Gravitational form factors of proton with light-front quark-diquark model

Sreeraj Nair

中国科学院近代物理研究所
Institute of Modern Physics, Chinese Academy of Sciences

In collaboration with D. Chakrabarti (IITK, India), C. Mondal (IMP, China), A. Mukherjee (IITB, India) and X. Zhao (IMP, China)

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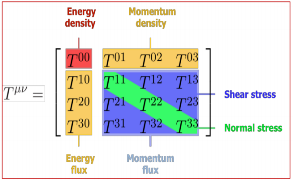
Nucleon gravitational form factors

Nucleon scattering by the classical gravitational field is described by the gravitational (energy momentum tensor) form factors (GFFs).

$$\langle P'|T_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B_i(q^2)\frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu}\Big] U$$

- Matrix elements of the energy momentum tensor (EMT) contain fundamental information about various mechanical properties.



The D-term ($D = 4C$)

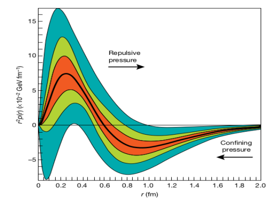
First measurement of the pressure distribution experienced by the quarks in the proton

Letter | Published: 16 May 2018

The pressure distribution inside the proton

V. D. Burkert, L. Elouadrhiri & F. X. Girod

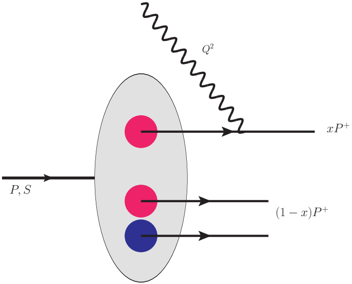
Nature **557**, 396–399(2018) | Cite this article



"The average peak pressure near the center is about 10^{35} pascals which is about 10 times greater than the pressure in the heart of a neutron star".

- $\sum_a A_a(0) = 1$ $\quad\quad \sum_a B_a(0) = 0$ $\quad\quad \sum_a \bar{C}_a(0) = 0$
- $D(0)$ is not constrained by general principles.
- $D(q^2)$ is related to the stress tensor and internal forces.

Light-Front Quark-Diquark Model (LFQDM)



- $p = \mid u(ud)\rangle + \mid d(uu)\rangle$
- a single quark and a scalar diquark state
- Light front wave functions are constructed from the *AdS/QCD soft wall model WFs*.
- The light-front wave functions $\implies \psi^{\lambda_N}_{\lambda_q q}(x, \mathbf{p}_\perp)$
  $\lambda_N \rightarrow$ nucleon helicity and $\lambda_q \rightarrow$ struck quark helicity

Light-front quark-diquark Model(LFQDM)

- The light-front wave functions:

$$\begin{aligned}
\psi^{+}_{+q}(x,\mathbf{p}_\perp) &= \varphi^{(1)}_q(x,\mathbf{p}_\perp),\\
\psi^{+}_{-q}(x,\mathbf{p}_\perp) &= -\frac{p^1+ip^2}{xM}\varphi^{(2)}_q(x,\mathbf{p}_\perp),\\
\psi^{-}_{+q}(x,\mathbf{p}_\perp) &= \frac{p^1-ip^2}{xM}\varphi^{(2)}_q(x,\mathbf{p}_\perp),\\
\psi^{-}_{-q}(x,\mathbf{p}_\perp) &= \varphi^{(1)}_q(x,\mathbf{p}_\perp),
\end{aligned}$$

—Gutsche et. al. PRD 89 (2014)

- Modified soft-wall AdS/QCD wave function for two particle bound state:

$$\varphi^{(\nu)}_i(x,\mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\,x^{a_i^\nu}(1-x)^{b_i^\nu}\exp\left[-\frac{\mathbf{p}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

with the AdS/QCD scale parameter $\kappa = 0.4\ GeV$

—G. F. de Teramond and S. J. Brodsky, arXiv:1203.4025 [hep-ph].

