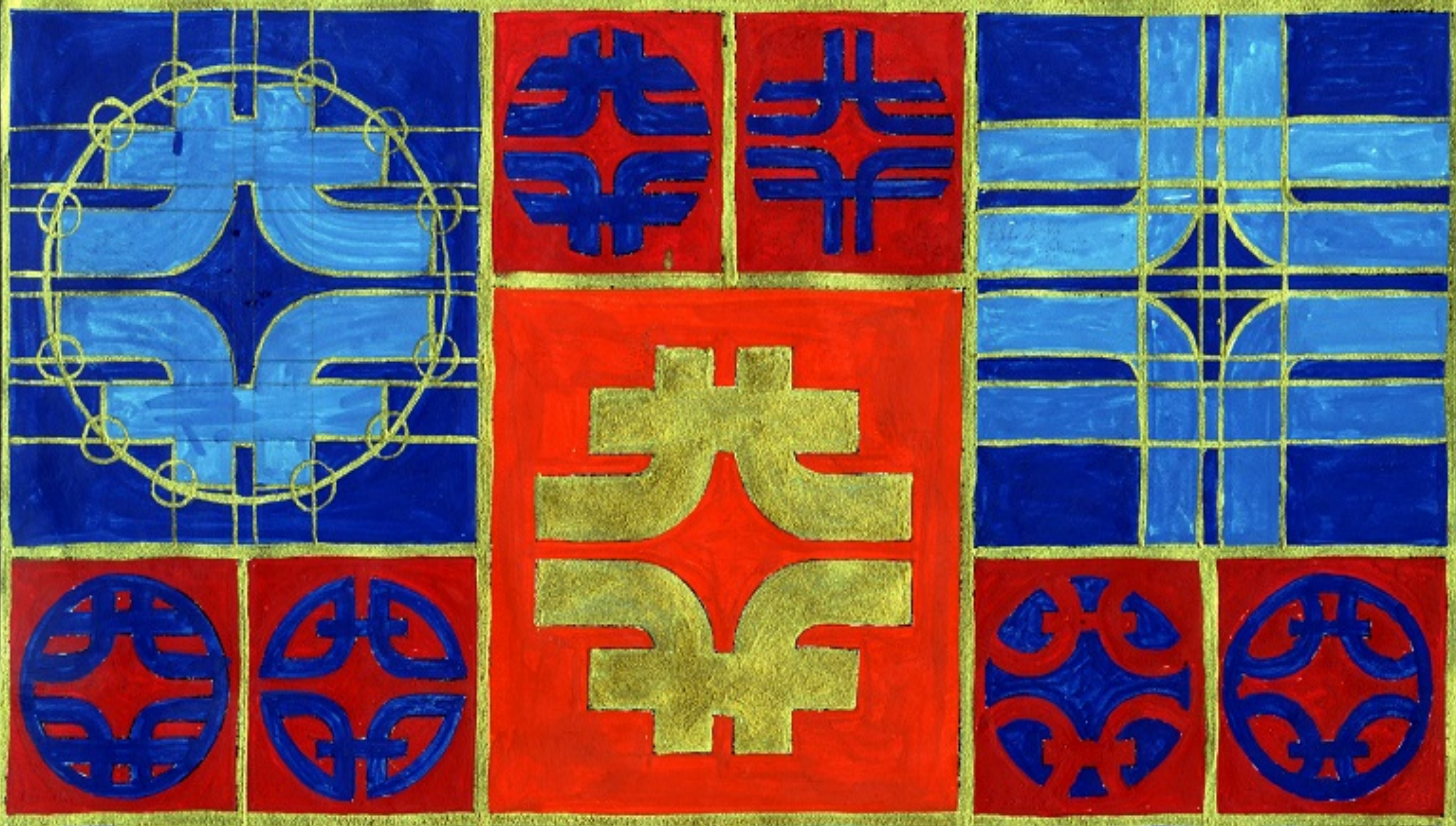


Lattice QCD for precision SM tests and new physics searches

William I. Jay (Fermilab) HADRON 2021 — 29 July 2021

Outline

- Why flavor? Why lattice QCD?
- Quark flavor physics: Experiments & lattice results (focus: b)
- Muon (g-2): Experiments & lattice results for HVP and HLbL
- Some exciting frontiers
- Summary

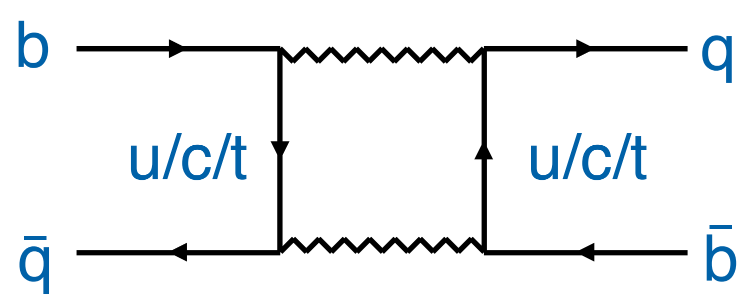
Impossible to be comprehensive in 30 min.

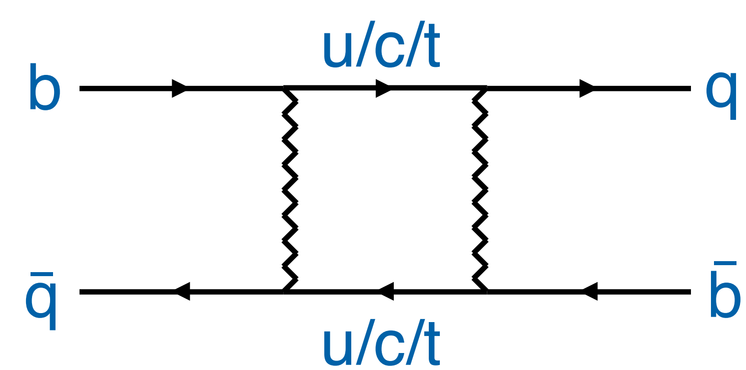
Apologies to those whose work has been omitted!

Precision Flavor Physics

A window to high scales

- **CP violation in neutral kaons**
  - **1964**: $(|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2} \rightarrow 3\pi$
  - CP violation anticipated 3rd quark generation
  - **1977**: B-mesons produced at Fermilab: p + Be ➞ $b\bar{b}$ ➞ $\mu^+\mu^-$
- **B^0-$\bar{B}^0$ mixing**
  - **1987**:





  - Loops with top quark dominate
  - Large mixing ⟹ top mass heavier than expected
  - **1995**: top quark discovered at Fermilab

Precision Flavor Physics

A window to high scales

We must know the fundamental parameters of the Standard Model precisely to be able recognize what a deviation looks like.

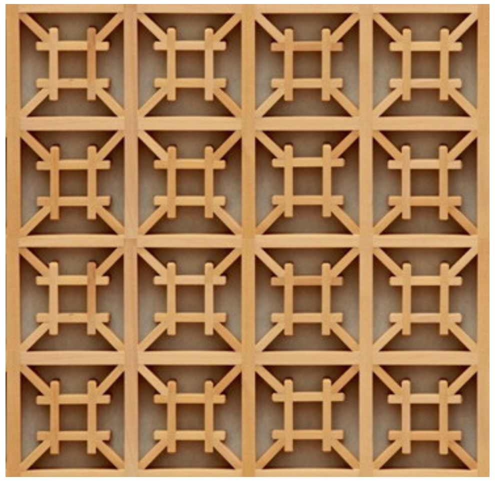
Determine the quark masses
Determine decay constants
Determine form factors
Calculate HVP, HLbL for (g-2)

Tackle using Lattice QCD

Lattice QCD

- Lattice QCD gives complete non-perturbative definition to the strong interactions
- This framework gives:

$$\mathcal{Z} = \int \mathcal{D}[\text{fields}]\, e^{-S_E[\text{fields}]}$$



- **Fundamental approximations:**
  - UV cutoff: lattice spacing *a* [target: a $\ll$ physical scales]
  - IR cutoff: finite spacetime volume $V = L^3 \times T$ [target: $1 \ll m_\pi L$]

- **Approximations of convenience:**
  - Often: Heavier-than-physical pions: $(m_\pi)^{\text{lattice}} > (m_\pi)^{\text{PDG}}$
  - Often: Isospin limit $m_u = m_d$
  - Often: QCD interactions only, no QED
  - Often: lighter-than-physical or static heavy quarks

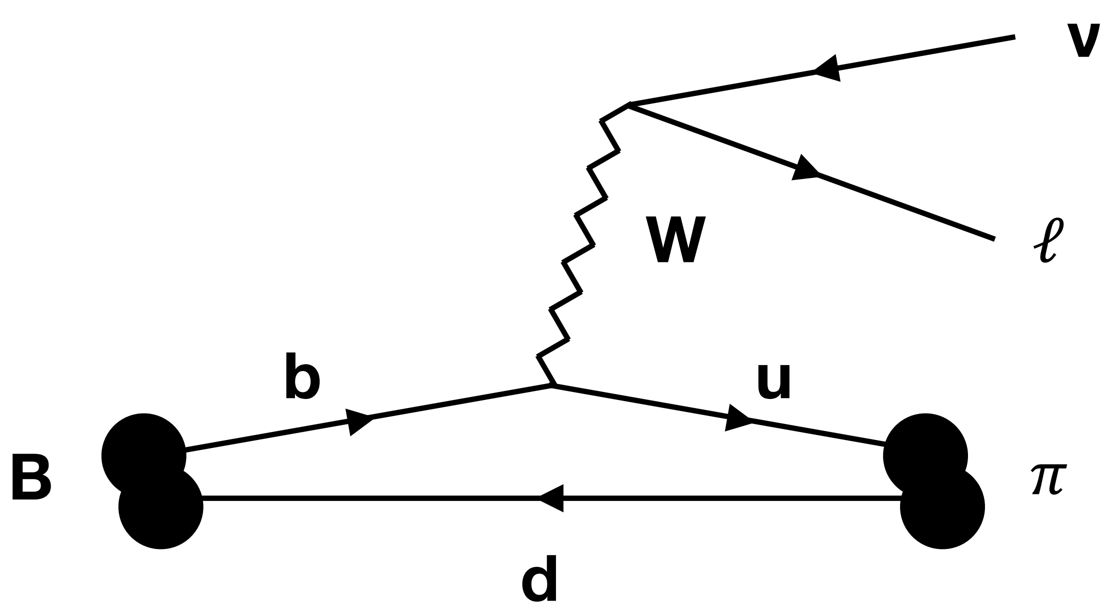
Lattice QCD is systematically improvable

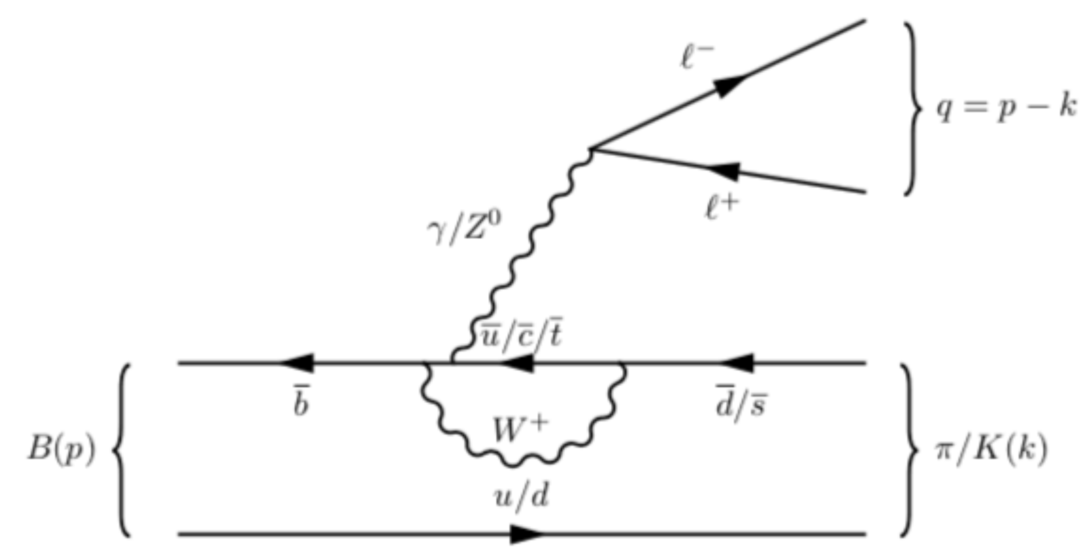
- All approximations admit theoretical descriptions via EFT
  - Cutoff dependence $\Leftrightarrow$ Symanzik effective theory
  - Finite-volume dependence $\Leftrightarrow$ Finite-volume χPT
  - Chiral extrapolation / interpolation $\Leftrightarrow$ χPT
  - Heavy quark extrapolation / interpolation $\Leftrightarrow$ HQET, NRQCD, etc…
  - Isospin breaking $\Leftrightarrow$ perturbative expansion of path integral
- Precise treatment of all systematic effects is key to modern high-precision lattice QCD studies

Quark Flavor and Lattice QCD

Two complementary roles

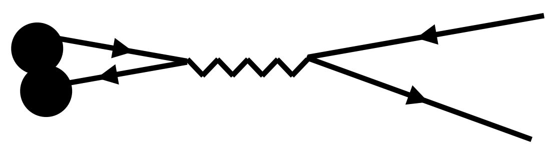
1. Determine CKM matrix elements from tree-level processes
2. Test the CKM paradigm of the Standard Model via rare decays





The CKM Matrix on the lattice

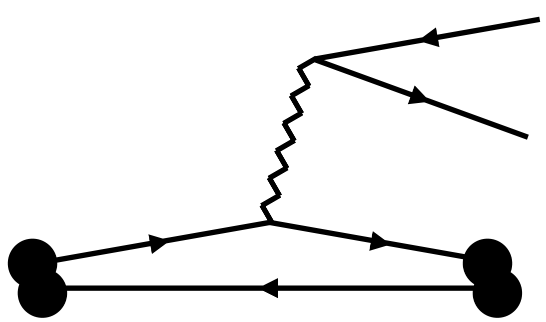
Leptonic decays



(Decay constants)

$$\langle 0|\, A^\mu \,|H(P)\rangle = i f_H\, p^\mu$$

Semi-leptonic decays



(Form factors)

$$f_J(p) \propto \langle \text{final}|\, J(p)\, |\text{initial}\rangle$$

$$\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\
 & K \to \pi\ell\nu & B \to \pi\ell\nu \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\
D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle &
\end{pmatrix}$$

"Gold-plated processes" $\Leftrightarrow$
Single hadron in initial state
Zero or one hadron in final state
All hadrons stable under QCD

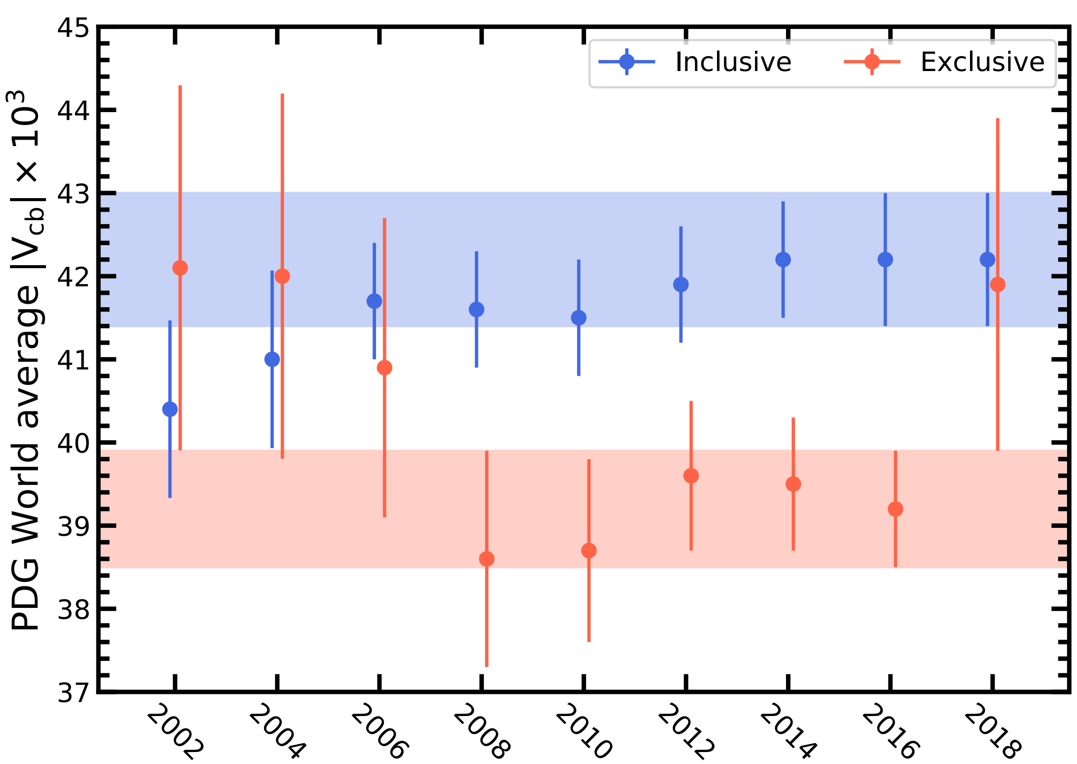
A Golden Age of Flavor

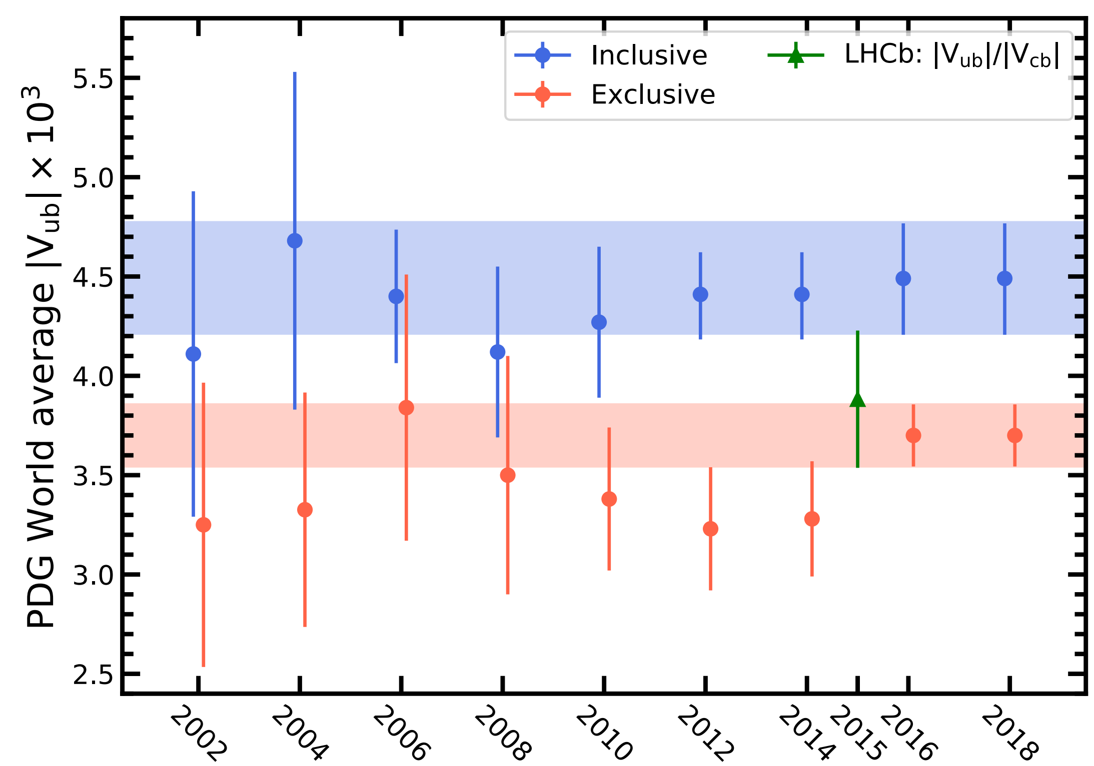
- Last 2 decades: BaBar; Belle; BES III; CDF, D0; ATLAS, CMS
- LHCb: pp at LHC
  - ~10^{12} b-hadrons to date (cf. ~10^{7} at LEP)
- Belle II: $e^{+}e^{-}$ around Υ(4s) ~ 10.5 GeV
  - Goal: 50 ab^{-1} (50x Belle), roughly 215 fb^{-1} to date
- Proposed tau-charm factories in China, Russia

Measuring CKM matrix elements

Tension in inclusive vs. exclusive determinations

- $|V_{cb}|$ from $B \to D^* \ell \nu$ has 3.3σ tension
- $|V_{cb}|$ from $B \to D \ell \nu$ has 2.0σ tension
- $|V_{ub}|$ from $B \to \pi \ell \nu$ has 2.8σ tension





Figures: Bouchard, Cao, Owen, arXiv:1902.09412

Testing Lepton Universality

$$R(D) = \mathcal{B}(B \to D\tau\bar{\nu}_\tau)/\mathcal{B}(B \to D\mu\bar{\nu}_\mu)$$

