Lattice QCD for Hadron Spectroscopy

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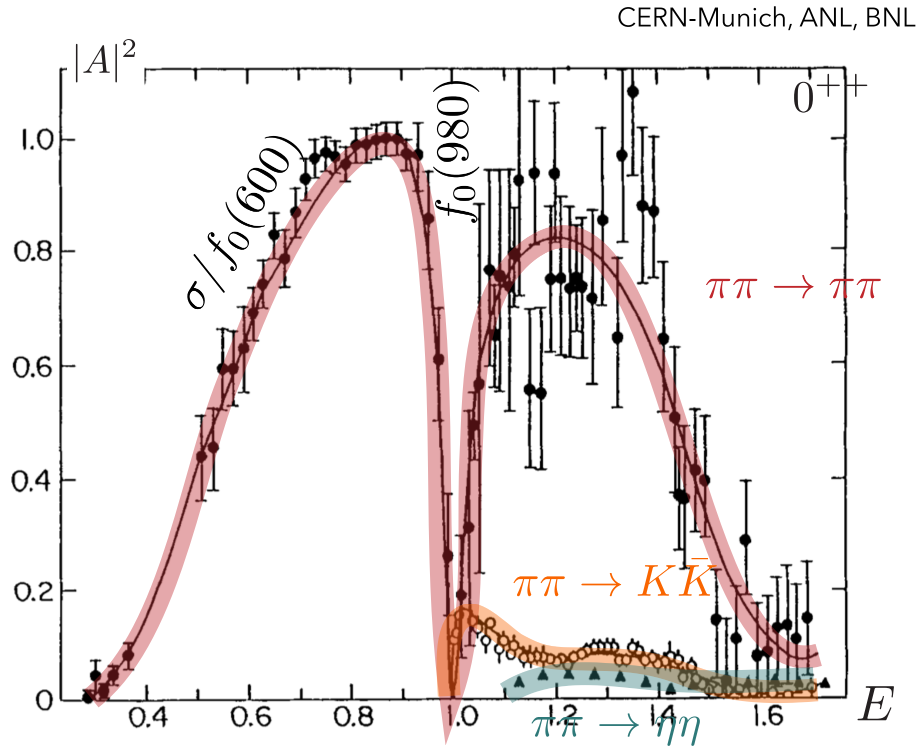
hadspec
hadspec.org

Hadron 2021
29th July

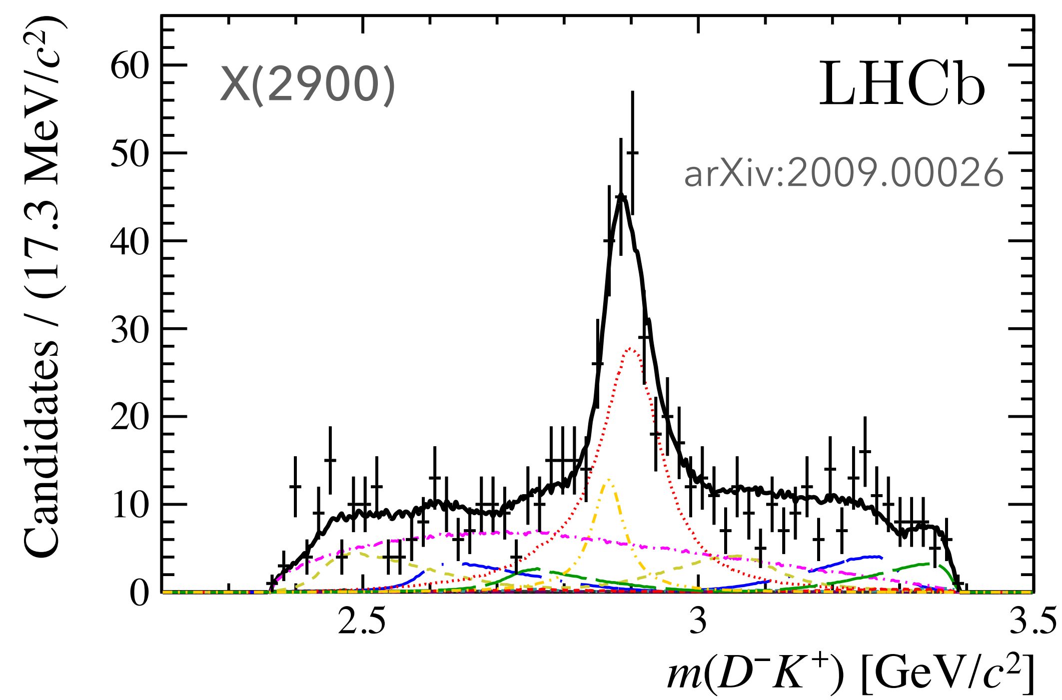
UNIVERSITY OF CAMBRIDGE

THE ROYAL SOCIETY

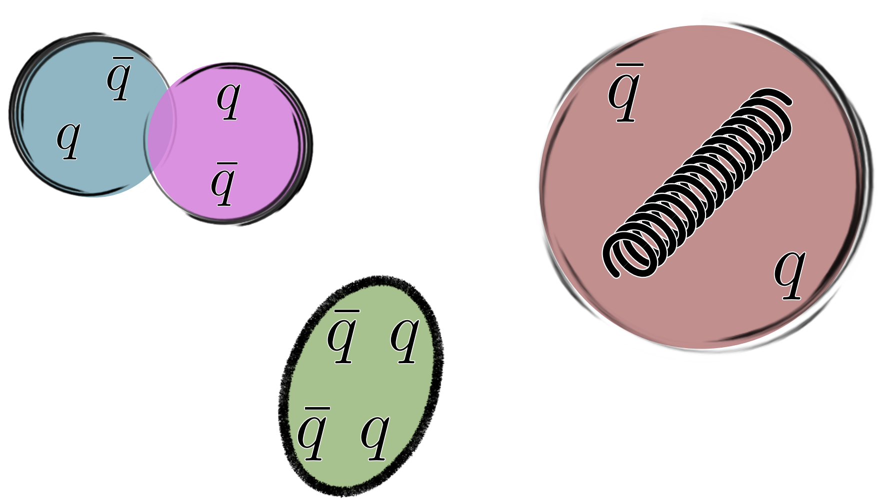
spectroscopy from first-principles is a hard problem



many puzzles - new and old



the quark model is a good guide for low-lying states



models can be useful, but what does QCD say?

provides a rigorous approach to hadron spectroscopy

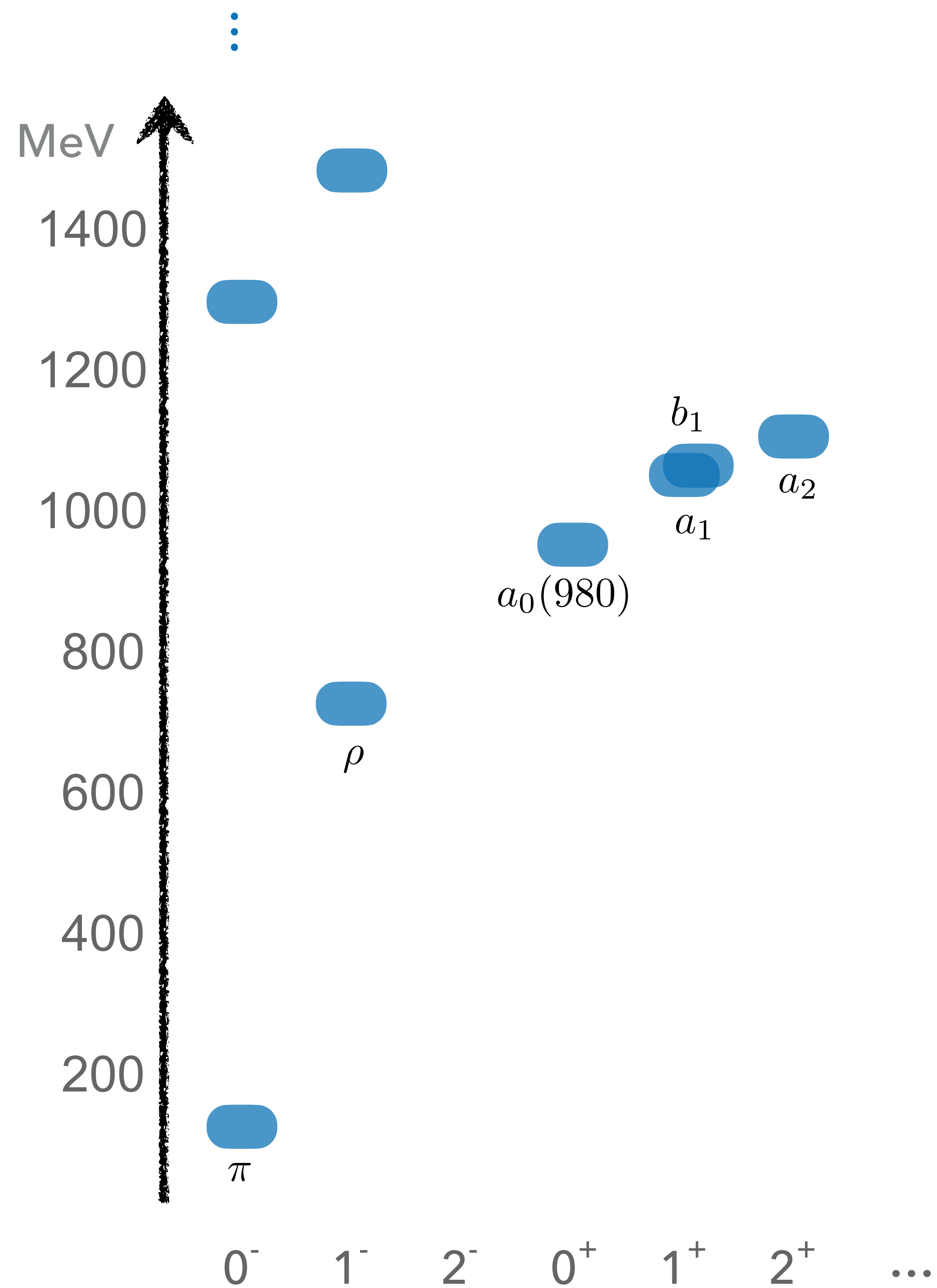
- as **rigorous** as possible
- **all** necessary **QCD** diagrams are computed
- **excited states** appear as **unstable resonances** in a scattering amplitude

tremendous progress in recent years
but not yet ready for precision comparisons

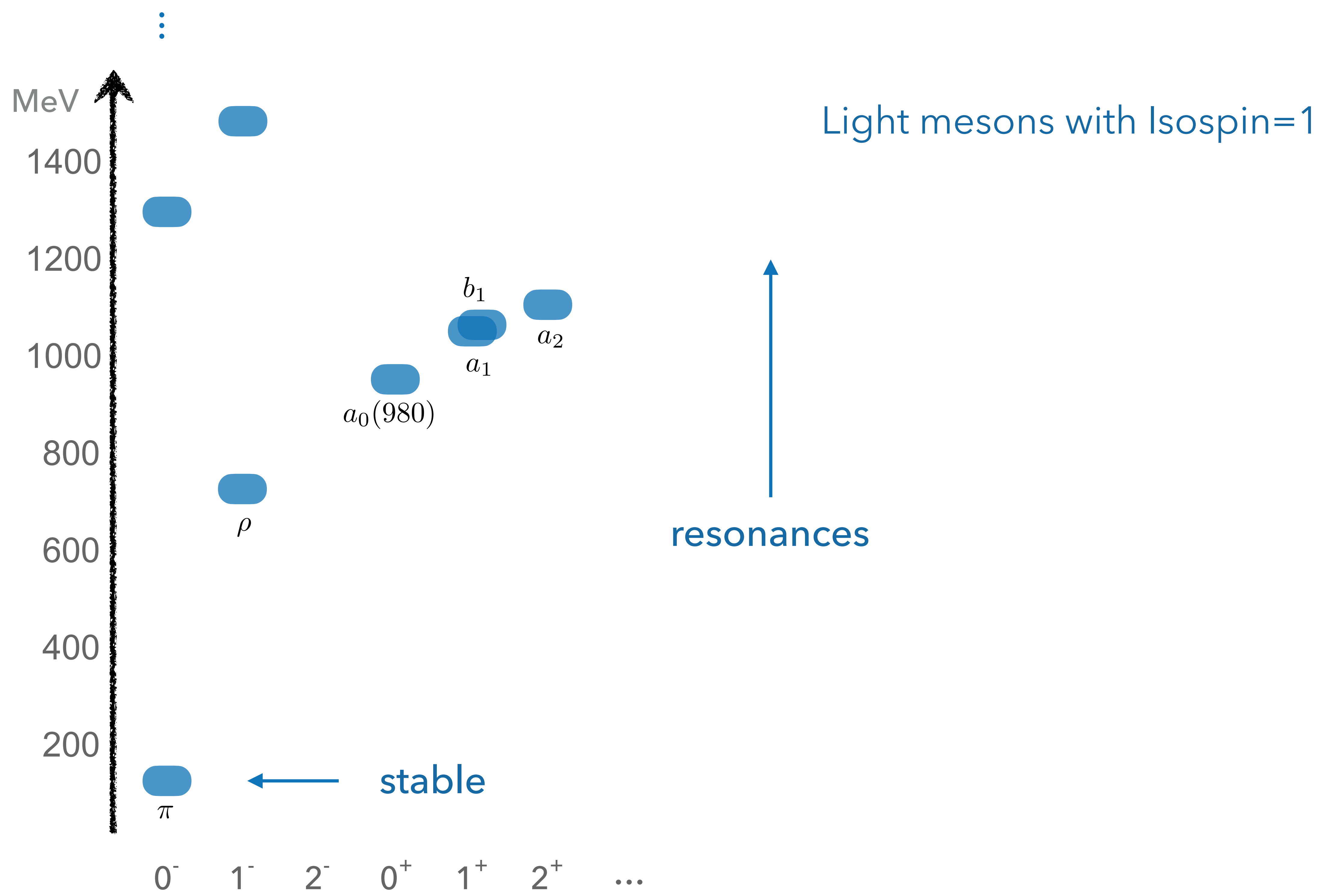
- physical pions are very light
- most interesting states can decay to **many** pions
- control of light-quark mass is a useful tool
- small effects not considered in general:

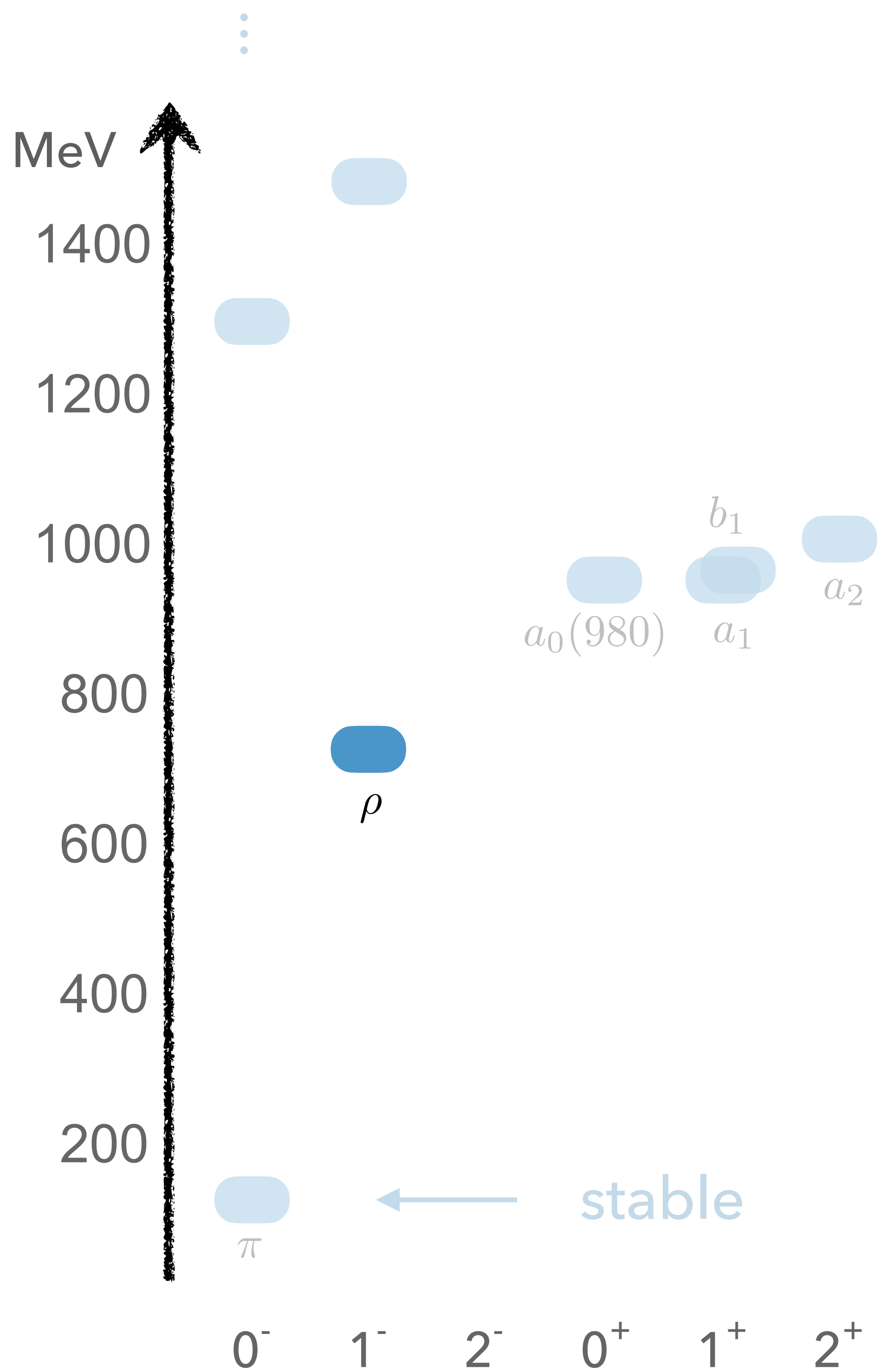
  finite lattice spacing, isospin breaking, EM interactions

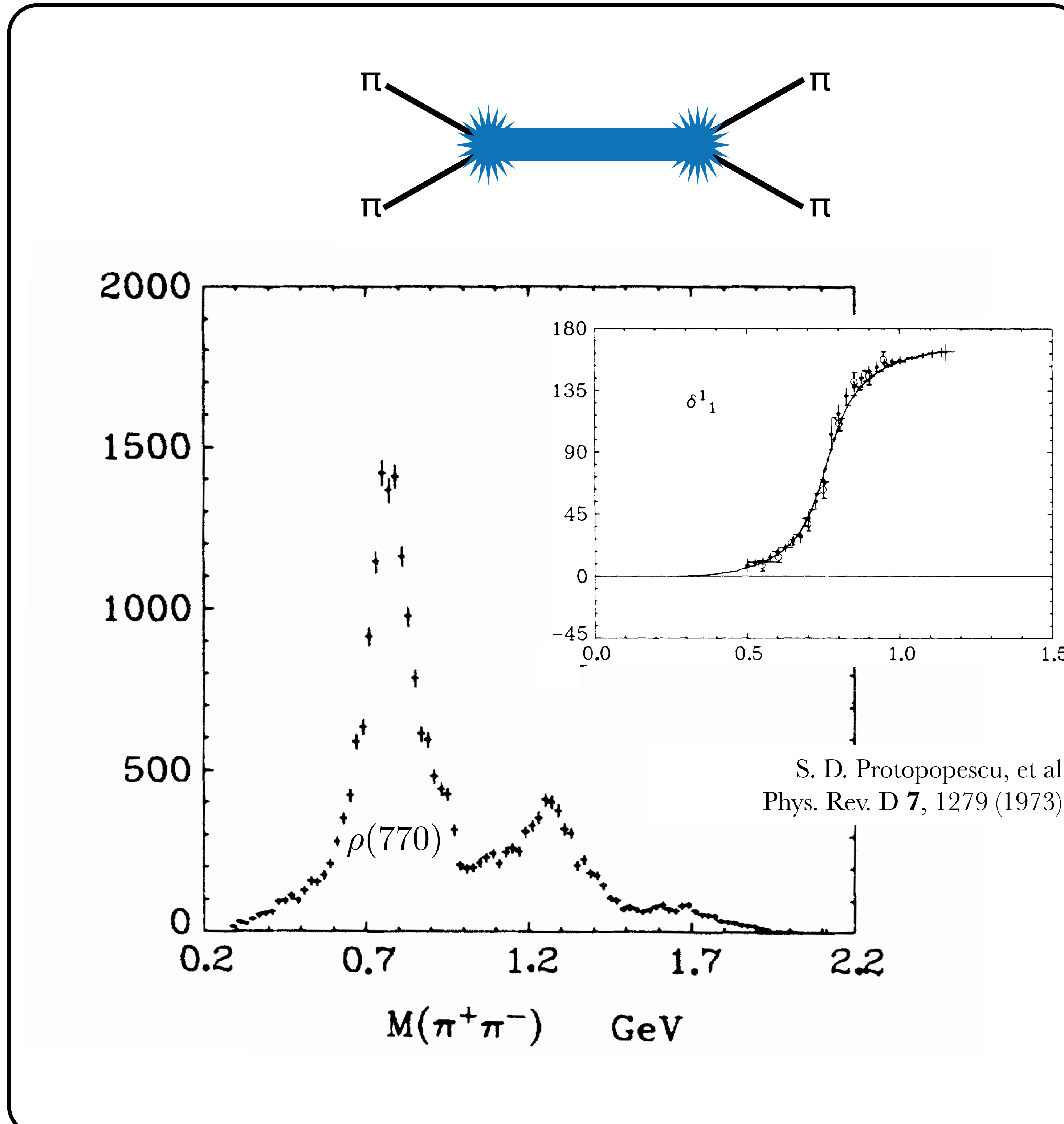
goal: what does QCD say about the excited hadron spectrum?



