

# Accessing $\pi$ GPDs through the Sullivan process: is it feasible?

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**Universidad  
de Huelva**

# Introduction

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# Introduction: GPDs and hadron's structure

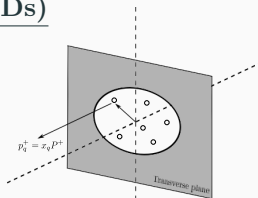
**Question:** *How can we gain insights into hadron's structure?*

## Generalised parton distributions (GPDs)

### **Probabilistic interpretation:**

probability amplitude of finding a parton at a given position in transverse plane carrying a momentum fraction “ $x$ ” of the hadron's averaged light-cone momentum.

[M.Burkardt-PRD:071503(62)2020]



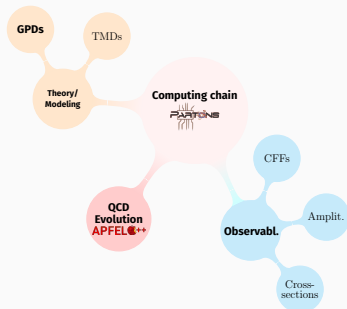
“3D picture” of hadrons

### **Properties:**

1. **Parametrize DVCS amplitudes through CFFs.**  
[X.Ji-PRL:610(78)1997]
2. PDFs as forward limit.
3. Electromagnetic and gravitational FFs as Mellin moments.

# Introduction

1. Generalised parton distributions:
  - “3D picture” of hadrons.
  - EMT gravitational form factors.
2. Pions: DCSB Nambu-Goldstone bosons:
  - Clear window onto emergence of hadronic mass.



## Two main questions guide this talk:

- Can we build “theoretically-complete” pion GPD models?
- Can we probe them in experiment?

Pion GPDs through Sullivan process.

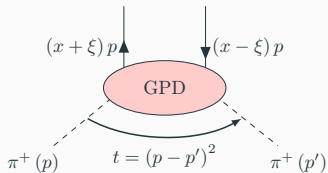
[D.Amrath et al.-EPJC:179(58)2008]

## **GPD modelling**

Can we build “theoretically-complete” pion GPD models?

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# GPD modelling: definition and properties



$x$ : Momentum fraction of  $p$ .

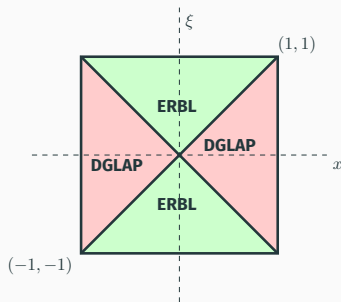
$\xi$ : Fraction of momentum longitudinally transferred.

$t$ : Momentum transfer.

## Kinematics:

[M. Diehl-Phys.Rept:41(388)2003]

- **DGLAP** ( $|x| > |\xi|$ ):  
Emits/takes a quark ( $x > 0$ )  
or antiquark ( $x < 0$ ).
- **ERBL**: ( $|x| < |\xi|$ ):  
Emits pair quark-antiquark.



# GPD modelling: definition and properties

- **Support:**

[M.Diehl et al.-PLB:359(428)1998]

$$(x, \xi) \in [-1, 1] \otimes [-1, 1]$$

- **Polynomiality:** Order- $m$  Mellin moments are degree- $(m + 1)$  polynomials in  $\xi$ .

[X.Ji-JPG:1181(24)1998, A.Radyushkin-PLB:81(449)1999]

$$\int_{-1}^1 dx x^m H(x, \xi, t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m+1} c_k^{(m)}(t) \xi^k$$

**Lorentz invariance**

- **Positivity:**

[P.V.Pobylitsa-PRD:114015(65)2002, B.Pire et al.-EPJC:103(8)1999]

$$|H^q(x, \xi, t=0)| \leq \sqrt{q \left( \frac{x+\xi}{1+\xi} \right) q \left( \frac{x-\xi}{1-\xi} \right)}, \quad |x| \geq \xi$$

**Positivity of Hilbert space norm**

- **Low energy soft-pion theorem**

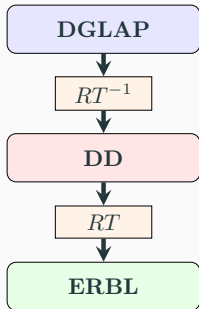
[M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015]

