_T \ll Q$  is the regime of TMDs of the pion That require fitting "region"  $p_T \sim k_T \ll Q$  differential pion-induced Drell-Yan cross section



Some earlier work on pion-hadron DY

- F. Ceccopieri, A. Courtoy, S. Noguera, S. Scopetta, EJP 2018 CSS Evolution w/ NJL
- Vladimirov JHEP 2019 Pion-induced DY processes within TMD factorization

# "More granular" $p_T \sim k_T \ll Q$ access to the Pion TMDs

#### **TMD** Factorization

$$\tilde{d}^2 \boldsymbol{b}_{\mathrm{T}} \over (2\pi)^2 \ e^{i \boldsymbol{p}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{W}(x_F, b_T, Q)$$

$$f(x_F, b_T, Q) = \sum_j H^{\mathrm{DY}}_{jar{\jmath}}(Q, \mu, a_s(\mu)) ilde{f}_{j/A}(x_A, b_{\mathrm{T}}; \zeta_A, \mu) ilde{f}_{ar{\jmath}/B}(x_B, b_{\mathrm{T}}; \zeta_B, \mu)$$

TMD Factorization

- **Collins Soper Sterman NPB 1985**
- **◆** Ji Ma Yuan PRD PLB ...2004, 2005
- +Aybat Rogers PRD 2011
- **Collins 2011 Cambridge Press**
- *Echevarria*, *Idilbi*, *Scimemi JHEP 2012*
- **+**... many more refs



## The Pion TMDs & Factorization

### **TMD Factorization**

 Mulders Tangerman NPB 1995

Boer Mulders PRD 1997

## Leading Power TMDPDFs



$$f(x, k_T, s_T) = \frac{1}{2} \left[ f_1^{\pi}(x, k_T^2) + \frac{s_T^i \epsilon^{ij} k_T^j}{m_{\pi}} h_1^{\pi \perp}(x, k_T^2) \right]$$

Factorization carried out Fourier  $b_T$  space FT TMDs  $\tilde{f}(x, b_T; \zeta, \mu)$ Real QCD need QFT definitions of TMDs LC & UV divergences TMD Evolution depends on rapidity  $\zeta$  and RGE scales  $\mu$ 

$$\frac{\mathrm{d}\sigma^W}{\mathrm{d}Q^2\,\mathrm{d}x_F\,\mathrm{d}p_{\mathrm{T}}^2} = \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} \ e^{i\boldsymbol{p}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}\tilde{W}(x_F,b_T,Q)$$

$$ilde{W}(x_F, b_T, Q) = \sum_j H^{\mathrm{DY}}_{j\bar{j}}(Q, \mu)$$







TMD Factorization

- Collins Soper Sterman NPB 1985
- ✦ Ji Ma Yuan PRD PLB ...2004, 2005
- +Aybat Rogers PRD 2011
- **Collins 2011 Cambridge Press**
- Echevarria, Idilbi, Scimemi JHEP 2012,

 $(a, a_s(\mu)) \tilde{f}_{j/A}(x_A, b_{\mathrm{T}}; \zeta_A, \mu) \tilde{f}_{\bar{\jmath}/B}(x_B, b_{\mathrm{T}}; \zeta_B, \mu)$ 

,



$$\tilde{f}_{j/H}^{\text{sub}}(x, \boldsymbol{b}; \boldsymbol{\mu}, \zeta_{\text{PDF}}) = \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \underbrace{\tilde{f}_{j/H}^{\text{unsub}}(x, \boldsymbol{b}; \boldsymbol{\mu}, \boldsymbol{n}_B(\boldsymbol{y}_B))}_{\hat{S}(\boldsymbol{b}; y_A, y_B)\tilde{S}(\boldsymbol{b}; y_s, y_B)} \sqrt{\frac{\tilde{S}(\boldsymbol{b}; y_A, y_B)\tilde{S}(\boldsymbol{b}; y_s, y_B)}{\tilde{S}(\boldsymbol{b}; y_s, y_B)}}$$

$$\hat{f}_{j/H}^{\text{unsub}}(x, \boldsymbol{b}_T; \boldsymbol{\mu}, y_P - y_B) = \int \frac{d\boldsymbol{b}^-}{2\pi} e^{-i\boldsymbol{x}P^+\boldsymbol{b}^-} \langle P|\bar{\psi}(\boldsymbol{y}_B)|^2 \frac{d\boldsymbol{b}^-}{2\pi} e^{-i\boldsymbol{x}P^+} \frac{d\boldsymbol{b}^-}{2\pi} e^{-i\boldsymbol{x}$$

