## The 3-D Structure of the Pion From Pion-induced Drell-Yan Scattering



## Leonard Gamberg

w/ Patrick Barry, Eric Moffat, Alexei Prokudin Wally Melnitchouk, Nobuo Sato, ..... "JAM 3-D"







## The Pion as bound state QCD

- Pion plays a central/"outsized" role in hadron physics
- As a nearly massless  $\bar{q}q$  bound state Goldstone boson
  - ★ Chiral symmetry is explicitly broken by small current quark masses

Píon pole condition from Bethe-Salpeter

$$m_{\pi}^2 = rac{m}{2 \, G_{\pi} \, M \, I(m_{\pi}^2)}$$

moving at close to speed of light

Gell Mann Reiner Oaks Relation

$$f_{\pi}^2 m_{\pi}^2 = \frac{1}{2} \left( m_u + m_d \right) \left\langle \bar{u}u + \bar{d}d \right\rangle$$

 $\star$  Mass without mass" bulk of pion mass due to quantum fluctuations of  $\bar{q}q$  pairs, gluons, & energy associated with quarks





## The Pion recent progress

Also @ high energies pion reveals unique spectrum of momentum distributions,  $f_{i/\pi}(x,\mu)$ • See talk of P.C. Barry Monday: DY & LN process as predicted from Collinear Factorization

$$\frac{\mathrm{d}\sigma^{\mathrm{DY}}}{\mathrm{d}Q^{2}\mathrm{d}y} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{\mathrm{DY}}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2})$$

2) 
$$\frac{\mathrm{d}^3 \sigma^{\text{LN}}}{\mathrm{d} x_B \,\mathrm{d} Q^2 \,\mathrm{d} x_L} = \frac{4\pi\alpha^2}{x_B \,Q^4} \left(1 - y_e + \frac{y_e^2}{2}\right) F_2^{\text{LN}}(x_B \,\mathrm{d} x_B \,\mathrm{d} x_B \,\mathrm{d} x_B)$$

understanding how these contrasting manifestations of the same  $\bar{q}q$  bound state arise dynamically at different energy scales from first principles remains a major challenge in QCD

 $f_{b/B}(x_b, \mu^2)$ 









## The Pion recent progress

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Jefferson Lab Angular Momentum (JAM) Barry, Sato, Melnitchouk, C.R. Ji, PRL 2018



FIG. 2. Pion valence (green), sea quark (blue) and gluon (red, scaled by 1/10) PDFs versus  $x_{\pi}$  at  $Q^2 = 10 \text{ GeV}^2$ , for the full DY + LN (dark bands) and DY only (light bands) fits. The bands represent  $1\sigma$  uncertainties, as defined in the standard Monte Carlo determination of the uncertainties [42] from the experimental errors. The model dependence of the fit is represented by the outer yellow bands.









Barry et al *Phys.Rev.D* 103 (2021), also included transverse momentum dependent Drell-Yan data in a global QCD analysis for the first time. These data were able to be described well by our O( $\alpha_s$ ) framework, and new pion PDFs were extracted. The inclusion of the pT-dependent data helped in reduce the uncertainties of the gluon distribution at large x





Jefferson Lab Angular Momentum (JAM) Cao Barry, Sato, Melnitchouk, arXiv:2103.02159

1) 
$$\frac{d\sigma^{DY}}{dQ^{2}dy} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(A) = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$\frac{d\sigma^{DY}}{dQ^{2}dy} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$\frac{\pi}{q} + \frac{\pi}{q} +$$



## Extended fit to include "large" p<sub>T</sub> Drell Yan data

### **Collinear Factorization CSS NPB 1985**

$$(x_a, x_b, y, p_{
m T}, Q^2, \mu^2) \, f_{a/A}(x_a, \mu^2) \, f_{b/B}(x_b, \mu^2)$$

 $p_T \lesssim Q$ 



Jefferson Lab Angular Momentum (JAM) Cao Barry, Sato, Melnitchouk, arXiv:2103.02159

New Monte Carlo QCD analysis of pion parton distribution functions, including, for the first time, large transverse momentum dependent pion-nucleus Drell-Yan cross sections together with  $P_T$ -integrated Drell-Yan & leading neutron electroproduction data from HERA.