D. Chakrabarti and CM, Eur. Phys. J. C **73**, 2671 (2013).

Nucleon gravitational form factors in LFQDM

$T^{\mu\nu}$ of a free quark inside the proton

$$T^{\mu\nu} = \frac{i}{2}[\bar{\psi}\gamma^{\mu}(\overrightarrow{\partial}^{\nu}\psi) - \bar{\psi}\gamma^{\mu}\overleftarrow{\partial}^{\nu}\psi]$$

$$\langle P', S'|T^{\mu\nu}(0)|P, S\rangle$$

μ	ν	S	S'	GFF
$+$	$+$	$\uparrow$	$\uparrow$	A
$+$	$+$	$\uparrow$	$\downarrow$	B
$-$	$\perp$	$\uparrow$	$\downarrow$	A, B, C
$+$	$-$	$\uparrow$	$\downarrow$	$A, B, C, \bar{C}$

- $A(Q^2)$ and $B(Q^2)$ are obtained from the $(++)$ component.
- $C(Q^2)$ and $\bar{C}(Q^2)$ are extracted from the $(-\perp)$ and $(+-)$ components.

Final analytic results

GFFs in terms of overlap integrals

- $A^q(Q^2) = \mathcal{I}_1^q(Q^2)$
- $B^q(Q^2) - = \mathcal{I}_2^q(Q^2)$
- $C^q(Q^2) = -\frac{1}{4Q^2}\left[2M^2\mathcal{I}_1^q(Q^2) - Q^2\mathcal{I}_2^q(Q^2) - \mathcal{I}_3^q(Q^2)\right]$
- $\bar{C}^q(Q^2) = -\frac{1}{4M^2}\left[\mathcal{I}_3^q(Q^2) - \mathcal{I}_4^q(Q^2)\right]$

where

$$\mathcal{I}_1^q(Q^2) = \int dxx\Big[{N_1}^2x^{2a_1}(1-x)^{2b_1+1} + {N_2}^2x^{2a_2-2}(1-x)^{2b_2+3}\frac{1}{M^2}\left(\frac{\kappa^2}{\log(1/x)} - \frac{Q^2}{4}\right)\Big]g(x)$$

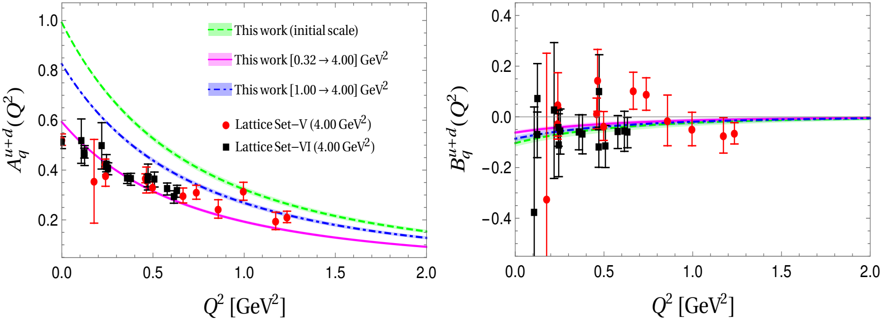
$$\mathcal{I}_2^q(Q^2) = \ 2\int dx\, N_1N_2x^{a_1+a_2}(1-x)^{b_1+b_2+2}g(x)$$

$$\mathcal{I}_3^q(Q^2) = \ 2\int dxN_2N_1x^{a_1+a_2-2}(1-x)^{b_1+b_2+2} \times \left[\frac{4(1-x)^2\kappa^2}{\log(1/x)} + Q^2(1-x)^2 - 4m^2\right]g(x)$$

$$\mathcal{I}_4^q(Q^2) = -2\int dx\, N_2N_1x^{a_1+a_2-2}(1-x)^{b_1+b_2+2}\left[\frac{\kappa^2(1-x)^2}{\log(1/x)} + \frac{Q^2(1-x)^2}{4} + m^2\right]g(x)$$

