Analysis tools for hadron spectroscopy

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LHCb Collaboration
JPAC Collaboration

HADRON2021 in memoriam Simon Eidelman

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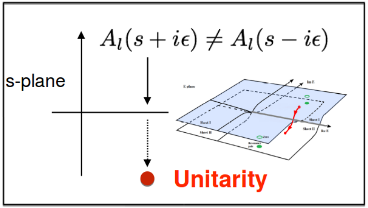
Tools in hadron spectroscopy

General principles of the scattering theory

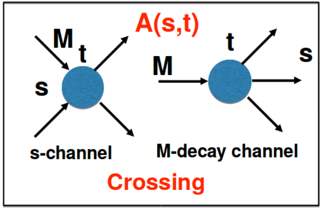
- Lorentz invariance = independence of the reference frame, known behavior under boosts and rotations
- Unitarity = constraint to imaginary part of scattering amplitude
- Analyticity = implementation of relevant, closest singularities
- Crossing = decay and scattering regions are analytically connected



$$A(s,t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

$$A_l(s) = \lim_{\epsilon \to 0} A_l(s+i\epsilon)$$



Joint Physics Analysis Center

Joint effort of theorists and experimentalists in support of studies of strong interaction

Collaboration with several experimental groups

Involved in ongoing projects with LHCb, COMPASS, GlueX, BESIII, and EIC.

The plan for the talk

Two theoretical frameworks:

- Dalitz-plot decomposition [MM et al.(JPAC), PRD 101, 034033 (2020)]: [1] and [2]
  (How to write a general decay amplitude for particles with spin)
- Three-body unitarity [MM et al.(JPAC), JHEP 08 (2019) 080]: [3]
  (Line shape of resonance decaying to three particles)

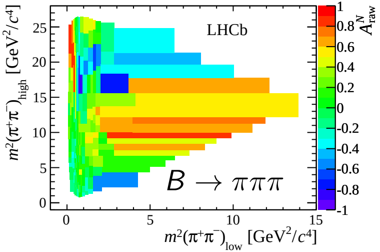
Applied in three LHCb analysis:

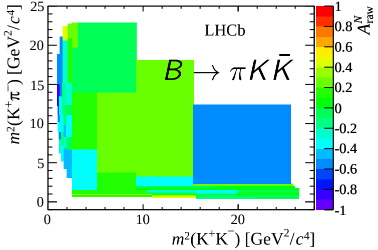
1. Search for CPV in $\Xi_b^- \to pK^-K^-$
2. Search for pentaquarks in $B_s^0 \to J/\psi p\bar{p}$
3. Lineshape analysis for $\Lambda_b^{**0} \to \Lambda_b^0 \pi^+\pi^-$

Amplitude analysis of $\Xi_b^- \to pK^-K^-$

CP violation in hadronic decays

CPV is well seen in $B \to hhh$:





Appearance of CPV effects:

$$B \to f: \quad A = \sum_i |A_i| e^{i(\delta_i + \phi_i)}$$

$$\bar{B} \to \bar{f}: \quad \bar{A} = \sum_i |A_i| e^{i(\delta_i - \phi_i)},$$

- Strong phase (δ) does not change under CP,
- Weak phase (ϕ) flip the sign.

$$\mathcal{A}_{\rm CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}$$

$$\to \sum_i |A_i||A_j| \sin(\delta_i - \delta_j) \sin(\phi_i - \phi_j)$$

no CPV in baryonic decays

In Λ_b^0 decay:

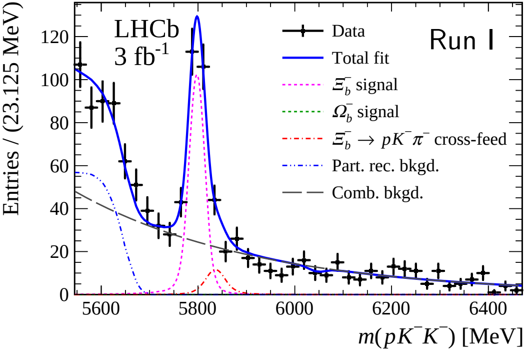
- pK^-, $p\pi^-$ [r], $K_S^0 p\pi^-$ [r], $\Lambda K^+ K^-$, $\Lambda K^+\pi^-$ [r]
- $p\pi^-\pi^-\pi^+$ [r], $p\pi^- K^- K^+$ [r], $pK^-\pi^+\pi^+$ [r], $pK^- K^+ K^+$ [r]

