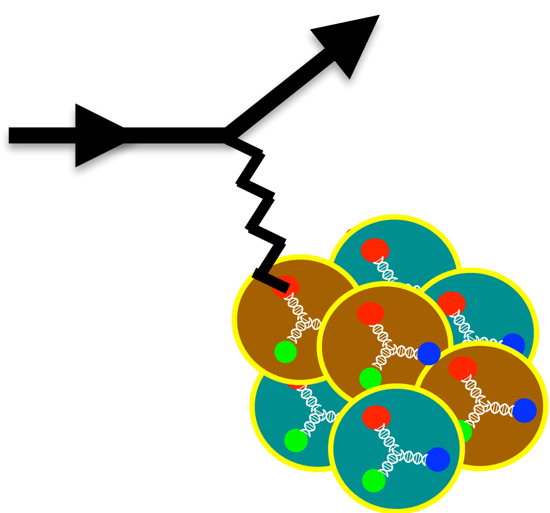
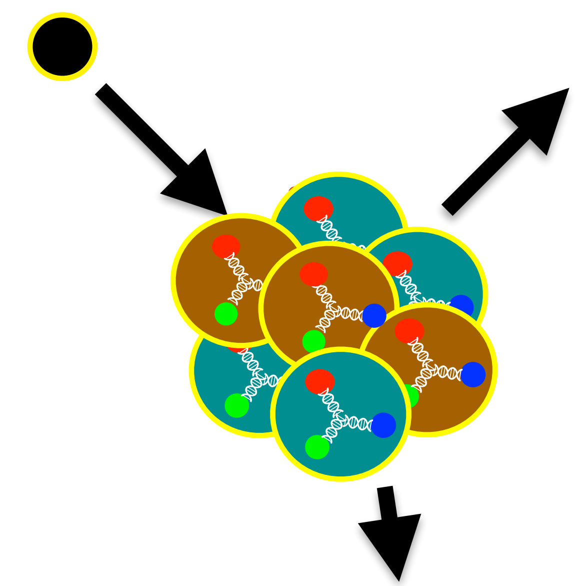
Nuclear matrix elements from lattice QCD for electroweak and beyond-Standard-Model processes



Michael Wagman



Davoudi, Detmold, Orginos, Parreño, Savage, Shanahan, MW, Phys. Rept. 900 (2021)

Hadron 2021

19th International Conference on Hadron Spectroscopy and Structure

July 26, 2021

Fermilab

NPLQCD

Thanks to my collaborators

Beane

Chang

Davoudi

Detmold

Illa

Murphy

Orginos

Parreño

Savage

Shanahan

Tiburzi

Wagman

Winter

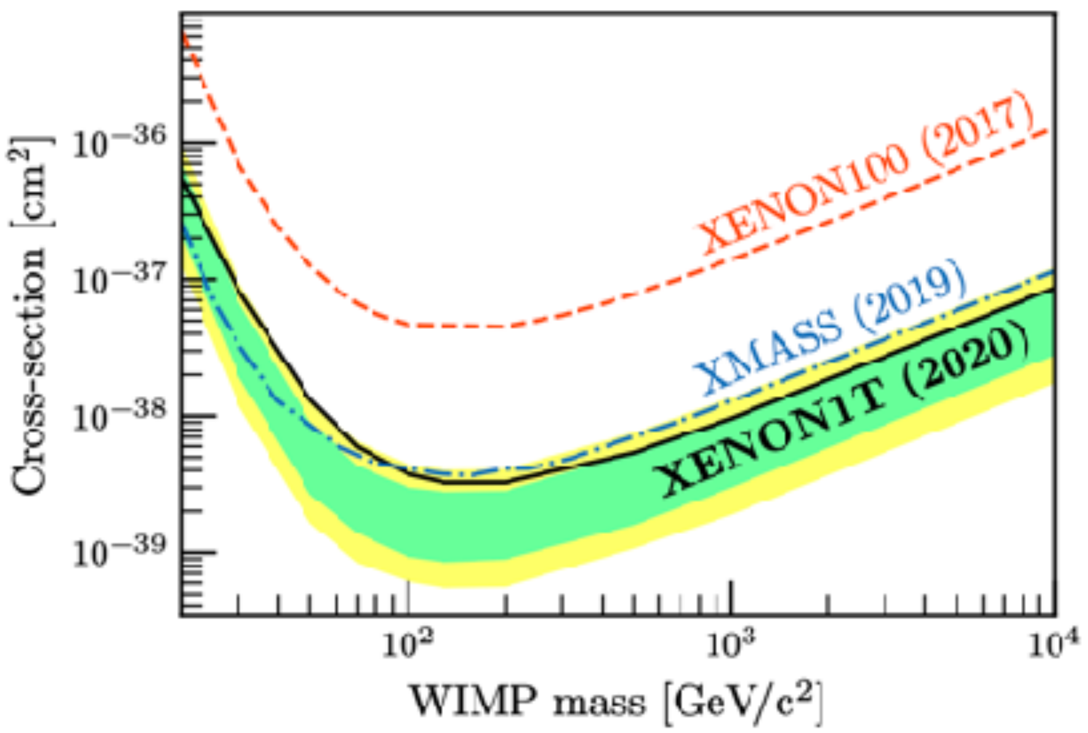
New physics and nuclei

Nuclei are abundant and useful experimental targets

Converting between nuclear- and nucleon-level cross-sections requires

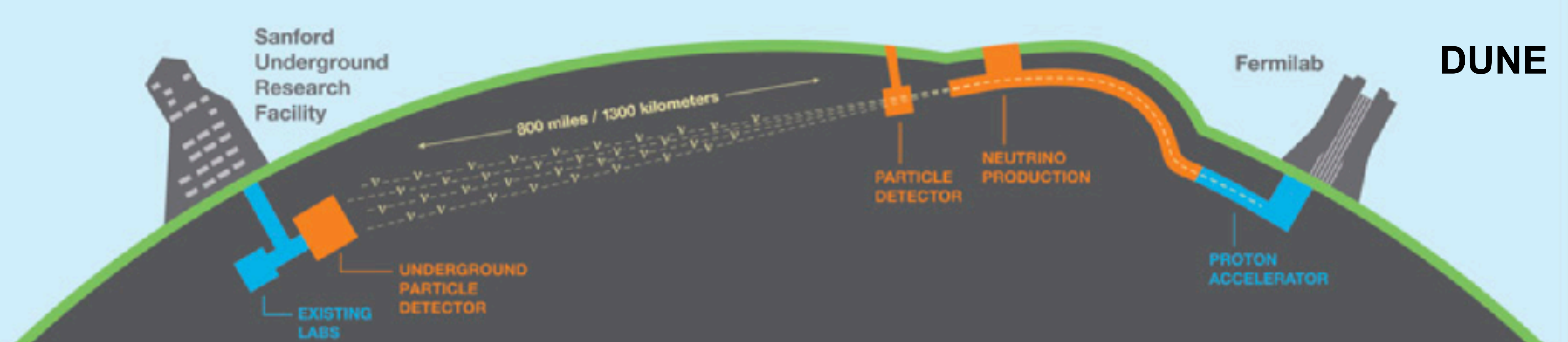
- Nuclear models (uncertainty quantification?)
- Direct LQCD calculations (impractical)
- LQCD informed EFT + modeling

Xenon1T constraint on dark matter-nucleus



New physics searches such as dark matter direct detection and $0\nu\beta\beta$ require nuclear matrix elements to relate experimental observables and theory parameters

Standard Model predictions with controlled uncertainties essential for next-generation accelerator neutrino experiments aiming for few-percent systematic uncertainties



Many-quark bound-state structure

The partonic structure of nuclei is noticeably different from the nucleon

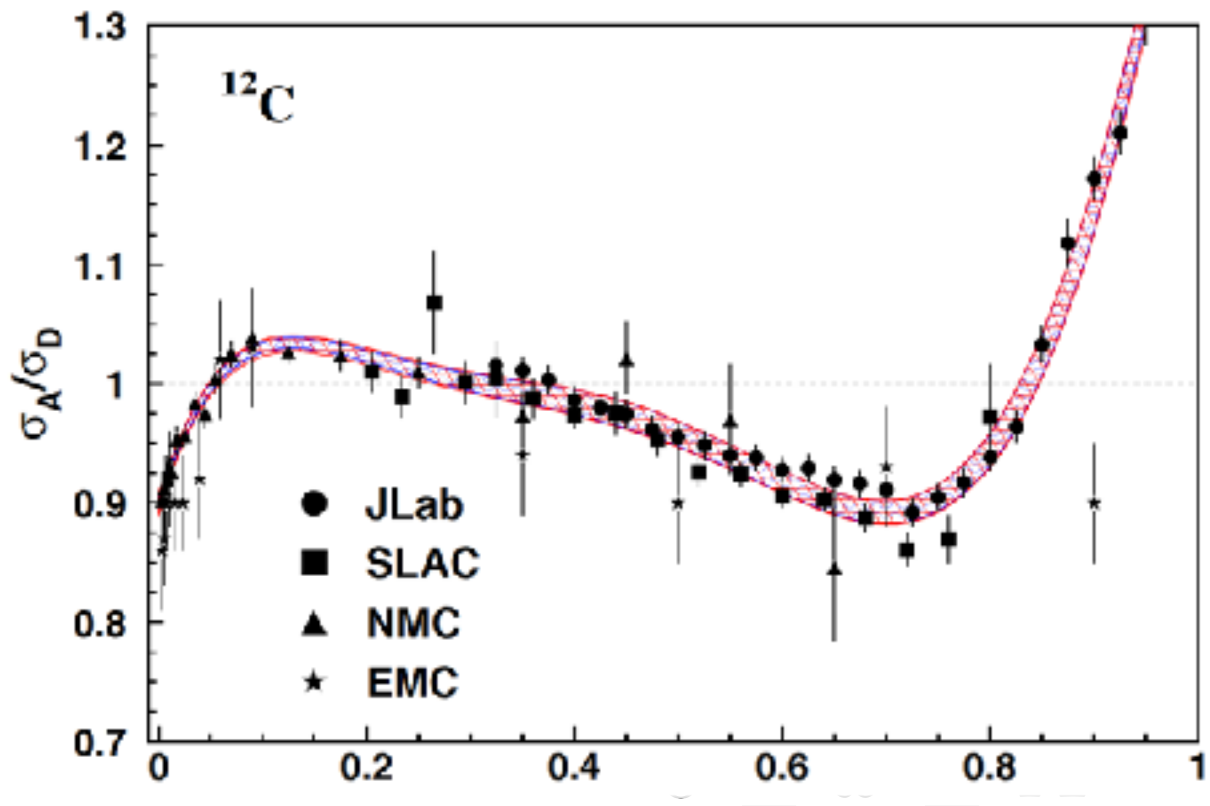
Aubert et al (EMC), Phys. Lett. 123B (1983)

The emergence of the EMC effect and it's analogs from QCD is not yet understood

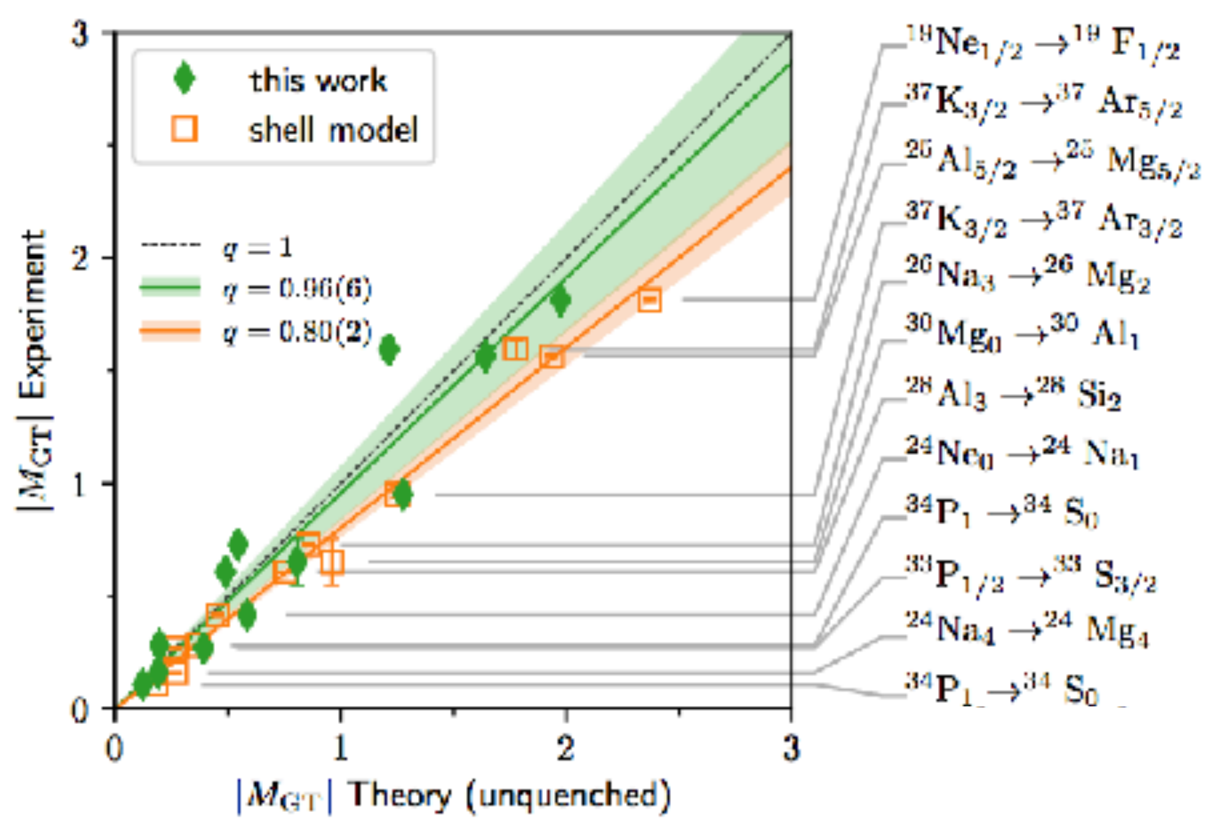
Review: Hen et al, Rev. Mod. Phys. 89 (2017)

LQCD results for light nuclei can constrain EFT and test models of EMC effect

Chen and Detmold, Phys. Lett. B 625 (2005)



Malace et al, Int. J. Mod. Phys. E 23 (2014)



Gysbers et al, Nature Phys. 15 (2019)

Electroweak reaction rates can be reproduced without "quenching" g_A in chiral EFT and nuclear model calculations including multi-nucleon correlations and currents

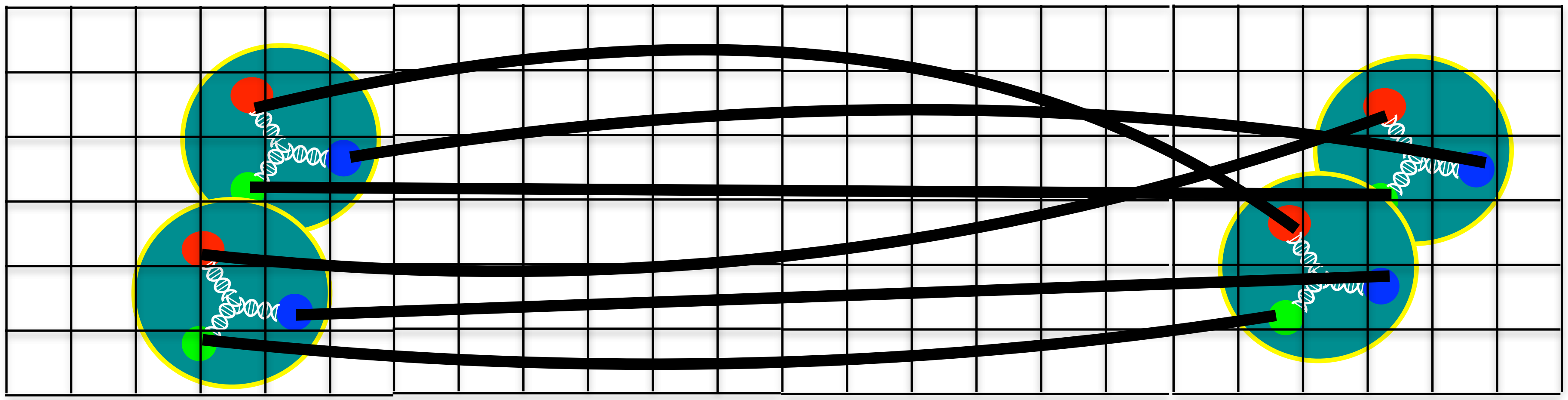
Pastore et al, PRC 97 (2018)

Gysbers et al, Nature Phys. 15 (2019)

Do nuclear effects on axial currents emerge from the Standard Model?