$R(D^*)$ similar

Combined 3.1σ tension with SM prediction

$R(K)$ 2.6σ

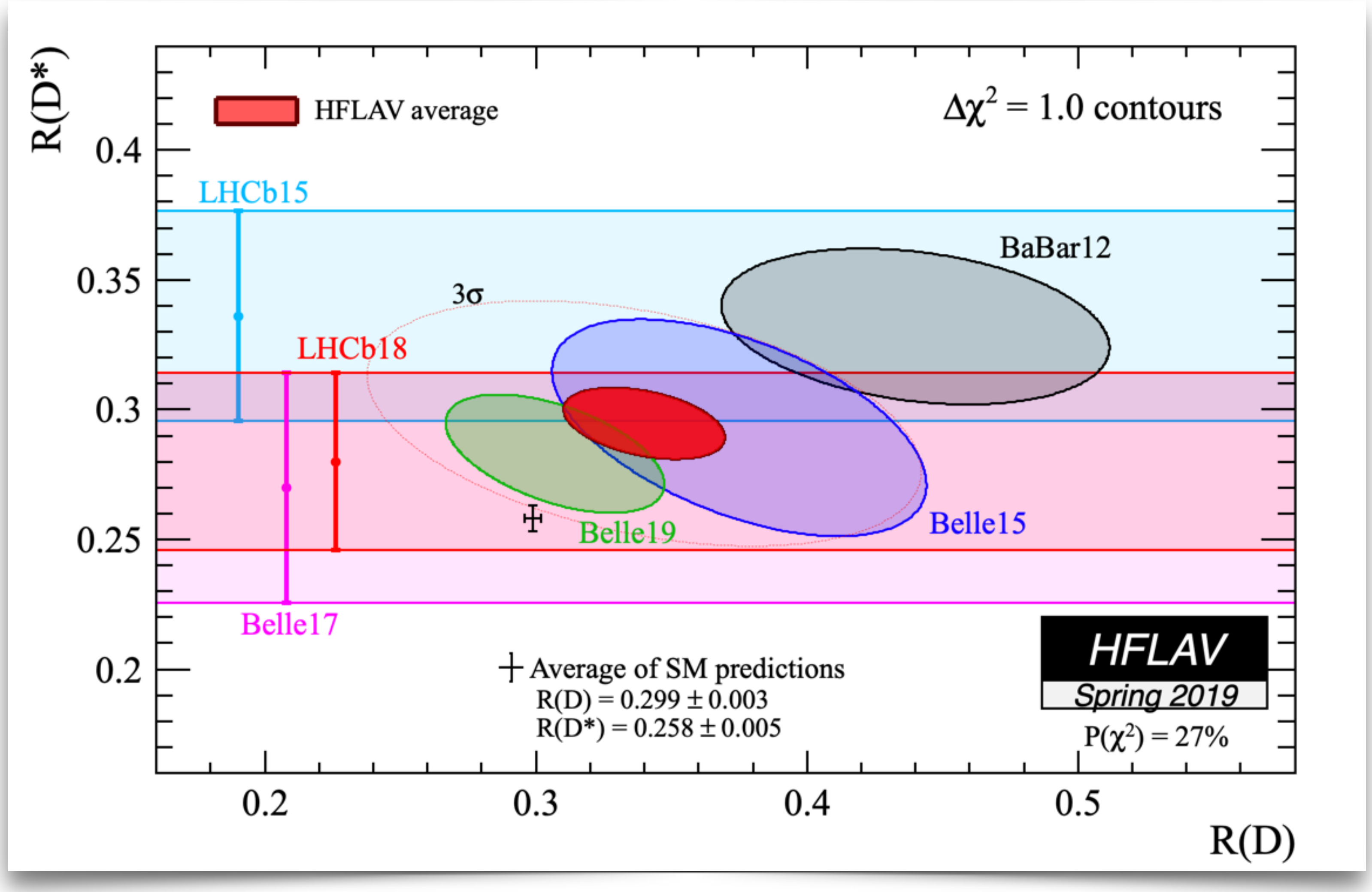


Figure: hflav.web.cern.ch

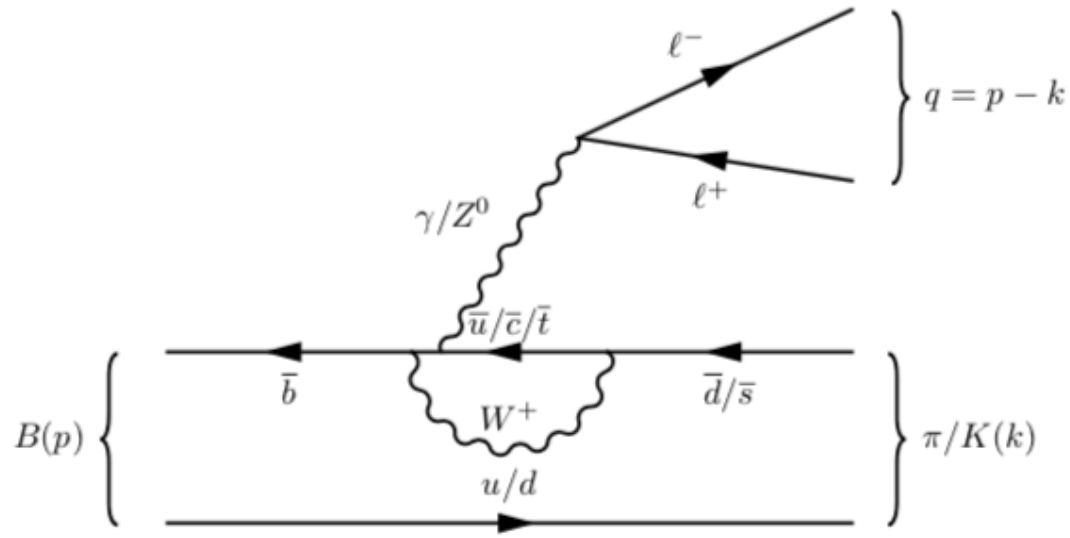
Studying Rare Decays

Rare processes are sensitive probes of high-scale physics

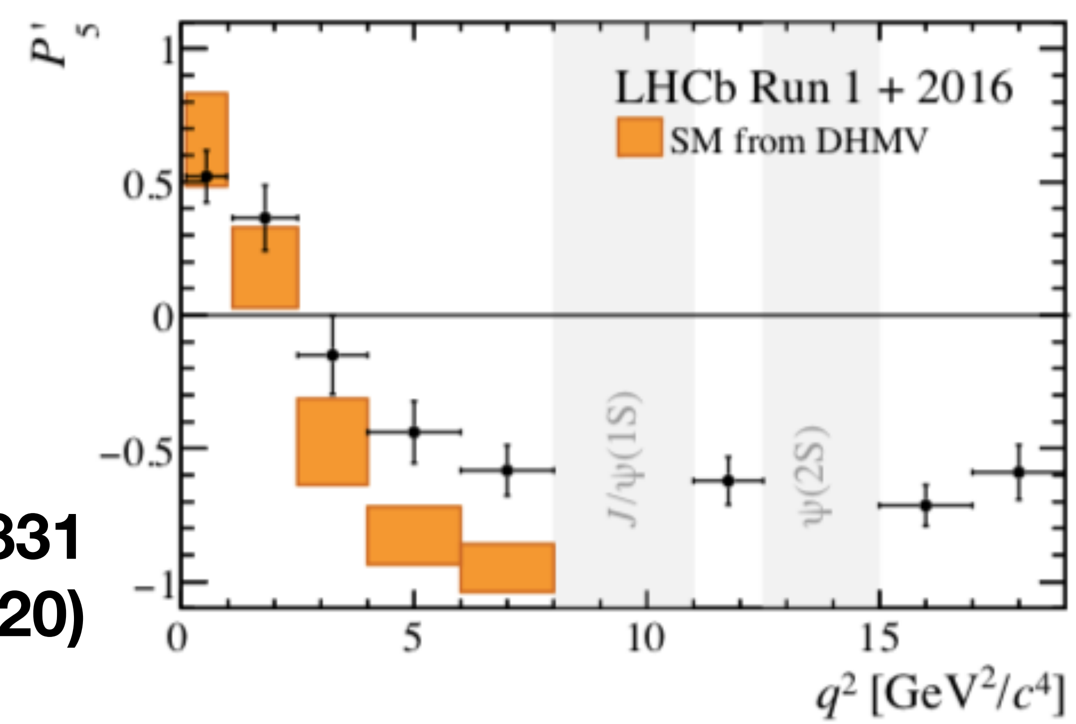
- New sources of CP violation?
- New RH currents?

$$B \longrightarrow K\ell^+\ell^-$$

$$B \longrightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$$



Persistent tension ~3σ tension with SM in angular distribution P_5'



arXiv:2003.04831
PRL 125, 011802 (2020)

Studying Rare Decays

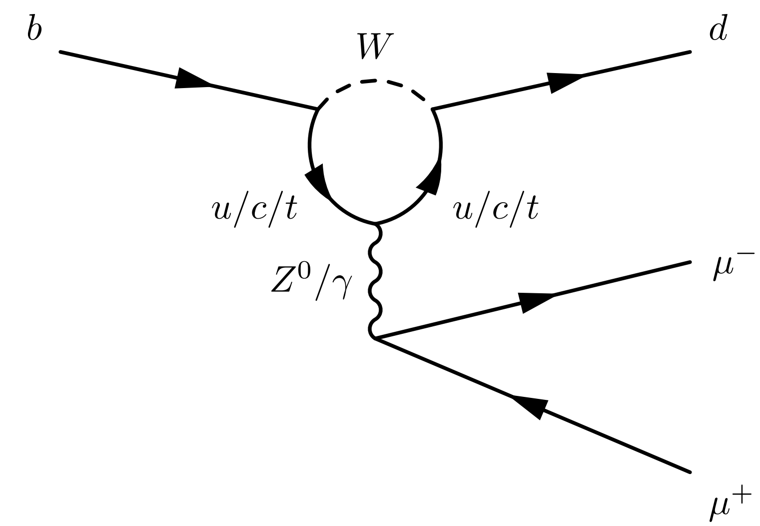
CKM-suppressed decays are a powerful new experimental handle

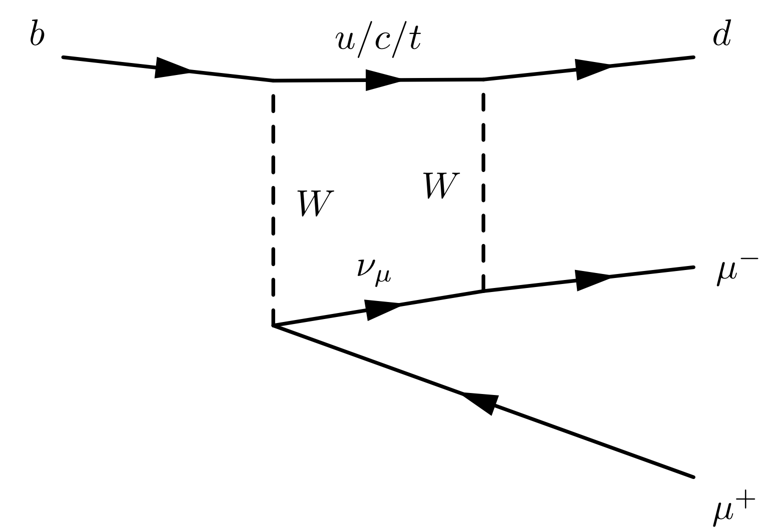
$b \to s\ell^+\ell^-$

$b \to d\ell^+\ell^-$ is 32 × rarer

In 2015, LHCb actually observed 94±12 events for

$$B \to \pi\mu^+\mu^-$$

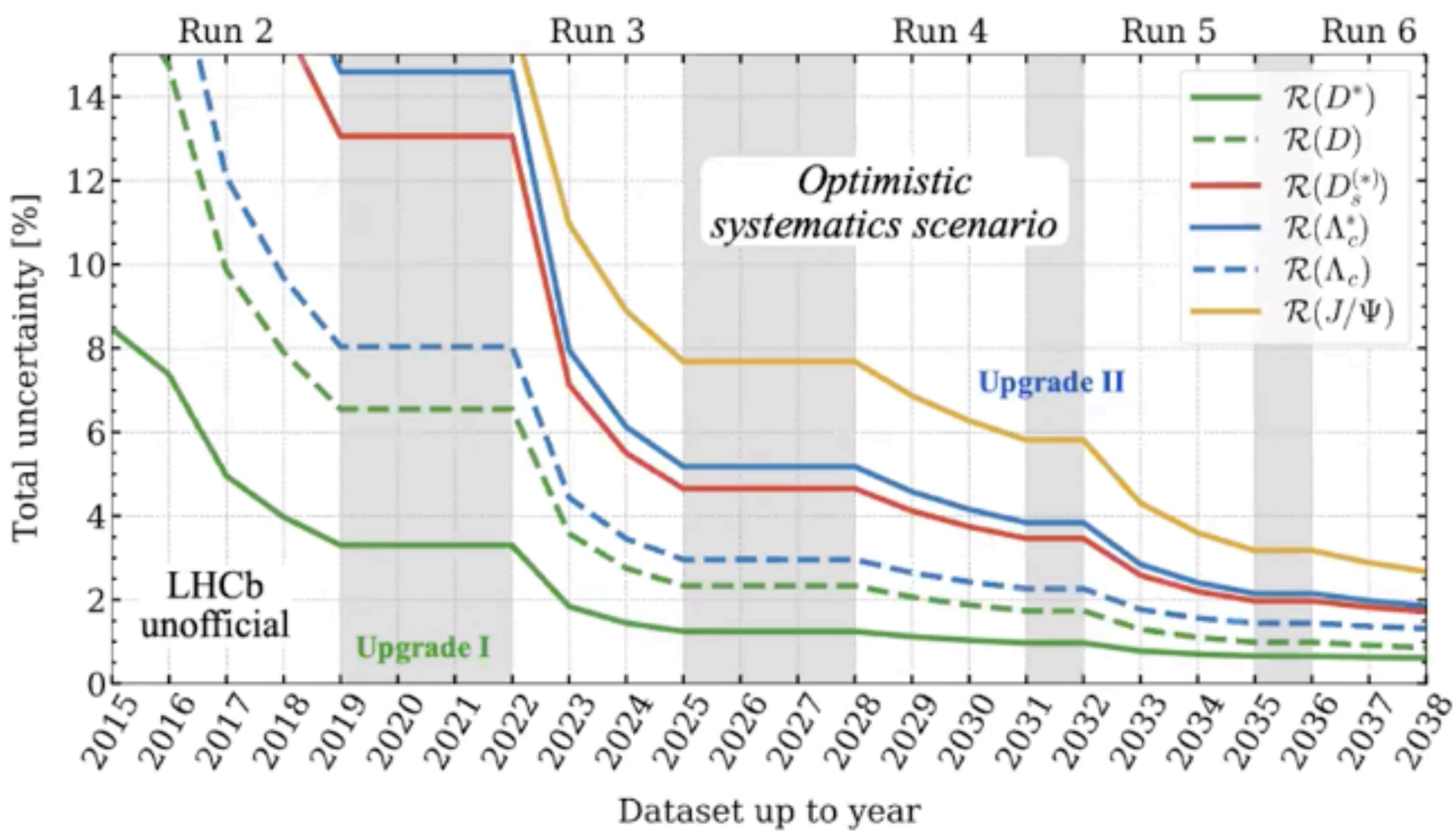




(No tensions yet, but an exciting unexplored region!)

LHCb in Run 3 and Run4

A bright future



Flavor physics and lattice QCD

- Overall precision around ~0.3%
- My take: f_K/f_π essentially solved as a “QCD-only” problem
- Current precision now requires systematic inclusion QED and strong-isospin breaking effects

CalLat: arXiv:2005.04795 PRD 102, 034507 (2020)

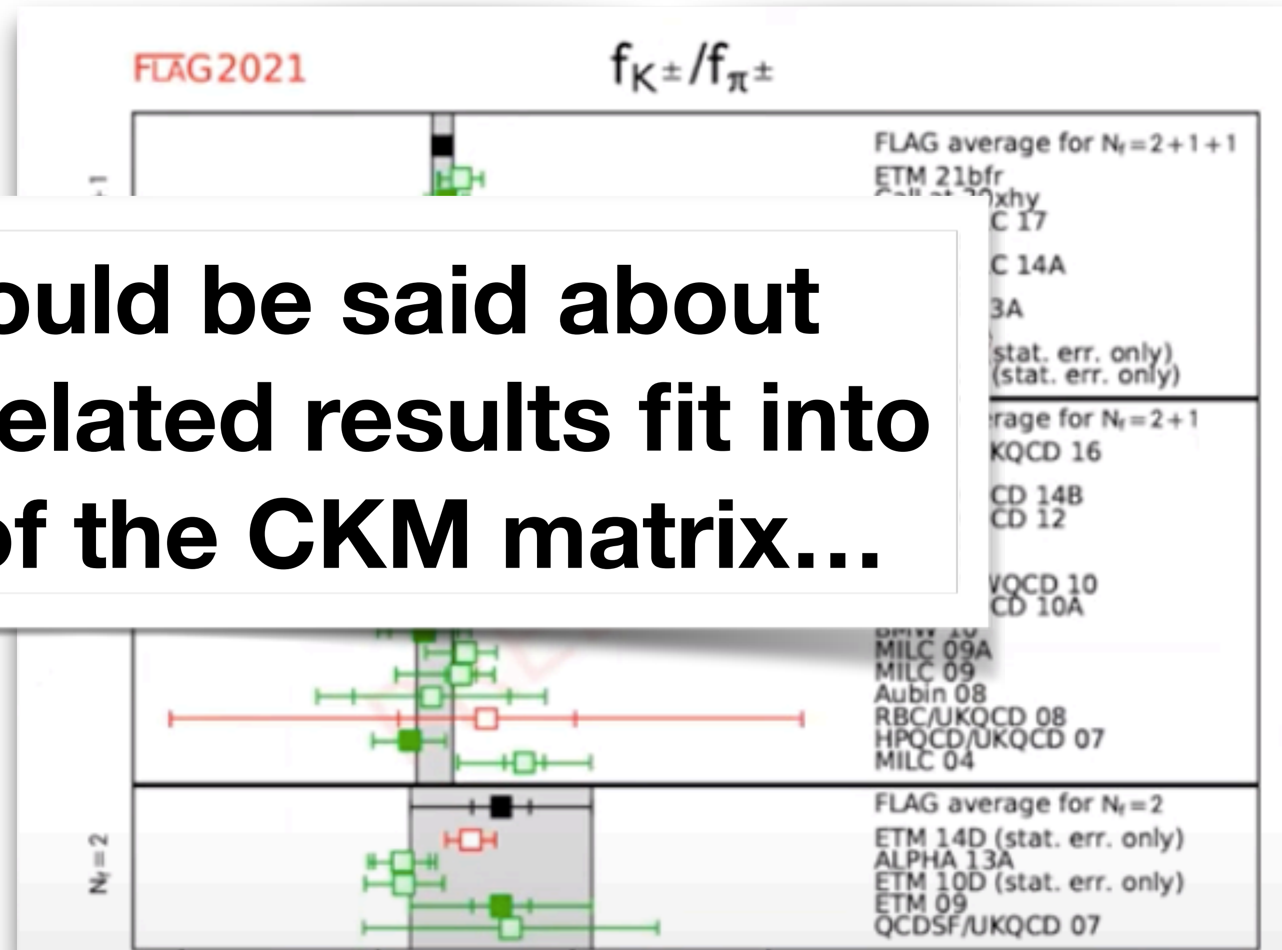
ETMC: arXiv:2104.06747

Credit: Shot from Ruth Van de Water @ Lattice 2021, FLAG working group members T. Kaneko, J. Simone, S. Simula, N. Tantalo for preliminary figure

Flavor physics and lattice QCD

- Overall precision around ~0.3%
- My take: f_K/f_π essentially solved as a "QC
- Current p systemat strong-isospin breaking effects

Much more could be said about how these and related results fit into unitarity tests of the CKM matrix…



CallLat: arXiv:2005.04795 PRD 102, 034507 (2020)

ETMC: arXiv:2104.06747

Credit: Shot from Ruth Van de Water @ Lattice 2021, FLAG working group members T. Kaneko, J. Simone, S. Simula, N. Tantalo for preliminary figure

Decays of heavy quarks

Some selected lattice results 2020 / 2021

The challenge of heavy quarks

Heavy quarks are hard: lattice artifacts grow like powers $(am_h)^n$ — especially tricky for masses near or above the cutoff

1. Use an effective theory for heavy quarks (b, sometimes c)
   - "FNAL interpretation," NRQCD, RHQ, Oktay-Kronfeld
   - Good: Solves problem with artifacts (am_h)
   - No free lunch: EFTs require matching, which introduces systematic effects

2. Use highly-improved relativistic light-quark action on fine lattices
   - Good: advantageous renormalization, continuum limit
   - No free lunch: simulations still need $am_h < 1$ and often an extrapolation to the physical bottom mass

The challenge of heavy quarks

- Many different treatments used in the literature:

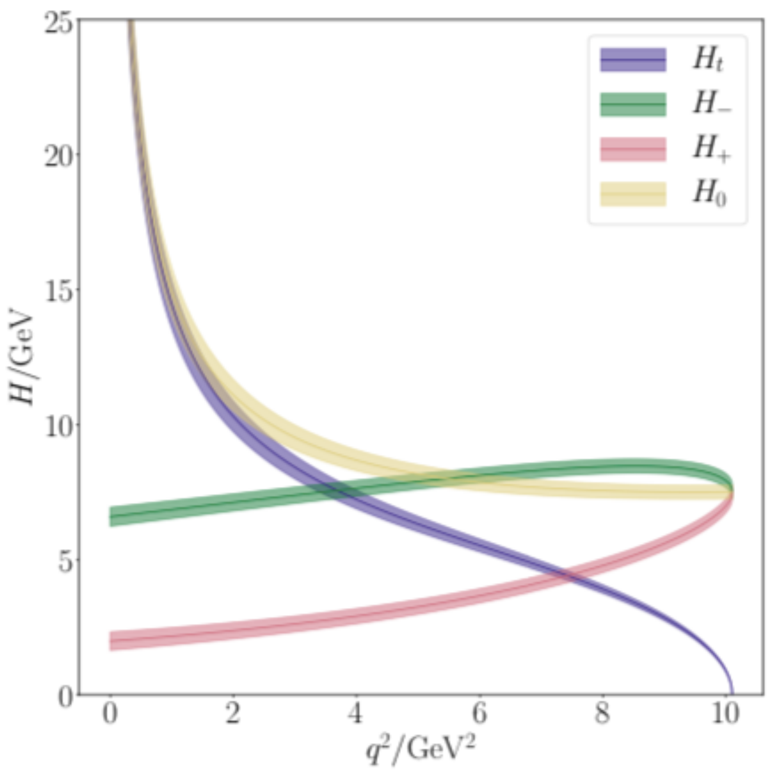
Group	Heavy valence		Sea	“Generation”
HPQCD	NRQCD	on	ASQTAD	I
HPQCD	NRQCD	on	HISQ	II
HPQCD	HISQ	on	HISQ	III
FNAL/MILC	Fermilab	on	ASQTAD	1
FNAL/MILC	Fermilab	on	HISQ	2
FNAL/MILC	HISQ	on	HISQ	3
JLQCD	Möbius DW	on	Möbius DW	
LANL/SWME	Oktay-Kronfeld	on	HISQ	
RBC/UKQCD	RHQ	on	DW	
ETMC	Twisted mass	on	Twisted mass	

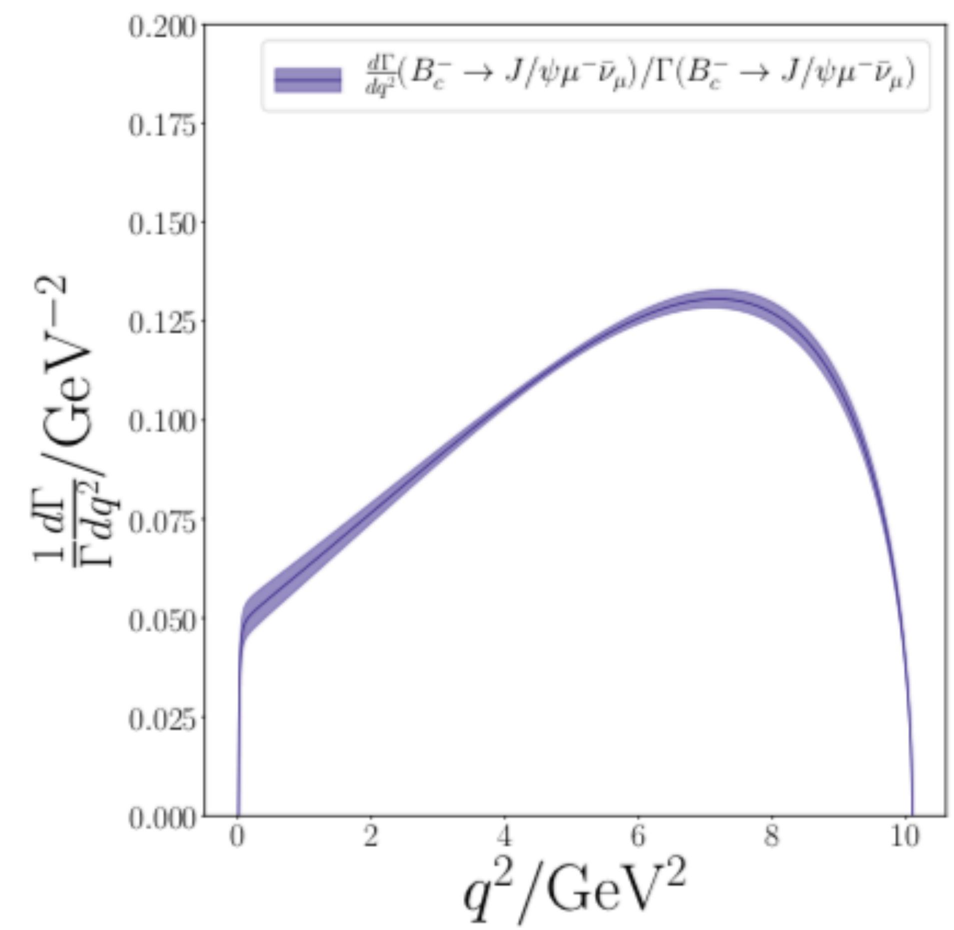
HPQCD: $B_c \to J/\psi$

"Generation III"

arXiv:2007.06957
PRD 102 (2020) 9, 094518
arXiv:2007.06956
PRL 125 (2020) 22, 222003

- Ensembles: 4x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.04 - 0.09] fm
- Valence quarks: all HISQ
- $\Gamma(B_c \to J/\psi \mu \bar{\nu}) / |\eta_{EW} V_{cb}|^2 = 1.73(12) \times 10^{13}\ s^{-1}$ [7%]
- $Br(B_c \to J/\psi \mu \bar{\nu}) = 0.0150(11)_{thy}(10)_{|\eta EW\ Vcb|}(3)_{lifetime}$
- $R(J/\psi)$=0.2582(38) [1.5%]





HPQCD: $f^T_{J/\psi}$

arXiv:2008.02024
PRL 125 (2020) 22, 222003

"Generation III"

- Ensembles: 4x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.04 - 0.09] fm
- Valence quarks: all HISQ
- $f^T_{J/\psi}$(2 GeV) = 0.3927(27) GeV in MSbar

VII. CONCLUSIONS

We have shown here that it is possible to renormalise lattice tensor currents to give accurate results for continuum matrix elements in the $\overline{\text{MS}}$ scheme using nonperturbative determination of intermediate renormalisation factors in momentum-subtraction schemes. A key requirement is that the nonperturbative renormalisation factors should be obtained at multiple values of the renormalisation scale, μ, so that μ-dependent nonperturbative (condensate) contamination of Z_T can be fitted and removed. This contamination would otherwise give a systematic error of 1.5% using the RI-SMOM scheme and 3% using the RI′-MOM scheme in our calculation.

