Hadron 2021

S-matrix Bootstrap for Effective Field Theories

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29/07/21

Pion scattering: arXiv: 1810.12849 with J. Penedones and P. Vieira

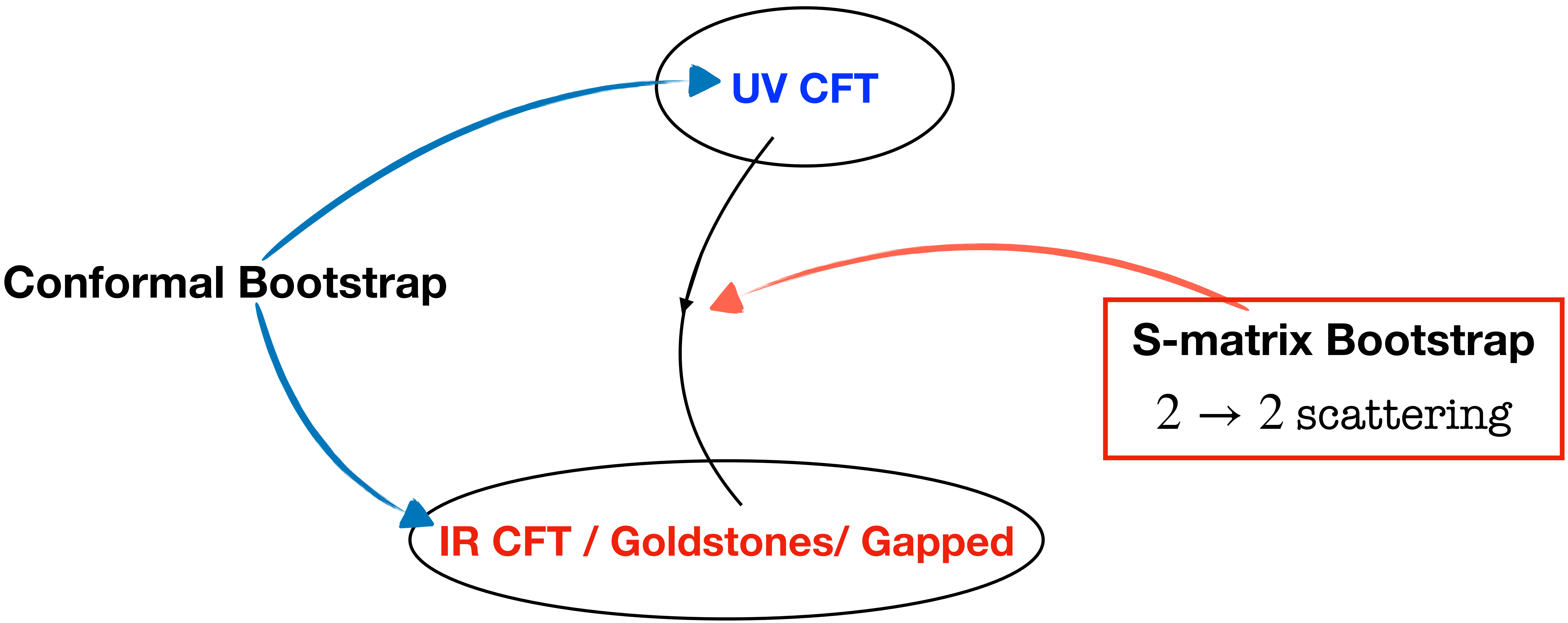
Flux-Tube phonons: arXiv: 1906.08098 with J. Elias-Miro', A. Hebbar , J. Penedones and P. Vieira; arXiv: 2106.07957 with J. Elias Miro'

Massless Pions: arXiv: 2011.02802 with J. Penedones and P. Vieira

Supergravitons: arXiv: 2102.02847 with J. Penedones and P. Vieira

TEL AVIV UNIVERSITY

Bootstrap philosophy

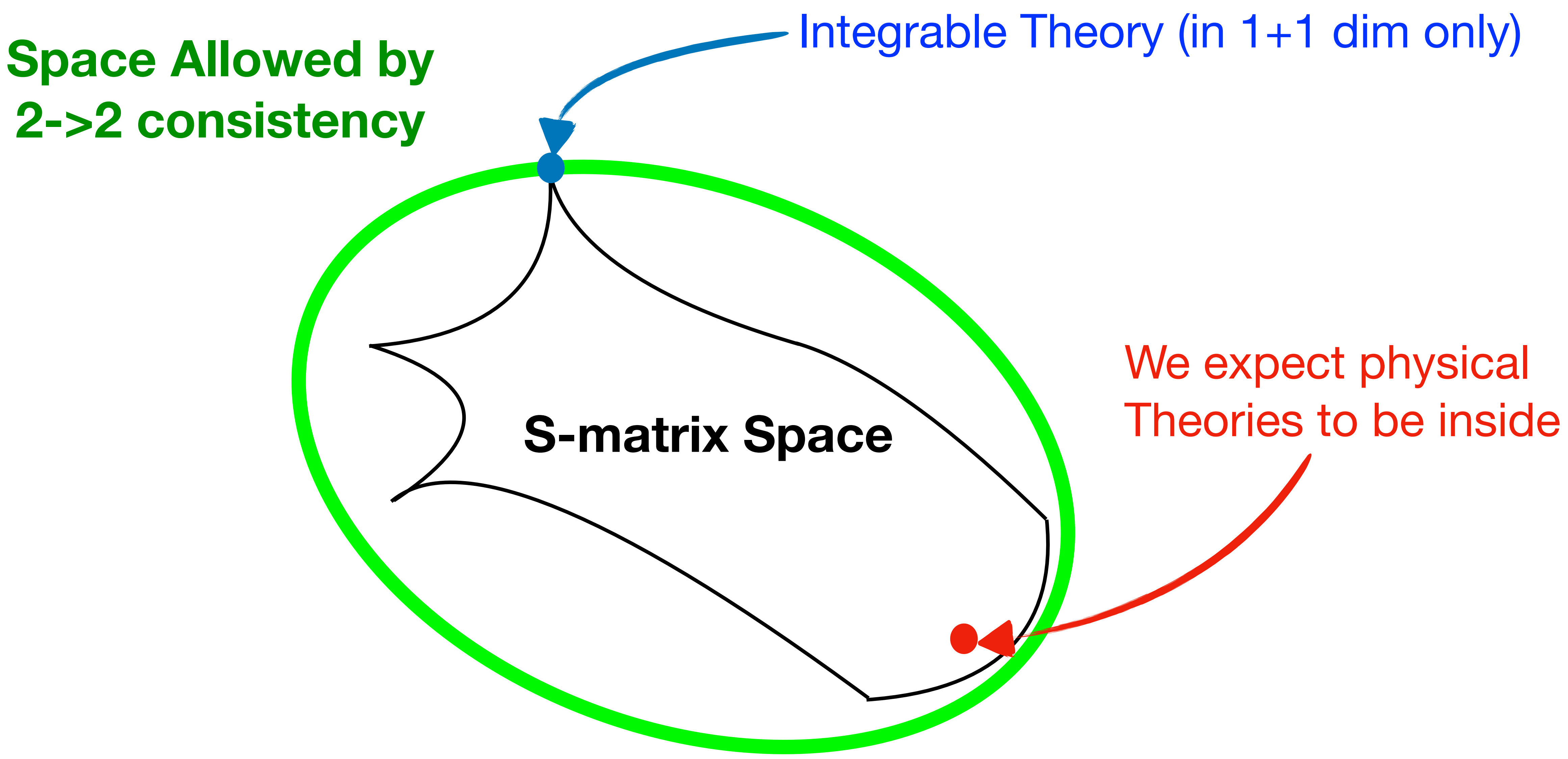


Constraining the outcomes of a scattering experiment using crossing, analyticity and unitarity.

Disclaimer: space of amplitudes consistent with $2 \to 2$ unitarity

Full Unitarity: $|\text{Prob}_{2\to 2}(s,\ell)|^2 + \text{positive} = 1$

$$\implies |\text{Prob}_{2\to 2}(s,\ell)|^2 \leq 1$$



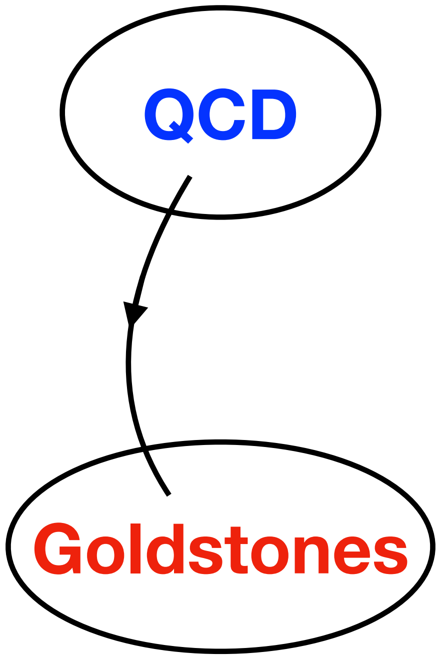
Plan of the Talk

1) Low energy χEFT: what to bootstrap?