**PCAC/Axial-Vector WTI**

## GPD modelling: covariant extension

**Covariant extension:** given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied. [N.Chouika et al.-EPJC:906(77)2017]

$$H(x, \xi, t) = \mathcal{R} [h(\beta, \alpha, t)] + \frac{1}{|\xi|} D^+ \left( \frac{x}{\xi}, t \right) + \text{sign}(\xi) D^- \left( \frac{x}{\xi}, t \right)$$



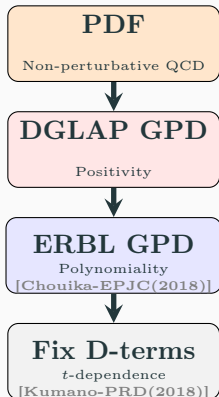
1. Build positive DGLAP GPD  $\Rightarrow$   
**How?**
2. Covariant extension: ERBL GPD
3. Soft pion theorem: fix  $D^\pm(\alpha, 0)$

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Pob.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓



# GPD modelling: from separable LFWFs to positive DGLAP GPDs

**Question:** *How can we build a positive DGLAP GPD?*



1. Overlap representation [M.Diehl-NPB:33(569)2001]

$$H^q(x, \xi, t)|_{|x| \geq \xi} = \int \frac{d^2 k_{\perp}}{16\pi^3} \Psi^{q*}(x_-, k_{\perp}^2, -) \Psi^q(x_+, k_{\perp}^2, +)$$

2. Assume factorisation of the LFWF

[J.-L.Zhang et al.-PLB:136158(815)2021]

$$\Psi^q(x, k_{\perp}^2) \propto \varphi(x) \phi(k_{\perp}^2)$$

↓ (Overlap rep.)

$$H^q(x, \xi, t)|_{|x| \geq \xi} = \sqrt{q \left( \frac{x - \xi}{1 - \xi} \right) q \left( \frac{x + \xi}{1 + \xi} \right)} \Phi(x, \xi, t)$$

↓ ( $t = 0$ )

$$H^q(x, \xi, 0)|_{|x| \geq \xi} = \sqrt{q \left( \frac{x - \xi}{1 - \xi} \right) q \left( \frac{x + \xi}{1 + \xi} \right)}$$

**Positivity saturated**

## Pion GPDs

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# Pion GPDs: Positive DGLAP GPDs

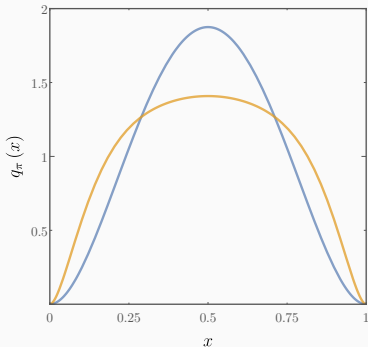
1. Under certain PTIR, chiral symmetry allows to factorize LFWF:

[J.-L.Zhang et al.-PLB:136158(815)2021]

$$\Psi_{\pi}^{\lambda_1 \lambda_2}(x, k_{\perp}^2) = \sqrt{q_{\pi}(x)} \frac{i^{\lambda_1 \lambda_2} M^2}{(k_{\perp}^2 + M^2)^2}$$

2. Pion GPD saturating positivity

$$H_{\pi}^q(x, \xi, t)|_{\text{DGLAP}} = \frac{\sqrt{q_{\pi}(x_-) q_{\pi}(x_+)}}{(1+z^2)^2} \left[ 3 + \frac{1-2z}{1+z} \frac{\text{arctanh}\left(\sqrt{\frac{z}{1+z}}\right)}{\sqrt{\frac{z}{1+z}}} \right]$$
$$z = -t(1-x)^2 / 4M^2(1-\xi^2)$$