Crucial for rare loop decays like B → K$\ell^+\ell^-$

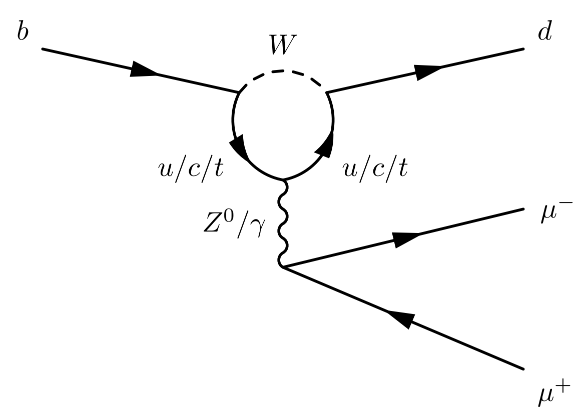


Table VII: Error budget for ratio J/ψ vector and tensor decay constants

	$f^T_{J/\psi}/f^V_{J/\psi}$
$(am_c)^2 \to 0$	0.11
$(\tilde{a}\mu)^2 \to 0$	0.27
Z_T	0.12
Z_V	0.14
Missing α_s^4 term	0.06
Statistics	0.41
Sea mistuning	0.04
Condensates	0.07
Total	0.54

FNAL/MILC B→D*

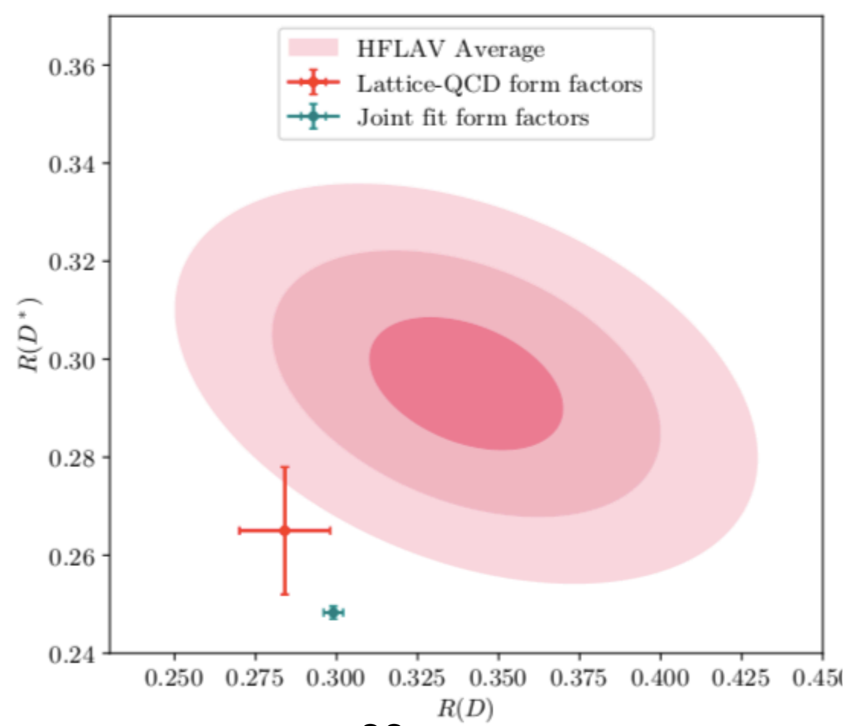
arXiv:2105.14019

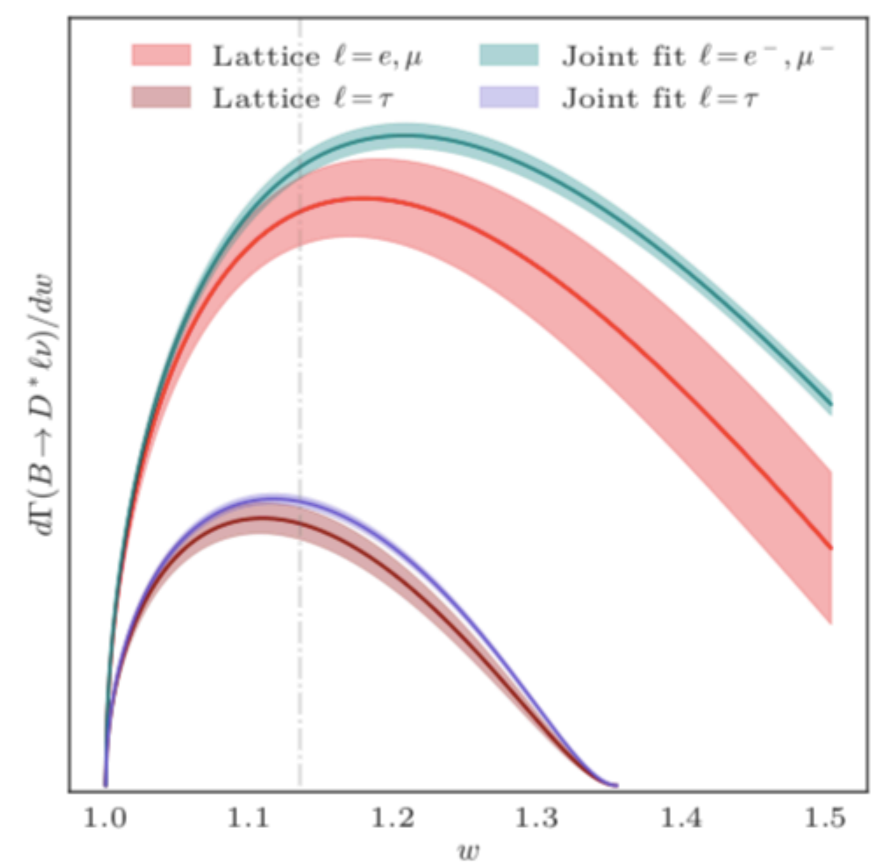
"Generation 1"

- Ensembles: 15x (N_f=2+1) MILC asqtad
- Lattice spacings: 5x in [0.045 - 0.15] fm
- Light valence quarks: asqtad
- Heavy valence charm/bottom: EFT (FNAL interpretation)
- Full physical q^2—world-first calculation away from $q^2=q^2_{max}$
- $|V_{cb}| = (38.40 \pm 0.66_{th} \pm 0.34_{exp})\times10^{-3}$
- R(D*)

  $= 0.265 \pm 0.013$

Results confirm tension between theory and experiment





HPQCD: $B_s \to D_s^*$

arXiv:2105.11433

"Generation III"

- Ensembles: 4x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.04 - 0.09] fm
- Valence quarks: all HISQ
- Full kinematic q^2
- $R(D_s^*) = 0.2442(79)_{\mathrm{latt}}(35)_{\mathrm{EM}}$
- $|V_{cb}| = 43.0(2.1)_{\mathrm{latt}}(1.7)_{\mathrm{exp}}(0.4)_{\mathrm{EM}} \times 10^{-3}$

"… a model-independent determination of $|V_{cb}|$ using $B_s \to D_s^*$ will require a reduction in uncertainty by a factor of ≈ 3 to reach the same precision as that quoted for the exclusive determination using $B \to D^*$ at zero-recoil."

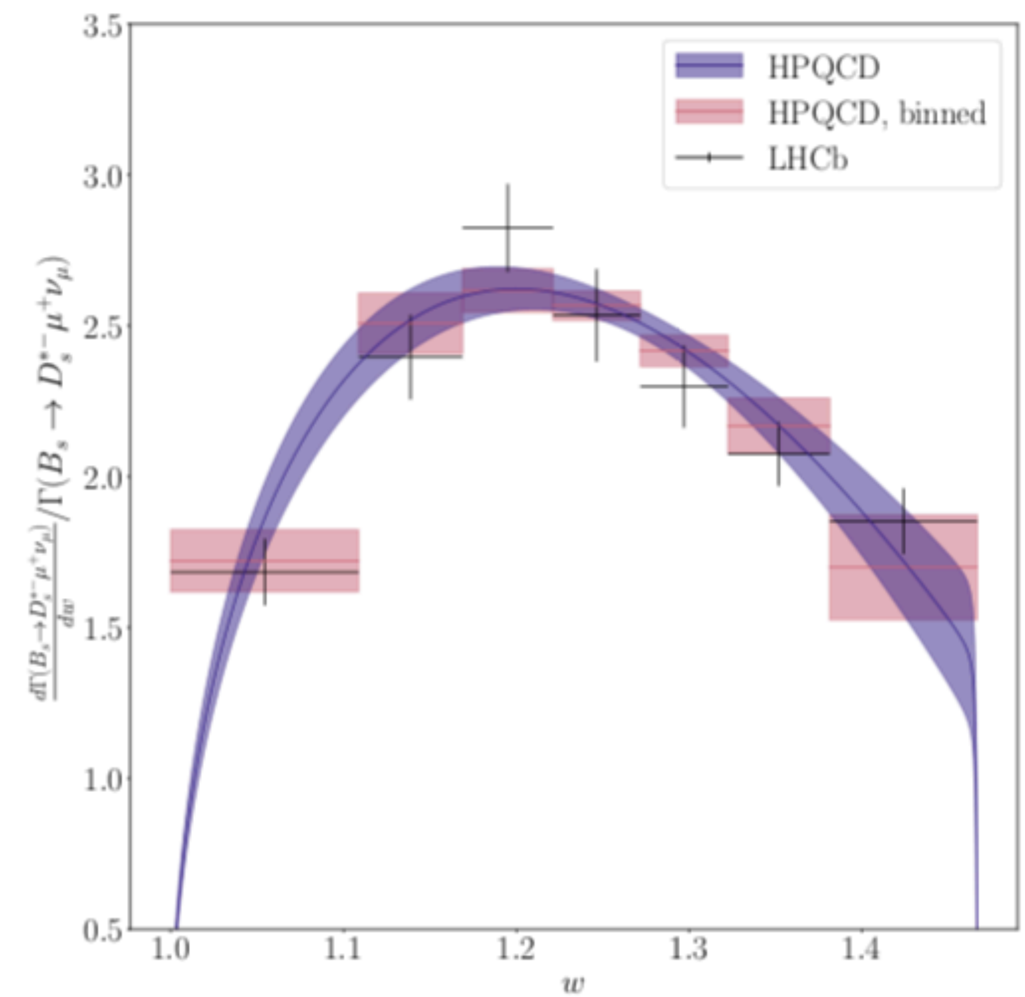
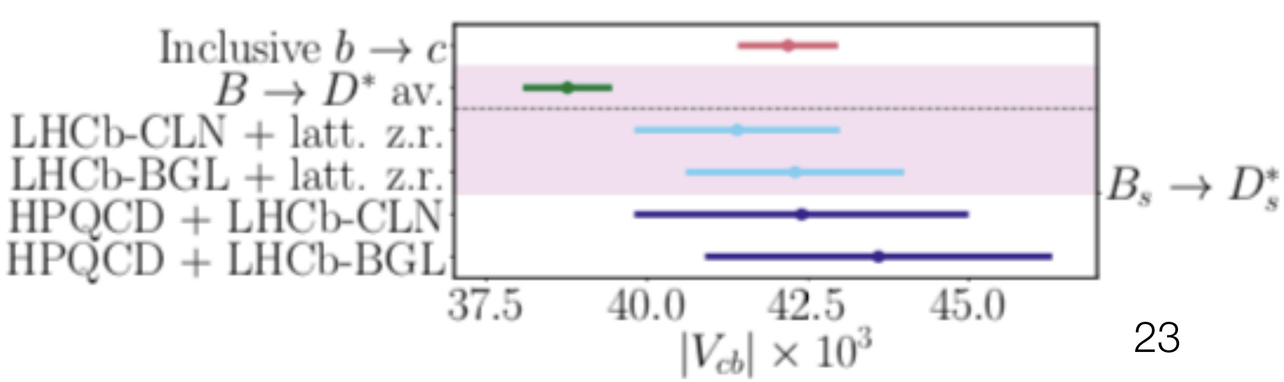


FIG. 11. The differential rate $d\Gamma/dw$ for $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ as a function of the recoil $w = v_{B_s} \cdot v_{D_s^*}$ and normalised by the total decay rate calculated from our form factors is given by the purple band. We also show our rate integrated across bins and measurements by LHCb [54].



Further work on B decays

Large community pursuing calculations with heavy quarks in a variety of discretization schemes

- JLQCD $B \to D^{(*)}$ preliminary results in 2019 with Möbius domain wall fermions
- LANL/SWME $B \to D^{(*)}$ preliminary results in 2019 with Oktay-Kronfeld heavy quarks on HISQ
- RBC/UKQCD $B_s \to D_s$ preliminary results in 2019 with domain wall up to charm and RHQ for bottom.
- ETMC (see “frontiers” later)

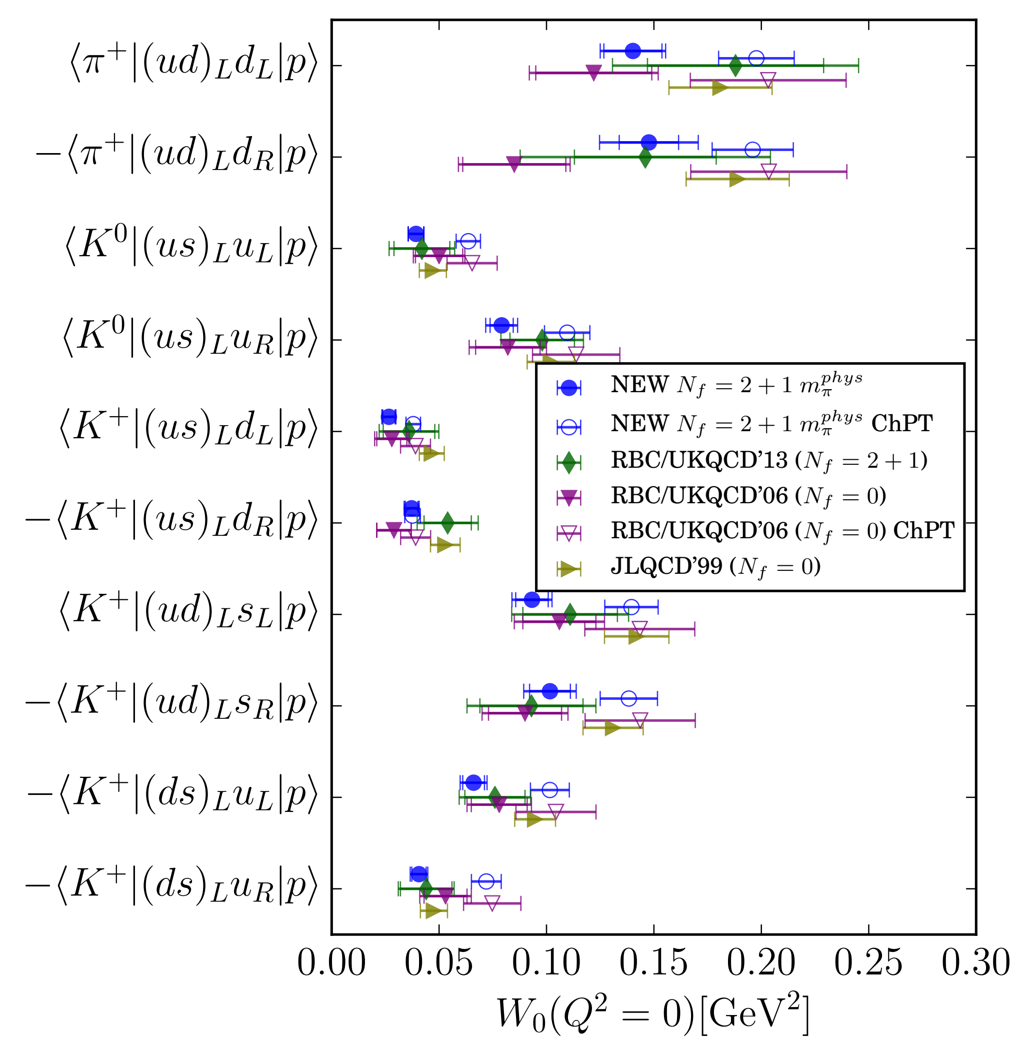
Proton Decay

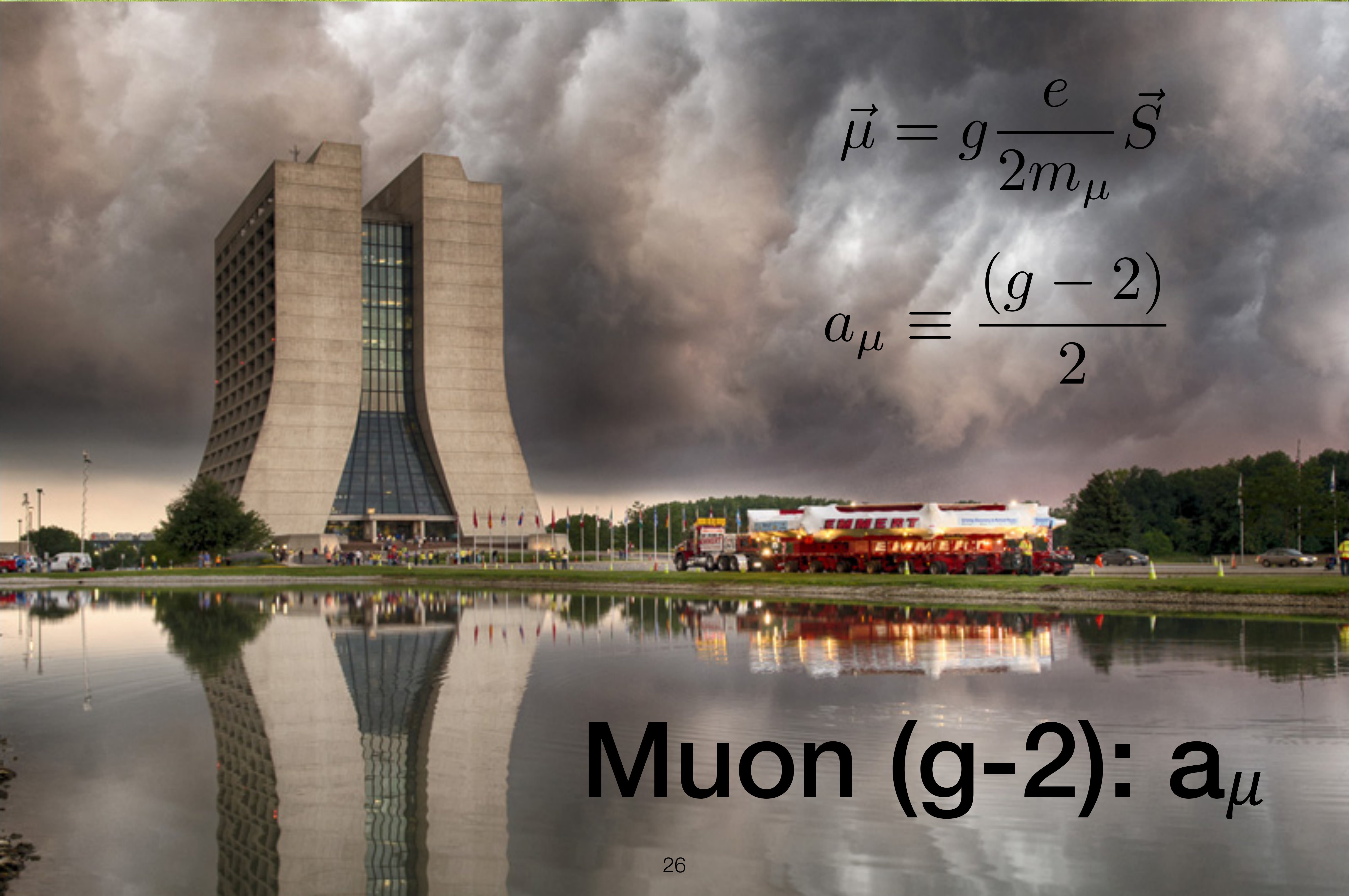
See talk by Sergey Syritsyn Friday 30 Jul 2021 at 11:55

$$p \to \bar{\nu} K^+$$

$$p \to e^+ \pi^0$$

- Experiments looking for proton decay set stringent limits on Grand Unified Theories.
- Reliable proton-to-meson matrix elements are needed from theory
- Martin and Stavenga had suggested that these matrix elements might be highly suppressed at physical quark masses (cf. arXiv:1110.2188)
- A new lattice calculation revisits these matrix elements with physical-mass pions and a chirally symmetric quarks
- Results are consistent previous direct (=lattice) and indirect (=χPT) determinations





$$\vec{\mu} = g\frac{e}{2m_\mu}\vec{S}$$

$$a_\mu \equiv \frac{(g-2)}{2}$$

Muon (g-2): a_μ

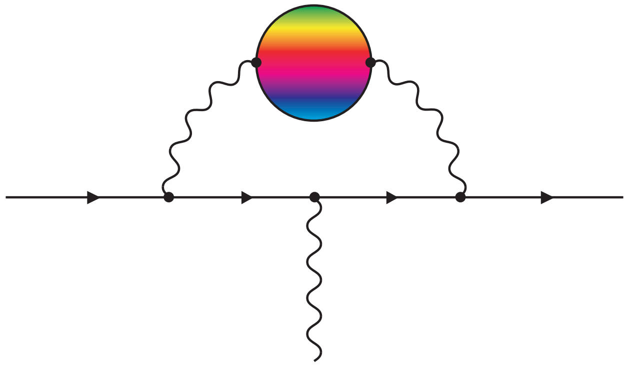
Measurements of (g-2)

- 2006—BNL E821: **a_μ(BNL) = 116 592 080 (63)** × **10^{-11}** [0.54 ppm]
- 2021—FNAL E989: **a_μ(FNAL) = 116 592 040 (54)** × **10^{-11}** [0.46 ppm]
- 2021—BNL+FNAL: **a_μ(EXP) = 116 592 061 (41)** × **10^{-11}** [0.35 ppm]

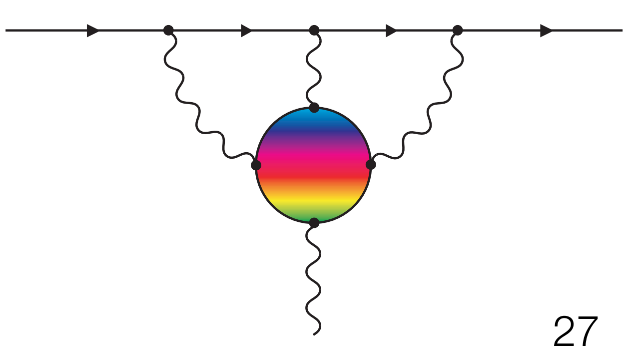
FNAL E989: PRL 126 (2021) 14, 141801
White paper: Phys.Rept. 887 (2020) 1-166