2) S-matrix Bootstrap in d=4 for massless particles

3) Numerical results

Scattering of Goldstones with $O(N)$ symmetry



QCD in the chiral limit flows to a theory of charged Goldstones in the IR

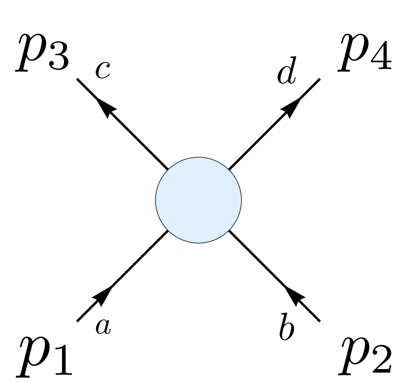
Derivative interaction $\Longrightarrow$ absence of IR divergences

$$\mathscr{L} = \frac{1}{4} f_\pi^2 \mathbf{tr}(\partial_\mu U^\dagger \partial^\mu U) + \ell_1 [\mathbf{tr}(\partial_\mu U^\dagger \partial^\mu U)]^2 + \ell_2 \mathbf{tr}(\partial_\mu U^\dagger \partial_\nu U)\mathbf{tr}(\partial^\mu U^\dagger \partial^\nu U) + \ldots$$

Universal Leading Order

Wilson coefficients: theory dependent

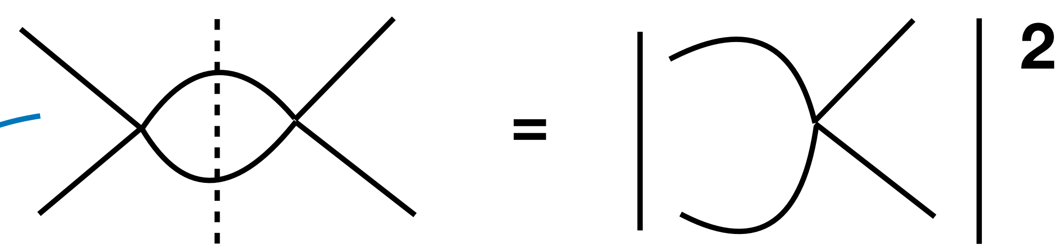
$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_1 - p_4)^2$$



$$= \frac{s}{f^2}\delta_{ab}\delta^{cd} + \frac{t}{f^2}\delta_a^c\delta_b^d + \frac{u}{f^2}\delta_a^d\delta_b^c$$

Observables to bound

Universal Leading Order $A(s\,|\,t,u)^{(1)} = \frac{s}{f^2} + \mathcal{O}(s^2)$



$$A(s\,|\,t,u)^{(2)} = \alpha\frac{s^2}{f^4} + \beta\frac{t^2+u^2}{f^4} - \frac{N-2}{32\pi^2}\frac{s^2}{f^4}\log\frac{-s}{f^2} - \frac{t-u}{96\pi^2 f^4}\left(t\log\frac{-t}{f^2} - u\log\frac{-u}{f^2}\right)$$

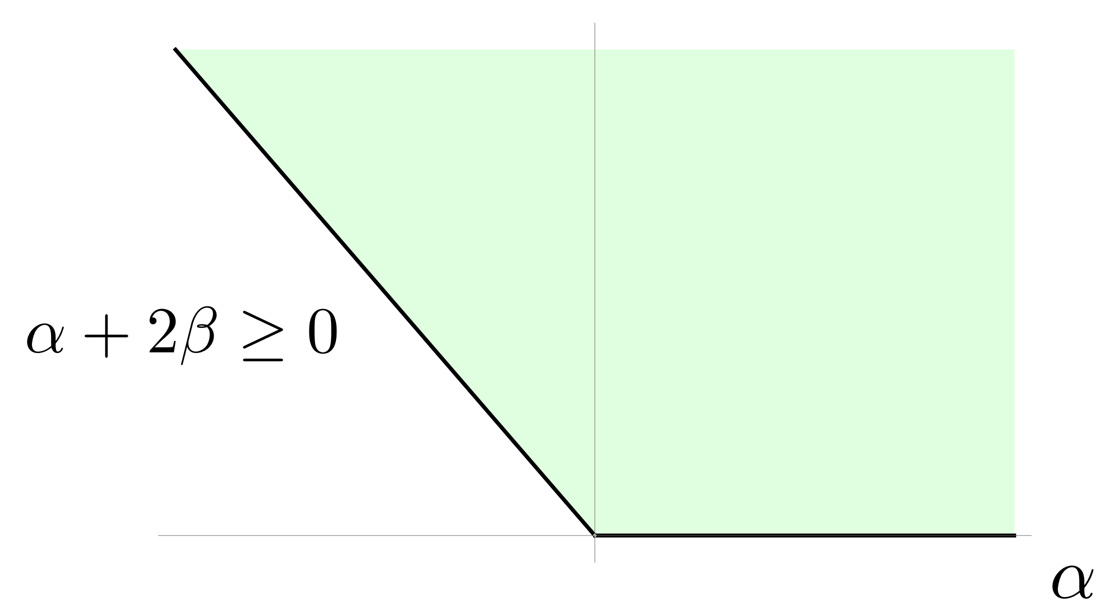
Theory dependent

Completely fixed by elastic unitarity saturation

Positivity Bounds

$$\text{Im}\, T(s, \theta = 0) > 0$$

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06



S-matrix Bootstrap algorithm

1) We write an **Ansatz** analytic and crossing symmetric

2) We match the low energy expansion of the ansatz with the EFT

3) We impose unitarity numerically

4) We look for the allowed region in the Wilson coefficient space

$$\{\alpha, \beta\}$$

Crossing and analyticity

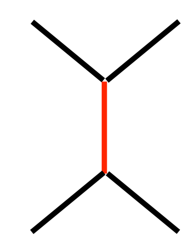
Crossing symmetry is trivial

$$T(s,t,u)_{ab}^{cd} = A(s\,|\,t,u)\delta_{ab}\delta^{cd} + A(t\,|\,s,u)\delta_a^c\delta_b^d + A(u\,|\,s,t)\delta_a^d\delta_b^c$$

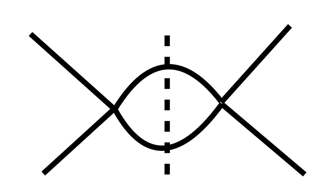
$$A(s|t,u) = A(s|u,t)$$

Analyticity: we assume Maximal analyticity

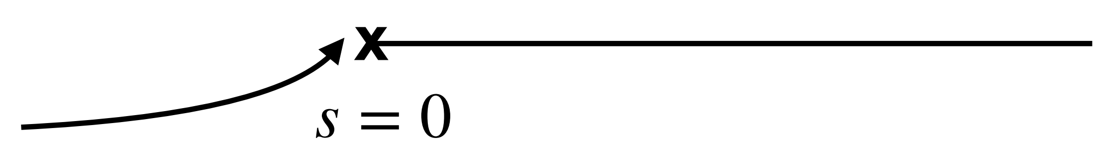
Only massless stable particles with even interactions →
no bound state poles!



Unitarity → discontinuity for s, t, u > 0



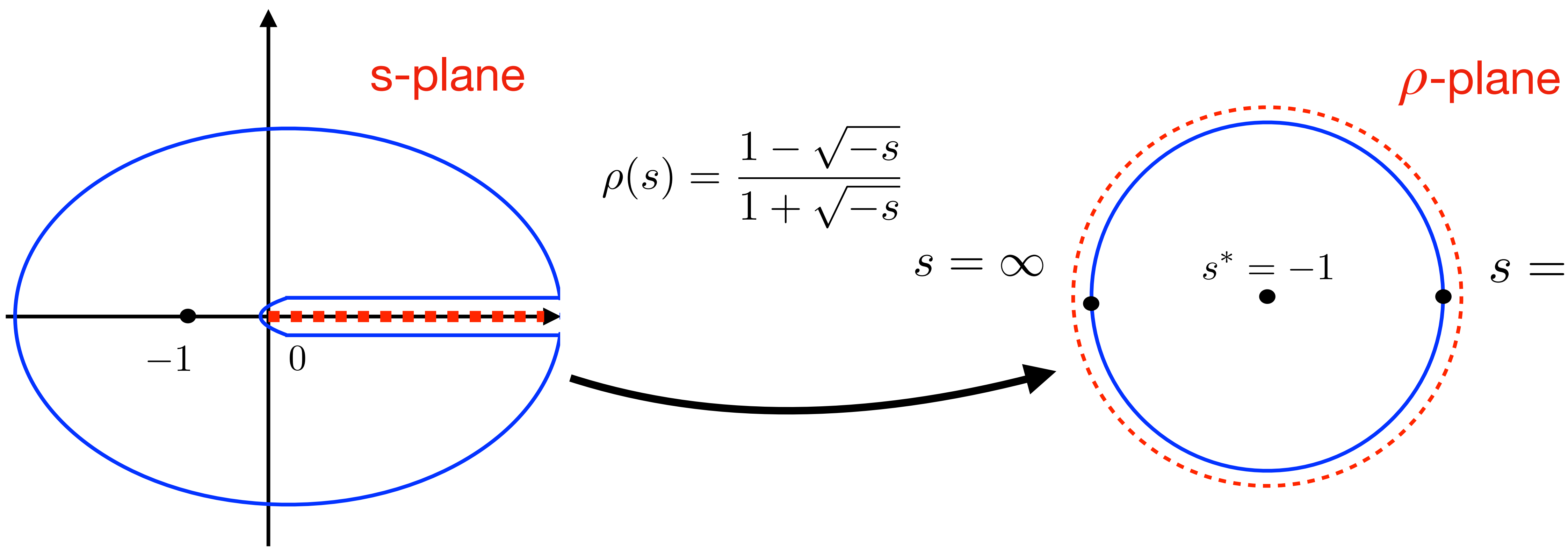
All normal thresholds collapse
at $s = 0$