Nuclear matrix elements

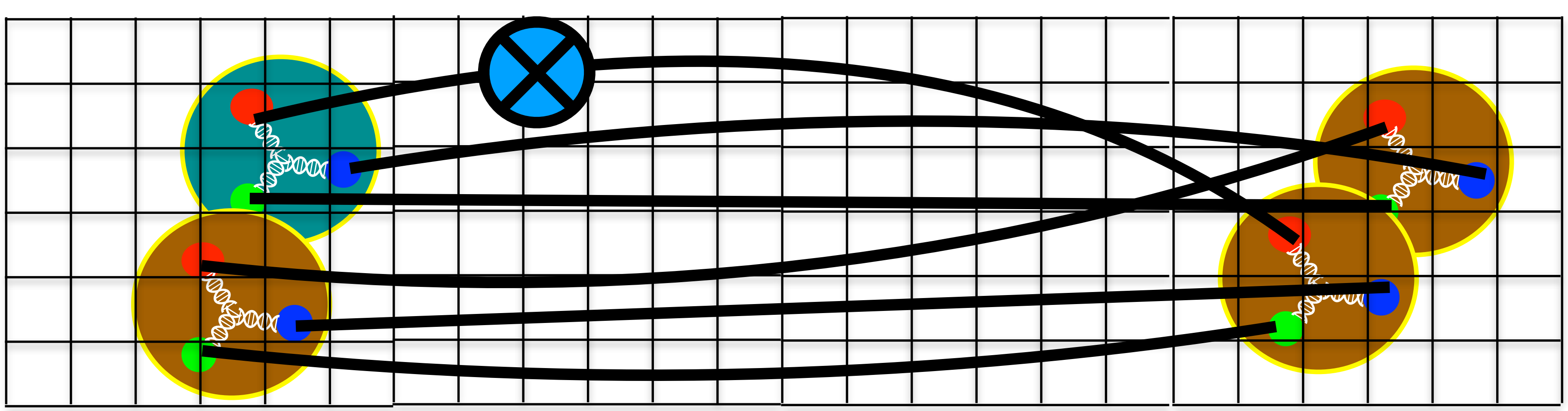
LQCD spectrum determined from 2-point correlation functions



$$C_A(t) = \langle A(t) A^\dagger(0) \rangle = \sum_n \langle 0 | A(0) e^{-Ht} | n \rangle \langle n | A^\dagger(0) | 0 \rangle + \ldots$$

$$= \sum_n |Z_n|^2 e^{-E_n t}$$

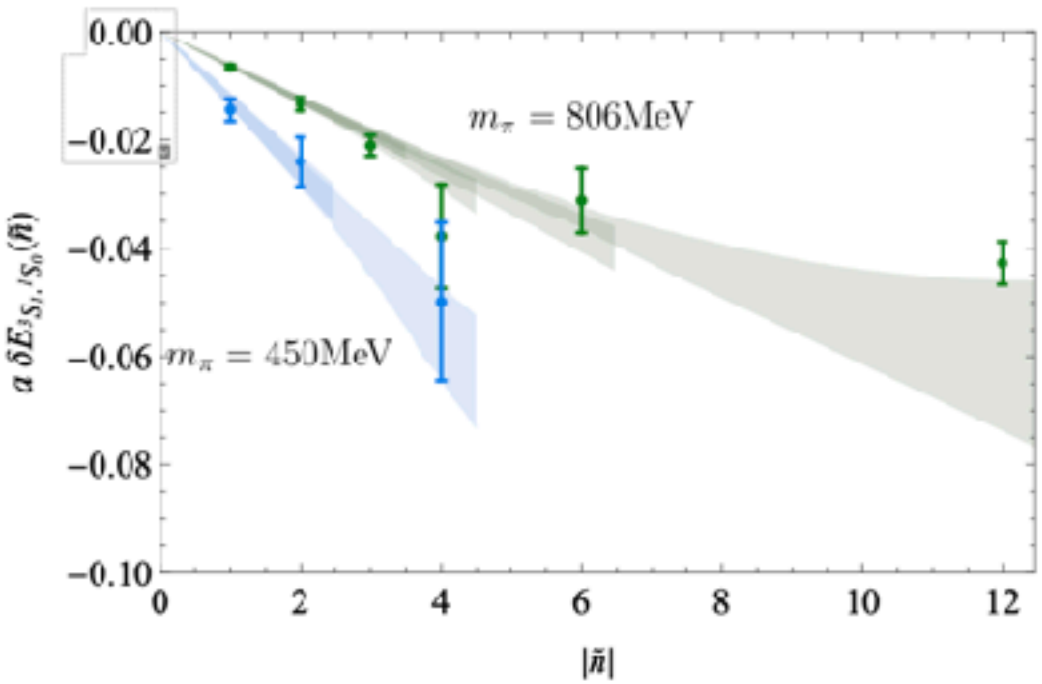
Nuclear matrix elements determined from 3-point correlation functions including a local operator insertion

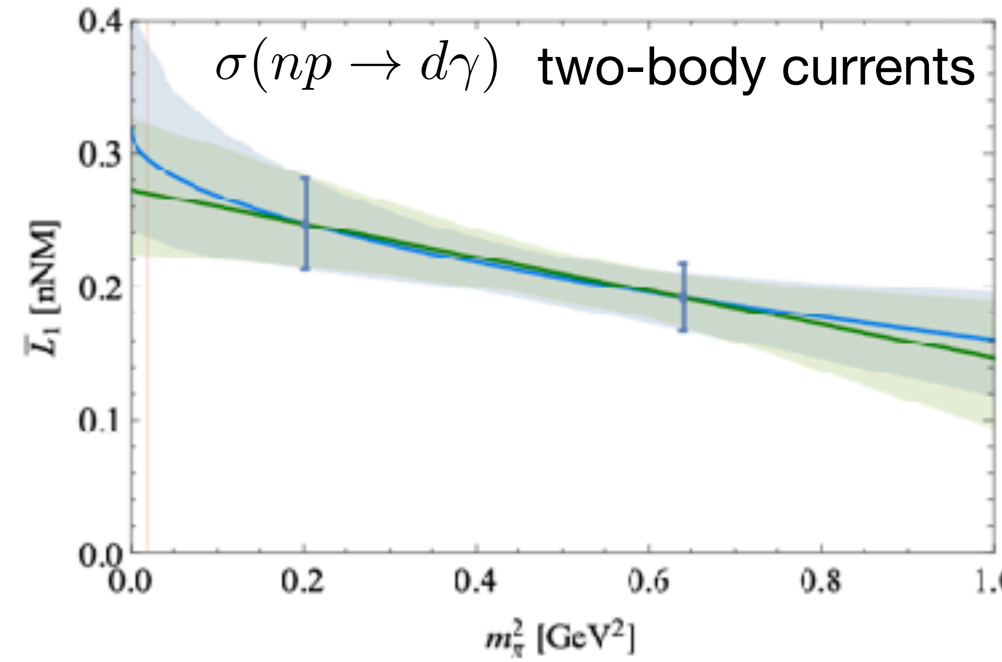


Electromagnetic structure of nuclei

First nuclear matrix elements computed from linear response to applied background magnetic fields, e.g. ${}^3S_1 - {}^1S_0$ energy splitting

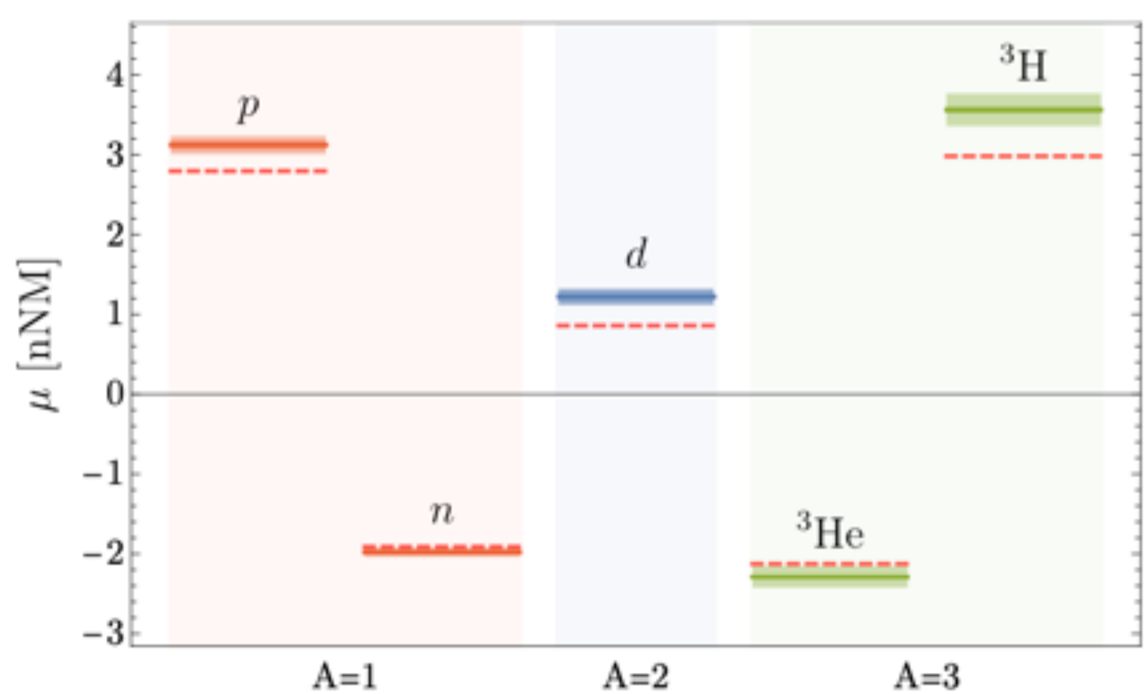
Beane et al [NPLQCD], PRL 115 (2015)

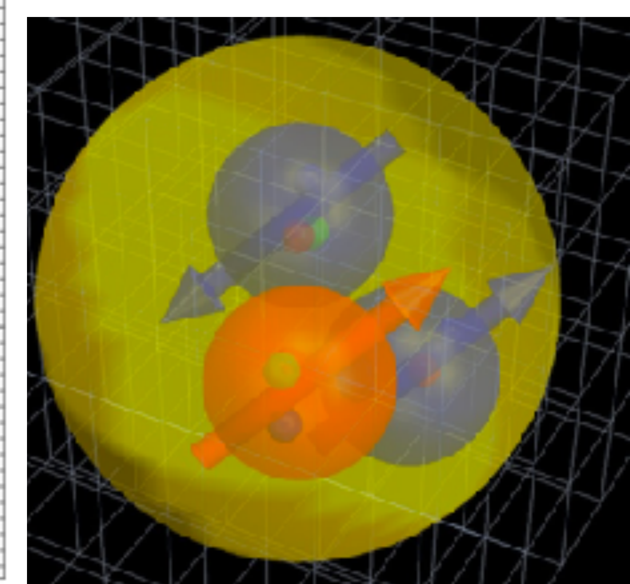




Magnetic moments of light nuclei computed with heavier-than-physical quark masses

Results show simple mass dependence, consistency with shell model expectations

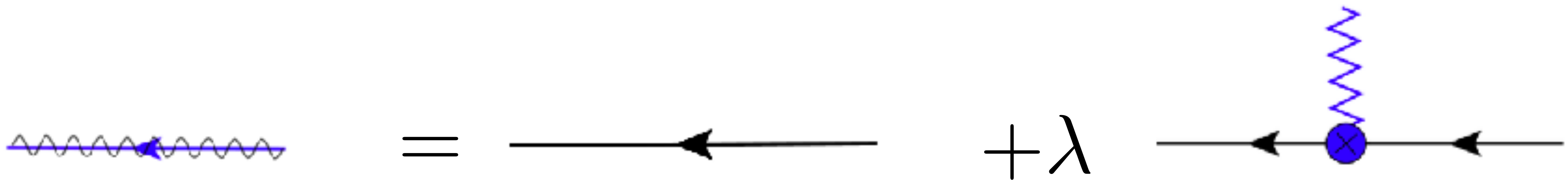




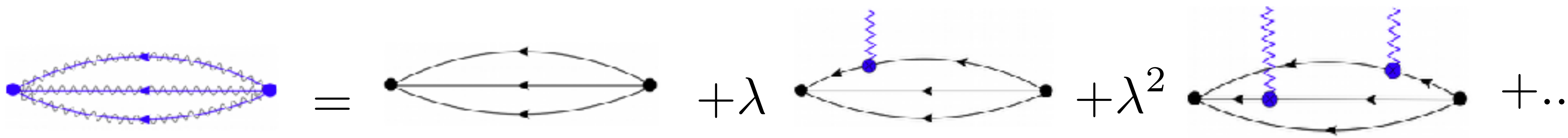
Beane et al [NPLQCD], PRL 113 (2014)

Fixed-order background fields

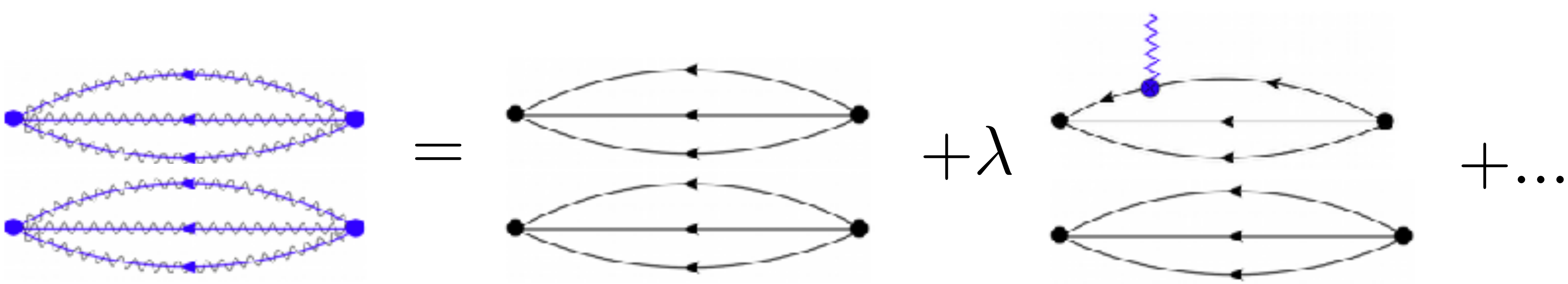
Basic input: linear combinations of quark propagators with propagators that contain a current inserted everywhere ("background field")



Correlation functions encode linear (quadratic, …) response to background field in linear (quadratic, …) terms of polynomial



Nuclear correlation functions with current insertions can be formed as straightforwardly as two-point functions, and desired responses extracted using linear algebra



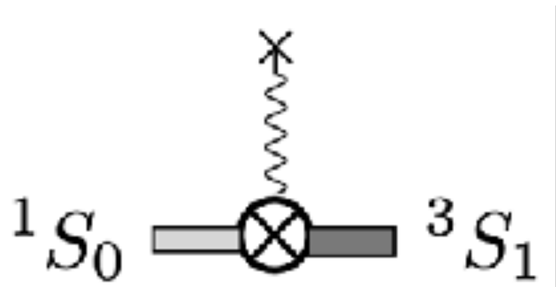
Savage, MW et al [NPLQCD], PRL 119 (2017) Tiburzi, MW et al [NPLQCD], PRD 96 (2017)

Electroweak reactions

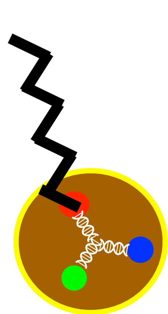
Axial current transition matrix element between spin-singlet and spin-triplet np systems computed using fixed-order background fields

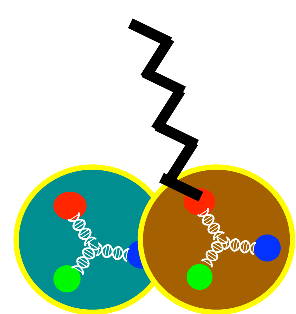
Savage, MW et al [NPLQCD], PRL 119 (2017)

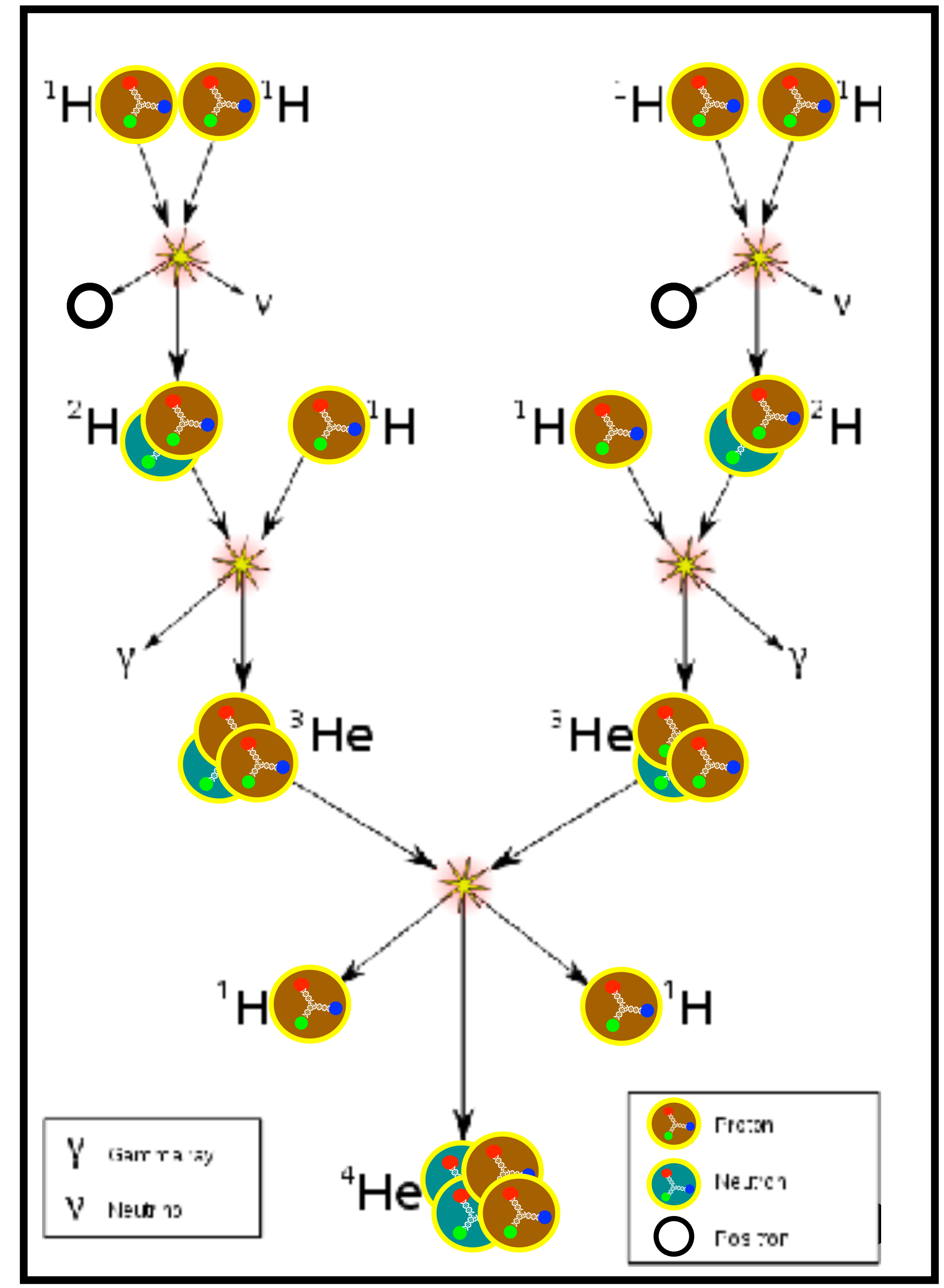
LQCD results matched to pionless EFT by computing same background-field correlation function using dibaryon-field formalism



