Semi-Exclusive Double Drell-Yan factorization and GTMDs

HADRON 2021 - Jul 2021

19th International Conference on Hadron Spectroscopy and Structure in memoriam Simon Eidelman

Patricia Andrea Gutiérrez García
Work in progress with Miguel Echevarría and Ignazio Scimemi
Complutense University of Madrid

Outline

- Introduction
- SCET Intro
- Semi-Exclusive DDY factorization
  - Current operator, Multipole expansion
  - Soft factor, 2TMD, GTMDs
- Color
- Zero-bin and Rapidity Divergences
- Conclusions

Motivation

- Understanding multi dimensional inner structure of strongly interacting systems
- GTMDs absorb both GPDs and TMDs
- *How to obtain GTMDs in a cross-section using **SCET**?*

$$\pi(p_b) + N(p_a, \lambda_a) \to \gamma_1^*(q_1, \lambda_1) + \gamma_2^*(q_2, \lambda_2) + N'(p'_a, \lambda'_a)$$

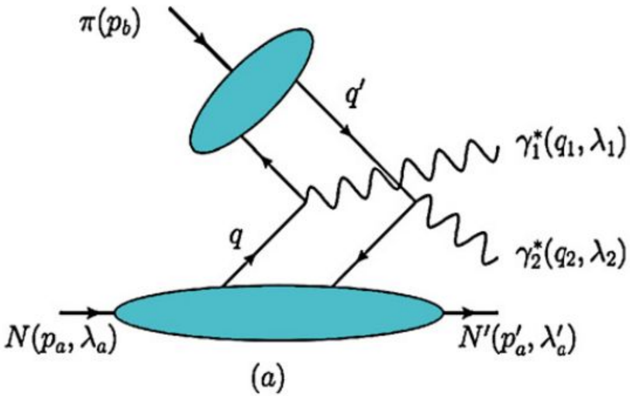


Figure 1: Exclusive DDY. Bhattacharya, S., Metz, A., & Zhou, J. (2017) Physics Letters B, 771, 396-400

Soft Collinear Effective Theory

- Light-cone coordinates:

$$p^\mu = (n\cdot p)\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\frac{n^\mu}{2} + p_\perp^\mu \equiv p_+^\mu + p_-^\mu + p_\perp^\mu \qquad \text{with} \qquad n_\mu = (1,0,0,1) \quad \text{and} \quad \bar{n}_\mu = (1,0,0,-1)\,.$$

- Dominant contributions from particles with collineal, anticollineal and soft momentum

$$\mathcal{L}(\phi) = \underbrace{\mathcal{L}(\phi_c)}_{\equiv\mathcal{L}_c} + \underbrace{\mathcal{L}(\phi_{\bar{c}})}_{\equiv\mathcal{L}_{\bar{c}}} + \underbrace{\mathcal{L}(\phi_s)}_{\equiv\mathcal{L}_s} + \mathcal{L}_{c+s}(\phi_c, \phi_{\bar{c}}, \phi_s)$$

- *Valid at small* $q_T << Q$

- Each of the dominant regions have a dedicated field

Soft Collinear Effective Theory

- Each field scales differently: **decoupling** of SCET modes

Hard	$q^\mu \sim Q(1,1,\lambda)$
Collinear	$k^\mu \sim Q(\lambda^2,1,\lambda)$
Anti-collinear	$k^\mu \sim Q(1,\lambda^2,\lambda)$
Soft	$k^\mu \sim Q(\lambda,\lambda,\lambda)$

$$\lambda = \frac{q_T}{p_a^+}$$

- After matching: only effective operator(s) joins the fields:

$$J_{\mu QCD} = \sum_{q_1} e_{q_1} \bar{\psi}\gamma^\mu\psi \longrightarrow J^\mu_{SCET} = \sum_q e_q C(Q^2/\mu^2)\bar{\chi}^q_{\bar{n}} S^{T\dagger}_{\bar{n}} \gamma^\mu S^T_n \chi^q_n$$

- Wilson Line (gauge invariance):

$$S^T_n = T_{sn} S_n$$

$$S_n(x) = P\exp\left[ig\int_{-\infty}^{0} dsn\cdot A^a_s(x+sn)t^a\right]$$

- Cross-section factorization

Semi-Exclusive DDY factorization

- Semi-Exclusive Double Drell-Yan cross-section:

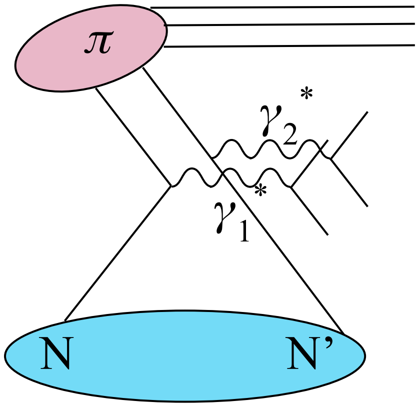
$$\frac{d\sigma}{d^4q_1 d^4q_2} \propto \sum_X \int dz_{1,2,3} e^{-iq_1 z_1 - iq_2 z_2 + iq_1 z_3} \langle \Pi N | \overline{T}\{J^{\dagger\alpha}(z_1) J^{\dagger\beta}(z_2)\} | X N' \rangle \times \langle X N' | T\{J^\mu(z_3) J^\nu(0)\} | \Pi N \rangle$$

- Matching to SCET current

- Particles assigned to sectors

$$N \to n, \Pi \to \bar{n}, X \to s, \bar{n}$$

$$\sum_X |X\rangle\langle X| = \sum_{X_{\bar{n}}} |X_{\bar{n}}\rangle\langle X_{\bar{n}}| \times \sum_{X_s} |X_s\rangle\langle X_s| = 1$$



- Decoupling of SCET sectors after fierzing:

$$\frac{d\sigma}{d^4q_1 d^4q_2} = \int d^4 z_{1,2,3} e^{-iq_1 z_1 - iq_2 z_2 + iq_1 z_3} \sum_\Gamma \sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \Phi_{DDY}(z) f_{pion}(z) f_{NN'}(0, z_3) f_{N'N}(z_1, z_2)$$

Semi-Exclusive DDY factorization

$$\frac{d\sigma}{d^4q_1 d^4q_2} = \int d^4 z_{1,2,3} e^{-iq_1 z_1 - iq_2 z_2 + iq_1 z_3} \sum_{\Gamma} \sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \Phi_{DDY}(z) f_{pion}(z) f_{NN'}(0, z_3) f_{N'N}(z_1, z_2)$$

- Multipole expansion + FTs properties lead to:

$$\frac{d\sigma}{dx_{1,2} dy_{1,2} d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma} \sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp} - i\vec{q}_{2\perp}\vec{b}_{2\perp} + i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_\perp\}) \tilde{f}_{pion}(\{y, \vec{b}_\perp\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x, \vec{b}_\perp\})$$

Factorized cross-section in impact parameter space with:

$$\tilde{f}(x; \vec{b}_\perp) = \int d^2\vec{k}_\perp e^{i\vec{k}_\perp \cdot \vec{b}_\perp} f(x; \vec{k}_\perp)$$

and

Φ_{DDY} - Soft factor with 8 Wilson lines

f_{pion} - Naive Double TMD

$f_{NN'} f_{N'N}$ - Naive Two GTMDs

SCET factorization: Soft factor

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma}\sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp}-i\vec{q}_{2\perp}\vec{b}_{2\perp}+i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_\perp\}) \tilde{f}_{pion}(\{y,\vec{b}_\perp\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x,\vec{b}_\perp\})$$

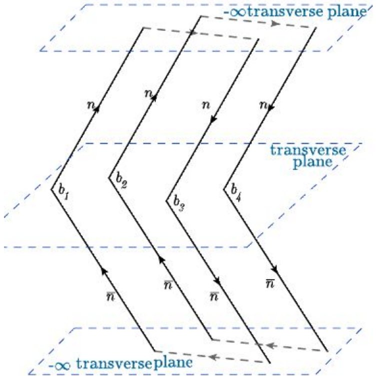


Figure 3: Double Wilson Loop Soft Factor.
Vladimirov, A. (2018) Journal of High Energy Physics, 2018(4), 1-46 [3]