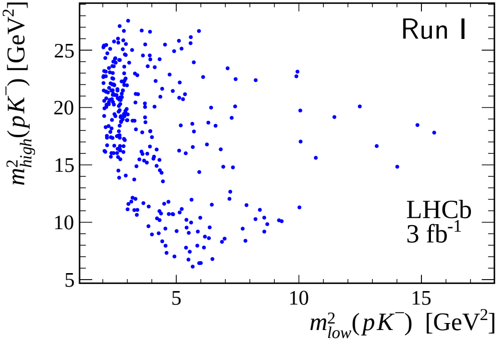
In Ξ_b^- decay:

- $pK^-\pi^+\pi^+$ [r], $pK^-\pi^+K^+$ [r]

Observation of $\Xi_b \to pK^-K^-$ decay

[arXiv:2104.15074]





- Run I + a part of Run II $\Rightarrow$ 5 fb^{-1}.
- 460 signal candidates,
  - + combinatorial background,
  - + $\Xi_b^- \to pK^-K^-$ misidentification.

- K^-K^- symmetry makes Dalitz plot (double entry) symmetric
- Only half is analysed (single entry)

Observation of $\Xi_b \to pK^-K^-$ decay

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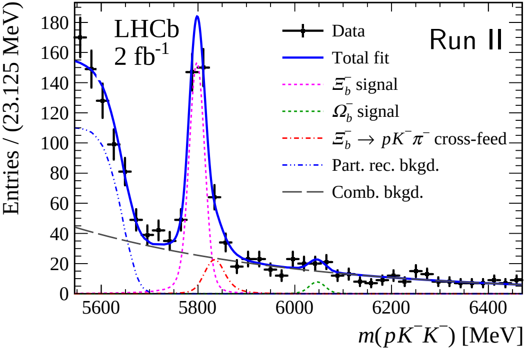

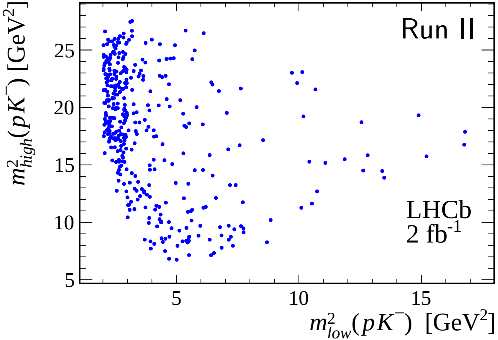

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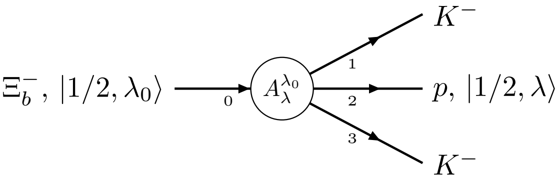
Ξ_b^- decay amplitude



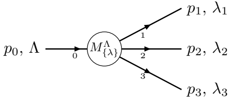
- Baryon decay (half-integer spin)
  $\Rightarrow$ 4π-fold azimuthal symmetry of the decay amplitude
- Two identical kaons in the final state
  $\Rightarrow$ Symmetry of the decay amplitude.
- Intermediate Σ, Λ states decaying to pK^-

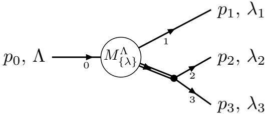
Conventional helicity approach

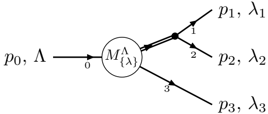
Complicated cases: particles with spin in isobar model [Hansen (1974)], [Herndon(1975)]

$$M^{\Lambda}_{\{\lambda\}} = M^{\Lambda}_{1,\{\lambda\}} + M^{\Lambda}_{2,\{\lambda\}} + M^{\Lambda}_{3,\{\lambda\}}$$

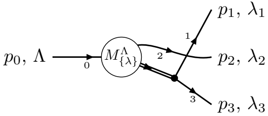
$$\underbrace{M^{\Lambda}_{\{\lambda\}}} = \underbrace{H_1 D(\phi_1, \theta_1, 0)\, D(\phi_{23}, \theta_{23}, 0) W_1(\dots)} + \underbrace{H_3 D(\phi_3, \theta_3, 0)\, D(\phi_{12}, \theta_{12}, 0) W_3(\dots)}$$







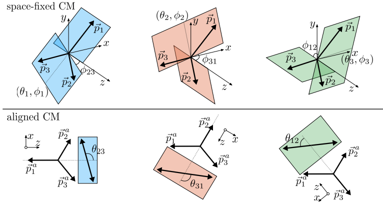
$$+ \underbrace{H_2 D(\phi_2, \theta_2, 0)\, D(\phi_{31}, \theta_{31}, 0) W_2(\dots)}$$



