Testing the molecular nature of $\phi(2170)$ (and K^* resonances with hidden charm)

Alberto Martínez Torres University of São Paulo 28/07/2021

In collaboration with: Brenda B. Malabarba (University of São Paulo), Xiu-Lei Ren (University of Mainz), Kanchan P. Khemchandani (Federal University of São Paulo), Li-Sheng Geng (Beihang University)

- $\phi(2170)$: observed by different collaborations (2006-2020) in processes like $\Gamma = 125 \pm 65 \text{ MeV}$
- Different theoretical models trying to explain its nature and properties:

$$s\bar{s}(n^{2S+1}L_J = 3^3S_1) \implies \Gamma \sim 300 \text{ MeV}$$

$$s\bar{s}(2^{3}D_{1}) \Longrightarrow \Gamma_{K^{*}(892)\bar{K}^{*}(892)}, \Gamma_{K^{*}(1410)\bar{K}}$$

$$s\bar{s}g \Longrightarrow \Gamma_{K^*(1410)\bar{K}} \gtrsim \Gamma_{K_1(1270)\bar{K}}$$
, Mode K

selection rule). Not supported by Lattice QCD and QCD Gaussian

sum rules.

Tetraquark \implies Difficulties in obtaining a compatible mass

1) Aubert et al., Phys. Rev. D74,091103(2006); Phys. Rev. D76, 012008(2007); 2) Ablikim et al., Phys. Rev. Lett, 100, 102003 (2008), Phys. Rev. D102, 012008 (2020); 3) Barnes et al., Phys. Rev. D68, 054014 (2003); 4) Ding and Yan, Phys. Lett. B657, 49(2007); Phys. Lett. B650, 390(2007); 5) Wang, Nucl. Phys. A791, 106(2007); 6) Dudek, Phys. Rev. D84, 074023 (2011);7) Ho et al., Phys. Rev. D100, 034012 (2019).

 $e^+e^- \to K^+K^-\pi^{+(0)}\pi^{-(0)}, J/\psi \to \eta K^+K^-\pi^+\pi^-, e^+e^- \to \phi \eta' \text{ (PDG: } M = 2160 \pm 80 \text{ MeV},$

 $_{\bar{K}} > \Gamma_{K(1460)\bar{K}}, \Gamma_{K_1(1400)\bar{K}}, \Gamma_{K_1(1270)\bar{K}}$ $K(1460)\overline{K}$ forbidden (spin

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BESIII collaboration (Phys.
Rev. Lett. 124, 112001 (2020):
K^+(1460)K^-, K_1^+(1400)K^-,
K^{*+}(1410)K^{-}, K_{1}^{+}(1270)K^{-},
K^{*+}(892)K^{*-}(892)
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- $e^+e^- \to K^+K^-\pi^{+(0)}\pi^{-(0)}, J/\psi \to \eta K^+K^-\pi^+\pi^-, e^+e^- \to \phi \eta' \text{ (PDG:}$ $M = 2160 \pm 80$ MeV, $\Gamma = 125 \pm 65$ MeV)



8) A. Martínez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale, E. Oset, Phys. Rev. D78, 074031 (2008); 9) M. Napsuciale, E. Oset, et al., Phys. Rev. D76, 074012 (2007). 10) J. A. Oller, E. Oset, Nucl. Phys. A620, 438 (1997); 11) L. Roca, E. Oset, J. Singh, Phys. Rev. D72, 014002 (2005); 12) L.S. Geng, E. Oset, L. Roca, J. A. Oller, Phys. Rev. D72, 014002 (2005).

• $\phi(2170)$: observed by different collaborations (2006-2020) in processes like

\implies Observed in the $\phi f_0(980)$ invariant mass distribution

- $e^+e^- \to K^+K^-\pi^{+(0)}\pi^{-(0)}, J/\psi \to \eta K^+K^-\pi^+\pi^-, e^+e^- \to \phi \eta' \text{ (PDG:}$ $M = 2160 \pm 80$ MeV, $\Gamma = 125 \pm 65$ MeV)



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• $\phi(2170)$: observed by different collaborations (2006-2020) in processes like

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• We have determined the decay width of $\phi(2170)$ to kaonic resonances: $K^{+}(1460)K^{-}, K_{1}^{+}(1400)K^{-}, K_{1}^{+}(1270)K^{-}, K^{*+}(892)K^{*-}(892).$

