Visualization of internal forces inside the proton in a classical relativistic model

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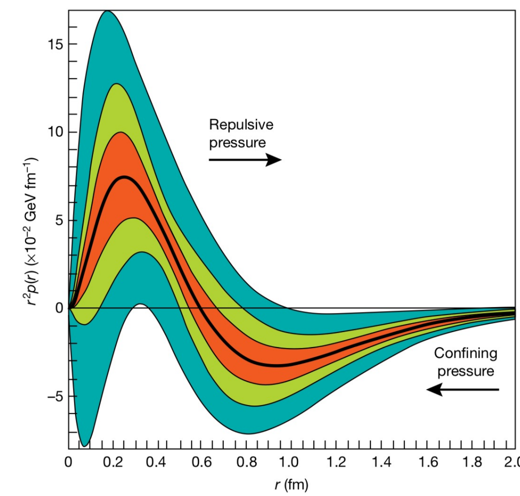
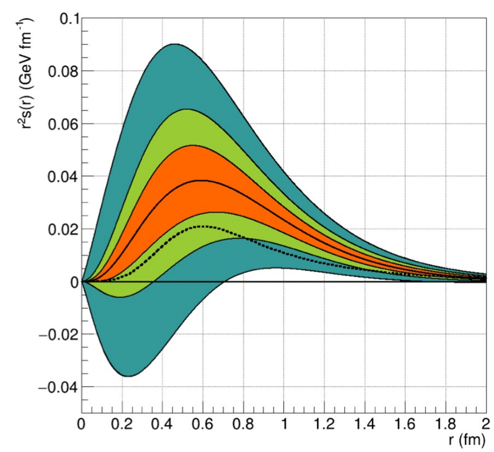
Motivation/Introduction

- Energy Momentum Tensor (EMT) contains information on mass, spin, and D-term
  - D-term describes internal forces inside the nucleon
- First measurement of pressure inside proton at Jefferson Lab, VA (top)
- Later measurement of shear force (bottom)





Sources:
Top graph: Burkert, V.D., Elouadrhiri, L. & Girod, F.X. The pressure distribution inside the proton. *Nature* **557**, 396–399 (2018) doi:10.1038/s41586-018-0060-z

Bottom graph: Burkert, V.D., Elouadrhiri, L. & Girod, F.X. Determination of shear forces inside the proton. arXiv:2104.02031v2 (2021)

Outline

- Introduce the classical model
- Results for s(r) and p(r)
- Review the D-term
- Results for D_{reg}
- Important conclusions/takeaways

The Classical Model

- system is defined by classical field equations ($\hbar$=c=1)

$$
\begin{aligned}
\left[(m - g_S\phi)(\partial_t + \vec{v}\cdot\vec{\nabla}_r) + m\,\vec{F}\cdot\vec{\nabla}_p\right]\Gamma(\vec{r},\vec{p},t) &= 0\,, \\
\partial_\alpha G^{\alpha\beta} + m_V^2 V^\beta &= g_V\, j^\beta, \\
(\Box + m_S^2)\phi &= g_S\,\rho\,, \\
\partial_\alpha F^{\alpha\beta} &= e\, j^\beta.
\end{aligned}
$$

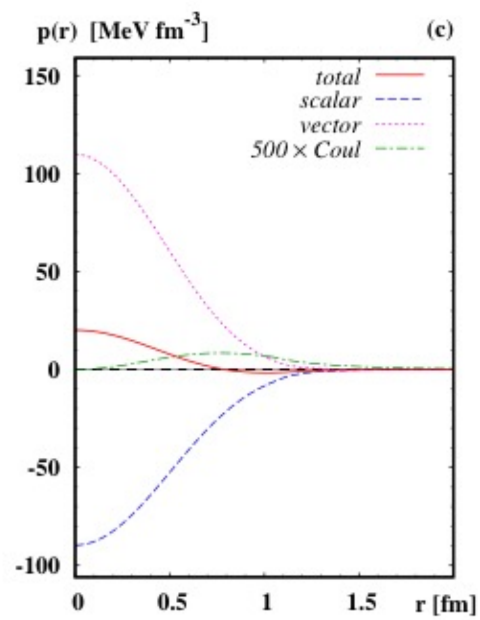
$$
\begin{aligned}
T^{\mu\nu} &= (m - g_S\,\phi)\rho u^\mu u^\nu + F^{\mu\rho}F_\rho{}^\nu + \frac{1}{4}\,g^{\mu\nu}\,F_{\kappa\rho}F^{\kappa\rho} + \partial^\mu\phi\,\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\,\partial^\rho\phi - \frac{1}{2}m_S^2\phi^2\right) \\
&+ G^{\mu\rho}G_\rho{}^\nu + m_V^2 V^\mu V^\nu + g^{\mu\nu}\left(\frac{1}{4}G_{\kappa\rho}G^{\kappa\rho} - \frac{1}{2}\,m_V^2 V_\rho V^\rho\right)
\end{aligned}
$$