- A special set of angles for every decay chain
- Consistency of quantization direction – **Wigner rotations**

$$W_i(\dots) = D^{j_1}(\tilde{\phi}^i_1, \tilde{\theta}^i_1, 0)\, D^{j_2}(\tilde{\phi}^i_2, \tilde{\theta}^i_2, 0)\, D^{j_3}(\tilde{\phi}^i_3, \tilde{\theta}^i_3, 0)$$

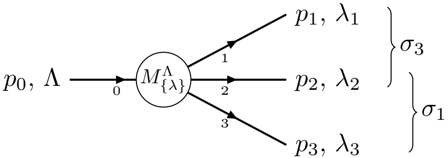
- Might give wrong answer for non-integer spin ($\phi \in [-\pi, \pi]$)



The Dalitz-Plot decomposition

[MM et al.(JPAC), PRD 101, 034033 (2020)]

Reformulation of the helicity approach



$$= \sum_{\nu} \underbrace{D^{J*}_{\Lambda\nu}(\phi_1, \theta_1, \phi_{23})}_{\text{Decay-plane orientation}} \times \underbrace{O^{\nu}_{\{\lambda\}}(\{\sigma\})}_{\text{Dalitz-plot function}}$$

Model-independent factorization of the overall rotation:

- Exploits properties of the Lorentz group (orientation – just three Euler angles)
- Dalitz-plot function depends entirely on 2 variables, $\{\sigma\} \equiv \{\sigma_1, \sigma_2, \sigma_3\}$
- No azimuthal phase factors in $O^{\nu}_{\{\lambda\}}$.

Gives significant benefits to

- Pentaquark analysis, Λ_b/Λ_c polarionation measurements, Baryonic decay chains,. . .

Dalitz-Plot function

[MM et al.(JPAC), PRD 101, 034033 (2020)]

Master formula $0 \to 1\,2\,3$ decay with arbitrary spins

$$O^{\nu}_{\{\lambda\}}(\{\sigma\}) = \sum_{(ij)k} \sum_{s}^{(ij)\to i,j} \sum_{\tau} \sum_{\{\lambda'\}} n_J n_s\, d^J_{\nu,\tau-\lambda'_k}(\hat{\theta}_{k(1)}) X_s^{\tau,\lambda'_k;\lambda'_i,\lambda'_j}(\sigma_k)\, d^s_{\tau,\lambda'_i-\lambda'_j}(\theta_{ij})$$
$$\times\, d^{j_1}_{\lambda'_1,\lambda_1}(\zeta^1_{k(0)})\, d^{j_2}_{\lambda'_2,\lambda_2}(\zeta^2_{k(0)})\, d^{j_3}_{\lambda'_3,\lambda_3}(\zeta^3_{k(0)}),$$

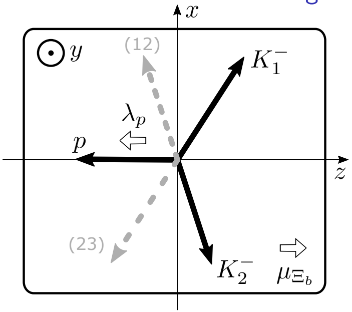
- Three decay chains, $(ij)k \in \{(12)3, (23)1, (31)2\}$.
- $\theta_{ij} = \theta_{ij}(\{\sigma\})$ is an isobar decay angle
- $\hat{\theta}_{k(1)} = \hat{\theta}_{k(1)}(\{\sigma\})$ is the particle-0 Wigner angle
- $\zeta^i_{k(0)} = \zeta^i_{k(0)}(\{\sigma\})$ is the particle-i Wigner angle

Applied in LHCb

- P_c in $\Lambda_b^0 \to J/\psi p K^-$
- P_{cs} in $\Xi_b^- \to J/\psi \Lambda K^-$
- $\Lambda_c^+/\Xi_c^+ \to pK^-\pi^+$
- Some analyses in progress

Symmetrization: $\underbrace{(K^- p)}_{\text{high}} K^- + K^- \underbrace{(pK^-)}_{\text{low}}$

[arXiv:2104.15074]



$$A_{\lambda_0,\lambda} = \sum_{\nu} D^{1/2}_{\lambda_0,\nu}(\alpha,\beta,\gamma) O^{\nu}_{\lambda}(m^2_{\text{high}}, m^2_{\text{low}})$$

The helicity amplitude with explicit permutation symmetry:

$$O^{\nu}_{\lambda} = T^{\nu}_{\lambda}(m^2_{\text{high}}, m^2_{\text{low}}) \;\boxed{+}\; (-1)^{\nu+\lambda} T^{\nu}_{\lambda}(m^2_{\text{low}}, m^2_{\text{high}})$$