- in solution to the \*Collins-Soper **\***RG equations



## **Renormalization and Evolution-** $\{\zeta, \mu\}$ TMD



Collins Soper Eq.

 $\frac{\partial \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$ 



 $rac{d ilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_k(lpha_s(\mu)$ 

\* RGE for TMD

 $d \ln ilde{f}_{j/H}(x,b_T;\mu$  $\tilde{\Phi}^{[\gamma^+]}(x,\vec{b}_T;Q^2,\mu_Q) = \tilde{f}_1(x,b_T;Q^2,\mu_Q) - iM\epsilon^{ij}$ 

Solve simultaneously and get evolved rend

$$\tilde{K}(b_T,\mu) \equiv \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{S(b_T, y_n, -\infty)}{S(b_T, y_n, -\infty)}$$

$$O(1)$$

$$\frac{(\mu,\zeta)}{\tilde{u}_{J}} = -\gamma_{F}(\alpha_{s}(\mu),\zeta/\mu) - \frac{(\mu,\zeta)}{M^{2}} - \frac{(\mu,\zeta)}{M^{2}} \frac{\tilde{f}_{L}}{\tilde{b}_{T}} \frac{\tilde{f}_{L}}{\tilde{b}_{T}}(x,b_{T};Q^{2},\mu_{Q}) \text{ormalized TMD}_{TMD} \rightarrow \zeta = Q^{2}, \quad \mu = \mu_{Q} \sim Q$$



$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/2}}$$



 $\mu_{b_*} = C_1/b_*(b_T)$ 



### • In small- $p_{\rm T}$ region, Use the CSS formalism for TMD evolution

$$\frac{d\sigma}{dQ^{2} dy dq_{T}^{2}} = \frac{4\pi^{2}\alpha^{2}}{9Q^{2}s} \sum_{j,j_{A},j_{B}} H_{j\bar{j}}^{DY}(Q,\mu_{Q},a_{s}(\mu_{Q})) \int \frac{d^{2}b_{T}}{(2\pi)^{2}} e^{i\boldsymbol{q}_{T}\cdot\boldsymbol{b}_{T}}$$
Aybat Rogers 201 & Collins 201  
$$\times e^{-\frac{g_{j/A}(x_{A},b_{T};b_{max})}{\int_{x_{A}}^{1} \frac{d\xi_{A}}{\xi_{A}}} f_{j_{A}/A}(\xi_{A};\mu_{b_{*}}) \tilde{V}_{j/j_{A}}^{PDF}\left(\frac{x_{A}}{\xi_{A}},b_{*};\mu_{b_{*}}^{2},\mu_{b_{*}},a_{s}(\mu_{b_{*}})\right)$$
$$\times e^{-\frac{g_{j/B}(x_{B},b_{T};b_{max})}{\int_{x_{B}}^{1} \frac{d\xi_{B}}{\xi_{B}}} f_{j_{B}/B}(\xi_{B};\mu_{b_{*}}) \tilde{C}_{\bar{j}/j_{B}}^{PDF}\left(\frac{x_{B}}{\xi_{B}},b_{*};\mu_{b_{*}}^{2},\mu_{b_{*}},a_{s}(\mu_{b_{*}})\right)$$
$$\times \exp\left\{-\frac{g_{K}(b_{T};b_{max})}{Q_{0}^{2}} + \tilde{K}(b_{*};\mu_{b_{*}}) \ln \frac{Q^{2}}{\mu_{b_{*}}^{2}} + \int_{\mu_{b_{*}}}^{\mu_{Q}} \frac{d\mu'}{\mu'} \left[2\gamma_{j}(a_{s}(\mu')) - \ln \frac{Q^{2}}{(\mu')^{2}}\gamma_{K}(a_{s}(\mu'))\right]\right\}$$

