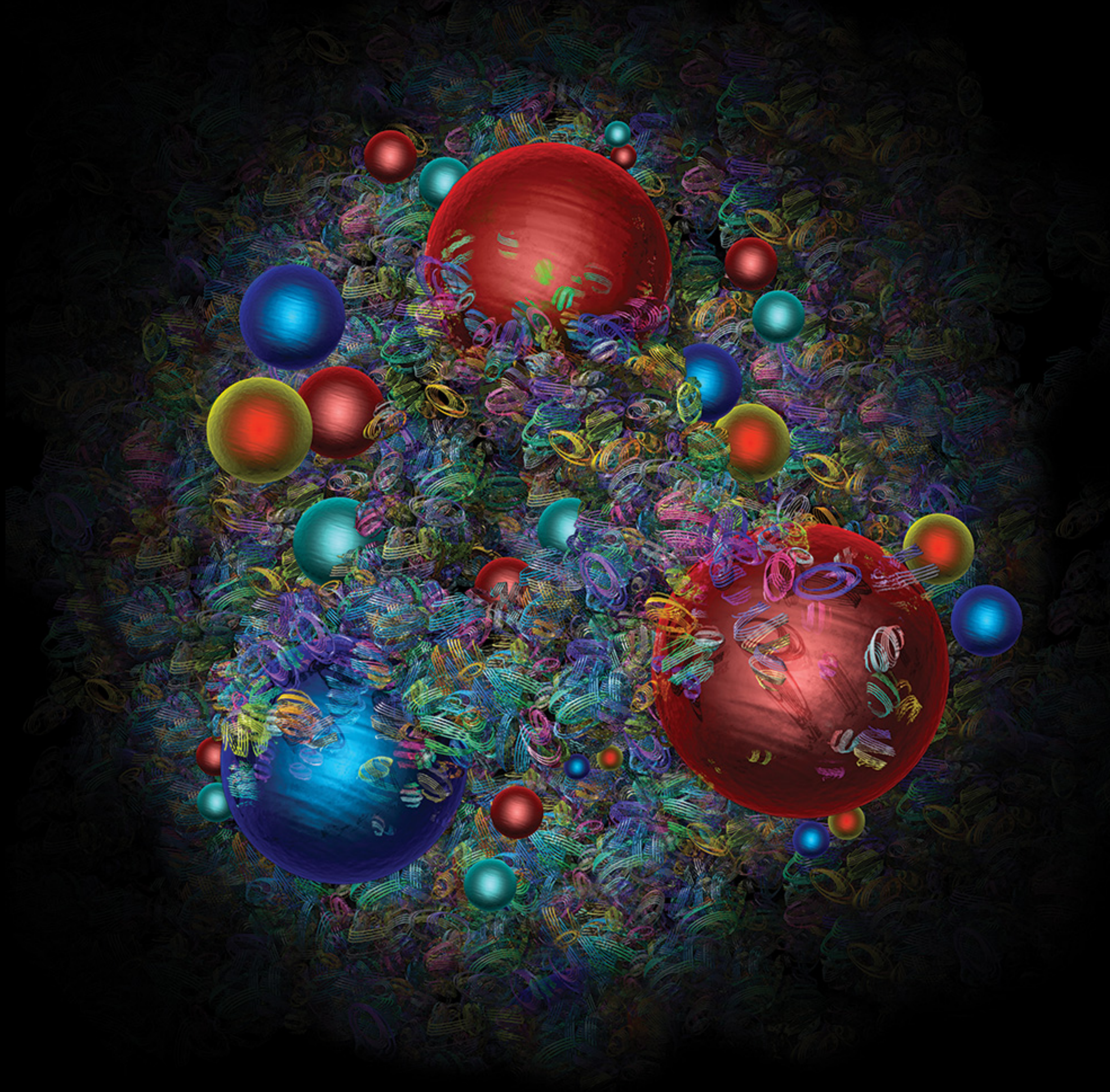
Hadron structure at small-x via unintegrated gluon densities

Michael Fucilla

Università della Calabria & INFN-gruppo collegato di Cosenza



HADRON 2021

Outline

- BFKL and unintegrated gluon densities

- Exclusive forward meson leptoproduction

- Conclusions and outlook

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- **BFKL and unintegrated gluon densities**

- Exclusive forward meson leptoproduction

- Conclusions and outlook

High-energy factorization and the UGD

Balitsky-Fadin-Kuraev-Lipatov (BFKL) resummation

* Leading-Logarithm-Approximation (LLA): $(\alpha_s \ln s)^n$
* Next-to-Leading-Logarithm-Approximation (NLLA): $\alpha_s(\alpha_s \ln s)^n$

Unintegrated gluon densities

* Definition $\mathscr{F}(x, \overrightarrow{k}),\qquad f_g(x, Q^2) = \int \frac{d^2 \overrightarrow{k}}{\pi \overrightarrow{k}^2} \mathscr{F}(x, \overrightarrow{k}) \theta(Q^2 - \overrightarrow{k}^2)$
* Evolution equation as a function of $\ln(s/Q^2) = \ln(1/x)$

$$\frac{\partial \mathscr{F}}{\partial \ln(1/x)} = \mathscr{F} \otimes \mathscr{K}$$

High-energy factorization and the UGD

Deep inelastic scattering

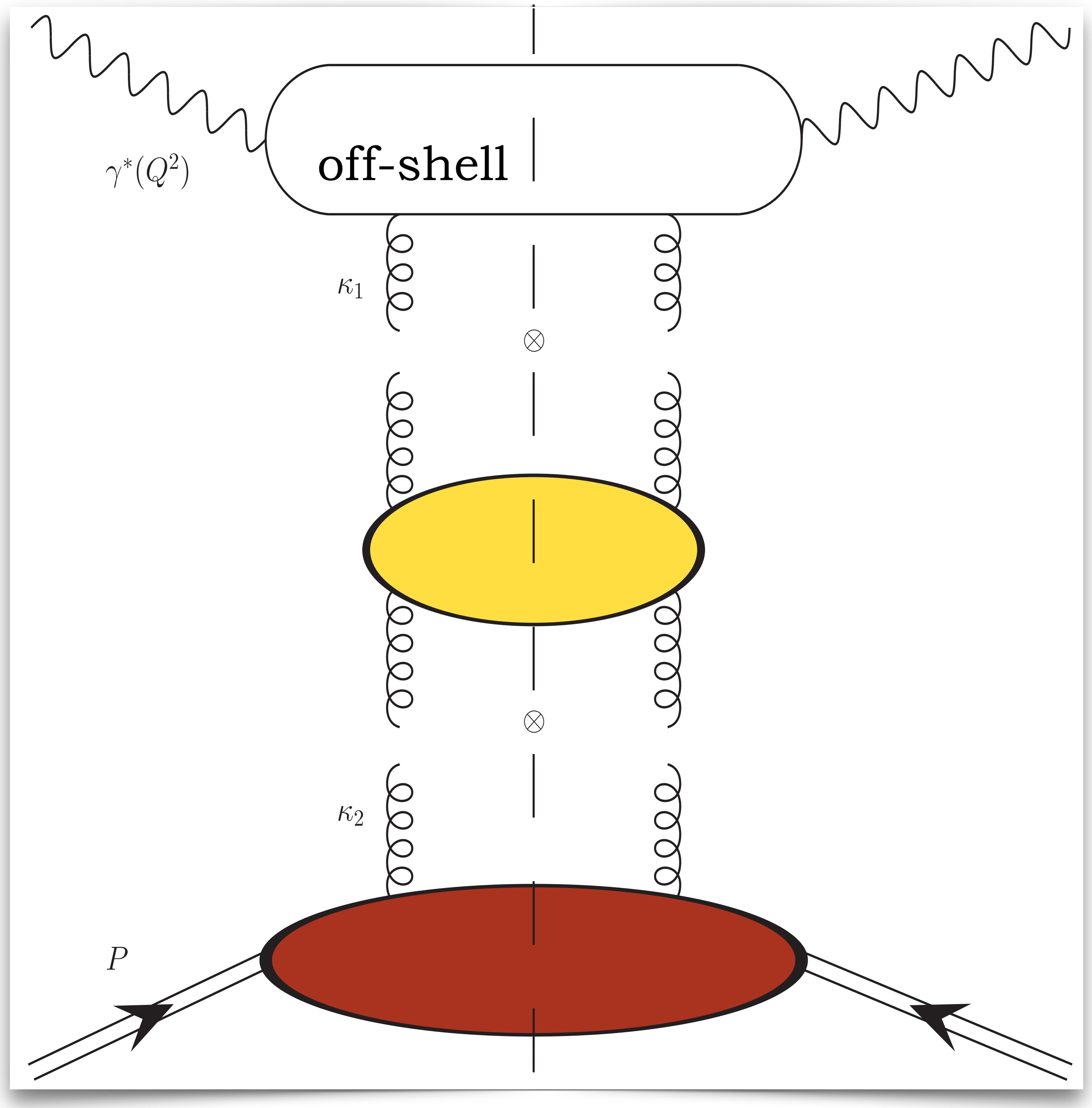
* **Total cross section**

$$\sigma_\lambda(x, Q^2) = \frac{\mathcal{G}}{(2\pi)^4} \int \frac{d^2\vec{k}_1}{\vec{k}_1^{\,2}} \int \frac{d^2\vec{k}_2}{\vec{k}_2^{\,2}} \Phi_\lambda(\vec{k}_1) F(x, \vec{k}_1, \vec{k}_2) \Phi_p(\vec{k}_2)$$

$$F(x, \vec{k}_1, \vec{k}_2) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} d\nu \left(\frac{\vec{k}_1^{\,2}}{\vec{k}_2^{\,2}}\right)^{i\nu} \frac{e^{in(\theta_1-\theta_2)}}{2\pi|\vec{k}_1||\vec{k}_2|} e^{\bar{\alpha}_s \chi_n(\nu) \ln\left(\frac{1}{x}\right)}$$

* **Growth at small-x**

$$F \sim \frac{x^{-\omega_0}}{\sqrt{\ln(1/x)}} \qquad \omega_0 = 4\bar{\alpha}_s \ln 2$$



$\Phi^{\gamma^* \to \gamma^*}$

$\otimes$

$\mathcal{G}_{\text{BFKL}}$

$\otimes$

$\Phi^{P}_{[\text{NP}]}$

High-energy factorization and the UGD

IR-safe colorless $\{\Phi^{\mathrm{i}\to\mathrm{o}}\}$

(Fadin-Martin theorem)

[V.S. Fadin, A.D. Martin (1999)]