**Two models:**

- Algebraic model

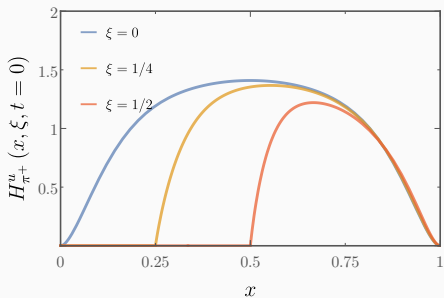
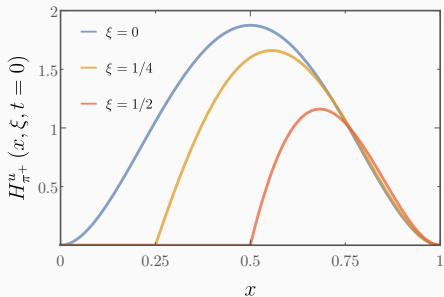
$$q_{\pi}(x) = 30x^2(1-x)^2$$

- Realistic model (DSE)

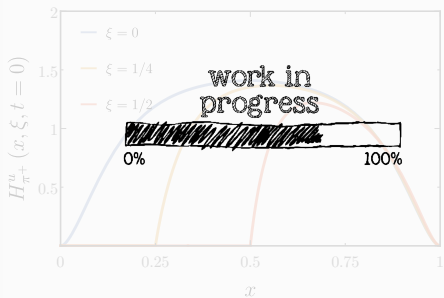
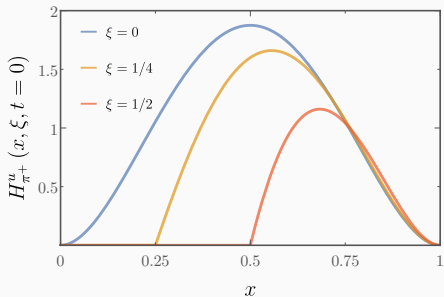
[M.Ding et al.-PRD:054014(101)2020]

$$q_{\pi}(x) = \mathcal{N}_q x^2(1-x)^2 \times \left[ 1 + \gamma x(1-x) + \rho \sqrt{x(1-x)} \right]$$

## Pion GPDs: Positive DGLAP GPDs



## Pion GPDs: Positive DGLAP GPDs



# Pion GPDs: covariant extension

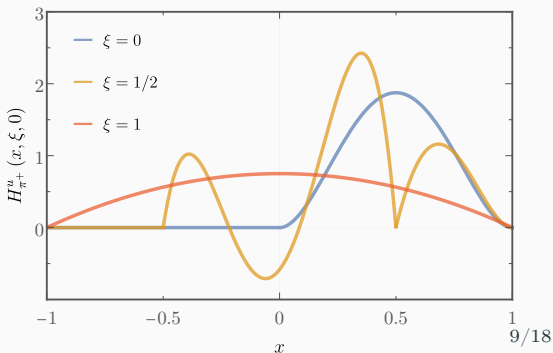
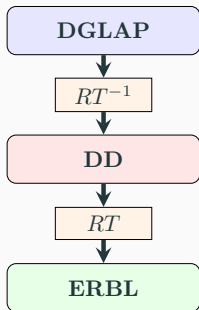
Covariant extension:

$$H^q(x, \xi, t) = \mathcal{R}[h(\beta, \alpha, t)] + \frac{1}{|\xi|} D^+ \left( \frac{x}{\xi}, t \right) + \text{sgn}(\xi) D^- \left( \frac{x}{\xi}, t \right)$$

Fix D-terms with soft pion theorem:

[M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015]

$$H_{\pi^+}^{I=0}(x, \xi, t) \Big|_{\xi=1, t=0} = H_{\pi^+}(x, \xi, t) - H_{\pi^+}(-x, \xi, t) \Big|_{\xi=1, t=0} = 0$$
$$H_{\pi^+}^{I=1}(x, \xi, t) \Big|_{\xi=1, t=0} = H_{\pi^+}(x, \xi, t) + H_{\pi^+}(-x, \xi, t) \Big|_{\xi=1, t=0} = \varphi\left(\frac{1+x}{2}\right)$$



# Phenomenology of pion GPDs

Can we probe them in experiment?

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# Phenomenology of pion GPDs: Sullivan process

We have established a way of building pion GPD models **fulfilling all of the QCD theoretical constraints**, so...