Dominant Theoretical Uncertainties

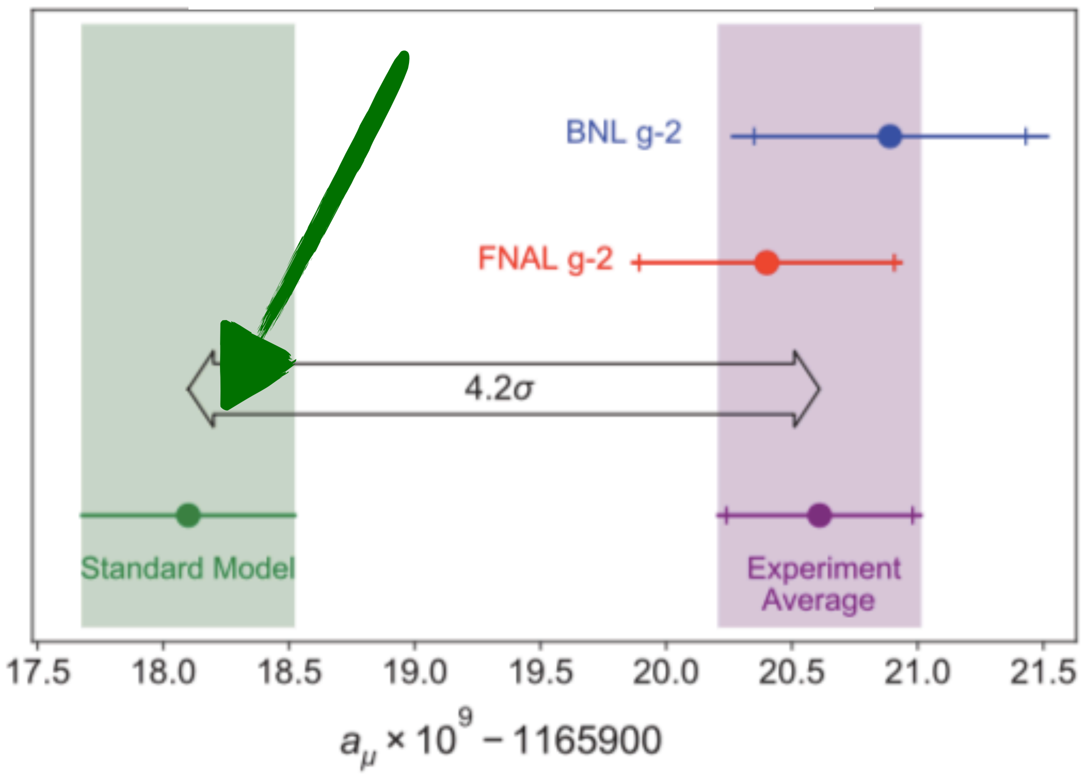
Hadronic vacuum polarization



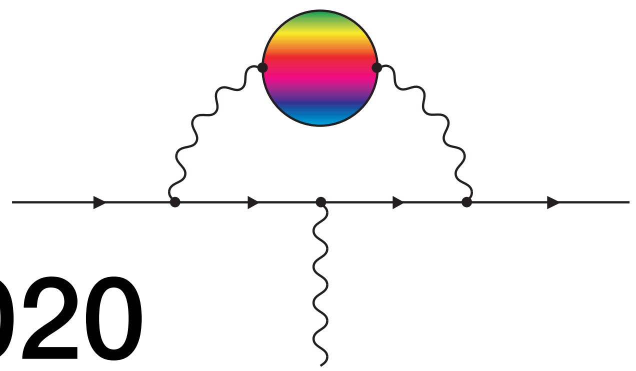
Hadronic "light-by-light"



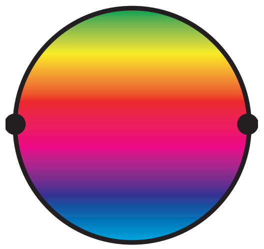
"Standard Model" = (g-2) Theory Initiative White Paper



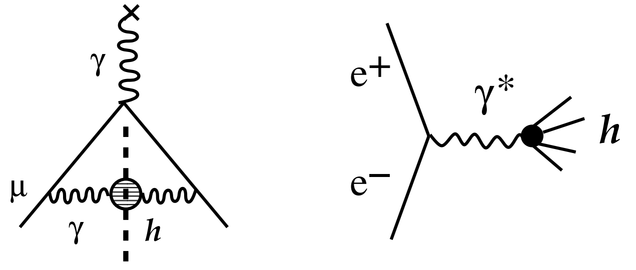
Theory for (g-2)

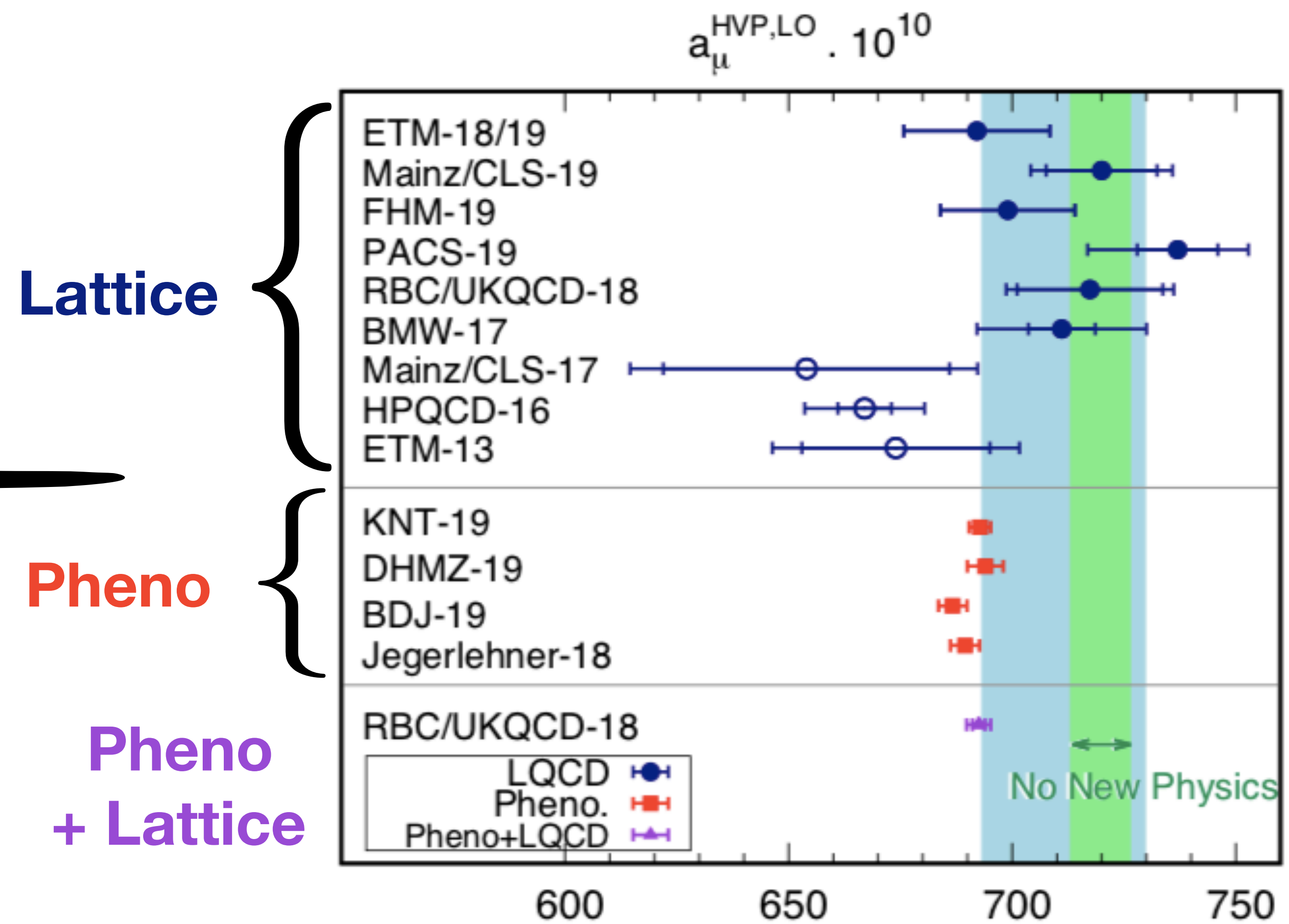


HVP: (g-2) Theory Initiative ca. Nov 2020

$a_\mu^{\mathrm{HVP,LO}} \sim \int dQ^2 \times$  $\times \mathrm{kernel}$

$$a_\mu^{\mathrm{HVP,LO}} \sim \text{“} \int \frac{\sigma_{tot}(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \text{”}$$



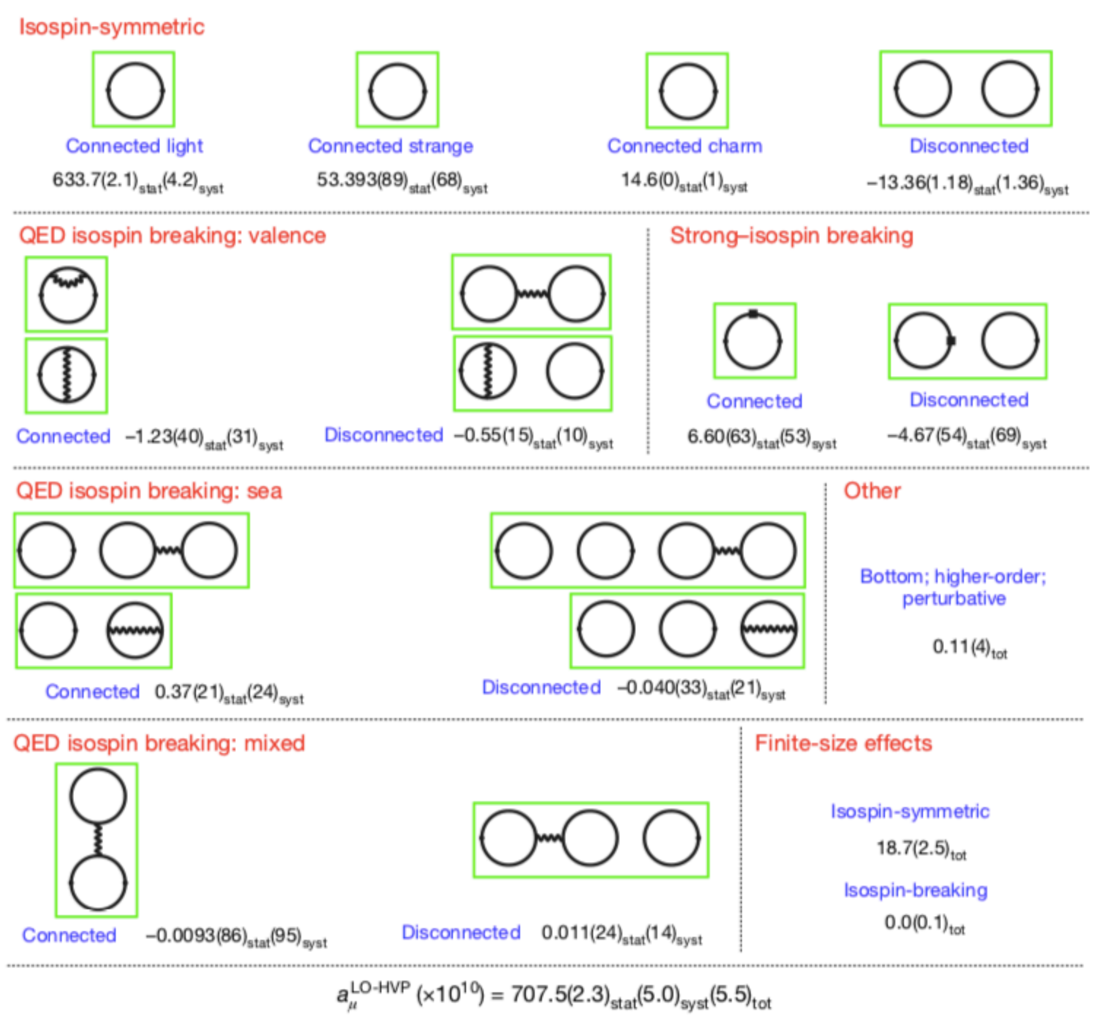


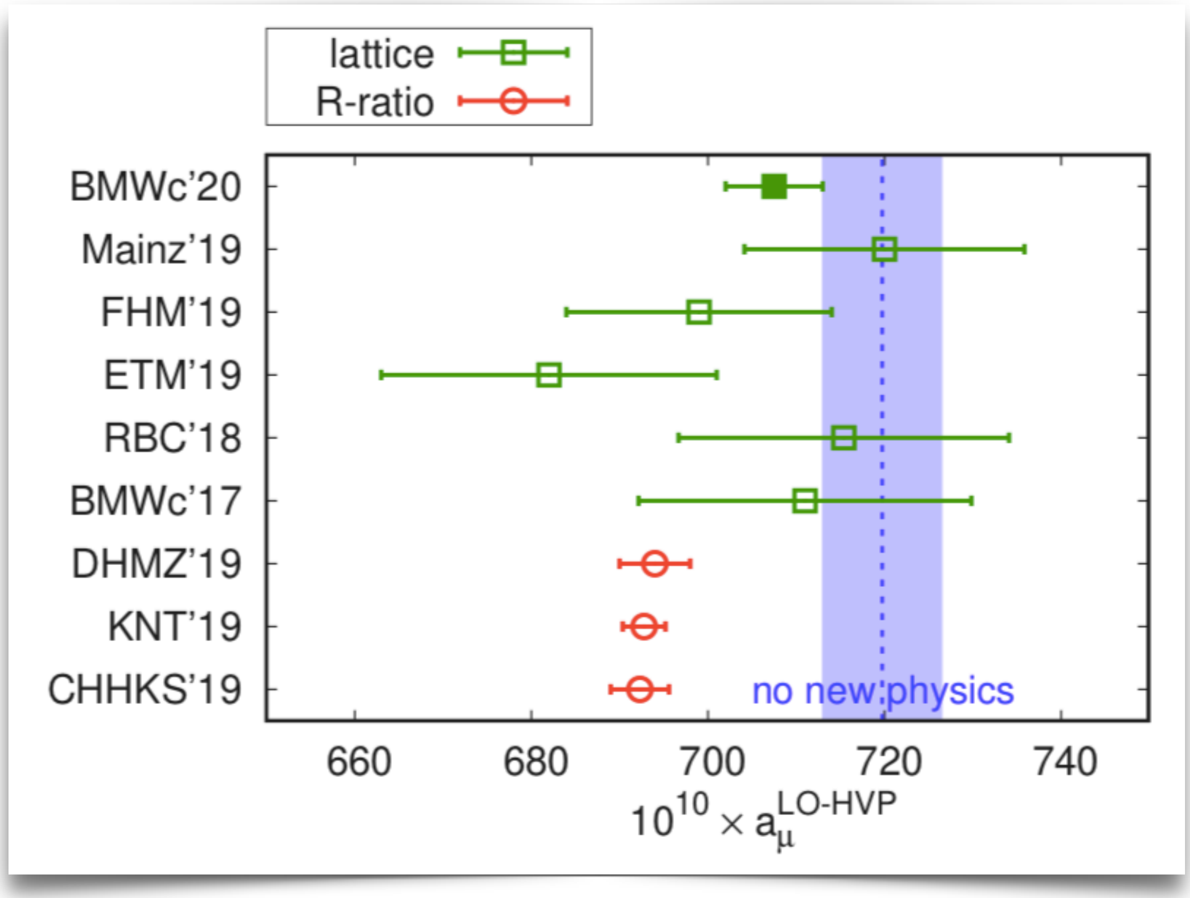
Phys.Rept. 887 (2020) 1-166

LQCD and a_μ^{HVP}

arXiv:2002.12347
Nature 593 (2021) 7857, 51-55

- BWMc has recently published a new lattice results for the HVP
- a_μ^{HVP}= 707.5(2.3)$_{stat}$(5.0)$_{syst}$[5.5] (0.8%)
- The final reported precision is quite impressive





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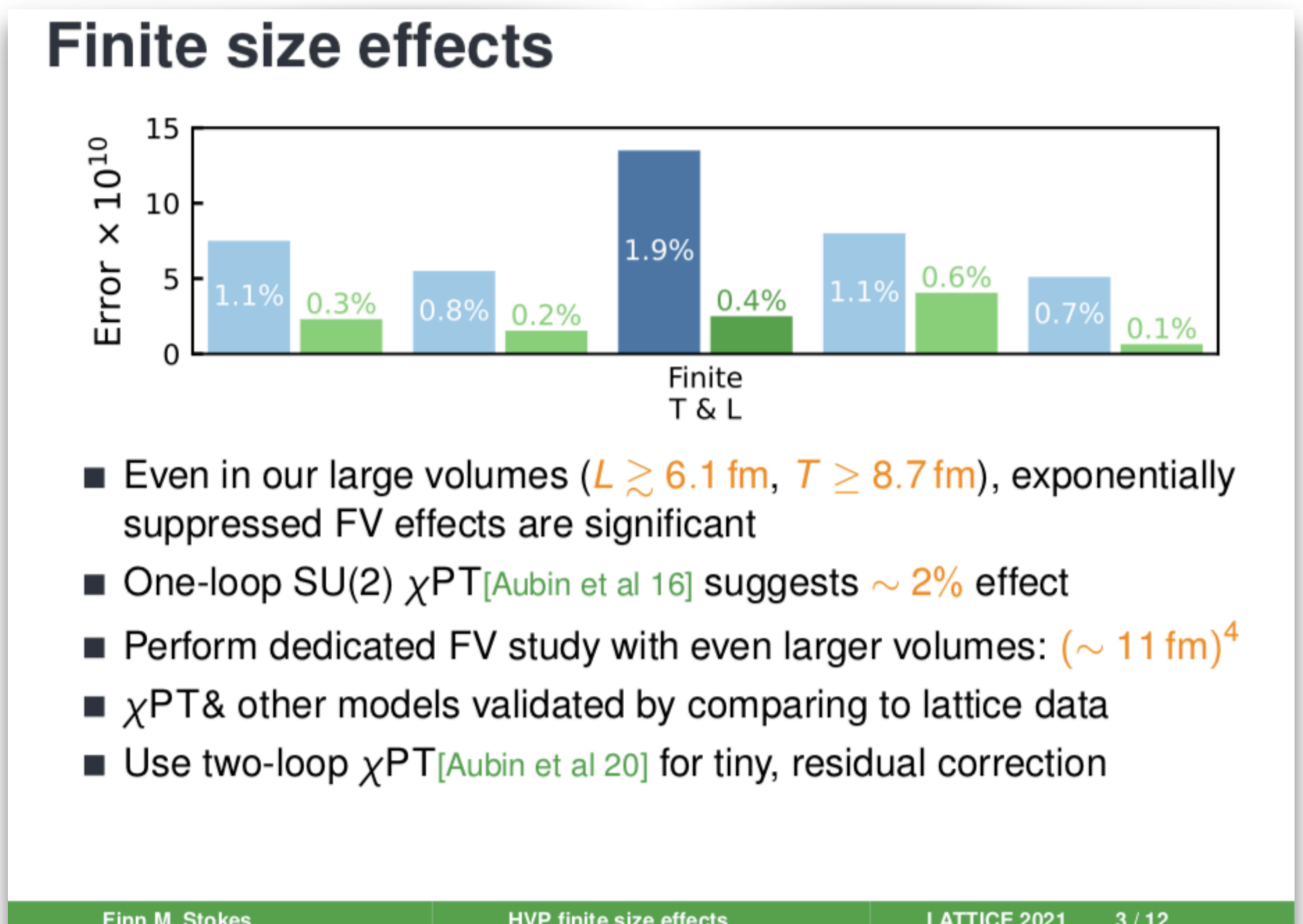
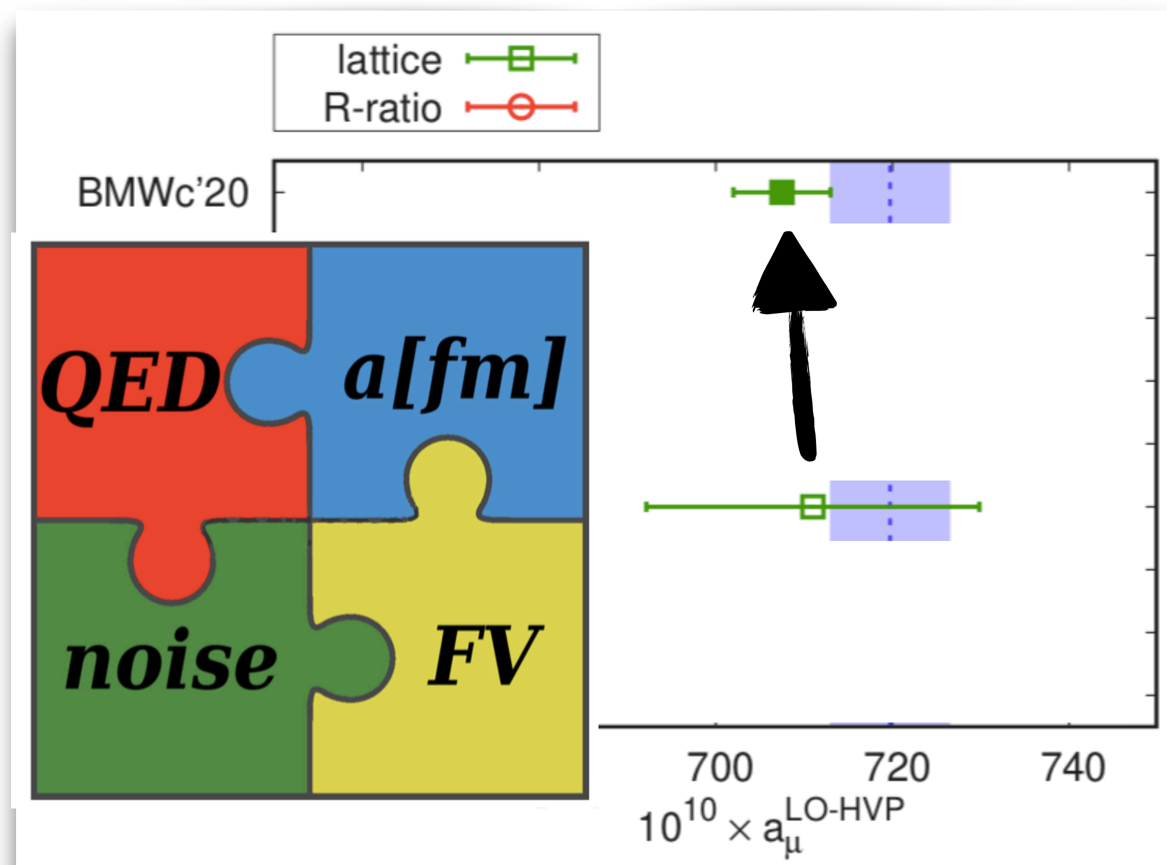
- The final reported precision is quite impressive