Results used to constrain LEC for two-body axial current operator in pionless EFT

g_A 

L_{1A} 

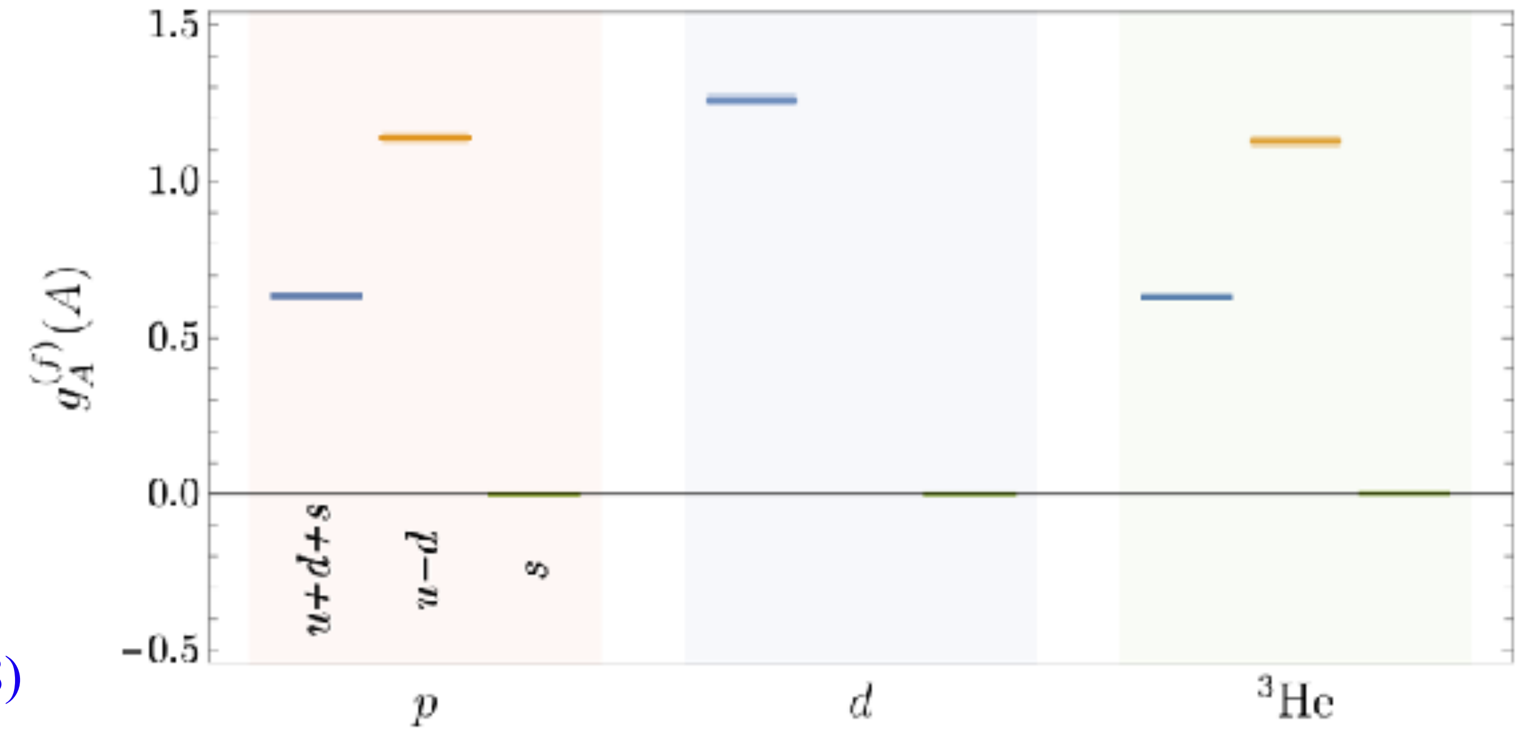


Same operator relevant for proton-proton fusion and other reactions, future LQCD calculations could improve phenomenological predictions

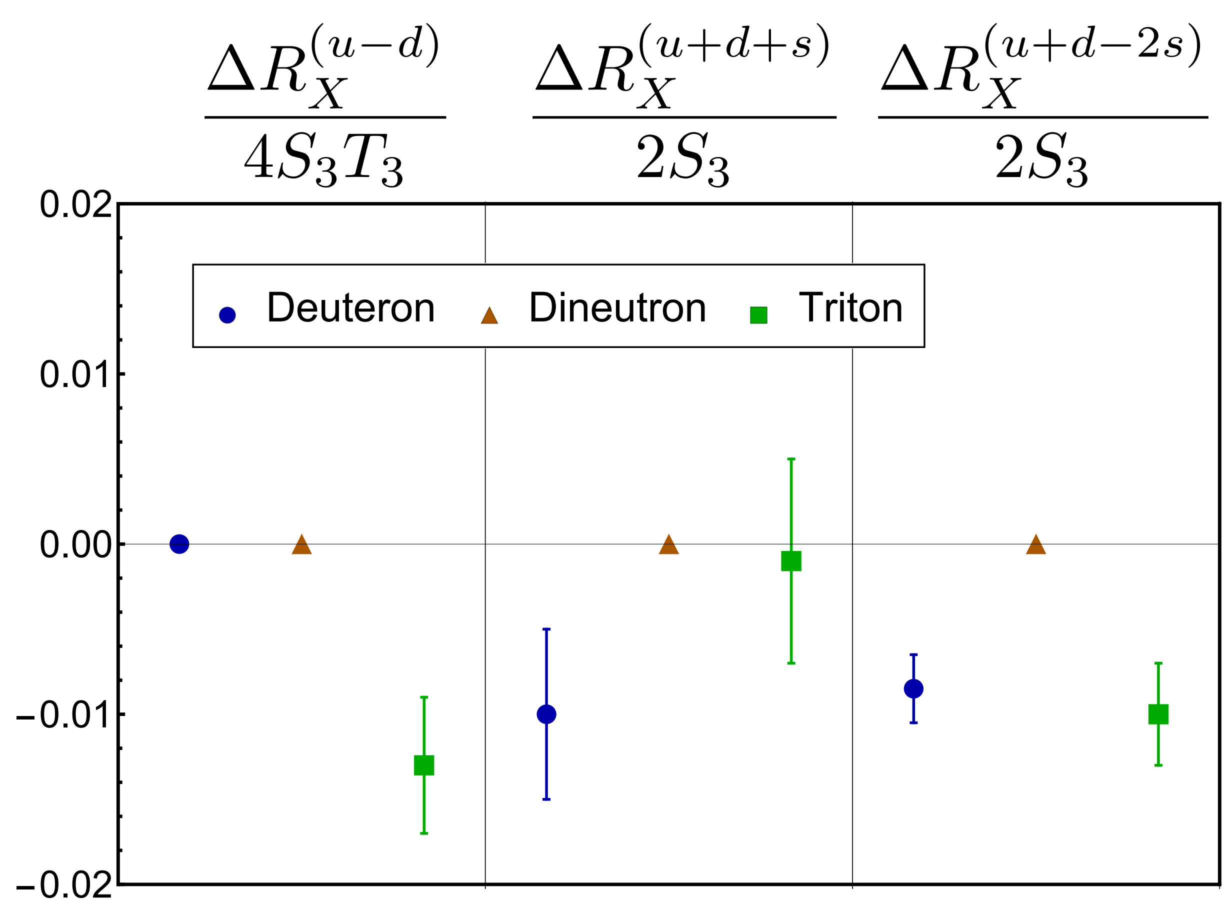
Axial matrix elements

Flavor decomposition of axial matrix elements of up to three nucleon systems computed with $m_\pi = 806$ MeV (1 lattice spacing / volume)

Chang, MW et al [NPLQCD], PRL 120 (2018)

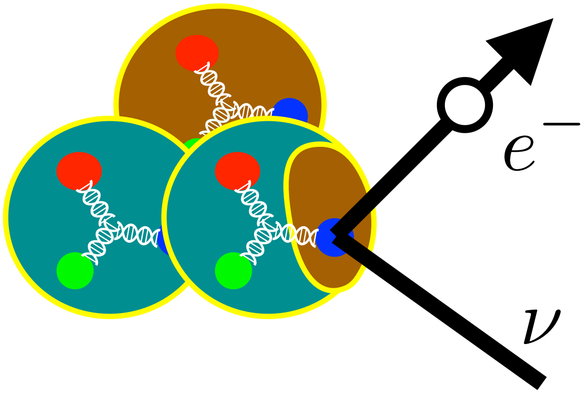


$N_f = 3,\ m_\pi = 806(9)$ MeV, $a = 0.145(2)$ fm



Fractional differences from naive shell model expectations show that multi-nucleon correlations lead to percent-level effects on axial charges of light nuclei for these quark masses

Triton β decay



Triton β - decay rate governed by Gamow-Teller matrix element

$$g_A({}^3\mathrm{H}) = |\langle {}^3\mathrm{He}|A_3^+|{}^3\mathrm{H}\rangle| = |\langle {}^3\mathrm{H}|A_3^+|{}^3\mathrm{H}\rangle|$$

Computed in ChEFT Baroni et al, PRC 98 (2018)

After fitting LECs to experimental triton β-decay rate predicts

$$\frac{|\langle {}^3\mathrm{He}|A_3^+|{}^3\mathrm{H}\rangle|}{g_A} = 0.951(13)$$

Deviations from 1 arise from two-body currents and multi-nucleon interactions

NLO calculations in pionless EFT relate nuclear effects to the two-body axial current coupling L_{1A} appearing in proton-proton fusion

De-Leon, Platter, Gazit (2016)

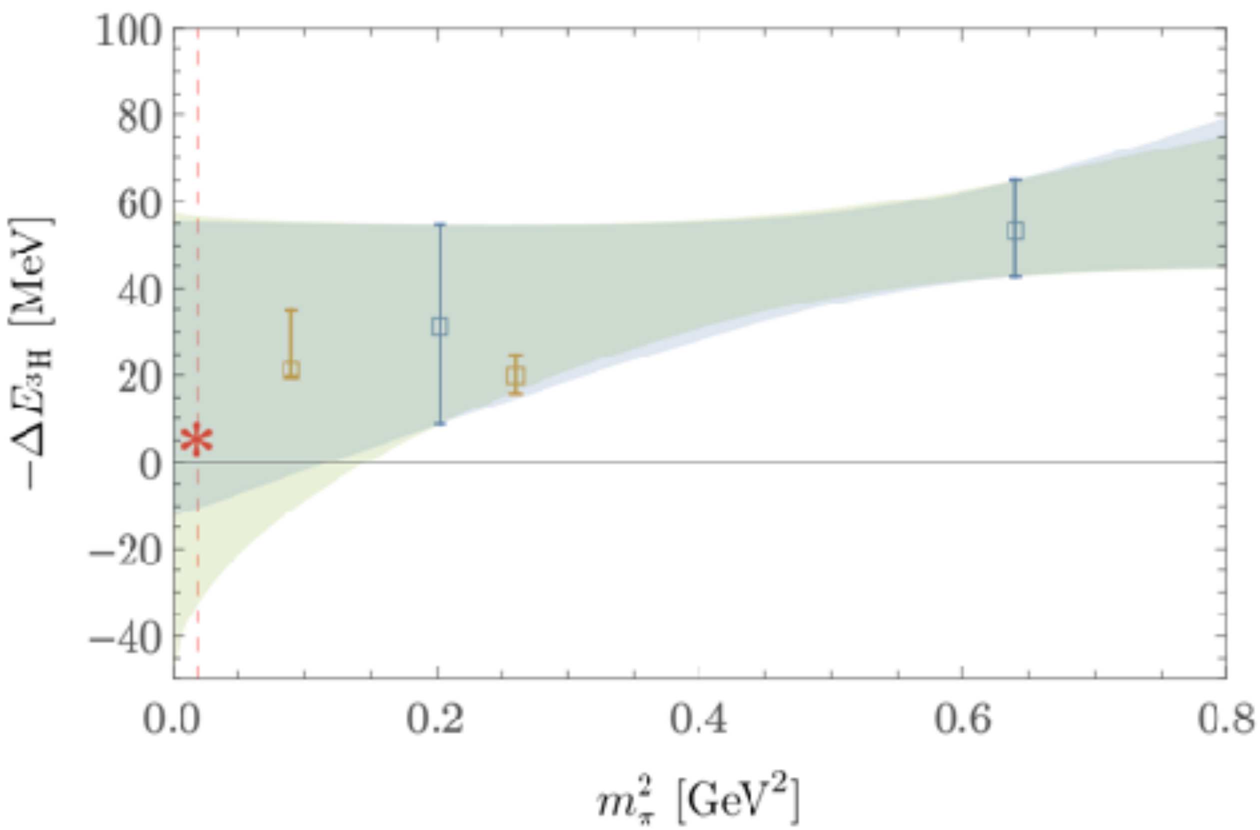
Triton β decay from LQCD

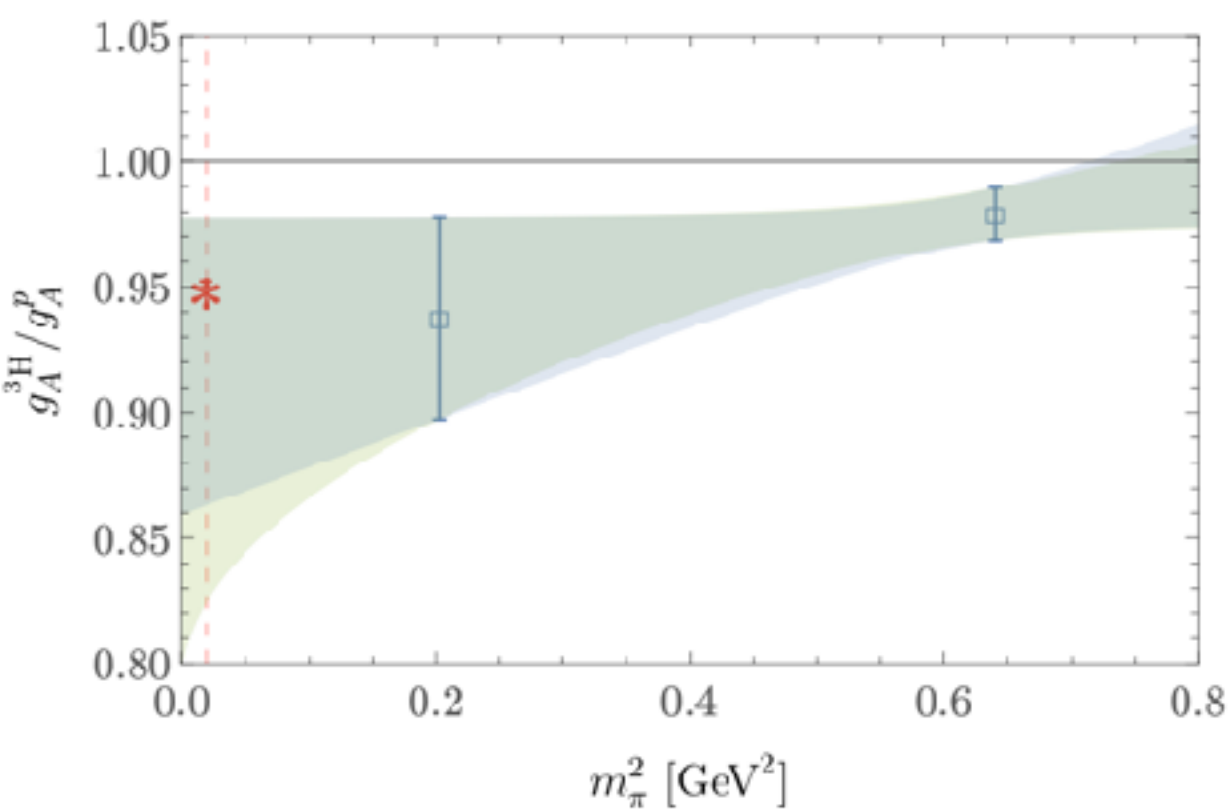
LQCD calculations of triton recently performed using $m_\pi = 450$ MeV