Light mesons with Isospin=1


MeV
1400
1200
1000
800
600
400
200
b_1
a_2
a_1
$a_0(980)$
ρ
π
stable
resonances
Light mesons with Isospin=1
0^-
1^-
2^-
0^+
1^+
2^+
...





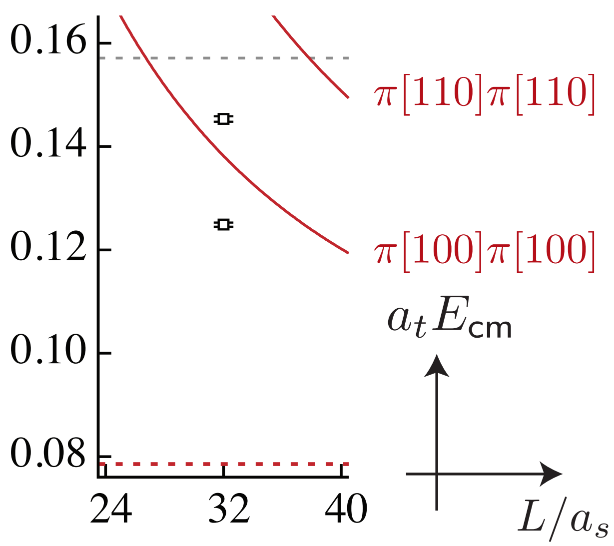
"most rigorous" quantity for unstable states $t \sim \dfrac{c^2}{s_{\rm pole} - s}$ $\sqrt{s_{\rm pole}} = m \pm \frac{i}{2}\Gamma$

