Tool kit for baryon amplitude analyses and polarization measurements

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Motivation: baryon amplitude analyses

- Spectroscopy
  - Baryon decays often feature complicated phase-space structures
  - Excellent place to search for new baryonic and exotic pentaquark states
- Parity-violation
  - Measurement of decay asymmetry parameters
  - Determines sensitivity to polarization (later in the talk)
- CP-violation
  - Still unobserved for baryons
  - Comparison of baryon-antibaryon amplitude models
  - Resonance interference patterns can enhance sensitivity

Motivation: Polarization

- Baryon amplitude analyses can be used to measure baryon polarization vector ***P***
- Polarization is excellent probe for baryon production physics (with no meson counterpart)
  - Strong production: anchor point for low-energy QCD
  - Weak production: additional observable for New Physics tests
- For heavy baryons, multibody decays provide best statistics
  - e.g. $\Lambda_c^+ \to pK^-\pi^+$ is the main decay channel of Λ_c^+ (BF = 6.28%) with no neutral hyperons (problematic for fixed-target experiments)
- Essential for heavy baryon electromagnetic dipole moments measurement via spin precession in bent crystals, see PRD **103** (2021) 072003 and Refs therein

The Tool Kit

Main tools for baryon amplitude analysis:

- Polarization system
- Amplitude model
- Amplitude fit framework

- Will also present tests and sensitivity studies

- Will be illustrated taking as example the $\Lambda_c^+ \to pK^-\pi^+$ decay: amplitude analysis from semileptonic decay production ongoing at LHCb

Polarization system

- Polarization components defined w.r.t. an external system
- Tailored according to production mechanism
  - Strong production: natural system is production plane formed by proton and baryon momenta: only orthogonal polarization allowed by parity conservation
  - Weak production: best system is helicity system reached from mother particle: z axis defined by baryon momentum in mother rest frame

- For Λ_c^+ produced in semileptonic decays:
  - $\hat{\boldsymbol{z}}_{\Lambda_c^+} = \hat{\boldsymbol{p}}(\Lambda_c^+)$
  - $\hat{\boldsymbol{x}}_{\Lambda_c^+}$ defined such that $x - z$ plane coincides with $\Lambda_c^+ - \mu$ one

Helicity formalism

- Considering amplitude models written in the helicity formalism
- Multi-body decays decomposed via single two-body $A \to BC$ amplitudes defined in terms of final-state helicities
- Structure:
- Complex coupling: encodes the decay dynamics, to be determined from fit
- Angular dependence: fixed from angular momentum conservation, expressed in terms of Wigner D-matrices
- Invariant mass dependence: parametrization of the A particle width

$$\mathcal{A}^{A\to BC}_{\lambda_B,\lambda_C} = \mathcal{H}^{A\to BC}_{\lambda_B,\lambda_C} \times D^{J_A}_{m_A,\lambda_B-\lambda_C}(\phi_B,\theta_B,0)^* \times \mathcal{R}(m^2_{BC})$$

Writing the amplitude model

Given the three-body decay $\Lambda_c^+ \to pK^-\pi^+$

- Amplitudes built for each intermediate resonance $\Lambda_c^+ \to Rh$
  - $\Lambda^* \to pK^-$, $\Delta^* \to p\pi^+$, $K^* \to K^-\pi^+$
  - multiplying two-body helicity amplitudes, e.g.

$$\mathcal{A}^{[R]}_{m_{\Lambda_c^+},\lambda_R,\lambda_p} = \mathcal{A}^{\Lambda_c^+\to R\pi^+}_{\lambda_R,0}\,\mathcal{A}^{R\to pK^-}_{\lambda_p,0}$$

- Total helicity amplitudes for definite initial and final particles helicities to be obtained summing over all intermediate resonance helicity states

$$\mathcal{A}_{m_{\Lambda_c^+},\lambda_p} = \sum_{i=1}^{N_R}\sum_{\lambda_{R_i}=-J_{R_i}}^{J_{R_i}} \mathcal{A}^{[R_i]}_{m_{\Lambda_c^+},\lambda_{R_i},\lambda_p}$$