Parreño, MW et al [NPLQCD] PRD 103 (2021)

Signal-to-noise problem makes calculations exponentially noisier at lighter quark masses

Results consistent with bound triton obtained on 3 volumes





Parreño, MW et al [NPLQCD] PRD 103 (2021)

Axial current matrix element calculations with $m_\pi = 450$ MeV permit preliminary extrapolation to physical point

Several systematic uncertainties remain, but encouraging agreement with experiment seen

Matching to finite-volume pionless EFT used to constrain L_{1A}

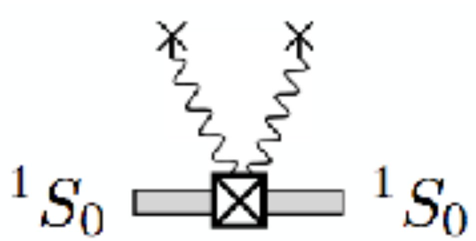
Detmold and Shanahan, PRD 103 (2021)

Double β decay

Doubly-weak reactions important but less well-understood phenomenologically

Double-beta decay reactions include additional two-body currents not present in single-beta decay

Isotensor axial polarizability:



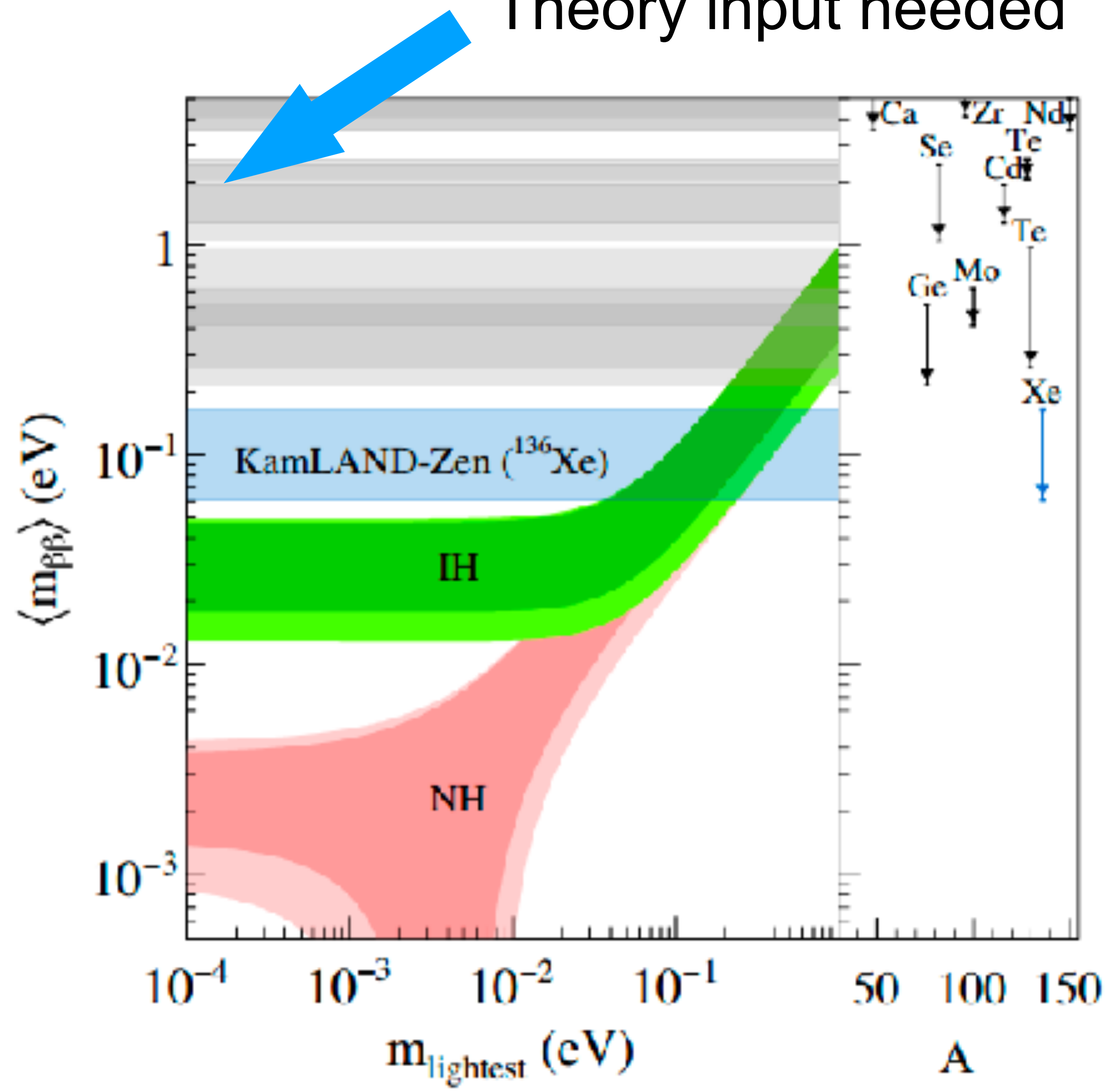
Shanahan, MW et al [NPLQCD], PRL 119 (2017)

Two-body currents needed at leading order in ChEFT

Cirigliano et al, PRL 120 (2018)

LQCD studies of light nuclei can be used to fix unknown LEC

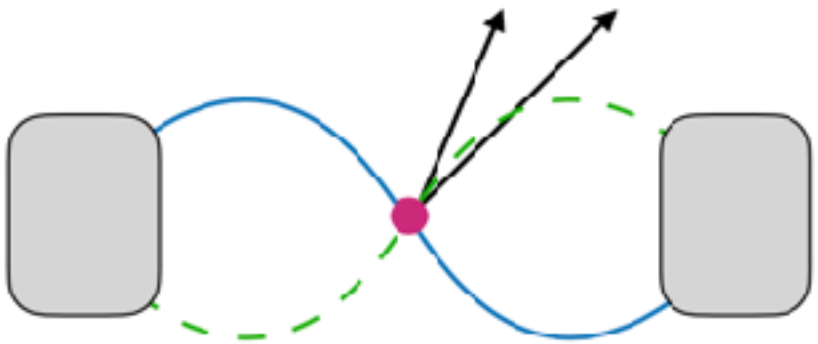
Theory input needed



$0\nu\beta\beta$ in LQCD

Matrix elements for $\pi^- \rightarrow \pi^+ e^- e^-$ arising from short-distance new physics mechanisms computed

Nicholson et al, PRL 121 (2018)

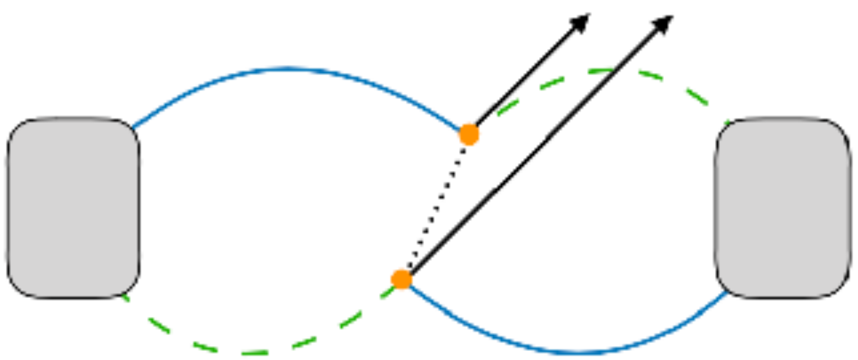


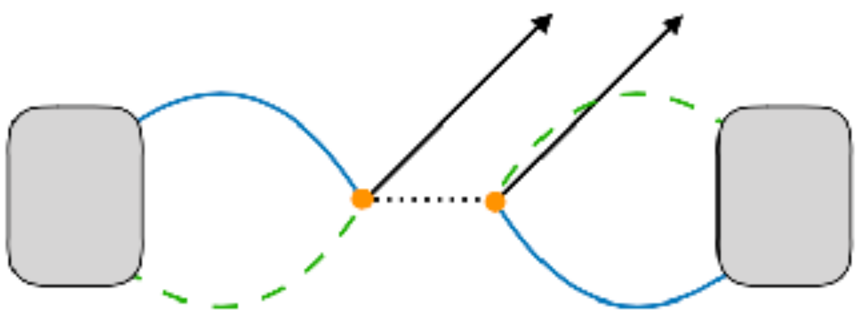
Cirigliano et al, arXiv:2003.08493

Same matrix elements enter nuclear processes in EFT

Long-distance Majorana neutrino exchange matrix elements computed for $\pi^- \pi^- \rightarrow e^- e^-$ and $\pi^- \rightarrow \pi^+ e^- e^-$

Feng et al, PRL 122 (2019) Tuo, Feng, and Jin, PRD 100 (2019) Detmold and Murphy, arXiv:2004.07404



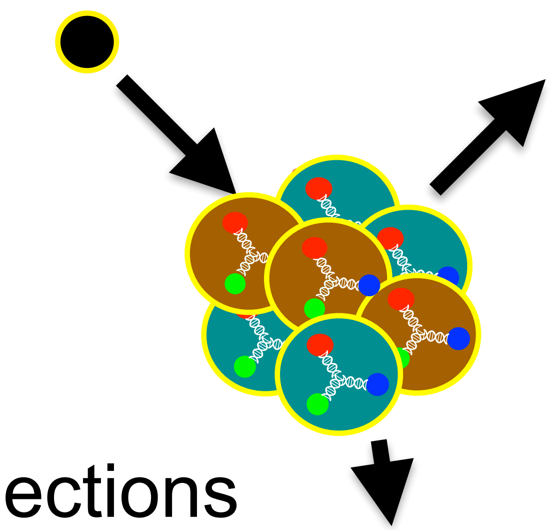


Although kinematically disallowed, can be matched to ChPT and used to constrain poorly known LEC

$$g_\nu^{\pi\pi}(\mu = 770 \text{ MeV}) = -11.96(31),$$
$$= -10.89(28)(33)_L(66)_a;$$
$$= -10.78(12)(51),$$

Dark Matter Direct Detection

Experiments look for nuclei recoiling from scattering with something invisible

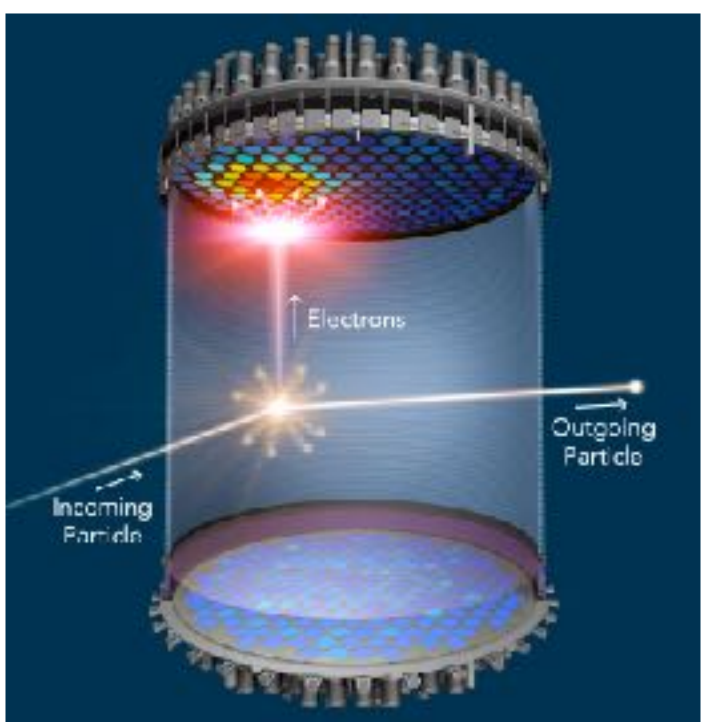


Heavy nuclei used to maximize sensitivity

QCD needed to relate DM-nucleus and DM-nucleon cross-sections and enable comparison between different experiments

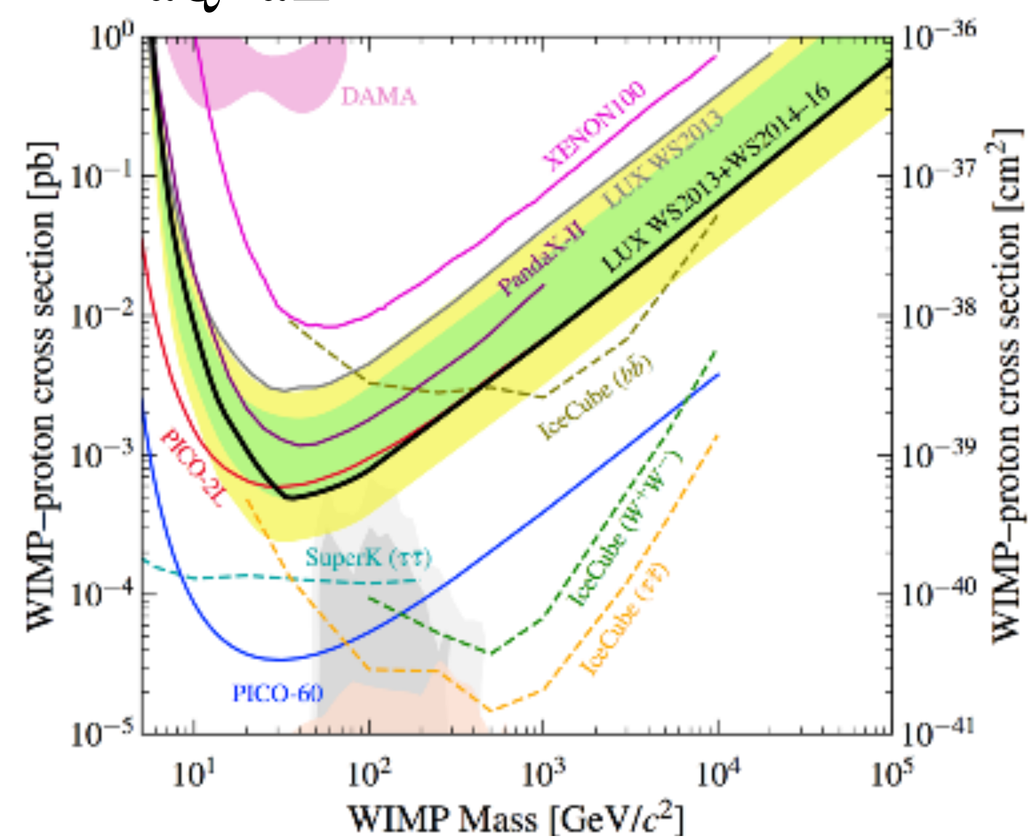
QCD

Impulse approximation: $$\frac{d\sigma^2(DM-A)}{dQ^2dE} = A^2\,\frac{d\sigma^2(DM-N)}{dQ^2dE} \quad + \quad ?$$



LUX

Impulse approximation



Akerib et al (LUX), PRL 118 (2017)

Scalar Currents

QCD effects reduce scalar isoscalar couplings (σ- terms) of A=2 by 1(1)% and ^{3}H by 4(1)% with $m_\pi = 806\ \mathrm{MeV}$

— Consistent with quark mass dependence of nuclear binding energies
NPLQCD, PRD 89 (2014)

Isovector QCD effects similar

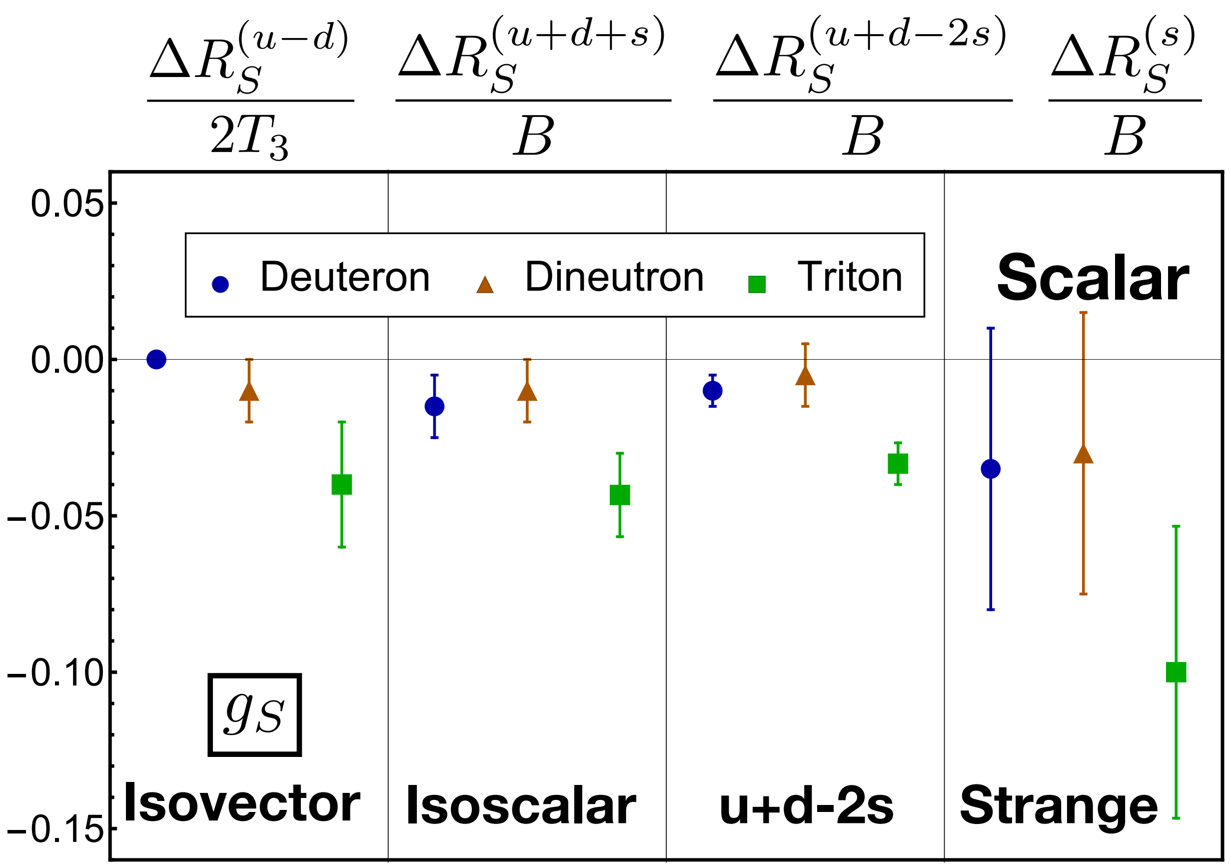
Scalar coupling to strange quarks reduced by 10(4)% in ^{3}H