• Perturbative content calculated from first principles of QFT

• Non-perturbative content (as for collinear case) to be fit to data

inciples of QFT ase) to be fit to data



- Perturbative content calculated from first principles of QFT
- Non-perturbative content (as for collinear case) to be fit

\*

$$egin{aligned} g_K(b_T; b_{max}) &= rac{g_2 b_{NP}^2}{2} \ln \left( 1 + rac{b_T^2}{b_{NP}^2} 
ight) = g_2 b_{NP}^2 \ln \left( 1 + rac{b_T^2}{b_{NP}^2} 
ight) = g_2 b_{NP}^2 \ln \left( 1 + rac{b_T^2}{b_{NP}^2} 
ight) \end{aligned}$$

$$g_k(b_T; b_{max}) = g_0 \left( 1 - \exp\left[ -\frac{C_F \alpha_s(\mu_{b*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2} \right] \right)$$

$$g_k(b_T; b_{max}) = \frac{g_2}{2} b_T b_*$$

 $g_k(b_T; b_{max})$ 

s of QFT <mark>be fit</mark>



Aidala, Field, Gamberg, Rogers PRD 2014 "Large  $b_T$ "

Collins Rogers PRD 2015 "Large  $b_T$  "

Vladimirov JHEP 2019 "Large  $b_T$  "



- Perturbative content calculated from first principles of QFT
- Non-perturbative content (as for collinear case) to be fit

$$\# g_j(x,b_T) = b_T^2 \left(rac{g_1}{2} + g_1 g_3 \log\left(rac{10xx_0}{(x+x_0)^2}
ight)$$

$$g_j(x, b_T) = \frac{(g_1 + g_3 x)b_T^2}{\sqrt{1 + |g_4|x^2 b_T^2}}$$

$$\tilde{f}(b_T, x, Q) = N \frac{\frac{M^{1+\nu}}{b_T \Gamma(\nu)} \left(\frac{2}{b_T}\right)^{1-\nu} k_{1-\nu} (Mb_T \frac{10xx_0}{(x+x_0)}) \times (1+Ab_T \frac{M^{1+\nu}}{(x+x_0)})}{\frac{M^{1+\nu}}{b_T_* \Gamma(\nu)} \left(\frac{2}{b_T_*}\right)^{1-\nu} k_{1-\nu} (Mb_T_* \frac{10xx_0}{(x+x_0)}) \times (1+Ab_T \frac{M^{1+\nu}}{(x+x_0)})}$$

 $g_j$ 

s of QF7 <mark>5e fit</mark>

 $\left( \right) \right)$ 

Aybat Rogers 2011

Vladimirov JHEP 2019

Qiu & Zhang PRL 2001

 $\frac{b_T + B/b_T)}{b_{T*} + B/b_{T*})}$ 

Bessel parametrization has large exponential behavior in  $b_{\rm T}$ 



Y = FO - AY

E615 DY Data ( E537 also) also need proton data E288 E605 E772

**Fixed Target Data only!** 

### 1) We have fit the NP parameters will show in simultaneous fit

### 2) Open TMD & collinear params to all pion data **Preliminary** Vladimirov's NP content

## Open proton and pion TMDs and pion collinear

Fit all pT-integrated DY, LN, low-pT DY data from pions and protons

Includes now a fit of the universal NP TMD parameter

reaction: DYpT
filters: Q<9
filters: xF<0.8
filters: pTmax<0.4*Qmin
reaction: DYpT