Deep inelastic scattering

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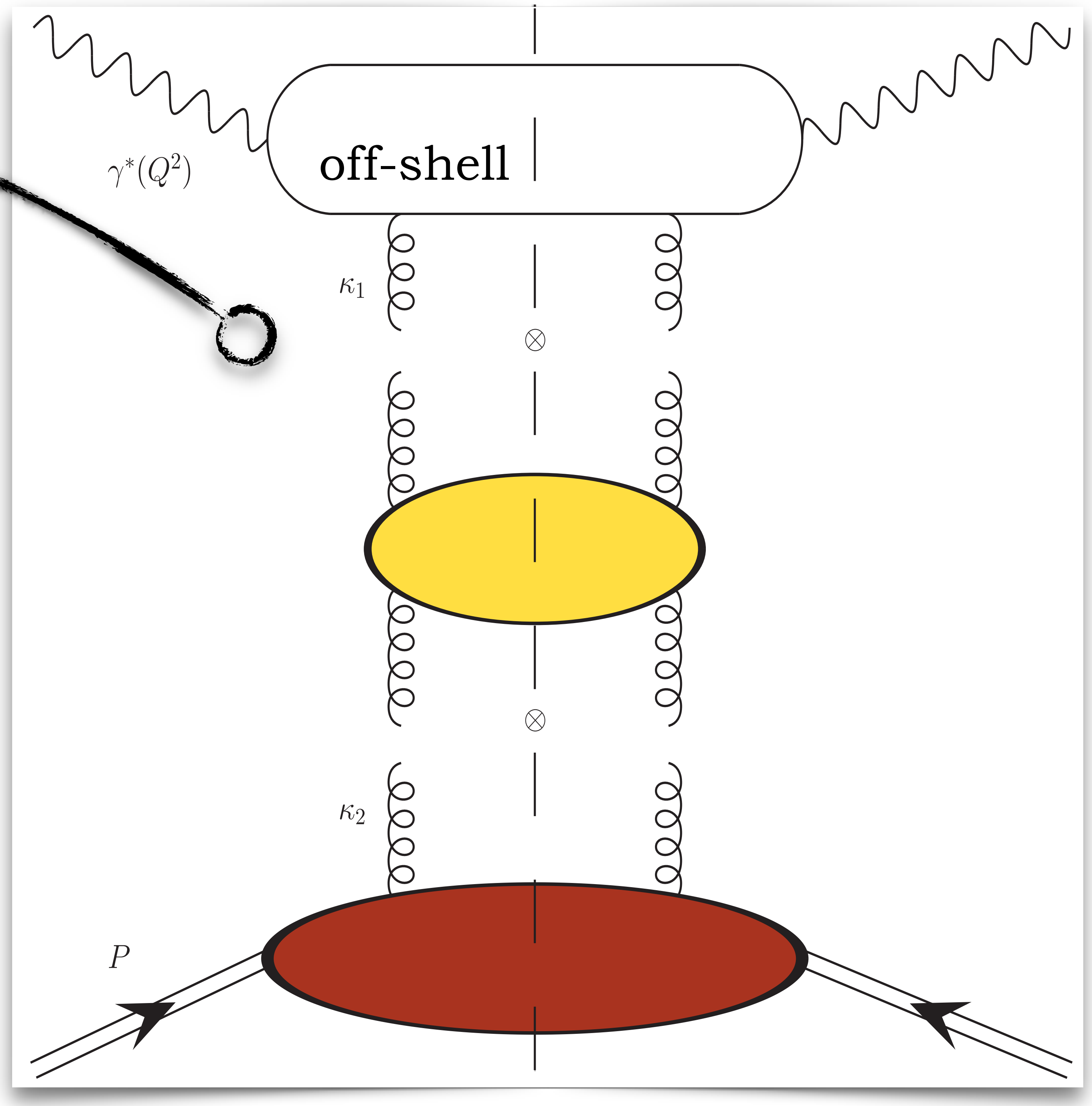

$$\Phi^{\gamma^* \to \gamma^*}$$

$$\otimes$$

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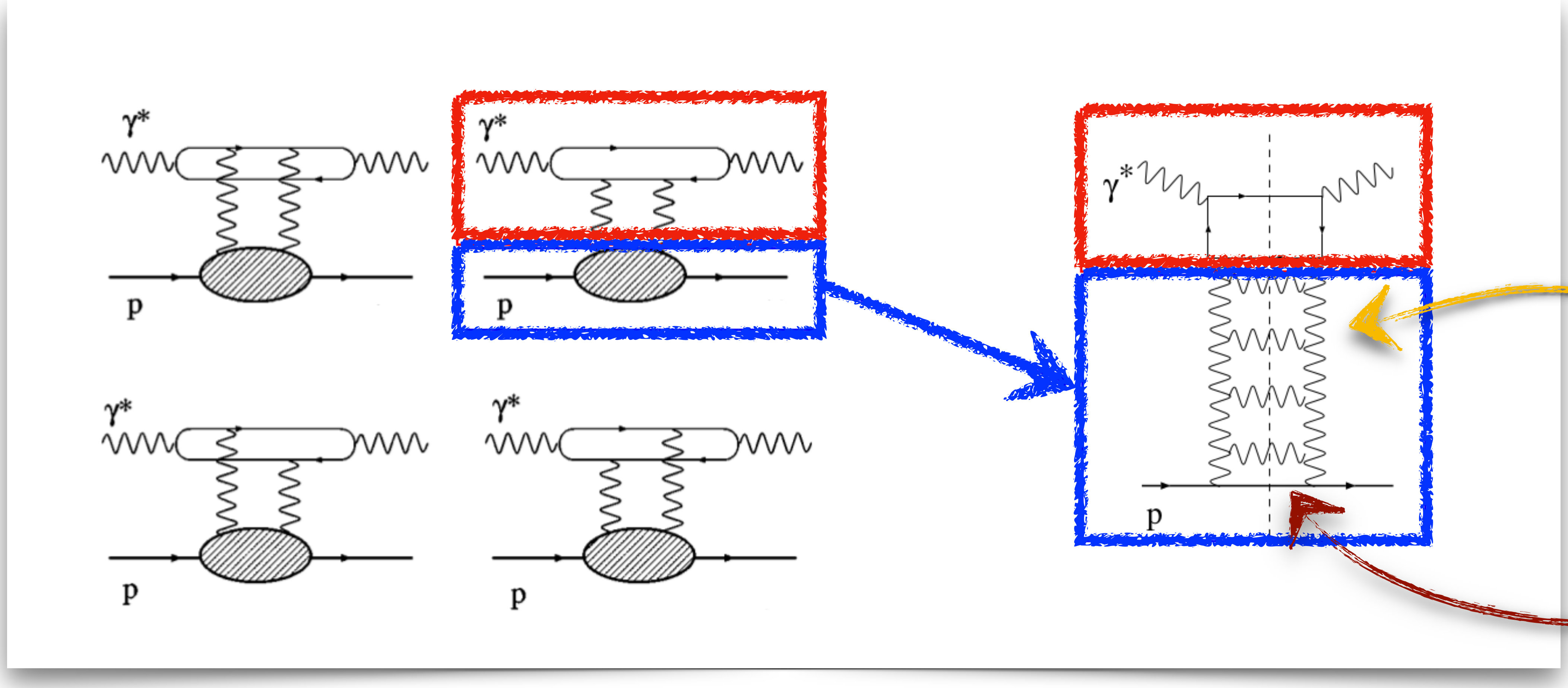
High-energy factorization and the UGD

Unintegrated gluon density

$$\sigma_\lambda(x, Q^2) = \frac{\mathcal{G}}{(2\pi)^4} \int \frac{d^2\vec{k}_1}{\vec{k}_1^2} \int \frac{d^2\vec{k}_2}{\vec{k}_2^2} \Phi_\lambda(\vec{k}_1) F(x, \vec{k}_1, \vec{k}_2) \Phi_p(\vec{k}_2) = \frac{\mathcal{G}}{(2\pi)^4} \int \frac{d^2\vec{k}_1}{\vec{k}_1^4} \Phi_\lambda(\vec{k}_1) \mathcal{F}(x, \vec{k}_1)$$

$$\mathcal{F}(x, \vec{k}_1) \equiv \frac{\vec{k}_1^2}{(2\pi)^3} \int \frac{d^2\vec{k}_2}{\vec{k}_2^2} \Phi_p(\vec{k}_2) F(x, \vec{k}_1, \vec{k}_2)$$



- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\rm tot}(\gamma^* p \to X) \propto \mathcal{I}m_s\{\mathcal{A}(\gamma^* p \to \gamma^* p)\} \equiv \Phi_{\gamma^* \to \gamma^*} \circledast \mathcal{F}(x, \kappa^2)$$

◇ $\mathcal{F}(x, \kappa^2)$ is the **unintegrated gluon distribution** (**UGD**) in the proton

▶ Small-x limit: **UGD** = [**BFKL gluon ladder**] ⊛ [**proton impact factor**]

◇ Takes into account the resummation of high-energy logs

◇ Describes the coupling of the gluon Green's function to the proton

▶ Proton impact factor is non-perturbative $\Longrightarrow$ UGD needs to be modeled!

High-energy factorization and the UGD

Features

* Small-x and large k_t
* Speaks the language of Reggeized gluon
* Inclusive or exclusive processes
* Double-log-approximation (DLA): $\alpha_s \ln(Q^2/Q_0^2)\ln(1/x)$

$$BFKL \longleftrightarrow DGLAP$$

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Unitarity violation and diffusion

* Violation of the Martin-Froissart buond: $\sigma_{\rm tot} \leq \dfrac{\pi\Delta^2}{2m_\pi^2}\ln^2 s$
* Diffusion to the infrared: $l_\perp e^{-k\sqrt{\Delta Y/2}} \lesssim k_\perp \lesssim l_\perp e^{k\sqrt{\Delta Y/2}}$

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$$\xrightarrow{\text{Non-linear evolution}} \quad BK/JIMWLK \text{ domain}$$