The Ansatz

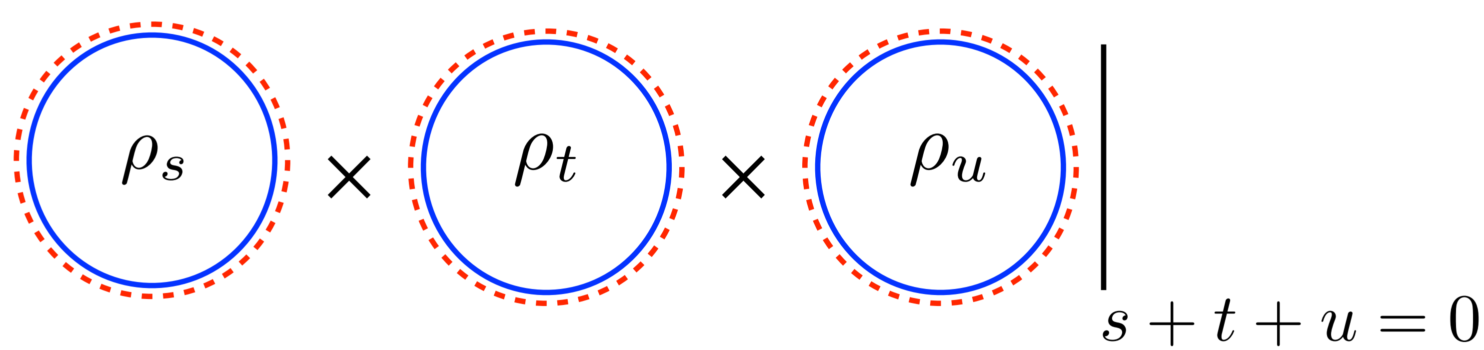
$\{\alpha, \beta\}$

$$A(s|t,u) = \texttt{low energy} + {\sum}' c_{ab}\rho(s)^a(\rho(t)^b + \rho(u)^b) + {\sum}' d_{ab}(\rho(t)^a\rho(u)^b + \rho(t)^b\rho(u)^a)$$



Analytic Extension $\{s, t, u\} \in \mathbb{C}^3/\text{cuts}$

We Taylor expand in a bigger space



Unitarity

Let's look at s-channel unitarity: project onto irreps of flavor and spin

Singlet $$A^{(0)}(s,t,u) = N_f A(s|t,u) + A(t|s,u) + A(u|s,t)$$

Antisymmetric $$A^{(1)}(s,t,u) = A(t|s,u) - A(u|s,t)$$

Symmetric traceless $$A^{(2)}(s,t,u) = A(t|s,u) + A(u|s,t)$$

Partial wave projections: a linear operation on the ansatz

d=4 $$\boxed{S_\ell^{(I)} = 1 + \frac{i}{64\pi}\int_{-1}^{1} P_\ell(x) A^{(I)}(s,x)} \qquad x = \cos\theta$$

$$|S_\ell^{(I)}(s)|^2 \leq 1 \text{ for } I = 0,1,2,\ \ell = 0,1,\ldots,\infty,\ s > 0$$

Set of infinite quadratic constraints

Numerical optimization problem

$$\min \beta, \qquad \text{with} \quad \alpha = \alpha^*$$

Over the space of crossing symmetric and analytic functions of s, t, u

$$|S_\ell^{(I)}(s)|^2 \leq 1, \qquad s > 0, \qquad \ell = 0, \dots, \infty$$

$A \supset \alpha s^2 + \beta(t^2 + u^2)$

Discretize unitarity on a grid of points

M_{max}

$$\ell = 0, \dots, L_{\text{max}}$$

To do Numerics we need to introduce some non-rigorous cutoffs!

For each N_{max} we want to have M_{max} and L_{max} very large!!

$$M_{\text{max}} = 200 \qquad L_{\text{max}} = 90 \qquad N_{\text{max}} = 12, \dots, 23$$

400 variables, 18x10^3 quadratic constraints, ~ 7 h per point on 40 cores for N_max=23

Bootstrap Summary

1) Crossing Symmetric Ansatz

$$T(s,t,u)_{ab}^{cd} = A(s\,|\,t,u)\delta_{ab}\delta^{cd} + A(t\,|\,s,u)\delta_a^c\delta_b^d + A(u\,|\,s,t)\delta_a^d\delta_b^c$$

1) Mandelstam Analyticity + Real Analyticity

$$A(s|t,u) = \sum_{n\leq m}^{\infty} a_{nm}\left(\rho_t^n\rho_u^m + \rho_t^m\rho_u^n\right) + \sum_{n,m}^{\infty} b_{nm}\left(\rho_t^n + \rho_u^n\right)\rho_s^m \qquad \rho(s) = \frac{1-\sqrt{-s}}{1+\sqrt{-s}}$$

2) Impose Low Energy Behavior

$$A(s\,|\,t,u)^{(2)} = \alpha\frac{s^2}{f^4} + \beta\frac{t^2+u^2}{f^4} - \frac{N-2}{32\pi^2}\frac{s^2}{f^4}\log\frac{-s}{f^2} - \frac{t-u}{96\pi^2 f^4}\left(t\log\frac{-t}{f^2} - u\log\frac{-u}{f^2}\right)$$

3) Check Unitarity Numerically

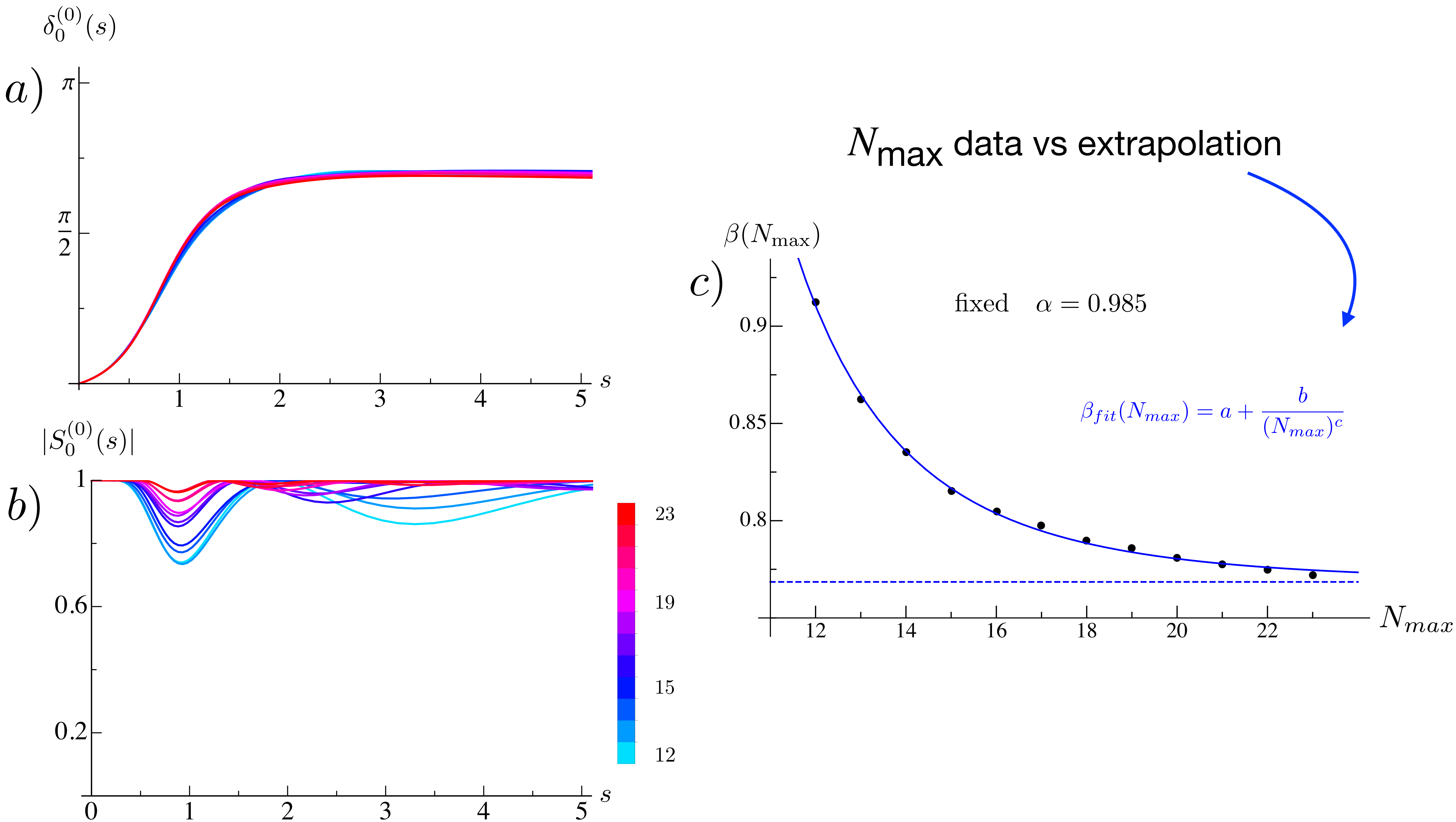
$$S_\ell^{(I)}(s) = 1 + i\int_{-1}^{1} P_\ell(x)A^{(I)}(s,x)dx \qquad \boxed{|S_\ell^{(I)}(s)|^2 \leq 1}$$