**Question:** *Can we probe those pion GPDs through experiment?*

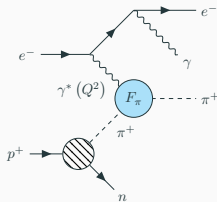
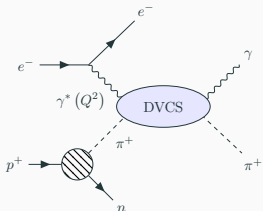
DVCS amplitudes are parametrized by hadron GPDs.

[X.Ji-PRD:7114(55)1997]

Sullivan process [J.D.Sullivan-PRD:1732(5)1972]

One pion exchange approximation: [D.Amrath et al.-EPJC:179(58)2008]

- $-t < 0, 6 \text{ GeV}^2$
  - $\sigma_L \gg \sigma_{\perp}$
- } Met at EIC [EICYR:phys.ins-det/2103.05419]



Employed for EFFs.

[G.M.Huber et al.-  
PRC:045203(78)2008]

Can we probe  
pion GPDs?

[D.Amrath et al.-  
EPJC:179(58)2008]



# Phenomenology of pion GPDs: Sullivan process

In fact... this has been advocated in the recent EIC-Yellow report

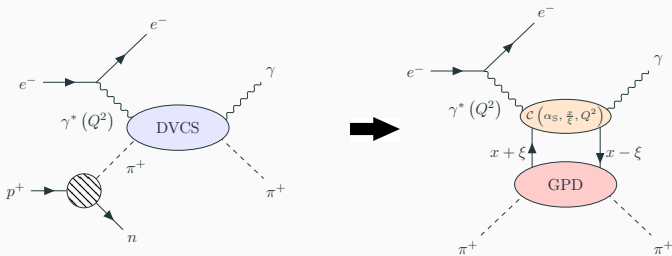
[EICYR:phys.ins-det/2103.05419]

Science Question	Key Measurement	Key Requirements
What are the quark and gluon energy contributions to the pion mass?	Pion structure function data over a range of $x$ and $Q^2$ .	<ul style="list-style-type: none"> <li>Need to uniquely determine <math>e + p \rightarrow e' + X + n</math> (low <math>-t</math>)</li> <li>CM energy range <math>\sim 10</math>-100 GeV</li> <li>Charged and neutral currents desirable</li> </ul>
Is the pion full or empty of gluons as viewed at large $Q^2$ ?	Pion structure function data at large $Q^2$ .	<ul style="list-style-type: none"> <li>CM energy <math>\sim 100</math> GeV</li> <li>Inclusive and open-charm detection</li> </ul>
What are the quark and gluon energy contributions to the kaon mass?	Kaon structure function data over a range of $x$ and $Q^2$ .	<ul style="list-style-type: none"> <li>Need to uniquely determine <math>e + p \rightarrow e' + X + \Lambda/\Sigma^0</math> (low <math>-t</math>)</li> <li>CM energy range <math>\sim 10</math>-100 GeV</li> </ul>
Are there more or less gluons in kaons than in pions as viewed at large $Q^2$ ?	Kaon structure function data at large $Q^2$ .	<ul style="list-style-type: none"> <li>CM energy <math>\sim 100</math> GeV</li> <li>Inclusive and open-charm detection</li> </ul>
Can we get quantitative guidance on the emergent pion mass mechanism?	Pion form factor data for $Q^2 = 10$ -40 (GeV/c) $^2$ .	<ul style="list-style-type: none"> <li>Need to uniquely determine exclusive process <math>e + p \rightarrow e' + \pi^+ + n</math> (low <math>-t</math>)</li> <li><math>e + p</math> and <math>e + D</math> at similar energies</li> <li>CM energy <math>\sim 10</math>-75 GeV</li> </ul>
What is the size and range of interference between emergent-mass and the Higgs-mass mechanism?	Kaon form factor data for $Q^2 = 10$ -20 (GeV/c) $^2$ .	<ul style="list-style-type: none"> <li>Need to uniquely determine exclusive process <math>e + p \rightarrow e' + K + \Lambda</math> (low <math>-t</math>)</li> <li>L/T separation at CM energy <math>\sim 10</math>-20 GeV</li> <li><math>\Lambda/\Sigma^0</math> ratios at CM energy <math>\sim 10</math>-50 GeV</li> </ul>
What is the difference between the impacts of emergent- and Higgs-mass mechanisms on light-quark behavior?	Behavior of (valence) up quarks in pion and kaon at large $x$ .	<ul style="list-style-type: none"> <li>CM energy <math>\sim 20</math> GeV (lowest CM energy to access large-<math>x</math> region)</li> <li>Higher CM energy for range in <math>Q^2</math> desirable</li> </ul>
What is the relationship between dynamically chiral symmetry breaking and confinement?	Transverse-momentum dependent Fragmentation Functions of quarks into pions and kaons.	<ul style="list-style-type: none"> <li>Collider kinematics desirable (as compared to fixed-target kinematics)</li> <li>CM energy range <math>\sim 20</math>-140 GeV</li> </ul>
<b>More speculative observables</b>		
What is the trace anomaly contribution to the pion mass?	Elastic $J/\Psi$ production at low $W$ off the pion.	<ul style="list-style-type: none"> <li>Need to uniquely determine exclusive process <math>e + p \rightarrow e' + J/\Psi + \pi^+ + n</math> (low <math>-t</math>)</li> <li>High luminosity (<math>\geq 10^{34}</math> cm<math>^{-2}</math> sec<math>^{-1}</math>)</li> <li>CM energy <math>\sim 70</math> GeV</li> </ul>
Can we obtain tomographic snapshots of the pion in the transverse plane? What is the pressure distribution in a pion?	Measurement of DVCS off pion target as defined with Sullivan process.	<ul style="list-style-type: none"> <li>Need to uniquely determine exclusive process <math>e + p \rightarrow e' + \gamma + \pi^+ + n</math> (low <math>-t</math>)</li> <li>High luminosity (<math>\geq 10^{34}</math> cm<math>^{-2}</math> sec<math>^{-1}</math>)</li> <li>CM energy <math>\sim 10</math>-100 GeV</li> </ul>
Are transverse momentum distributions universal in pions and protons?	Hadron multiplicities in SIDIS off a pion target as defined with Sullivan process.	<ul style="list-style-type: none"> <li>Need to uniquely determine SIDIS off pion <math>e + p \rightarrow e' + h + X + n</math> (low <math>-t</math>)</li> <li>High luminosity (<math>10^{34}</math> cm<math>^{-2}</math> sec<math>^{-1}</math>)</li> <li><math>e + p</math> and <math>e + D</math> at similar energies desirable</li> <li>CM energy <math>\sim 10</math>-100 GeV</li> </ul>