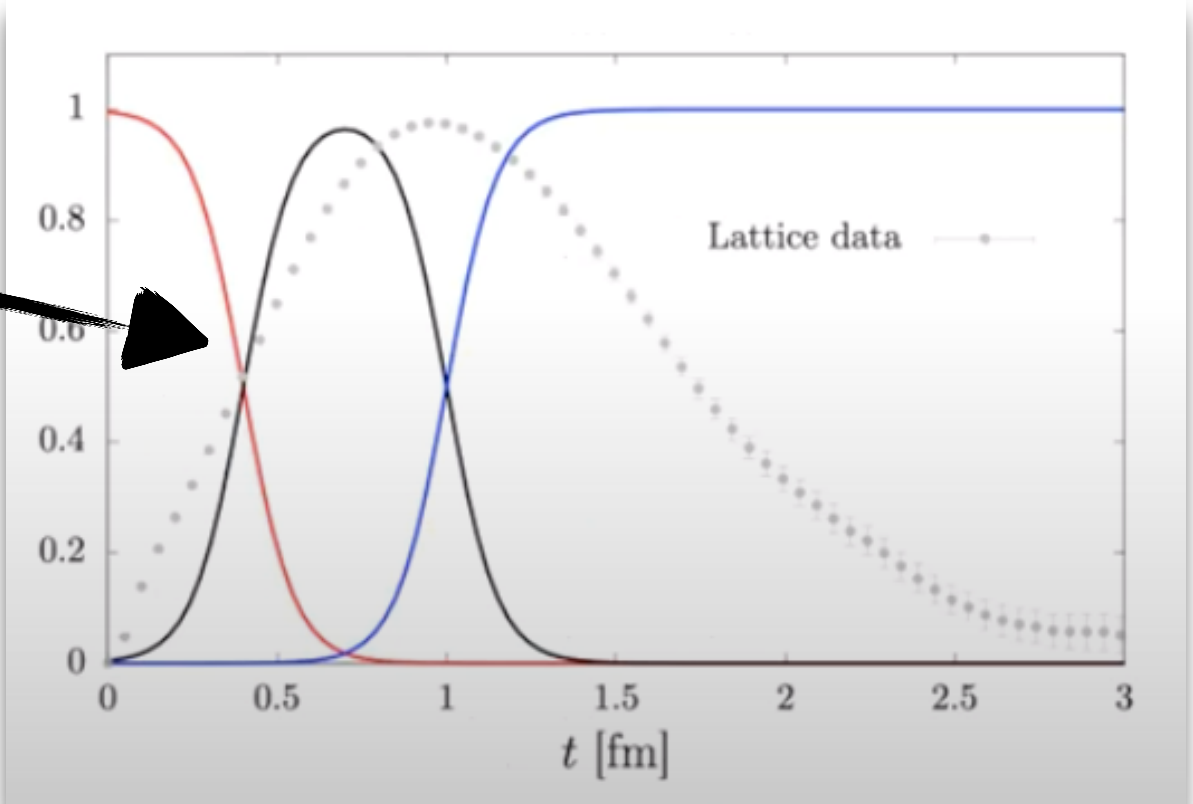


LQCD and a_μ^{HVP}

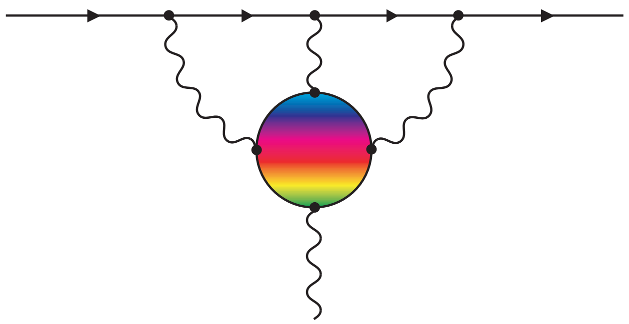
arXiv:2002.12347
Nature 593 (2021) 7857, 51-55

- Intermediate lattice results provide powerful consistency checks
- Example: “Windows” for and isolating and studying systematics effects
  - Bernecker and Meyer, Eur.Phys.J. A47 (2011) 148, arXiv:1107.4388
  - RBC/UKQCD: PRL 121 (2018) 2, 022003, arXiv:1801.07224
- Idea: convolute lattice integrand with a smooth “filter” to isolate regions of interest
- Some lattice groups already have already obtained values with commensurate precision

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

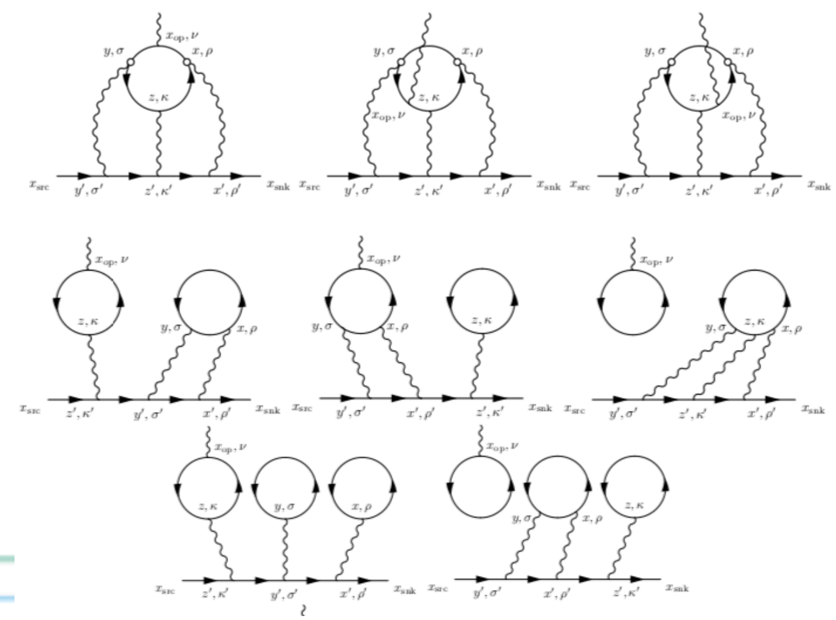


LQCD and a_μ^{HLbL}

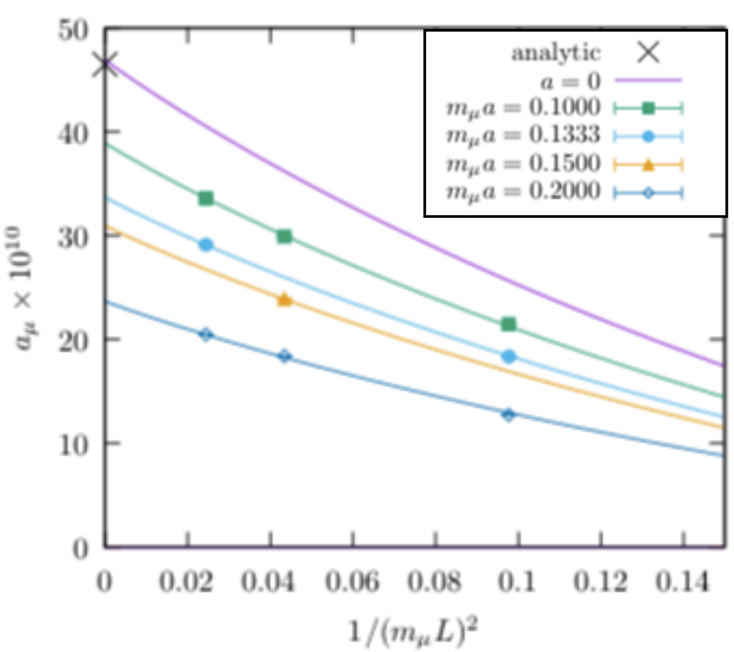


- In 2019, RBC completed the first calculation of a_μ^{HLbL} with fully controlled systematic errors
- Finite-volume QED_L prescription for photons
- Result: $a_\mu^{HLbL} = 7.87(3.06)(1.77)\times 10^{-10}$

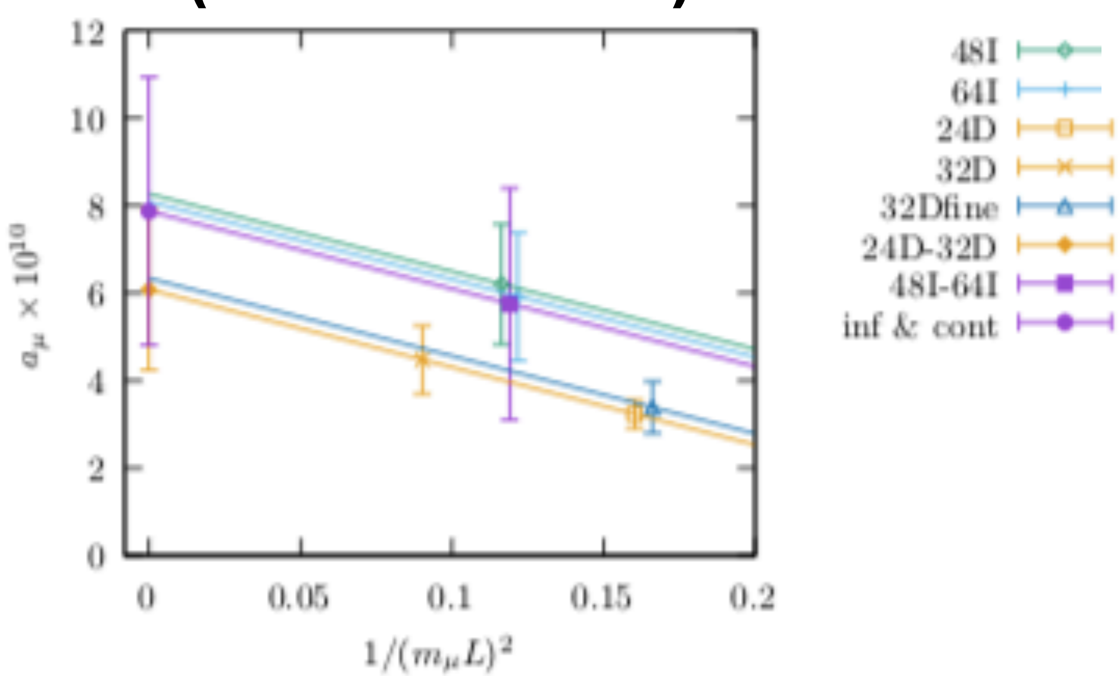
arXiv:1911.08123
PRL 124 (2020) 13, 132002



QED cross check



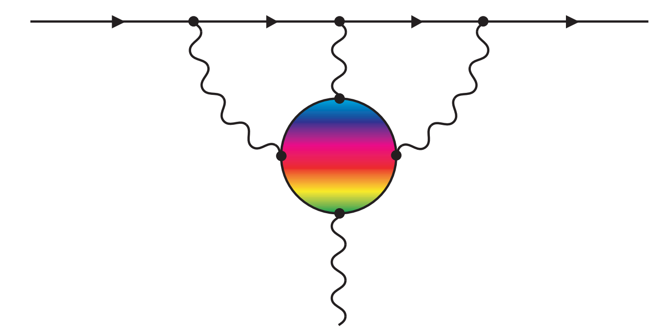
QCD total result (conn + disconn)



	con	discon	tot
a_μ	24.16(2.30)	-16.45(2.13)	7.87(3.06)
sys hybrid $\mathcal{O}(a^2)$	0.20(0.45)	0	0.20(0.45)
sys $\mathcal{O}(1/L^3)$	2.34(0.41)	1.72(0.32)	0.83(0.56)
sys $\mathcal{O}(a^4)$	0.88(0.31)	0.71(0.28)	0.95(0.92)
sys $\mathcal{O}(a^2\log(a^2))$	0.23(0.08)	0.25(0.09)	0.02(0.11)
sys $\mathcal{O}(a^2/L)$	4.43(1.38)	3.49(1.37)	1.08(1.57)
sys strange con	0.30	0	0.30
sys sub-discon	0	0.50	0.50
sys all	5.11(1.32)	3.99(1.29)	1.77(1.13)

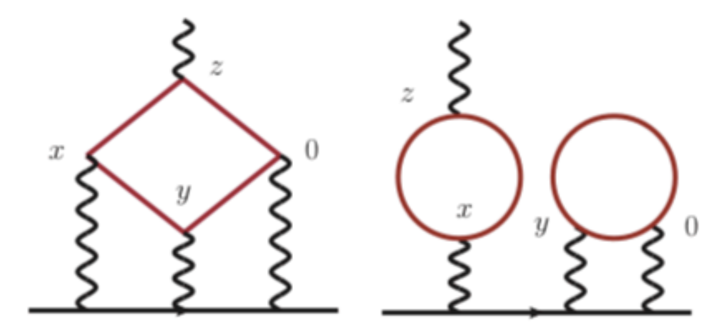
TABLE III. Central value and various systematic errors, use the hybrid continuum limit for the connected diagrams. Numbers in parentheses are statistical error for the corresponding values.

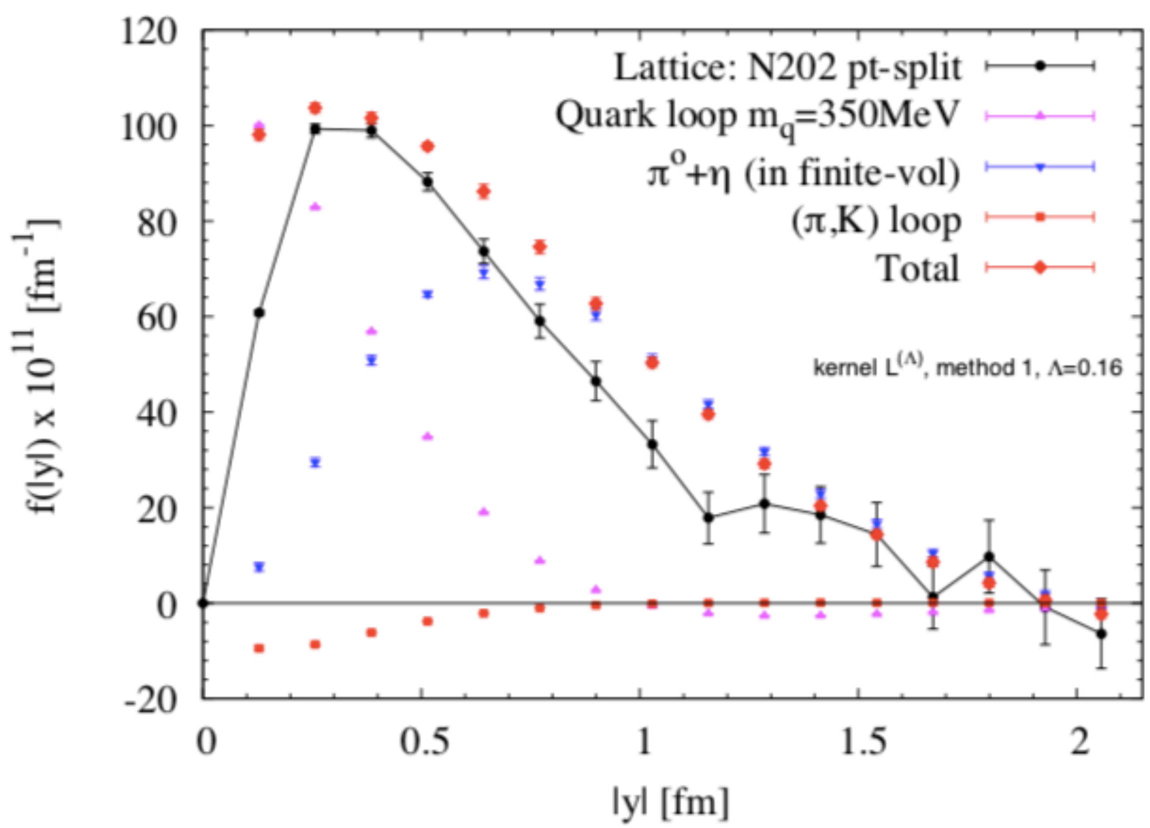
LQCD and a_μ^{HLbL}

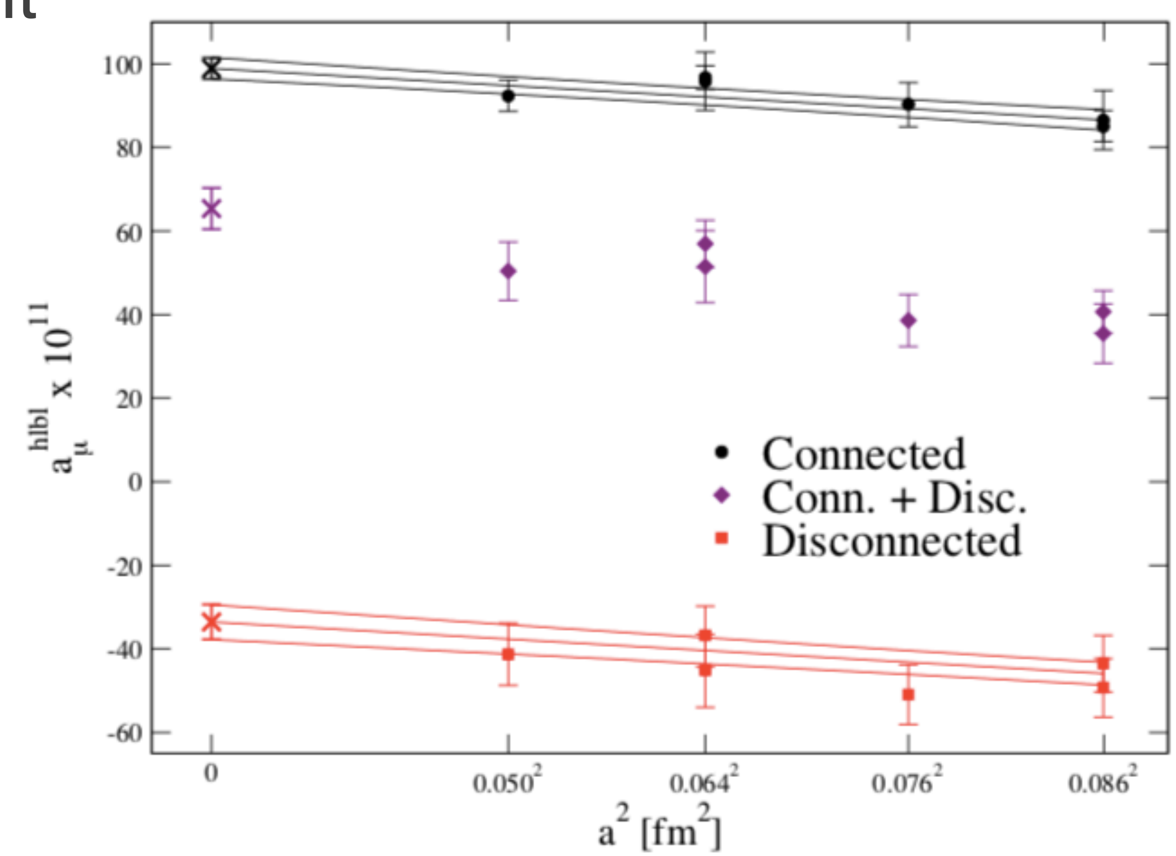


- Mainz group released their first result in 2020
- QED via ctm, infinite volume Euclidean scheme
- Calculation done at SU(3)$_f$ point: $m_\pi = m_K = 420$ MeV
- $a_\mu^{HLbL} = 65.4(4.9)(6.6)\times10^{-11}$ at SU(3)$_f$ point
- $a_\mu^{HLbL} = 104.0(20.8)\times10^{-11}$ at physical point

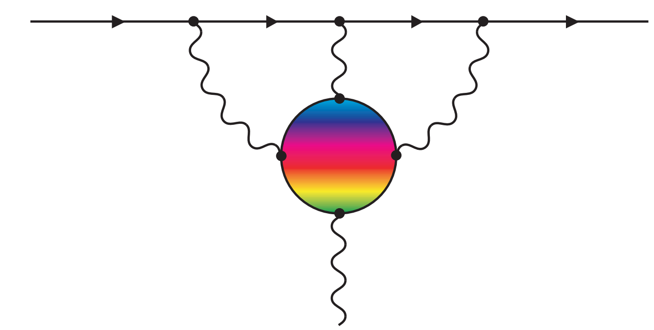
arXiv:2006.16224
Eur.Phys.J.C 80 (2020) 9, 869





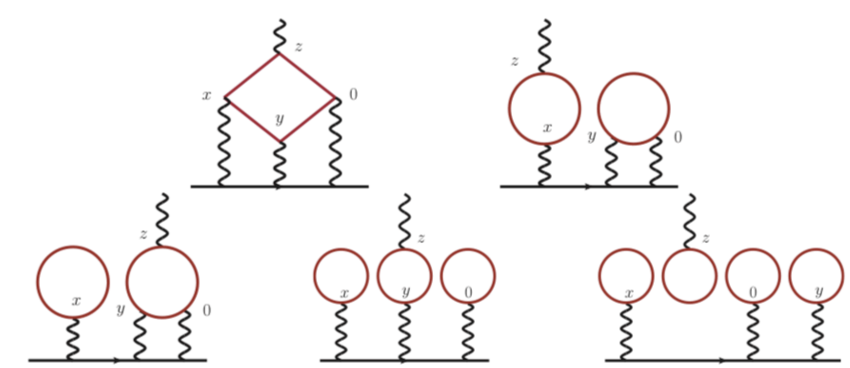


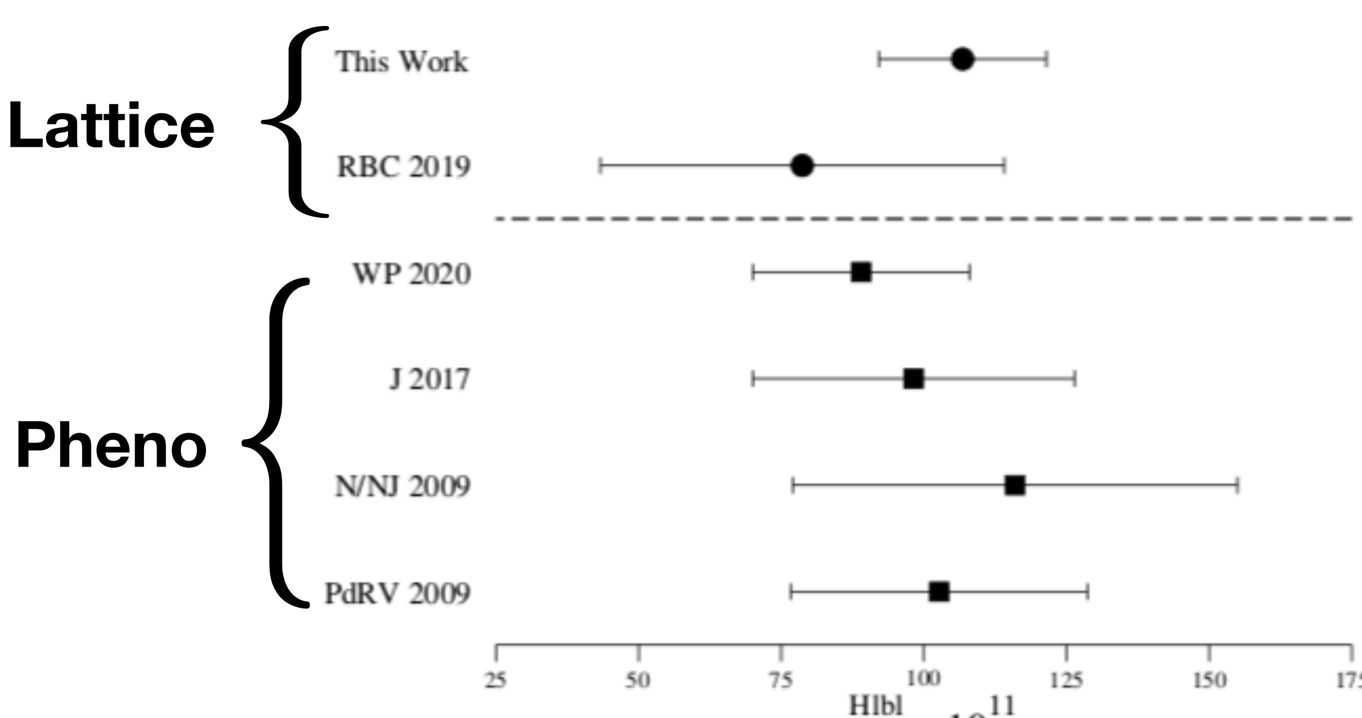
LQCD and a_μ^{HLbL}

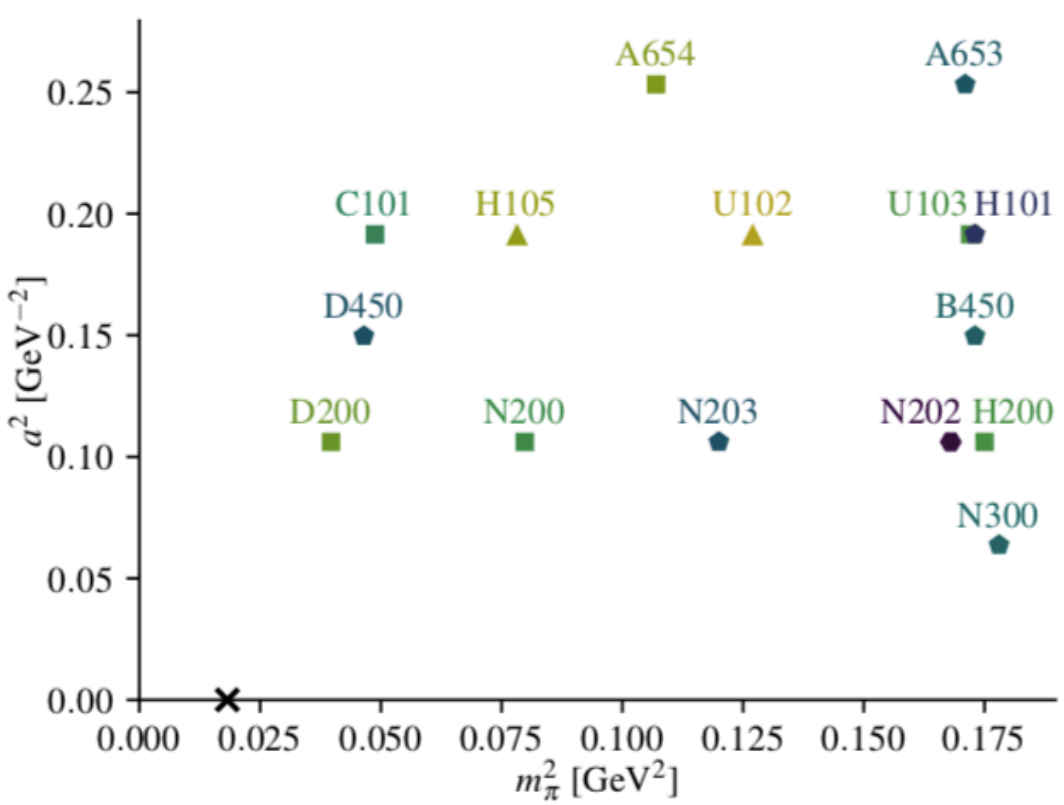


- Recent result from the Mainz group away from $SU(3)_f$
- Good agreement with previous determinations
- $a_\mu^{HLbL} = 106.8(14.7)\times10^{-11}$
- “It now appears conclusive that the hadronic light-by-light contribution cannot explain the current tension between theory and experiment for the muon.”