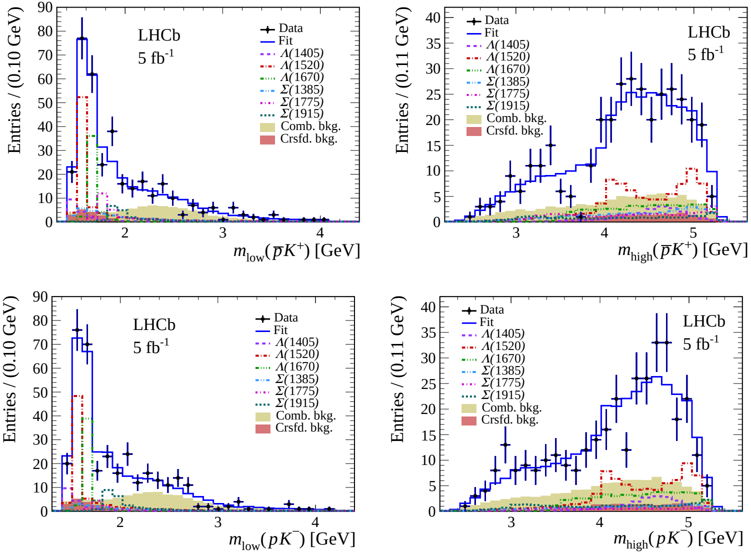
T^{ν}_{λ} is the resonance decay amplitude $\Xi_b^- \to R^0(\to K^- p)K^-$.

$(-1)^{\nu+\lambda}$: permutation of kaons $K_1^- \leftrightarrow K_3^-$ flips the x-z plane $\Rightarrow R_z(\pi)$

Clearly $\boxed{+}$ in covariant: $A = \bar{u}(p,\lambda)\left[\frac{\not p_{R_1} + m_{\text{high}}}{2m^2_{\text{high}}} + \frac{\not p_{R_3} + m_{\text{low}}}{2m^2_{\text{low}}}\right] u(p_b,\nu) \Rightarrow$ check!

Fit to $\Xi_b^- \to pK^-K^-$. CP test!

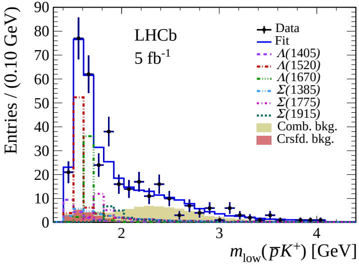
[arXiv:2104.15074]

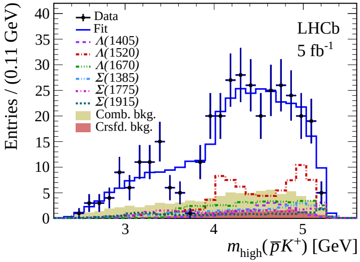


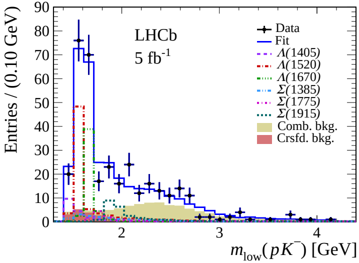
Fractions:
$\Sigma(1385) \to 0.26(11)(17)$
$\Lambda(1405) \to 0.19(6)(7)$
$\Lambda(1520) \to 0.76(9)(8)$
$\Lambda(1670) \to 0.45(7)(13)$
$\Sigma(1775) \to 0.22(8)(9)$
$\Sigma(1915) \to 0.26(9)(21)$
errors: (stat.)(syst.)

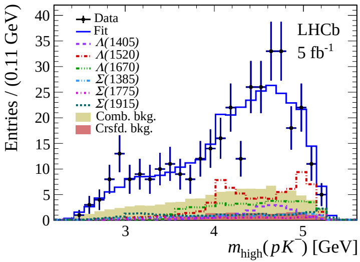
Fit to $\Xi_b^- \to pK^-K^-$. CP test!

[arXiv:2104.15074]









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errors: (stat.)(syst.)

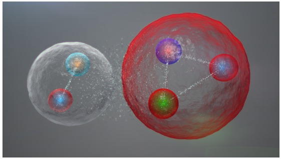
Asymmetries $A_{\mathrm{CP}} \times 10^{-2}$:

$\Sigma(1385) \to -27(34)(73)$
$\Lambda(1405) \to -1(24)(32)$
$\Lambda(1520) \to -5(9)(8)$
$\Lambda(1670) \to +3(14)(10)$
$\Sigma(1775) \to -47(26)(14)$
$\Sigma(1915) \to +11(26)(22)$
errors: (stat.)(syst.)