#### JAM20 Pion PDFs

3) 
$$\frac{d\sigma^{\mathrm{DY}}}{dQ^2dydp_T} = \sum_{a,b} \int \mathrm{d}x_a \,\mathrm{d}x_b \,H_{a,b}^{\mathrm{FO}}(x_a, x_b, y, p_{\mathrm{T}}, Q_{\mathrm{T}})$$

## Extended fit to include "large" p<sub>T</sub> Drell Yan data



Pion valence quark, sea quark, and gluon (scaled by a factor of 1/10) PDFs extracted from the JAM Monte Carlo analysis of DY (pT-integrated and pT-dependent) and leading neutron data at a scale of  $\mu^2 = 10 \text{ GeV}^2$ 





2)









#### Jefferson Lab Angular Momentum (JAM) Cao Barry, Sato, Melnitchouk, Phys. Rev. D 103 (2021)

New Monte Carlo QCD analysis of pion parton distribution functions, including, for the first time, large transverse momentum dependent pion-nucleus Drell-Yan cross sections



3) 
$$\frac{d\sigma^{\mathrm{DY}}}{dQ^2 dy dp_T} = \sum_{a,b} \int \mathrm{d}x_a \,\mathrm{d}x_b \,H_{a,b}^{\mathrm{FO}}(x_a, x_b, y, p_{\mathrm{T}}, Q_{\mathrm{T}})$$

Extended fit to include "large" p<sub>T</sub> Drell Yan data

To describe the transverse momentum "region"  $p_T \sim k_T \ll Q$  is the regime of TMDs of the pion That require fitting "region"  $p_T \sim k_T \ll Q$  differential pion-induced Drell-Yan cross section



Some earlier work on pion-hadron DY

- F. Ceccopieri, A. Courtoy, S. Noguera, S. Scopetta, EJP 2018 CSS Evolution w/ NJL
- Vladimirov JHEP 2019 Pion-induced DY processes within TMD factorization

# "More granular" $p_T \sim k_T \ll Q$ access to the Pion TMDs

#### **TMD** Factorization

$$\tilde{d}^2 \boldsymbol{b}_{\mathrm{T}} \over (2\pi)^2 \ e^{i \boldsymbol{p}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \tilde{W}(x_F, b_T, Q)$$

$$f(x_F, b_T, Q) = \sum_j H^{\mathrm{DY}}_{jar{\jmath}}(Q, \mu, a_s(\mu)) ilde{f}_{j/A}(x_A, b_{\mathrm{T}}; \zeta_A, \mu) ilde{f}_{ar{\jmath}/B}(x_B, b_{\mathrm{T}}; \zeta_B, \mu)$$

TMD Factorization

- **Collins Soper Sterman NPB 1985**
- **◆** Ji Ma Yuan PRD PLB ...2004, 2005
- +Aybat Rogers PRD 2011
- **Collins 2011 Cambridge Press**
- *Echevarria*, *Idilbi*, *Scimemi JHEP 2012*
- **+**... many more refs



## The Pion TMDs & Factorization

### **TMD Factorization**

 Mulders Tangerman NPB 1995

Boer Mulders PRD 1997

## Leading Power TMDPDFs



$$f(x, k_T, s_T) = \frac{1}{2} \left[ f_1^{\pi}(x, k_T^2) + \frac{s_T^i \epsilon^{ij} k_T^j}{m_{\pi}} h_1^{\pi \perp}(x, k_T^2) \right]$$

Factorization carried out Fourier  $b_T$  space FT TMDs  $\tilde{f}(x, b_T; \zeta, \mu)$ Real QCD need QFT definitions of TMDs LC & UV divergences TMD Evolution depends on rapidity  $\zeta$  and RGE scales  $\mu$ 

$$\frac{\mathrm{d}\sigma^W}{\mathrm{d}Q^2\,\mathrm{d}x_F\,\mathrm{d}p_{\mathrm{T}}^2} = \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} \ e^{i\boldsymbol{p}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}\tilde{W}(x_F,b_T,Q)$$

$$ilde{W}(x_F, b_T, Q) = \sum_j H^{\mathrm{DY}}_{j\bar{j}}(Q, \mu)$$







TMD Factorization

- Collins Soper Sterman NPB 1985
- ✦ Ji Ma Yuan PRD PLB ...2004, 2005
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- **Collins 2011 Cambridge Press**
- Echevarria, Idilbi, Scimemi JHEP 2012,

 $(a, a_s(\mu)) \tilde{f}_{j/A}(x_A, b_{\mathrm{T}}; \zeta_A, \mu) \tilde{f}_{\bar{\jmath}/B}(x_B, b_{\mathrm{T}}; \zeta_B, \mu)$ 