— Dominant coupling in some BSM models

QCD results can test EFT power counting / nuclear models used to describe larger nuclei

Hoferichter, Klos, Menéndez, Schwenk, PRD 94 (2016)

Fieguth et al, PRD 97 (2018)



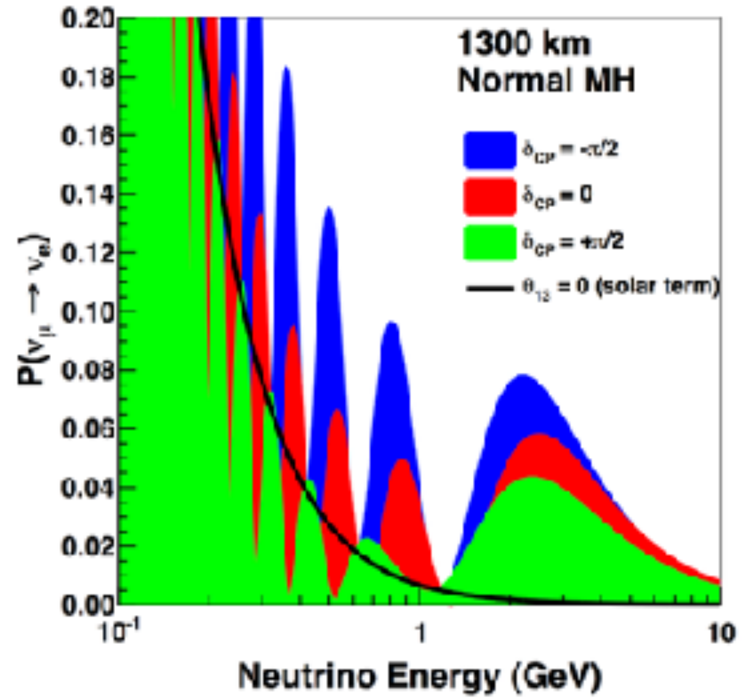
Chang, MW et al [NPLQCD], PRL 120 (2018)

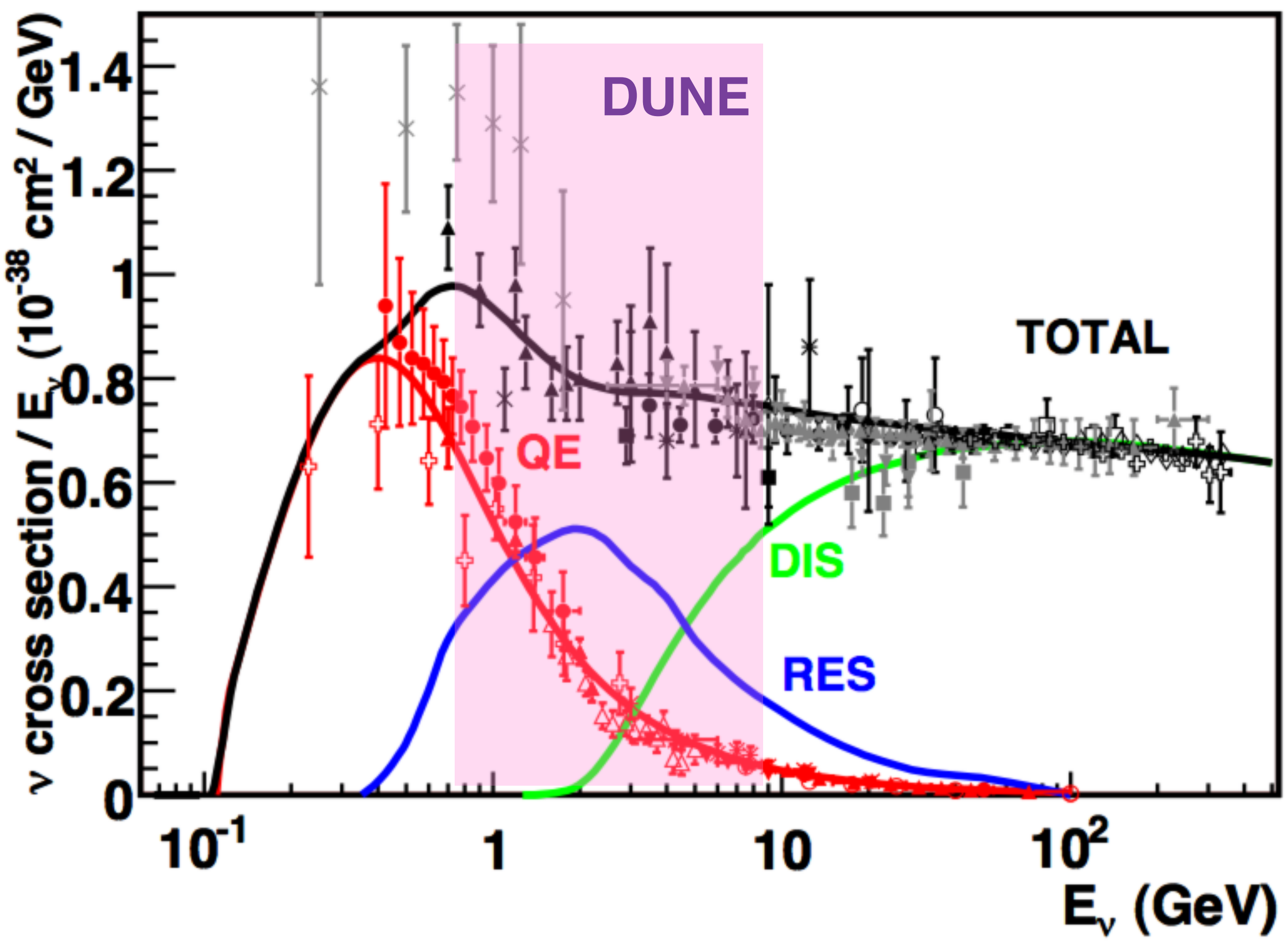
Neutrino-nucleus scattering

Neutrino nucleus scattering at scales ~ 1 GeV is a nonperturbative QCD process

νA cross-sections must be known precisely (few-percent-level?) to achieve design sensitivity to *CP* violation at DUNE

Acciarri et al (DUNE) arXiv 1512.06148





Formaggio, Zeller, Rev. Mod. Phys. 84 (2012)

Accelerator neutrino flux covers a wide range of energies with different dominant physics processes:

- Quasi-elastic
- Resonance production
- Transition region
- Deep inelastic scattering

Poorly known isovector EMC effect may be relevant for neutrino experiments including NuTeV anomaly

Cloët, Bentz, Thomas, PRL 102 (2009)

Hadron PDF moments from LQCD

Moments of PDFs calculable from matrix elements of local operators, e.g. momentum fraction of parton i in hadron h

$$\langle x \rangle_h^i = \int_{-A}^{A} dx\; x\; q_h^i(x) = \frac{M_h}{S[p_\mu p_\nu](2J+1)} \sum_{\lambda=-J}^{J} \langle h, \mathbf{p}, \lambda | T^i_{\mu\nu} | h, \mathbf{p}, \lambda \rangle$$

symmetrize and subtract trace

traceless part of stress-energy tensor

Stress-energy tensor

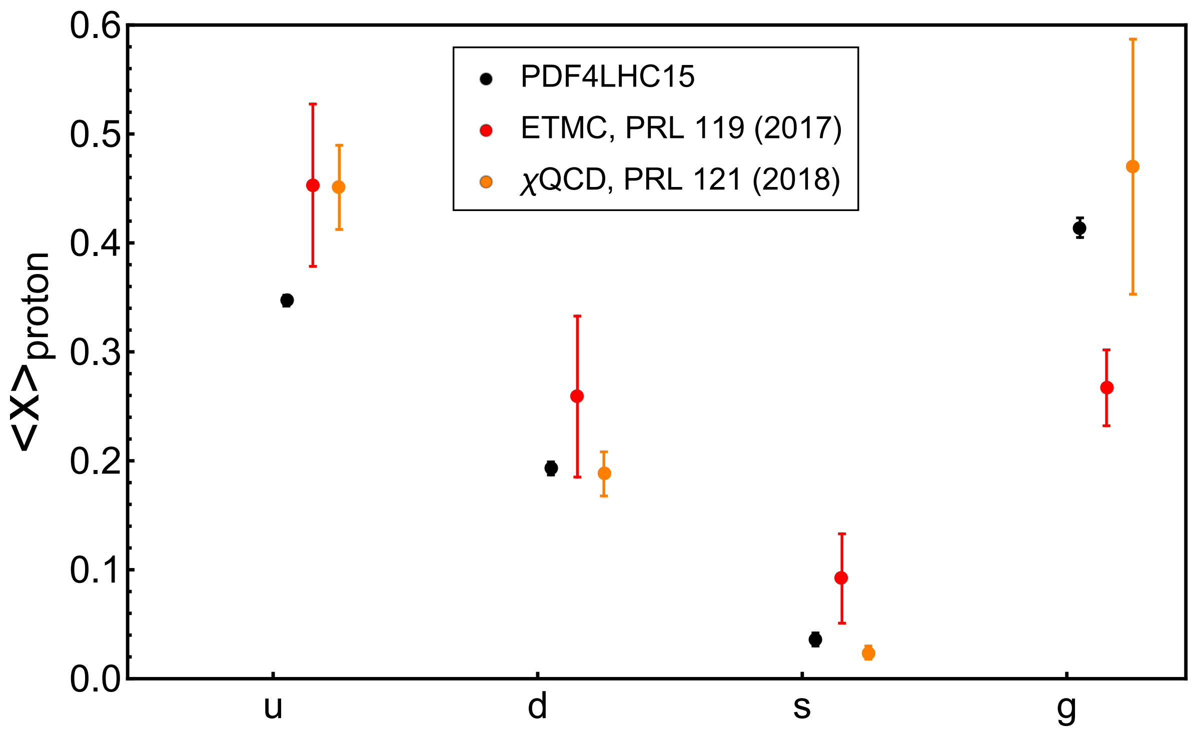
$$T^q_{\mu\nu} = S\left[\frac{1}{2}\overline{q}(i\overleftrightarrow{D}_\mu)\gamma_\nu q\right]$$

$$T^g_{\mu\nu} = S[G_\mu{}^\alpha G_{\alpha\nu}]$$

$\langle x \rangle^q_{\text{proton}}$ and $\langle x \rangle^g_{\text{proton}}$ calculated in LQCD by several groups

Review: Lin et al, Prog. Part. Nucl. Phys. 100 (2018)

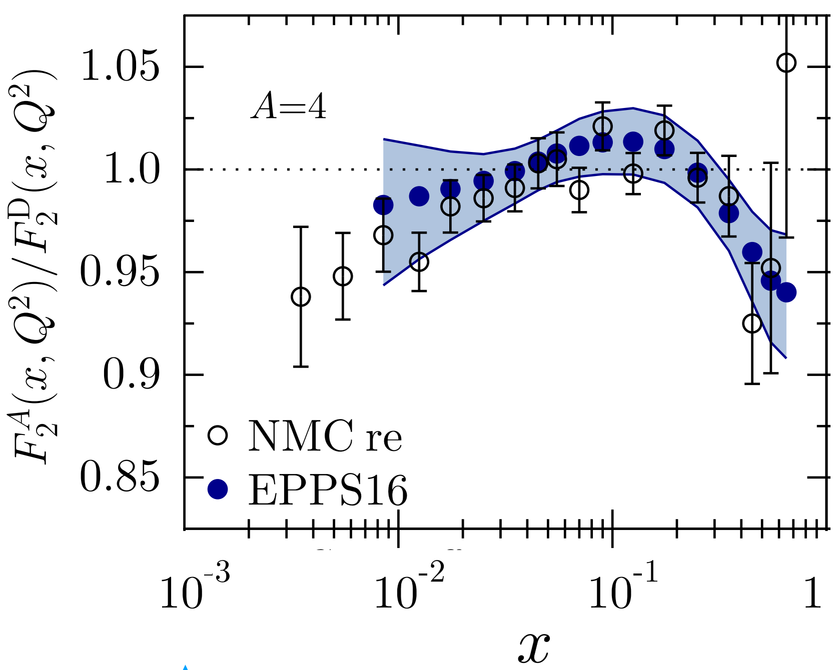
LQCD calculations with physical m_π are in reasonable agreement with momentum fractions predicted by phenomenological PDFs



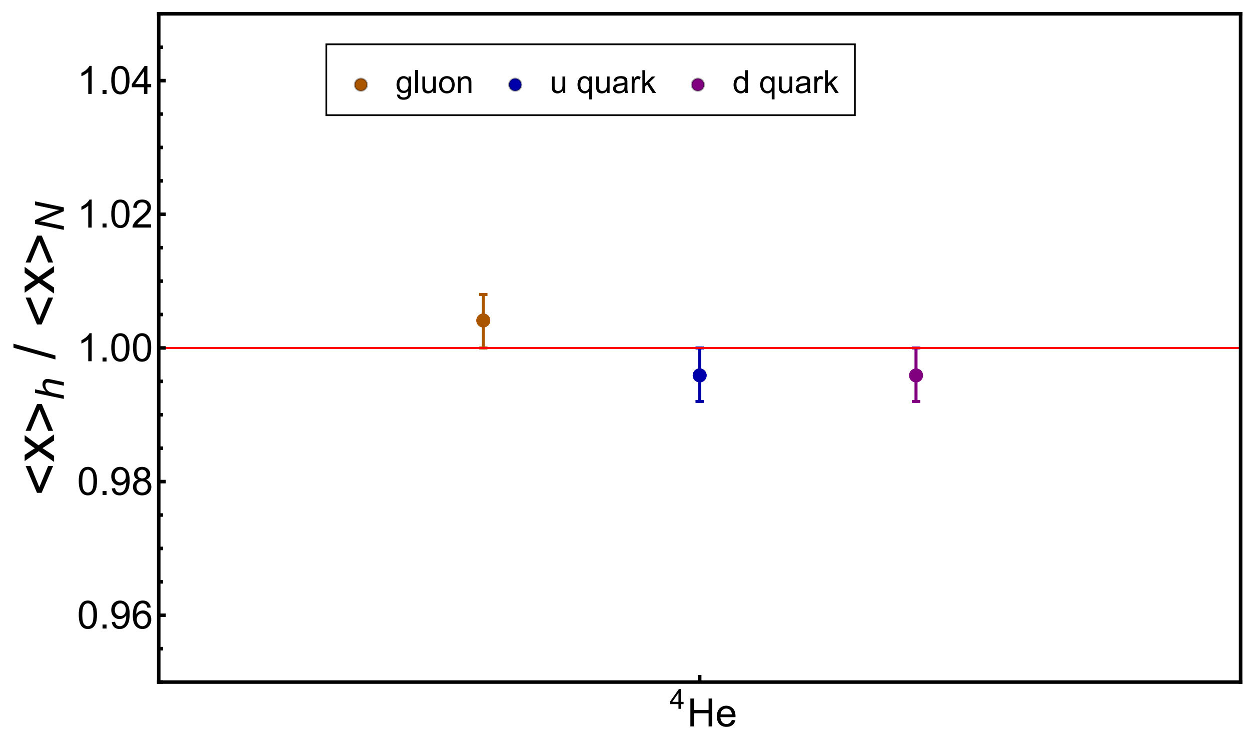
Nuclear momentum fractions

Nuclear matrix elements of $T^q_{\mu\nu}$ and $T^g_{\mu\nu}$ probe x-integrated EMC effect. Phenomenological expectations?