4) Solve the optimization problem $\{\alpha, \beta\}$

Min β for fixed α: numerics

For each α, we fix N_{max} and take L_{max} very large

We increase N_{max} until convergence

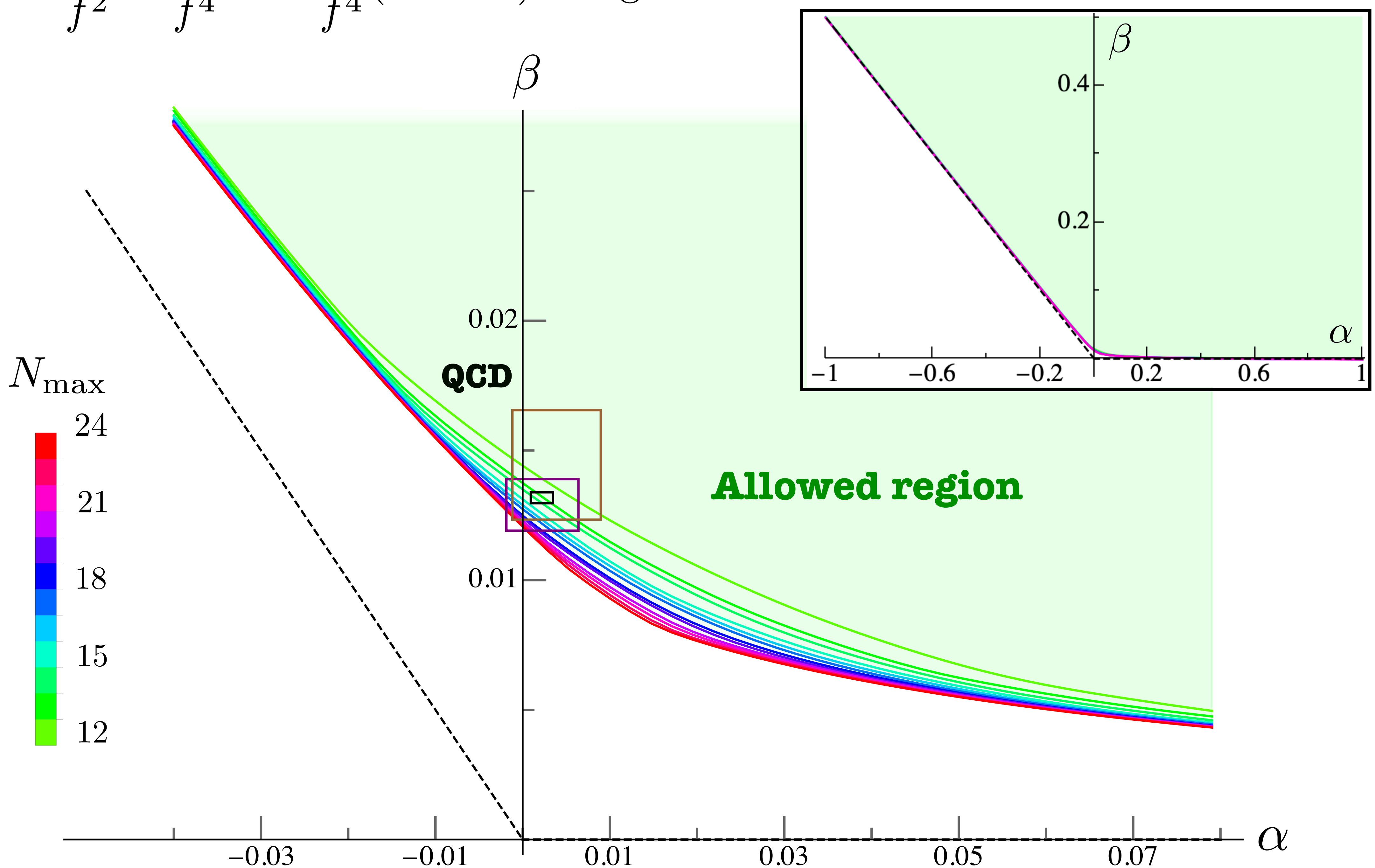


Allowed (α, β) space

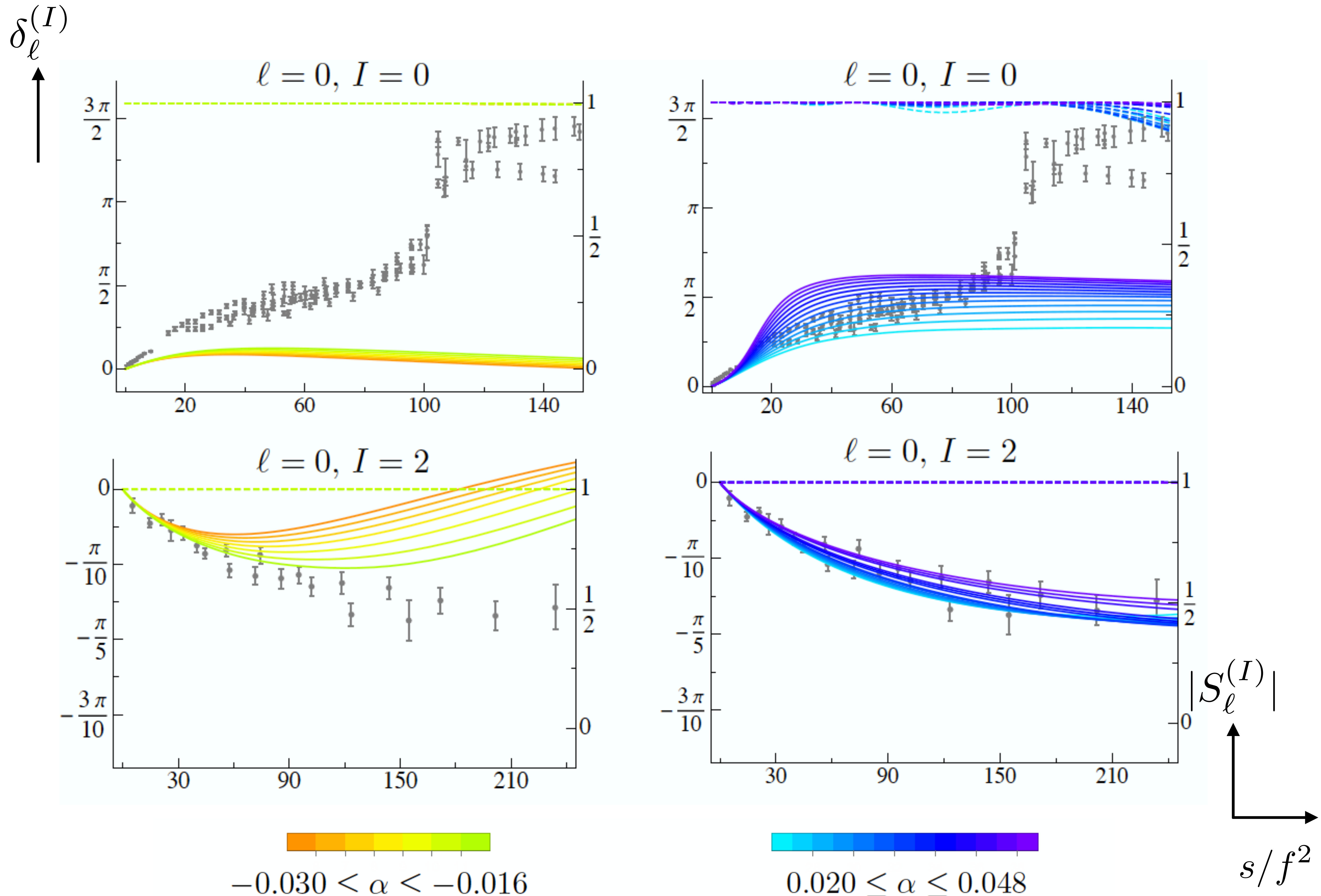
Amplitude = low energy expansion + UV ignorance