$$g(x) = \exp\left[-\frac{\log(1/x)}{\kappa^2}\frac{Q^2}{4}\right]$$

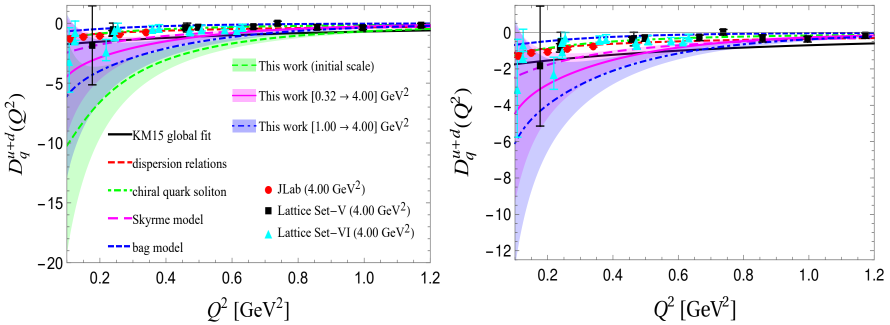
Results : $A(Q^2)$ and $B(Q^2)$



- Evolution done using HOPPET toolkit to compare with lattice results.
- $A(Q^2)$ and $B(Q^2)$ are consistent with the lattice QCD results for the lower initial scale.

—D. Chakrabarti, C. Mondal, A. Mukherjee, SN and X. Zhao, Phys. Rev. D **102**, 113011 (2020).

Results : $D(Q^2) = 4C(Q^2)$

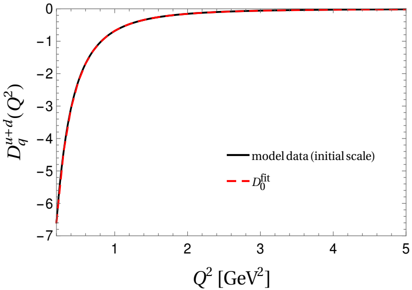


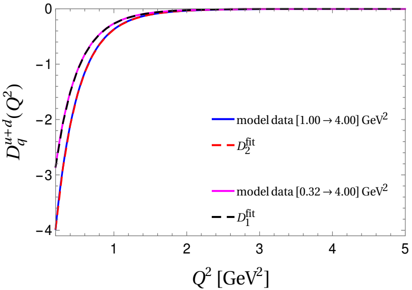
- qualitative behavior of $D(Q^2)$ is compatible with lattice and the experimental data.
- error band $\rightarrow$ 2% uncertainty in model parameter.

—D. Chakrabarti, C. Mondal, A. Mukherjee, SN and X. Zhao, Phys. Rev. D **102**, 113011 (2020).

Fit Function for $D(Q^2)$

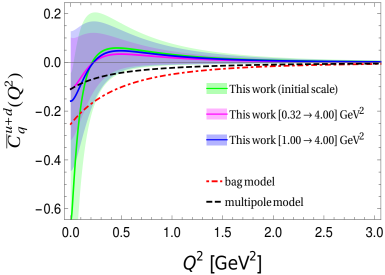
$$D^q_{\rm fit}(Q^2) = 4C^q_{\rm fit}(Q^2) = \frac{a_q}{(1+b_q Q^2)^{c_q}}$$





Parameters	μ^2(GeV^2)	a_q	b_q	c_q
$D_0^{\rm fit}$	initial scale	−18.8359	−2.2823	2.7951
$D_1^{\rm fit}$	[0.32 → 4]	−5.5861	−0.29724	11.6641
$D_2^{\rm fit}$	[1.00 → 4]	−7.77884	0.291081	11.884

Results : $\bar{C}(Q^2)$



Approaches/Models	$\bar{C}_q^{u+d}(0)$
This work ($\sqrt{0.32}$ GeV → 2 GeV)	-0.109
This work (1.00 GeV → 2 GeV)	-0.159
IFF (2 GeV) [1]	-0.11
Asymptotic (∞ GeV) [2]	-0.15
QCDSR-I (1 GeV) [3]	$\times 10^{-2}$
QCDSR-II (1 GeV) [3]	$\times 10^{-2}$
IP [4]	1.4×10^{-2}

- $\bar{C}(Q^2)$ is negative at low Q^2 (< 0.22 GeV^2) like other models.
- distinctly different behavior in the region of $Q^2 > 0.22$ GeV^2, where it exhibits positive distribution.
- The positive distribution decreases with QCD evolution.