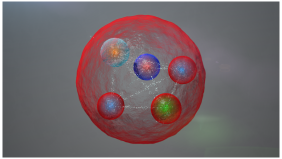
Amplitude analysis of $B_s^0 \to J/\psi p\bar{p}$

[LHCb-PAPER-2021-018 (NEW! in preparation)]

Studies of pentaquarks in LHCb



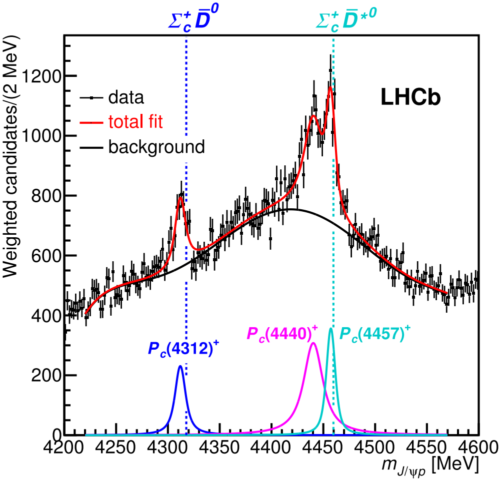
Molecule:
interacting color-singlets



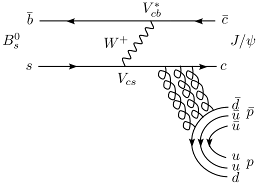
Tightly-bound:
interacting color-octets

- P_c $\Lambda_b^0 \to (J/\psi p)K^-$ [PRL 122 (2019) 22, 222001] (AmAn is ongoing)
- P_c $\Lambda_b^0 \to (J/\psi p)\pi^-$ [PRL 117 (2016) 082003] (full statistics AmAn is ongoing)
- "P_{cs}" $\Xi_b^- \to J/\psi \Lambda K^-$ [Sci.Bull. 66 (2021) 1278-1287]
- "P_c" $B_s^0 \to J/\psi p\bar{p}$ [LHCb-PAPER-2021-018]

$$\Lambda_b^0 \to J/\psi p K^-$$

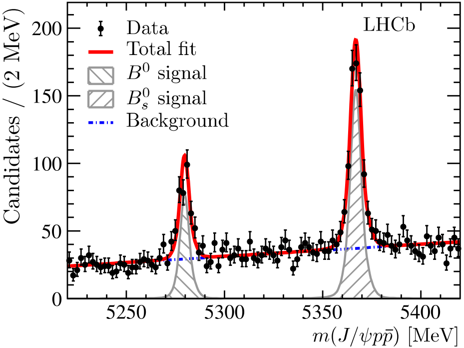


$B_s^0 \to J/\psi p\bar{p}$



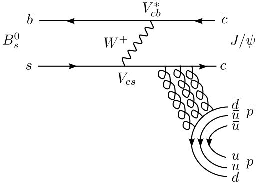
[PRL 122, 191804 (2019)]

- Pentaquarks(?)
- Glueballs(?)



$B_s^0 \to J/\psi p\bar{p}$

[LHCb-PAPER-2021-018 (in preparation)]



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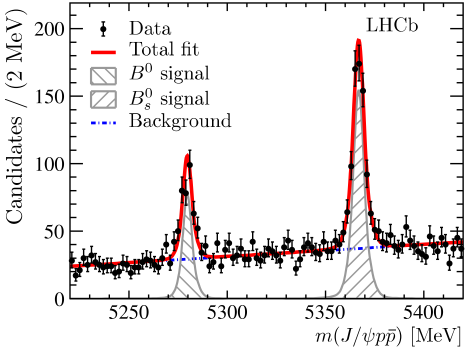
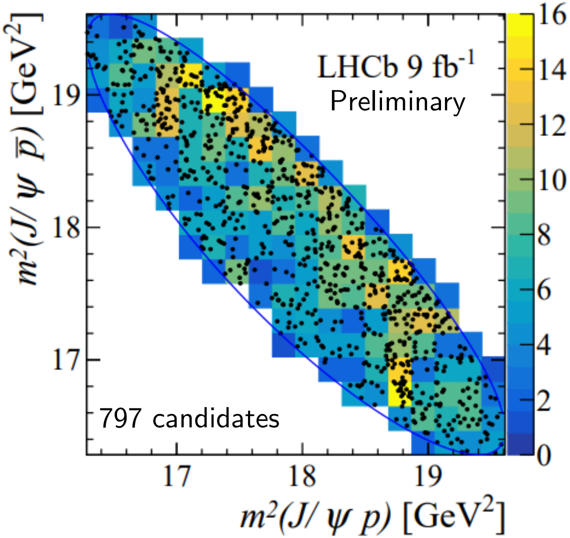