arXiv:2104.02632







Lattice (g-2) @ LATTICE21

JULY 26-30 2021, ZOOM/GATHER@MIT

Finite size effects in the leading hadronic vacuum polarisation contribution to $(g-2)_\mu$	Dr Finn M. Stokes	13:00 - 13:15
Continuum extrapolation of the hadronic vacuum polarization	Kalman Szabo	13:15 - 13:30
Hadronic vacuum polarization of the muon on 2+1+1-flavor HISQ ensembles: an update.	Shaun Lahert	13:30 - 13:45
The muon g-2 with four flavors of staggered quarks	Prof. Christopher Aubin	13:45 - 14:00
High precision scale setting on the lattice	Lukas Varnhorst	14:00 - 14:15
Hadronic vacuum polarization contribution to the muon g-2 from the Mainz collaboration	Dr Gerardin Antoine	05:00 - 05:15
Consistency of lattice and R-ratio determinations of the HVP: Update from RBC/UKQCD	Christoph Lehner	05:15 - 05:30
Window contributions to the muon hadronic vacuum polarization with twisted-mass fermions	Davide Giusti	05:30 - 05:45
Multi-level computation of the hadronic vacuum polarization contribution to $(g_\mu-2)$	Leonardo Giusti	05:45 - 06:00
Leading isospin breaking effects in the HVP contribution to a_{μ} and to the running of α	Andreas Risch	06:00 - 06:15
QED and strong isospin corrections in the hardonic vacuum polarization contribution to the anomalous magnetic moment of the muon	Letizia Parato	06:15 - 06:30
Pseudoscalar transition form factors and the hadronic light-by-light contribution to the muon g-2	Willem Verplanke	06:30 - 06:45
Pion Pole Contribution to HLbL from Twisted Mass Lattice QCD at the physical point	Sebastian Andreas Burri	06:45 - 07:00
Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD: a complete calculation	En-Hung Chao	07:00 - 07:15
Cutoff effects in short-distance quantities in lattice QCD	Tim Harris	07:15 - 07:30
Hadronic contributions to the running of electromagnetic and weak couplings	Miguel Teseo San José Pérez	07:30 - 07:45
HVP contribution to Running Coupling and Electroweak Precision Science	Kohtaroh Miura	07:45 - 08:00

≥ 17 talks at lattice conference earlier this week

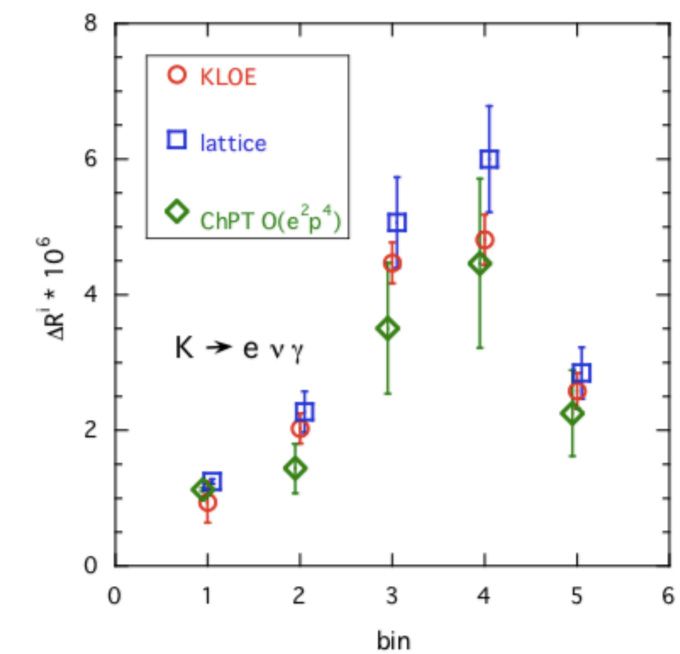
The field is quickly moving. Stay tuned for new results!

Frontiers of lattice QCD

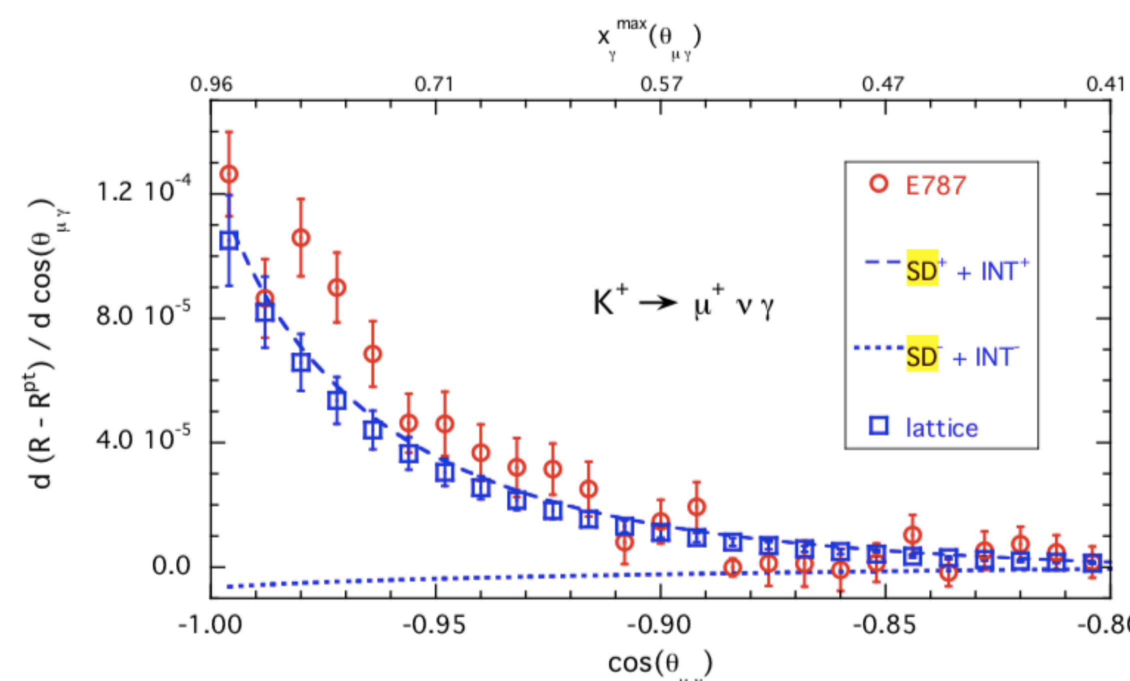
Radiative Decays: P→$\ell\nu\gamma$

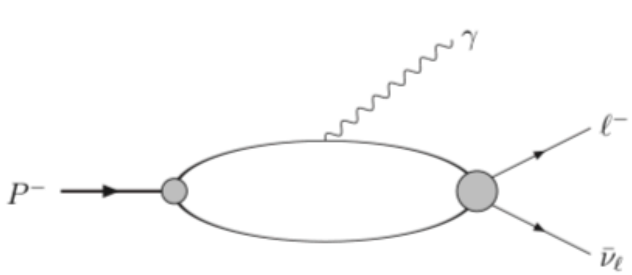
arXiv:2006.05358
PRD 103 (2021) 1, 014502
arXiv:2012.02120
PRD 103 (2021) 5, 053005

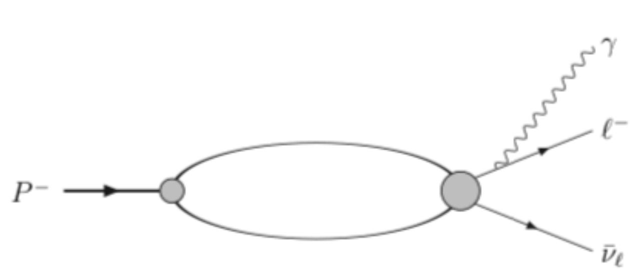
- Rome-Southampton
- Ensembles: 12x (N_f=2+1+1) ETMC
- a ≈ 0.6, 0.8, 0.9 fm, m_π = 230 - 450 MeV
- Finite-volume QED_L prescription for photons
- Computed structure-dependent V, A form factors
- Compared to KLOE, PIBETA, E787, ISTRA+, OKA
- Agreement with KLOE data (K →e$\nu\gamma$)
- Tension, e.g., with FNAL E787 (K →$\mu\nu\gamma$)



“We are able to separate unambiguously and non-pertubatively the point-like contribution, from the structure-dependent, IR-safe, terms in the amplitude.





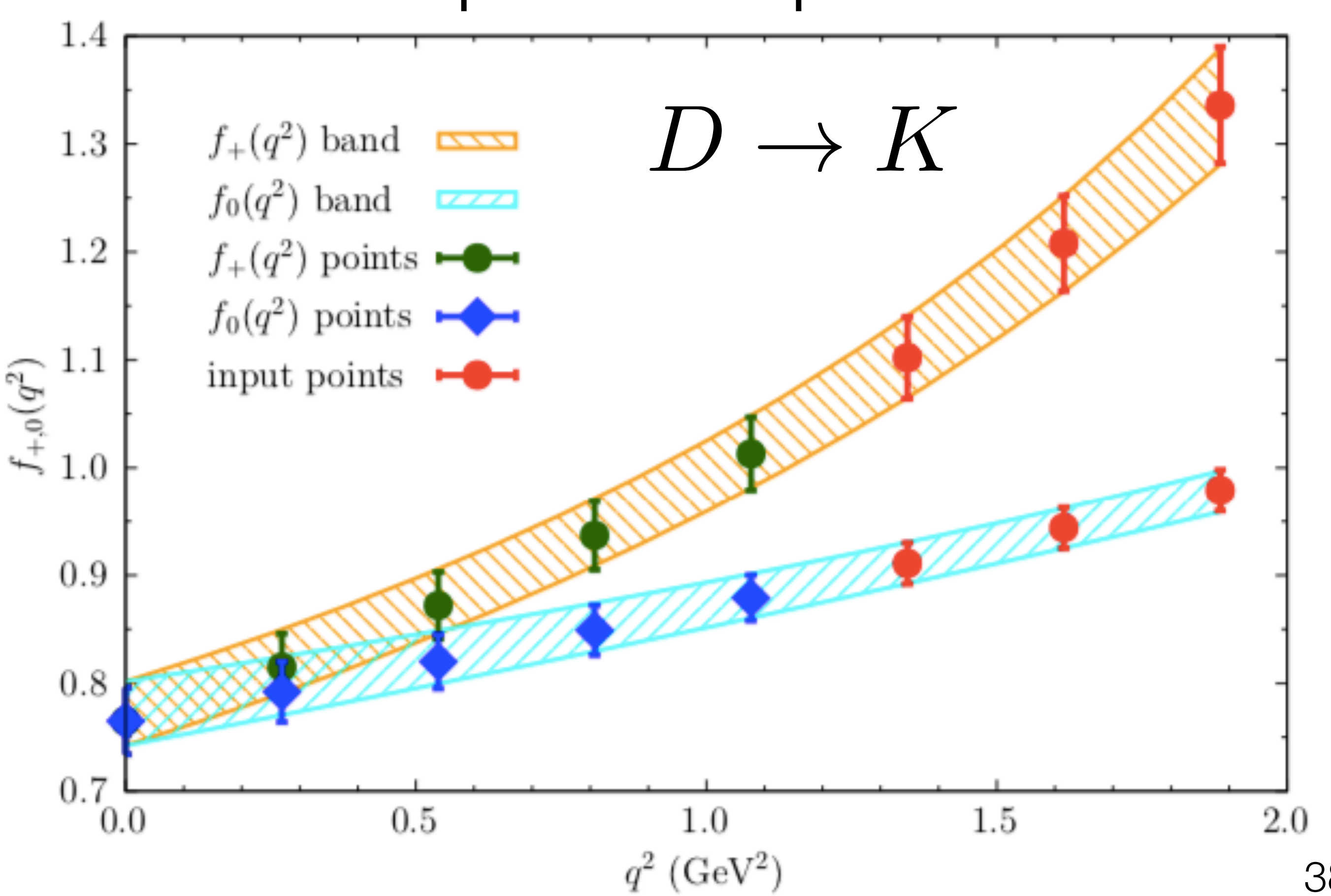


Unitarity and analyticity

arXiv:2105.02497
arXiv:2105.07851

Constraining semileptonic decays

- Recent work from S. Simula, L. Vittorio and colleagues
- Motivation: computing full q^2 dependence is very demanding for exclusive B-meson semileptonic decays
- Idea: Unitarity and analyticity constrain the q^2 behavior
- New: Dispersion relations applied to lattice QCD correlation functions allow $f(q^2 \approx q^2_{max})$ to be extended to the full kinematic range [1990s: used perturbation theory]
- New: Improved / simplified treatment of systematic effects



$$B \to D^{(*)}$$

$$\chi_{0^+}(m_b^{phys}) = 7.58\,(59) \cdot 10^{-3}\,,$$

$$\chi_{1^-}(m_b^{phys}) = 6.72\,(41) \cdot 10^{-4}\ \mathrm{GeV}^{-2}$$

$$\chi_{0^-}(m_b^{phys}) = 2.58\,(17) \cdot 10^{-2}\,,$$

$$\chi_{1^+}(m_b^{phys}) = 4.69\,(30) \cdot 10^{-4}\ \mathrm{GeV}^{-2}$$

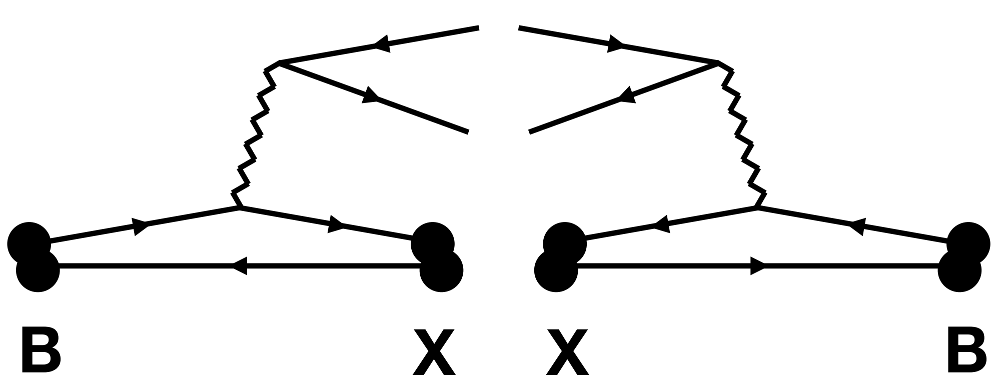
Results within ~2.5σ of PT
~5-10% agreement for 1^-, 0^-
~20% agreement for 0^+, 1^+

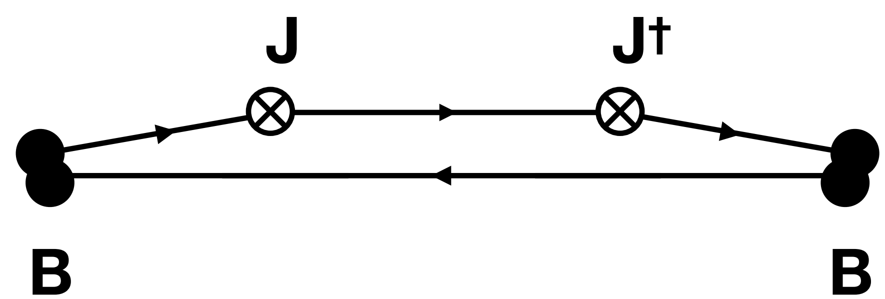
Inclusive Processes

arXiv:2005.13730
PRL 125 (2020) 3, 032001

- Recall: Long-standing tension between exclusive (fixed final state, e.g., D) and inclusive (sum over all final states)
- Presently, inclusive rates are calculated via the OPE using perturbation theory
- Calculating both using a single theoretical framework is very attractive
- Main idea: compute forward-scattering matrix elements using lattice QCD and relate them to the sum over all states:

$$d\sigma \propto \mathcal{M}\mathcal{M}^\dagger$$

$\sum_X$ 



Summary

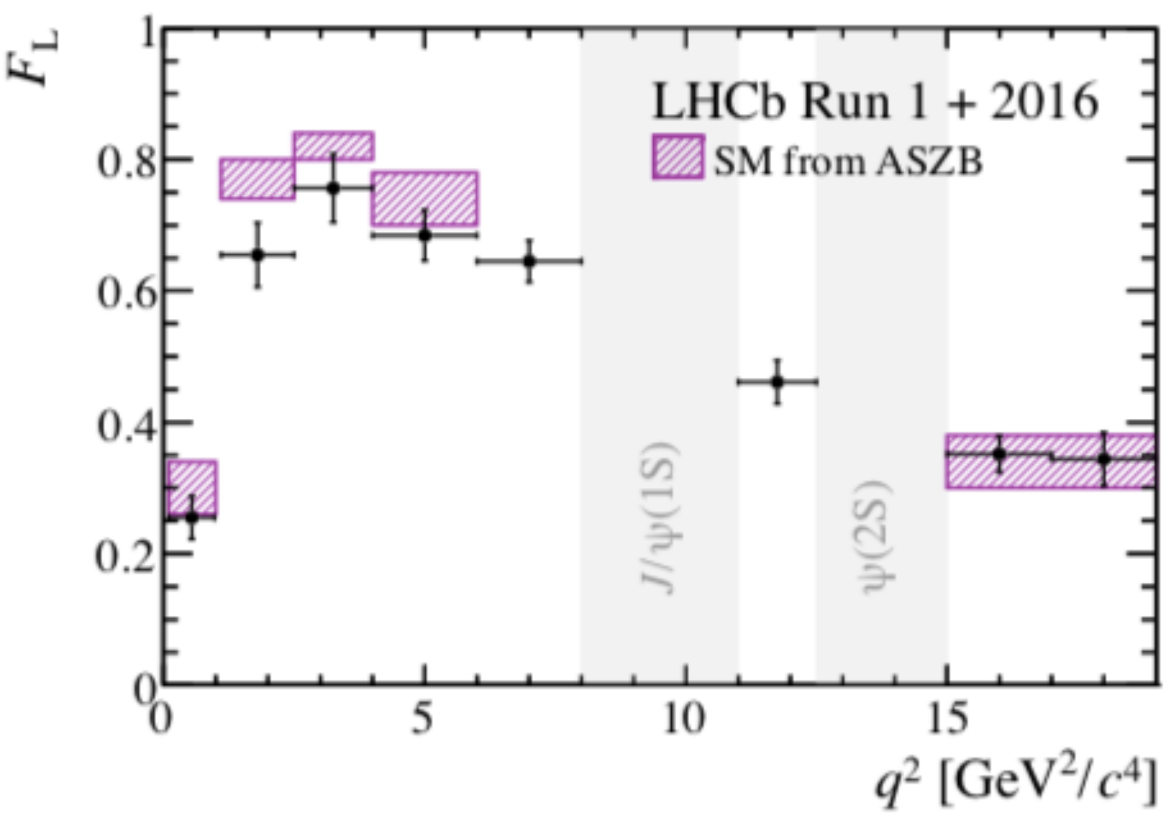
- Lattice QCD for flavor physics has become a mature field producing high-precision results
- In several cases, the “QCD-only” problems have been successfully tackled
- Further precision often now requires systematic inclusions of QED effects and strong isospin breaking
- Lattice QCD will play a pivotal role over the next few years in interpreting upcoming experimental results
- Progress is being driven by theoretical insight, algorithmic progress, and improved computing power