Let us see if that would be feasible in a future electron-ion collider.

# Phenomenology of pion GPDs:

**Goal:** Employ our GPD models to analyse DVCS contribution the Sullivan process in the one-pion exchange approximation.



## Phenomenology

1.  $\sigma$ -DVCS

[D.Amrath et al.-EPJC(2008)]

2. Asymmetry

3. ...

## QCD Evolution

**APFEL++**

[V.Bertone et al.]  
-CPCComm(2014),

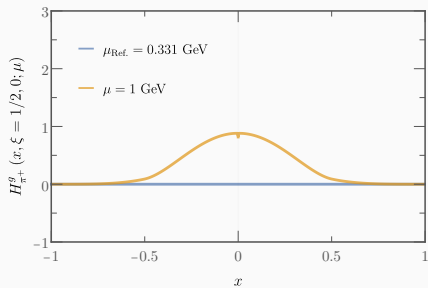
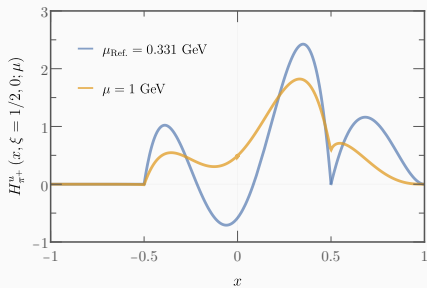
[V.Bertone et al.:  
hep-ph/1708.00911]