$m_\pi = 239$ MeV

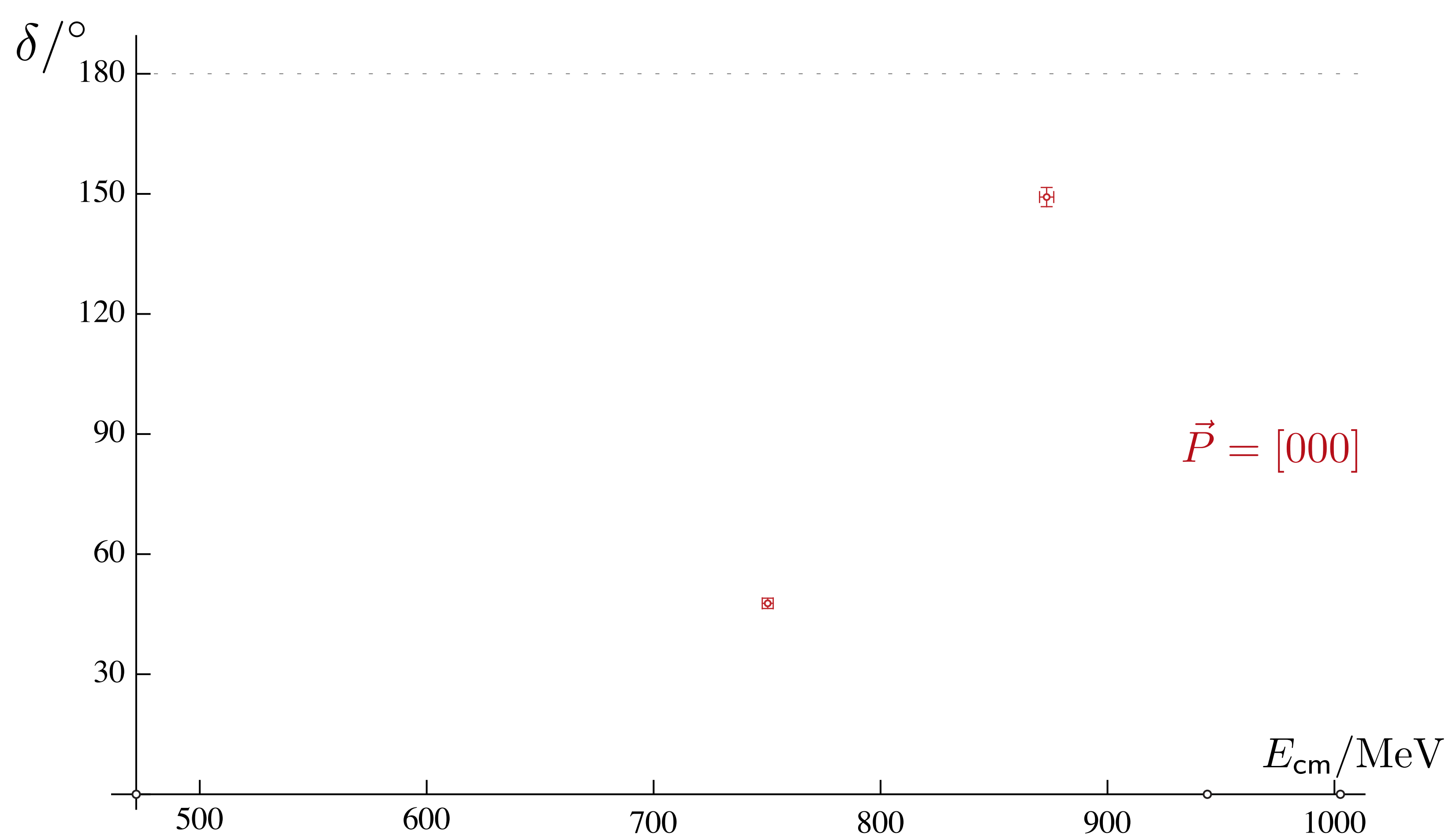
L/a_s=32

momentum is quantized in a finite volume $\vec{p} = \frac{2\pi\vec{n}}{L}$

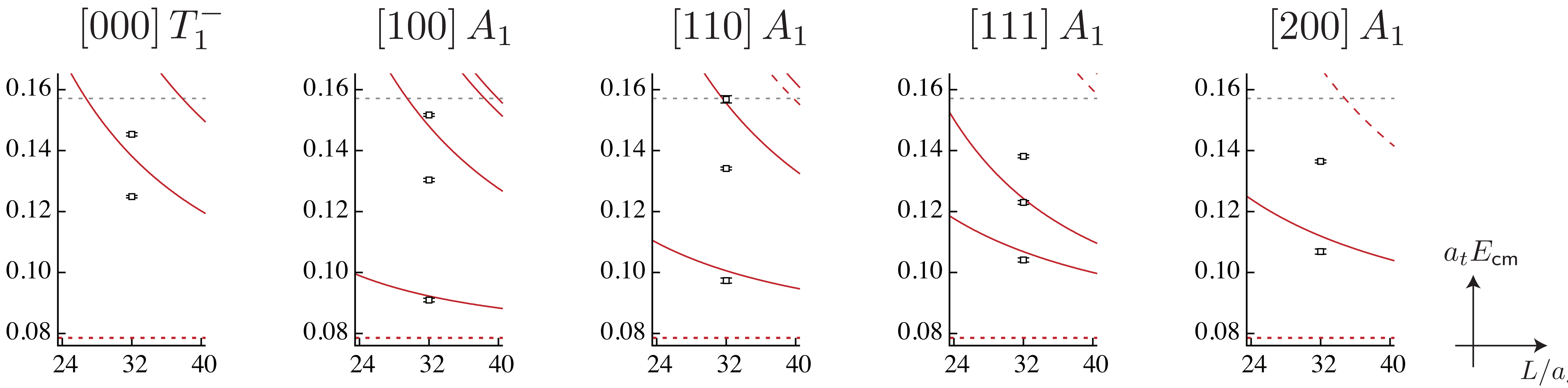
$[000]\, T_1^-$



at-rest

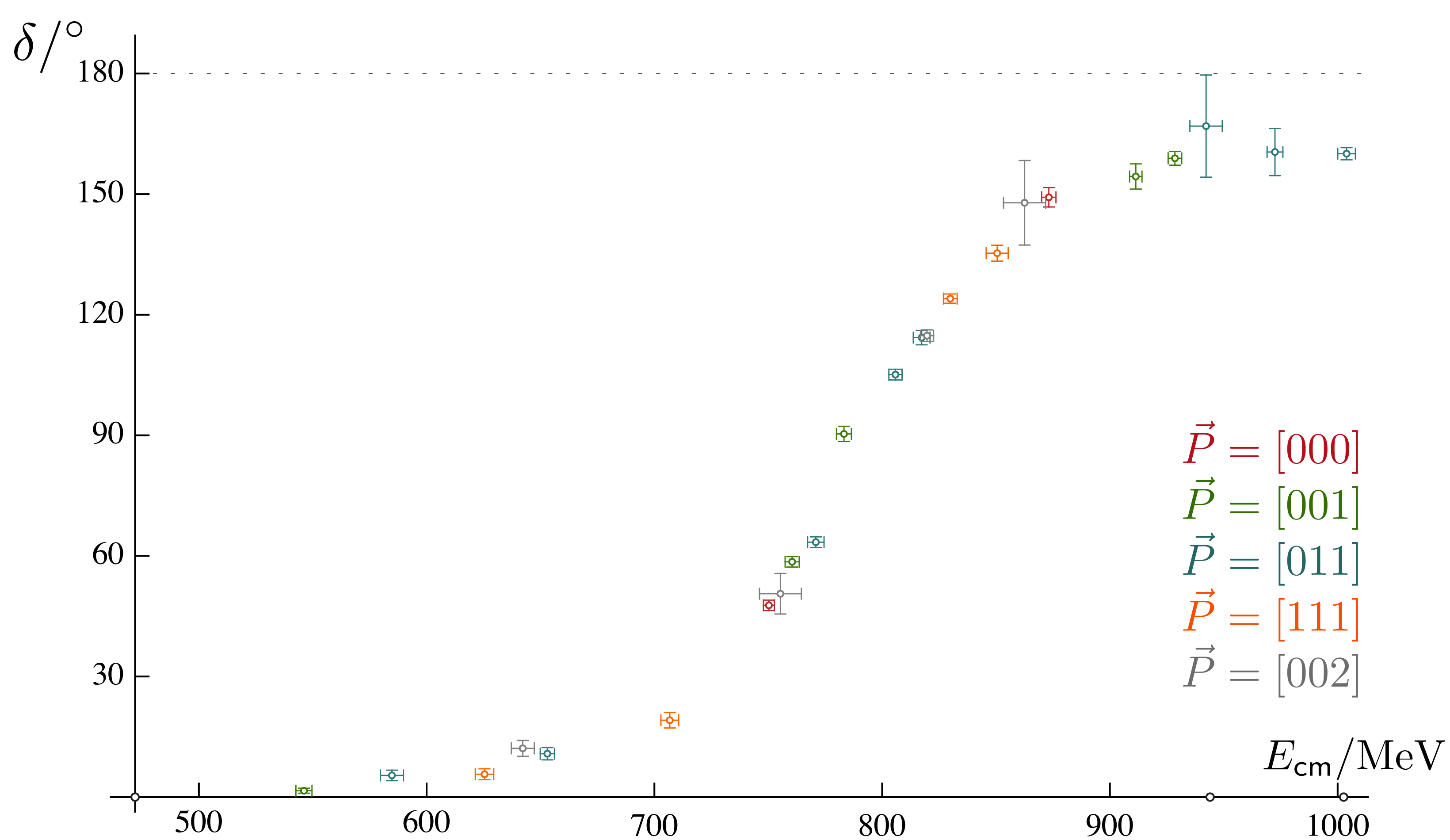


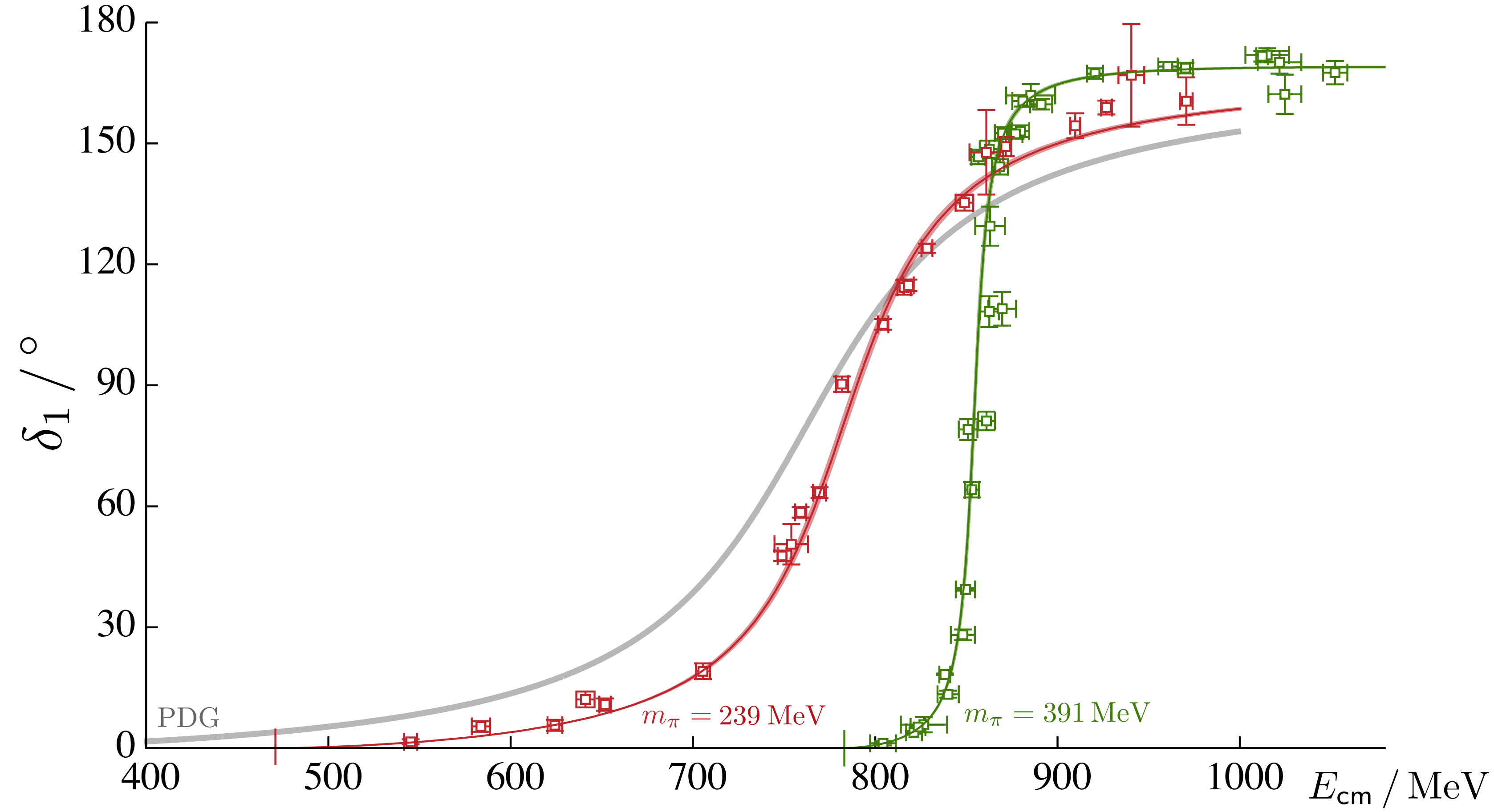
$m_\pi = 239$ MeV



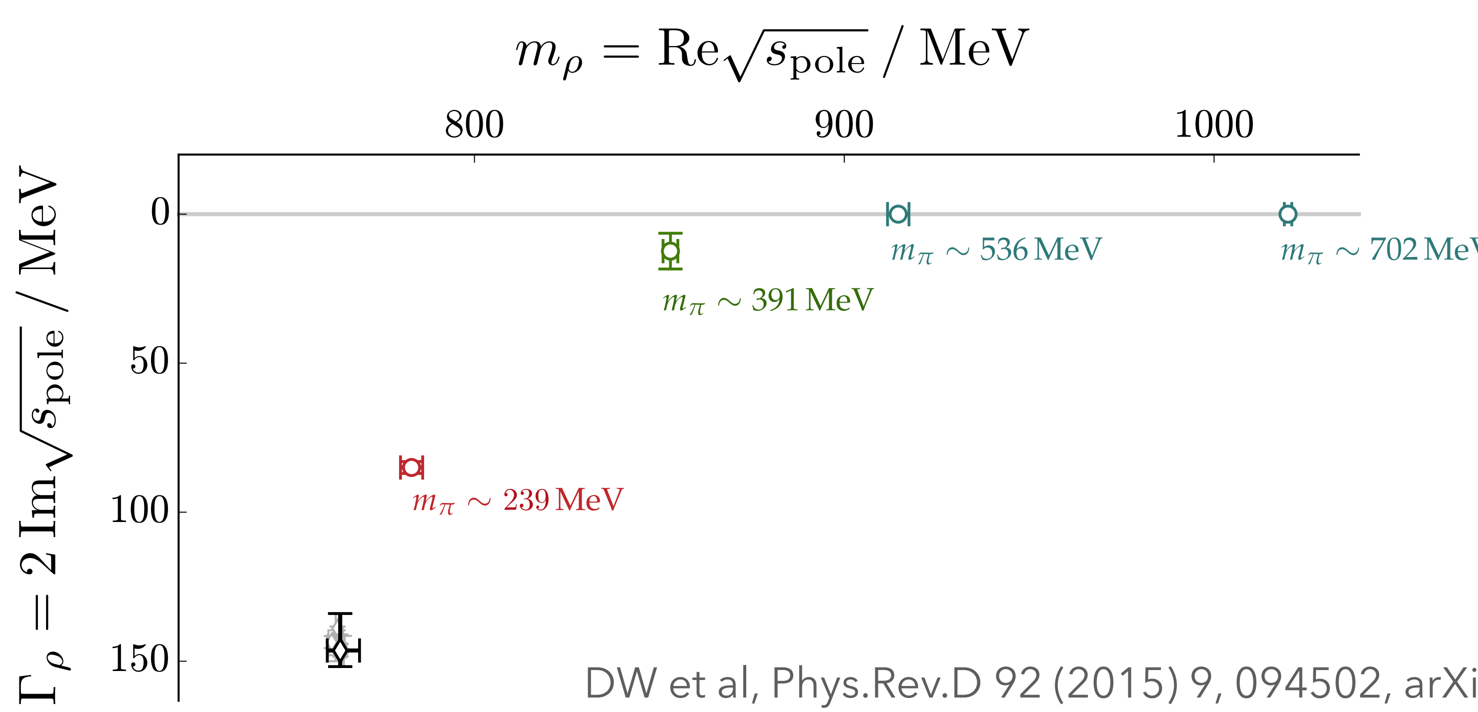
at-rest

non-zero overall momentum

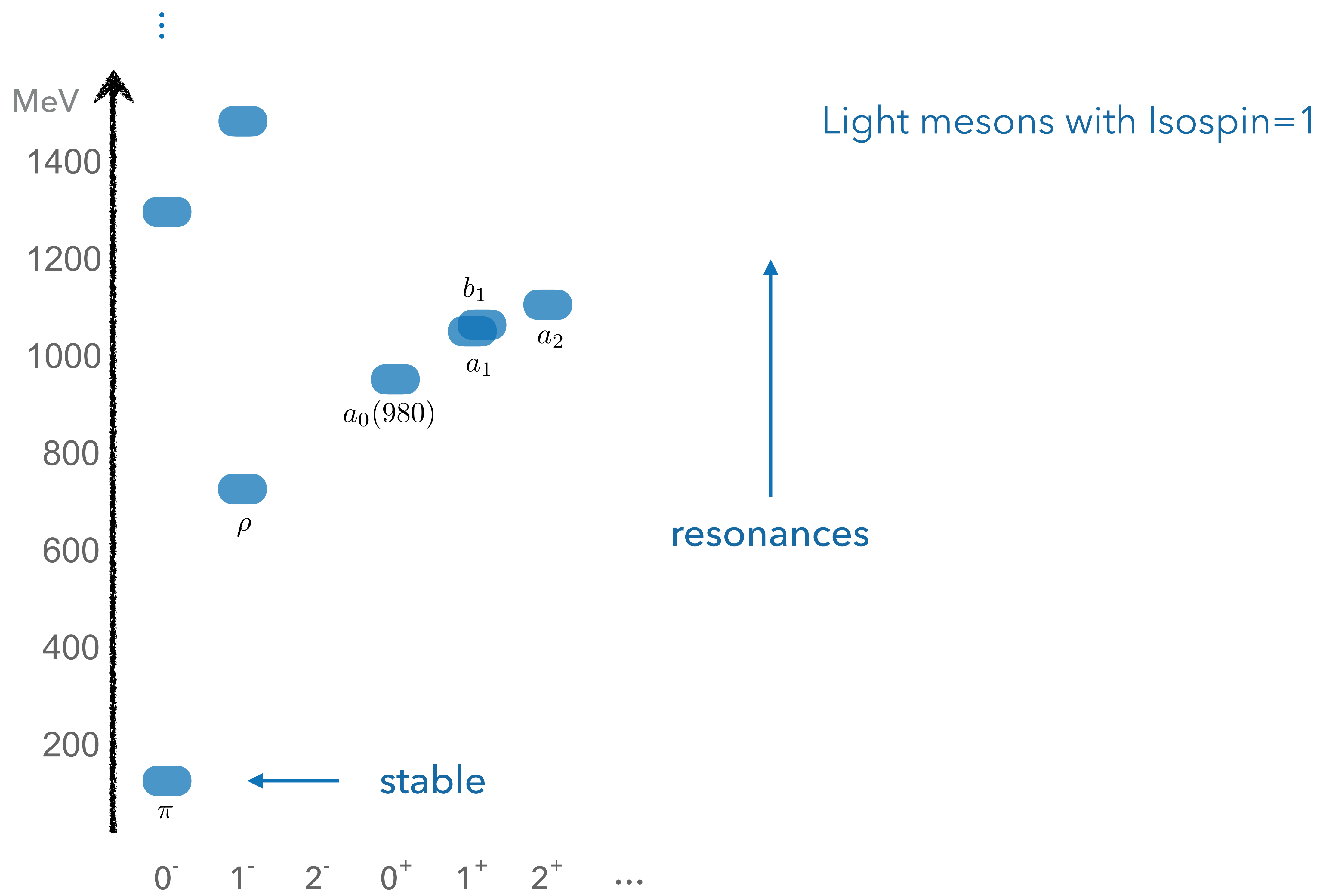




$$t \sim \frac{c^2}{s_{\text{pole}} - s}$$



DW et al, Phys.Rev.D 92 (2015) 9, 094502, arXiv: 1507.02599
J. Dudek et al, Phys.Rev.D 87 (2013) 3, 034505, arXiv:1212.0830


MeV
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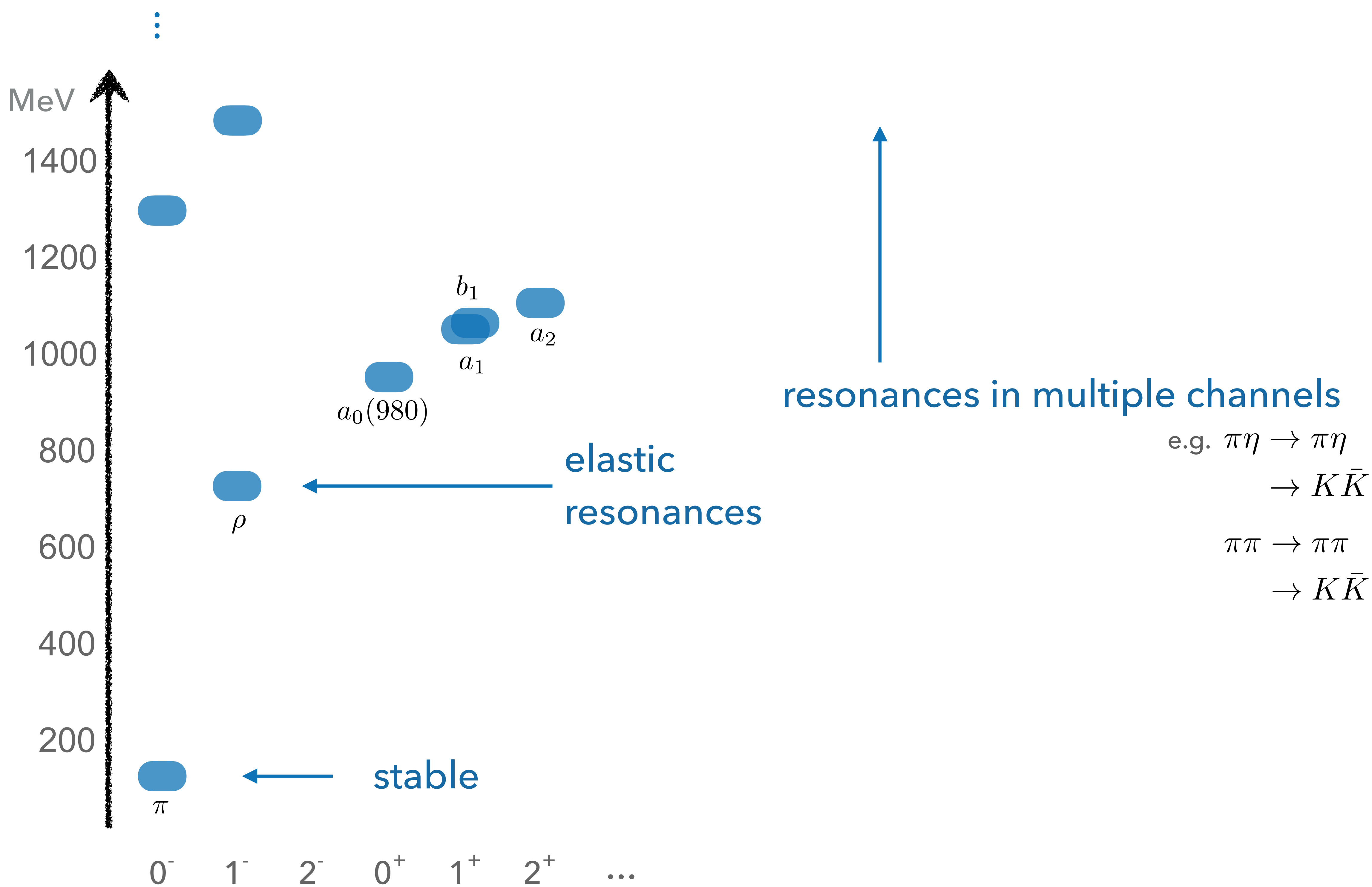
resonances
Light mesons with Isospin=1
0^- 1^- 2^- 0^+ 1^+ 2^+ ...


MeV
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400
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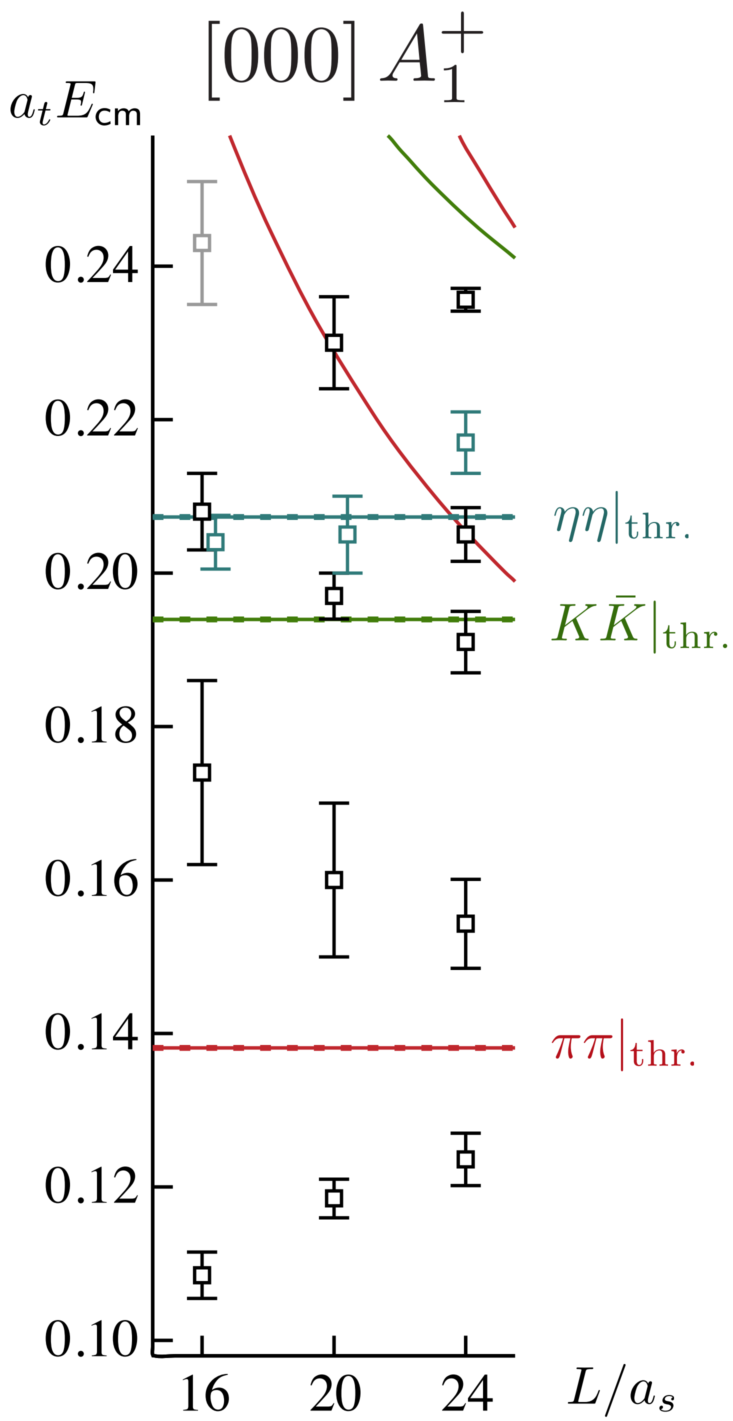
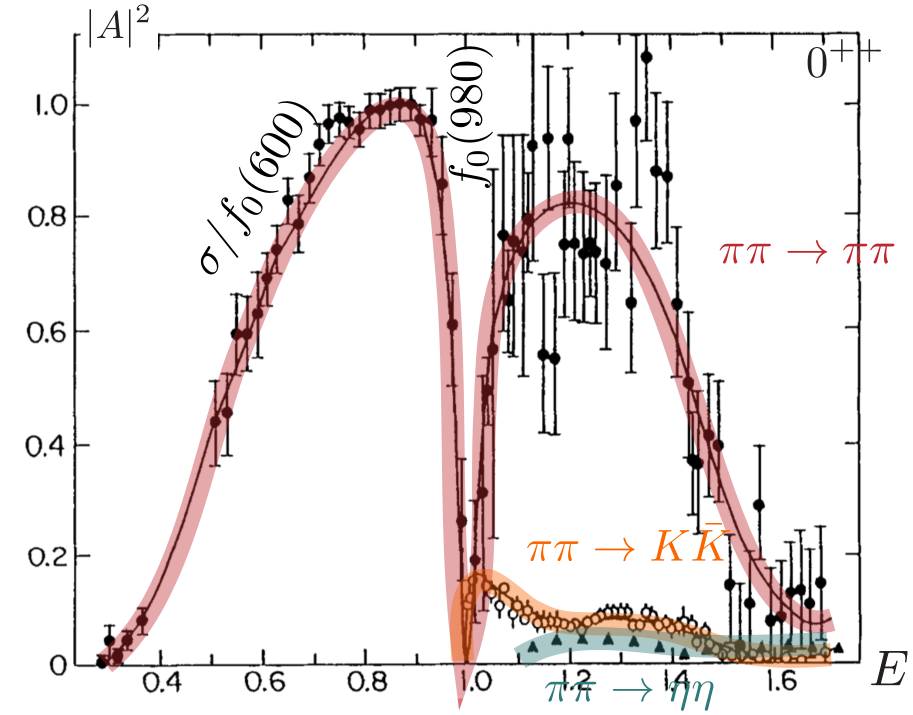
b_1
a_2
a_1
$a_0(980)$
ρ
π
0⁻ 1⁻ 2⁻ 0⁺ 1⁺ 2⁺ …
stable
elastic resonances
resonances in multiple channels
e.g. $\pi\eta \to \pi\eta$
$\to K\bar{K}$
$\pi\pi \to \pi\pi$
$\to K\bar{K}$

$m_\pi = 391$ MeV



CERN-Munich, ANL, BNL



R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

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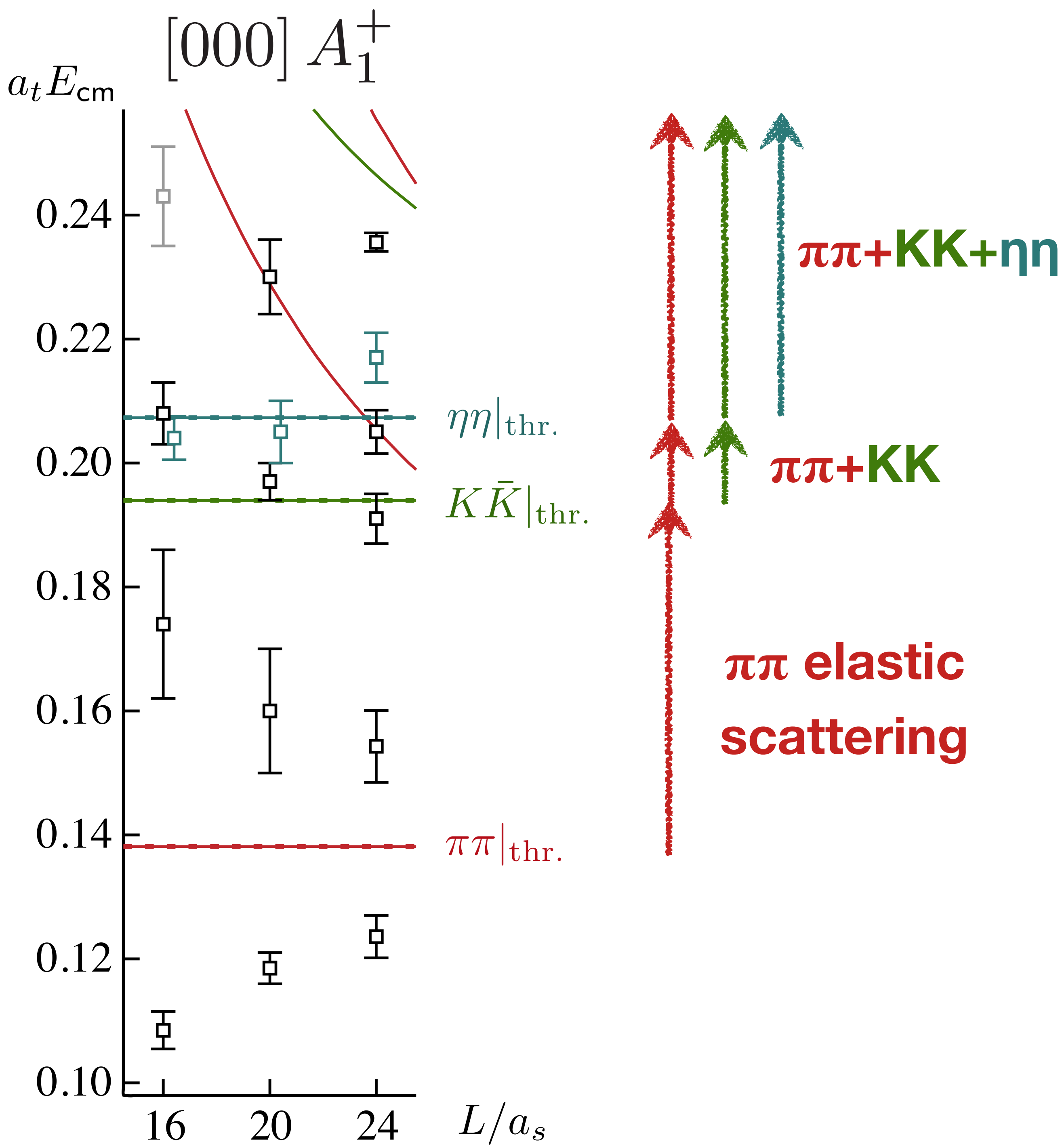

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} & \pi\pi \to \eta\eta \\ & K\bar{K} \to K\bar{K} & K\bar{K} \to \eta\eta \\ & & \eta\eta \to \eta\eta \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ & K\bar{K} \to K\bar{K} \end{pmatrix}$$

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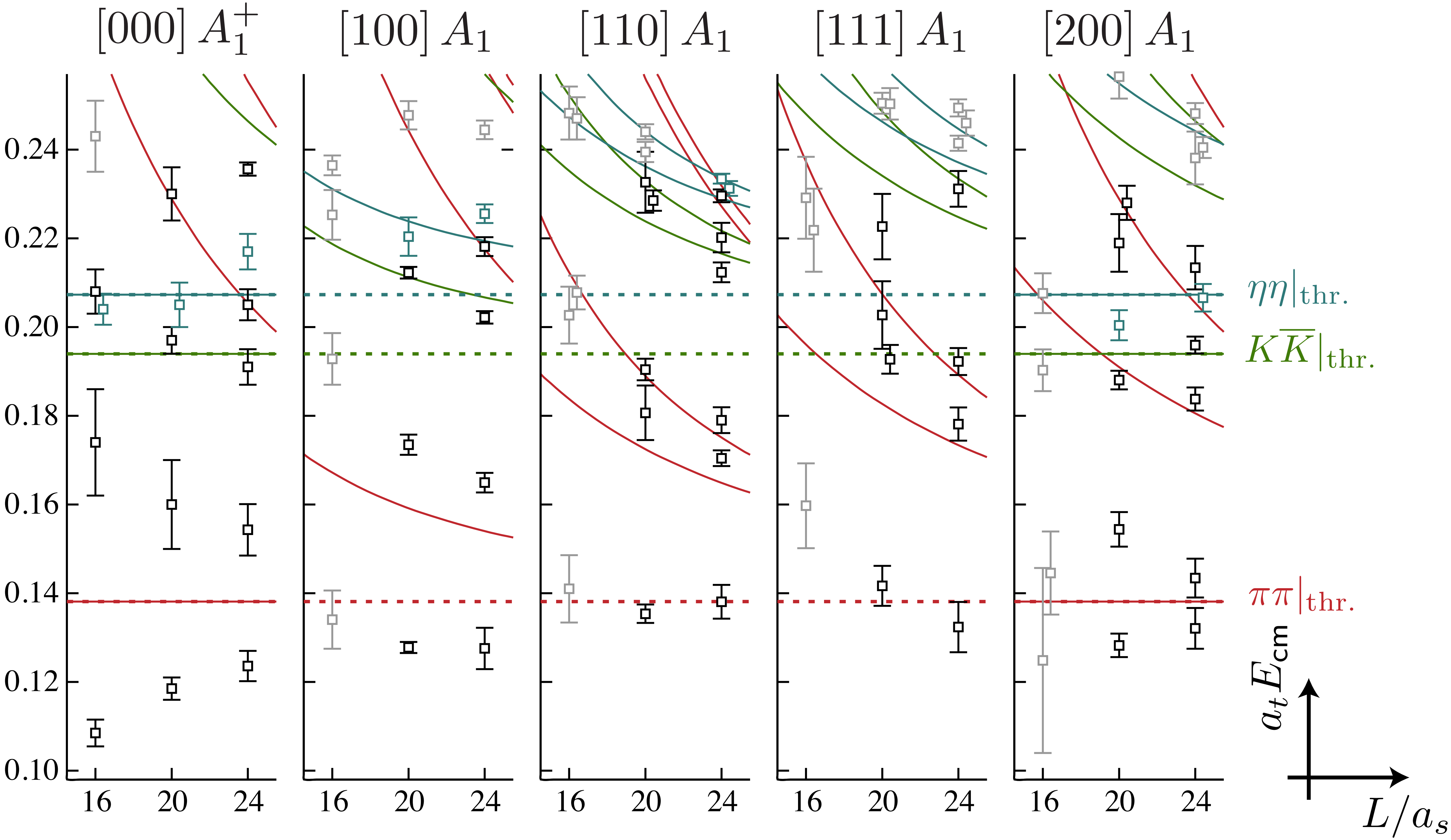
R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