,



$$\tilde{f}_{j/H}^{\text{sub}}(x, \boldsymbol{b}; \boldsymbol{\mu}, \zeta_{\text{PDF}}) = \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \underbrace{\tilde{f}_{j/H}^{\text{unsub}}(x, \boldsymbol{b}; \boldsymbol{\mu}, \boldsymbol{n}_B(\boldsymbol{y}_B))}_{\hat{S}(\boldsymbol{b}; y_A, y_B)\tilde{S}(\boldsymbol{b}; y_s, y_B)} \sqrt{\frac{\tilde{S}(\boldsymbol{b}; y_A, y_B)\tilde{S}(\boldsymbol{b}; y_s, y_B)}{\tilde{S}(\boldsymbol{b}; y_s, y_B)}}$$

$$\hat{f}_{j/H}^{\text{unsub}}(x, \boldsymbol{b}_T; \boldsymbol{\mu}, y_P - y_B) = \int \frac{d\boldsymbol{b}^-}{2\pi} e^{-i\boldsymbol{x}P^+\boldsymbol{b}^-} \langle P|\bar{\psi}(\boldsymbol{y}_B)|^2 \frac{d\boldsymbol{b}^-}{2\pi} e^{-i\boldsymbol{x}P^+} \frac{d\boldsymbol{b}^-}{2\pi} e^{-i\boldsymbol{x}$$

- in solution to the \*Collins-Soper **\***RG equations



## **Renormalization and Evolution-** $\{\zeta, \mu\}$ TMD



Collins Soper Eq.

 $\frac{\partial \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$ 



 $rac{d ilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_k(lpha_s(\mu)$ 

\* RGE for TMD

 $d \ln ilde{f}_{j/H}(x,b_T;\mu$  $\tilde{\Phi}^{[\gamma^+]}(x,\vec{b}_T;Q^2,\mu_Q) = \tilde{f}_1(x,b_T;Q^2,\mu_Q) - iM\epsilon^{ij}$ 

Solve simultaneously and get evolved rend

$$\tilde{K}(b_T,\mu) \equiv \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{S(b_T, y_n, -\infty)}{S(b_T, y_n, -\infty)}$$

$$O(1)$$

$$\frac{(\mu,\zeta)}{\tilde{u}_{J}} = -\gamma_{F}(\alpha_{s}(\mu),\zeta/\mu) - \frac{(\mu,\zeta)}{M^{2}} - \frac{(\mu,\zeta)}{M^{2}} \frac{\tilde{f}_{L}}{\tilde{b}_{T}} \frac{\tilde{f}_{L}}{\tilde{b}_{T}}(x,b_{T};Q^{2},\mu_{Q}) \text{ormalized TMD}_{TMD} \rightarrow \zeta = Q^{2}, \quad \mu = \mu_{Q} \sim Q$$



$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/2}}$$



 $\mu_{b_*} = C_1/b_*(b_T)$ 



### • In small- $p_{\rm T}$ region, Use the CSS formalism for TMD evolution

$$\frac{d\sigma}{dQ^{2} dy dq_{T}^{2}} = \frac{4\pi^{2}\alpha^{2}}{9Q^{2}s} \sum_{j,j_{A},j_{B}} H_{j\bar{j}}^{DY}(Q,\mu_{Q},a_{s}(\mu_{Q})) \int \frac{d^{2}b_{T}}{(2\pi)^{2}} e^{i\boldsymbol{q}_{T}\cdot\boldsymbol{b}_{T}}$$
Aybat Rogers 201 & Collins 201  
$$\times e^{-\frac{g_{j/A}(x_{A},b_{T};b_{max})}{\int_{x_{A}}^{1} \frac{d\xi_{A}}{\xi_{A}}} f_{j_{A}/A}(\xi_{A};\mu_{b_{*}}) \tilde{V}_{j/j_{A}}^{PDF}\left(\frac{x_{A}}{\xi_{A}},b_{*};\mu_{b_{*}}^{2},\mu_{b_{*}},a_{s}(\mu_{b_{*}})\right)$$
$$\times e^{-\frac{g_{j/B}(x_{B},b_{T};b_{max})}{\int_{x_{B}}^{1} \frac{d\xi_{B}}{\xi_{B}}} f_{j_{B}/B}(\xi_{B};\mu_{b_{*}}) \tilde{C}_{\bar{j}/j_{B}}^{PDF}\left(\frac{x_{B}}{\xi_{B}},b_{*};\mu_{b_{*}}^{2},\mu_{b_{*}},a_{s}(\mu_{b_{*}})\right)$$
$$\times \exp\left\{-\frac{g_{K}(b_{T};b_{max})}{Q_{0}^{2}} + \tilde{K}(b_{*};\mu_{b_{*}}) \ln \frac{Q^{2}}{\mu_{b_{*}}^{2}} + \int_{\mu_{b_{*}}}^{\mu_{Q}} \frac{d\mu'}{\mu'} \left[2\gamma_{j}(a_{s}(\mu')) - \ln \frac{Q^{2}}{(\mu')^{2}}\gamma_{K}(a_{s}(\mu'))\right]\right\}$$