Studying Rare Decays

Rare processes are sensitive probes of high-scale physics

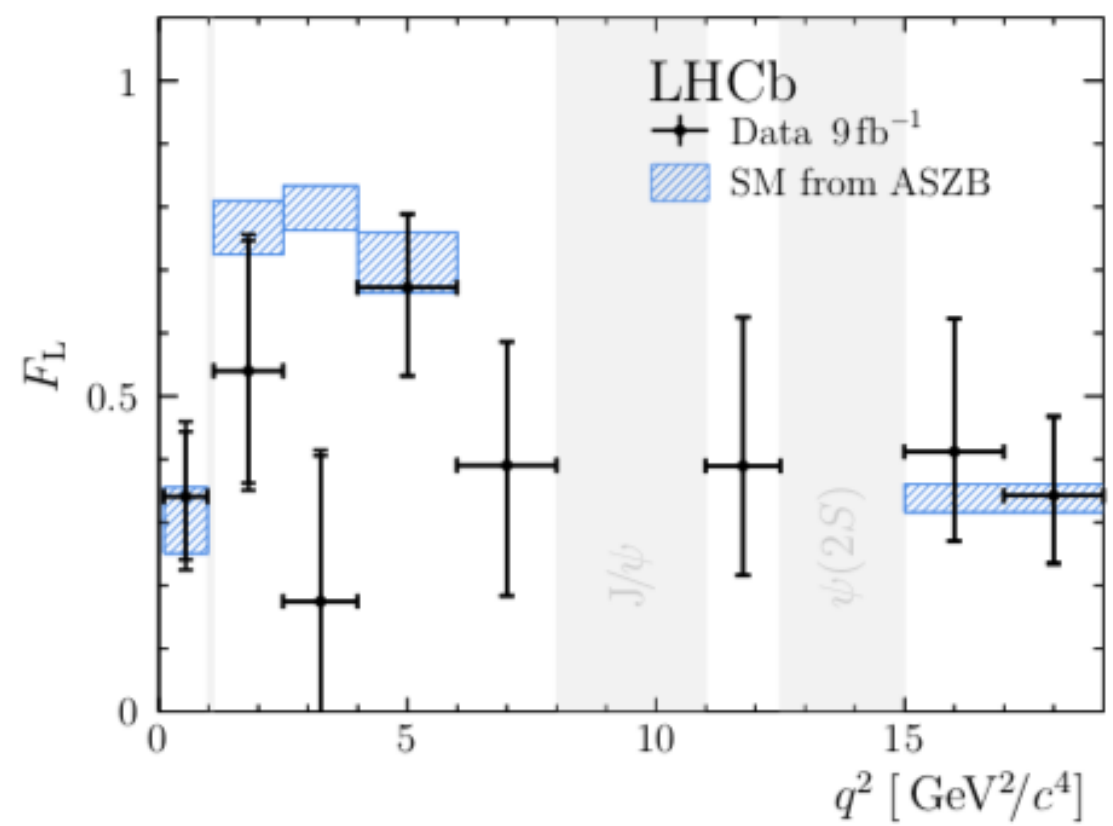
F_L = vector meson longitudinal polarization fraction

$$B^0 \to K^{*0}\mu^+\mu^-$$



arXiv:2003.04831
PRL 125, 011802 (2020)

$$B^+ \to K^{*+}\mu^+\mu^-$$

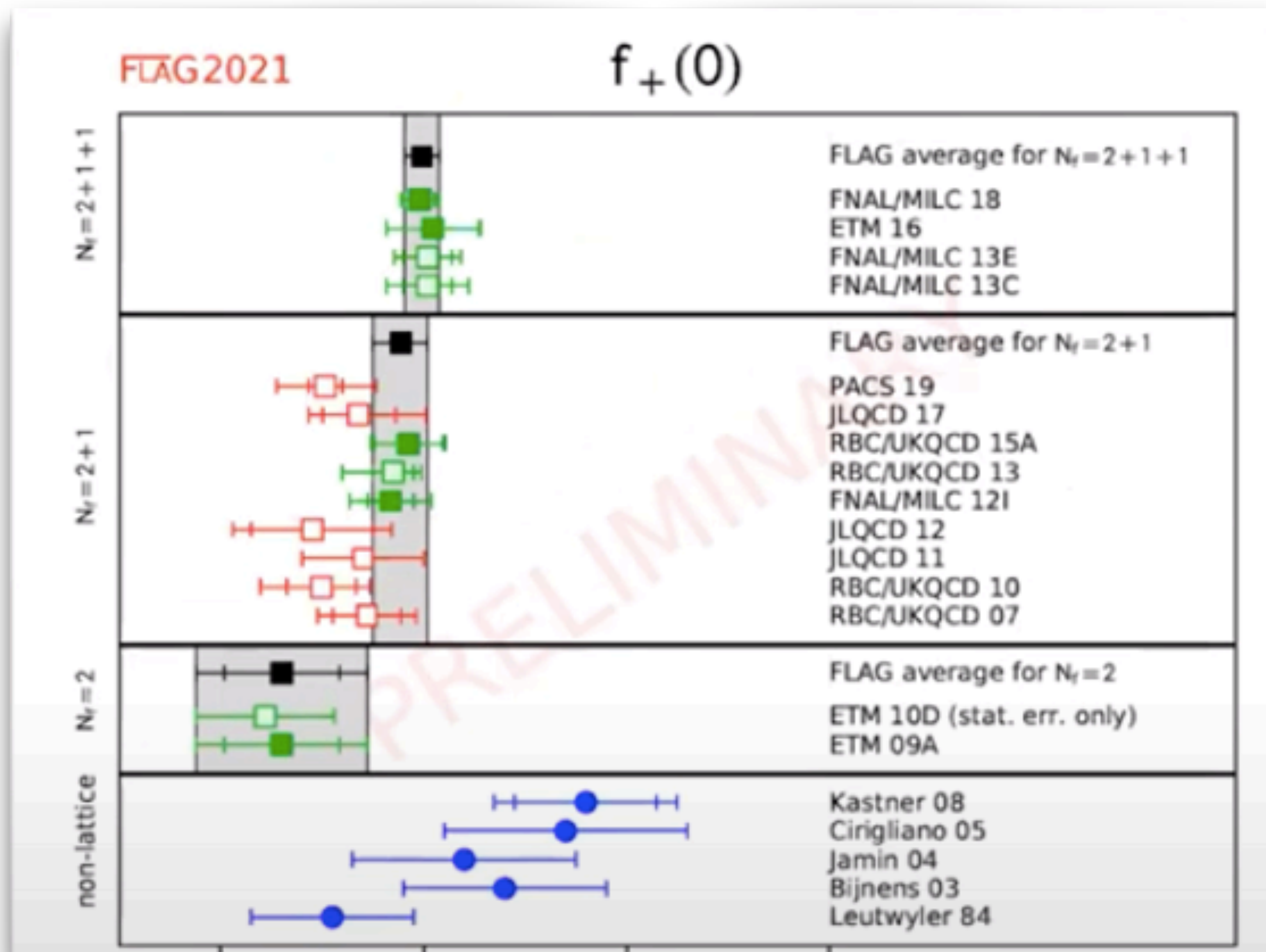


arXiv:2012.13241
PRL 126, 161802 (2021)

Flavor physics and lattice QCD

A few recent success stories

- New result since FLAG 2019 with ~0.2% precision
- Milestone: $|V_{us}|$ from $K_{\ell 3}$ decay now commensurate with f_K/f_π and $|V_{ud}|$ from nuclear beta decay
- See also: Ruth Van de Water @ Lattice 2021



FNAL/MILC: arXiv:1809.02827
PRD 99, 114509 (2019)

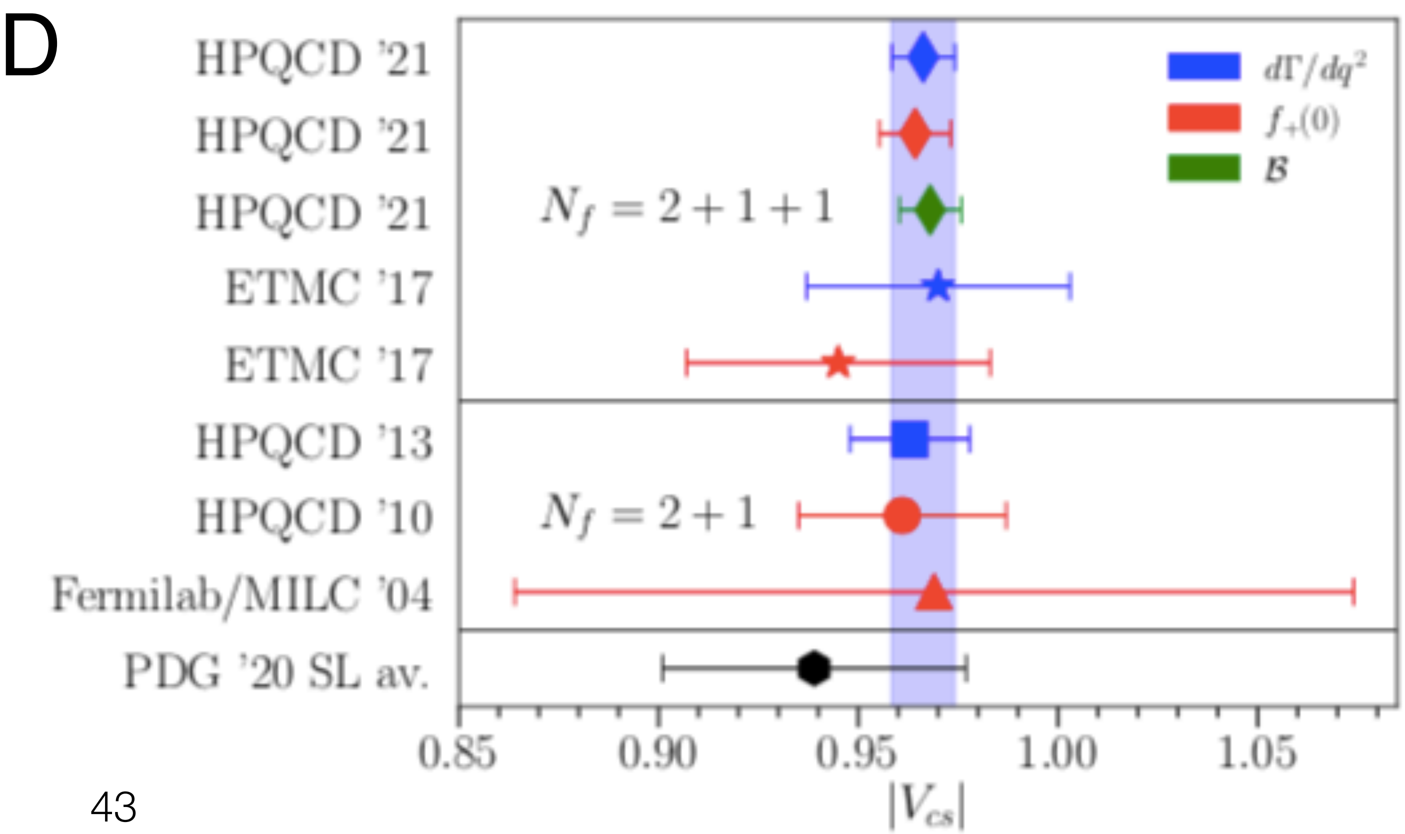
Credit: Shot from Ruth Van de Water @ Lattice 2021, FLAG working group members T. Kaneko, J. Simone, S. Simula, N. Tantalo for preliminary figure

HPQCD: D→K$\ell\nu$

arXiv:2104.09883

"Generation III"

- Ensembles: 8x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.45 - 0.15] fm
- Valence quarks: all HISQ
- Full kinematic q^2
- $|V_{cs}| = 0.9663(53)_{\rm latt}(39)_{\rm exp}(19)_{\eta {\rm EW}}\ (40)_{\rm EM}$ [0.8%]
- $R_{\mu/e} = 0.9779(2)_{\rm latt}(50)_{\rm EM}$
  - EM = short-distance QED



HPQCD: $B_c \to B_{s(d)}$

arXiv:2007.06957
PRD 102 (2020) 9, 094518

"Generation II + III"

- Ensembles: 6x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.06 - 0.15] fm
- Light/charm valence quarks: HISQ
- Valence bottom: NRQCD, HISQ
- Full physical q^2
- $\Gamma(B_c^+ \to B_s^0 \bar{\ell} \nu)$
  $= 26.25(90)_{\text{CKM}}(83)_{\text{latt}} \times 10^9\ \text{s}^{-1}$
- $\Gamma(B_c^+ \to B^0 \bar{\ell} \nu)$
  $= 1.650(61)_{\text{CKM}}(84)_{\text{latt}} \times 10^9\ \text{s}^{-1}$

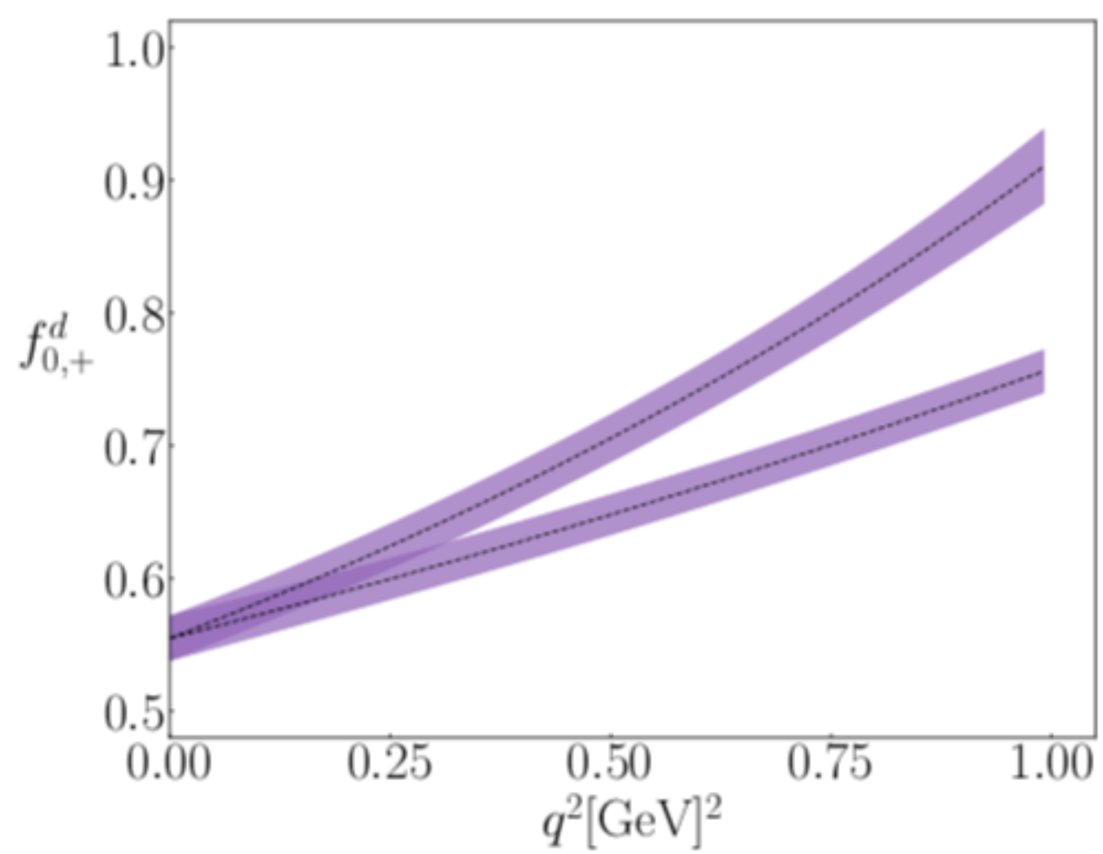


FIG. 27: Final form factors from the chained fits of f_0 (below) and f_+ (above) for $B_c^+ \to B^0 \bar{\ell} \nu_\ell$ in the physical-continuum limit, plotted against the entire range of physical q^2. This fit is described in Sec. IV D.

HPQCD B→D

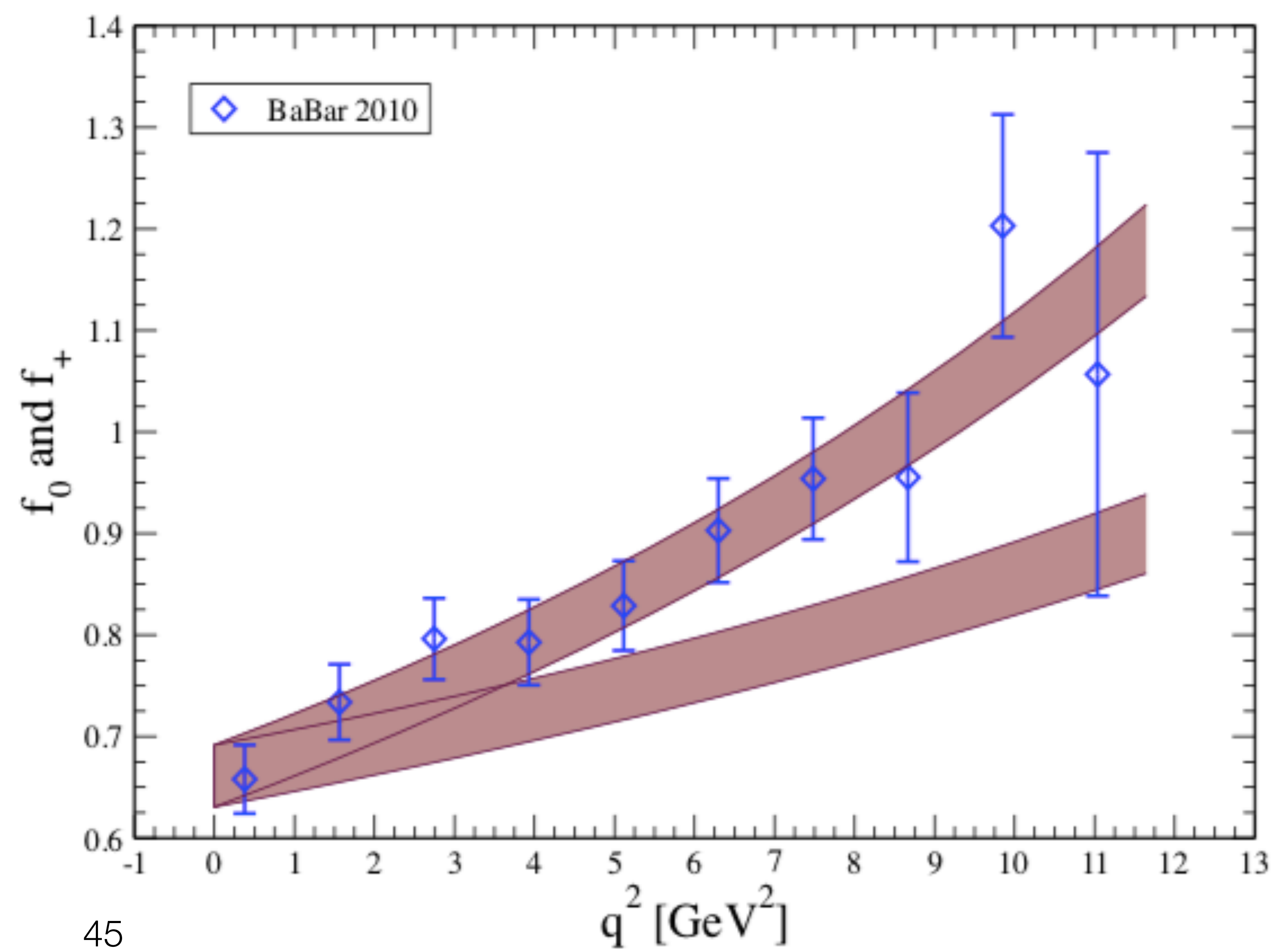
arXiv:1505.03925
PRD 92 (2015) 5, 054510

"Generation I"

- Ensembles: 5x (N_f=2+1) MILC asqtad
- Lattice spacings: 2 x in [0.09, 0.12] fm
- Light valence and charm: HISQ
- Heavy b: NRQCD
- Full physical q^2
- R(D) = 0.300(8)
- G(1) = 1.035(40)

TABLE VI. Error budget table for $R(D)$.

Type	Partial errors [%]
lattice statistics	1.24
chiral extrapolation	0.28
discretization	1.08
kinematic	1.61
matching	1.03
finite size effect	0.1
total	2.54



HPQCD $B_s \rightarrow D_s$

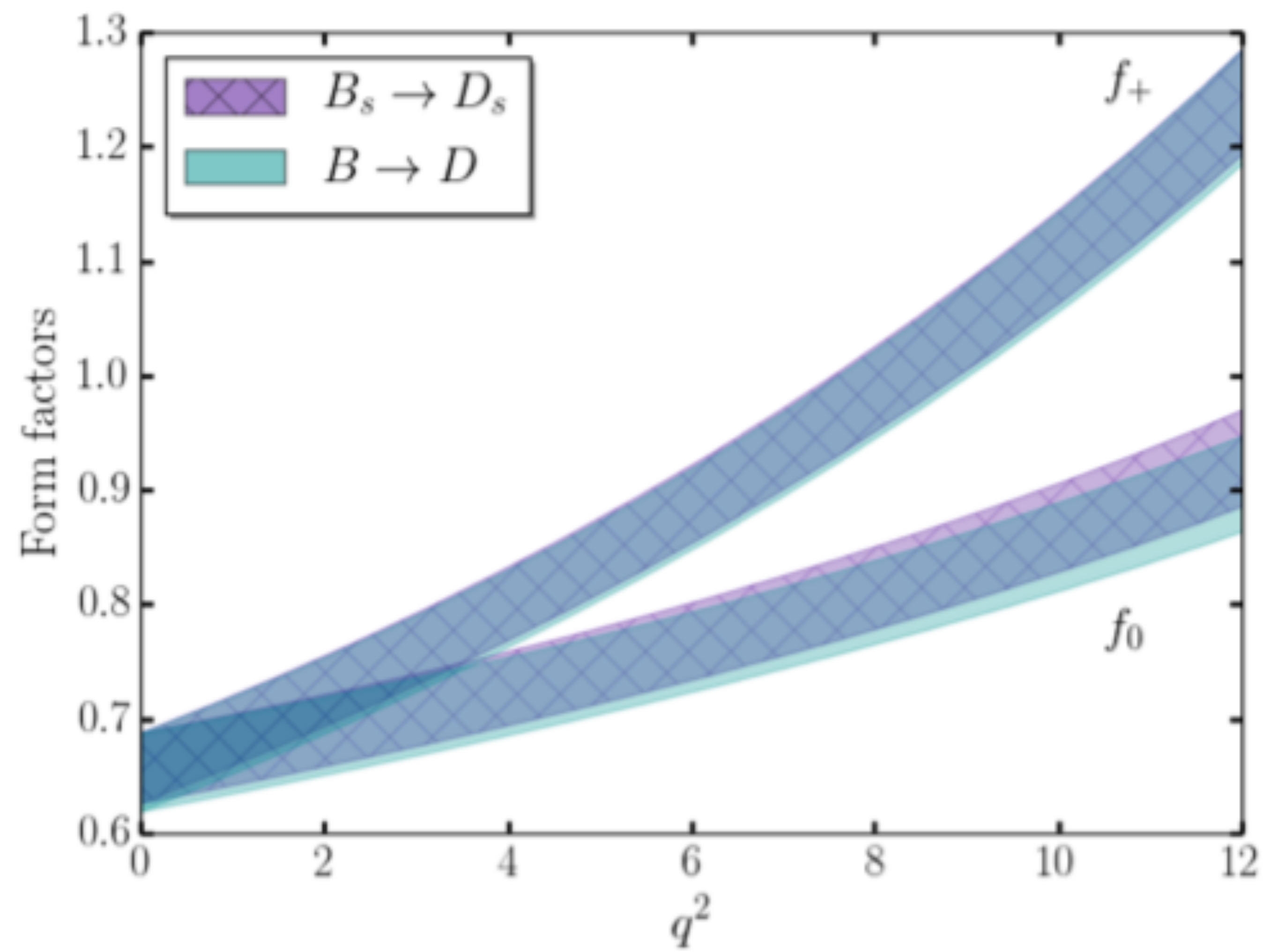
arXiv:1703.09728
PRD 95 (2017) 11, 114506

"Generation I"

- Ensembles: 5x (N_f=2+1) MILC asqtad
- Lattice spacings: [0.09, 0.12] fm
- Light valence and charm: HISQ
- Heavy b: NRQCD
- Full physical q^2
- G(1)=1.068(40)

TABLE VIII. Error budget for the form factors at zero momentum transfer, $f_0(0) = f_+(0)$, for the $B_s \rightarrow D_s \ell \nu$ semileptonic decay. We describe each source of uncertainty in more detail in the accompanying text.

Type	Partial uncertainty (%)
Statistical	1.22
Chiral extrapolation	0.80
Quark mass tuning	0.66
Discretization	2.47
Kinematic	0.71
Matching	2.21
total	3.70



HPQCD $B_{(s)} \rightarrow D_{(s)}^{\star}$

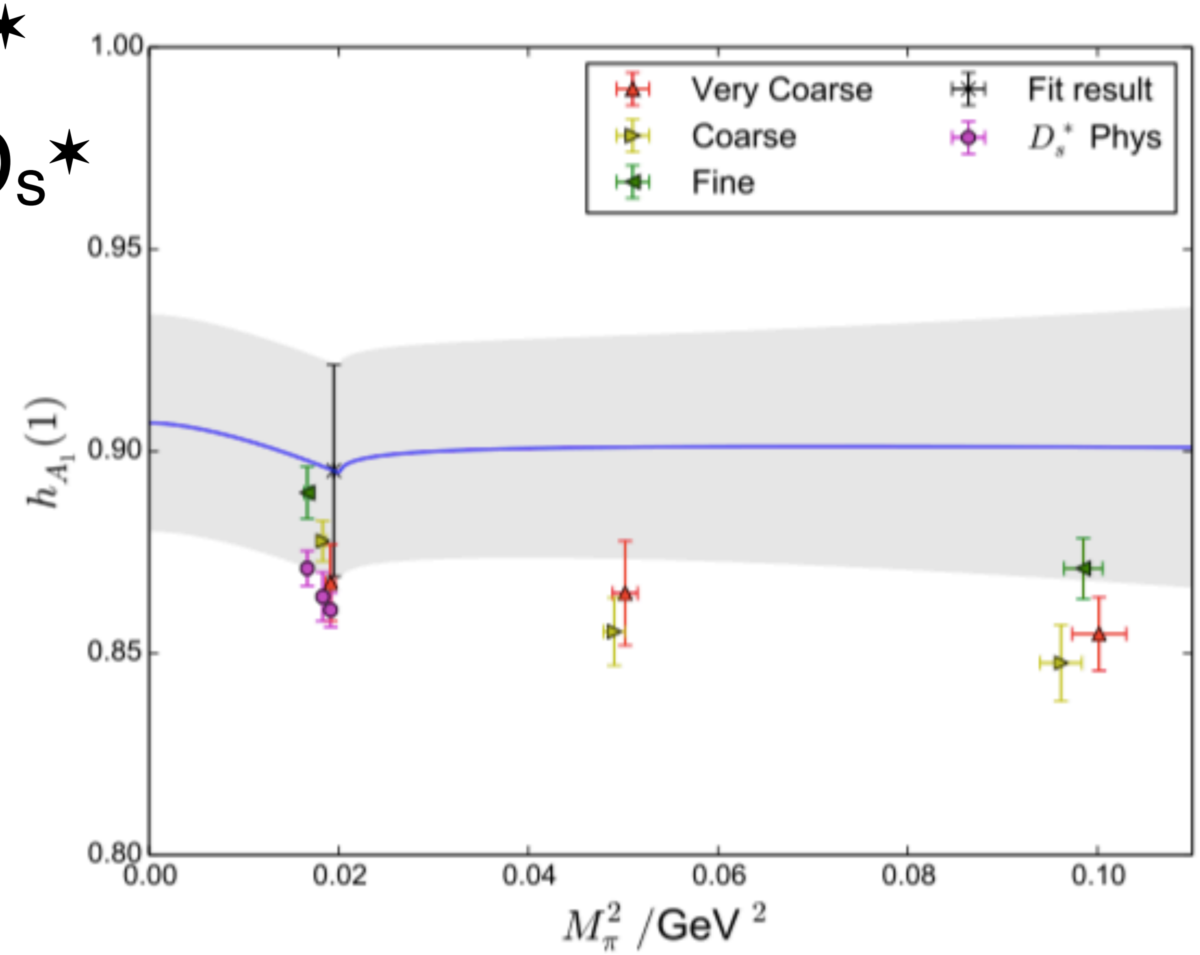
arXiv:1711.11013
PRD 97 (2018) 5, 054502

“Generation II”

- Ensembles: 8x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.09, 0.12, 0.15] fm
- Light valence and charm: HISQ
- Heavy b: NRQCD
- $h_{A1}(1)$ = 0.895(10)(24), $B \rightarrow D^{\star}$
- $h_{A1}(1)$ = 0.883(12)(28), $B_s \rightarrow D_s^{\star}$
- Zero recoil only (w=1)

TABLE IX: Partial errors (in percentages) for $h_{A_1}^{(s)}(1)$.