Global fits to available data constraining nuclear PDFs (charged lepton DIS, neutrino DIS, Drell-Yan, …) performed by multiple groups: EPPS, nCTEQ, DSSZ, …



Integrate nuclear and nucleon PDFs

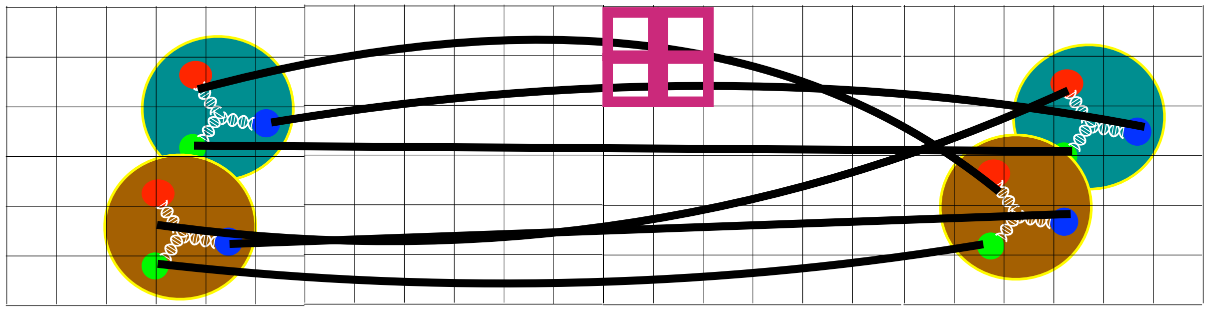


Eskola, Paakkinen, Paukkunen, Salgado, Eur. Phys. J. C 163 (2017)

Experiment	Observable	Collisions	Data points	χ^2	References
SLAC E139	DIS	e^-He(4), e^-D	21	12.2	[72]
CERN NMC 95, re	DIS	μ^-He(4), μ^-D	16	18.0	[73]
CERN NMC 95	DIS	μ^-Li(6), μ^-D	15	18.4	[74]
CERN NMC 95, Q^2 dep	DIS	μ^-Li(6), μ^-D	153	161.2	[74]
SLAC E139	DIS	e^-Be(9), e^-D	20	12.9	[72]
CERN NMC 96	DIS	μ^-Be(9), μ^-C	15	4.4	[75]

Scarcity of data for large x dominates uncertainties on moments of nuclear PDFs

Gluon momentum fractions



Gluon operators purely disconnected, contractions trivial but noise problematic

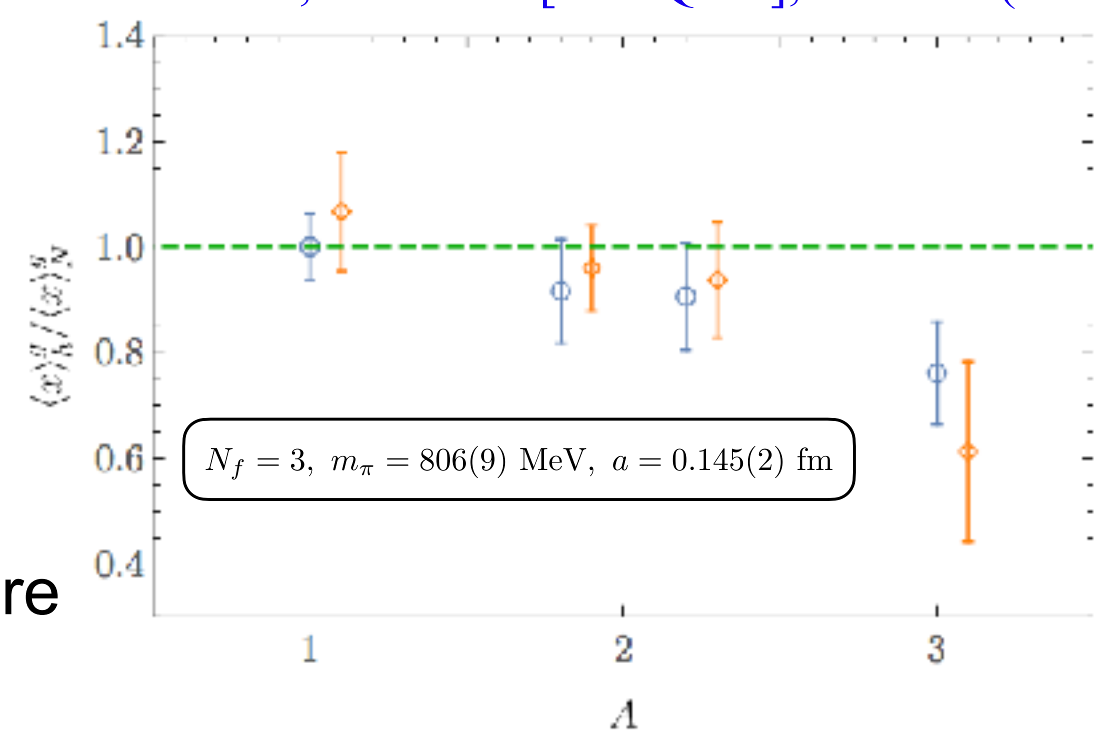
Gluon stress-energy tensor discretized, projected onto irreps of cubic group. Multiple irreps with same continuum limit used

Gradient flowed gluon operators used to reduce noise, high statistics still necessary $(N_{cfg} = 508) \times (N_{src} = 416)$

$$T^{g}_{\mu\nu} \to G_{3\alpha}G_{3\alpha} - G_{4\alpha}G_{4\alpha}$$

$$G_{\mu\nu} \propto \mathrm{Im}\left(\boxplus\right)$$

Winter, MW et al [NPLQCD], PRD 96 (2017)



Operator mixing between gluon and flavor-singlet quark momentum fraction neglected

Gluon transversity operators probe gluon structure in spin 1+ systems not present in nucleon

Jaffe and Manohar, Phys. Lett. B 223 (1989)

Detmold and Shanahan, PRD 94 (2016)

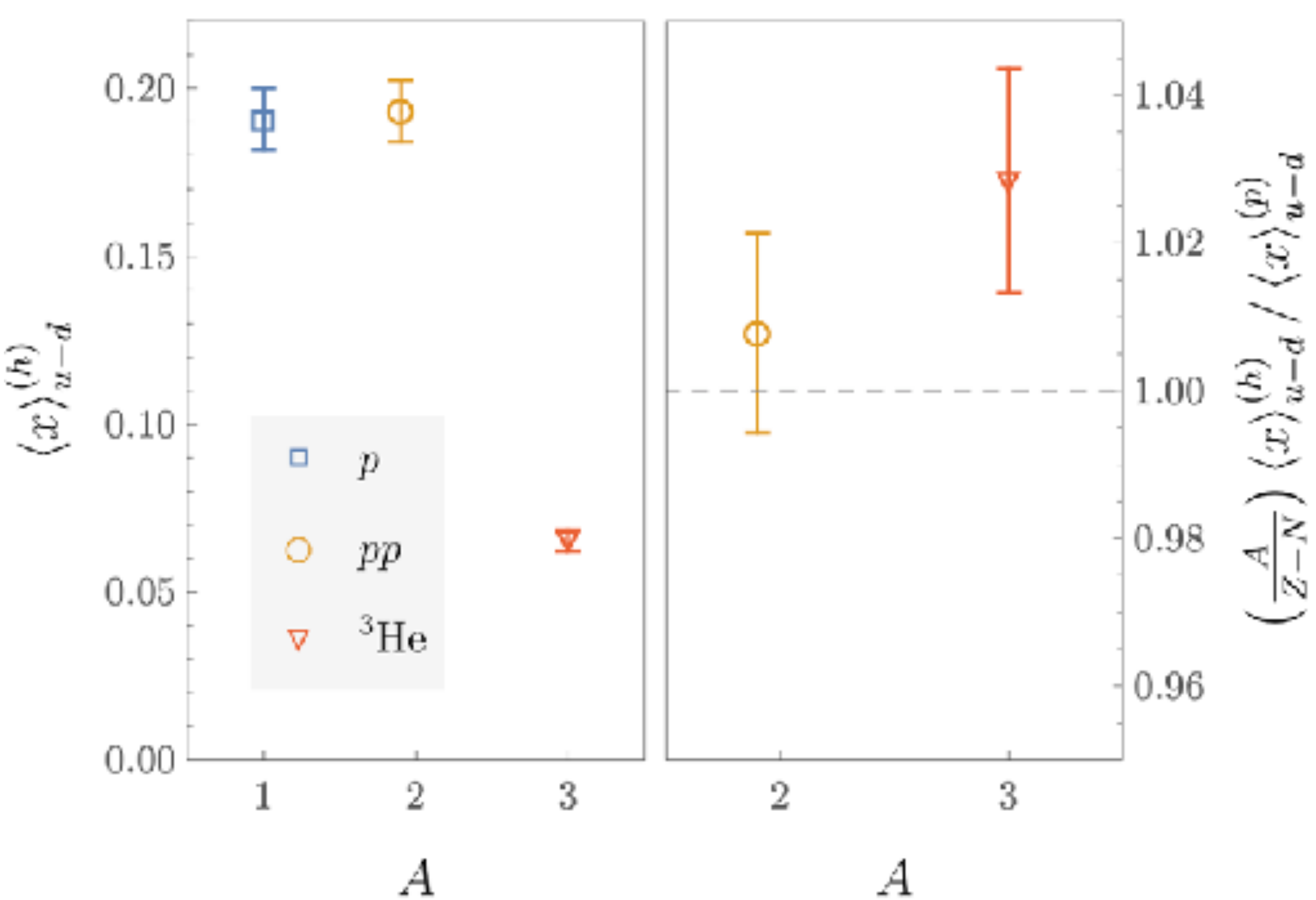
Deuteron has small but non-zero gluon transversity, magnitude consistent with large Nc expectations Winter, MW et al [NPLQCD], PRD 96 (2017)

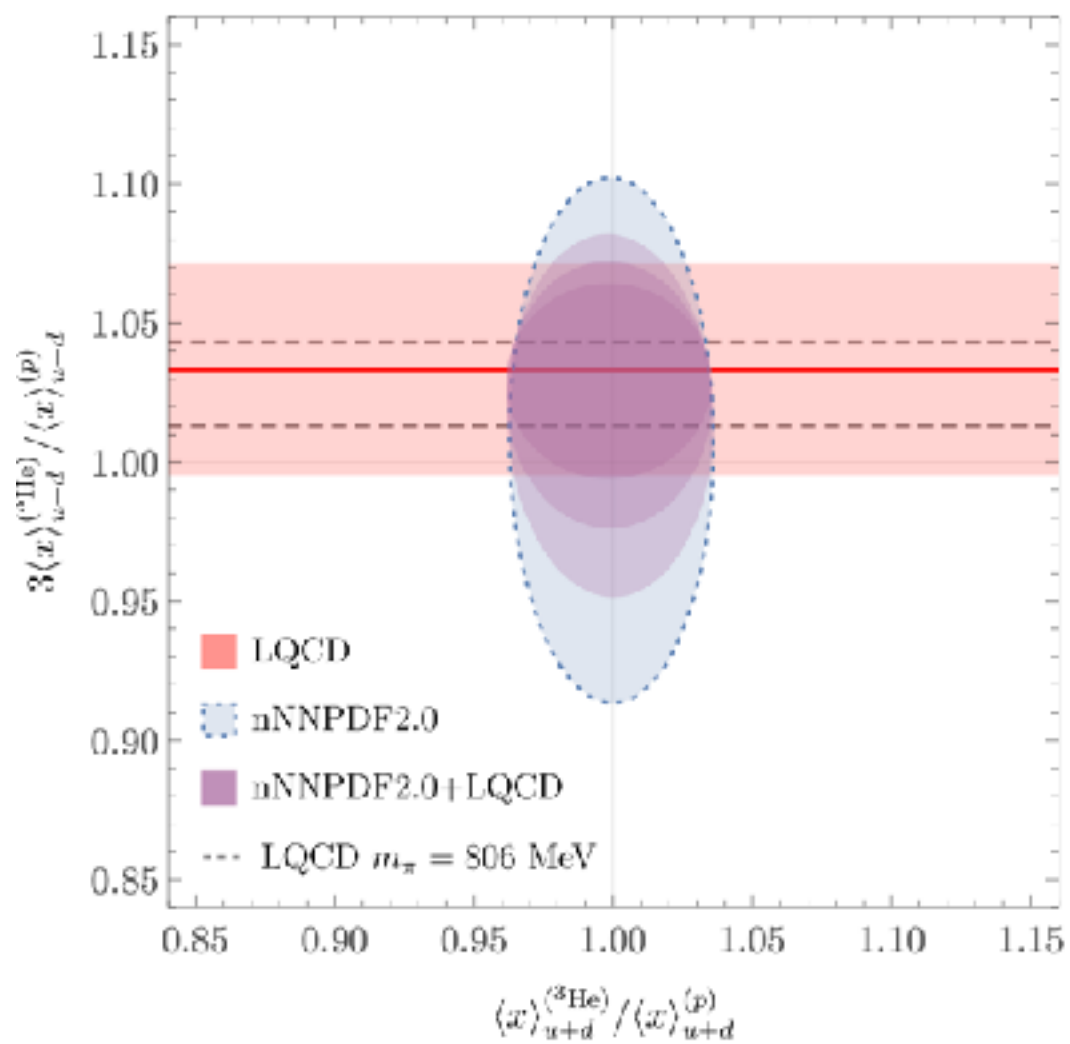
Quark momentum fractions

First calculation of isovector quark momentum fractions of light nuclei performed using fixed-order background field method

Detmold, MW et al [NPLQCD] PRL 126 (2021)

Discretized operator: $T^q_{\mu\nu} \to \{\overline{q}(x)\gamma_4[q(x+\hat{4}) - q(x)] - [\overline{q}(x+\hat{4}) - \overline{q}(x)]\gamma_4 q(x)\} - \{4 \leftrightarrow 3\}$





Although systematic uncertainties are not fully controlled (one lattice spacing, volume, quark mass, …) demonstrates potential for LQCD to usefully constrain nuclear PDFs

Systematic uncertainties

Several systematic uncertainties remain to be quantified in detail

- Heavier than physical quark masses only
- One lattice spacing
- Excited-state effects

Systematic uncertainties

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- One lattice spacing
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Gap between ground and two-nucleon finite-volume “scattering” states becomes small for large volumes, ground-state dominance relies on overlap factors

$$Z_0 e^{-E_0 t}\left(1+\frac{Z_1}{Z_0}e^{-\delta t}+\dots\right) \qquad \delta \sim \frac{4\pi^2}{ML^2}$$

For non-positive-definite correlation functions, cancellations between the ground and excited-state could in principle conspire to form a “false plateau”

See e.g. Iritani et al, JHEP 10 (2016)

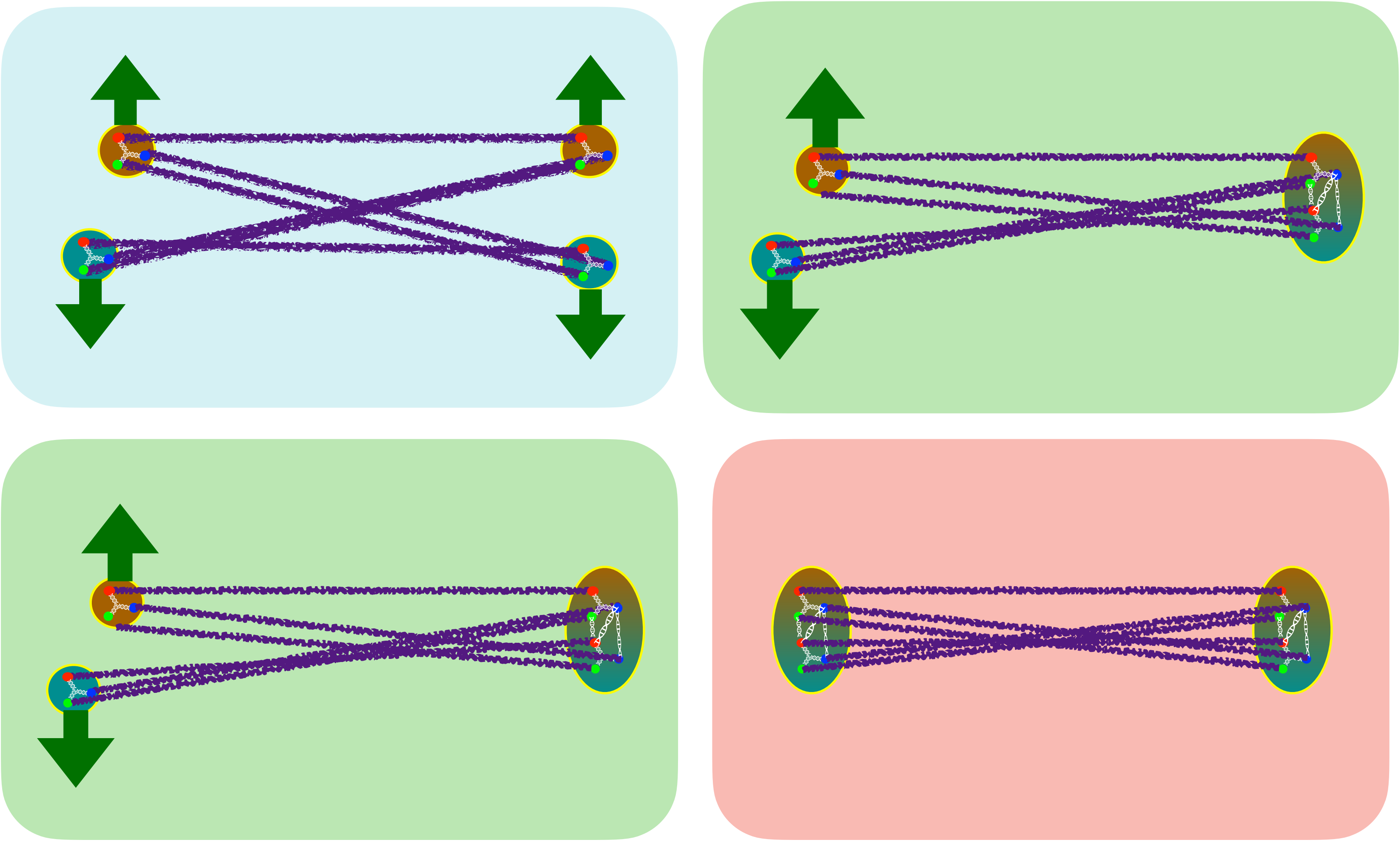
First studies using positive-definite correlation functions (enabled by distillation / stochastic LapH) give results that suggest tensions with previous studies