Pressure Inside the Proton

- The stress tensor can be decomposed as:

$$T^{ij} = \left(e_r^i e_r^j - \frac{1}{3}\delta^{ij} \right) s(r) + p(r)\delta^{ij}$$

$$\begin{aligned} p_{\rm scal}(r) &= -\frac{1}{6}\,\phi'(r)^2 - \frac{1}{2}\,m_S^2\phi(r)^2\,, \\ p_{\rm vect}(r) &= \frac{1}{6}\,V_0'(r)^2 + \frac{1}{2}\,m_V^2 V_0(r)^2\,, \\ p_{\rm Coul}(r) &= \frac{1}{6}\,A_0'(r)^2\,, \end{aligned}$$



$$\int dr\; r^2 p_i(r) = \begin{cases} -10.916\,\text{MeV} & \text{for} \quad i = \text{scalar}, \\ 10.891\,\text{MeV} & \text{for} \quad i = \text{vector}, \\ 0.025\,\text{MeV} & \text{for} \quad i = \text{Coulomb}. \end{cases}$$

Stability Inside the Proton

- Model results for p(r) and s(r):

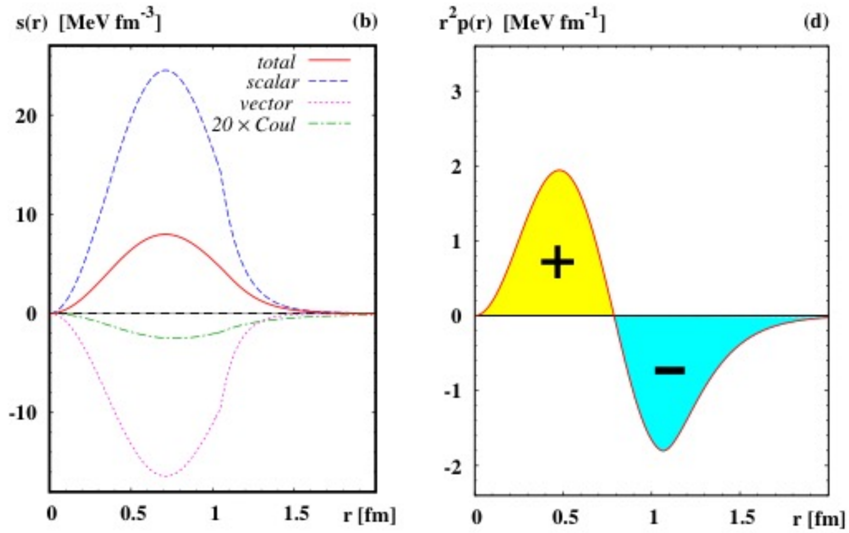
$$s(r) = \phi'(r)^2 - V_0'(r)^2 - A_0'(r)^2\,,$$
$$p(r) = \frac{1}{6}A_0'(r)^2 - \frac{1}{6}\phi'(r)^2 - \frac{1}{2}m_S^2\phi^2 + \frac{1}{6}V_0'(r)^2 + \frac{1}{2}m_V^2 V_0^2$$