Uncertainty	$h_{A_1}(1)$	$h^s_{A_1}(1)$	$h_{A_1}(1)/h^s_{A_1}(1)$
α_s^2	2.1	2.5	0.4
$\alpha_s\Lambda_{QCD}/m_b$	0.9	0.9	0.0
$(\Lambda_{QCD}/m_b)^2$	0.8	0.8	0.0
a^2	0.7	1.4	1.4
$g_{D^*D\pi}$	0.2	0.03	0.2
Total systematic	2.7	3.2	1.7
Data	1.1	1.4	1.4
Total	2.9	3.5	2.2



HPQCD $B_s \to D_s^*$

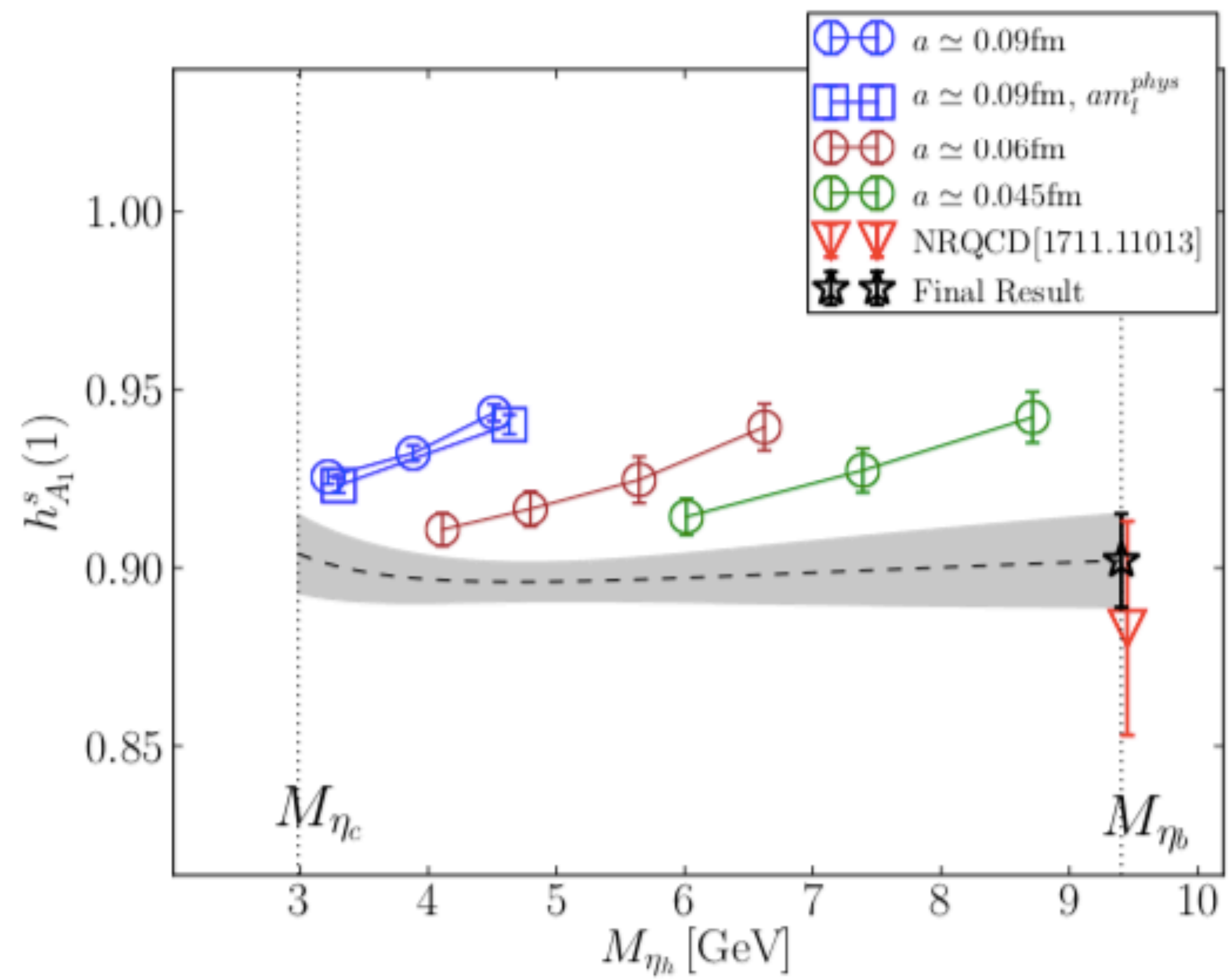
arXiv:1904.02046
PRD 99 (2019) 11, 114512

"Generation III"

- Ensembles: 4x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.04, 0.06, 0.09] fm
- Valence quarks: all HISQ
- $m_h a < 0.8$ [close-to-physical b at 0.04 fm]
- Zero recoil only (w=1)
- $h_{A1}^s(1) = 0.9020(96)(90)$

TABLE IV: Error budget for $h^s_{A_1}(1)$. Errors are given as a percentage of the final answer. The mass mistuning error includes that from valence strange and sea light and strange quarks; we find that taking a ± 10 MeV uncertainty in the physical value of the η_b mass has a negligible effect.

Source	% Fractional Error
Statistics $+Z_A$	1.06
$a \to 0$	0.73
$m_h \to m_b$	0.69
mass mistuning	0.20
Total	1.45



HPQCD $B_s \to D_s^\star$

arXiv:1904.02046
PRD 99 (2019) 11, 114512

“Generation III”

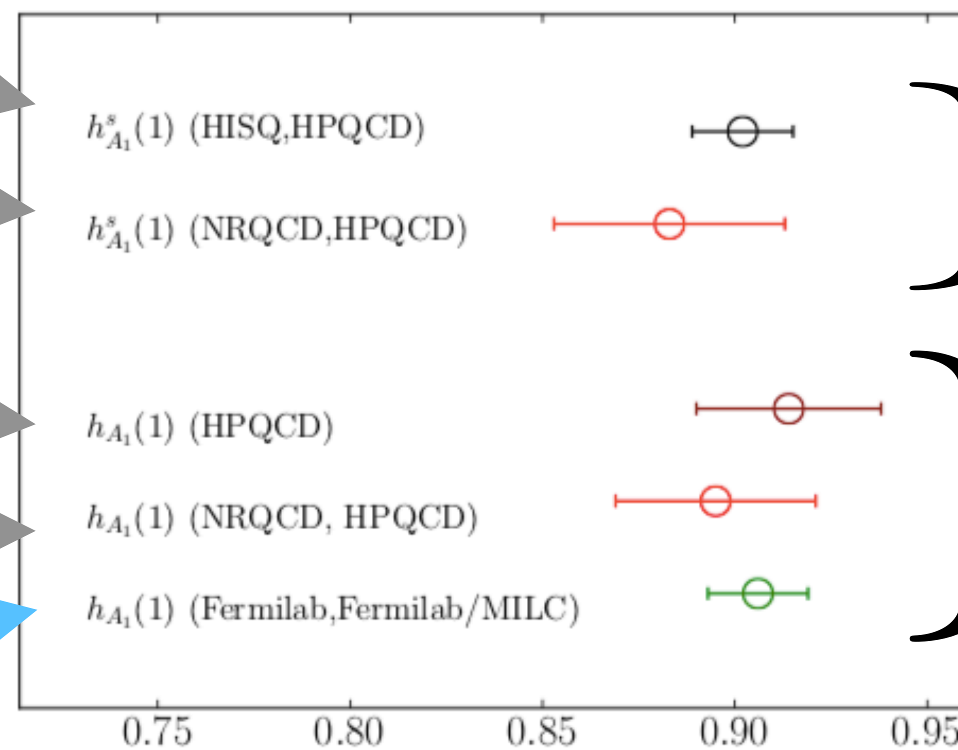
“Generation III”

“Generation II”

“Generation III”

“Generation II”

“Generation 1”



$B_s \to D_s^\star$

$B \to D^\star$

FIG. 6: Comparison of lattice QCD results for $h^s_{A_1}(1)$ and $h_{A_1}(1)$. Our results for $h^{(s)}_{A_1}(1)$ are marked ‘(HISQ, HPQCD)’ and for $h_{A_1}(1)$ are marked ‘(HPQCD)’. Those marked ‘(NRQCD,HPQCD)’ are from [41] and the value marked ‘(Fermilab, Fermilab/MILC)’ is from [39].

HPQCD $B_s \to D_s$

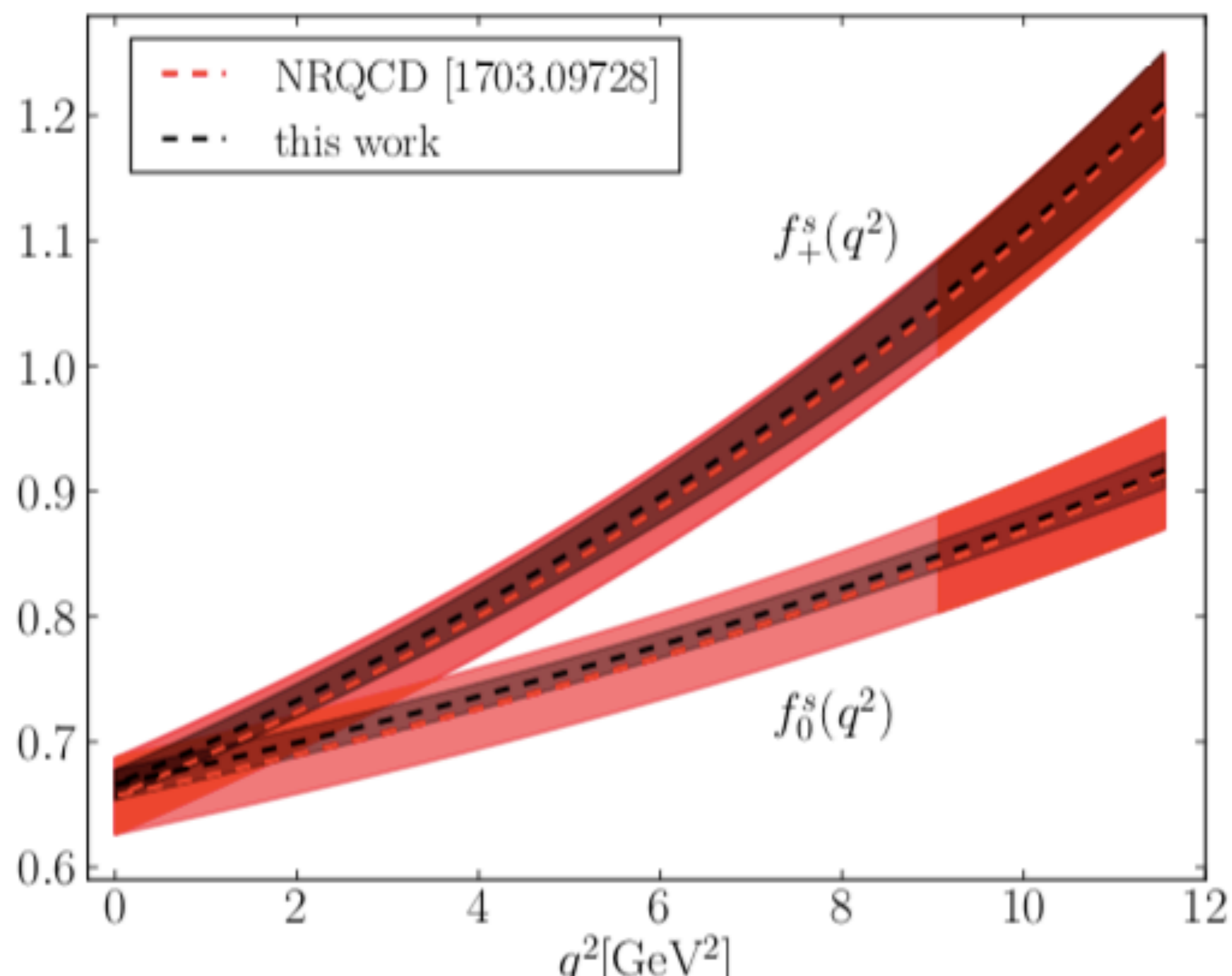
arXiv:1906.00701
PRD 101 (2020) 7, 074513

"Generation III"

- Ensembles: 4x (N_f=2+1+1) MILC HISQ
- Lattice spacings: [0.04, 0.06, 0.09] fm
- Valence quarks: all HISQ
- $m_h a < 0.8$
- Full physical q^2
- $R(D_s) = 0.2987(46)$

Source	% Fractional Error
Statistics	1.11
$m_h \to m_b$ and $a \to 0$	1.20
Quark mistuning	0.58
Total	1.73

TABLE VI: Error budget for $f_0^s(q^2_{\max})$.



FNAL/MILC B→D*

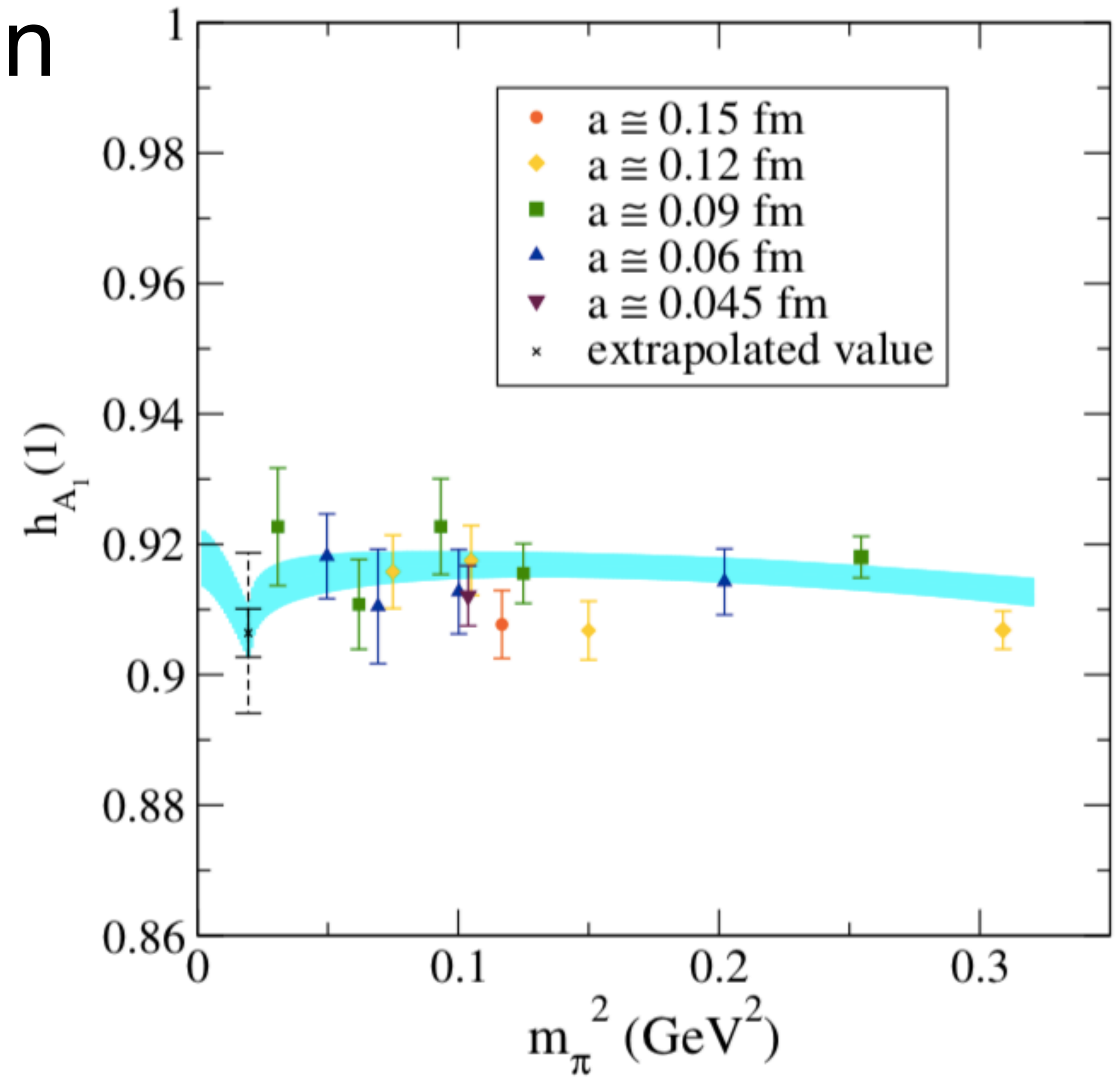
arXiv:1403.0635
PRD 89 (2014) 11, 114504

"Generation 1"

- Ensembles: 15x (N_f=2+1) MILC asqtad
- Lattice spacings: 5 x in [0.045 - 0.15] fm
- Light valence: asqtad staggered
- Heavy b/c: FNAL interpretation
- Zero recoil only (w=1)
- $h_{A1}(1)=F(1) = 0.906(4)(13)$

TABLE X. Final error budget for $h_{A_1}(1)$

Uncertainty	$h_{A_1}(1)$
Statistics	0.4%
Scale (r_1) error	0.1%
χPT fits	0.5%
$g_{D^* D\pi}$	0.3%
Discretization errors	1.0%
Perturbation theory	0.4%
Isospin	0.1%
Total	1.4%



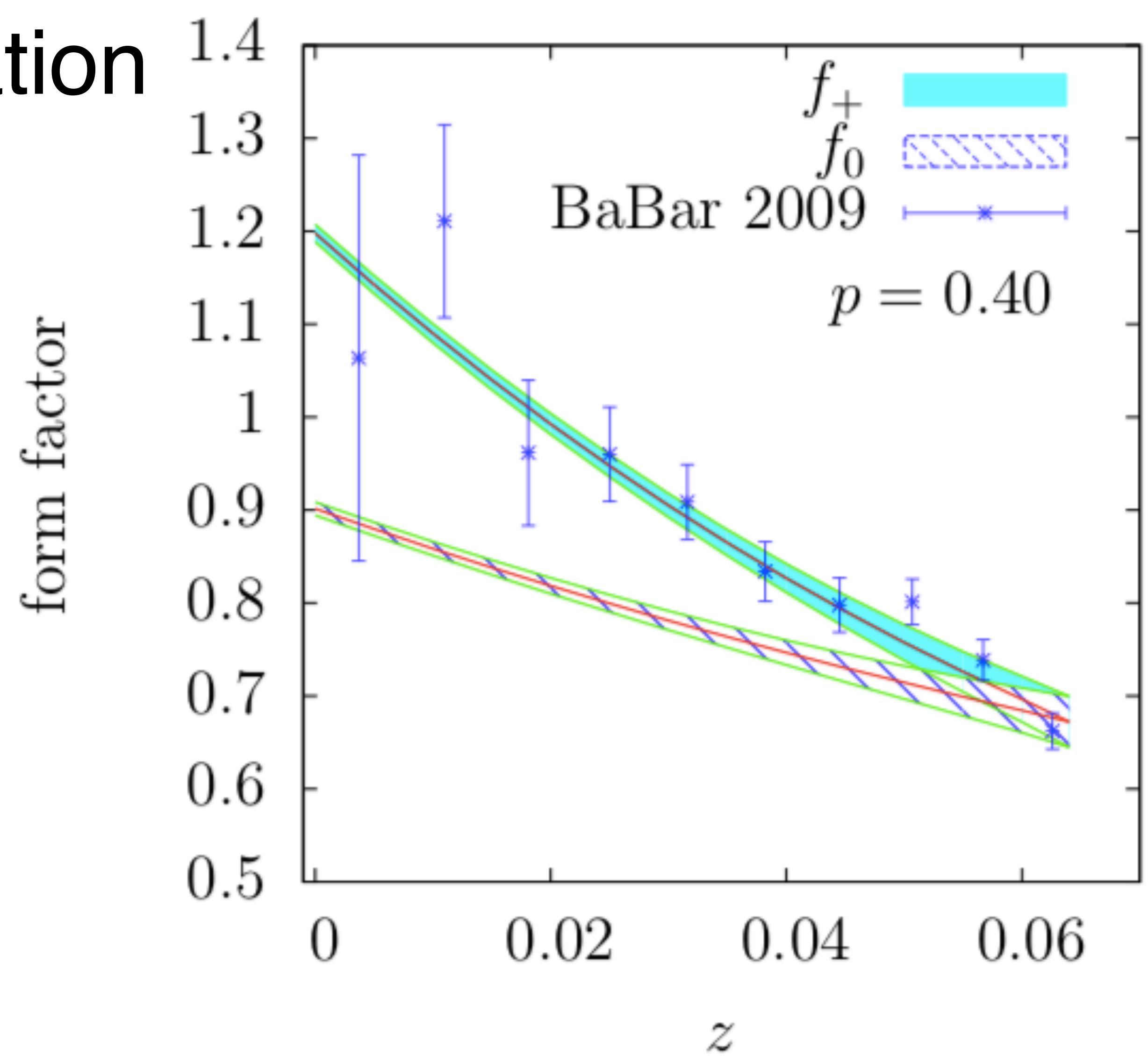
FNAL/MILC B→D

arXiv:1503.07237
PRD 92 (2015) 3, 034506

"Generation 1"

- Ensembles: 14x (N_f=2+1) MILC asqtad
- Lattice spacings: 4 x in [0.045 - 0.12] fm
- Light valence: asqtad staggered
- Heavy b/c: FNAL interpretation
- Full physical q²
- R(D) = 0.299(11)
- G(1) = 1.054(4)(8)

Source	$f_+(\%)$	$f_0(\%)$
Statistics+matching+χPT cont. extrap.	1.2	1.1
(Statistics)	(0.7)	(0.7)
(Matching)	(0.7)	(0.7)
(χPT/cont. extrap.)	(0.6)	(0.5)
Heavy-quark discretization	0.4	0.4
Lattice scale r_1	0.2	0.2
Total error	1.2	1.1



FNAL/MILC $B_{(s)} \to D_{(s)}^{(*)}$

- 2nd Generation:
  - FNAL on (N_f=2+1+1) MILC HISQ
  - Plan: joint correlated analysis on $B \to D$ and $B \to D^*$
  - Analysis underway by Alex Vaquero

"Generation 2"

- 3rd Generation:
  - HISQ on (Nf=2+1+1) MILC HISQ
  - Complete set: scalar, vector, and tensor currents
  - Broad range of momenta across kinematic range
  - $B_{(s)} \to D_{(s)}$ [+ many others, e.g., $B_{(s)}/D_{(s)} \to K/\pi$)
  - Analysis underway by WJ, Andrew Lytle

"Generation 3"