Francis et al, PRD 99 (2019) Hörz et al, PRC 103 (2021) Green et al, arXiv:2103.01054

The variational method

Correlation-function matrices for an interpolator set including both local “hexaquark” and bilocal “dibaryon” operators can generalize calculations performed to date

Variational bounds on energy spectrum obtained by diagonalizing these matrices



Although application of variational methods to multi-nucleon systems has long been advocated, it has only recently become computationally feasible through methods such as distillation and propagator sparsening

Peardon et al PRD 80 (2009)

Morningstar et al PRD 83 (2011)

Detmold, MW et al, arXiv:1908.07050

Li et al, PRD 103 (2021)

Hexaquark operators

Known from $\pi\pi$ scattering studies near the ρ resonance that local and nonlocal operators can be nearly orthogonal

Dudek et al, PRD 87 (2013)

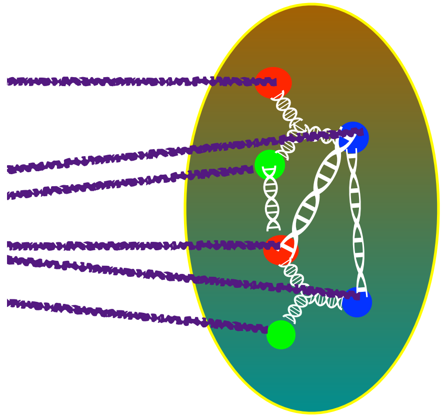
Wilson et al, PRD 92 (2015)

$$\overline{q}(x)\Gamma q(x) \qquad \pi(\vec{p}_1)\pi(\vec{p}_2)$$

Calculations with $t \sim$ few fm neglecting one type of operator show plateau-like behavior but energy spectra with “missing levels” (compared to more complete calculations)

Analog of $\overline{q}(x)\Gamma q(x)$ operators - local (up to Gaussian smearing) “hexaquark”

$$H_{0\mathbf{n}s}(t) = \sum_{\vec{x}\in\Lambda_{\mathcal{S}}} \psi_{\mathbf{n}}^{[H]}(\vec{x})\varepsilon^{abcdef}\frac{1}{2}\Big[p_{1s}^{abc}(\vec{x},t)n_{2s}^{def}(\vec{x},t) - p_{2s}^{abc}(\vec{x},t)n_{1s}^{def}(\vec{x},t) + n_{1s}^{abc}(\vec{x},t)p_{2s}^{def}(\vec{x},t) - n_{2s}^{abc}(\vec{x},t)p_{1s}^{def}(\vec{x},t)\Big]$$



Quark exchange symmetries very useful for reducing the number of weights

Detmold and Orginos, PRD 87 (2013)

$$2880 \rightarrow 21$$

Dibaryon operators

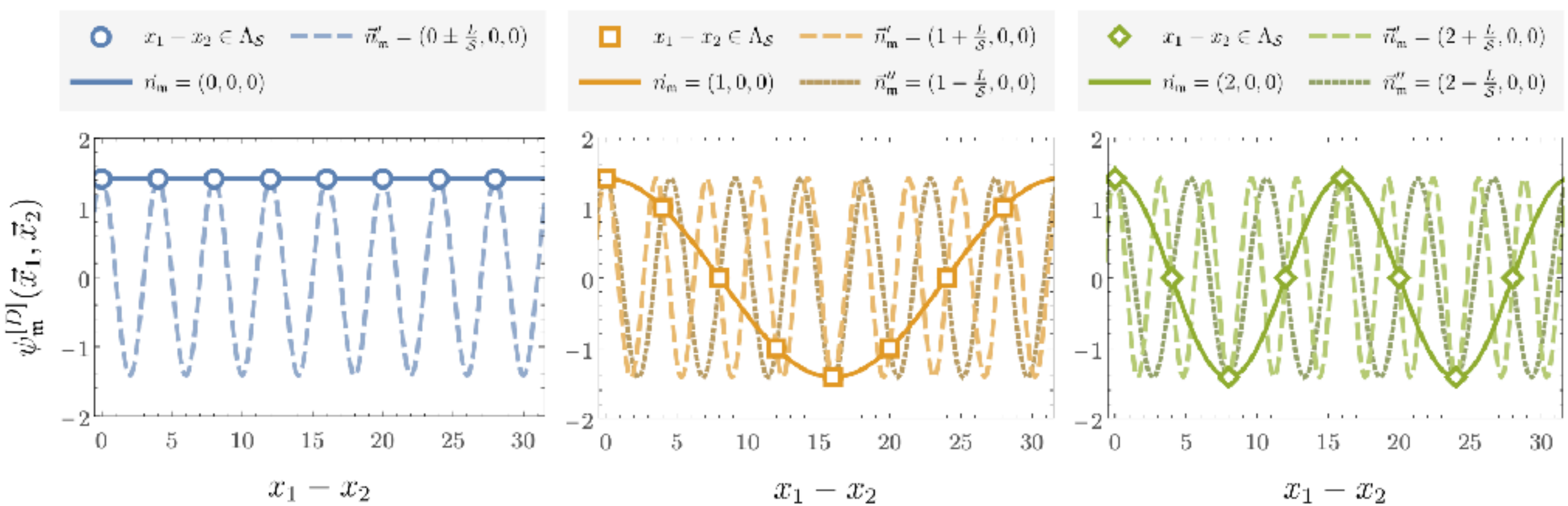
Non-interacting two-baryon FV energy eigenstates involve color singlet baryons

$$D_{\rho \mathfrak{m} s}(t) = \sum \psi_{\mathfrak{m}}^{[D]}(\vec{x}_1, \vec{x}_2) \sum v^{\rho}_{\sigma\sigma'} \frac{1}{\sqrt{2}} \left[p_{\sigma s}(\vec{x}_1, t) n_{\sigma' s}(\vec{x}_2, t) + (-1)^{1-\delta_{\rho 0}} n_{\sigma s}(\vec{x}_1, t) p_{\sigma' s}(\vec{x}_2, t) \right]$$

With plane-wave product wave functions

$$\psi_{\mathfrak{m}}^{[D]}(\vec{x}_1, \vec{x}_2) = e^{i\vec{k}_{\mathfrak{m}} \cdot (\vec{x}_1 - \vec{x}_2)} \qquad \vec{k}_{\mathfrak{m}} = \frac{2\pi \vec{n}_{\mathfrak{m}}}{L}$$

Quark propagator sparsening leads to incomplete Fourier projection and mixing with higher modes, but these are negligible compared to other excited states



Quasi-local operators

What about loosely bound systems like the deuteron?

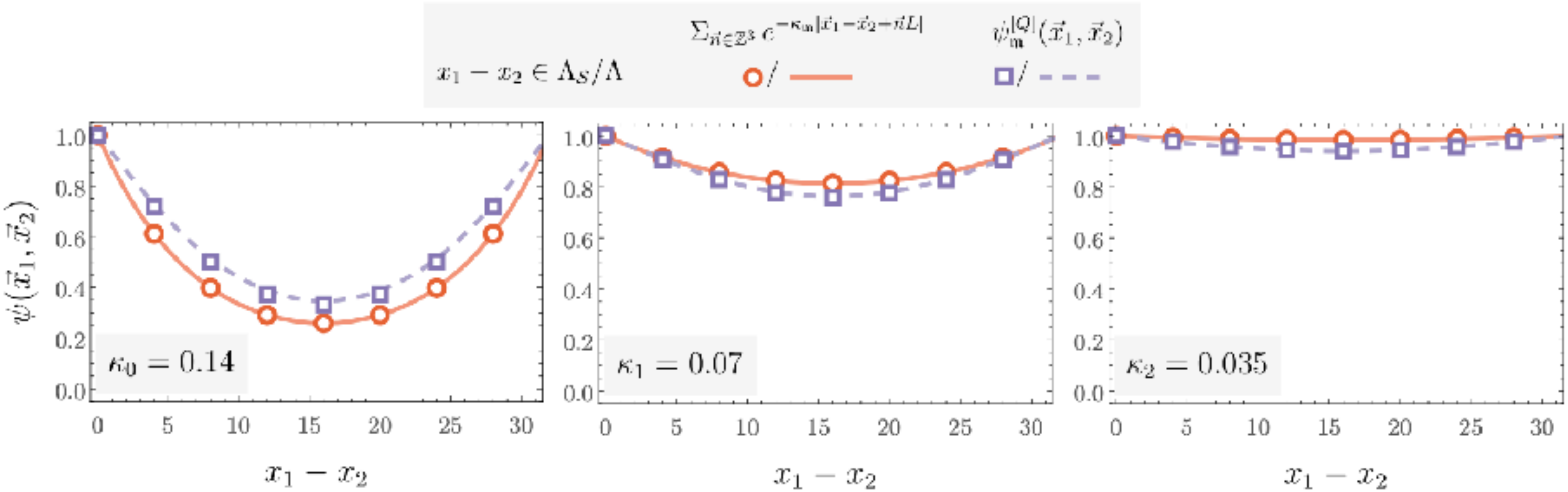
Finite-volume EFT wavefunction: $$\sum_{\vec{n}\in\mathbb{Z}_3} e^{-\kappa|\vec{x}_1-\vec{x}_2+n\vec{L}|}\left(\frac{\mathcal{A}}{|\vec{x}_1-\vec{x}_2+\vec{n}L|}+\dots\right)$$

See e.g. Koning, Lee, and Hammer, Annals Phys. 327, 1450 (2012)

Briceño, Davoudi, Lee and Savage, PRD 88 (2013)

Doesn't factorize into product of single-baryon wavefunctions, no baryon blocks…

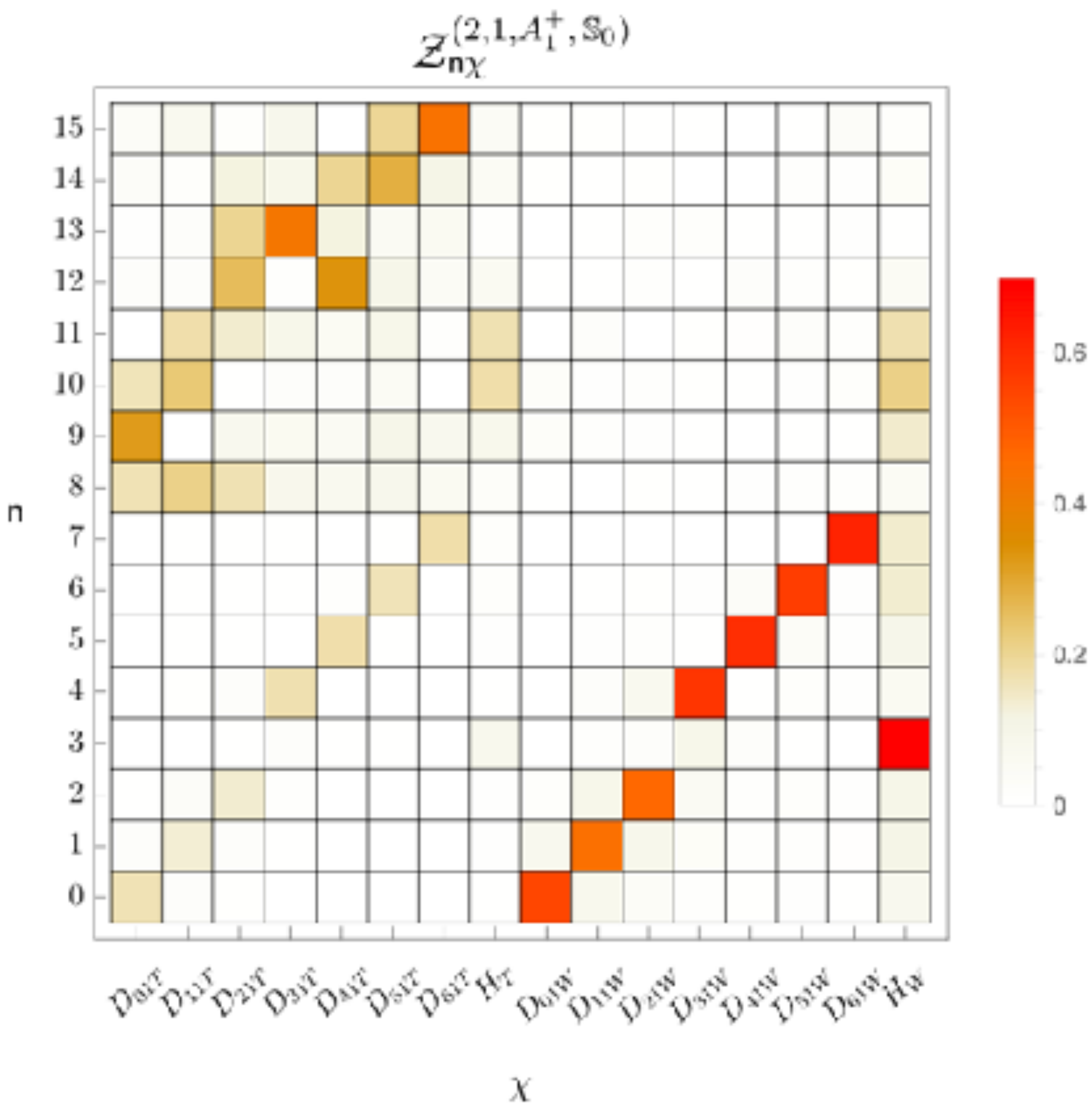
Factorizable approximation: $$\psi_{\mathfrak{m}}^{[D]}(\vec{x}_1,\vec{x}_2)=\sum_{\tau\in\mathbb{T}_{\mathcal{S}}} e^{-\kappa_{\mathfrak{m}}|\tau(\vec{x}_1)-\vec{R}|}e^{-\kappa_{\mathfrak{m}}|\tau(\vec{x}_2)-\vec{R}|}$$

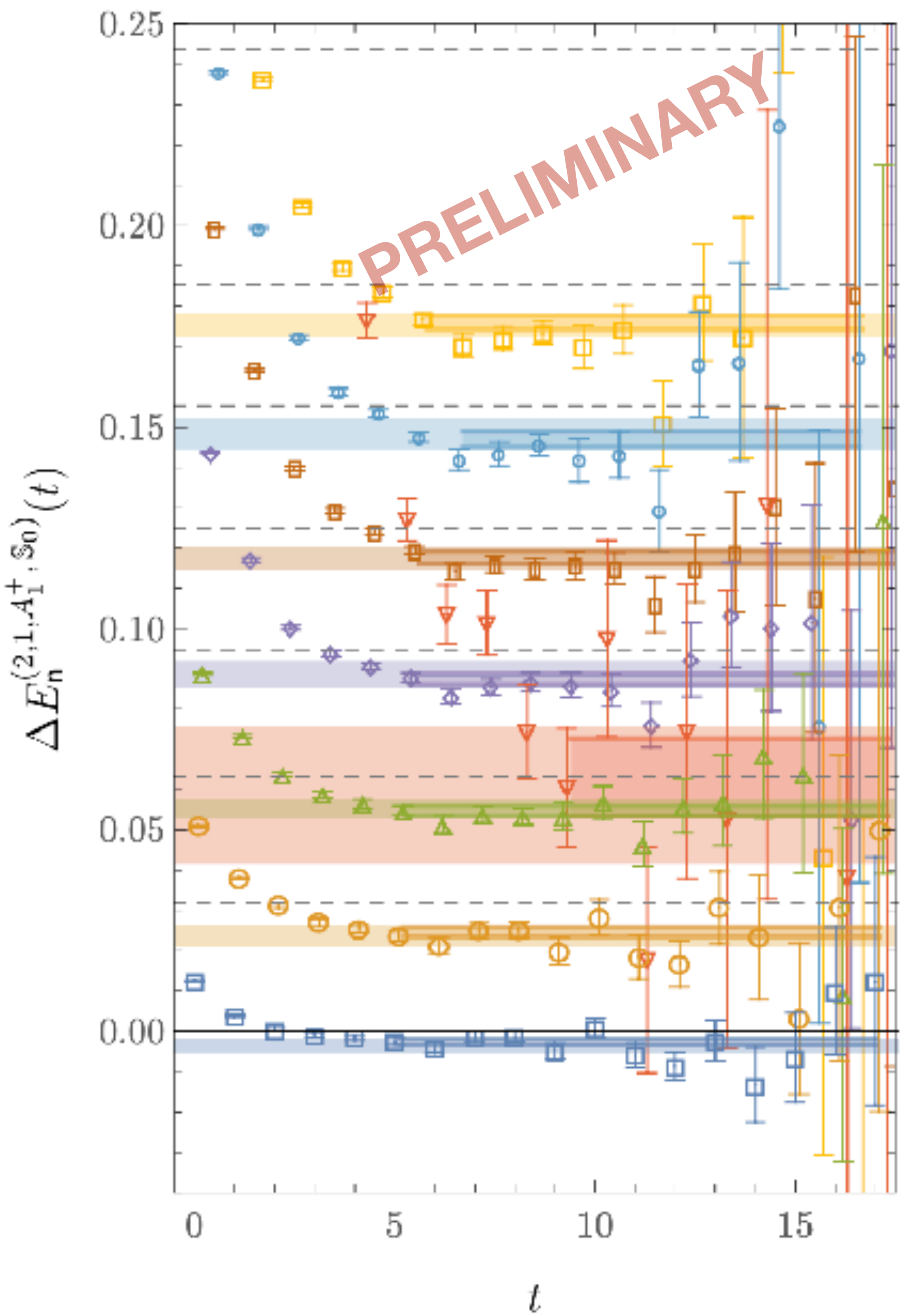


Towards variational studies of nuclei

Diagonalization of correlation-function matrices can be used to remove excited-state contamination from states strongly overlapping with other operators