$m_\pi = 391$ MeV

local q$\overline{\text{q}}$ & 2-hadron operators
conservatively 57 energy levels
dominated by S-wave interactions



R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L))\right] = 0$$

Lüscher et al

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

determinant condition:

- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations

- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \to \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Chew-Mandelstam phase space:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$

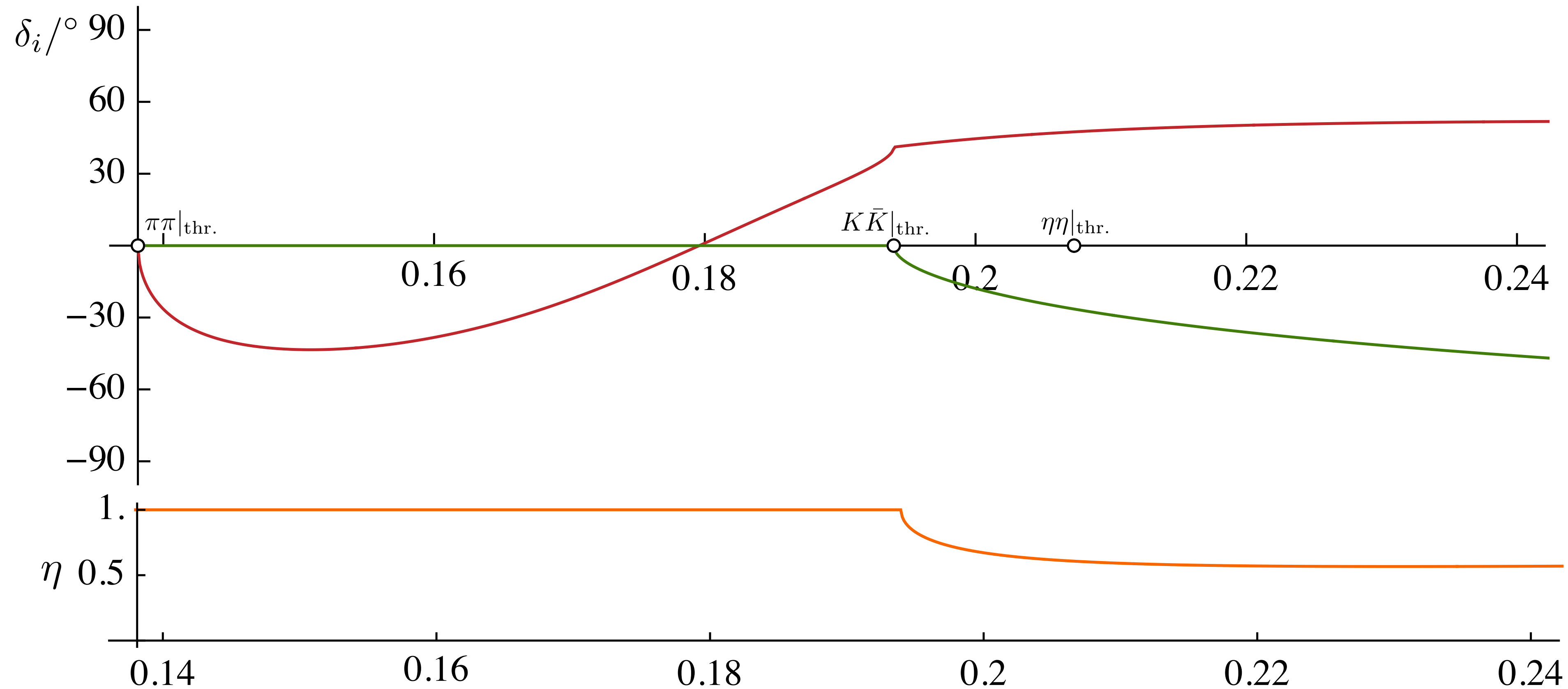
use a dispersion relation to generate a real part from iρ

- K is real for real energies
- we use a broad selection of K-matrices
- neglects left-hand cut

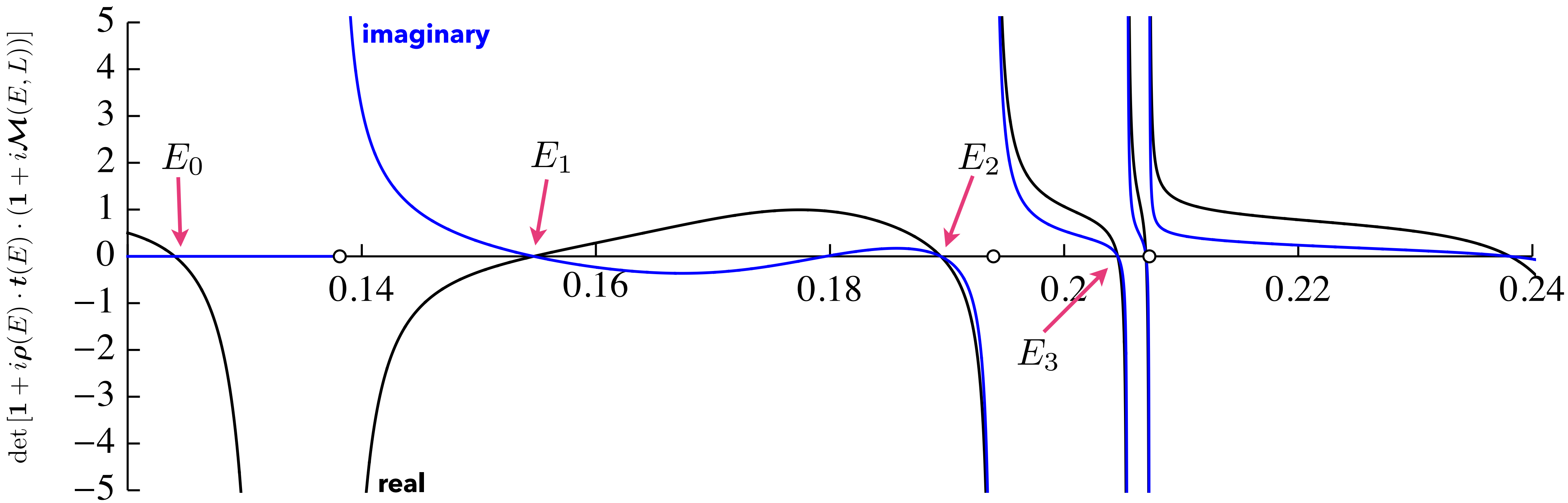
$$t_{11} = \frac{1}{2i\rho_1}\left(\eta e^{2i\delta_1} - 1\right)$$

$$t_{12} = \frac{1}{2\sqrt{\rho_1\rho_2}}(1-\eta^2)^{\frac{1}{2}} e^{i\delta_1 + i\delta_2}$$

$$S_{ii} = \eta e^{2i\delta_i}$$

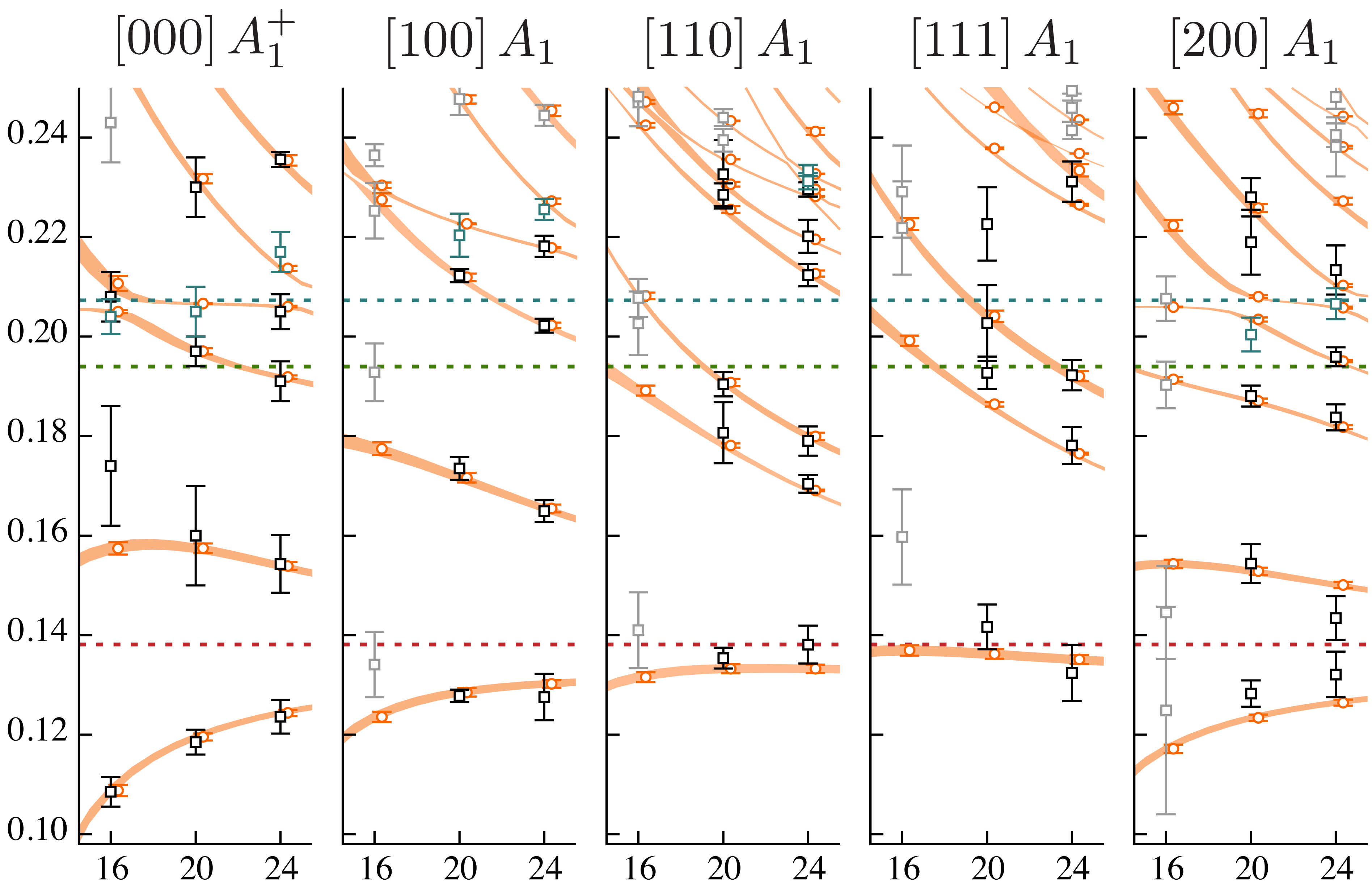


we can identify the solutions



$m_\pi = 391$ MeV

$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$



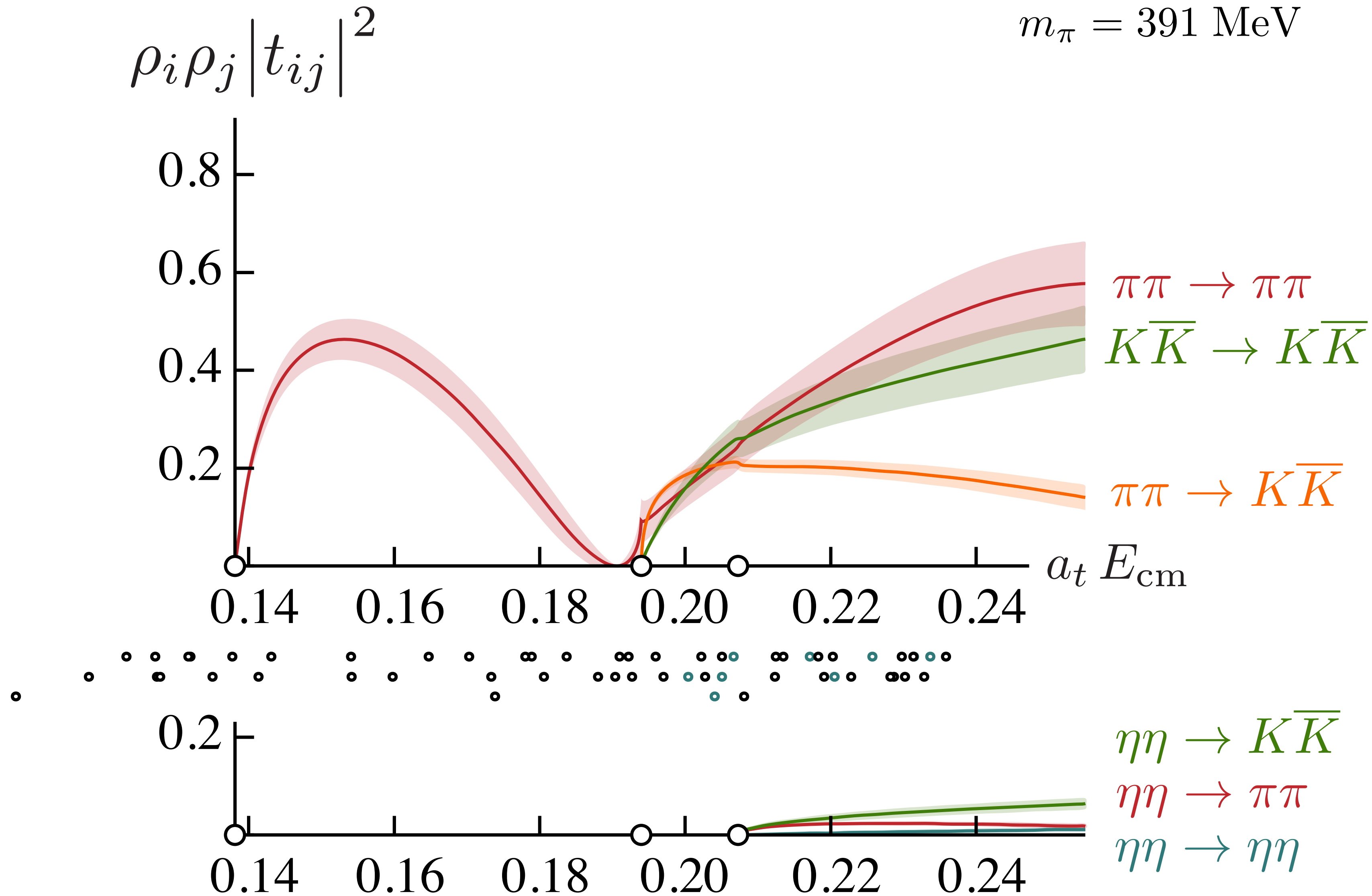
$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

An example S-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a + bs & c + ds & e \\ c + ds & f & g \\ e & g & h \end{pmatrix}$$