idx	col	obs	tar	npts	chi2	chi2/npts	nchi2
12881	E288	Ed3sigma/dp3	рр	52.00	86.43	1.66	0.01
12882	E288	Ed3sigma/dp3	рр	54.00	73.36	1.36	0.01
12883	E288	Ed3sigma/dp3	рр	51.00	95.59	1.87	0.09
10605	E605	Ed3sigma/dp3	рр	25.00	63.89	2.56	0.25
10772	E772	Ed3sigma/dp3	рр	39.00	105.65	2.71	0.04
1001	E615	d2sigma/dpTdm	pi-p	59.00	210.18	3.56	4.93
1002	E615	d2sigma/dpTdx	pi-p	48.00	112.10	2.34	0.01
15371	E537	d2sigma/dpT2dm	pi-p	61.00	50.59	0.83	1.77
15372	E537	d2sigma/dpT2dx	pi-p	45.00	43.28	0.96	0.49
reaction: dy-pi	on						

filters: Q2>4.16\*\*2 filters: Q2<59.0 filters: xF>0 filters: xF<0.9 reaction: dy-pion

			Ldi	npts	chi2	chi2/npts
10001	E615	dsig/drtau/dxF	pi-W	61.00	50.23	0.82
10002	NA10	dsig/drtau/dxF	pi-W	36.00	22.34	0.62
10003	NA10	dsig/drtau/dxF	pi-W	20.00	21.05	1.05

idx	col	obs	tar	npts	chi2	chi2/npts	nchi2
1000	H1	F2LN	ep->e'nX	58.00	21.72	0.37	13.57
2000	ZEUS	R	ep->e'nX	50.00	69.84	1.40	0.36

### 2) Open TMD & collinear params to all pion data **Preliminary**

## Open proton and pion TMDs and pion collinear

idx	dist	type	value
1	pdf-pion	g1 N	3.75e-01
2	pdf-pion	g1 a	-1.03e+00
3	pdf-pion	g1 b	2.65e+00
4	pdf-pion	u1 a	-9.12e-01
5	pdf-pion	u1 b	5.49e+00
6	pdf-pion	ubv1 a	-3.13e-01
7	pdf-pion	ubv1 b	9.70e-01
8	p->pi,n	lambda	1.37e+00

9	universal	g2	9.61e-0
10	tmd	g1	2.55e-(
11	tmd	g3	7.81e-0
12	tmd	g4	-5.27e+
13	tmd-pion	g1	4.65e-0
14	tmd-pion	g3	5.89e-0
15	tmd-pion	g4	-2.24e-

16	DYpT	12881	1.03e+00
17	DYpT	12882	1.03e+00
18	DYpT	12883	1.08e+00
19	DYpT	10605	1.07e+00
20	DYpT	10772	9.80e-01

21	DYpT	1001	1.36e+00
22	DYpT	1002	9.81e-01
23	DYpT	15371	1.17e+00
24	DYpT	15372	9.09e-01
25	dy-pion	10001	1.00e+00
26	dy-pion	10002	8.27e-01
27	dy-pion	10003	7.74e-01
28	In	1000	1.18e+00
29	In	2000	9.76e-01



### 2) Open TMD & collinear params to all pion data **Preliminary**

nb. fixed target DY experiments, the expected constraints will be on the collinear valence distribution.

## Open proton and pion TMDs and pion collinear

PDFs look somewhat	0.5
different with the inclusion of pT data.	0.4
Gluon is larger at higher x, while sea is smaller	$\begin{pmatrix} 0.3\\ (x)fx\\ 0.2 \end{pmatrix}$
Valence has some reduction at larger x	0.1
	0.0



### 2) Open TMD & collinear params to all pion data **Preliminary**

## Open proton and pion TMDs and pion collinear



### 2) Open TMD & collinear params to all pion data **Preliminary**





2) Open TMD & collinear params to all pion data

 $10^{-34}$ 



2) Open TMD & collinear params to all pion data **Preliminary** 



### Open TMD & collinear params to all pion data **Preliminary**

## Simultaneous fit for CSS and Bessel

- 1)
- 2) Fit collinear and TMD to pion DYpT data
- 3) Fit collinear and TMD to all data