• Perturbative content calculated from first principles of QFT

• Non-perturbative content (as for collinear case) to be fit to data

inciples of QFT ase) to be fit to data



- Perturbative content calculated from first principles of QFT
- Non-perturbative content (as for collinear case) to be fit

\*

$$egin{aligned} g_K(b_T; b_{max}) &= rac{g_2 b_{NP}^2}{2} \ln \left( 1 + rac{b_T^2}{b_{NP}^2} 
ight) = g_2 b_{NP}^2 \ln \left( 1 + rac{b_T^2}{b_{NP}^2} 
ight) = g_2 b_{NP}^2 \ln \left( 1 + rac{b_T^2}{b_{NP}^2} 
ight) \end{aligned}$$

$$g_k(b_T; b_{max}) = g_0 \left( 1 - \exp\left[ -\frac{C_F \alpha_s(\mu_{b*}) b_T^2}{\pi g_0(b_{max}) b_{max}^2} \right] \right)$$

$$g_k(b_T; b_{max}) = \frac{g_2}{2} b_T b_*$$

 $g_k(b_T; b_{max})$ 

s of QFT <mark>be fit</mark>



Aidala, Field, Gamberg, Rogers PRD 2014 "Large  $b_T$ "

Collins Rogers PRD 2015 "Large  $b_T$  "

Vladimirov JHEP 2019 "Large  $b_T$  "



- Perturbative content calculated from first principles of QFT
- Non-perturbative content (as for collinear case) to be fit

$$\# g_j(x,b_T) = b_T^2 \left(rac{g_1}{2} + g_1 g_3 \log\left(rac{10xx_0}{(x+x_0)^2}
ight)$$

$$g_j(x, b_T) = \frac{(g_1 + g_3 x)b_T^2}{\sqrt{1 + |g_4|x^2 b_T^2}}$$

$$\tilde{f}(b_T, x, Q) = N \frac{\frac{M^{1+\nu}}{b_T \Gamma(\nu)} \left(\frac{2}{b_T}\right)^{1-\nu} k_{1-\nu} (Mb_T \frac{10xx_0}{(x+x_0)}) \times (1+Ab_T \frac{M^{1+\nu}}{(x+x_0)})}{\frac{M^{1+\nu}}{b_T_* \Gamma(\nu)} \left(\frac{2}{b_T_*}\right)^{1-\nu} k_{1-\nu} (Mb_T_* \frac{10xx_0}{(x+x_0)}) \times (1+Ab_T \frac{M^{1+\nu}}{(x+x_0)})}$$

 $g_j$ 

s of QF7 <mark>5e fit</mark>

 $\left( \right) \right)$ 

Aybat Rogers 2011

Vladimirov JHEP 2019

Qiu & Zhang PRL 2001

 $\frac{b_T + B/b_T)}{b_{T*} + B/b_{T*})}$ 

Bessel parametrization has large exponential behavior in  $b_{\rm T}$ 



Y = FO - AY

E615 DY Data ( E537 also) also need proton data E288 E605 E772

**Fixed Target Data only!** 

### 1) We have fit the NP parameters will show in simultaneous fit

### 2) Open TMD & collinear params to all pion data **Preliminary** Vladimirov's NP content

## Open proton and pion TMDs and pion collinear

Fit all pT-integrated DY, LN, low-pT DY data from pions and protons

Includes now a fit of the universal NP TMD parameter

reaction: DYpT
filters: Q<9
filters: xF<0.8
filters: pTmax<0.4*Qmin
reaction: DYpT