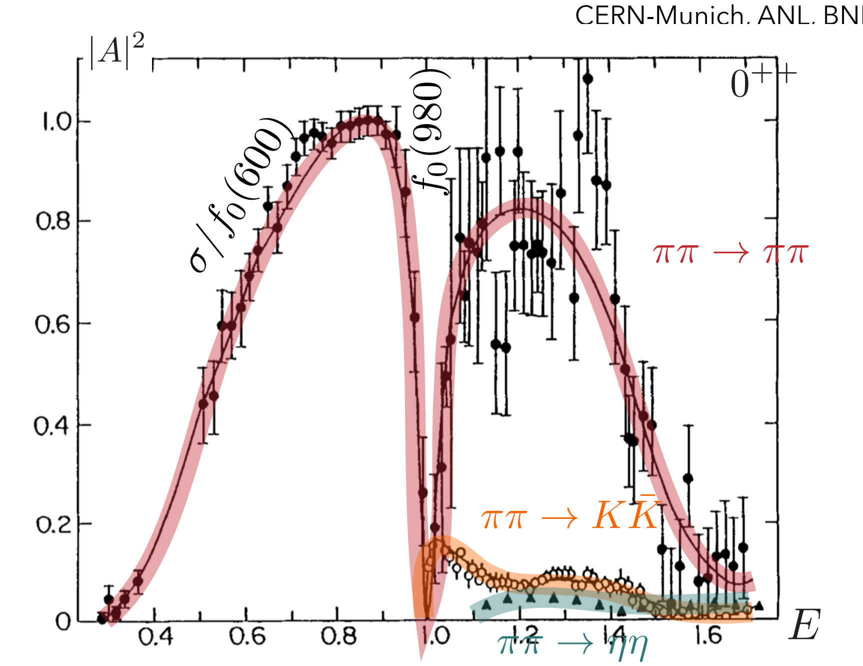
$$s = E_{\mathsf{cm}}^2$$



$$\chi^2/N_{\rm dof} = \frac{44.0}{57-8} = 0.90$$

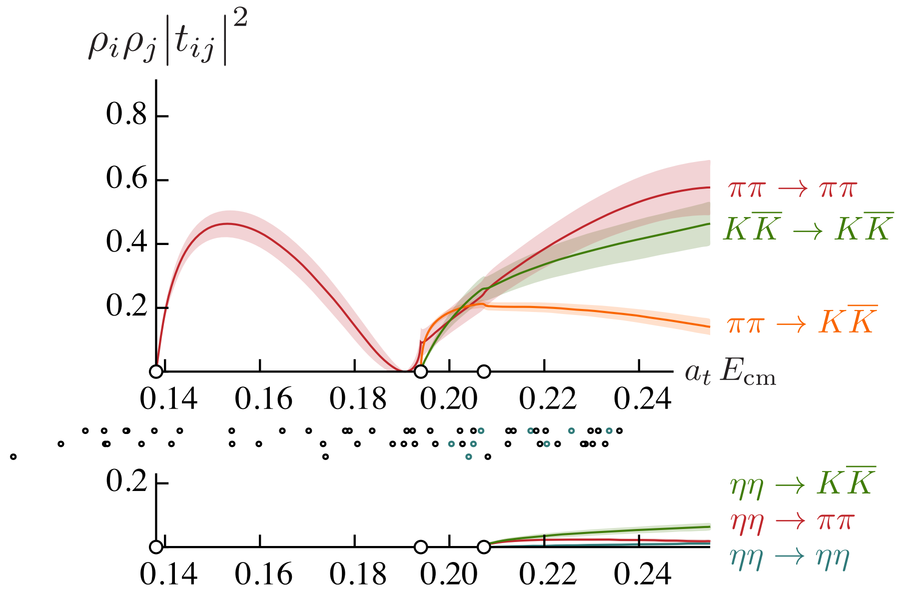
57 energy levels

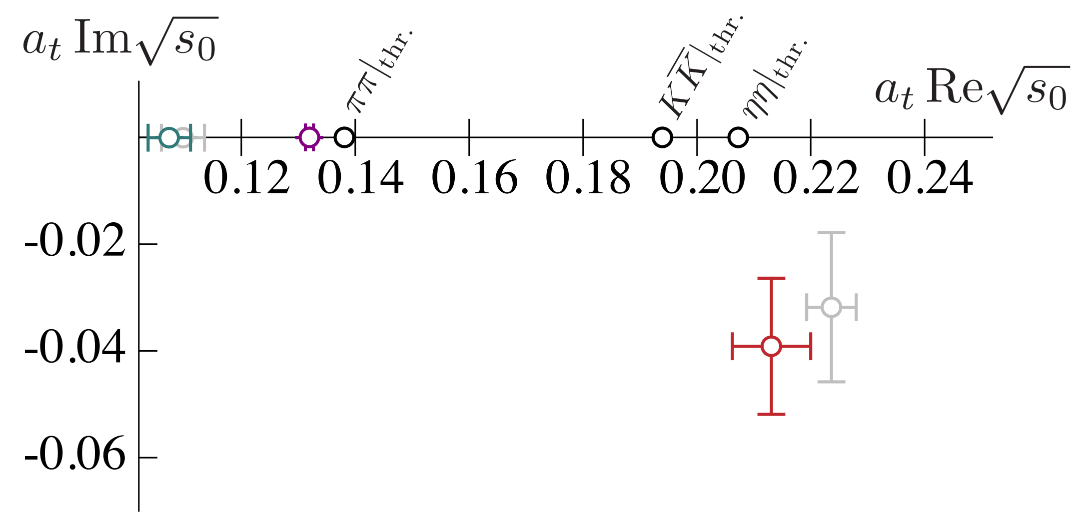
CERN-Munich. ANL. BNL

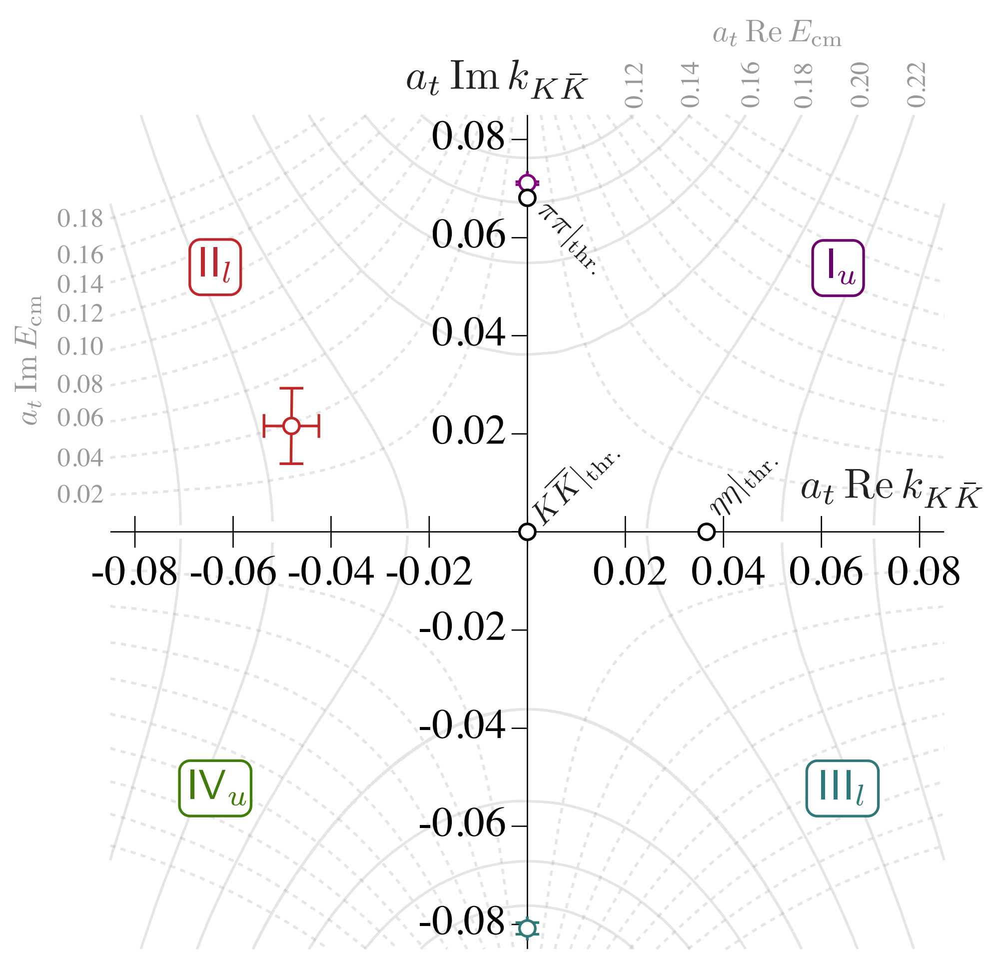


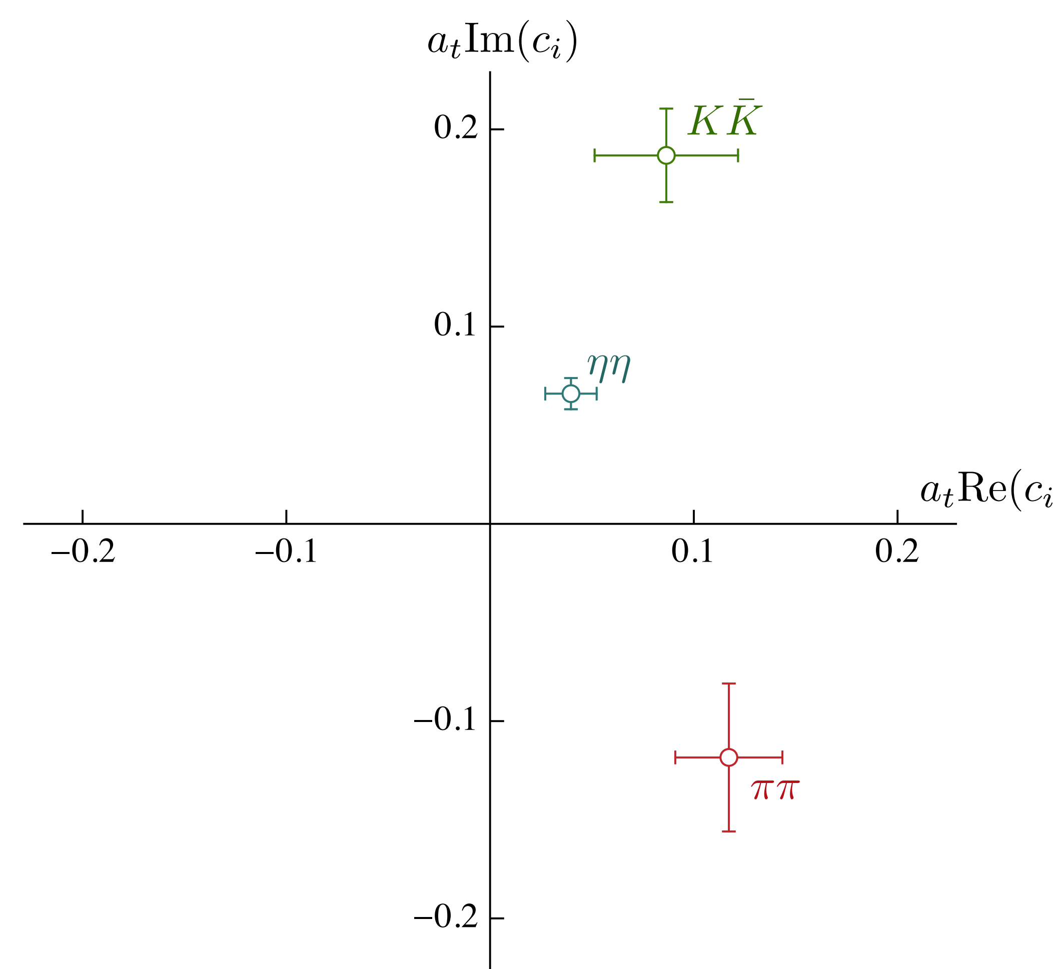
$m_\pi = 391$ MeV

a pole in multiple channels: $t_{ij} \sim \dfrac{c_i c_j}{s_{\rm pole} - s}$ $\sqrt{s_{\rm pole}} = m \pm \frac{i}{2}\Gamma$









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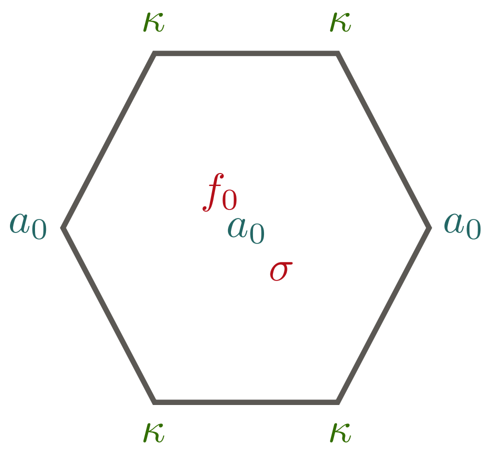

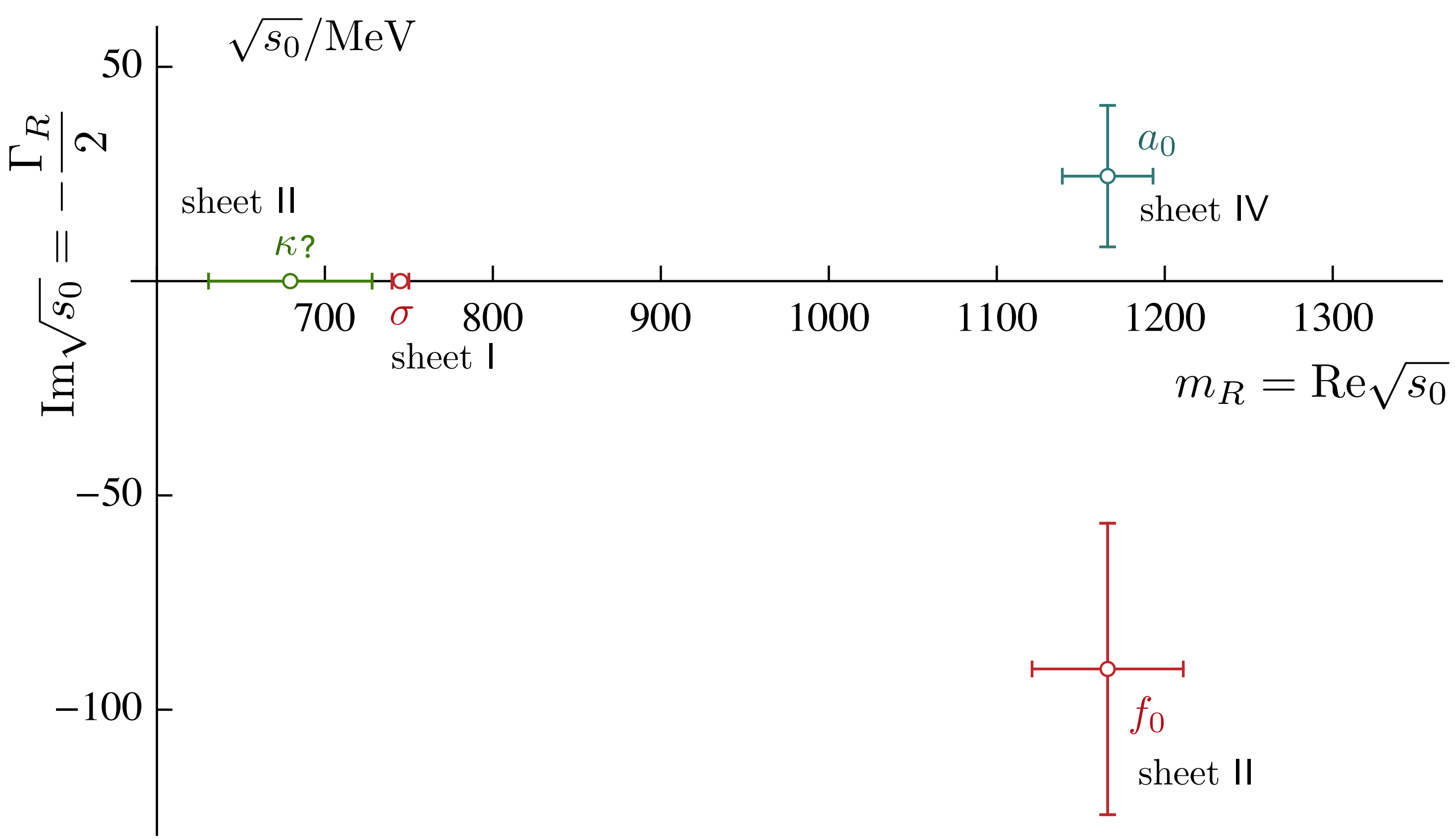

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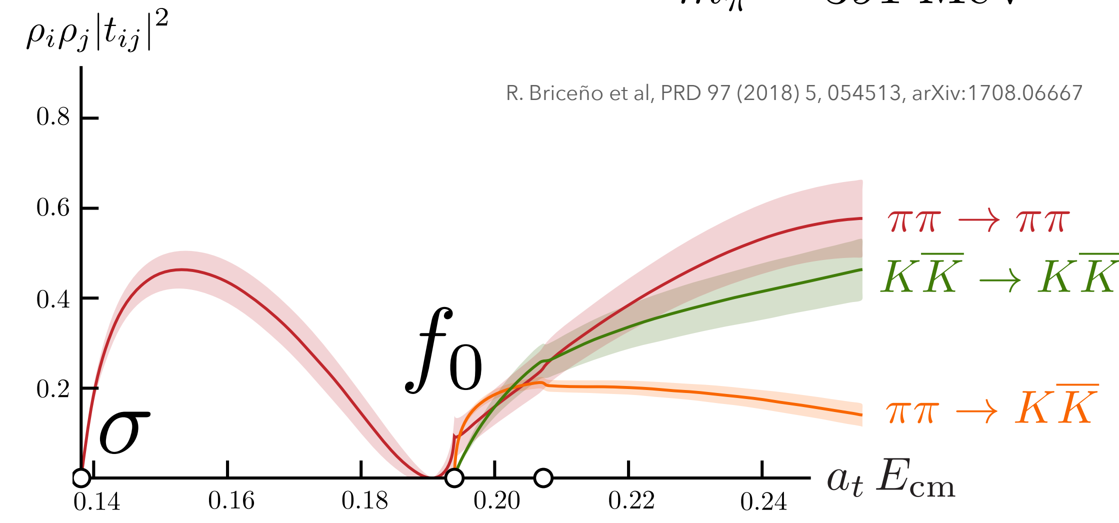

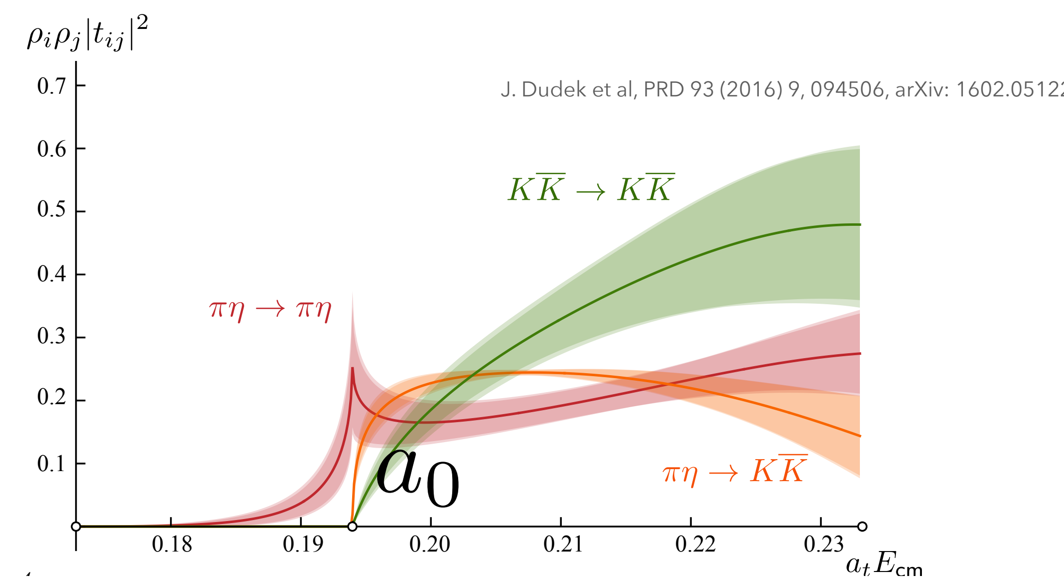

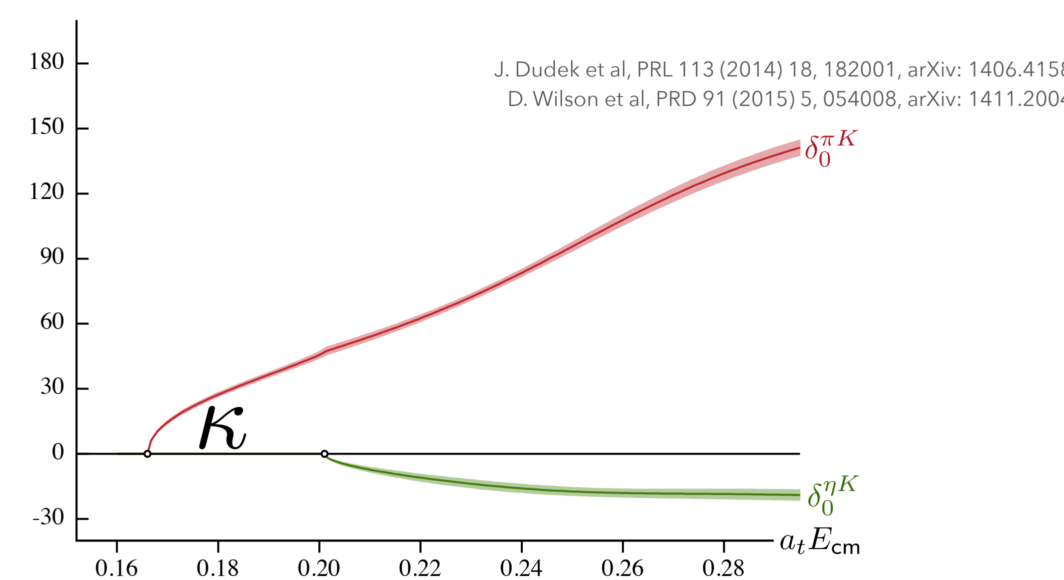

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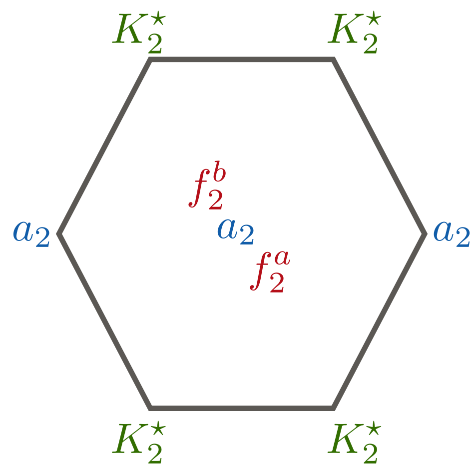
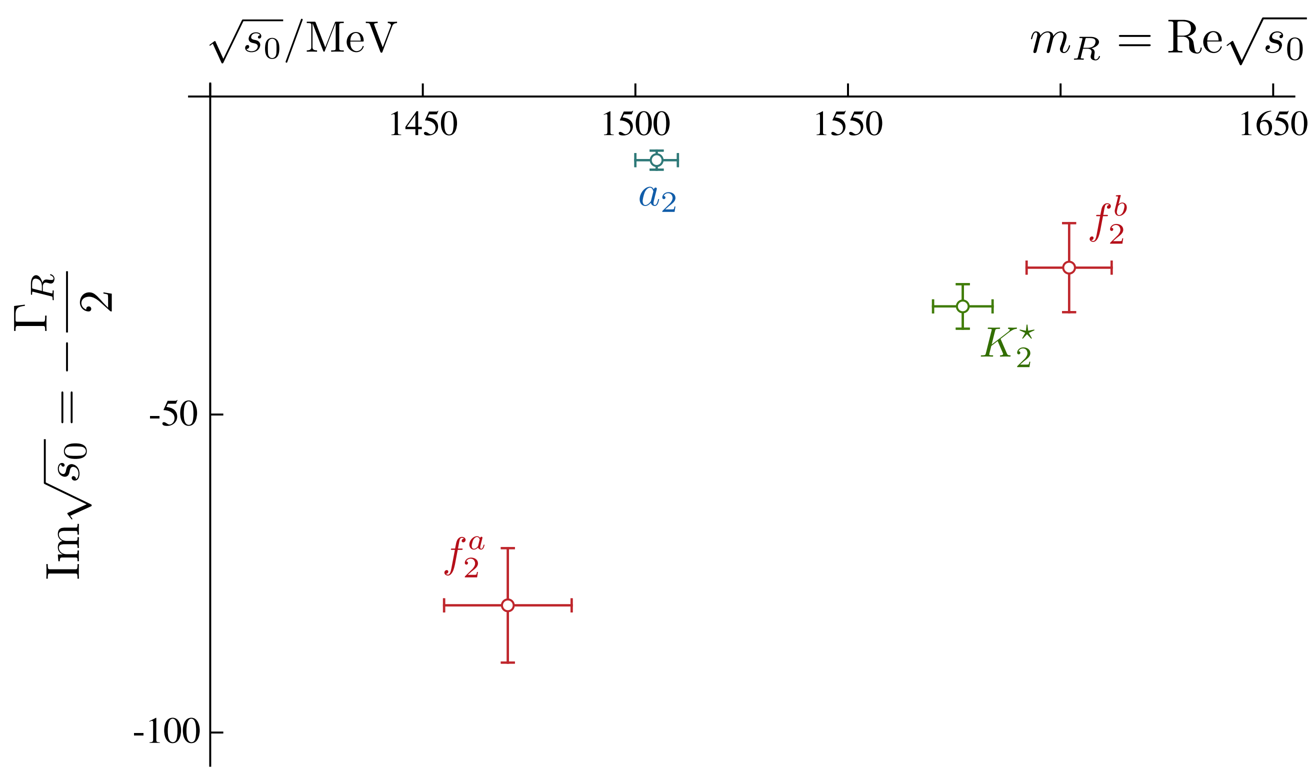