- p(r) and s(r) are not independent

$$\frac{n-1}{r}s(r) + \frac{n-1}{n}s'(r) + p'(r) = 0$$

- von Laue condition:

$$\int_0^\infty dr\; r^2 p(r) = 0\,.$$



Features Observed in Prior EMT Studies

- $s(r)$ is positive at all r
- $p(r)$ has one node at r_0 with $p(r) > 0$ for $r < r_0$ and $p(r) < 0$ for $r > r_0$
- Normal force (per unit area) is always positive:

$$\frac{2}{3}s(r) + p(r) > 0$$

New Features Observed in Our Study

- Long-distance behavior of the densities:

$$
\begin{aligned}
T_{00}(r) &= \frac{1}{2}\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} + \dots \\
s(r) &= -\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} + \dots \\
p(r) &= \frac{1}{6}\,\frac{\alpha}{4\pi}\,\frac{\hbar c}{r^4} + \dots
\end{aligned}
$$

- We obtain three new features:
  - A node in s(r)
  - A second node in p(r)
  - A node in the normal force

What is the D-term?[1]

- “The last global unknown property”
- Most fundamental information corresponds to form factor at zero momentum transfer
    - For the nucleon, these are: Q, μ, g_A , g_p, M, J, and D

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N' \vert J^\mu_{\mathbf{em}} \vert N \rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\text{C}$
				$\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N' \vert J^\mu_{\mathbf{weak}} \vert N \rangle$	$\longrightarrow$	$g_A = 1.2694(28)$
				$g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\mathbf{grav}} \vert N \rangle$	$\longrightarrow$	$m = 938.272013(23)\,\text{MeV}/c^2$
				$J = \frac{1}{2}$
				$\boldsymbol{D} = \boldsymbol{?}$

- Experimental values exist for all properties except the D-term

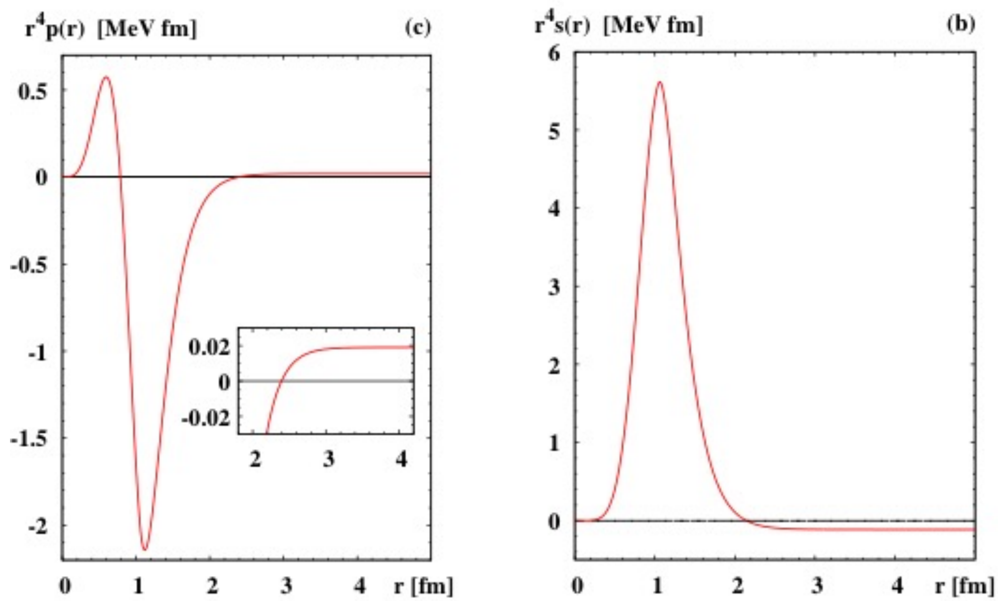
[1] This information is from “Forces inside hadrons: pressure, surface tension, mechanical radius, and all that” by M. V. Polyakov and P. Schweitzer; arXiv:1805.06596v3 [hep-ph] 14 Sep 2018

The Divergence of the D-term

- Two equivalent definitions:

$$D_s = -\frac{2(n-1)}{n(n+2)} M \int d^n r \, r^2 s(r) \,,$$

$$D_p = M \int d^n r \, r^2 p(r) \,,$$



Regularized Result (D_{reg})