Amplitude analysis

$$4\,\text{dim.}: \qquad A \to \overbrace{▧(m^2_{J/\psi p}, m^2_{J/\psi \bar{p}})}^{\text{Dalitz plot}} \times \overbrace{▧(\theta_{J/\psi}, \phi_{J/\psi})}^{J/\psi\to\mu^+\mu^-)}.$$

Three decay chains: $▧ = ▧^{X}(m^2_{p\bar{p}}) + ▧^{P_c^+}(m^2_{J/\psi p}) + ▧^{P_c^-}(m^2_{J/\psi \bar{p}})$

Symmetries of the decay

- Resonances in $J/\psi p$ and $J/\psi \bar{p}$ are CP-conjugated
- Initial state (B_s) is not flavor-tagged $\Rightarrow$ symmetric model
- Non-resonant $p\bar{p}$ with $J_X^P = 1^{--}$ is null-hypothesis (CP-even).
- Combination of P_c^+ and P_c^- is CP-odd $\Rightarrow$ total interference vanishes
- For $J^P_{P_c} = 1/2^+, 3/2^-$, interference of $P_c^+ + P_c^-$ with X vanishes on Dalitz (PC vs PV decays)

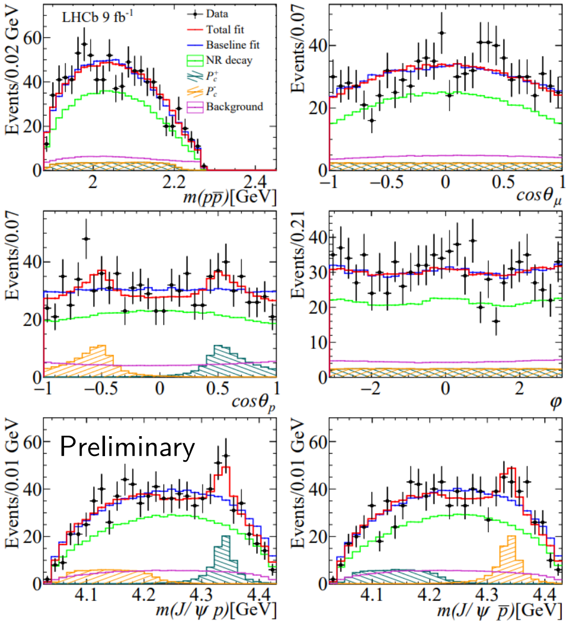
Results

- (Two) new P_c^+ states are observed:

$$m_{P_c} = 4337^{+7}_{-4}\,(\text{stat})^{+2}_{-2}\,(\text{syst})\,\text{MeV},$$
$$\Gamma_{P_c} = 29^{+26}_{-12}\,(\text{stat})^{+14}_{-14}\,(\text{syst})\,\text{MeV},$$

  with significance $> 3\,\sigma$.
- No evidence for narrow $P_c^+(4312)$ from Λ_b^0 decay
- No evidence for the glueball state $f_J(2220)$
- $J^P \in \{1/2^\pm, 3/2^\pm\}$ are tested. No strong preference is found.

See also Liupan talk on Friday

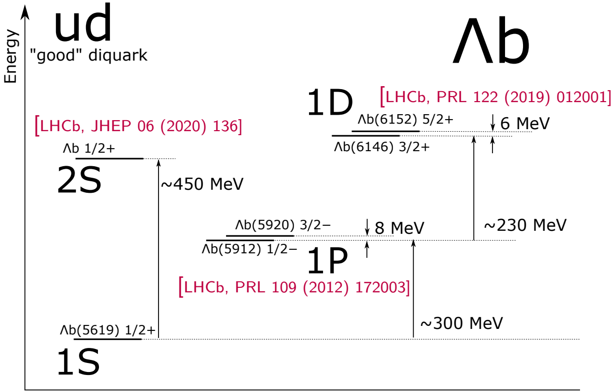


Three-body dynamics in decays of Λ_b^{**0}

[LHCb, JHEP 06 (2020) 136]

Excited Λ_b resonances

Spectroscopy of the good diquark

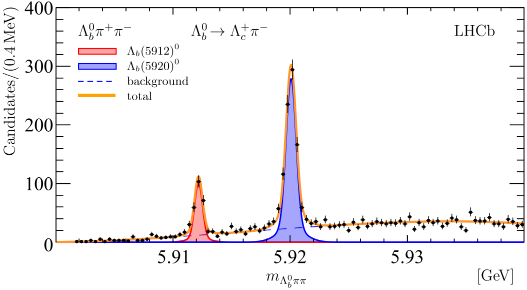


Diquark picture, $Q(qq)$ with (qq) having $J^P = 0^+$, works very well for yet observed states.