$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$



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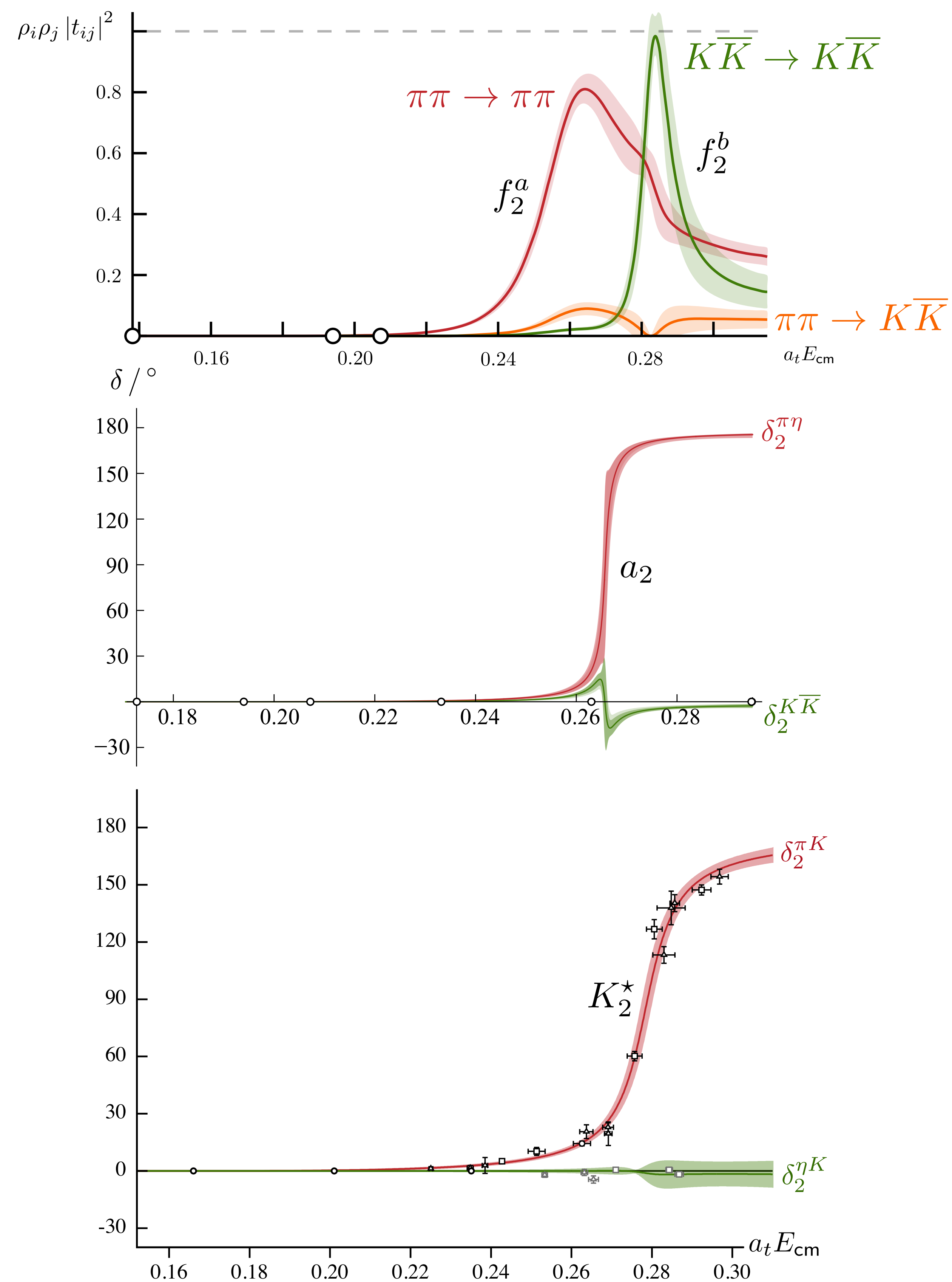
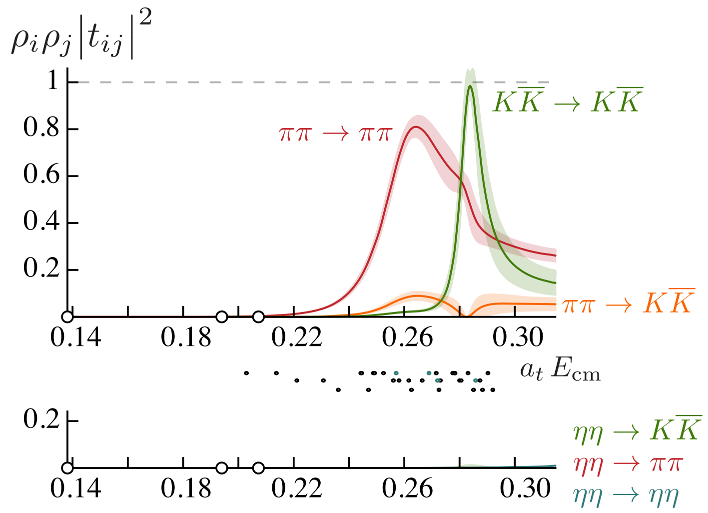
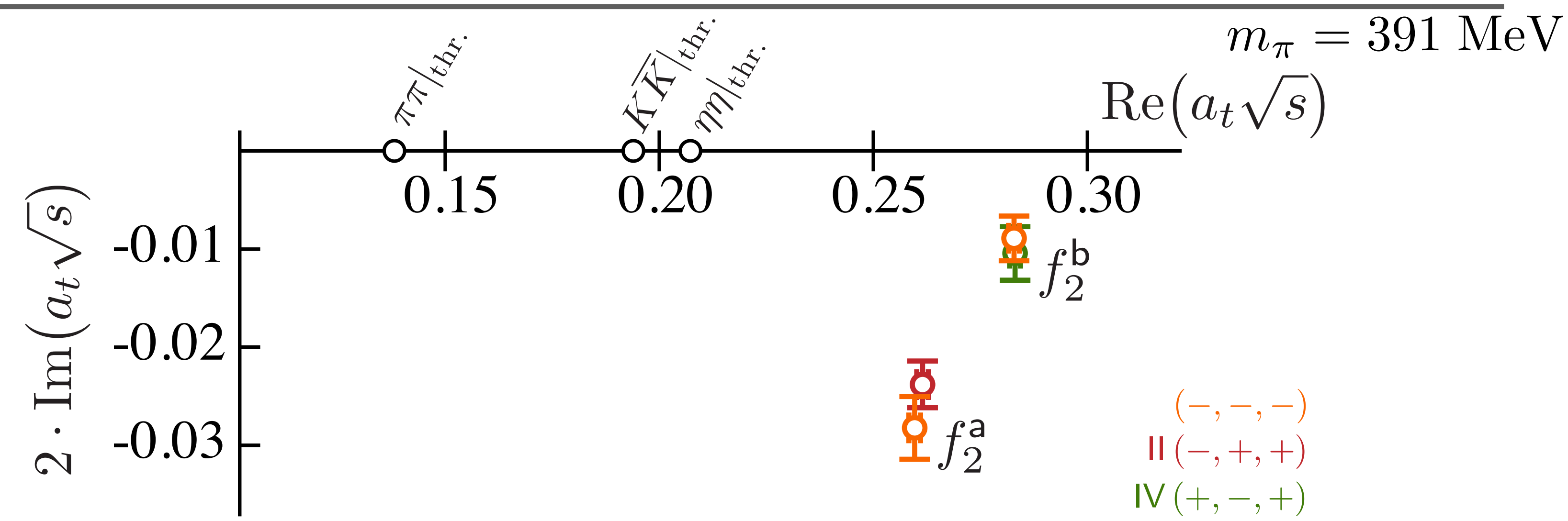
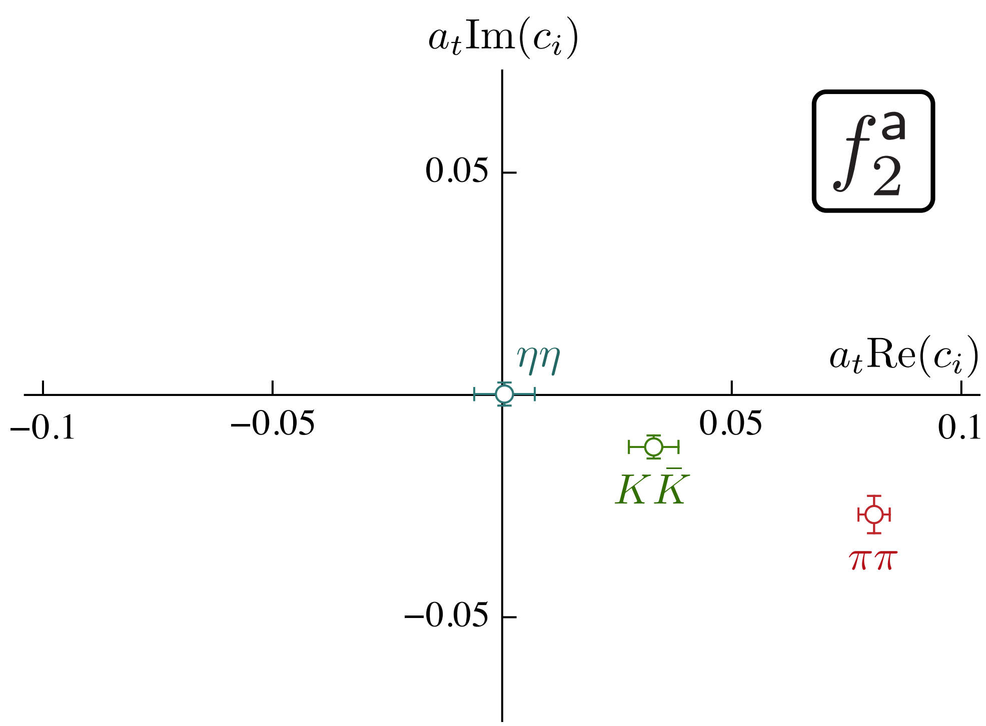
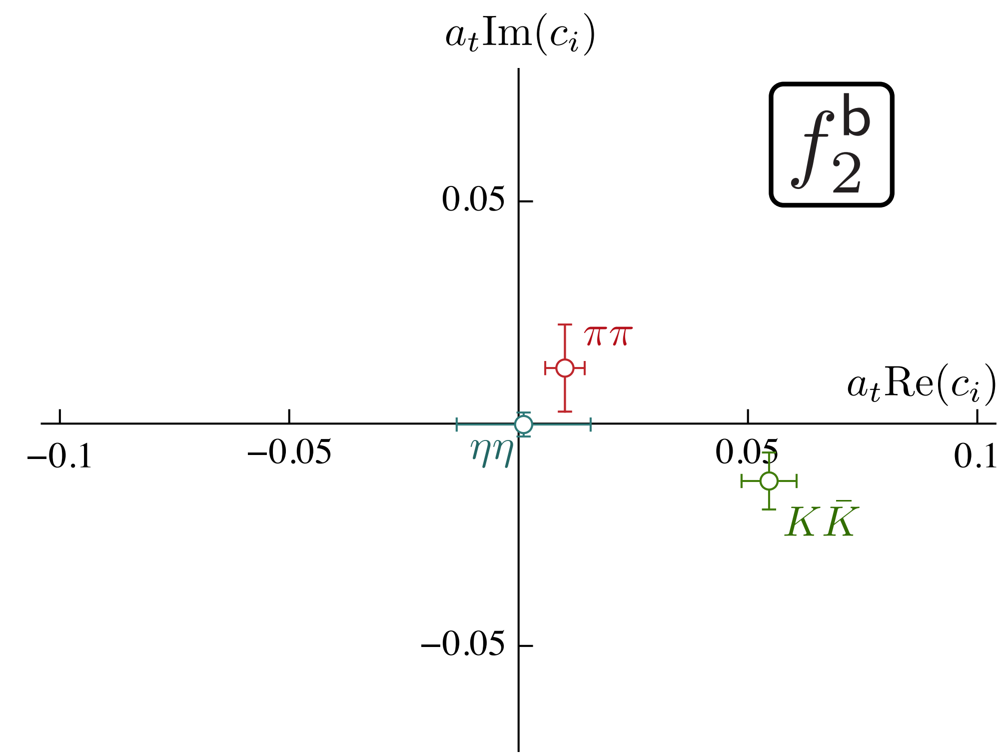








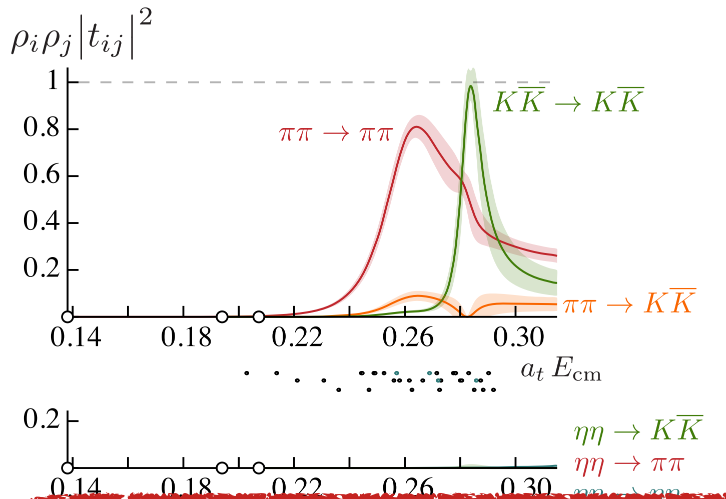


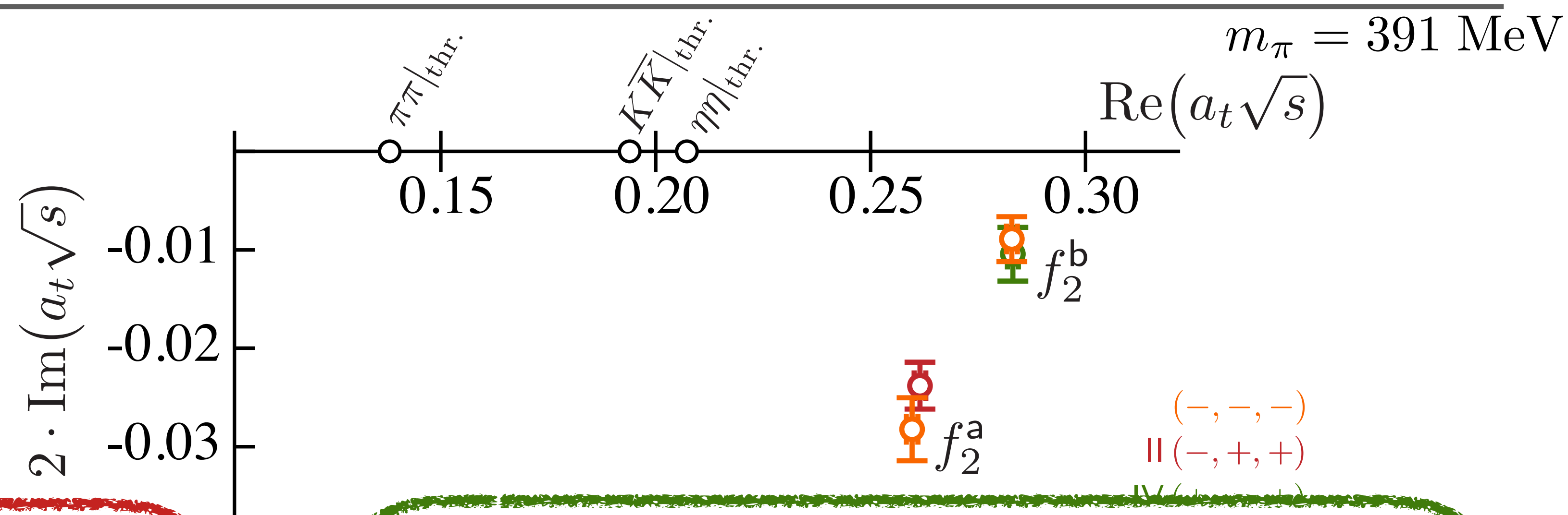
$f_2^a: \quad \sqrt{s_0} = 1470(15) - \frac{i}{2}\,160(18)$ MeV

$\mathrm{Br}(f_2^a \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^a \to K\overline{K}) \sim 12\%$

$f_2^b: \quad \sqrt{s_0} = 1602(10) - \frac{i}{2}\,54(14)$ MeV

$\mathrm{Br}(f_2^b \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^b \to K\overline{K}) \sim 92\%$



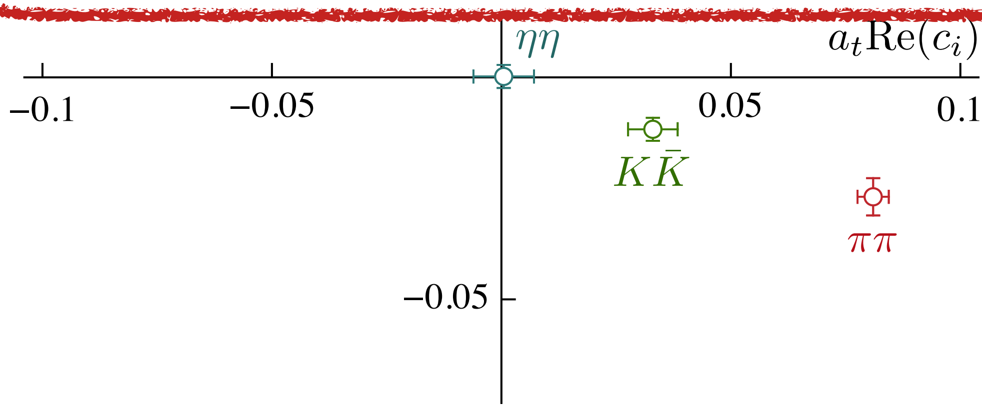


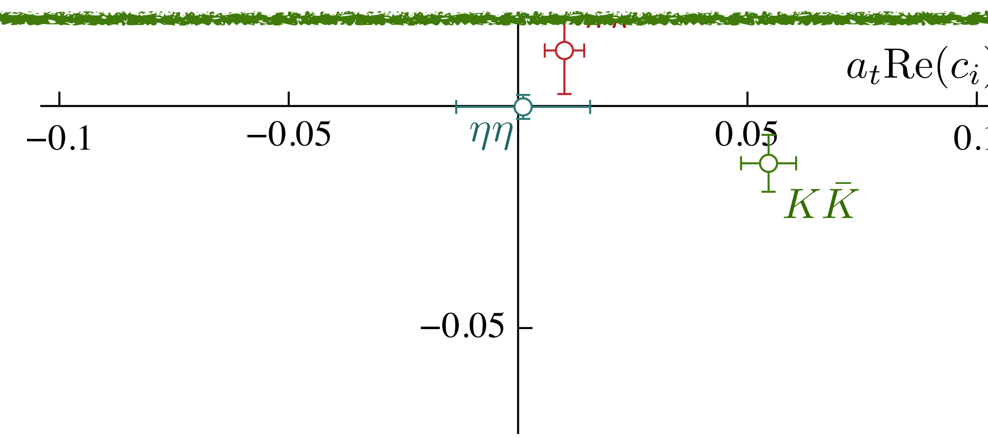
$f_2(1270)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	$(84.2\ ^{+2.9}_{-0.9})$ %
Γ_2	$\pi^+\pi^-2\pi^0$	$(7.7\ ^{+1.1}_{-3.2})$ %
Γ_3	$K\bar{K}$	$(4.6\ ^{+0.5}_{-0.4})$ %
Γ_4	$2\pi^+2\pi^-$	(2.8 ± 0.4) %

$f_2'(1525)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\bar{K}$	(88.7 ± 2.2) %
Γ_2	$\eta\eta$	(10.4 ± 2.2) %
Γ_3	$\pi\pi$	$(8.2 \pm 1.5) \times 10^{-3}$
Γ_4	$K\bar{K}^*(892)+$ c.c.	