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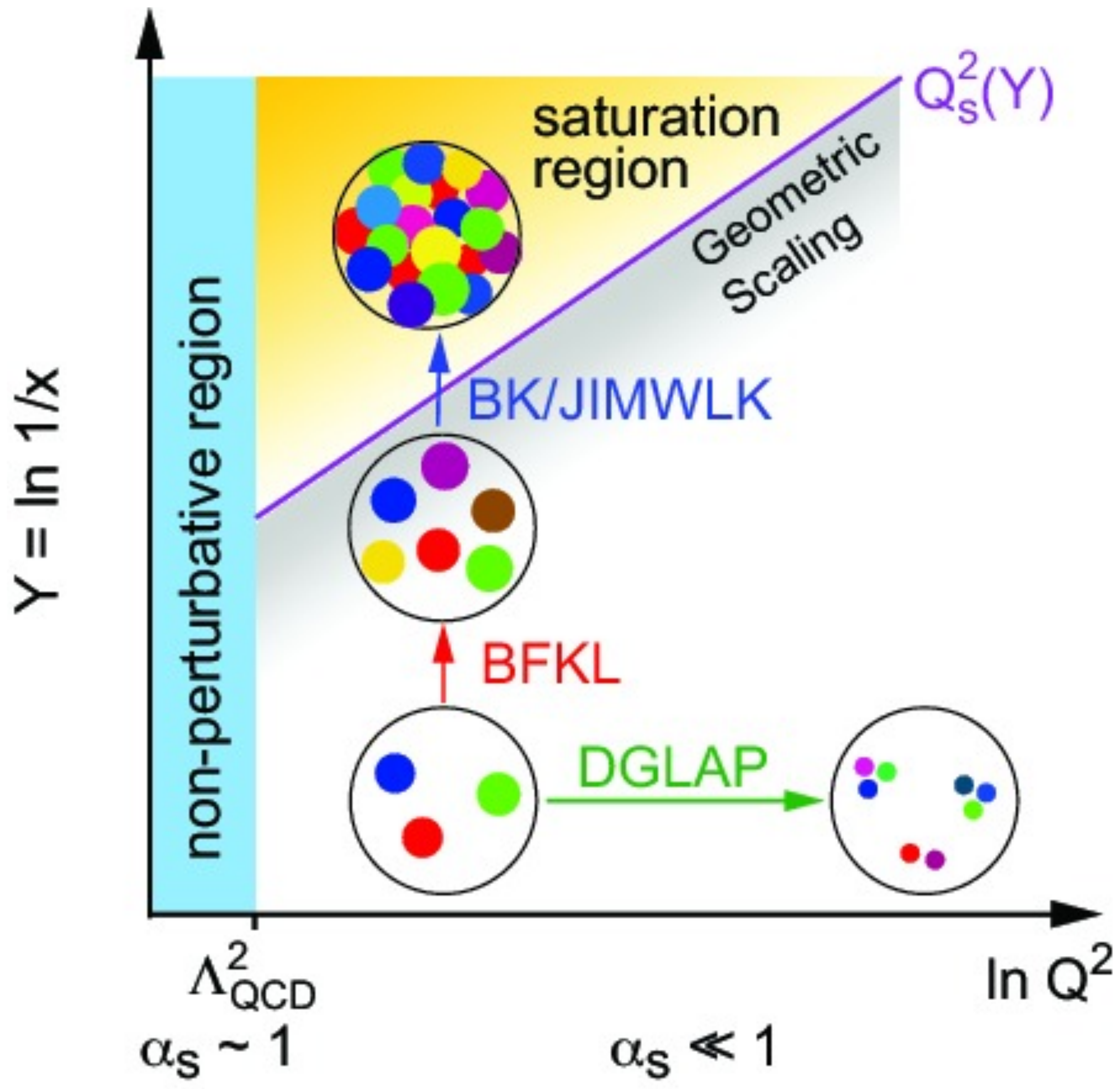

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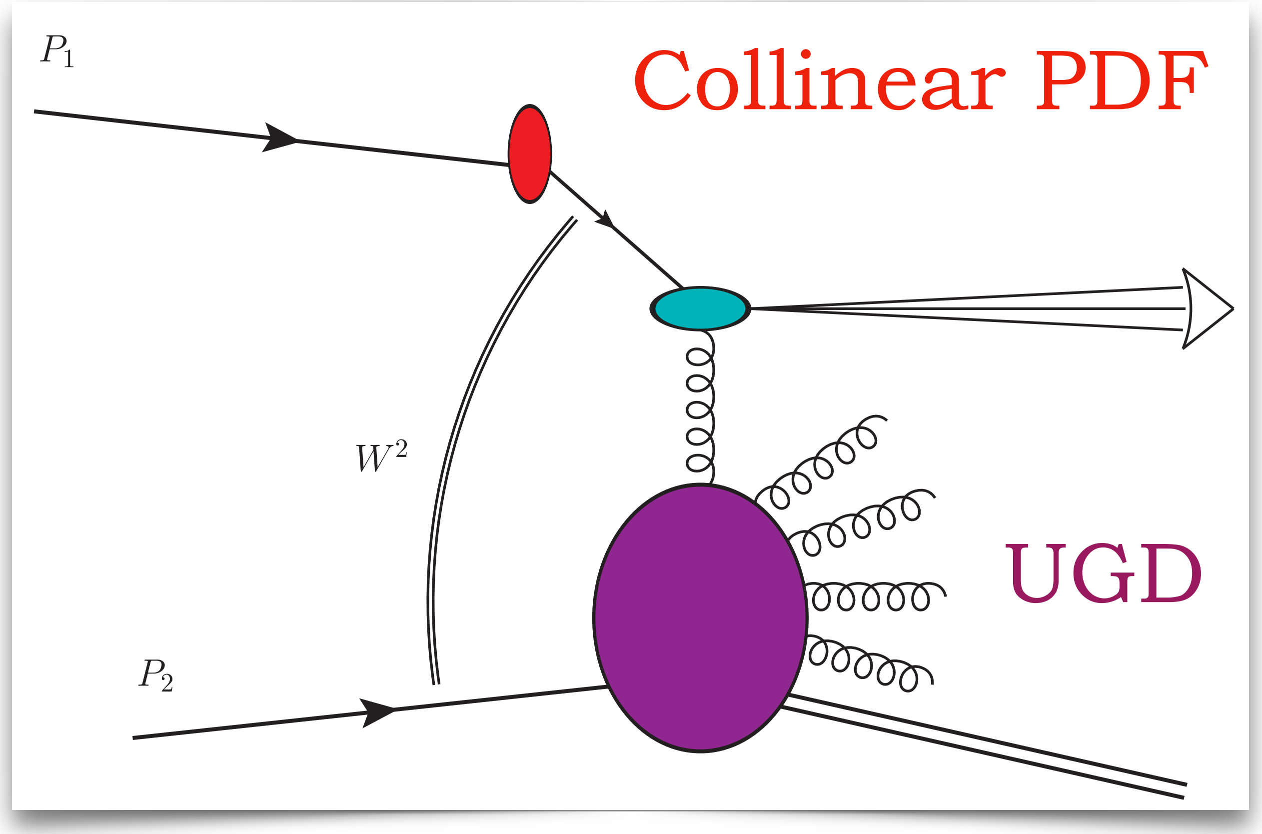
Nonlinear evolution $\longrightarrow$ *BK/JIMWLK* domain

Hybrid or pure factorization?

Forward emissions

- *Asymmetric* config. $\leftrightarrow$ fast parton + small-x gluon
- Hybrid **high-energy**/**collinear** factorization



- *Distinctive signals* of small-x dynamics **expected**
- Phenomenology:
  forward jet, Drell-Yan, Higgs or vector meson

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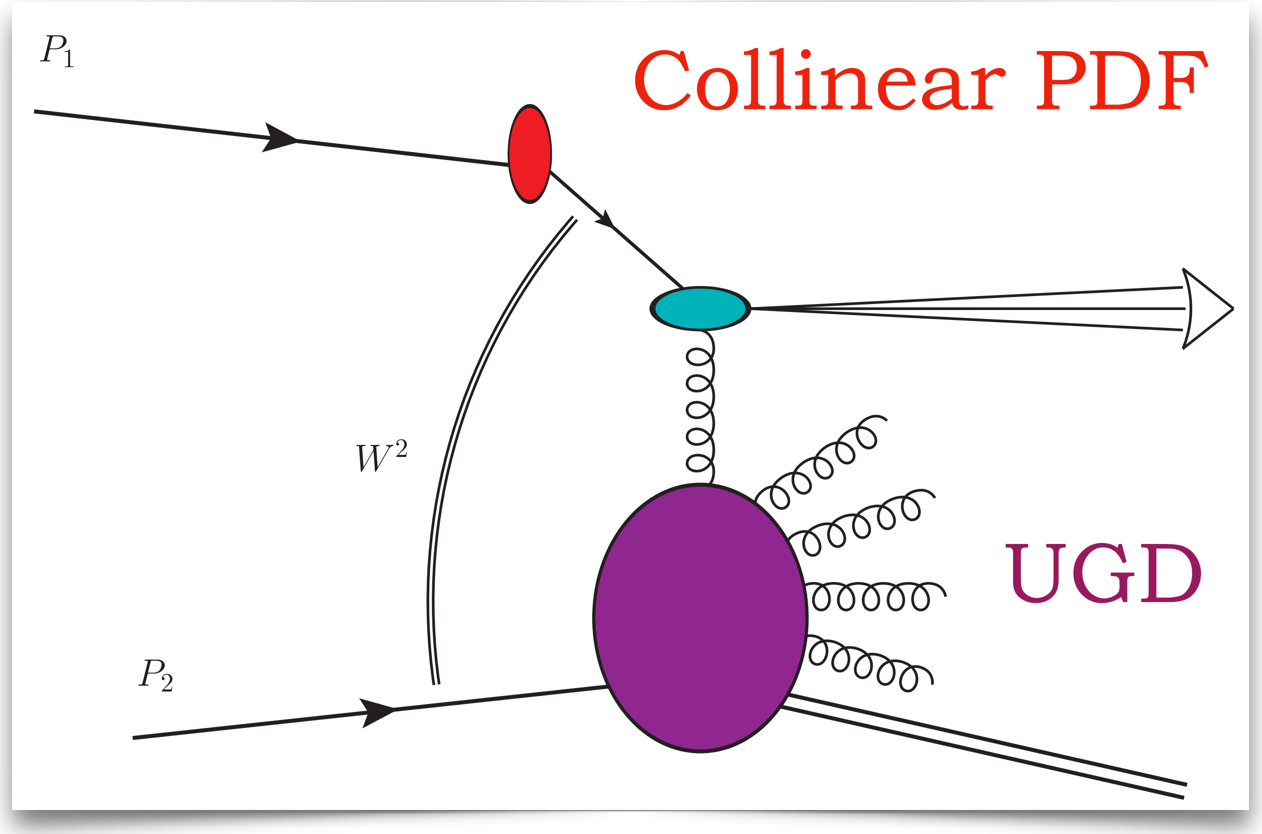