idx	col	obs	tar	npts	chi2	chi2/npts	nchi2
12881	E288	Ed3sigma/dp3	рр	52.00	86.43	1.66	0.01
12882	E288	Ed3sigma/dp3	рр	54.00	73.36	1.36	0.01
12883	E288	Ed3sigma/dp3	рр	51.00	95.59	1.87	0.09
10605	E605	Ed3sigma/dp3	рр	25.00	63.89	2.56	0.25
10772	E772	Ed3sigma/dp3	рр	39.00	105.65	2.71	0.04
1001	E615	d2sigma/dpTdm	pi-p	59.00	210.18	3.56	4.93
1002	E615	d2sigma/dpTdx	pi-p	48.00	112.10	2.34	0.01
15371	E537	d2sigma/dpT2dm	pi-p	61.00	50.59	0.83	1.77
15372	E537	d2sigma/dpT2dx	pi-p	45.00	43.28	0.96	0.49
reaction: dy-pi	on						

filters: Q2>4.16\*\*2 filters: Q2<59.0 filters: xF>0 filters: xF<0.9 reaction: dy-pion

			Ldi	npts	chi2	chi2/npts
10001	E615	dsig/drtau/dxF	pi-W	61.00	50.23	0.82
10002	NA10	dsig/drtau/dxF	pi-W	36.00	22.34	0.62
10003	NA10	dsig/drtau/dxF	pi-W	20.00	21.05	1.05

idx	col	obs	tar	npts	chi2	chi2/npts	nchi2
1000	H1	F2LN	ep->e'nX	58.00	21.72	0.37	13.57
2000	ZEUS	R	ep->e'nX	50.00	69.84	1.40	0.36

### 2) Open TMD & collinear params to all pion data **Preliminary**

## Open proton and pion TMDs and pion collinear

idx	dist	type	value
1	pdf-pion	g1 N	3.75e-01
2	pdf-pion	g1 a	-1.03e+00
3	pdf-pion	g1 b	2.65e+00
4	pdf-pion	u1 a	-9.12e-01
5	pdf-pion	u1 b	5.49e+00
6	pdf-pion	ubv1 a	-3.13e-01
7	pdf-pion	ubv1 b	9.70e-01
8	p->pi,n	lambda	1.37e+00

9	universal	g2	9.61e-0
10	tmd	g1	2.55e-(
11	tmd	g3	7.81e-0
12	tmd	g4	-5.27e+
13	tmd-pion	g1	4.65e-0
14	tmd-pion	g3	5.89e-0
15	tmd-pion	g4	-2.24e-

16	DYpT	12881	1.03e+00
17	DYpT	12882	1.03e+00
18	DYpT	12883	1.08e+00
19	DYpT	10605	1.07e+00
20	DYpT	10772	9.80e-01

21	DYpT	1001	1.36e+00
22	DYpT	1002	9.81e-01
23	DYpT	15371	1.17e+00
24	DYpT	15372	9.09e-01
25	dy-pion	10001	1.00e+00
26	dy-pion	10002	8.27e-01
27	dy-pion	10003	7.74e-01
28	In	1000	1.18e+00
29	In	2000	9.76e-01



### 2) Open TMD & collinear params to all pion data **Preliminary**

nb. fixed target DY experiments, the expected constraints will be on the collinear valence distribution.

## Open proton and pion TMDs and pion collinear

PDFs look somewhat	0.5
different with the inclusion of pT data.	0.4
Gluon is larger at higher x, while sea is smaller	$\begin{pmatrix} 0.3\\ (x)fx\\ 0.2 \end{pmatrix}$
Valence has some reduction at larger x	0.1
	0.0



### 2) Open TMD & collinear params to all pion data **Preliminary**

## Open proton and pion TMDs and pion collinear



### 2) Open TMD & collinear params to all pion data **Preliminary**





2) Open TMD & collinear params to all pion data

 $10^{-34}$ 



2) Open TMD & collinear params to all pion data **Preliminary** 



### Open TMD & collinear params to all pion data **Preliminary**

## Simultaneous fit for CSS and Bessel

- 1)
- 2) Fit collinear and TMD to pion DYpT data
- 3) Fit collinear and TMD to all data