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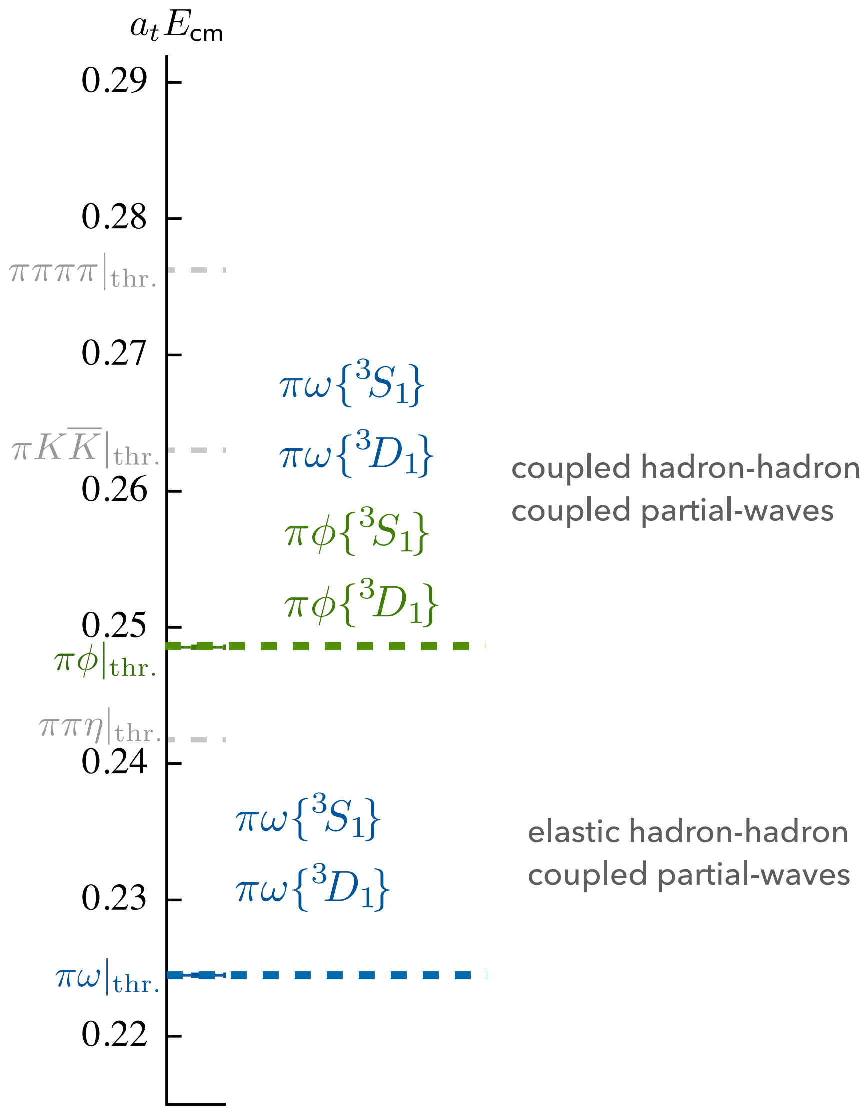
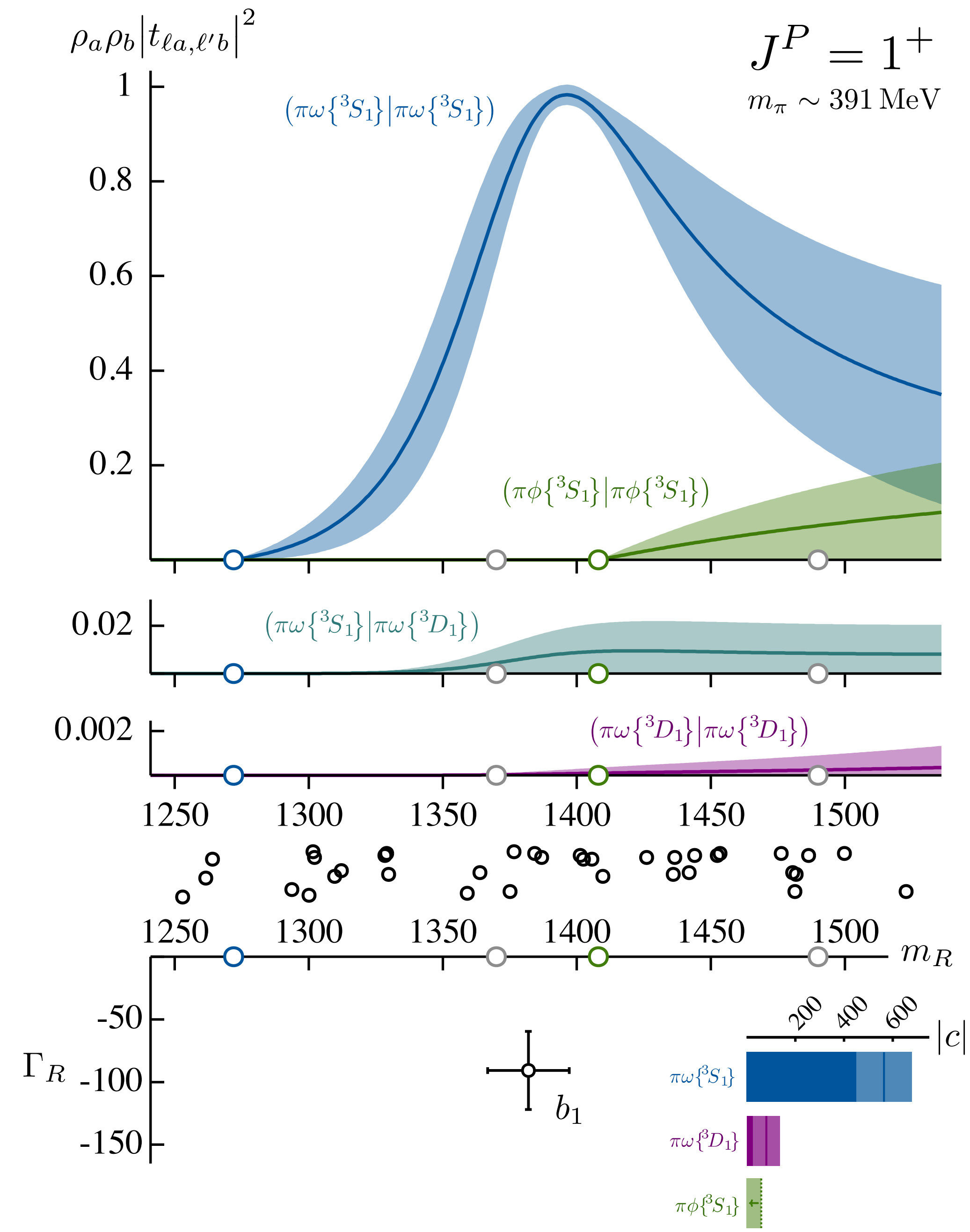
scattering with spinning particles

A. J. Woss, et al, PRD100 (2019) 5, 054506, arXiv: 1904.04136

stable vector ω scattering





$b_1(1235)$ $I^G(J^{PC}) = 1^+(1^{+-})$

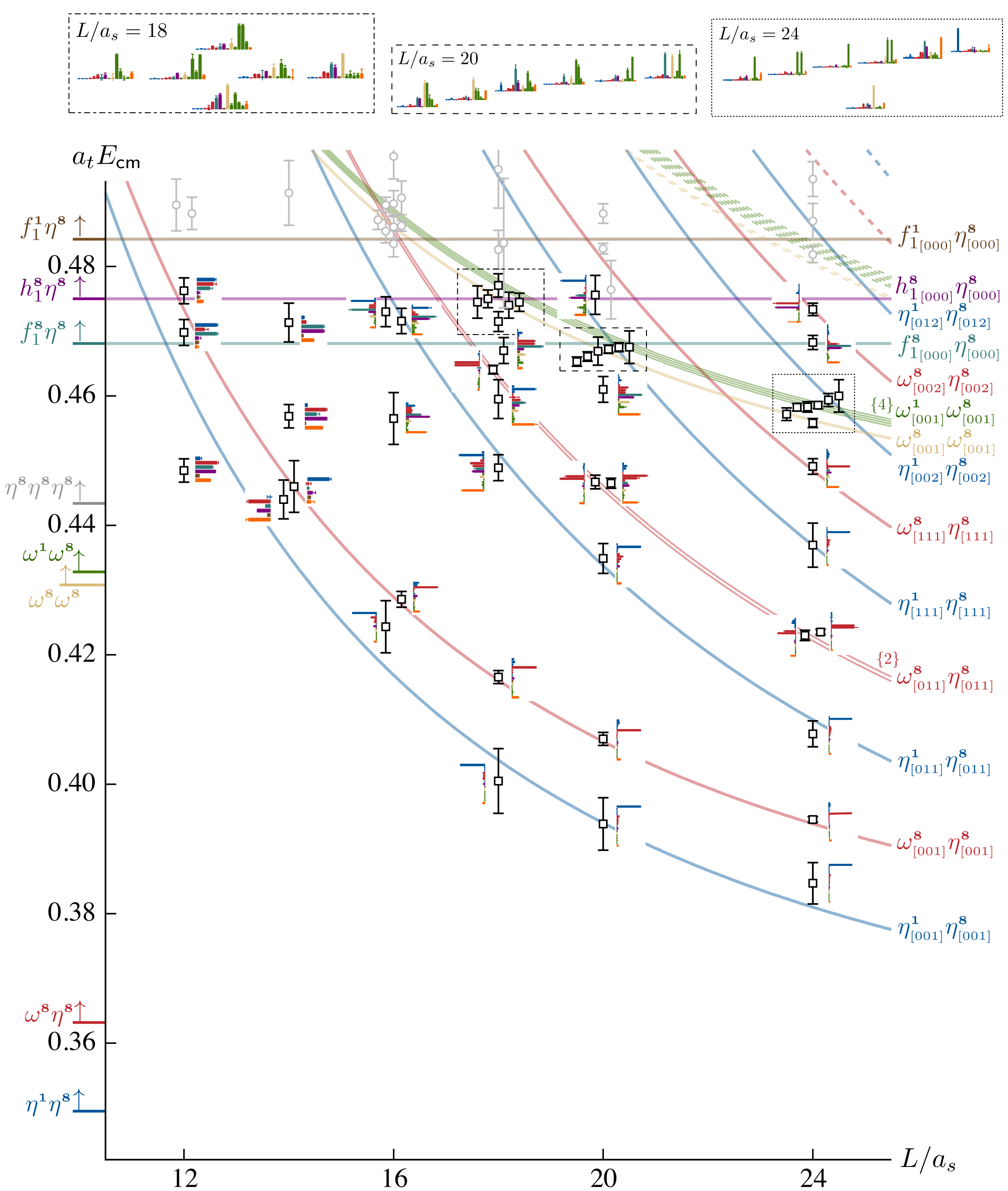
Mass $m = 1229.5 \pm 3.2$ MeV (S = 1.6)
Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

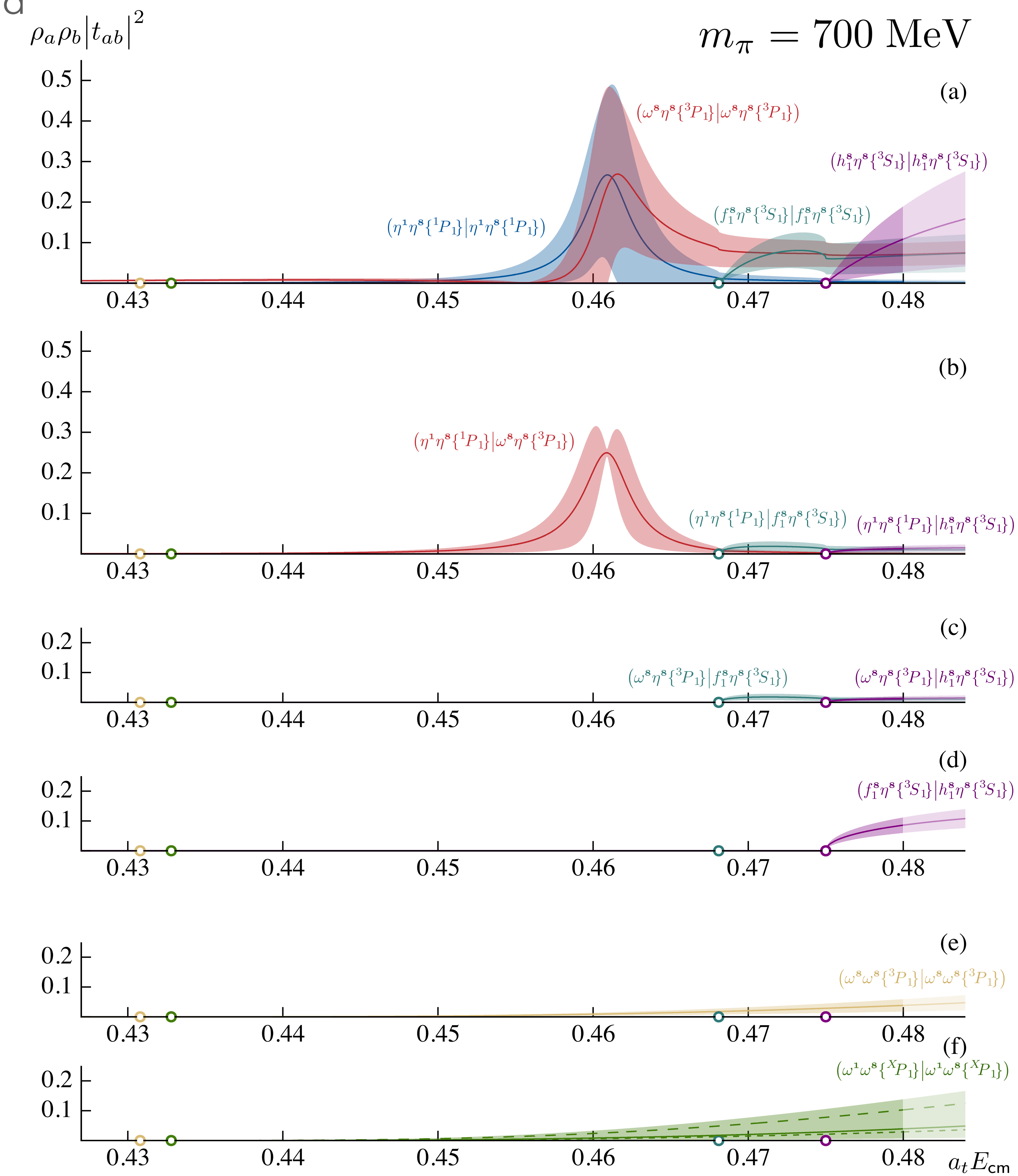
$b_1(1235)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\omega\pi$	dominant		348
[D/S amplitude ratio $= 0.277 \pm 0.027$]			

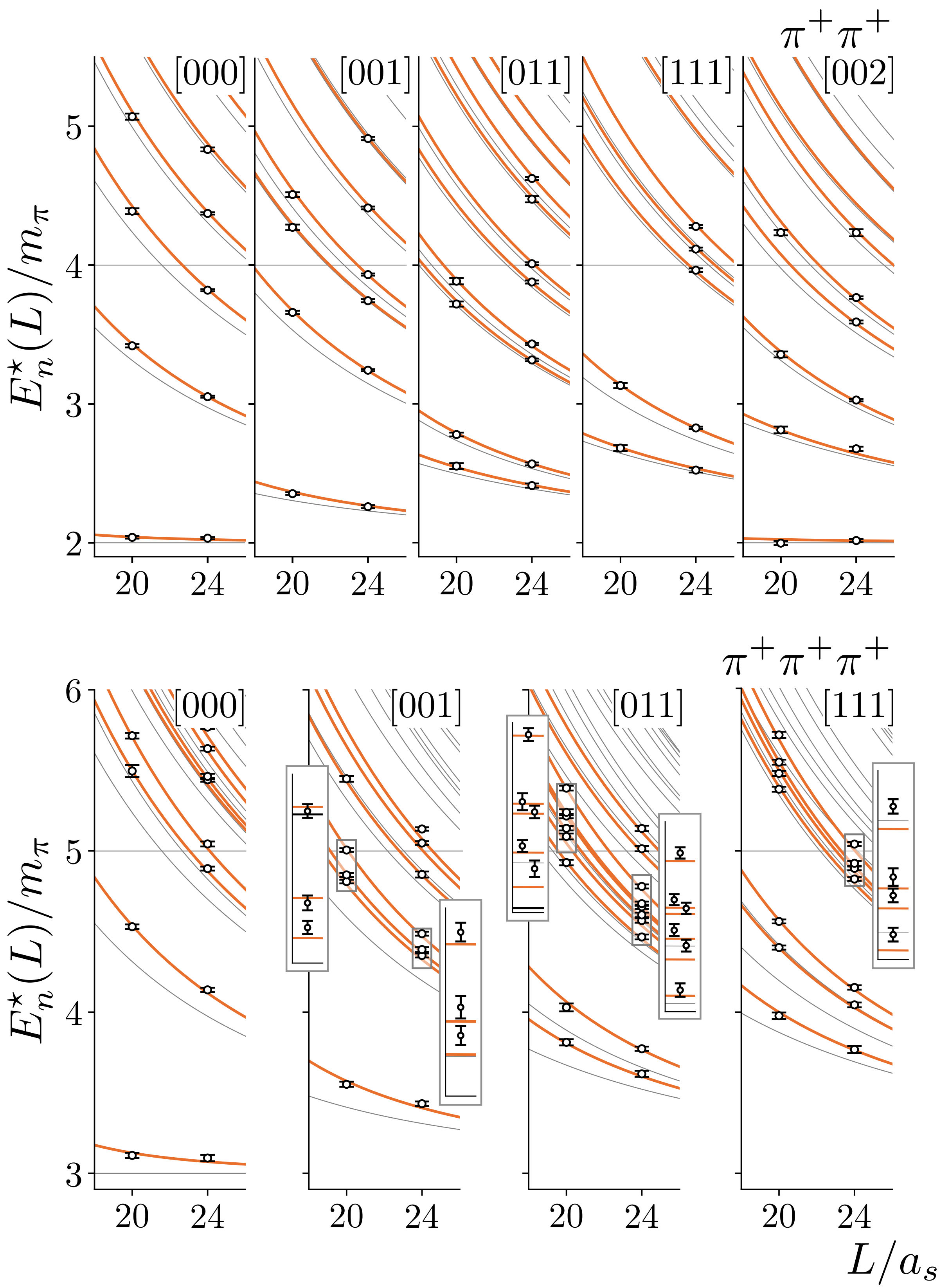
A. J. Woss et al, PRD103 (2021) 5, 054502 arXiv:2009.10034