- We can compute the D-term in terms of:

$$D(\zeta) = \zeta\, D_p + (1-\zeta)\, D_s$$

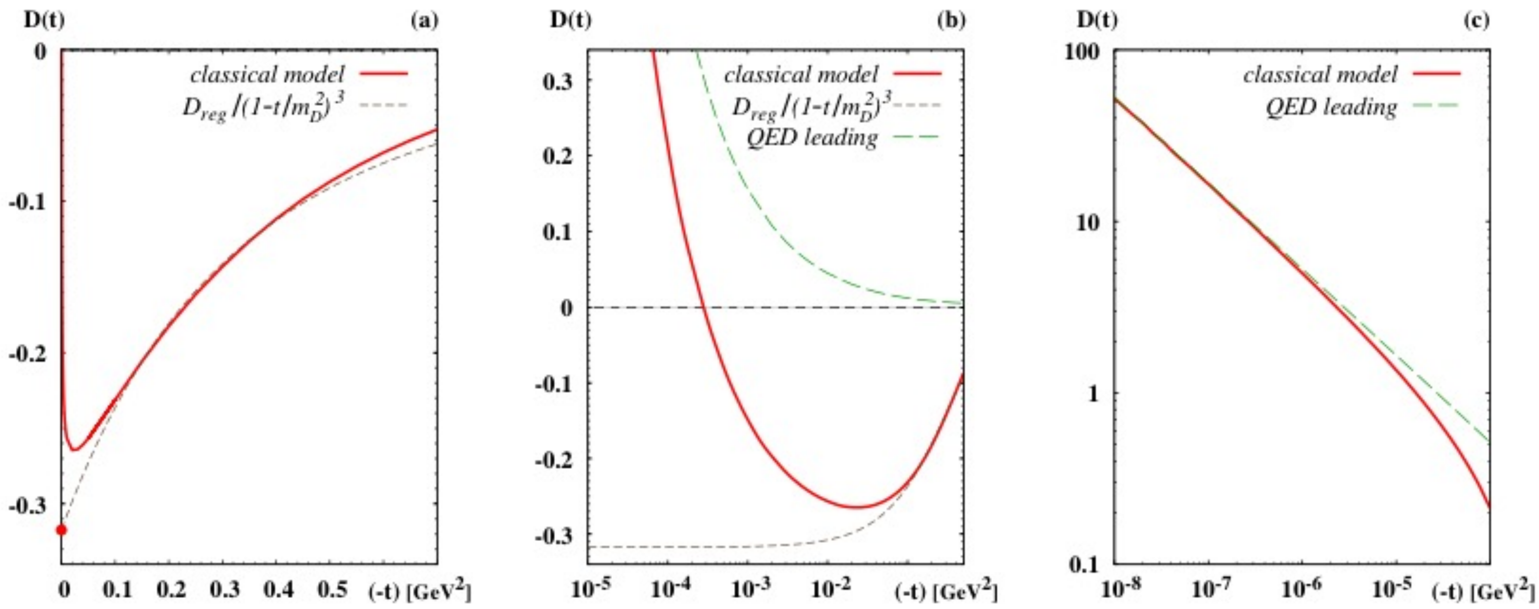
- D is divergent except when $\zeta = \zeta_{reg}$
- $s(r) = a_s/r^N$ and $p(r) = a_p/r^N$
- Using s(r) and p(r) relation,

$$a_p/a_s = -(n-1)(N-n)/(nN)$$

- We obtain: $$\zeta_{\rm reg} = \frac{2N}{n(n+2-N)} \qquad D_{\rm reg} = D(\zeta_{\rm reg}) = M \int d^3r\, r^2\, \frac{4}{9}\Big[6p(r) + s(r)\Big] = -0.317\,(\hbar c)^2.$$

Comparison to Other Studies

- Results are model-independent for r >> 3 fm



Conclusion/Future Studies

- Used classical model to calculate D-term
  - Found that it is divergent at long distances

- A(0) = 1; J(0) = ½
- How do we calculate D-term in a system with long-range forces?
  - One solution is our regularization scheme

- Since QED effects on D(t) become apparent at (-t) << 0.1 GeV^2, are they measurable?