Fit both proton and pion to only DYpT data -- get proton parameters

### Open TMD & collinear params to all pion data **Preliminary**

## Bessel and CSS -- fit proton and pion to DYpT

reaction: DYpT filters: Q<9 filters: xF<0.8 filters: pTmax<0.4\*Qmin reaction: DYpT

idx	col	obs	tar	npts	chi2	chi2/npts	rchi2	nchi2
12881	E288	Ed3sigma/dp3	рр	52.00	105.90	2.04	0.00	0.03
12882	E288	Ed3sigma/dp3	pp	54.00	70.32	1.30	0.00	0.07
12883	E288	Ed3sigma/dp3	рр	51.00	87.94	1.72	0.00	0.02
10605	E605	Ed3sigma/dp3	pp	25.00	62.44	2.50	0.01	0.02
10772	E772	Ed3sigma/dp3	pp	39.00	124.68	3.20	0.68	2.10
1001	E615	d2sigma/dpTdm	pi-p	59.00	202.03	3.42	0.00	3.35
1002	E615	d2sigma/dpTdx	pi-p	48.00	155.19	3.23	0.00	0.13
15371	E537	d2sigma/dpT2dm	pi-p	61.00	49.93	0.82	0.00	1.51
15372	E537	d2sigma/dpT2dx	pi-p	45.00	41.67	0.93	0.00	0.62

dist	type	value
universal	g0	6.44e-02
tmd	CA	5.18e+00
tmd	СВ	1.33e-03
tmd	М	1.98e+00
tmd	nu	3.17e+00
tmd	x0	5.30e-02
tmd-pion	CA	3.22e+00
tmd-pion	СВ	9.28e-02
tmd-pion	М	1.02e+00
tmd-pion	nu	1.85e+00
tmd-pion	x0	6.62e-02
DYpT	12881	9.56e-01
DYpT	12882	9.33e-01
DYpT	12883	9.63e-01
DYpT	10605	9.77e-01
DYpT	10772	8.55e-01
DYpT	1001	1.29e+00
DYpT	1002	9.41e-01
DYpT	15371	1.16e+00
DYpT	15372	8.98e-01
	dist universal tmd tmd tmd tmd tmd-pion tmd-pion tmd-pion tmd-pion tmd-pion DYpT DYpT DYpT DYpT DYpT DYpT DYpT DYpT	dist         type           universal         g0           tmd         CA           tmd         CB           tmd         M           tmd         nu           tmd         x0           tmd-pion         CA           tmd-pion         CA           tmd-pion         CA           tmd-pion         M           tmd-pion         M           tmd-pion         M           tmd-pion         N           tmd-pion         N           tmd-pion         N           tpp         12881           DYpT         12882           DYpT         12883           DYpT         10605           DYpT         1001           DYpT         1001           DYpT         1002           DYpT         15371           DYpT         15372

### Open TMD & collinear params to all pion data **Preliminary**

### Bessel and CSS -- fit collinear and NP pion to pion data

#### **Collinear PDFs**

Noticable change in gluon distributions at lower x

Valence does not change too much



### Open TMD & collinear params to all pion data **Preliminary**

### Bessel and CSS -- fit collinear and NP pion to pion data

#### Click to add text

idx	dist	type	value
1	pdf-pion	g1 N	3.13e-01
2	pdf-pion	g1 a	-1.90e+00
3	pdf-pion	g1 b	1.49e-08
4	pdf-pion	u1 a	-9.54e-01
5	pdf-pion	u1 b	6.89e+00
6	pdf-pion	ubv1 a	-2.91e-01
7	pdf-pion	ubv1 b	1.01e+00
8	p->pi,n	lambda	1.40e+00

9	tmd-pion	CA	1.00e+01
10	tmd-pion	СВ	4.29e-01
11	tmd-pion	М	2.71e+00
12	tmd-pion	nu	1.74e+00
13	tmd-pion	x0	2.40e-02

14	DYpT	1001	1.20e+00
15	DYpT	1002	8.61e-01
16	dy-pion	10001	1.01e+00
17	dy-pion	10002	8.33e-01
18	dy-pion	10003	7.80e-01
19	In	1000	1.17e+00
20	In	2000	9.72e-01

Open TMD & collinear params to all pion data

## Bessel and CSS -- fit collinear and NP pion to pion data





![](_page_28_Figure_6.jpeg)

## Next step(!) Toward 3-D Pion structure

- 3-D imagining / TMDs using Drell Yan ×  $\sigma(p_T) \sim W(p_T) + Y(p_T)$  - Collins Soper Sterman NPB 1985
- Goal to use  $p_T(q_T)$  data over full range &
- simultaneous fit of pion pdfs & TMDs
- Cross section different "regions"-"two scales"
- *W* valid for  $\Lambda_{QCD} \sim p_T \ll Q$  TMD factorization
- **FO** valid for  $\Lambda_{OCD} \ll p_T \sim Q$  Collinear factorization