C. Thomas Wed 28th July Meson Spectroscopy Parallel

- first computation of the decays of an exotic hybrid



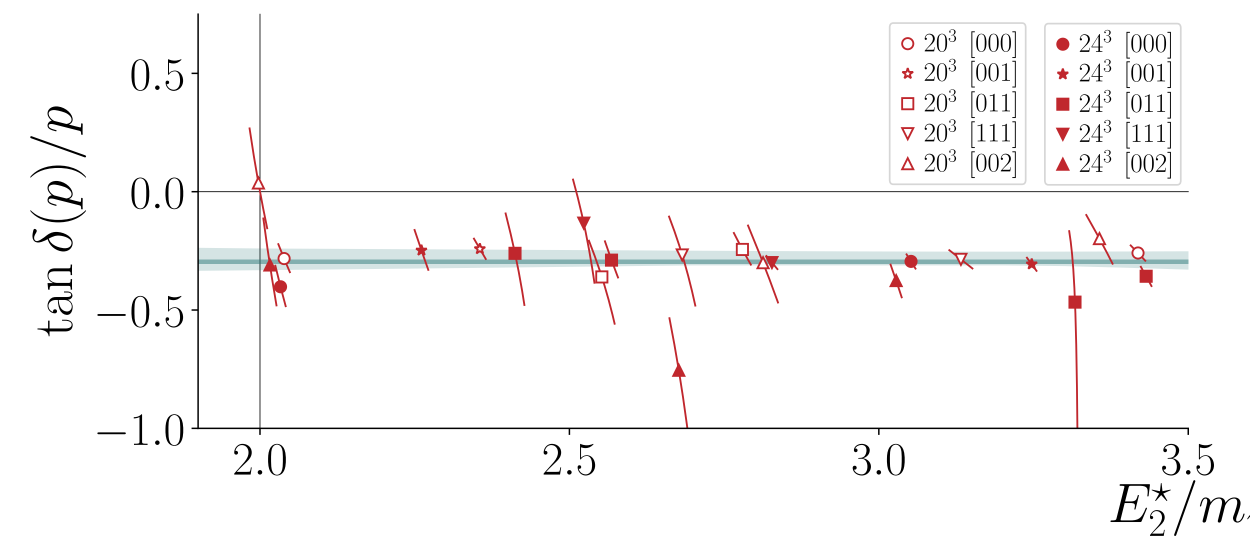


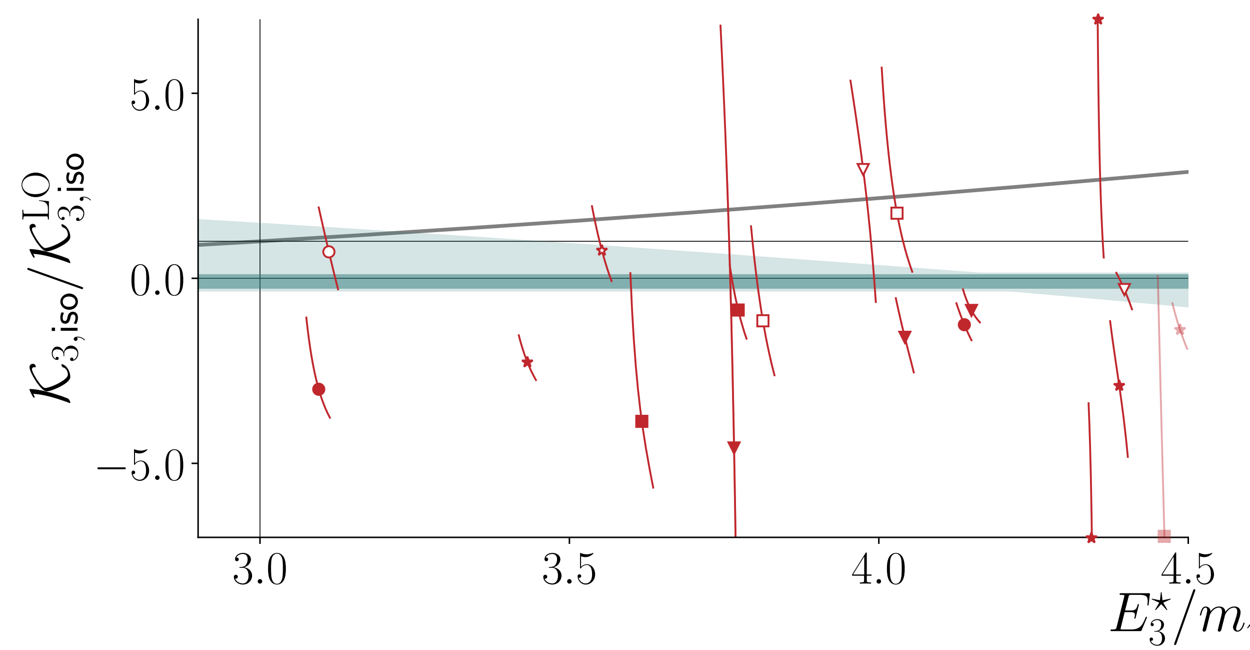


M. T. Hansen et al, PRL 126 (2021) 012001, arXiv:2009.04931

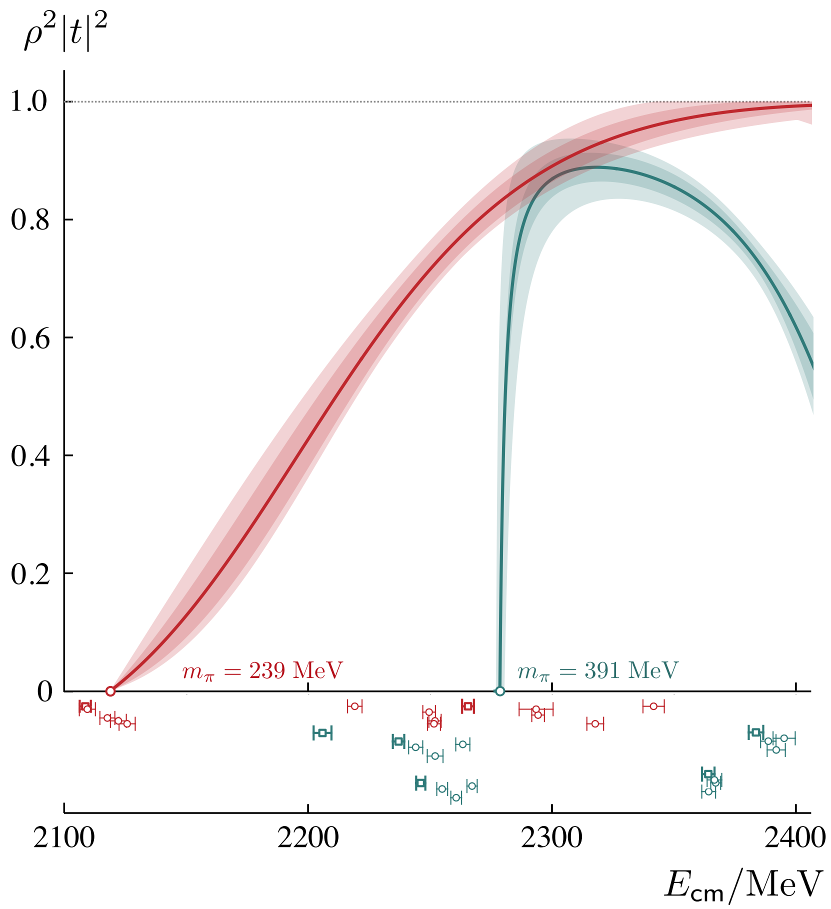
M. T. Hansen Wed 28th July Meson Spectroscopy Parallel

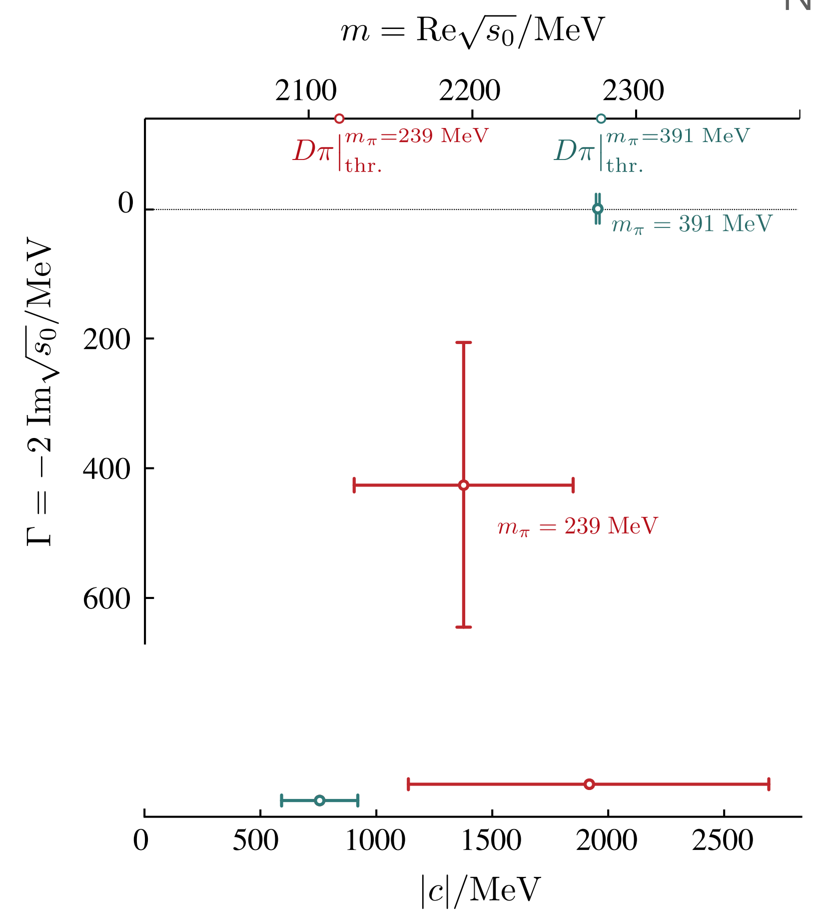
+ F. Romero-Lopez - next talk



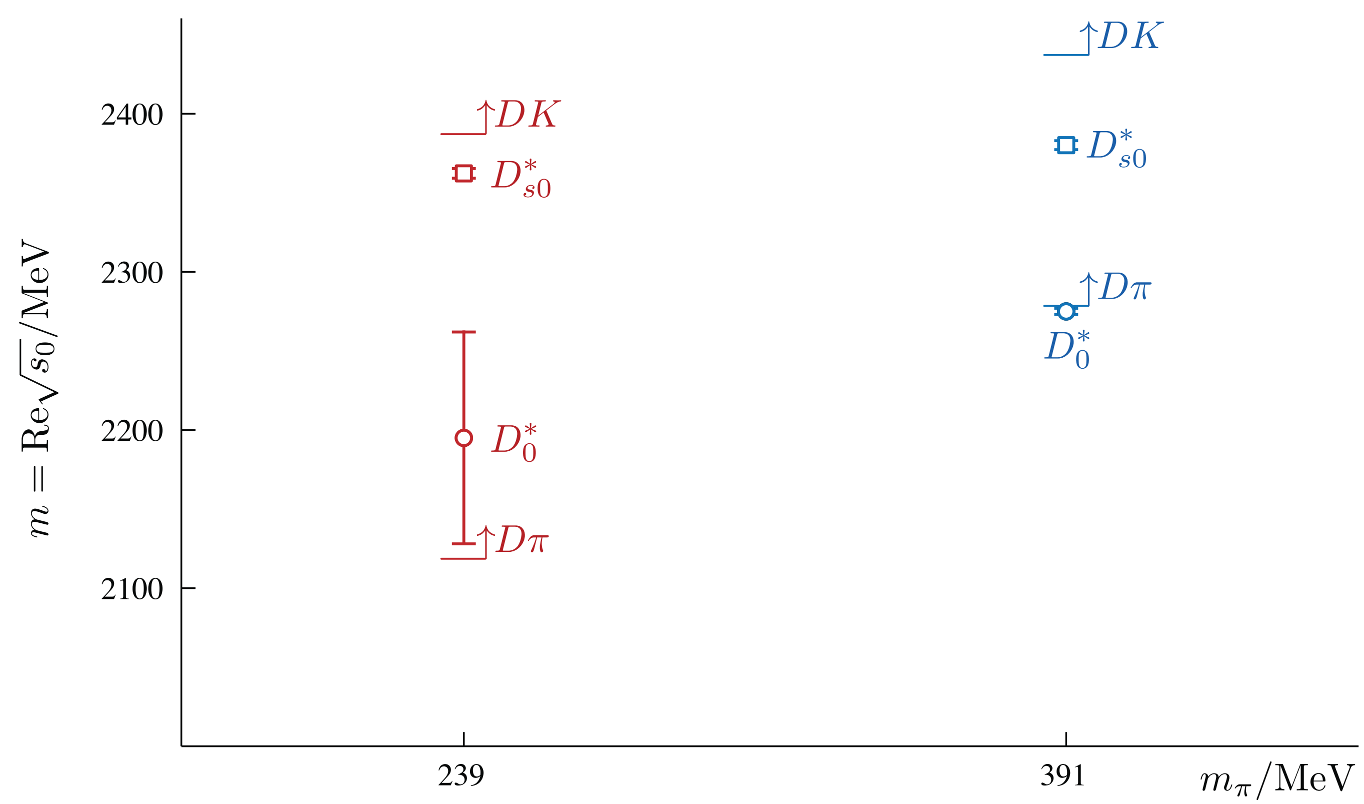


G. Cheung et al, JHEP 02 (2021) 100 arXiv: 2008.06432
L. Gayer, N. Lang et al, arXiv:2102.04973
N. Lang 27th July Meson Spectroscopy Parallel

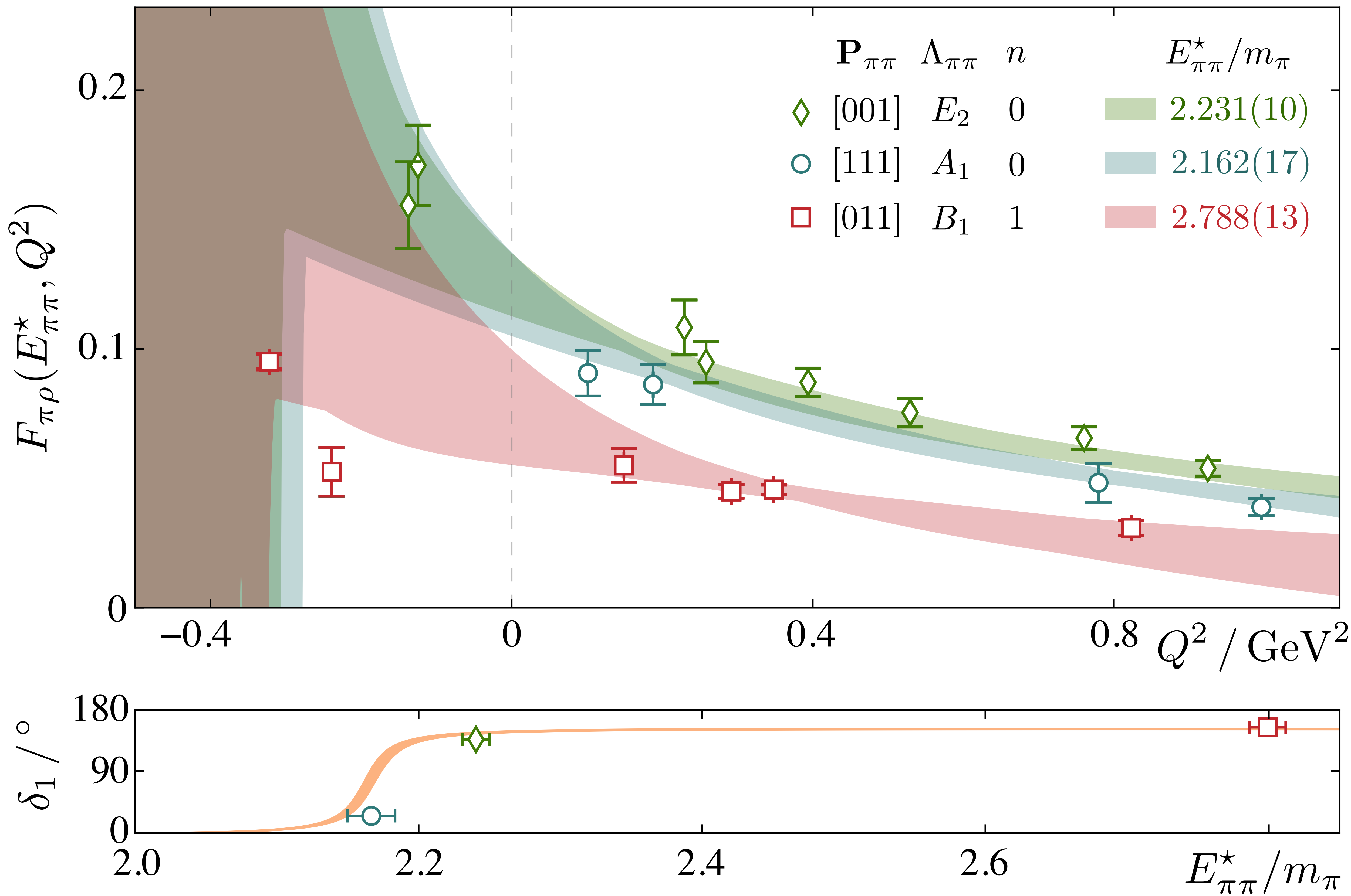




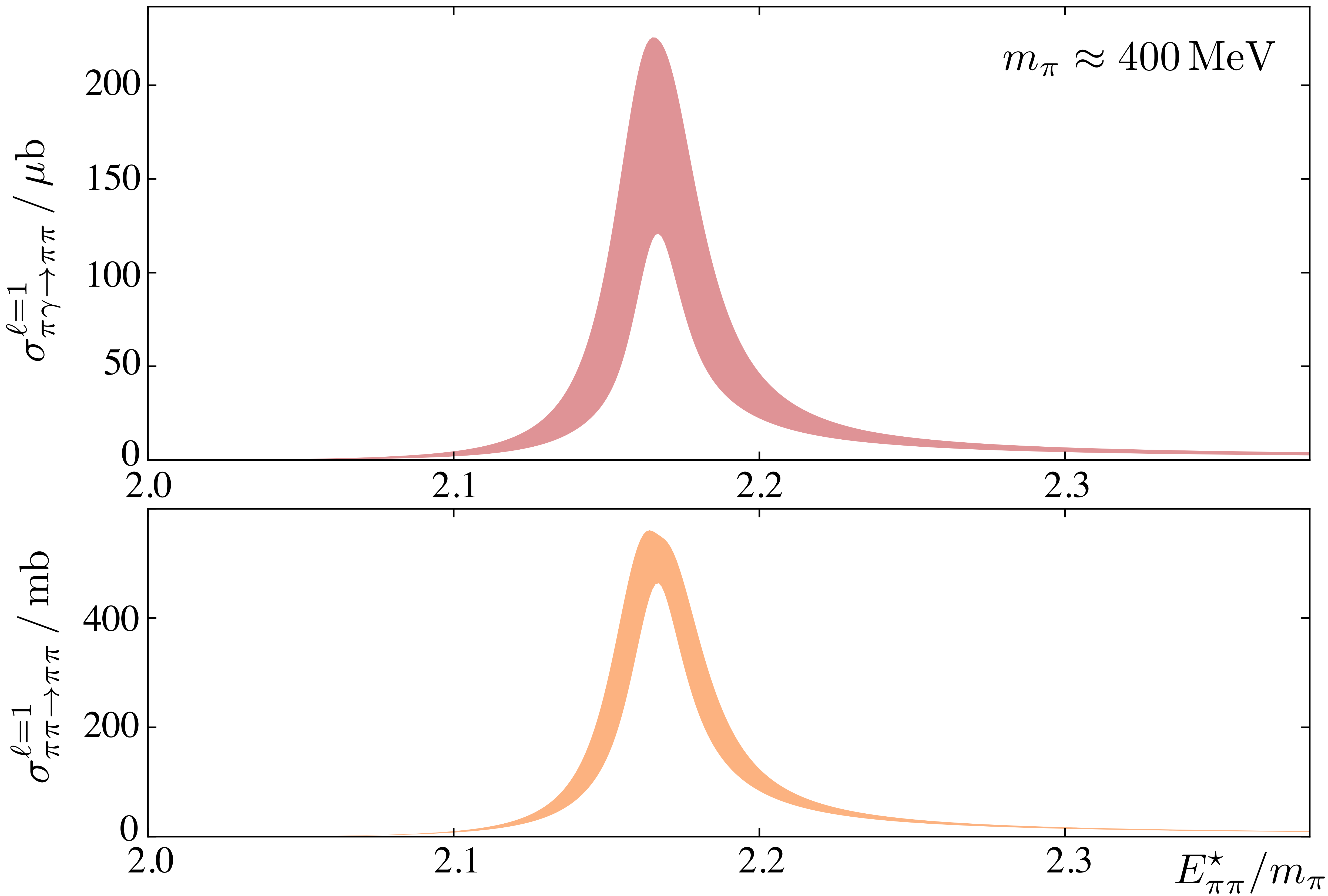
suggestive of a much lighter D_0^* compared with the D_{s0}^*



Briceno et al, PRL 115 (2015) 242001



Briceno et al, PRL 115 (2015) 242001



Lattice QCD provides a first-principles tool
to do hadron spectroscopy

These methods are widely applicable
- coupled-channel scattering
- baryons
- charm quarks, beauty quarks
- form factors, radiative transitions (incl. resonances)

...

Control of 3+ body effects needed for
- lighter pion masses
- higher resonances