[1] C. Lorcé, H. Moutarde and A. P. Trawinski . Eur. Phys. J. C **79**, 89 (2019)

[2] Y. Hatta, A. Rajan and K. Tanaka, JHEP **12**, 008 (2018)
[3] K. Azizi and U. Ozdem, Eur. Phys. J. C **80**, 104 (2020)

[4] M. V. Polyakov and H. D. Son, JHEP **09**, 156 (2018)

Mechanical Properties

Pressure and shear forces inside the nucleon are given by

$$p(b) = \frac{1}{6M_n}\frac{1}{b^2}\frac{d}{db}b^2\frac{d}{db}\tilde{D}(b), \qquad s(b) = -\frac{1}{4M_n}b\frac{d}{db}\frac{1}{b}\frac{d}{db}\tilde{D}(b).$$

Normal and tangential forces: F_n and F_t, given by

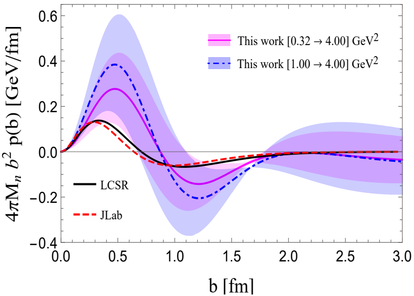
$$\begin{aligned} F_n(b) =& 4\pi M_n b^2\left(p(b) + \frac{2}{3}s(b)\right), \\ F_t(b) =& 4\pi M_n b^2\left(p(b) - \frac{1}{3}s(b)\right). \end{aligned}$$

where

$$\tilde{D}(b) = \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2}\, e^{i\vec{q}_\perp.\vec{b}_\perp} D(Q^2).$$

—I. Anikin, Phys. Rev. D 99, 094026 (2019).

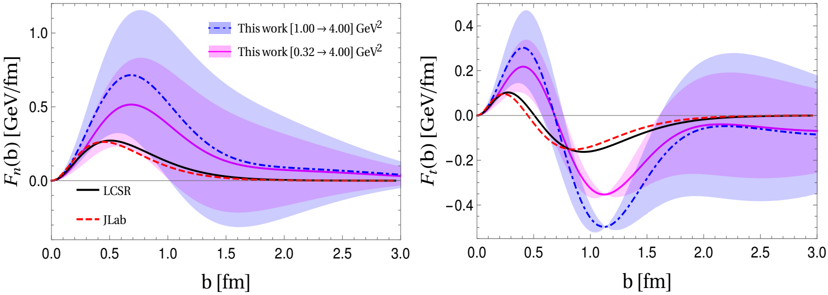
Results : Pressure and Shear in LFQDM



- overall qualitative behavior is in agreement with LCSR [1] and Jlab [2].
- $p(b)$ has a positive core and a negative tail which ensures the mechanical stability of the system.
- $s(b)$ (central line) is positive in all region of b.

[1] I. Anikin, Phys. Rev. D 99, 094026 (2019)

[2] V. Burkert, L. Elouadrhiri and F. Girod, Nature 557, no.7705, 396-399 (2018)



- $F_n(b)$ (central line) is always positive.
- $F_t(b)$ has a positive core (repulsive force) surrounded by a negative tail (attractive force).
- the qualitative behavior is in agreement with LCSR and Jlab.

——D. Chakrabarti, C. Mondal, A. Mukherjee, SN and X. Zhao, Phys. Rev. D **102**, 113011 (2020).