Final particle spin matching issue

- But, for multibody decays featuring different decay chains, the definition of the final state helicity is different
  - For $\Lambda_c^+ \to pK^-\pi^+$ the proton helicity differs for Λ^*, Δ^*, K^*
- Different definitions must be matched to a reference one before summing amplitudes
- We addressed recently this problem for generic multi-body particle decays, AHEP (2020) 6674595
  - Obtained a set of final particle spin rotations transforming the different helicity definitions into canonical spin states
  - Supersedes method used in previous analyses

Decay amplitude

- Decay amplitude for a $A \to R(\to 1,2)3$ topology
  - (below) decay amplitude for final particle helicities
  - (right) decay amplitude rotated to final particle canonical spin states

$$\mathcal{A}^{A\to R,3\to 1,2,3}_{m_A,\lambda_1^R,\bar{\lambda}_2^R,\bar{\lambda}_3^R}(\Omega) = \sum_{\lambda_R} \mathcal{H}^{R\to 1,2}_{\lambda_1^R,\bar{\lambda}_2^R} D^{*s_R}_{\lambda_R,\lambda_1^R+\bar{\lambda}_2^R}(\phi_1^R,\theta_1^R,0) \times \mathcal{H}^{A\to R,3}_{\lambda_R,\bar{\lambda}_3^R} D^{*s_A}_{m_A,\lambda_R+\bar{\lambda}_3^R}(\phi_R,\theta_R,0).$$

$$\begin{aligned}\mathcal{A}^{A\to R,3\to 1,2,3}_{m_A,m_1,m_2,m_3}(\Omega) = &\sum_{\lambda_1^R,\mu_1^R,\nu_1^R} D^{s_1}_{m_1,\nu_1^R}(\alpha_1^{W,R},\beta_1^{W,R},\gamma_1^{W,R}) \\ &\times D^{s_1}_{\nu_1^R,\mu_1^R}(\phi_R,\theta_R,0) \\ &\times D^{s_1}_{\mu_1^R,\lambda_1^R}(\phi_1^R,\theta_1^R,0) \\ \times &\sum_{\lambda_2^R,\mu_2^R,\nu_2^R} D^{s_2}_{m_2,\nu_2^R}(\alpha_2^{W,R},\beta_2^{W,R},\gamma_2^{W,R}) \\ &\times D^{s_2}_{\nu_2^R,\mu_2^R}(\phi_R,\theta_R,0) \\ &\times D^{s_2}_{\mu_2^R,\lambda_2^R}(\phi_1^R,\theta_1^R,0) \\ \times &\sum_{\lambda_3^R} D^{s_3}_{m_3,\lambda_3^R}(\phi_R,\theta_R,0) \\ &\times \mathcal{A}^{A\to R,3\to 1,2,3}_{m_A,\lambda_1^R,\bar{\lambda}_2^R,\bar{\lambda}_3^R}(\Omega)\end{aligned}$$

Baryon polarization

- Generic baryon polarization described by density matrix
- For the $\Lambda_c^+ \to pK^-\pi^+$ case: Λ_c^+ spin 1/2 density matrix

$$\rho_{\Lambda_c^+} = \frac{1}{2}\left(\mathcal{I} + \vec{P}\cdot\vec{\sigma}\right) = \frac{1}{2}\begin{pmatrix} 1+P_z & P_x - iP_y \\ P_x + iP_y & 1-P_z \end{pmatrix}$$