Causal and Unitary

$$A(s|t,u) = \frac{s}{f^2} + \frac{\alpha}{f^4}s^2 + \frac{\beta}{f^4}(t^2+u^2) + \text{logarithms} + \dots$$

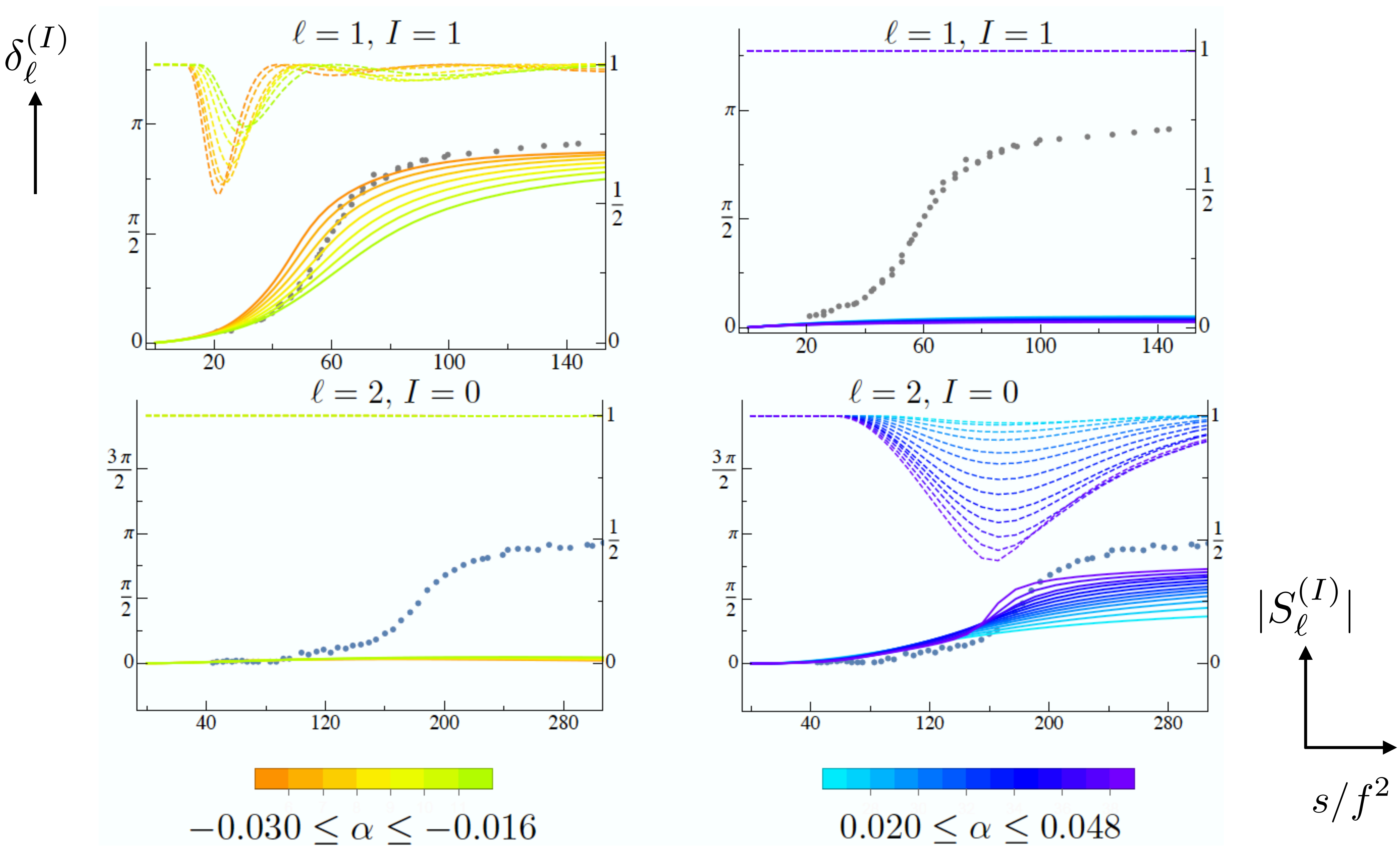


Phase shifts along the boundary: Spin=0

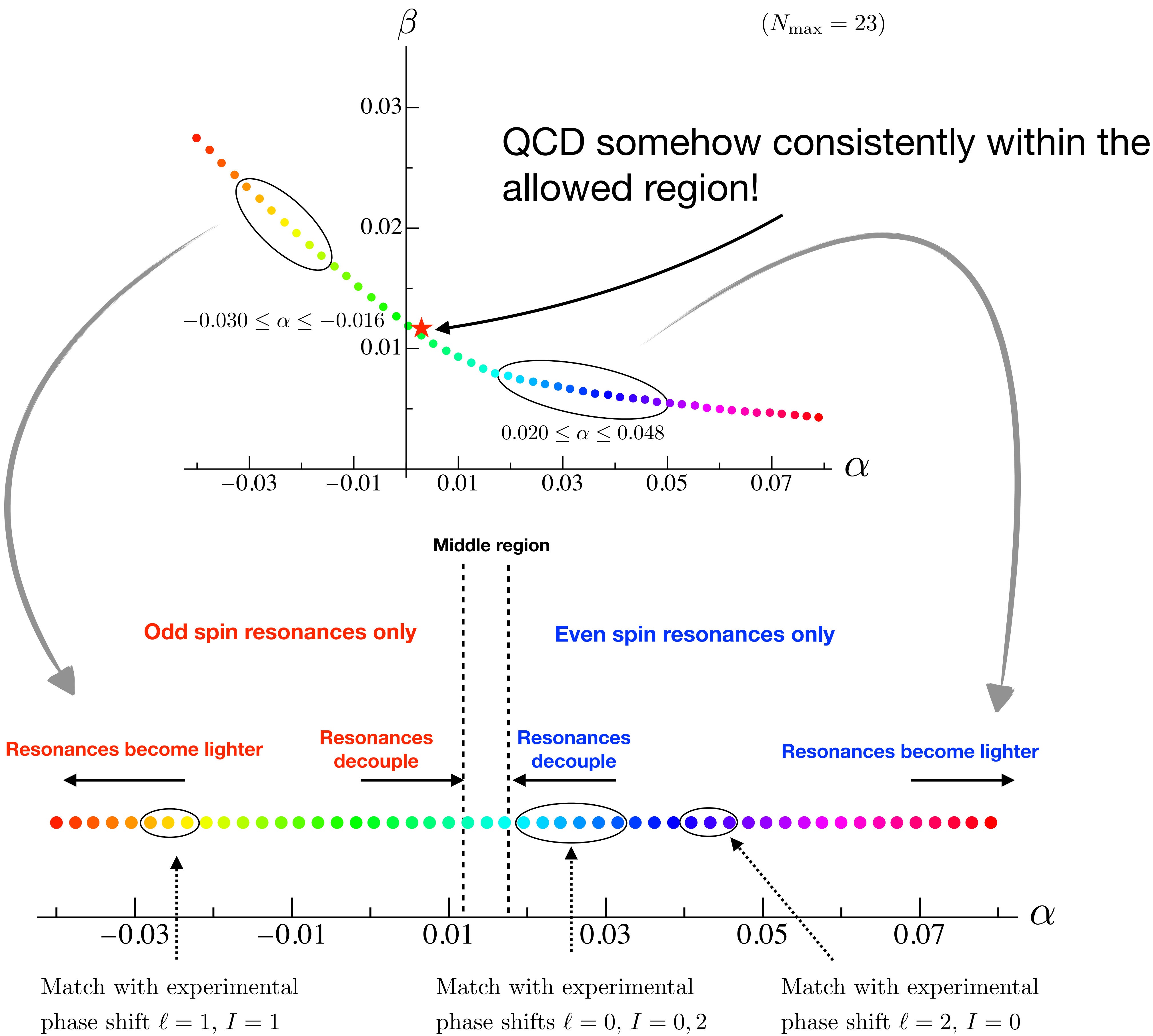


Higher spins

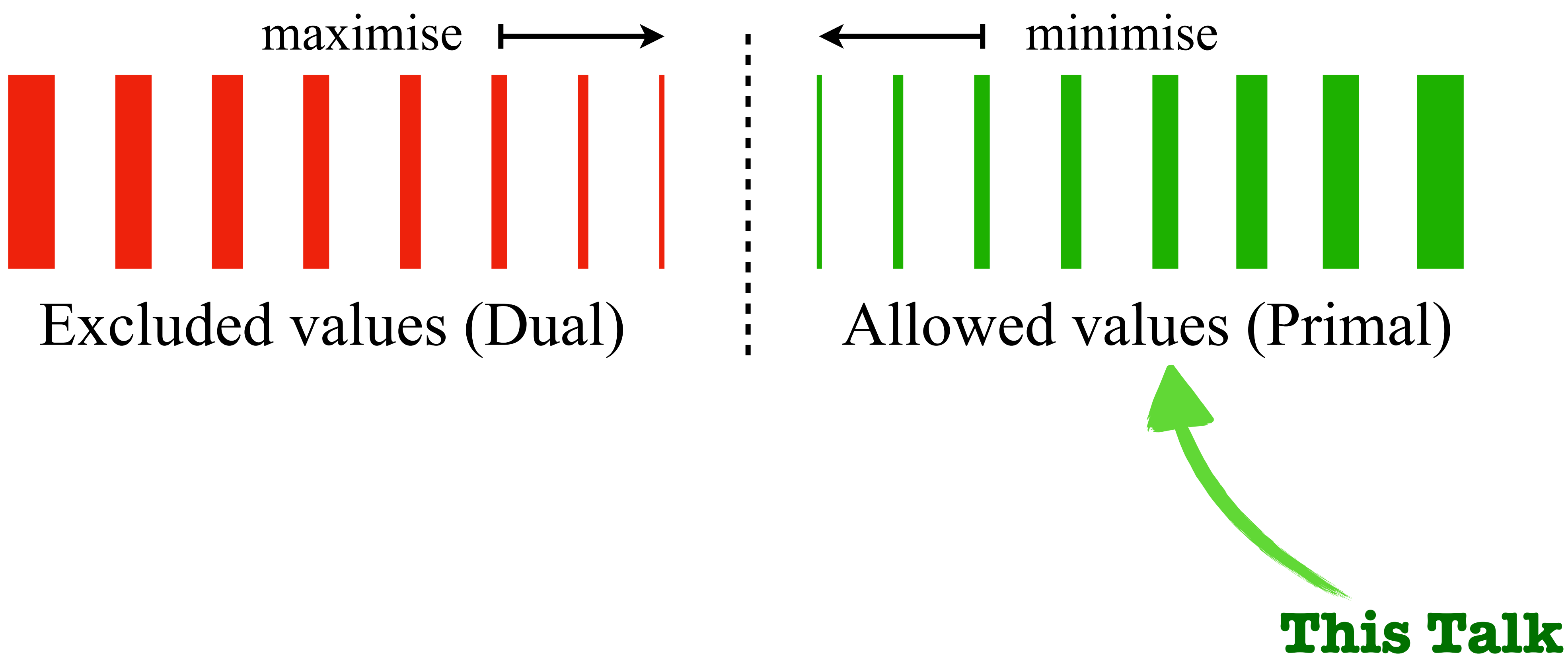
Clear Spin 1, 2! Evidence for Spin 3, 4.
For higher spins we would need an insane amount of time to converge



Walking along the boundary



Outlook and work in progress 1