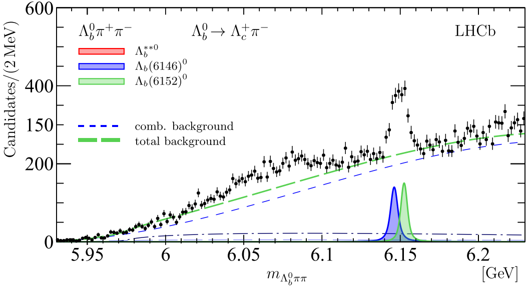
$\Lambda_b^0\pi^+\pi^-$ spectrum

[LHCb, JHEP 06 (2020) 136]

Low-mass part of the spectrum



High-mass part of the spectrum



The broad structure:

- An elastic resonance in a system $\Lambda_b^0\pi^+\pi^-$
- Three coupled quasi-stable channels: $\Lambda_b^0 f_0$, $\Sigma_b^\pm\pi^\mp$, and $\Sigma_b^{*\pm}\pi^\mp$

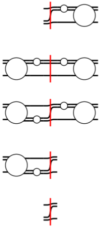
$\Rightarrow$ an excellent opportunity to explore the three-body unitarity

Three-body unitarity constraint

[MM et al.(JPAC), JHEP 08 (2019) 080]

Unitairity equation for the reduced amplitude

$$
\begin{aligned}
&\mathcal{T}(\sigma', s, \sigma) - \mathcal{T}^\dagger(\sigma', s, \sigma) = \\
&\quad 2i\,\frac{1}{\lambda_s^{1/2}(\sigma')}\int_{\sigma^-(\sigma',s)}^{\sigma^+(\sigma',s)} \mathrm{d}\sigma_3'\, t(\sigma_3')\mathcal{T}(\sigma_3', s, \sigma) \\
&\quad + \frac{i}{3}\int_4^{(\sqrt{s}-1)^2} \frac{\mathrm{d}\sigma''}{2\pi}\mathcal{T}^\dagger(\sigma', s, \sigma'')\, t(\sigma'')t^\dagger(\sigma'')\,\rho(\sigma'')\rho_s(\sigma'')\,\mathcal{T}(\sigma'', s, \sigma) \\
&\quad + 2i\iint_{\phi(\sigma_2'',\sigma_3'',s)>0} \frac{\mathrm{d}\sigma_2''\mathrm{d}\sigma_3''}{2\pi s}\,\mathcal{T}(\sigma', s, \sigma_2'')t(\sigma_2'')t(\sigma_3'')\mathcal{T}(\sigma_3'', s, \sigma) \\
&\quad + \frac{2i}{3}\,\frac{1}{\lambda_s^{1/2}(\sigma)}\int_{\sigma^-(\sigma,s)}^{\sigma^+(\sigma,s)} \mathrm{d}\sigma_2\,\mathcal{T}^\dagger(\sigma', s, \sigma_2)t^\dagger(\sigma_2) \\
&\quad + 6i\,\frac{2\pi s}{\lambda_s^{1/2}(\sigma')\lambda_s^{1/2}(\sigma)}\,\theta(\phi(\sigma', \sigma, s)).
\end{aligned}
$$



Related works:

- [Mai et al., EPJA 53 (2017) 9, 177]
- [Jackura et al.(JPAC), EPJC 79 (2019) 1, 56]

Three-main aspects:

- Two-body rescattering effects
  → influence of subchannel resonances to each-other lineshape
- Genuine three-to-three dynamics, generated states (the ladder)

[o] Lineshape of the regular states decaying to three particles

Unitairy resonance kernel

[MM et al.(JPAC), JHEP 08 (2019) 080]

side reminder

- A two-body resonance amplitude

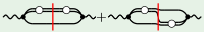
$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - im\Gamma(s)} = \frac{g^2}{m^2 - s - ig^2\Phi_2(s)/2}$$

$\Phi_2(s)$ is a two-body phase-space

A resonance in three-body system

An approximate-three-body unitarity

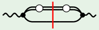
$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - ig^2/2\,[\;\cdots\;]}$$



contains effect of the subchannel-resonances **interference**

The quasi-two-body approximation

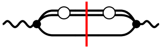
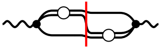
$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - ig^2/2\,[\;\cdots\;]}$$



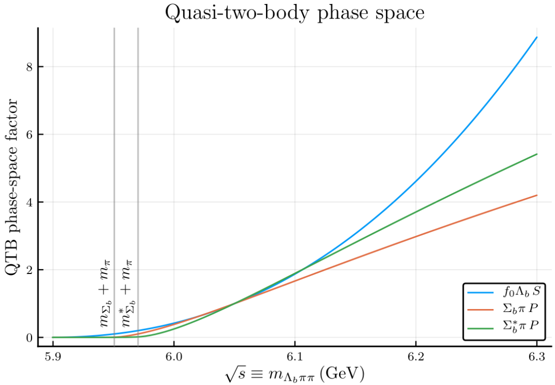
naively accounts for the subchannel-resonance decay