Fit both proton and pion to only DYpT data -- get proton parameters

### Open TMD & collinear params to all pion data **Preliminary**

## Bessel and CSS -- fit proton and pion to DYpT

reaction: DYpT filters: Q<9 filters: xF<0.8 filters: pTmax<0.4\*Qmin reaction: DYpT

idx	col	obs	tar	npts	chi2	chi2/npts	rchi2	nchi2
12881	E288	Ed3sigma/dp3	рр	52.00	105.90	2.04	0.00	0.03
12882	E288	Ed3sigma/dp3	pp	54.00	70.32	1.30	0.00	0.07
12883	E288	Ed3sigma/dp3	рр	51.00	87.94	1.72	0.00	0.02
10605	E605	Ed3sigma/dp3	pp	25.00	62.44	2.50	0.01	0.02
10772	E772	Ed3sigma/dp3	pp	39.00	124.68	3.20	0.68	2.10
1001	E615	d2sigma/dpTdm	pi-p	59.00	202.03	3.42	0.00	3.35
1002	E615	d2sigma/dpTdx	pi-p	48.00	155.19	3.23	0.00	0.13
15371	E537	d2sigma/dpT2dm	pi-p	61.00	49.93	0.82	0.00	1.51
15372	E537	d2sigma/dpT2dx	pi-p	45.00	41.67	0.93	0.00	0.62

dist	type	value
universal	g0	6.44e-02
tmd	CA	5.18e+00
tmd	СВ	1.33e-03
tmd	М	1.98e+00
tmd	nu	3.17e+00
tmd	x0	5.30e-02
tmd-pion	CA	3.22e+00
tmd-pion	СВ	9.28e-02
tmd-pion	М	1.02e+00
tmd-pion	nu	1.85e+00
tmd-pion	x0	6.62e-02
DYpT	12881	9.56e-01
DYpT	12882	9.33e-01
DYpT	12883	9.63e-01
DYpT	10605	9.77e-01
DYpT	10772	8.55e-01
DYpT	1001	1.29e+00
DYpT	1002	9.41e-01
DYpT	15371	1.16e+00
DYpT	15372	8.98e-01
	dist universal tmd tmd tmd tmd tmd-pion tmd-pion tmd-pion tmd-pion tmd-pion DYpT DYpT DYpT DYpT DYpT DYpT DYpT DYpT	dist         type           universal         g0           tmd         CA           tmd         CB           tmd         M           tmd         nu           tmd         x0           tmd-pion         CA           tmd-pion         CA           tmd-pion         CA           tmd-pion         M           tmd-pion         M           tmd-pion         M           tmd-pion         N           tmd-pion         N           tmd-pion         N           tpp         12881           DYpT         12882           DYpT         12883           DYpT         10605           DYpT         1001           DYpT         1001           DYpT         1002           DYpT         15371           DYpT         15372

### Open TMD & collinear params to all pion data **Preliminary**

### Bessel and CSS -- fit collinear and NP pion to pion data

#### **Collinear PDFs**

Noticable change in gluon distributions at lower x

Valence does not change too much



### Open TMD & collinear params to all pion data **Preliminary**

### Bessel and CSS -- fit collinear and NP pion to pion data

#### Click to add text

idx	dist	type	value
1	pdf-pion	g1 N	3.13e-01
2	pdf-pion	g1 a	-1.90e+00
3	pdf-pion	g1 b	1.49e-08
4	pdf-pion	u1 a	-9.54e-01
5	pdf-pion	u1 b	6.89e+00
6	pdf-pion	ubv1 a	-2.91e-01
7	pdf-pion	ubv1 b	1.01e+00
8	p->pi,n	lambda	1.40e+00

9	tmd-pion	CA	1.00e+01
10	tmd-pion	СВ	4.29e-01
11	tmd-pion	М	2.71e+00
12	tmd-pion	nu	1.74e+00
13	tmd-pion	x0	2.40e-02

14	DYpT	1001	1.20e+00
15	DYpT	1002	8.61e-01
16	dy-pion	10001	1.01e+00
17	dy-pion	10002	8.33e-01
18	dy-pion	10003	7.80e-01
19	In	1000	1.17e+00
20	In	2000	9.72e-01