Gapless EFTs in 1+1 dimensions

Elias-Miro', ALG: arXiv: 2106.07957

Rigorous bounds in gapped QFTs

ALG, Sever: arXiv: 2106.10257

Rigorous bounds in gapped EFTs:
Positivity vs Bootstrap

Elias-Miro', ALG: work in progress

Outlook and work in progress 2

Other EFTs: Maximal SUGRA in 10 D

Maximal SUGRA in 11 D and M-theory

ALG, Penedones, Vieira: arXiv: 2106.10257
ALG, Penedones, Vieira: work in progress

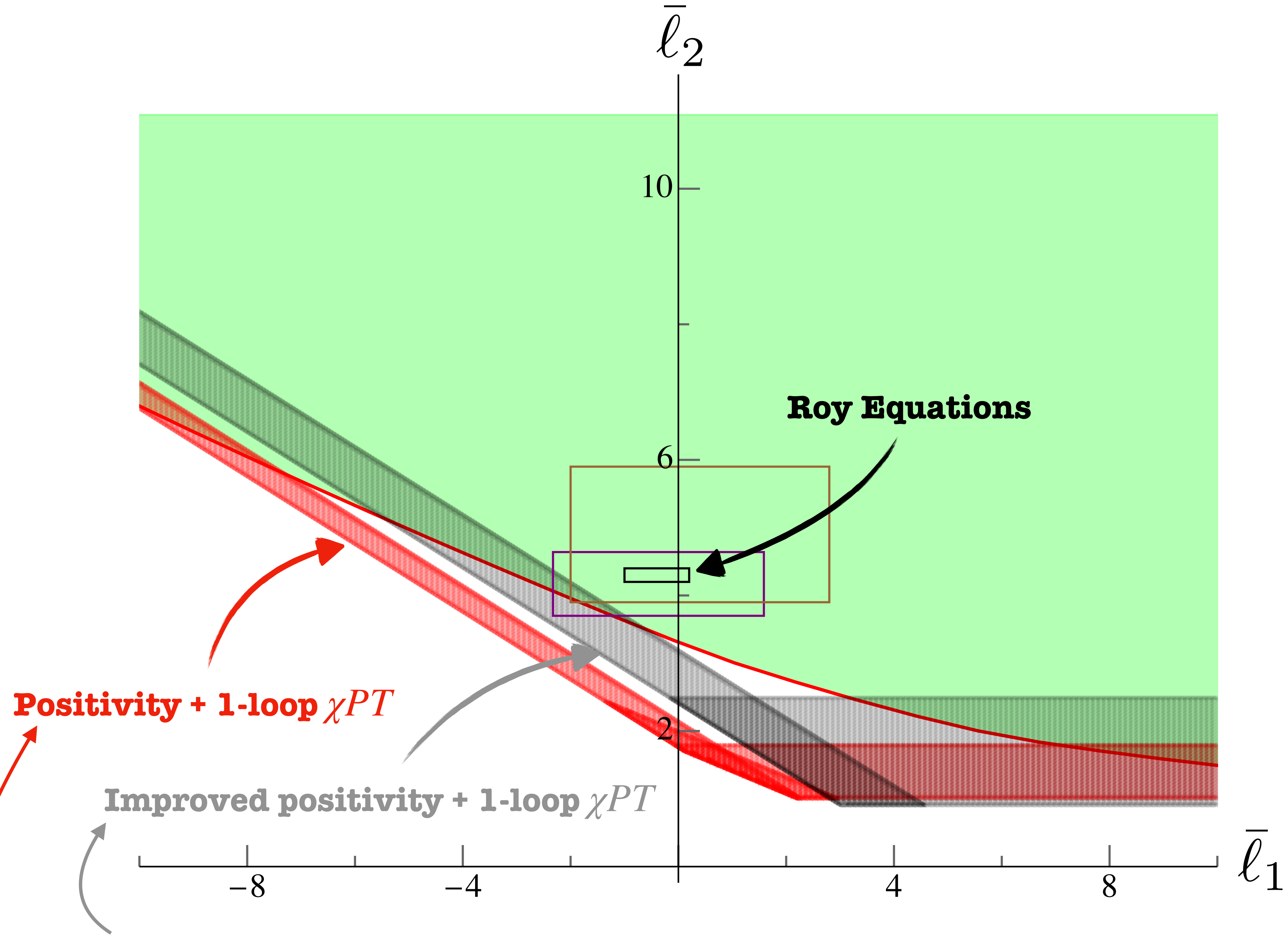
Fit experimental data using the S-matrix Bootstrap?