## Entails the study of TMD factorization & Matching in Drell Yan

![](_page_30_Picture_0.jpeg)

Collins Soper Sterman NPB 1985 **Collins 2011 Cambridge Press** 

**Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD 2016 - improved matching** 

$$\frac{d\sigma(m \lesssim q_T \lesssim Q, Q)}{dy dq^2 dp_T^2} = \frac{d\sigma^W(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \lesssim q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \lesssim Q} - \frac{d\sigma^FO}{dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \ll Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2} \bigg|_{m \ll q_T \iff Q} + \frac{d\sigma^{FO}(q_T, Q)}{dy$$

$$\equiv W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(rac{m}{Q}
ight)^c$$

$$\equiv W(p_T, Q) + Y(p_T, Q) + O\left(\frac{m}{Q}\right)^c$$

### • Cross section in terms of different "regions"

- *W* valid for  $q_T \sim k_T \ll Q$  TMD factorization
- **FO** valid for  $k_T \ll p_T \sim Q$  Collinear factorization
- ASY subtracts d.c. & in principle
- $ASY \rightarrow W, p_T \rightarrow \infty$  and  $ASY \rightarrow FO, p_T \rightarrow 0$

## Beyond the W term MATCHING $p_T$ in CSS

![](_page_30_Figure_12.jpeg)

Drill down  $\rightarrow$ 

## A first glimpse matching W + Y

![](_page_31_Figure_1.jpeg)

Key Elements

• Normalization control Thru W term •  $S_{NP}$ , while  $S_{pert}$  preserved  $ilde{f}_1(x, b_T, Q^2, \mu_Q) \sim \left[ ilde{C}^{f_1}\left( x/\hat{x}, m{b}_*; \mu_{b_*}^2, \mu_{b_*}, lpha(\mu_{b_*}) 
ight) \otimes f_1(\hat{x}, \mu_{b_*}) 
ight]$ 

 $\times \exp\left[-S_{pert}(\mu_{b_*}(b_T);\mu_{b_*},Q^2) - S_{NP}(b_T,Q)\right]$ 

• FO from Barry et al.  $f_{i/\pi}(x,\mu)$  @  $\mu = p_T/2$ 

![](_page_31_Picture_7.jpeg)

## Results – E288 pp data

- Perform simultaneous fit of small- $p_{\rm T}$  TMDs to E288 (*pp*) and E615  $(\pi A)$  data
- FO is prediction using collinear PDFs and scale  $\mu = p_T/2$

![](_page_32_Figure_4.jpeg)

Hard scale  $\mu \sim C p_T$ ,  $C \sim 1$ 

A first glimpse of matching W + Y Hard scale  $\mu \sim p_T$ 

![](_page_33_Figure_0.jpeg)

 $\frac{d\sigma(m \leq q_T \leq Q, Q)}{dy dq^2 dp_T^2} = W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(\frac{m}{Q}\right)^c$ 

## Summary

- We've used pdfs extracted from JAM 20 to describe the full  $p_T$  cross section for DY
- First simultaneous fit of pion collinear PDFs & TMDs
- Generally diagnosed challenges of matching  $\sigma(p_T) \sim W(p_T) + Y(p_T)$  $\star$  Entails the study of factorization & Matching Perform simultaneous fit of pion (and nucleon/nuclear) pdfs from

$$1) \quad \frac{d\sigma^{DY}}{dQ^{2}dy} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{DY}(x_{a}, x_{b}, y, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$2) \quad \frac{d^{3}\sigma^{LN}}{dx_{B} \, dQ^{2} \, dx_{L}} = \frac{4\pi\alpha^{2}}{x_{B} \, Q^{4}} \left(1 - y_{e} + \frac{y_{e}^{2}}{2}\right) F_{2}^{LN}(x_{B}, Q^{2}, x_{L})$$

$$3) \quad \frac{d\sigma^{DY}}{dQ^{2} dy dp_{T}} = \sum_{a,b} \int dx_{a} \, dx_{b} \, H_{a,b}^{FO}(x_{a}, x_{b}, y, p_{T}, Q^{2}, \mu^{2}) \, f_{a/A}(x_{a}, \mu^{2}) \, f_{b/B}(x_{b}, \mu^{2})$$