Open TMD & collinear params to all pion data

## Bessel and CSS -- fit collinear and NP pion to pion data







## Next step(!) Toward 3-D Pion structure

- 3-D imagining / TMDs using Drell Yan ×  $\sigma(p_T) \sim W(p_T) + Y(p_T)$  - Collins Soper Sterman NPB 1985
- Goal to use  $p_T(q_T)$  data over full range &
- simultaneous fit of pion pdfs & TMDs
- Cross section different "regions"-"two scales"
- *W* valid for  $\Lambda_{QCD} \sim p_T \ll Q$  TMD factorization
- **FO** valid for  $\Lambda_{OCD} \ll p_T \sim Q$  Collinear factorization

## Entails the study of TMD factorization & Matching in Drell Yan



Collins Soper Sterman NPB 1985 **Collins 2011 Cambridge Press** 

**Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD 2016 - improved matching** 

$$\frac{d\sigma(m \lesssim q_T \lesssim Q, Q)}{dy dq^2 dp_T^2} = \frac{d\sigma^W(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \lesssim q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \lesssim Q} - \frac{d\sigma^FO}{dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy$$

$$\equiv W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(rac{m}{Q}
ight)^c$$

$$\equiv W(p_T, Q) + Y(p_T, Q) + O\left(\frac{m}{Q}\right)^c$$

### • Cross section in terms of different "regions"

- *W* valid for  $q_T \sim k_T \ll Q$  TMD factorization
- **FO** valid for  $k_T \ll p_T \sim Q$  Collinear factorization
- ASY subtracts d.c. & in principle
- $ASY \rightarrow W, p_T \rightarrow \infty$  and  $ASY \rightarrow FO, p_T \rightarrow 0$

## Beyond the W term MATCHING $p_T$ in CSS



Drill down  $\rightarrow$ 

## A first glimpse matching W + Y



Key Elements

• Normalization control Thru W term •  $S_{NP}$ , while  $S_{pert}$  preserved  $ilde{f}_1(x, b_T, Q^2, \mu_Q) \sim \left[ ilde{C}^{f_1}\left( x/\hat{x}, m{b}_*; \mu_{b_*}^2, \mu_{b_*}, lpha(\mu_{b_*}) 
ight) \otimes f_1(\hat{x}, \mu_{b_*}) 
ight]$ 

 $\times \exp\left[-S_{pert}(\mu_{b_*}(b_T);\mu_{b_*},Q^2) - S_{NP}(b_T,Q)\right]$ 

• FO from Barry et al.  $f_{i/\pi}(x,\mu)$  @  $\mu = p_T/2$ 



## Results – E288 pp data

- Perform simultaneous fit of small- $p_{\rm T}$  TMDs to E288 (*pp*) and E615  $(\pi A)$  data
- FO is prediction using collinear PDFs and scale  $\mu = p_T/2$



Hard scale  $\mu \sim C p_T$ ,  $C \sim 1$ 

A first glimpse of matching W + Y Hard scale  $\mu \sim p_T$ 



 $\frac{d\sigma(m \leq q_T \leq Q, Q)}{dy dq^2 dp_T^2} = W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(\frac{m}{Q}\right)^c$ 

## Summary

- We've used pdfs extracted from JAM 20 to describe the full  $p_T$  cross section for DY
- First simultaneous fit of pion collinear PDFs & TMDs
- Generally diagnosed challenges of matching  $\sigma(p_T) \sim W(p_T) + Y(p_T)$  $\star$  Entails the study of factorization & Matching Perform simultaneous fit of pion (and nucleon/nuclear) pdfs from

$$1) \quad \frac{d\sigma^{DY}}{dQ^{2}dy} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$2) \quad \frac{d^{3}\sigma^{LN}}{dx_{B} \, dQ^{2} \, dx_{L}} = \frac{4\pi\alpha^{2}}{x_{B} \, Q^{4}} \left(1 - y_{e} + \frac{y_{e}^{2}}{2}\right) F_{2}^{LN}(x_{B}, Q^{2}, x_{L})$$

$$3) \quad \frac{d\sigma^{DY}}{dQ^{2} dy dp_{T}} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{FO}(x_{a}, x_{b}, y, p_{T}, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(W+Y) \quad ) \quad \frac{d\sigma}{dQ^{2} \, dx_{F} \, dp_{T}^{2}} = \frac{d\sigma^{W}}{dQ^{2} \, dx_{F} \, dp_{T}^{2}} + \frac{d\sigma^{DY}}{dQ^{2} \, dx_{F} \, dp_{T}^{2}} - ASY (\text{double count})$$