ALG, Penedones, Vieira: work in progress

Will we ever go beyond 2—>2? Including 2->3 processes in massless EFTs

Fantastic challenge

Comparison with positivity bounds



Wang, Guo, Zhang, Zhou: arXiv: 2004.03992

Manohar, Mateu: arXiv: 0801.3222

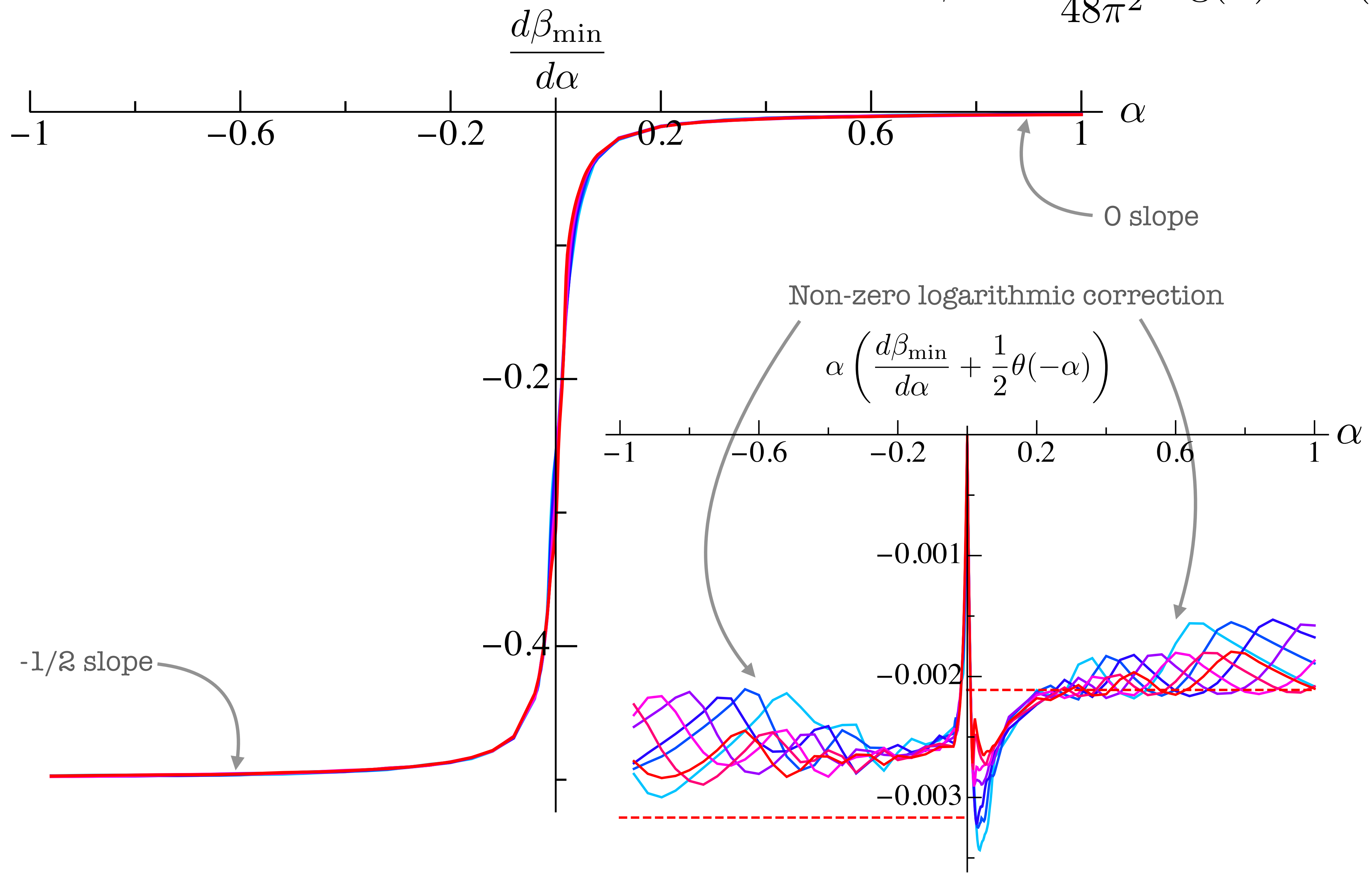
Log violations of positivity

There are asymptotic logarithmic corrections!

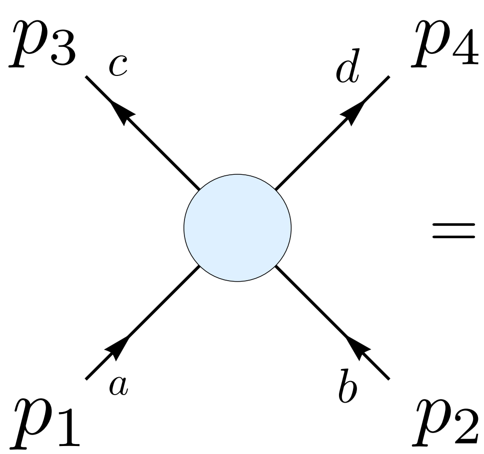
$$\alpha + 2\beta \geq -\frac{1}{16\pi^2}\log(-\alpha) + \mathcal{O}(1)$$

Consequence of the unitarity log's

$$\beta \geq -\frac{1}{48\pi^2}\log(\alpha) + \mathcal{O}(1)$$



Pseudo-goldstones scattering



$A(s|t,u)$

$$= \frac{s-m^2}{f^2}\delta_{ab}\delta^{cd} + \frac{t-m^2}{f^2}\delta_a^c\delta_b^d + \frac{u-m^2}{f^2}\delta_b^c\delta_a^d + \mathcal{O}(p^4, m^4, m^2p^2)$$

Tree-level amplitude:

$\ell = 0,\ I = 0$	$\mathcal{T}_0^{(0)} = \frac{2s-m^2}{32\pi f^2}$	$s_0 = \frac{1}{2}m^2$
$\ell = 0,\ I = 2$	$\mathcal{T}_0^{(2)} = \frac{2m^2-s}{16\pi f^2}$	$s_2 = 2m^2$
$\ell = 1,\ I = 1$	$\mathcal{T}_1^{(1)} = \frac{s-4m^2}{96\pi f^2}$	$s_1 = 4m^2$

They will receive Higher loop corrections

Chiral: zeros in $\mathcal{T}_\ell^{(I)}(s_I) = 0$

Weak Coupling Condition

Low energy data and bootstrap problem

Expansion at threshold: s=4

$$\mathrm{Re}[\mathcal{T}_\ell^{(I)}] = k^{2\ell}(a_\ell^{(I)} + k^2 b_\ell^{(I)} + \mathcal{O}(k^4))$$

Scattering Lengths

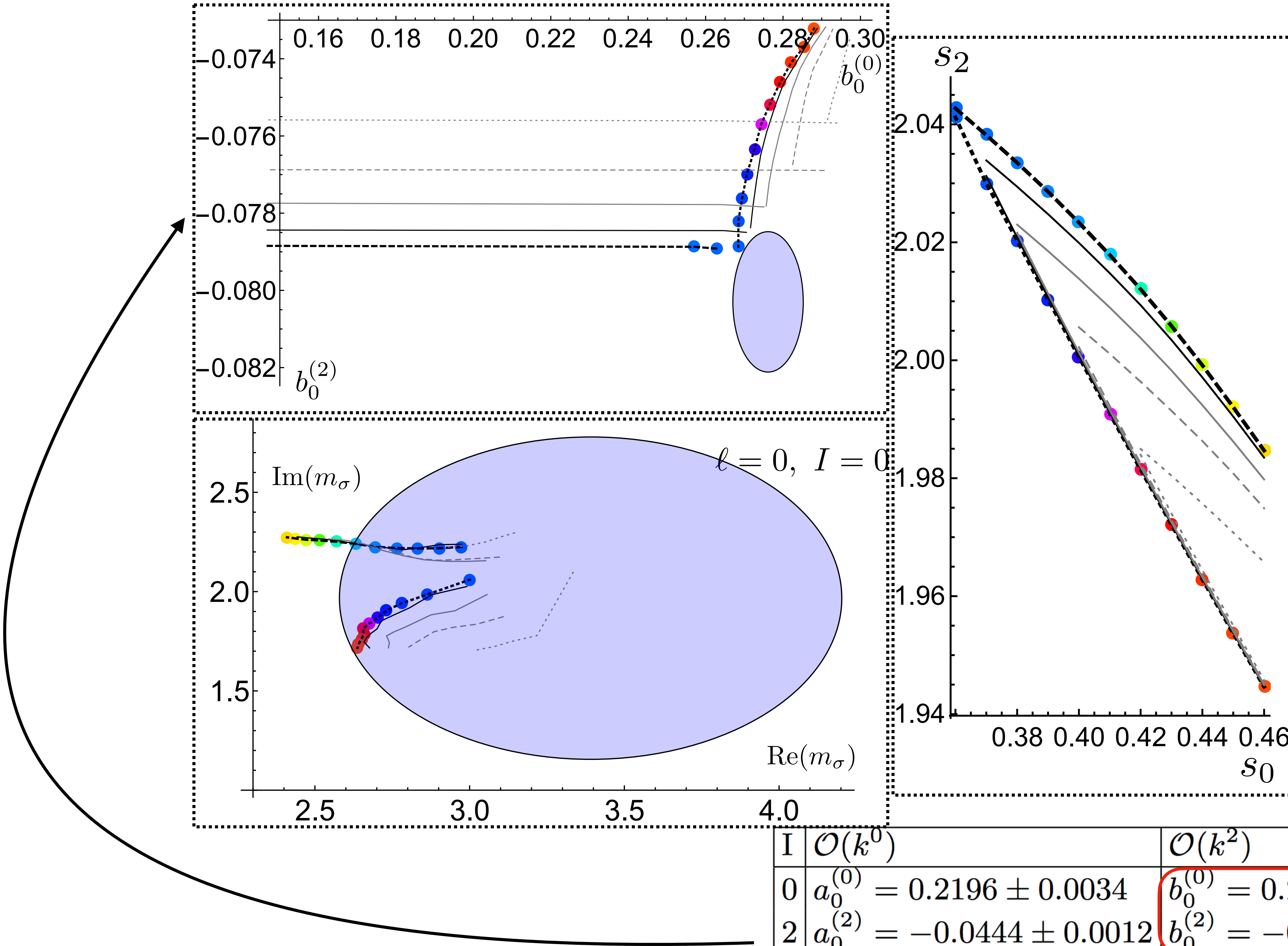
Effective ranges (scattering slopes)