Mechanical properties

Two-dimensional Galilean energy density, radial pressure, tangential pressure, isotropic pressure, and pressure anisotropy

$$\mu_a(b) = M_n\left\{\frac{A_a(b)}{2} + \bar{C}_a(b) + \frac{1}{4M_n^2}\frac{1}{b}\frac{d}{db}\left(b\frac{d}{db}\left[\frac{B_a(b)}{2} - 4C_a(b)\right)\right]\right\}$$

$$\sigma_{r,a}(b) = M_n\left\{-\bar{C}_a(b) + \frac{1}{M_n^2}\frac{1}{b}\frac{dC_a(b)}{db}\right\}$$

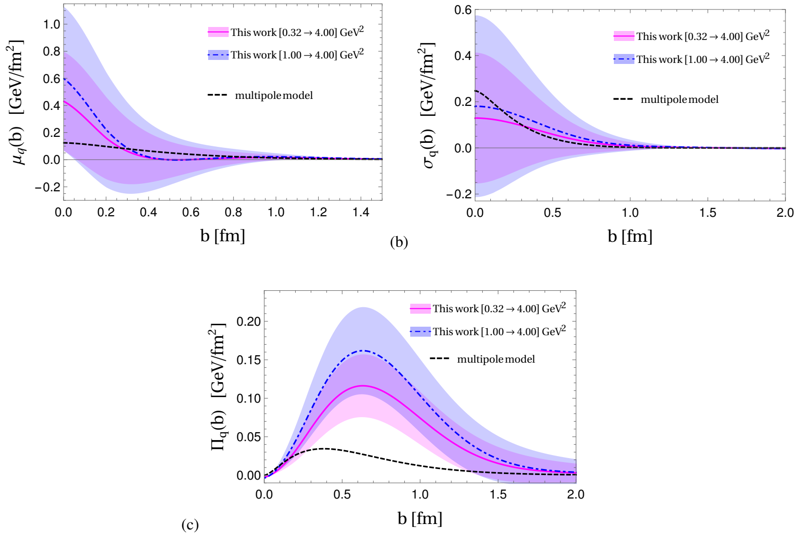
$$\sigma_{t,a}(b) = M_n\left\{-\bar{C}_a(b) + \frac{1}{M_n^2}\frac{d^2C_a(b)}{db^2}\right\}$$

$$\sigma_a(b) = M_n\left\{-\bar{C}_a(b) + \frac{1}{2}\frac{1}{M_n^2}\frac{1}{b}\frac{d}{db}\left(b\frac{dC_a(b)}{db}\right)\right\}$$

$$\Pi_a(b) = M_n\left\{-\frac{1}{M_n^2}b\frac{d}{db}\left(b\frac{dC_a(b)}{db}\right)\right\}$$

C. Lorcé, H. Moutarde and A. P. Trawiń ski, Eur. Phys. J. C 79, no. 1, 89 (2019).

Results: Mechanical properties



Plot of (a) energy density (b) isotropic pressure, and (c) pressure anisotropy

Conclusion

- We calculated the GFFs of the proton in a light-front quark-diquark model.
- $A(Q^2)$ and $B(Q^2)$ are comparable with lattice QCD results.
- We observe that the qualitative nature of $D(Q^2)$ in this model agrees with the JLab data and lattice QCD results.
- The pressure $p(b)$, shear $s(b)$, normal force $F_n(b)$ and tangential force $F_t(b)$ distributions are consistent with the experimental observation and other theoretical predictions.

Conclusion

- We calculated the GFFs of the proton in a light-front quark-diquark model.
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Thank You

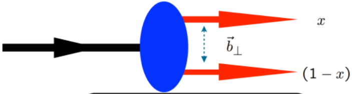
Backup

Light Front Holography

Light-Front QCD

Fixed $\tau = t + z/c$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$



$$\zeta^2 = x(1-x) b_\perp^2$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \, \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

ζ, ϕ

$m_q = 0$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential!

Light Front QCD

- LF Hamiltonian eigenvalue equation

$$H_{LF}|\psi(P)\rangle = \mathcal{M}^2|\psi(P)\rangle,$$

with $H_{LF} = P^\mu P_\mu = P^+P^- - P_\perp^2$, where $P^\pm = P^0 \pm p^3$.