- Same Soft factor than in DPS

$$\tilde{\Phi}_{DDY}(\vec{b}_{1\perp},\vec{b}_{2\perp},\vec{b}_{3\perp}) = \left\langle 0|S_n^{T\dagger}(\vec{b}_{1\perp})S_{\bar{n}}^T(\vec{b}_{1\perp})S_{\bar{n}}^{T\dagger}(\vec{b}_{2\perp})S_n^T(\vec{b}_{2\perp})S_{\bar{n}}^{T\dagger}(\vec{b}_{3\perp})S_n^T(\vec{b}_{3\perp})S_n^{T\dagger}(0)S_{\bar{n}}^T(0)|0\right\rangle$$

SCET factorization: 2TMD

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma}\sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp}-i\vec{q}_{2\perp}\vec{b}_{2\perp}+i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_\perp\}) \tilde{f}_{pion}(\{y,\vec{b}_\perp\}) \tilde{f}_{NN'} \tilde{f}_{N'N}(\{x,\vec{b}_\perp\})$$

- Pure Double TMDPDF (unsubtracted, with rapidity divergences):

Buffing, M. G., Diehl, M., & Kasemets, T. (2018). Journal of High Energy Physics, 2018(1), 1-112 [4]

$$\tilde{f}_{pion}(\{y,\vec{b}_\perp\}) = \prod_j \int \frac{dr_j^+}{2\pi} e^{-ir_j^+ y_j p_b^-} \left\langle \Pi | \bar{\chi}_{\bar{n}}(r_2^+,0^-,\vec{b}_{2\perp})\Gamma^\mu \chi_{\bar{n}}(r_1^+,0^-,\vec{b}_{1\perp})\bar{\chi}_{\bar{n}}(r_3^+,0^-,\vec{b}_{3\perp})\Gamma'^\beta \chi_{\bar{n}}(0)|\Pi\right\rangle$$

where $j = 1,\ldots,3$ and $y_3 = -y_1$

SCET factorization: GTMDs

- Factorized cross-section:

$$\frac{d\sigma}{dx_{1,2}dy_{1,2}d\vec{q}_{1,2\perp}} \propto \sum_{\Gamma}\sum_{\Gamma'} C_\Gamma C_{\Gamma'} H(Q^2/\mu^2) \int \frac{d^2\vec{b}_{1,2,3\perp}}{(2\pi)^6} e^{-i\vec{q}_{1\perp}\vec{b}_{1\perp}-i\vec{q}_{2\perp}\vec{b}_{2\perp}+i\vec{q}_{1\perp}\vec{b}_{3\perp}} \tilde{\Phi}_{DDY}(\{\vec{b}_\perp\})\tilde{f}_{pion}(\{y,\vec{b}_\perp\})\tilde{f}_{NN'}\tilde{f}_{N'N}(\{x,\vec{b}_\perp\})$$

where:

$$\tilde{f}(x;\vec{b}_\perp) = \int d^2\vec{k}_\perp e^{i\vec{k}_\perp\cdot\vec{b}_\perp} f(x;\vec{k}_\perp)$$

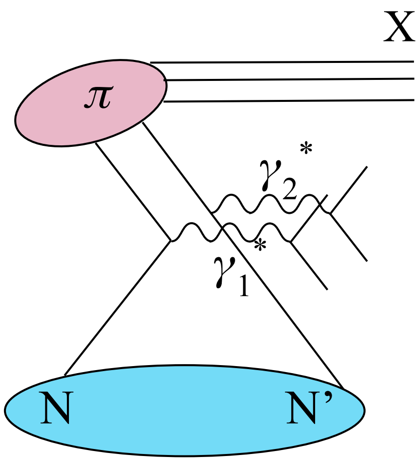
- Two pure GTMDs evaluated in different positions (unsubtracted, with rapidity divergences)

One-loop calculation in Echevarría Et. al. Physics Letters B, 759, 336-341

$$f_{pp',\lambda\lambda'}^{[\Gamma],0}(P,\Delta,x,\vec{k}_\perp) = \frac{1}{2}\int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} e^{i(z^-k^+/2-\vec{z}\cdot\vec{k}_\perp)} \langle p',\lambda'|\bar{q}W_n(-z/2)\Gamma W_n^\dagger q(z/2)|p,\lambda\rangle|_{z^+=0}$$