Example: QCD values

I	$\mathcal{O}(k^0)$	$\mathcal{O}(k^2)$
0	$a_0^{(0)} = 0.2196 \pm 0.0034$	$b_0^{(0)} = 0.276 \pm 0.006$
2	$a_0^{(2)} = -0.0444 \pm 0.0012$	$b_0^{(2)} = -0.0803 \pm 0.0012$
1		$a_1^{(1)} = 0.038 \pm 0.002$

Bootstrap problem:
Determine the space of chiral zeros,
imposing the experimental values of the scattering lengths

The shore of the peninsula



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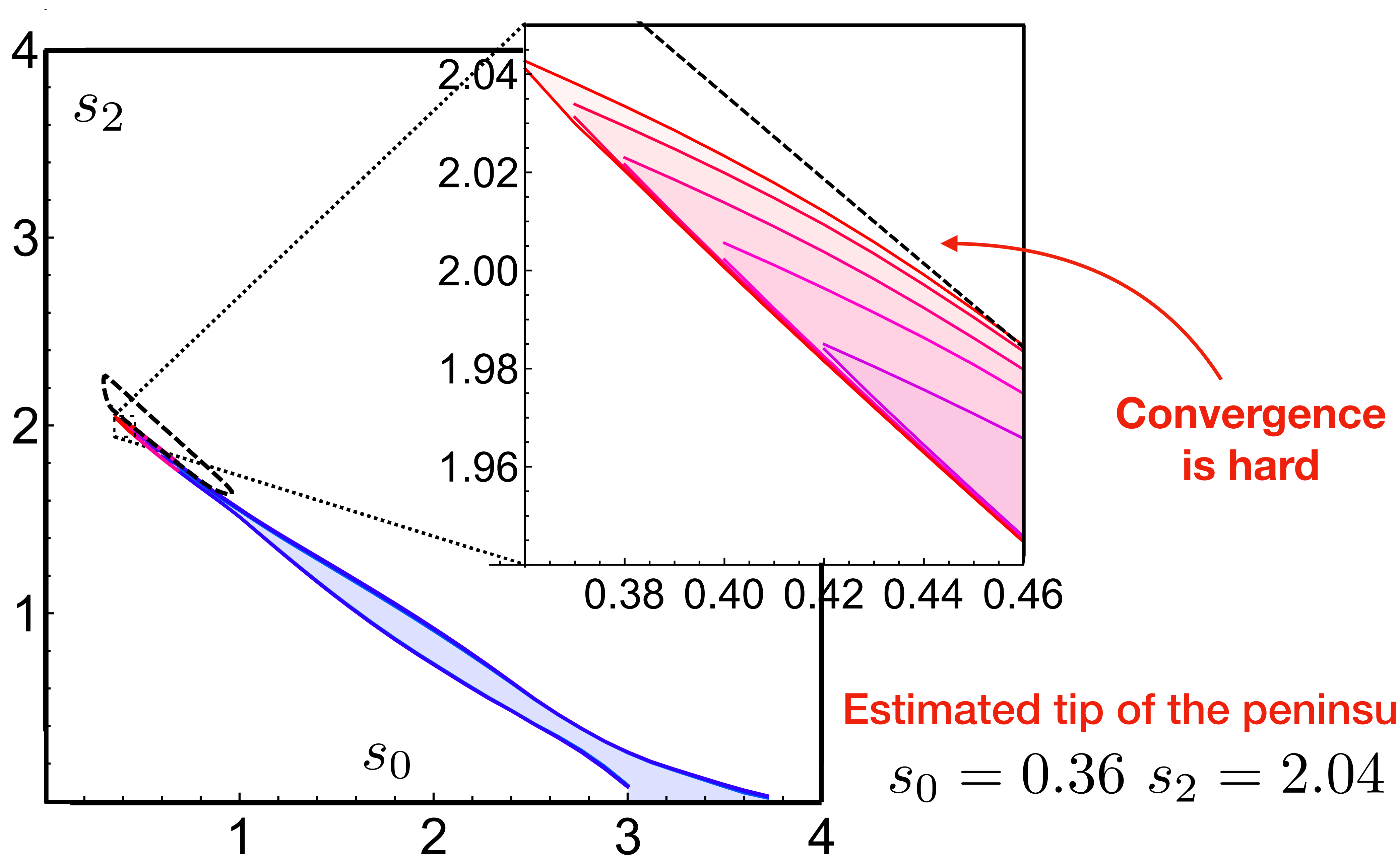
Pion peninsula

We also impose the ρ resonance at $s_\rho = 30 + 6i$
+ experimental value of scattering lengths

1810.12849 ALG, Penedones, Vieira



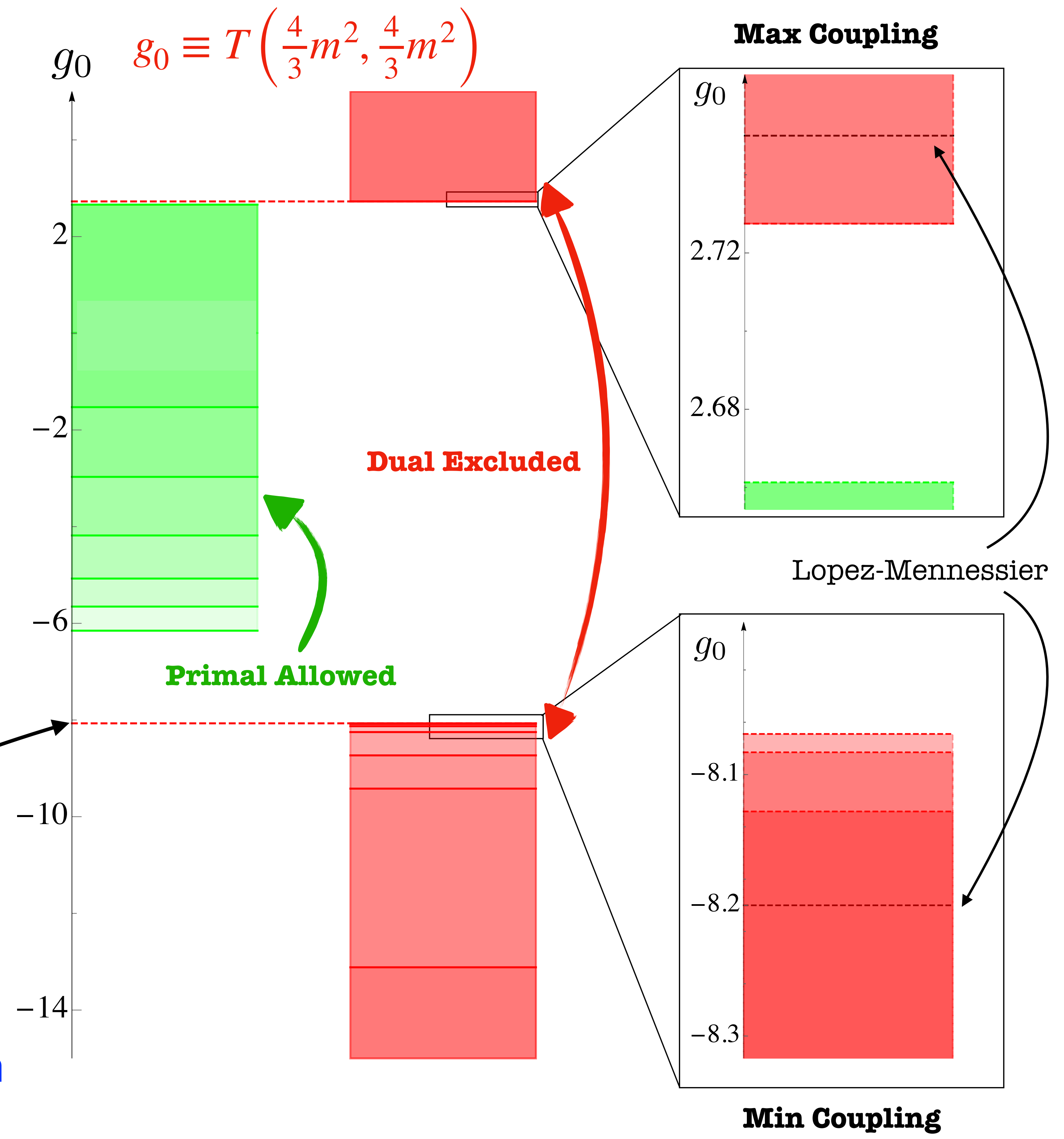
Estimated tip of the peninsula
$s_0 = 0.36 \; s_2 = 2.04$

Rigorous bounds: quartic coupling in 4 dimensions

Numerical framework to derive exclusion bounds!

We use only the proven analyticity properties and unitarity in the elastic region

2106.10257 ALG, Sever



Theory saturating the Froissart bound?

1504.01328 Nastase, Sonnenschein