$$(W+Y) \quad ) \quad \frac{d\sigma}{dQ^{2} \, dx_{F} \, dp_{T}^{2}} = \frac{d\sigma^{W}}{dQ^{2} \, dx_{F} \, dp_{T}^{2}} + \frac{d\sigma^{DY}}{dQ^{2} \, dx_{F} \, dp_{T}^{2}} - ASY (\text{double count})$$

$$W(x_{F}, p_{T}, Q) \rightarrow \tilde{f}(x, b_{T}, Q^{2}, \mu_{Q}) \sim \left[\tilde{C}^{f}(x/\hat{x}, b_{*}; \mu_{b_{*}}^{2}, \mu_{b_{*}}, \alpha(\mu_{b_{*}})) \otimes f_{1}(\hat{x}, \mu_{b_{*}})\right] \times \exp\left[-S_{pert}(\mu_{b_{*}}(b_{T}); \mu_{b_{*}}, Q^{2}) - S_{NP}(b_{T}, Q)\right]$$

4

![](_page_35_Picture_0.jpeg)

## Extras

#### **First Studies**

Vladimirov JHEP 2019 Pion-induced DY processes within TMD factorization

![](_page_36_Figure_2.jpeg)

Pion TMDPDF for d-quark in b-space. (Right) Pion TMDPDF for d-quark in kT -space.

$$\frac{\mathrm{d}\sigma^W}{\mathrm{d}Q^2\,\mathrm{d}x_F\,\mathrm{d}p_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2s}\sum_j H_{j\bar{\jmath}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q))$$

$$\times \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} \ e^{i\boldsymbol{p}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \ \tilde{f}_{j/\pi}(x_A, b_{\mathrm{T}}; Q^2, \mu_Q) \ \tilde{f}_{\bar{\jmath}/H}(x_B, b_{\mathrm{T}}; Q^2, \mu_Q)$$

#### E615 DY Data

Figure 5. Comparison of the theory prediction (solid line) to E615 differential in Q. The dashed line is the theoretical prediction after the addition of systematic shifts  $d_i$ . The values of the  $\chi^2$ and  $d_i$  are calculated for the each Q-bin with 16% correlated error. The vertical dashed line shows the estimation of the boundary for TMD factorization approach.

![](_page_36_Figure_9.jpeg)

![](_page_36_Figure_10.jpeg)

#### JHEP 2019 Vladimirov Pion-induced DY processes within TMD factorization

![](_page_37_Figure_1.jpeg)

Pion TMDPDF for d-quark in b-space. (Right) Pion TMDPDF for d-quark in kT -space. The bands are the  $1\sigma$  uncertainty band related to the data error-bands and calculated by the replica method.

Extracted from E615 DY Data

#### **COMMENTS**

#### Alot of engineering in the non perturbative TMD parametrization and modeling $\chi^2$ on normalization

Vladimirov JHEP 2019 Bacchetta et al. 2019 Pavia 19 (DY TMD analysis) Scimemi & Vladimirov JHEP 2020 (DY + SIDIS TMD analysis)

We diagnose W+FO @ Leading order simple NP

#### Bethe-Salpeter LC Wave Function methods for TMDs

![](_page_37_Figure_10.jpeg)

FIG. 2. Upper panel: DSE result for the time-reversal even uquark TMD of the pion,  $f_{\pi}^{u}(x, k_{T}^{2})$ , at the model scale of  $\mu_0^2 = 0.52 \text{ GeV}^2$ . Lower panel: Analogous result evolved to a scale of  $\mu = 6$  GeV using TMD evolution with the  $b^*$  prescription and  $g_2 = 0.09$  GeV [51]. The TMDs are given in units of GeV<sup>-2</sup> and  $k_T^2$  in GeV<sup>2</sup>.

Chao Shi and Ian C. Cloët

![](_page_37_Picture_15.jpeg)

![](_page_37_Picture_16.jpeg)

![](_page_37_Picture_17.jpeg)

## **Recent work** FO DY calculations

#### The predicted cross sections fall significantly short of the available data even at the highest accessible values of $q_T$

#### DIFFICULTIES IN THE DESCRIPTION OF DRELL-YAN PROCESSES ...

![](_page_38_Figure_3.jpeg)

FIG. 6. E288. Experimental data vs NLO QCD predictions for y = 0.4 and different invariant mass bins.

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![](_page_38_Figure_8.jpeg)