$P = (p + p')/2$ $\Delta = p - p'$ $k^+ = xp^+$

Color



- Consider color structure

$$f_{pion}^{[d_4,\ldots,d_1]} \propto \left\langle \Pi | \bar{\chi}_{\bar{n}}^{d_2}(r_2^+, 0^-, \vec{b}_{2\perp}) \Gamma^{\mu} \chi_{\bar{n}}^{d_1}(r_1^+, 0^-, \vec{b}_{1\perp}) \bar{\chi}_{\bar{n}}^{d_3}(r_3^+, 0^-, \vec{b}_{3\perp}) \Gamma'^{\beta} \chi_{\bar{n}}^{d_4}(0) | \Pi \right\rangle$$

$$f_{NN'} f_{N'N}^{[a_4,\ldots,a_1]} \propto \left\langle N | \bar{\chi}_{n}^{a_1}(0^+, r_1^-, \vec{b}_{1\perp}) \Gamma^{\alpha} \chi_{n}^{a_2}(0^+, r_2^-, \vec{b}_{2\perp}) | N' \right\rangle \left\langle N' | \bar{\chi}_{n}^{a_4}(0) \Gamma'^{\nu} \chi_{n}^{a_3}(0^+, r_3^-, \vec{b}_{3\perp}) | N \right\rangle$$

$$\Phi_{DDY}^{a_1,\ldots,a_4,b_4,\ldots,b_1}(\{b\}) = \left\langle 0 | S_n^{T\dagger a_1} S_{\bar{n}}^{T d_1}(\vec{b}_{1\perp}) S_{\bar{n}}^{T\dagger d_2} S_n^{T a_2}(\vec{b}_{2\perp}) S_{\bar{n}}^{T\dagger d_3} S_n^{T a_3}(\vec{b}_{3\perp}) S_n^{T\dagger a_4} S_{\bar{n}}^{T d_4}(0) | 0 \right\rangle$$

Color

- Obtain valid color structures using projectors

- Cross-section:

$$d\sigma \propto \left(f^{[1]}_{pion}(\{y,\vec{b}_\perp\}), f^{[8]}_{pion}(\{y,\vec{b}_\perp\}) \right) \times \begin{pmatrix} \Phi_{11}(\{b\}) & \Phi_{18}(\{b\}) \\ \Phi_{81}(\{b\}) & \Phi_{88}(\{b\}) \end{pmatrix} \times \begin{pmatrix} f^{[1]}_{NN'} f^{[1]}_{N'N}(\{x,\vec{b}_\perp\}) \\ 0 \end{pmatrix}$$

- Singlet term:

$$d\sigma^{[1,11,1]} \propto f^{[1]}_{pion}(\{y,\vec{b}_\perp\})\Phi_{11}(\{b\}) f^{[1]}_{NN'} f^{[1]}_{N'N}(\{x,\vec{b}_\perp\})$$

- Up until now: un-subtracted terms

Zero bin subtraction

$$d\sigma^{[1,11,1]} \propto f^{[1]}_{pion}(\{y, \vec{b}_\perp\})\Phi_{11}(\{b\})f^{[1]}_{NN'}f^{[1]}_{N'N}(\{x, \vec{b}_\perp\})$$

- Remove overlapping between regions

- Anti-collinear term without overlap with soft region (pure):

$$f_{pion}(r_l^+, 0^-, \vec{b}_{j\perp}) = \frac{\hat{f}_{pion}(r_l^+, 0^-, \vec{b}_{j\perp})}{\Phi_{11}(0^+, 0^-, \vec{b}_{j\perp})}$$

- Collinear term without overlap with soft region (pure):

$$f_{NN'}(r_1^-, \vec{b}_{1\perp}, r_2^-, \vec{b}_{2\perp})f_{N'N}(r_3^-, \vec{b}_{3\perp}) = \frac{\hat{f}_{NN'}(r_1^-, \vec{b}_{1\perp}, r_2^-, \vec{b}_{2\perp})\hat{f}_{N'N}(r_3^-, \vec{b}_{3\perp})}{\Phi_{DY}(\vec{b}_{1\perp}, \vec{b}_{2\perp})\Phi_{DY}(\vec{b}_{3\perp})}$$

where:

$$\Phi_{DY}(b_i, b_j) = \left\langle 0|S_n^{T\dagger}(b_i)S_{\bar{n}}^T(b_i)S_{\bar{n}}^{T\dagger}(b_j)S_n^T(b_j)|0\right\rangle$$