$$W(x_{F}, p_{T}, Q) \rightarrow \tilde{f}(x, b_{T}, Q^{2}, \mu_{Q}) \sim \left[\tilde{C}^{f}(x/\hat{x}, b_{*}; \mu_{b_{*}}^{2}, \mu_{b_{*}}, \alpha(\mu_{b_{*}})) \otimes f_{1}(\hat{x}, \mu_{b_{*}})\right] \times \exp\left[-S_{pert}(\mu_{b_{*}}(b_{T}); \mu_{b_{*}}, Q^{2}) - S_{NP}(b_{T}, Q)\right]$$

4



## Extras

#### **First Studies**

Vladimirov JHEP 2019 Pion-induced DY processes within TMD factorization



Pion TMDPDF for d-quark in b-space. (Right) Pion TMDPDF for d-quark in kT -space.

$$\frac{\mathrm{d}\sigma^W}{\mathrm{d}Q^2\,\mathrm{d}x_F\,\mathrm{d}p_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2s}\sum_j H_{j\bar{\jmath}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q))$$

$$\times \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} \ e^{i\boldsymbol{p}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \ \tilde{f}_{j/\pi}(x_A, b_{\mathrm{T}}; Q^2, \mu_Q) \ \tilde{f}_{\bar{\jmath}/H}(x_B, b_{\mathrm{T}}; Q^2, \mu_Q)$$

#### E615 DY Data

Figure 5. Comparison of the theory prediction (solid line) to E615 differential in Q. The dashed line is the theoretical prediction after the addition of systematic shifts  $d_i$ . The values of the  $\chi^2$ and  $d_i$  are calculated for the each Q-bin with 16% correlated error. The vertical dashed line shows the estimation of the boundary for TMD factorization approach.





#### JHEP 2019 Vladimirov Pion-induced DY processes within TMD factorization



Pion TMDPDF for d-quark in b-space. (Right) Pion TMDPDF for d-quark in kT -space. The bands are the  $1\sigma$  uncertainty band related to the data error-bands and calculated by the replica method.

Extracted from E615 DY Data

#### **COMMENTS**

#### Alot of engineering in the non perturbative TMD parametrization and modeling $\chi^2$ on normalization

Vladimirov JHEP 2019 Bacchetta et al. 2019 Pavia 19 (DY TMD analysis) Scimemi & Vladimirov JHEP 2020 (DY + SIDIS TMD analysis)

We diagnose W+FO @ Leading order simple NP

#### Bethe-Salpeter LC Wave Function methods for TMDs



FIG. 2. Upper panel: DSE result for the time-reversal even uquark TMD of the pion,  $f_{\pi}^{u}(x, k_{T}^{2})$ , at the model scale of  $\mu_0^2 = 0.52 \text{ GeV}^2$ . Lower panel: Analogous result evolved to a scale of  $\mu = 6$  GeV using TMD evolution with the  $b^*$  prescription and  $g_2 = 0.09$  GeV [51]. The TMDs are given in units of GeV<sup>-2</sup> and  $k_T^2$  in GeV<sup>2</sup>.

Chao Shi and Ian C. Cloët







## **Recent work** FO DY calculations

#### The predicted cross sections fall significantly short of the available data even at the highest accessible values of $q_T$

#### DIFFICULTIES IN THE DESCRIPTION OF DRELL-YAN PROCESSES ...



FIG. 6. E288. Experimental data vs NLO QCD predictions for y = 0.4 and different invariant mass bins.

Alessandro Bacchetta,<sup>1,2,\*</sup> Giuseppe Bozzi,<sup>1,2,†</sup> Martin Lambertsen,<sup>3,‡</sup> Fulvio Piacenza,<sup>1,2,§</sup> Julius Steiglechner,<sup>3,∥</sup> and Werner Vogelsang<sup>3,¶</sup>

#### PHYS. REV. D 100, 014018 (2019)