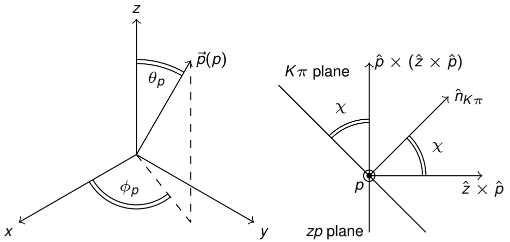
- Decay rate takes the from

$$\begin{aligned} p(\Omega, \vec{P}) \propto \sum_{m_p=\pm 1/2} \Big[&(1+P_z)|\mathcal{A}_{1/2,m_p}(\Omega)|^2 + (1-P_z)|\mathcal{A}_{-1/2,m_p}(\Omega)|^2 \\ &+ (P_x - iP_y)\mathcal{A}^*_{1/2,m_p}(\Omega)\mathcal{A}_{-1/2,m_p}(\Omega) \\ &+ (P_x + iP_y)\mathcal{A}_{1/2,m_p}(\Omega)\mathcal{A}^*_{-1/2,m_p}(\Omega) \Big] \end{aligned}$$

Baryon 3-body phase space

- Described by 5 variables
  - 2 two-body invariant mass "Dalitz" variables, describing decay dynamics
  - 3 decay orientation angles, describing decay plane orientation w.r.t. polarization system

- For $\Lambda_c^+ \to pK^-\pi^+$ decay, 5 uniform phase space variables can be chosen as
  - $\Omega = (m^2_{pK^-}, m^2_{K^-\pi^+}, \cos\theta_p, \phi_p, \chi)$
  - Angles being the polar and azimuthal proton angles, and the signed angle between $p, \hat{\boldsymbol{z}}_{\Lambda_c^+}$ and K, π planes

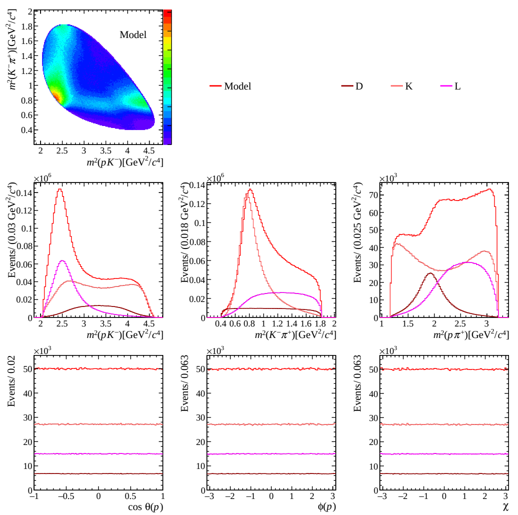


Amplitude fit

- We developed the fitting code for $\Lambda_c^+ \rightarrow pK^-\pi^+$ amplitude analysis basing on TensorFlowAnalysis package, by A. Poluektov and LHCb collaborators
- Exploits the machine-learning framework TensorFlow to build amplitude models
  - Based on computer algebra paradigm: users just specify the computational graph, with calculation, optimization and compilation for different architectures run automatically
  - Easier building of complicated amplitude models, otherwise prohibitive to build from basic, hard-coded functions
  - Caching of decay amplitude parts to speed up minimization process
- Minimization exploits the good-old MINUIT package
  - TF methods not suitable for physics purposes, not providing uncertainty estimates

Amplitude model tests

- We performed different tests for amplitude model implementation
- An important one was checking properties of the decay rate following from rotational invariance, valid **irrespective of the decay model** considered
  - Decay rate is isotropic in decay orientation angles for zero polarisation (right)
  - Invariant mass distributions are independent of the polarisation
- These tests showed inadequacy of previous spin matching method