Subtraction of rapidity divergences

$$d\sigma^{[1,11,1]} \propto f^{[1]}_{pion}(\{y,\vec{b}_\perp\})\Phi_{11}(\{b\})f^{[1]}_{NN'}f^{[1]}_{N'N}(\{x,\vec{b}_\perp\})$$

- Color summed cross-section term with **subtracted** functions:

$$d\sigma^{[1,11,1]}_{DDY}{}_{sub}(\{b\}) \propto H(Q^2/\mu^2)\frac{\hat{f}^{[1]}_{pion}}{\Phi_{11}}\Phi_{11}(\{b\})\frac{\hat{f}^{[1]}_{NN'}\hat{f}^{[1]}_{N'N}(\{x,\vec{b}_\perp\})}{\Phi_{DY}(\vec{b}_{1\perp},\vec{b}_{2\perp})\Phi_{DY}(0,\vec{b}_{3\perp})} =$$

$$= H(Q^2/\mu^2)\frac{\hat{f}^{[1]}_{pion}}{\sqrt{\Phi_{11}}}\frac{\sqrt{\Phi_{11}}}{\sqrt{\Phi_{DY}(\vec{b}_{1\perp},\vec{b}_{2\perp})}\sqrt{\Phi_{DY}(0,\vec{b}_{3\perp})}}\frac{\hat{f}^{[1]}_{NN'}\hat{f}^{[1]}_{N'N}(\{x,\vec{b}_\perp\})}{\sqrt{\Phi_{DY}(\vec{b}_{1\perp},\vec{b}_{2\perp})}\sqrt{\Phi_{DY}(0,\vec{b}_{3\perp})}} =$$

$$= H(Q^2/\mu^2) \times 2TMD \times \Phi_{New} \times GTMD * GTMD'$$

- New ratio of soft factors term

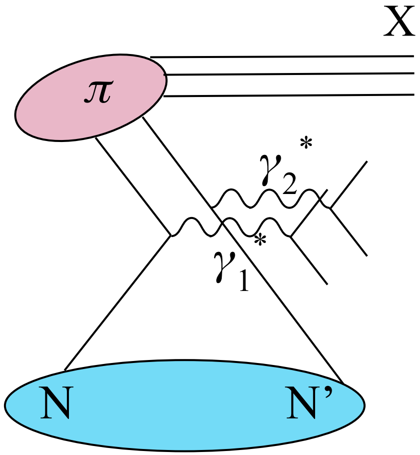
Conclusions

- First factorization of Exclusive DDY cross-section into functions with different scales

- Exclusive DDY gives access to GTMDs (+ DTMD!)

- New ratio of soft factors not present before: important for pheno!

- Future work:
  - Keep studying color structures
  - Check explicitly cancellation of rapidity divergences
  - Polarizations
  - ...



THANK YOU FOR YOUR ATTENTION!

Extra slides

Color

- Consider DPS color structure

- DPS projectors to get singlet states:

$$I_1 = \frac{\delta_{a_1 a_4}\delta_{a_2 a_3}}{N_c^2} \qquad\qquad I_8 = \frac{2t^A_{a_1 a_4} t^A_{a_2 a_3}}{N_c\sqrt{N_c^2 - 1}}$$

- Soft factor:

$$\Phi_{MN}(\{b\}) = I_M \times \Phi_{DDY}^{[a_1,\ldots,a,4][b_1,\ldots,b_4]}(\{b\}) \times I_N$$

with:

$$\Phi_{DDY}^{a_1,\ldots,a_4,b_4,\ldots,b_1}(\{b\}) = \left\langle 0 | S_n^{T\dagger a_1} S_{\bar{n}}^{T d_1}(\vec{b}_{1\perp}) S_{\bar{n}}^{T\dagger d_2} S_n^{T a_2}(\vec{b}_{2\perp}) S_{\bar{n}}^{T\dagger d_3} S_n^{T a_3}(\vec{b}_{3\perp}) S_n^{T\dagger a_4} S_{\bar{n}}^{T d_4}(0) | 0 \right\rangle$$

$$\Rightarrow \qquad \Phi_{DDY}(\{b\}) = \begin{pmatrix} \Phi_{11}(\{b\}) & \Phi_{18}(\{b\}) \\ \Phi_{81}(\{b\}) & \Phi_{88}(\{b\}) \end{pmatrix}$$