## LO/NLO CFFs

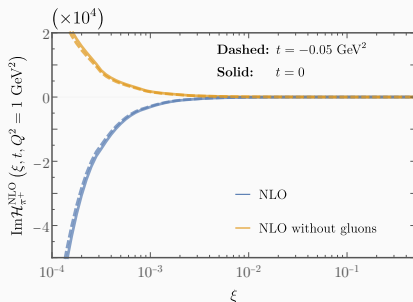
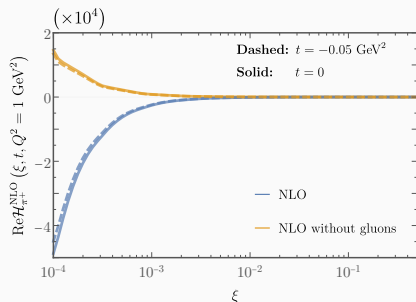
**PARTONS**

[B.Berthou et al.  
-EPJC(2018)]

# Phenomenology of pion GPDs: QCD evolution



# Phenomenology of pion GPDs: Compton Form Factors

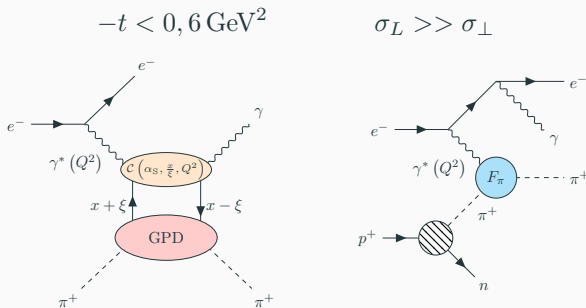


Gluon dominance makes essential at least NLO accuracy in any phenomenological analysis of DVCS at an EIC.

# Phenomenology of pion GPDs: DVCS and Sullivan process

Can we measure DVCS?

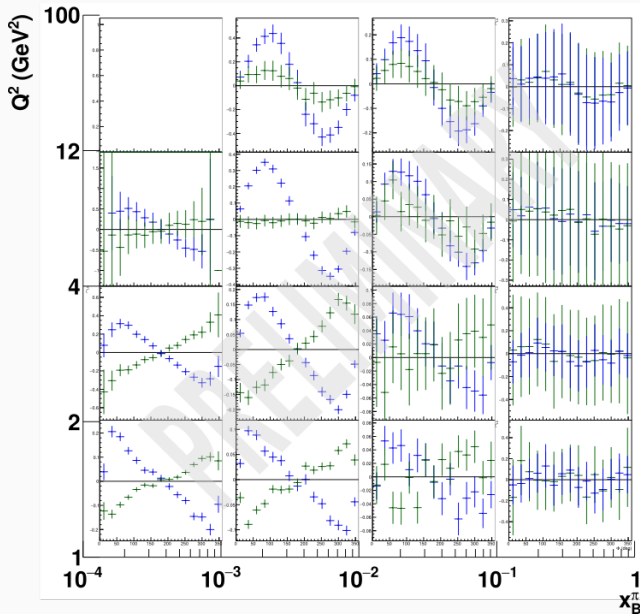
One pion exchange approximation: [D.Amrath et al.-EPJC:179(58)2008]



Changing lepton polarisation one can (formally) access interference between DVCS and BH amplitudes.

*Is it experimentally feasible?*

# Phenomenology of pion GPDs: Asymmetry (EIC)



Legend:

L-beam: 18 GeV

H-beam: 275 GeV

Blue: LO

Green: NLO

**Non-zero  
asymmetry:**

optimism about  
measuring DVCS  
on pions at future  
EIC.

## **Summary and perspectives**

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# Summary and perspectives

## Summary

1. Pion GPD models fulfilling every theoretical constraint
  - Polynomiality: Covariant extension.
  - Positivity
  - PCAC/AV-WTI: Soft pion theorem.
  - Agreement with experimental data for EFFs and GFFs.
2. PARTONS implements complete computing chain
  - From GPDs to DVCS CFFs
  - From DVCS CFFs to observables
3. DVCS on virtual pions influenced by gluon content
  - Higher order analysis needed for phenomenology.
4. Pion structure to be tested at future electron-ion colliders
  - Insights into EHM could be gained experimentally.



## Perspectives

*Currently at work*

1. Exploit realistic and phenomenological pion PDF
2. Extension to EicC.
3. Comparison with previous studies
  - Original paper by D. Amrath, M. Diehl and J. P. Lansberg  
[D.Amrath et al.-EPJC:179(58)2008]

*Forthcoming developments*

4. Extension of the computing chain
  - Higher order analysis
  - Baryons

**Thank you!**