Each energy level dominantly overlaps with one operator structure, sub-dominant operators collectively 30%





Interpolating-operator dependence

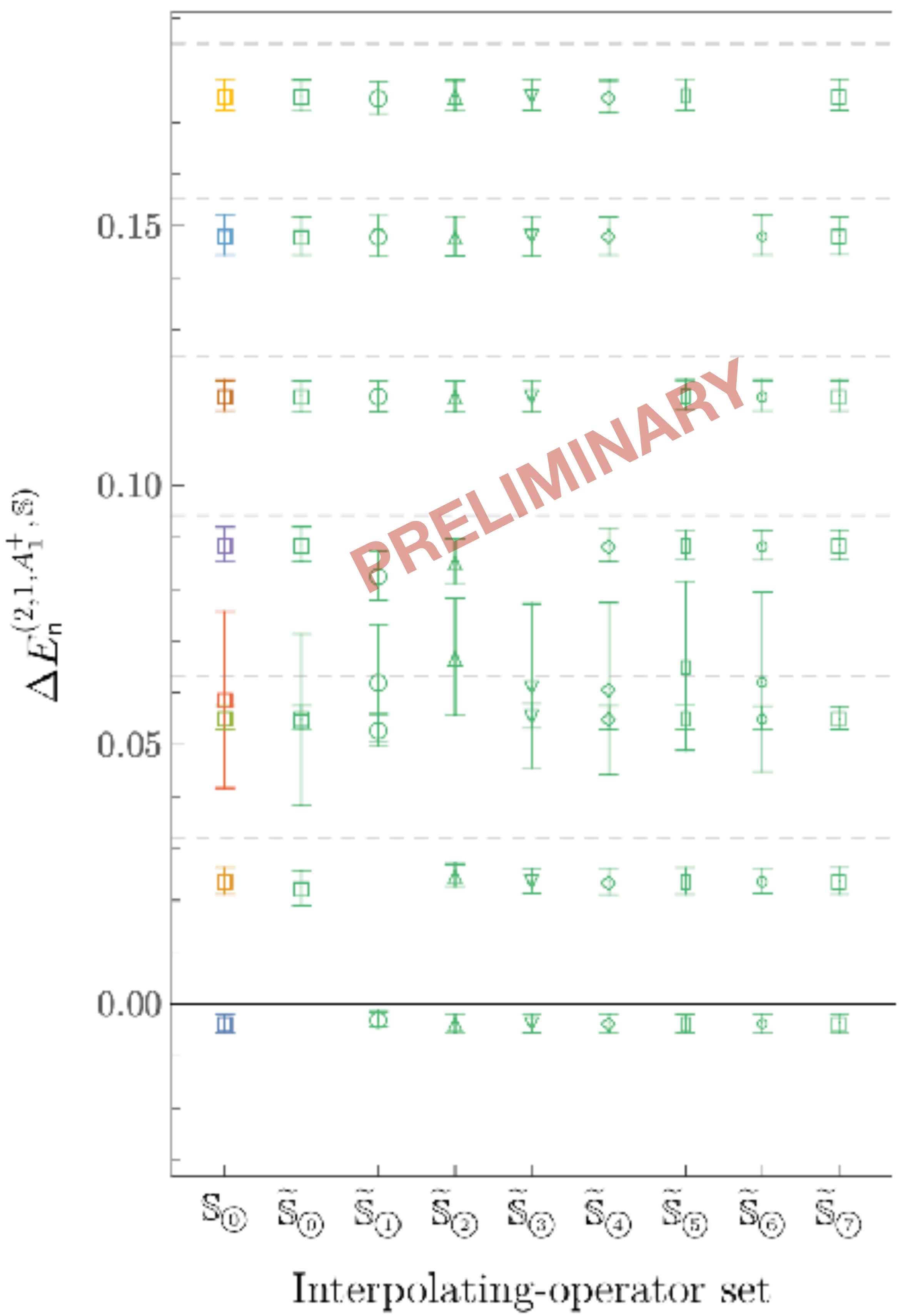
Removing the operator structure with maximum overlap on to a given energy level leads to “missing energy levels”

Even with 10s of interpolating operators, possible to “miss” ground-state

— valid lower bound on ground-state energy, but best-fit results can differ by 5+ σ

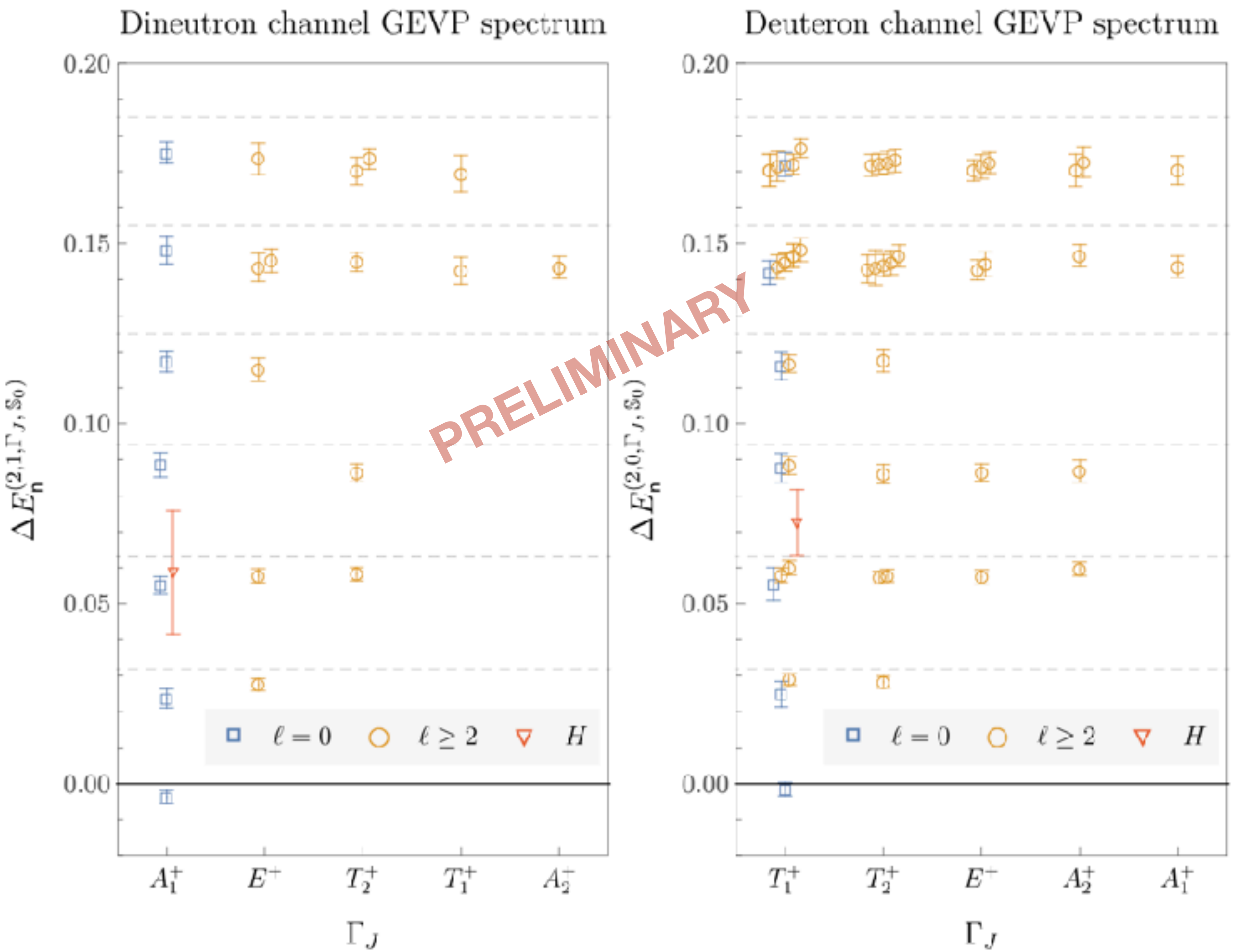
Consistent with various dibaryon and hexaquark operators being approximately orthogonal

Much larger ($t \gtrsim 1/\delta \sim 5 \text{ fm}$) source/sink separations would be needed to resolve spectrum using interpolating-operator set missing dominant operators



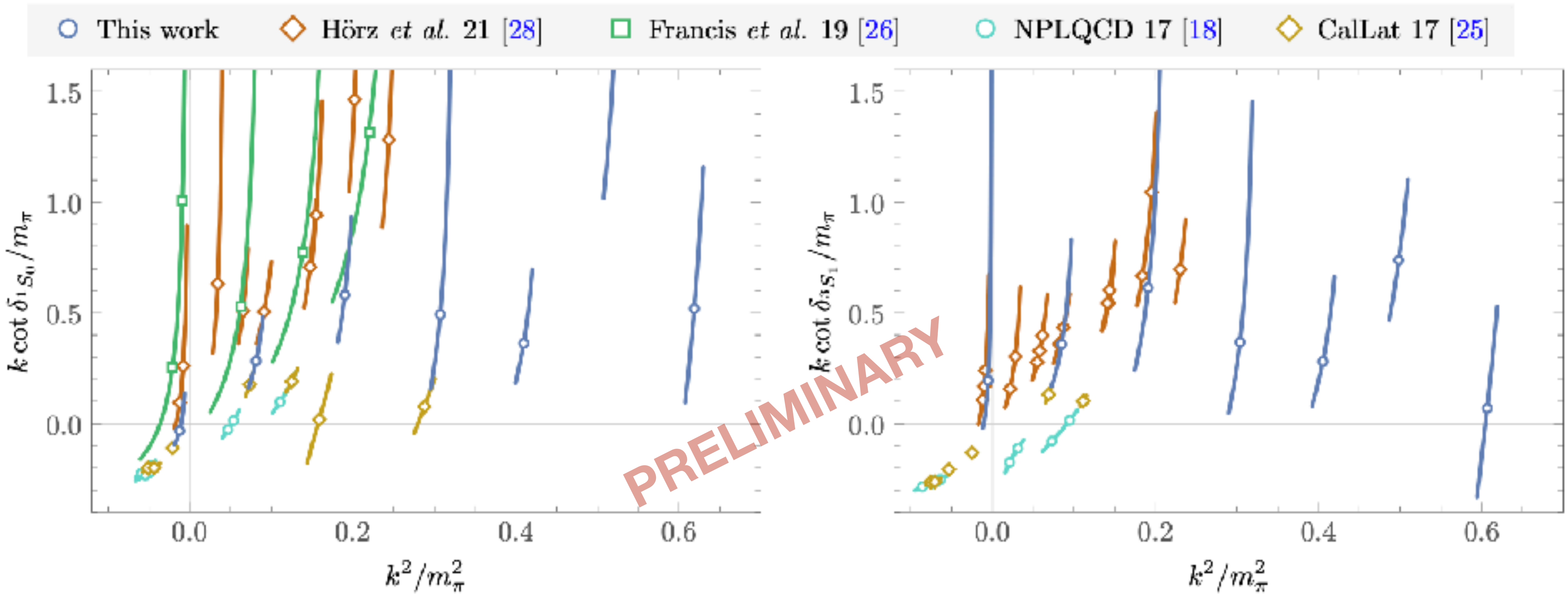
Variational energy spectrum

For a given interpolating-operator set, two-nucleon finite-volume energy spectrum can be extracted in various cubic irreps associated *S*-wave, *D*-wave, and higher-partial-wave interactions



Towards variational studies of nuclei

Finite-volume spectrum can be mapped to *S*-wave, *P*-wave and higher-partial-wave scattering phase shifts using generalizations of Lüscher's quantization condition



Recent calculations using dibaryon-dibaryon correlation functions give results consistent with variational methods, but there is tension with previous results using fits to dibaryon-hexaquark correlation functions only

Further studies are needed to conclusively determine the structure of two-nucleon ground states with heavier-than-physical quark masses

Outlook

LQCD calculations of nuclear matrix elements can constrain EFTs and nuclear models relevant for precise predictions of

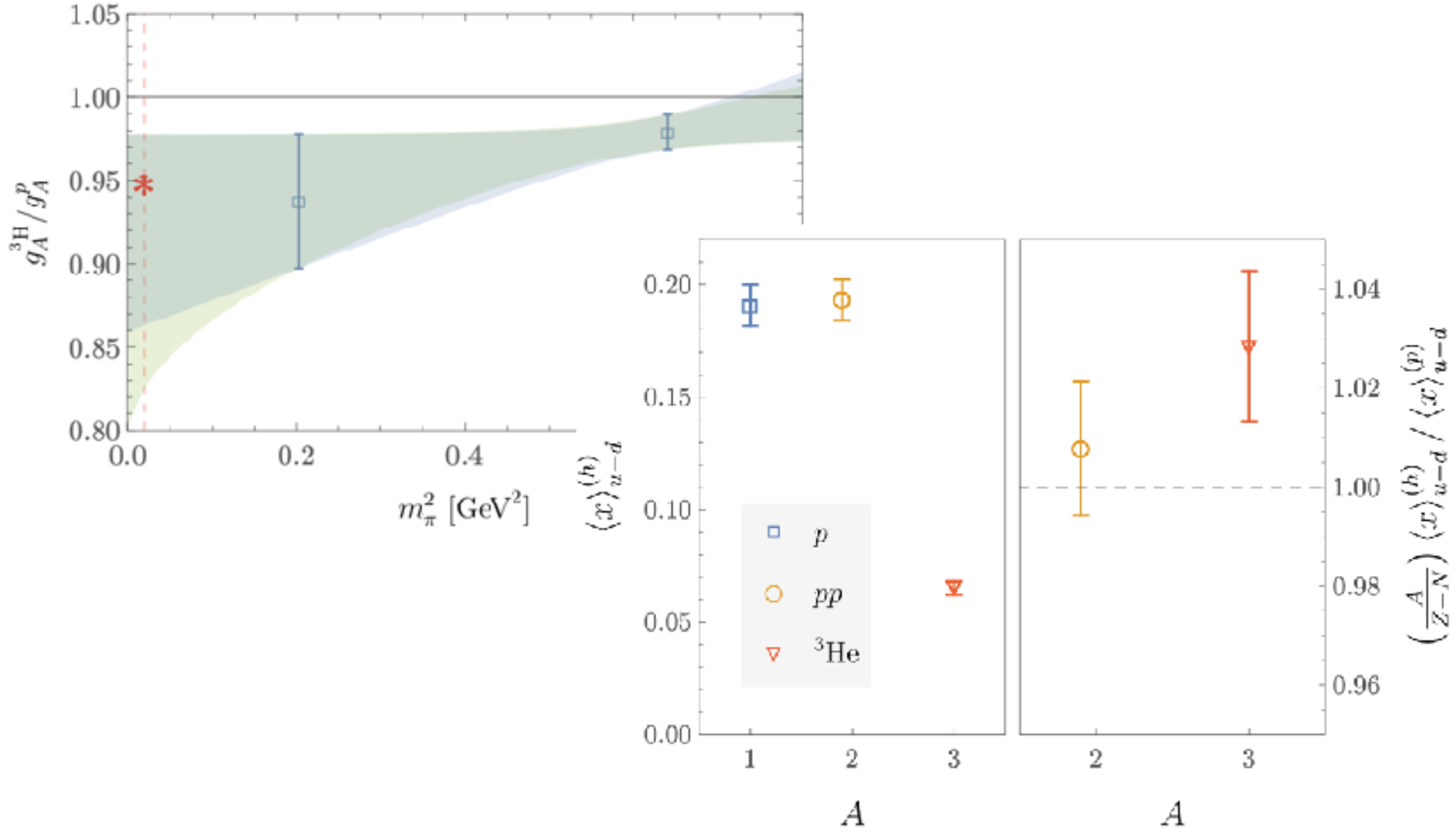
— electroweak reaction rates,

— double-beta decay,

— dark matter direct detection

— neutrino-nucleus scattering

…



Exploratory LQCD calculations of nuclear matrix elements pave the way for controlled predictions but still face hard-to-quantify systematic uncertainties arising from excited-state effects, discretization effects, finite-volume effects, …

Further studies with additional interpolating-operator structures and lattice spacings / volumes needed to conclusively resolve structure of two-nucleon spectrum at heavier-than-physical quark masses