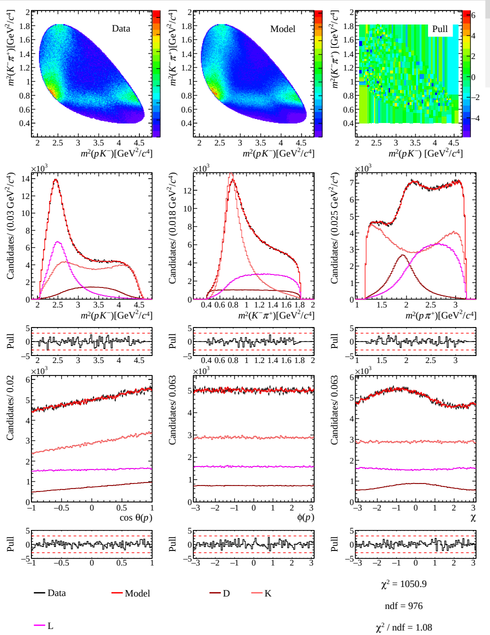
Sensitivity study for $\Lambda_c^+ \to pK^-\pi^+$

- We demonstrated the possibility to simultaneously measure $\Lambda_c^+ \to pK^-\pi^+$ amplitude model and Λ_c^+ polarization in AHEP (2020) 7463073 via:
  - Analytical study of the constraints posed by the amplitude fit to the decay rate
  - Amplitude fit to pseudodata generated with toy $\Lambda_c^+ \to pK^-\pi^+$ description
- Results:
  - Interference effects among different decay chain contributions are crucial: they give sensitivity to single complex helicity coupling values and to the polarization magnitude
  - A non-zero polarization is needed to determine entirely the amplitude model: it gives sensitivity to the parity-violating part of the decay amplitude

Pseudodata toy fit

- Set toy amplitude model with interfering K^*, Λ^*, Δ^* resonances
- Generated pseudo-data for $P_z = 0.5$ and zero polarisation
- Complex couplings, polarisation and resonance parameters retrieved from amplitude fit

- $P_z = 0.5$ (left): all parameters correctly measured within uncertainties
- $P = 0$: measured resonance parameters and polarisation, complex couplings not fully determined
  - Fit fractions correctly measured



Sensitivity to polarization

- Given the decay rate written as $p(\Omega) = f(\Omega) + Pg(\Omega)$
- Sensitivity to polarization can be measured as

$$S^2 = \int \frac{g^2}{f + P_0 g} d\Omega$$

- Cast into an effective decay asymmetry parameter $\alpha = \sqrt{3}S$, $0 < \alpha < 1$
- Generalising definition of two-body α parameters to multi-body decays
- Preliminary studies show $\Lambda_c^+ \to pK^-\pi^+$ decays have a large sensitivity to polarization, $\alpha \approx 0.65$, PRD **103** (2021) 072003
- Considering the large BF, $\Lambda_c^+ \to pK^-\pi^+$ is the best channel to access Λ_c^+ polarization

Summary

- Presented the main tools needed for baryon amplitude analyses
  - Polarization system: defined according to production process
  - Amplitude model: outlined method to write generic multi-body decays in helicity formalism
  - Amplitude fit framework: described our strategy based on TensorFlowAnalysis package

- Presented example tests for amplitude model implementation
- Presented sensitivity study for $\Lambda_c^+ \to pK^-\pi^+$ case
  - Demonstrated simultaneous extraction of amplitude model and baryon polarization
  - Large sensitivity to polarization estimated, making $\Lambda_c^+ \to pK^-\pi^+$ best probe for Λ_c^+ polarization

Thanks for your attention!

References:

- D. Marangotto, *Amplitude analysis and polarisation measurement of the Λ_c^+ baryon in the $pK^-\pi^+$ final state for the electromagnetic dipole moment determination*, PhD Thesis, `https://cds.cern.ch/record/2713231`
- D. Marangotto, *Helicity Amplitudes for Generic Multibody Particle Decays Featuring Multiple Decay Chains*, Advances in High Energy Physics (2020) 6674595, doi:10.1155/2020/6674595, arXiv:1911.10025.
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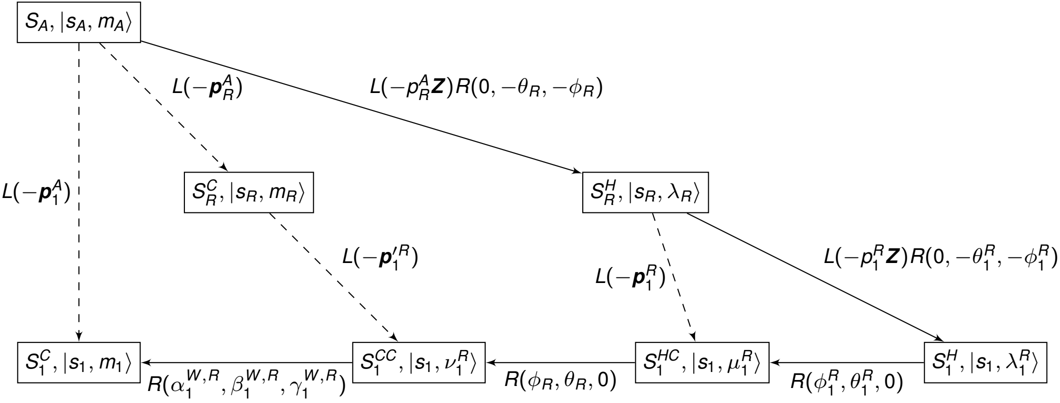
- S. Aiola et. al., *Progress towards the first measurement of charm baryon dipole moments*, Phys. Rev. D. **103** (2021) 072003, doi:10.1103/PhysRevD.103.072003, arXiv:2010.11902.

Backup Slides

Matching of final particle spin states, $A \to R \to 1$

- Apply Wigner rotation $R(\alpha_1^{W,R}, \beta_1^{W,R}, \gamma_1^{W,R}) = L(-\boldsymbol{p}_1^A)L(\boldsymbol{p}_R^A)L(\boldsymbol{p}_1'^R)$: obtain 1 canonical states reached from S_A, $|s_1, m_1\rangle$ states, S_1^C system



Λ_c^+ polarization and time-reversal

- Polarization components defined as

$$
\begin{aligned}
P_z &\propto \boldsymbol{P} \cdot \hat{\boldsymbol{p}}(\Lambda_c^+) \\
P_x &\propto \boldsymbol{P} \cdot \left[\left(\boldsymbol{p}(\Lambda_c^+) \times \boldsymbol{p}(\mu^-)\right) \times \hat{\boldsymbol{p}}(\Lambda_c^+)\right] \\
P_y &\propto \boldsymbol{P} \cdot \left[\boldsymbol{p}(\Lambda_c^+) \times \boldsymbol{p}(\mu^-)\right] ,
\end{aligned} \tag{1}
$$

- Longitudinal (P_z) and transverse (P_x) polarization are T-even, while normal (P_y) polarization is T-odd
- P_y can be produced only by
  - T-violation: $\mathcal{O}(CPV)$, tiny in $b \to c$ transitions
  - Final state interactions: only EM btw Λ_c^+ and μ^-, should be $\mathcal{O}(1\%)$ at max

- Reference: Sozzi, Discrete symmetries and CPV

Maximum-likelihood fit

- Fit parameters determined minimizing

$$-\log \mathcal{L}(\omega) = -\sum_{i=1}^{N} \log \left[p(\Omega_i|\omega) + \frac{p_{bkg}(\Omega_i) I(\omega)}{\epsilon(\Omega_i)} \frac{n_{bkg}}{n_{sig}} \right] + N \log I(\omega) + \mathrm{constant}$$

- Efficiency-corrected normalization computable using flat phase-space events simulated through full detector reconstruction

$$I(\omega) = \int p(\Omega|\omega)\epsilon(\Omega)d\Omega = \int p(\Omega|\omega)d\Omega' = \sum_{i=1}^{N_{MC}} p(\Omega_i|\omega)$$

- Background and efficiency parametrisation over phase space affect background term only
- For clean decays like $\Lambda_c^+ \to pK^-\pi^+$, $\frac{n_{bkg}}{n_{sig}} \ll 1$