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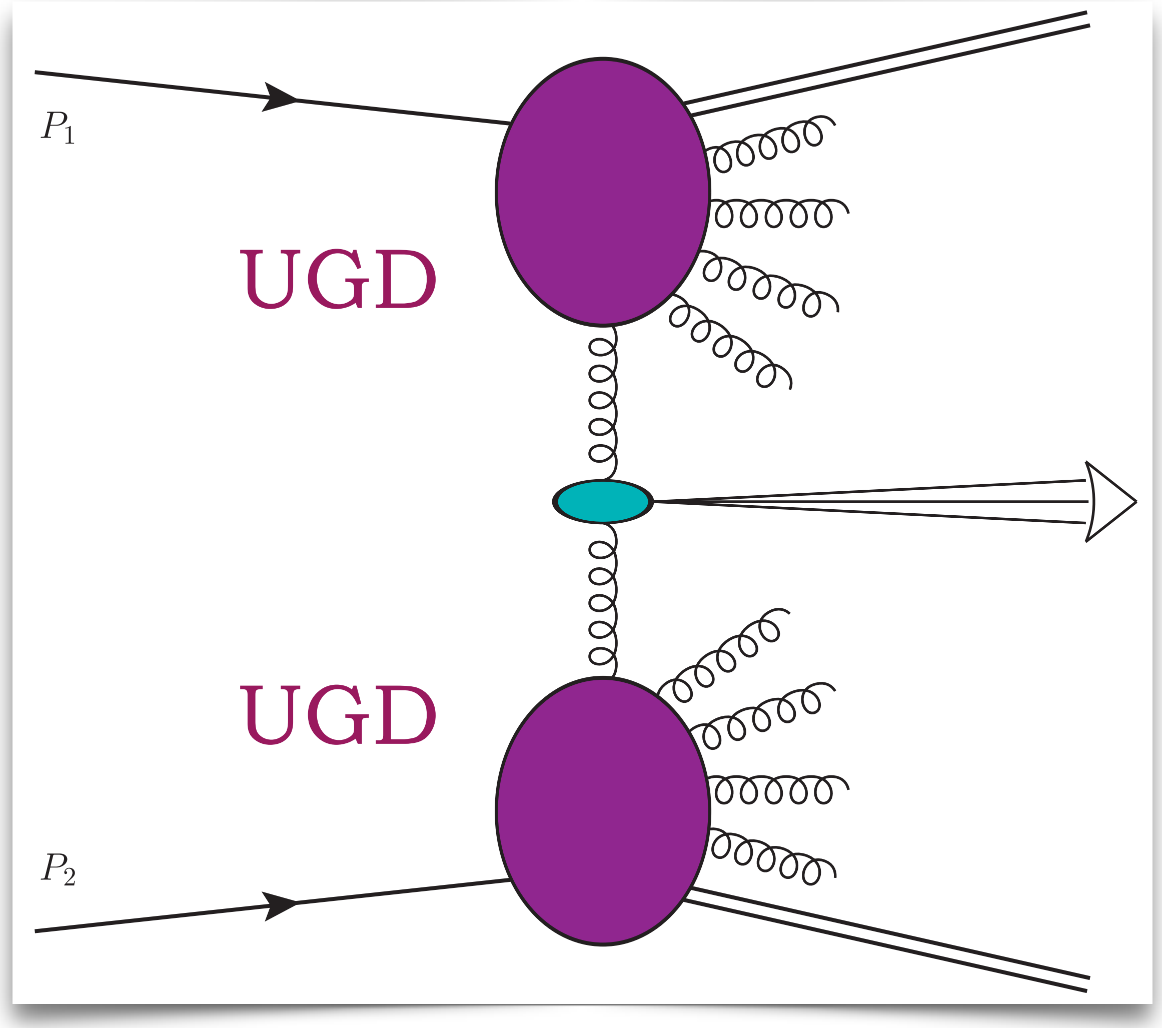
Central emissions

* *Gluon induced* ↔ small-x gluons
* Pure **high-energy** factorization



* Small-x dynamics to **enhance** f.o. description
* Phenomenology:
  central jet, Higgs or vector meson

Outline

BFKL and unintegrated gluon densities

Exclusive forward meson leptoproduction

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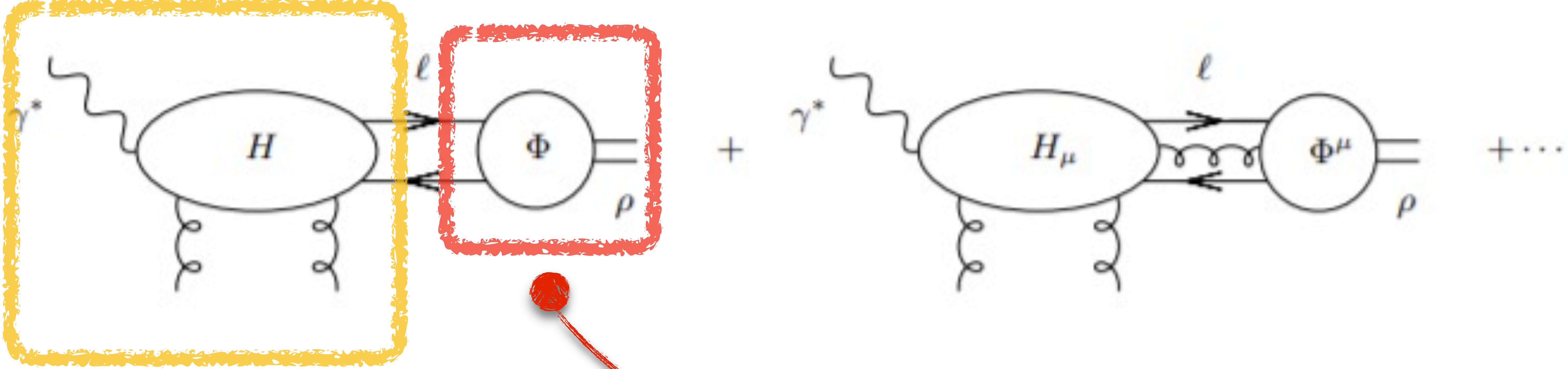
ρ-meson leptoproduction

[I. V. Anikin, D. Yu. Ivanov, B. Pire, L. Szymanowski and S. Wallon(2011)]

Impact factor

* Amplitude twist-expansion

$$A = \int d^4l \, Tr[H(l)\Phi(l)] + \int d^4l_1 \int d^4l_2 \, Tr[H_\mu(l_1, l_2)\Phi^\mu(l_1, l_2)] + \ldots$$

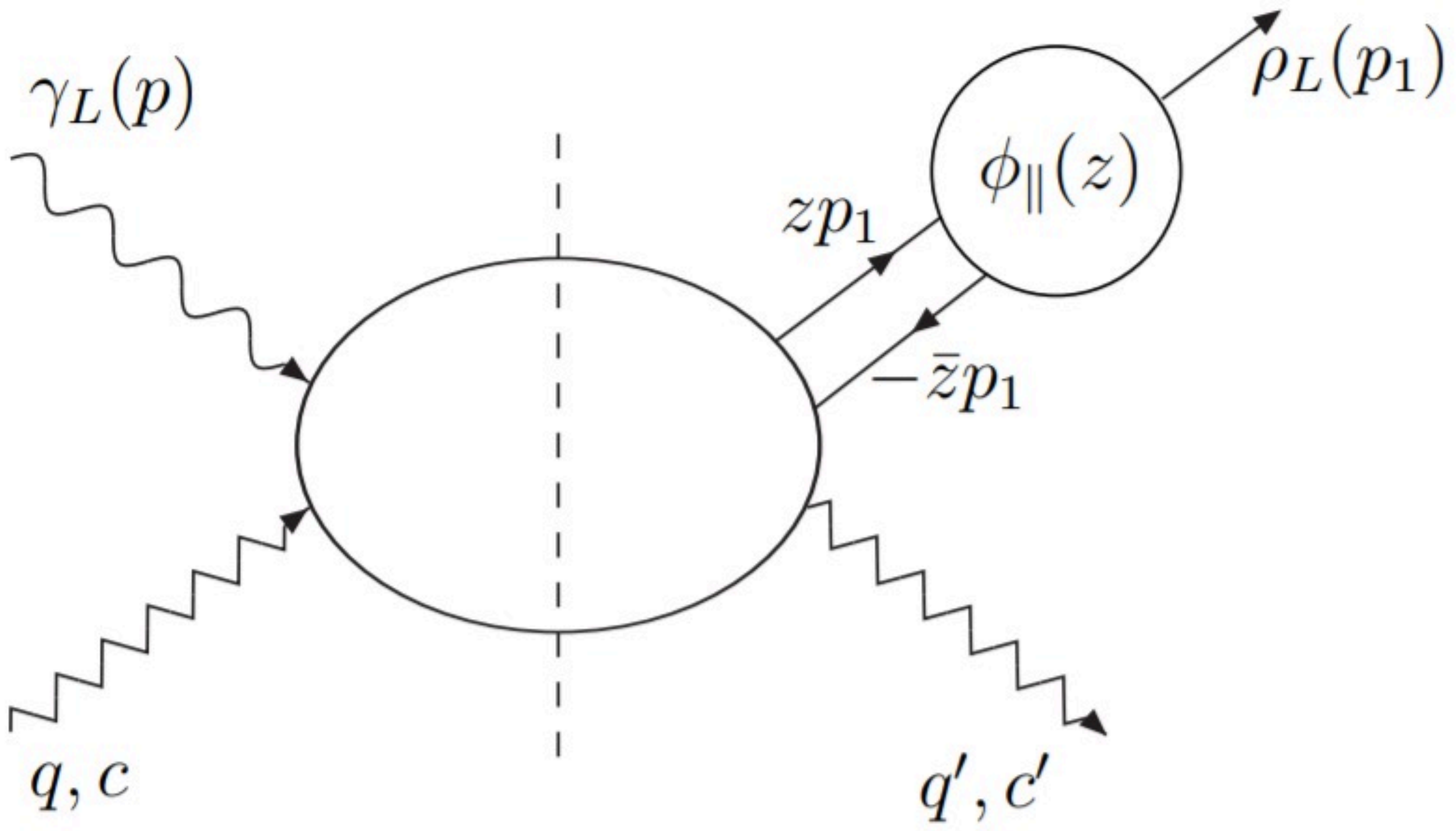


Hard coefficient function

Distribution amplitude

* Amplitude factorization achived by a Taylor expansion of the hard part

ρ-meson leptoproduction



- **Longitudinal case:** $\gamma_L \to \rho_L$
  - Starts the leading twist (twist two)
  - Known up to next-to-leading order
  - LO expression