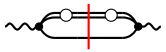
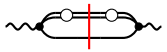
+ dispersion integral to ensure analytic structure (important close to threshold)

Appliction to $\Lambda_b^0\pi^+\pi^-$. Self-energy function

$\rho(s) =$  $+$ 

$$= \begin{pmatrix} g_{\Lambda_b^0 f_0} \\ g_{\Sigma_b^\pm \pi^\mp} \\ g_{\Sigma_b^{*\pm}\pi^\mp} \end{pmatrix}^\dagger \begin{pmatrix} \Lambda_b^0 f_0 | \Lambda_b^0 f_0 & \Lambda_b^0 f_0 | \Sigma_b^\pm \pi^\mp & \Lambda_b^0 f_0 | \Sigma_b^{*\pm}\pi^\mp \\ \Sigma_b^\pm\pi^\mp | \Lambda_b^0 f_0 & \Sigma_b^\pm \pi^\mp | \Sigma_b^\pm \pi^\mp & \Sigma_b^\pm\pi^\mp | \Sigma_b^{*\pm}\pi^\mp \\ \Sigma_b^{*\pm}\pi^\mp | \Lambda_b^0 f_0 & \Sigma_b^{*\pm}\pi^\mp | \Sigma_b^\pm \pi^\mp & \Sigma_b^{*\pm}\pi^\mp | \Sigma_b^{*\pm}\pi^\mp \end{pmatrix} \begin{pmatrix} g_{\Lambda_b^0 f_0} \\ g_{\Sigma_b^\pm \pi^\mp} \\ g_{\Sigma_b^{*\pm}\pi^\mp} \end{pmatrix}$$



$\sim \sqrt{}$  $\begin{pmatrix} 1 & z & z_* \\ \cdot & 1+y & x \\ \cdot & \cdot & 1+y_* \end{pmatrix} \sqrt{}$ 

with $x(m^2) = -0.014 + 0.004i$
$z(m^2) = -0.05 - 0.12i$,
$z_*(m^2) = -0.01 - 0.07i$,
$y(m^2) = 0.003$, and
$y_*(m^2) = -0.02$

$\Rightarrow$ **rescattering effects are negligible**

Fit to the data

[LHCb, JHEP 06 (2020) 136]

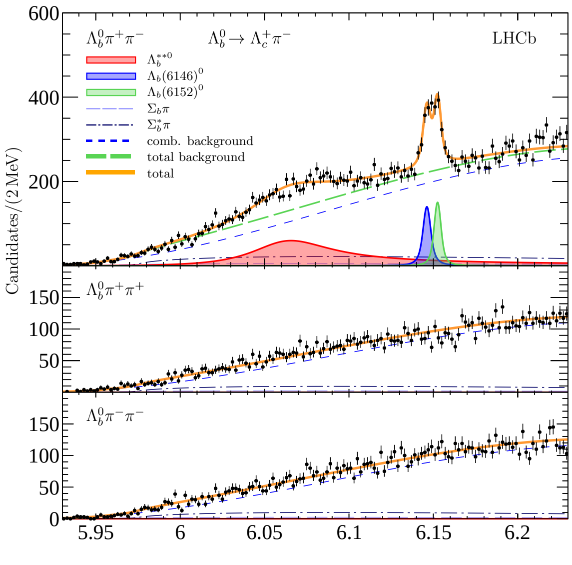
Parameters of the new $\Lambda_b^0(6072)$:

$$m = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2\,\mathrm{MeV}$$
$$\Gamma = 72 \pm 11 \pm 2\,\mathrm{MeV}$$

Advance parametrization slightly impacts the width measurement with respect to the naive BW

- smoothly goes underneath $\Sigma_b\pi$ threshold
- can describe Dalitz plot distribution

Currently, is being applied to move to complex examples [news are soon], . . .



Summary

Amplitude analysis is a critical tool for the further progress

- Cooperation between theory and experiment will speed up the progress
- JPAC is active in both supporting experimental analyses and leading the phenomonological investigations.
  - see talk on XYZ at EIC by D. Winney on Monday [link]
  - see talk on glueball search by A. Rodas on Monday [link]
  - see talk on COMPASS physics by L. Bibrzycki on Wednesday [link]
  - see talk on photoproduction by A. Blin on Monday [link]

Wide range of applications of the advanced amplitudes in LHCb. Just a few examples:

- Search for CP violation in baryonic decays
- Search for Pentaquarks in angular analysis
- Lineshape analysis for Λ_b^0 spectroscopy

Thank you for the attention