- Hadronic state $|\psi\rangle$ is an expansion in multiparticle Fock eigenstates $|n\rangle : |\psi\rangle = \sum_n \psi_n |n\rangle$.
- Using the frame $P = \left(P^+, \mathcal{M}^2/P^+, \vec{0}_\perp\right)$ where $P^2 = P^+P^-$, we get

$$\begin{aligned}\mathcal{M}^2 &= \sum_n \int [dx_i]\,[d^2k_{\perp i}] \sum_q \left(\frac{k_{\perp q}^2 + m_q^2}{x_q}\right)|\psi_n(x_i, k_{\perp i})|^2 \\ &+ \text{(interactions)},\end{aligned}$$

- the momentum fraction $x_i = k_i^+/P^+$, Momentum conservation requires $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n k_{\perp i} = 0$.

- To simplify the discussion we will consider a two-parton hadronic bound state.
- In the limit of zero quark mass $m_q \to 0$ and in the co-ordinate or impact space

$$\mathcal{M}^2 = \int_0^1 \frac{dx}{x(1-x)} \int d^2 b_\perp \, \psi^*(x, b_\perp) \left(-\nabla^2_{b_\perp}\right) \psi(x, b_\perp) + \text{(interactions)}.$$

- Write : $\zeta^2 = x(1-x) b_\perp^2, \qquad \psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$
- Laplacian : $\nabla^2_\zeta = \frac{1}{\zeta} \frac{d}{d\zeta} \left(\zeta \frac{d}{d\zeta}\right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2},$
- light-front wave equation for ϕ

$$\mathcal{M}^2 \phi(\zeta) = \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right) \phi(\zeta), \qquad L = L_z.$$

- The longitudinal mode is normalized as $\int_0^1 \frac{X^2(x)}{x(1-x)} = 1.$

AdS/CFT $\leftrightharpoons$ Light Front QCD

- AdS/CFT : gravity theory in (d+1)-dim AdS space $[x^M = (x^d, z)]$ is dual to a conformal theory in the d-dim boundary $[x^M = (x^d)]$.
- QCD is not conformal theory!
- As QCD is not the scale invariant theory one needs to break the conformal invariance.
- A boundary condition in the extra dimension z in AdS_{d+1} breaks the conformal invariance and allows QCD mass scale and confinement.

AdS/CFT $\leftrightharpoons$ Light Front QCD

- Confinement in AdS : 1. Hard-wall 2. Soft-wall
- In the hard-wall model, an IR cutoff is set at $\boxed{z_0 = 1/\Lambda_{QCD}}$. The effective potential in a hard wall

$$U(z) = \begin{cases} 0 & \text{if} \quad z \leq \frac{1}{\Lambda_{\rm QCD}}, \\ \infty & \text{if} \quad z > \frac{1}{\Lambda_{\rm QCD}}. \end{cases}$$

- quarks propagate freely in the hadronic interior upto $z_0 = 1/\Lambda_{QCD}$.
- In soft-wall model, no cutoff in z but a confining potential in z like $\boxed{U(z) \sim \kappa^4 z^2}$ is introduced.
- The form of the effective confining potential is unique,

—Brodsky,Teramond, Dosch: Nuovo, Cim. C 036, 265 (2013)

Light Front Holographic mapping

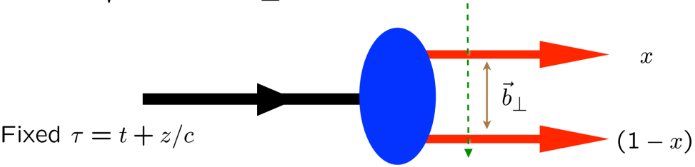
de Teramond

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$\psi(x, \vec{b}_\perp)$ ⟷ $\phi(z)$

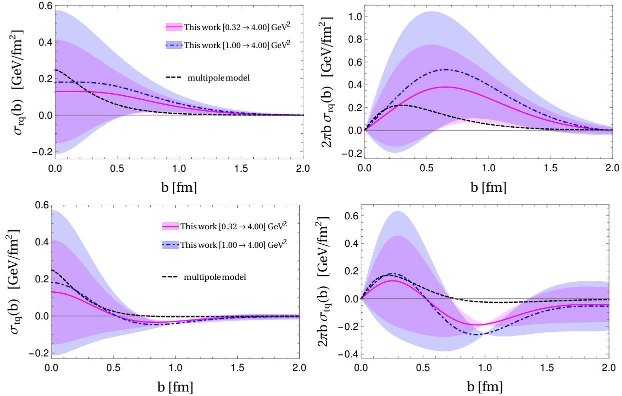
$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$ ⟷ z