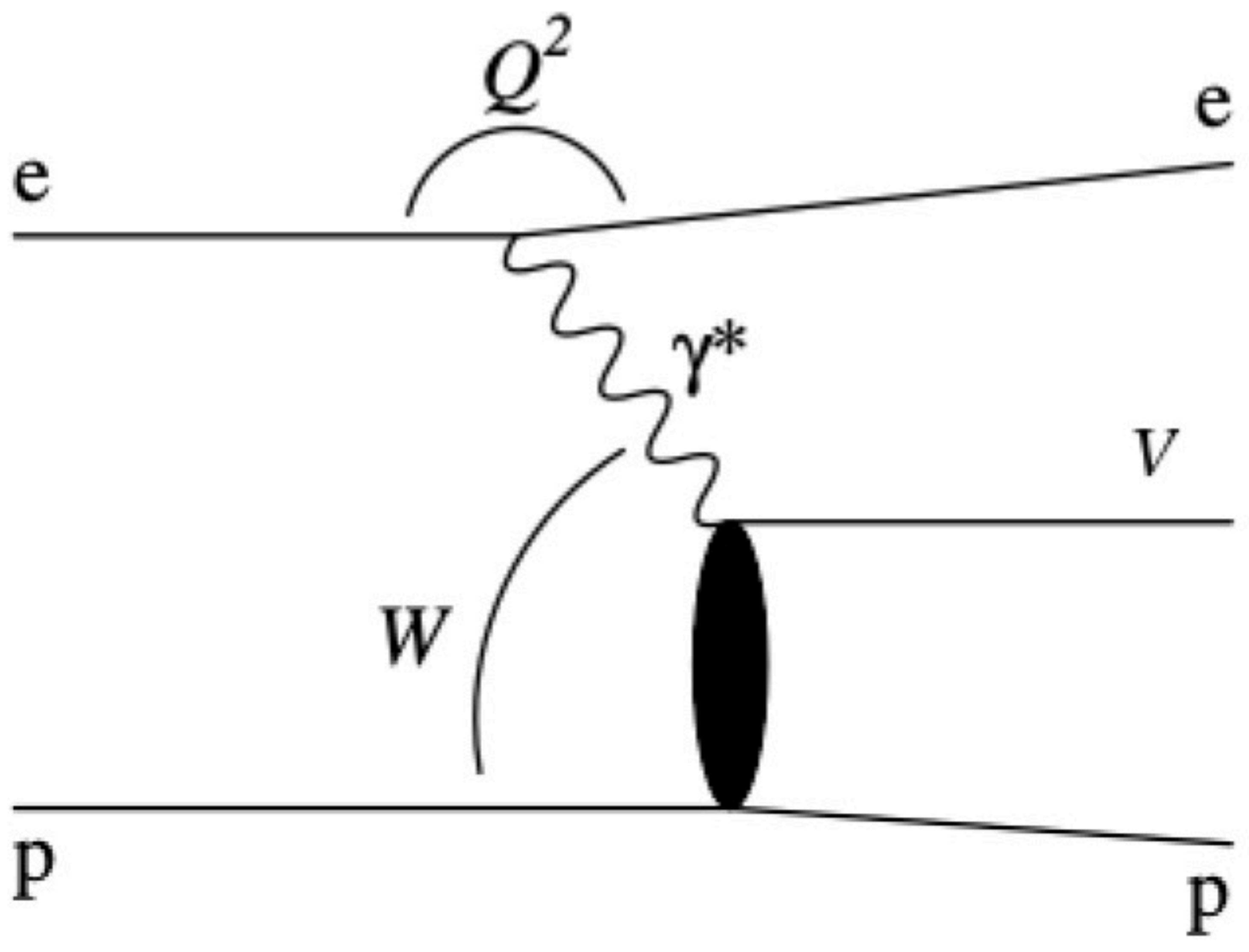
$$\Phi_{\gamma_L \to \rho_L}(k, Q, \mu^2) = 2B \frac{\sqrt{N_c^2 - 1}}{Q N_c} \int_0^1 dy \varphi_1(y; \mu^2) \left(\frac{\alpha}{\alpha + y\bar{y}} \right)$$

$$\alpha = \frac{k^2}{Q^2} \, , \, B = 2\pi\alpha_s \frac{e}{\sqrt{2}} f_\rho \, , \, \varphi_1 \to DA$$

- **Transverse case:** $\gamma_T \to \rho_T$
  - Starts at the next-to-leading twist (twist three)
  - Known up to leading order

ρ-meson leptoproduction

Electron-proton collision $e + p \longrightarrow e + \gamma^* + p \longrightarrow e + \rho + p$



* Exclusive reaction
* High-energy regime

$$s \equiv W^2 \gg Q^2 \gg \Lambda^2 \longrightarrow \text{small } x = \frac{Q^2}{W^2}$$

* Photon virtuality Q is the **hard scale**
* Process solved in helicity

Available data

* Hera

$$2.5 \text{ GeV}^2 < Q^2 < 60 \text{ GeV}^2$$

$$35 \text{ GeV} < W < 180 \text{ GeV}$$

* Zeus

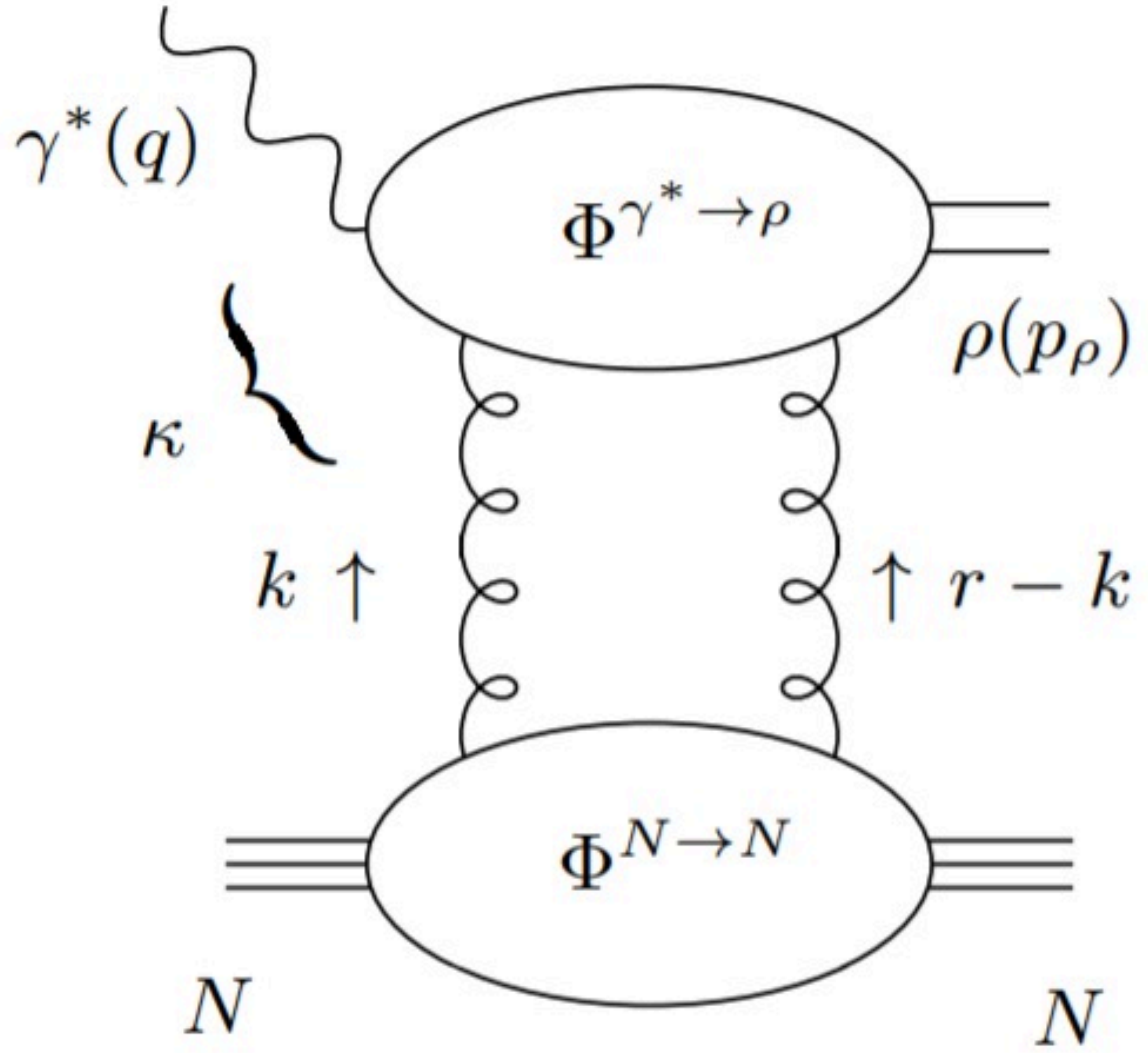
$$2 \text{ GeV}^2 < Q^2 < 60 \text{ GeV}^2$$

$$32 \text{ GeV} < W < 180 \text{ GeV}$$

ρ-meson leptoproduction

Leading helicity amplitudes $T_{\lambda_\rho \lambda_\gamma}(W^2, Q^2)$

- $Im_s\{A(\gamma^* p \to VP)\}$ dominates
- $T_{00} \gg T_{11} \gg T_{10} \gg T_{01} \gg T_{-11}$
- Small-size dipole mechanism $\longrightarrow$ k_T-factorization



$$T_{\lambda_V \lambda_\gamma}(s, Q^2) = is \int \frac{d^2k}{(k^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \to V(\lambda_V)}(k^2, Q^2) \mathscr{F}(x, k^2) , \qquad x = \frac{Q^2}{s}$$

- $V = \rho, \phi$ via **distribution amplitude (DAs):** $\varphi(y) = \varphi^{WW}(y) + \varphi^{gen}(y)$

ρ-meson leptoproduction

- **Wandzura-Wilczek (WW) approximation** → genuine terms neglected

$$T_{11} = is\frac{2BC}{Q^2}\int\frac{d^2k}{(k^2)^2}\mathscr{F}(x,k^2)\int_0^1\frac{dy}{(y\bar{y}+\tau)}\varphi_+^{WW}(y,\mu^2)\frac{\alpha(\alpha+2y\bar{y}+2\tau)}{(\alpha+y\bar{y}+\tau)^2}+o(\tau^2)$$

$$T_{00} = is\frac{4BC}{Q}\int\frac{d^2k}{(k^2)^2}\mathscr{F}(x,k^2)\int_0^1 dy\frac{\bar{y}y}{(y\bar{y}+\tau)}\varphi_+^{as}(y,\mu^2)\frac{\alpha}{(\alpha+y\bar{y}+\tau)}$$

$$\tau = \frac{m_q^2}{Q^2}\,, \qquad C = \sqrt{4\pi\alpha_{em}}$$

- **Generalized massive formula:**
  * $\tau = 0 \rightarrow$ no quark mass $\rightarrow \rho$-production
  * $\tau \neq 0 \rightarrow$ with quark mass $\rightarrow \phi$-production

[A. D. Bolognino, A. Szczurek, W. Schafër]

- **Vector meson-DAs** employed:
  * **asymptotic** $\varphi_1^{as}(y) \rightarrow a_2(\mu^2) = 0$
  * $\varphi_+^{WW}(y,\mu^2) = (2y-1)\varphi_{1T}^{WW}(y,\mu^2) + \varphi_{AT}^{WW}(y,\mu^2)$

ρ-meson leptoproduction

Models of unintegrated gluon density (UGD)

- **ABIPSW:** x-independent model $\mathcal{F}(x,k^2) = \frac{A}{(2\pi)^2 M^2}\left[\frac{k^2}{k^2+M^2}\right]$

  [I. V. Anikin et al. (2011)]

- **Toy model:** gluon momentum derivative $\mathcal{F}(x,k^2) = \frac{d(xg(x,k^2))}{d\ln k^2}$

- **IN:** soft-hard model $\mathcal{F}(x,k^2) = \mathcal{F}_{soft}(x,k^2) + \mathcal{F}_{hard}(x,k^2)$

  [I. P. Ivanov and N. N. Nikolaev (2002)]

- **HSS:** $\mathcal{F}(x,k^2) = \Phi_P \otimes \mathcal{G}_{BFKL}$

  [M. Hentschinski, A. Sabio Vera, C. Salas (2013)]

- **WMR:** angular ordering of gluon emissions

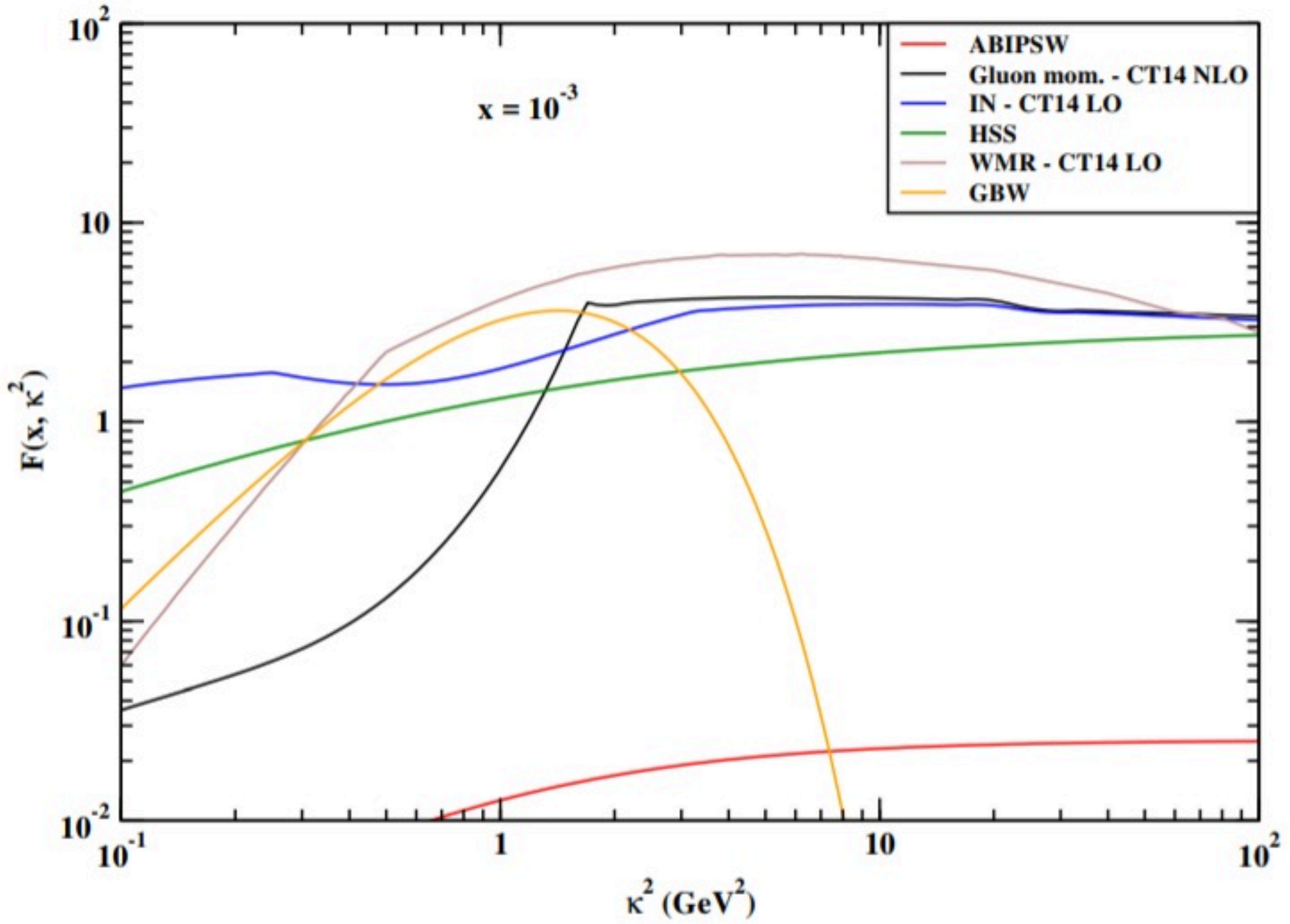
  [G. Watt, A. D. Martin, M. G. Ryskin (2003)]

- **GBW:** FT of dipole cross section

  [K. J. Golec-Biernat, M. Wüsthoff (1998)]

- **BCRT:** small-x improved unpolarized gluon TMD

  [A. Bacchetta, F.G. Celiberto, M. Radici, P. Taels (2020)]



ρ-meson leptoproduction at HERA

$$\sigma_L(\gamma^* p \to Vp) = \frac{1}{16\pi b(Q^2)} \frac{|T_{00}(s,Q^2)|^2}{W^2}$$

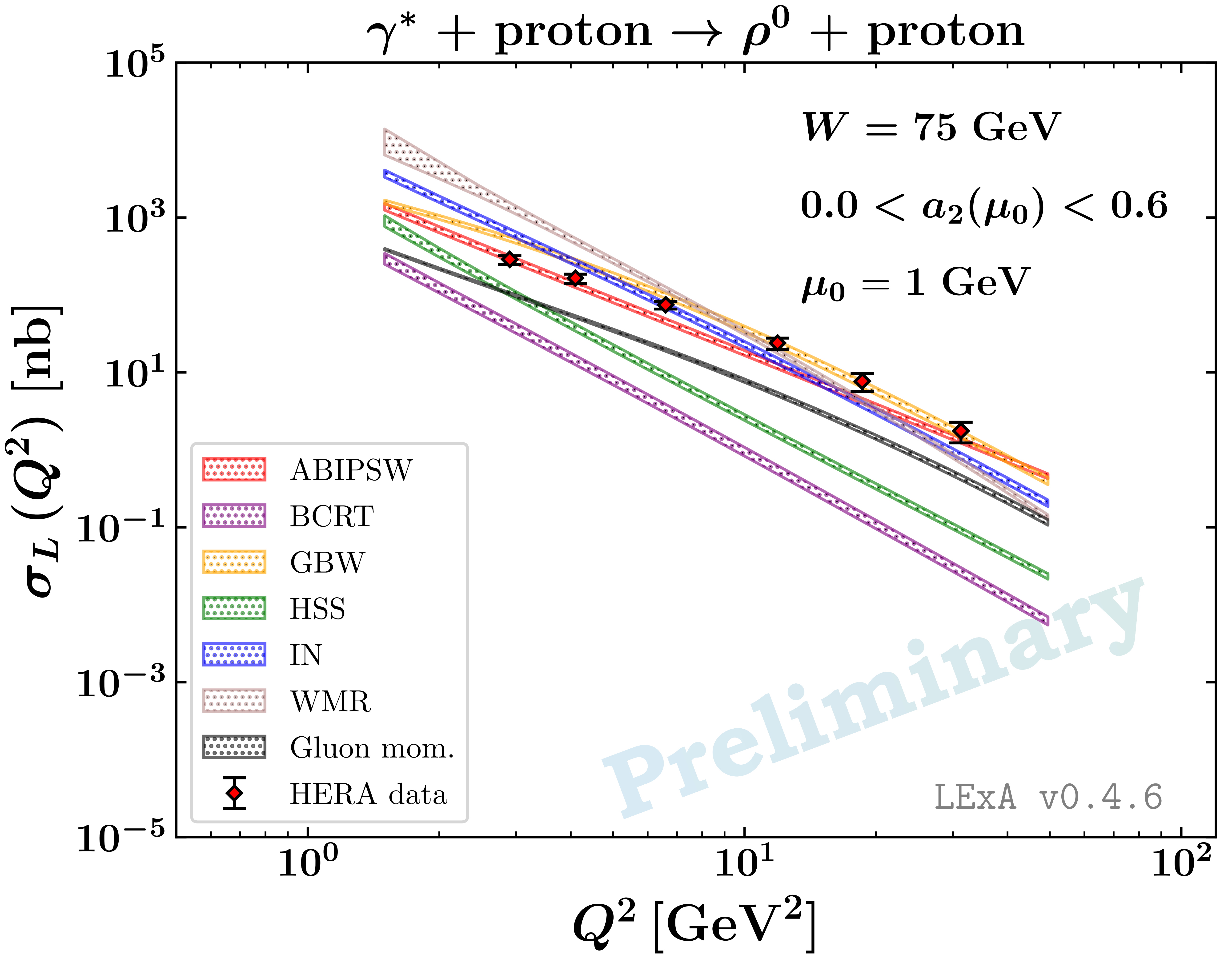
$$\sigma_T(\gamma^* p \to Vp) = \frac{1}{16\pi b(Q^2)} \frac{|T_{11}(s,Q^2)|^2}{W^2}$$

* $b(Q^2)$**-slope** for light vector mesons

$$b(Q^2) \approx \beta_0 - \beta_1 \ln\left[\frac{Q^2 + m_V^2}{m_{J/\Psi}^2}\right] + \frac{\beta_2}{Q^2 + m_V^2}$$

* For ρ-meson:

$$\beta_0 = 6.5\ \mathrm{GeV}^{-2}\,,\ \beta_1 = 1.2\ \mathrm{GeV}^{-2}\,,\ \beta_2 = 1.1\ \mathrm{GeV}^{-2}$$



[A. D. Bolognino, F. G. Celiberto, D. Yu. Ivanov, A. Papa, A. Szczurek, W. Schafër]

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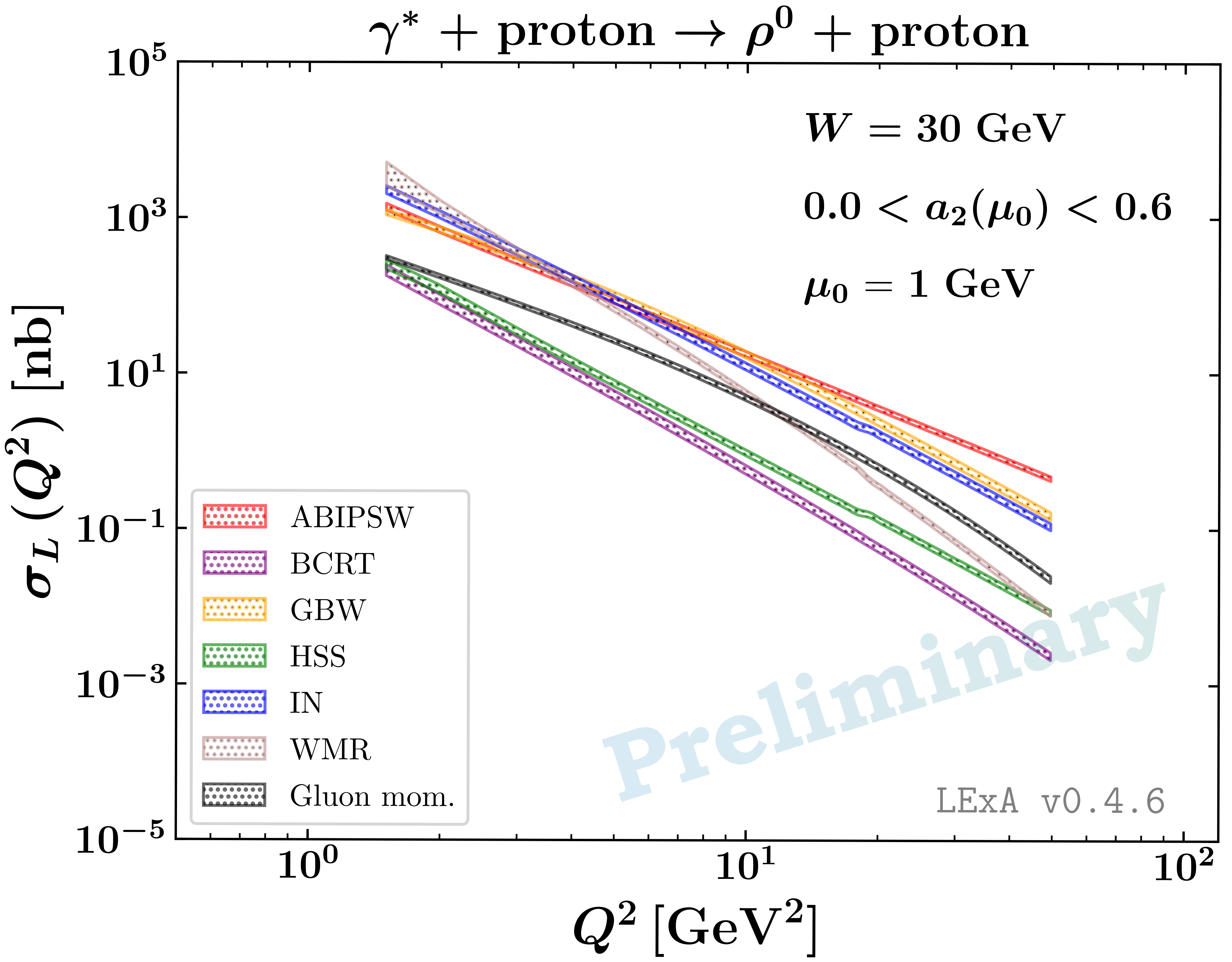