$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Results: Mechanical properties



Results: Mechanical properties

The pressure and the energy density in the center of nucleon are directly related to the GFFs

$$p_0 = -\frac{1}{24\pi^2 M_n}\int_0^\infty \mathrm{d}Q^2\; Q^3\; D(Q^2),$$
$$\mathcal{E} = \frac{M_n}{4\pi^2}\int_0^\infty \mathrm{d}Q^2 \left(A(Q^2) + \frac{Q^2}{4M_n^2} D(Q^2)\right),$$

respectively, while the mechanical radius can be obtained by

$$\langle r^2_{\mathrm{mech}}\rangle = 6D_{\mathrm{fit}}(0)\Big[\int_0^\infty \mathrm{d}Q^2\; D(Q^2)\Big]^{-1}.$$

Results: Mechanical properties

The mechanical properties: pressure, energy density, and mechanical radius of nucleon.

Approaches/Models	p_0 [GeV/fm^3]	$\mathcal{E}$ [GeV/fm^3]	$\langle r^2_{\text{mech}} \rangle$ [fm^2]
This work	0.29	3.21	0.74
QCDSR set-I (1 GeV)	0.67	1.76	0.54
QCDSR set-II (1 GeV)	0.62	1.74	0.52
Skyrme model	0.47	2.28	-
modified Skyrme model	0.26	1.445	-
χQSM	0.23	1.70	-
Soliton model	0.58	3.56	-
LCSM-LO	0.84	0.92	0.54

Results

Approaches/Models	$A_q^{u+d}(0)$	$J_q(0) = \frac{1}{2}[A_q^{u+d}(0) + B_q^{u+d}(0)]$	$D_{\rm fit}^{u+d}(0) = 4C_{\rm fit}^{u+d}(0)$	$\bar{C}_q^{u+d}(0)$
This work ($\sqrt{0.32}$ GeV $\to$ 2 GeV)	0.593	0.269	-5.586	-0.109
This work (1.00 GeV $\to$ 2 GeV)	0.825	0.369	-7.778	-0.159
LQCD (2 GeV) [?]	0.675	0.34	-	-
LQCD (2 GeV) [?]	0.547	0.33	-0.80	-
LQCD (2 GeV) [?]	0.553	0.238	-1.02	-
LQCD (2 GeV) [?]	0.520	0.213	-1.07	-
LQCD (2 GeV) [?]	0.572	0.226	-	-
LQCD (2 GeV) [?]	0.565	0.314	-	-
χPT (2 GeV) [?]	0.538	0.24	-1.44	-
IFF (2 GeV) [?]	0.55	0.24	-1.28	-0.11
Asymptotic (∞ GeV) [?]	-	0.18	-	-0.15
QCDSR-I (1 GeV) [?]	0.79	0.36	-1.832	-2.1 $\times 10^{-2}$
QCDSR-II (1 GeV) [?]	0.74	0.30	-1.64	-2.5 $\times 10^{-2}$
Skyrme [?]	1	0.5	-3.584	-
Skyrme [?]	1	0.5	-2.832	-
χQSM [?]	1	0.5	-1.88	-
χQSM [?]	1	0.5	-4.024	
χQSM [?]	-	-	-3.88	-
AdS/QCD Model I [?]	0.917	0.415	-	-
AdS/QCD Model II [?]	0.8742	0.392	-	-
LCSR-LO [?]	-	-	-2.104	-
KM15 fit [?]	-	-	-1.744	-
DR [?]	-	-	-1.36	-
JLab data [?]	-	-	-1.688	-
IP [?]	-	-	-	1.4 $\times$ 10^{-2}