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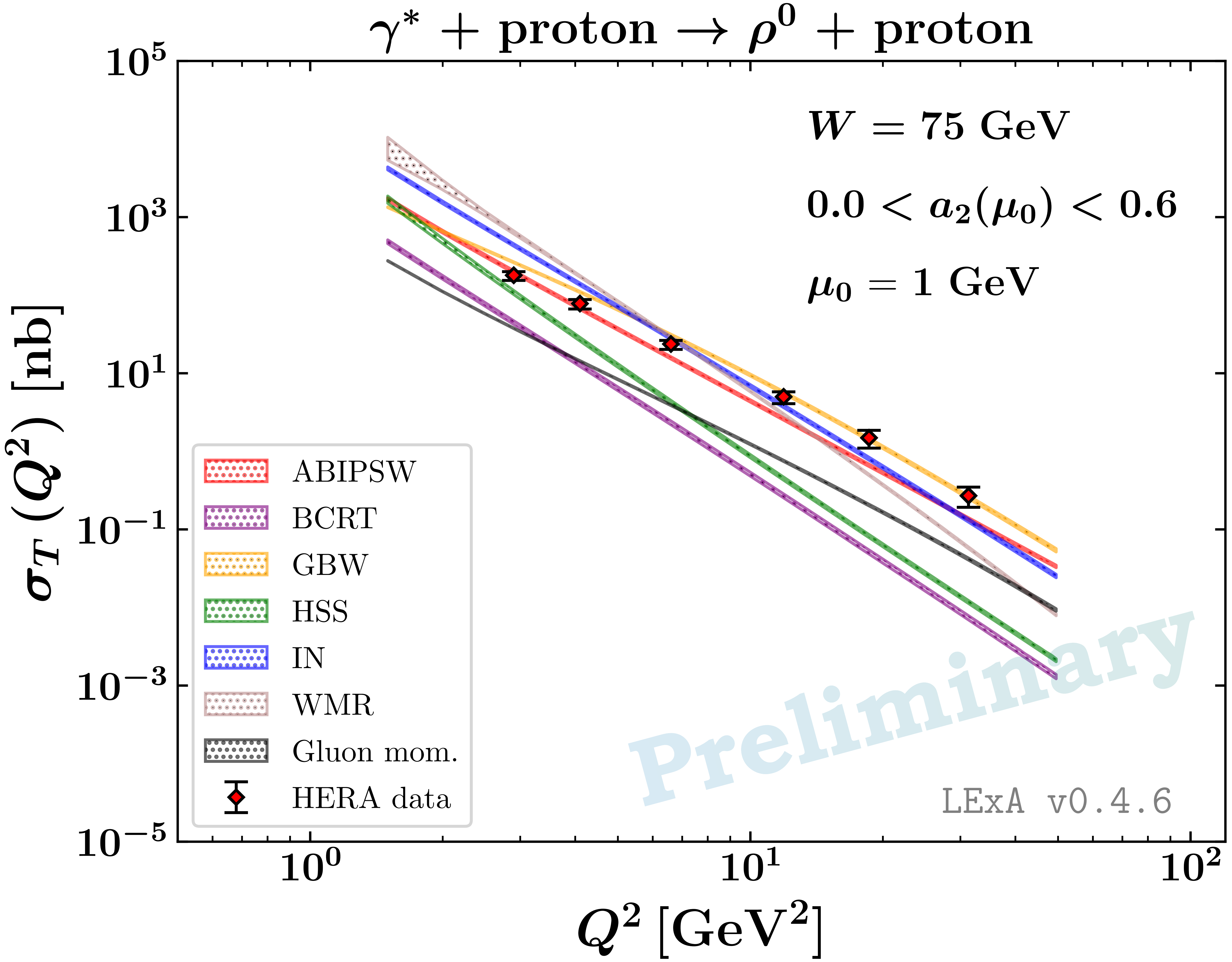

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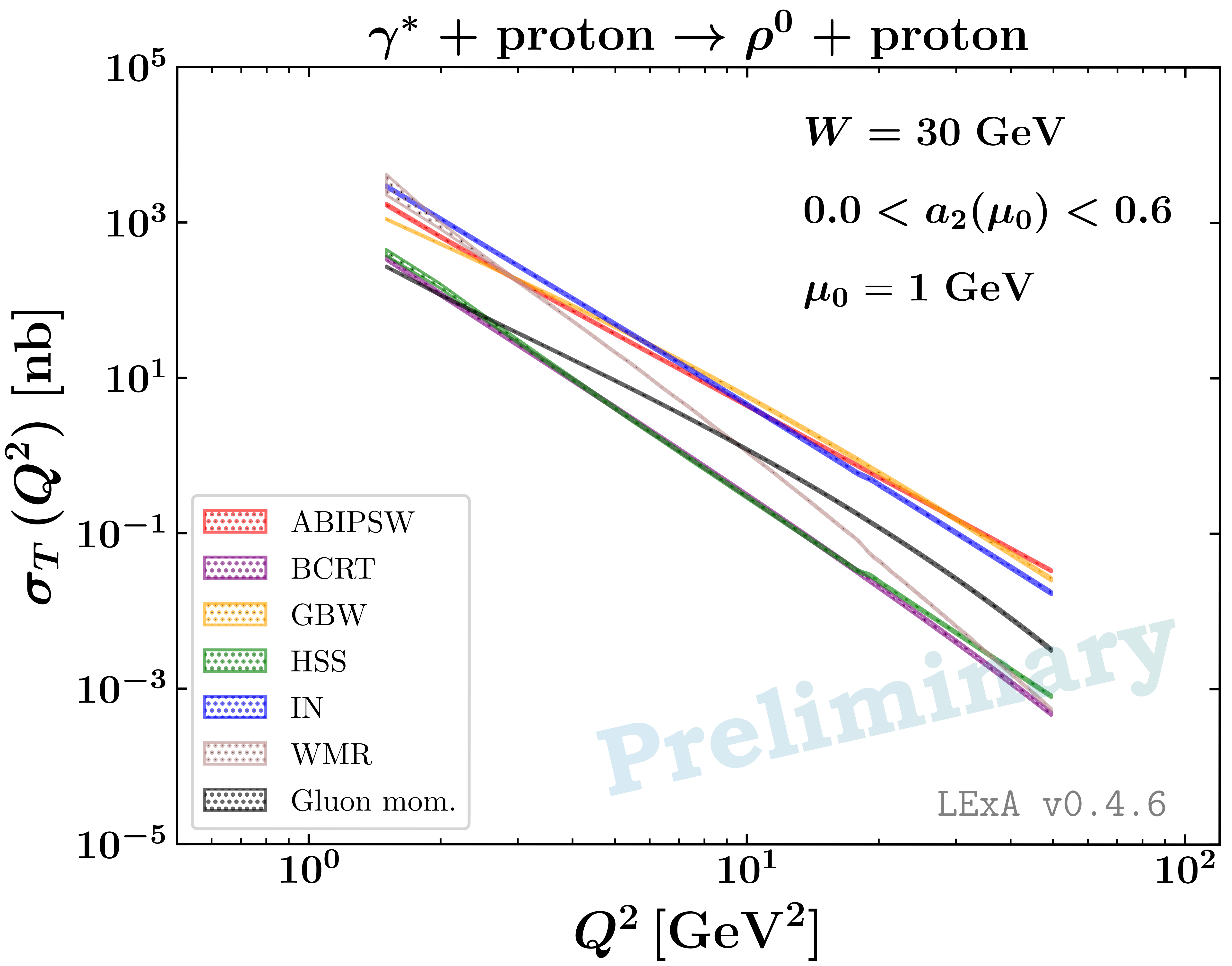

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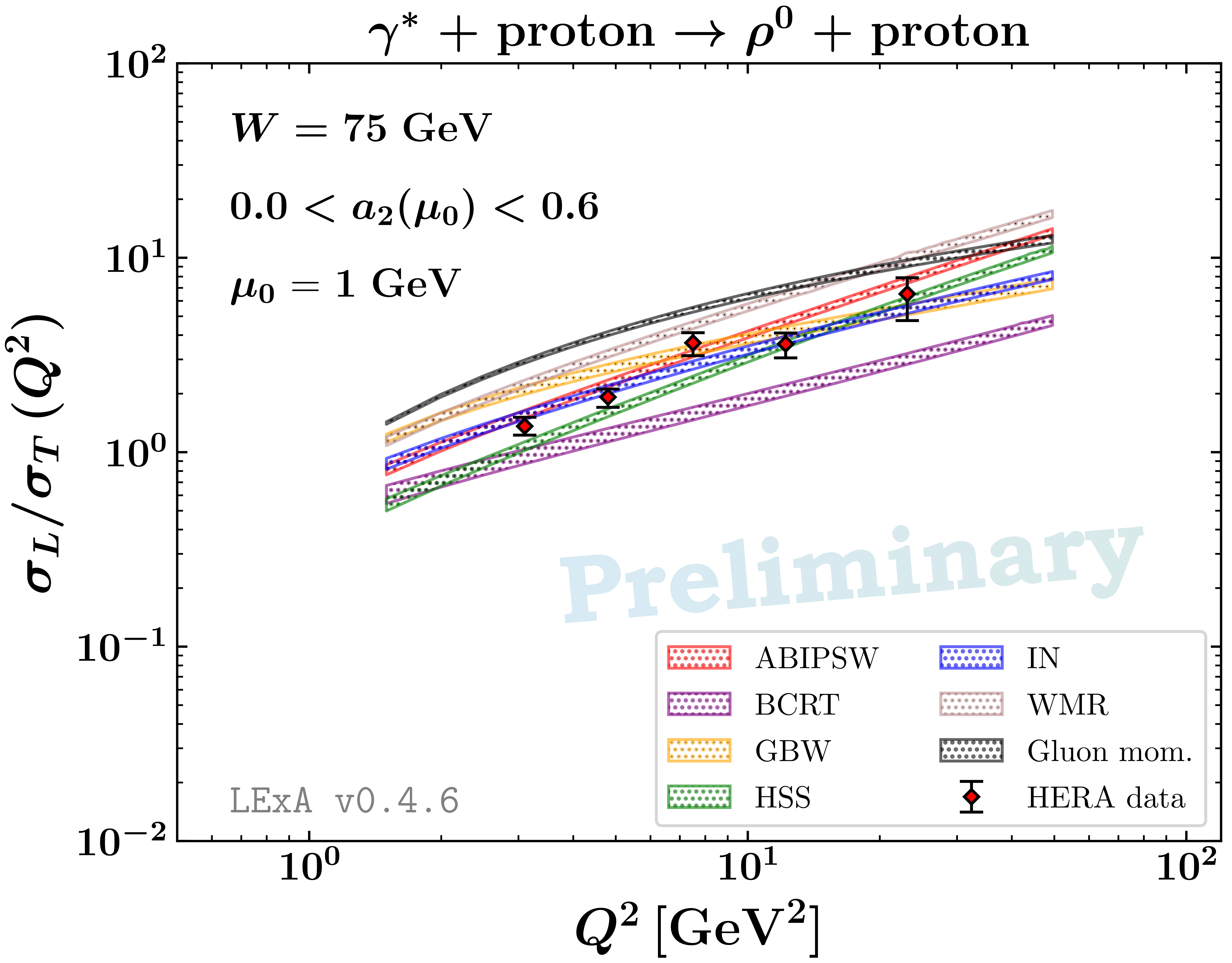

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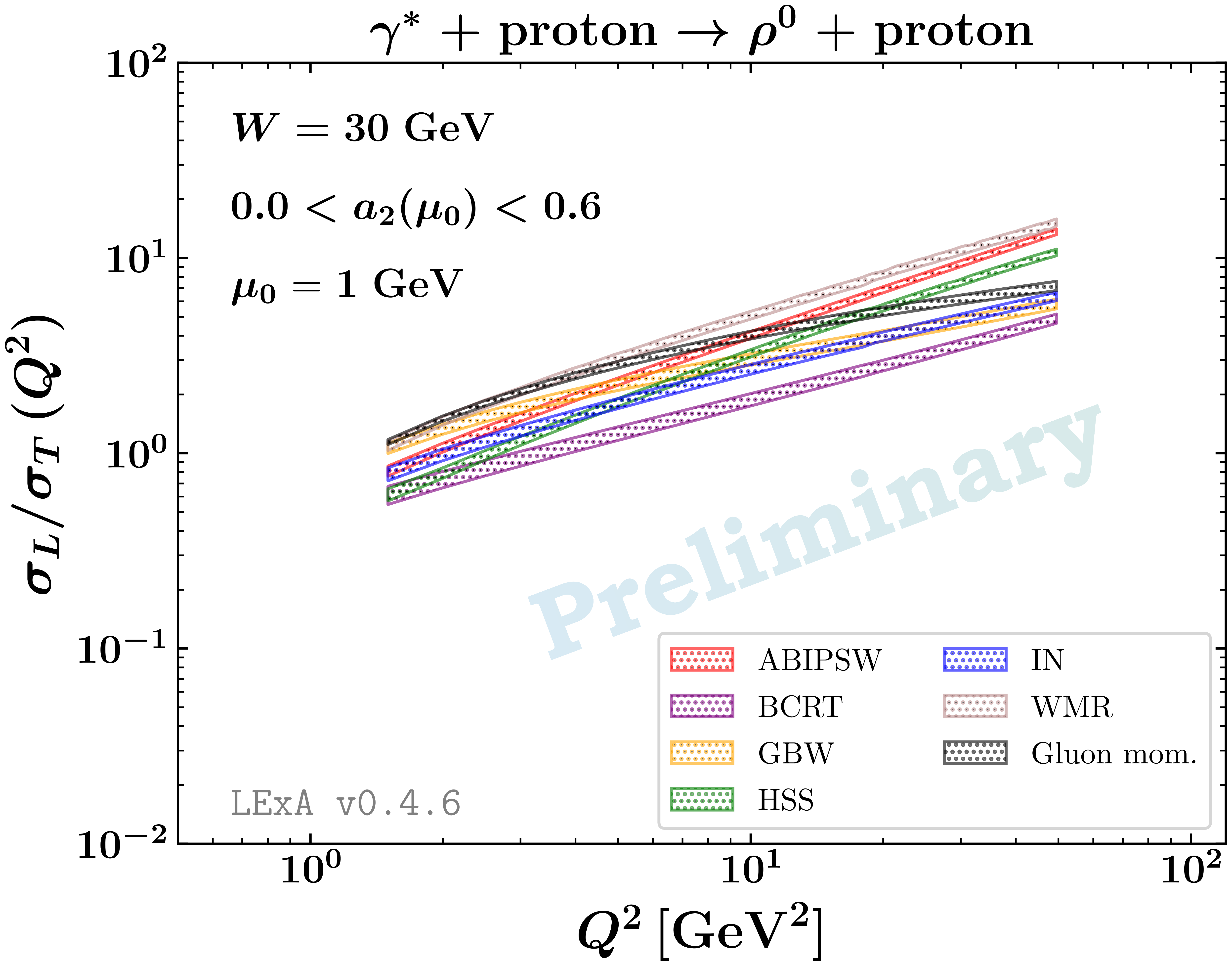

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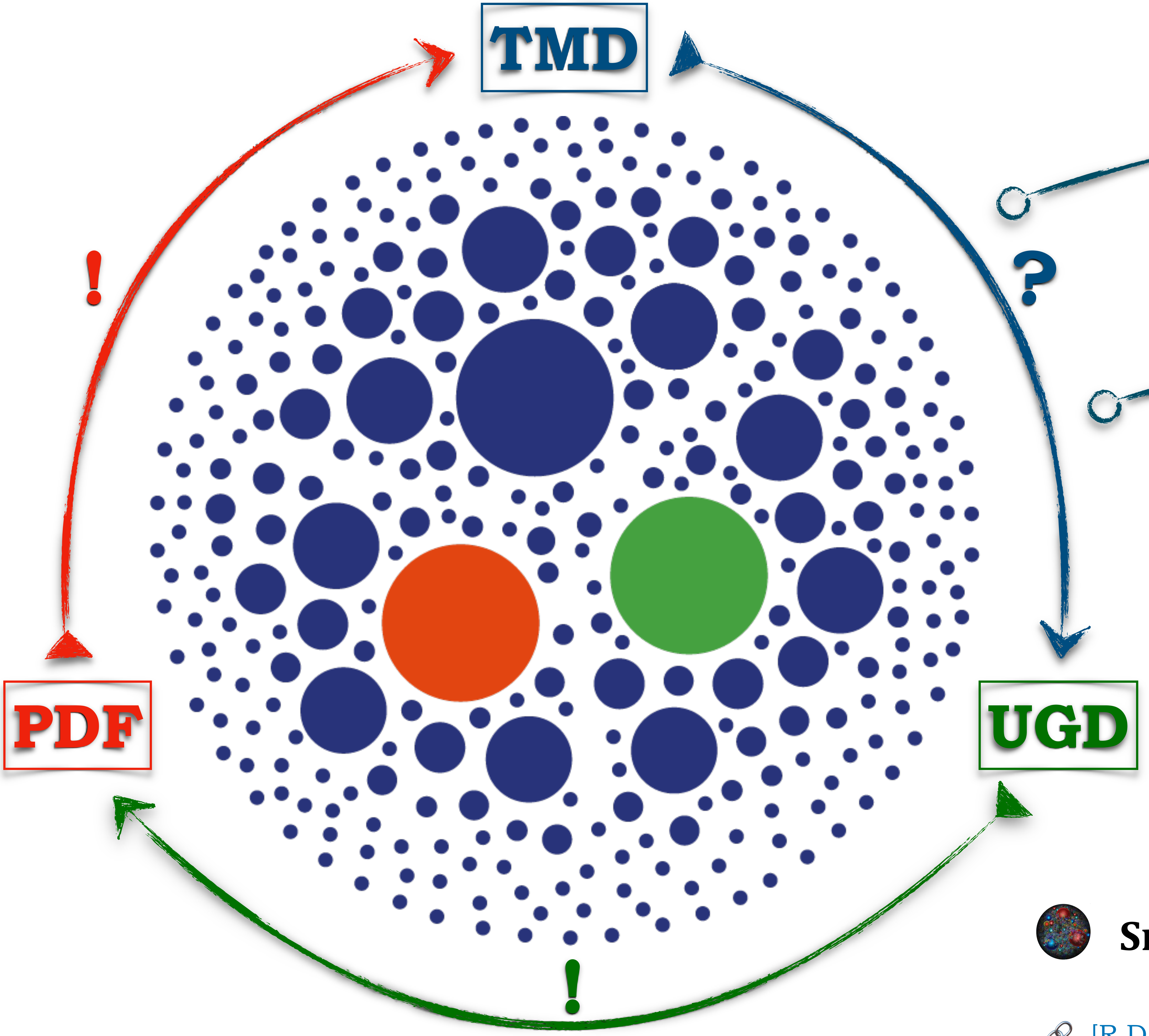
Conclusions and summary

- **Unintegrated gluon densities are essential for the description of high-energy QCD**
- **Vector meson leptoproduction is a suitable tool for the investigation of the UGD**
  - * Impact factors for both longitudaly and transversly polarized ρ-meson are known
  - * Hera data are available and predictions for future studies at the EIC has been built
- **None of models is able to reproduce the entire HERA Q^2-spectrum**
  - * UGD model extraction from fits
  - * Towards a unification of formalism

Towards a unification of formalisms



M. Nefedov's talk

F. Celiberto's talk

- **TMD evolution vs BFKL evolution**

  [M. Hentschinski (2021)] [M. Nefedov (2021)]

  [M. Hentschinski, A. Kusina, K. Kutak, M. Serino (2018)]

- **Small-x input to gluon TMDs**

  [A. Bacchetta, F.G. Celiberto, M. Radici, P. Taels (2020)]

- **Small-x resummed collinear PDFs**

  [R.D. Ball, V. Bertone, M. Bonvini, S. Marzani, J. Rojo, L. Rottoli (2018)]

Thanks for the attention!