K(1460) K

- $K_1(1270) \& K_1(1400)$
 - $\pi K^*(892), \phi K, \rho K \implies K_1(1270), \text{ two poles}$ $z_1 = 1195 - i123$ MeV $z_2 = 1284 - i73 \text{ MeV}$
 - Mixing scheme $\implies K_{1A}, K_{1B}$ belonging to the nonet of axials (mixing angles $29^\circ - 62^\circ$)
 - Phenomenological approach \implies Use of the data available in the PDG on their radiative decays
- 13) Brenda B. Malabarba, Xiu-Lei Ren, K. P. Khemchandani, A. Martínez Torres, Phys. Rev. D103, 016018 (2021); 14) A. Martínez Torres, D. Jido, Y. Kanada-En'yo; 15) Albaladejo, Oller, Roca, Phys. Rev. D82, 094019 (2010); 16) Shinmura, Hara, Yamada, JPS COnf. Proc, 26, 023003 (2019); 17) Filikhin, Kezerashvili, Suslov, et al., Phys. Rev. D102, 094027 (2020);18) Zhang, Hanhart, Meissner, Ju-Jun, arxiv: 2107.03168 [hep-ph]; 19) Palomar, Roca, Oset, Vacas, Nucl. Phys. A729, 743 (2003);







20) Brenda B. Malabarba, Xiu-Lei Ren, K. P. Khemchandani, A. Martínez Torres, Phys. Rev. D103, 016018 (2021)

$$-igV^{\mu}[\partial_{\mu}\Phi,\Phi],$$

 $\Phi \equiv \text{Matrix pseudoscalar meson octet fields} \quad {}^{t}K_{1}^{+} \rightarrow \phi K^{+} = g_{K_{1}^{+}} \rightarrow \phi K^{\epsilon}K_{1}^{+} \cdot \epsilon_{\phi}$

- **1)** $K_1(1270)$ as a molecular state.
- **2)** $K_1(1270)$ & $K_1(1400)$ as a mixture of
 - K_{1A}, K_{1B}
- 3) Phenomenological approach.



 $f_0(980)$ as a *s*-wave *KK*, $\pi\pi$ state:

$$t_{f_0 \to K^+ K^-} = g_{f_0 \to K^+ K^-}$$























• BESIII collaboration: $\phi(2170) \rightarrow K^*(892)\bar{K}^*(892)$ is suppressed





RESULTS	Larger coupline to πK^*	Mass g closer 1270 I	r to MeV					
$\phi(2170) \to K^+(1460)$	<i>K</i> ⁻		4	b(2170)	$\rightarrow K_1^+(12)$	270) <i>K</i> ⁻		
Form factor Decay width								
$\frac{1}{1} \sum_{i=1}^{n} \frac{1}{i} \sum_{i=1}^{n} \frac{1}$	= Form factor				Decay width			
Heaviside- Θ 1.5 \pm 0.5			Model A		Model B		Model C	
Monopole 1.3 ± 0.4		Poles z_1, z_2	Pole z_1	Pole z_2		Solution \mathbb{S}_1	Solution \mathbb{S}_2	Solu
Exponential 1.3 ± 0.5	Heaviside- Θ	1.5 ± 0.3	0.6 ± 0.1	0.22 ± 0.04	0.12 ± 0.04	1.6 ± 0.4	17 ± 3	41
	- Monopole	0.8 ± 0.2	0.3 ± 0.1	0.12 ± 0.02	0.07 ± 0.02	0.9 ± 0.2	9 ± 2	23
$d(2170) \rightarrow K^{+}(1400)$	K - Exponential	1.0 ± 0.2	0.4 ± 0.1	0.15 ± 0.03	0.09 ± 0.02	1.1 ± 0.3	11 ± 2	28
Form factor Decay width Model B Mod	lel C			Sizeable contributio	ns			
Heavise- Θ 2.6 \pm 0.5 15	± 4	Model	A: Mo	lecular	model fo	or $K_1(12)$	270)	
Monopole 1.9 ± 0.4 11	± 3	Model B: Mixing angle						
Exponential 2.1 ± 0.4 12	± 3	iviodel		enomen	lological	raulati	ve deca	ays)





RESULTS

• BESIII collaboration: $\mathscr{B}r\Gamma_R^{e^+e^-}$

 $\mathscr{B}r \equiv \text{Branching fraction } \phi(2170) \rightarrow RK^{-}$

$$B_1 \equiv \frac{\Gamma_{\phi_R \to K^+(1460)K^-}}{\Gamma_{\phi_R \to K_1^+(1400)K^-}} =$$

$$B_{1} \equiv \frac{\Gamma_{\phi_{R} \to K^{+}(1460)K^{-}}}{\Gamma_{\phi_{R} \to K^{+}(1400)K^{-}}} = \frac{\beta r [\phi_{R} \to K^{+}(1460)K^{-}]}{\beta r [\phi_{R} \to K^{+}(1400)K^{-}]} \Gamma_{R}^{e^{+}e^{-}} = 3.0 \pm 3.8,$$

$$B_{2} \equiv \frac{\Gamma_{\phi_{R} \to K^{+}(1460)K^{-}}}{\Gamma_{\phi_{R} \to K^{+}(1270)K^{-}}} = \frac{\beta r [\phi_{R} \to K^{+}(1460)K^{-}]}{\beta r [\phi_{R} \to K^{+}(1270)K^{-}]} \Gamma_{R}^{e^{+}e^{-}} = \begin{cases} 4.7 \pm 3.3, \text{ Solution} \\ 98.8 \pm 7.8, \text{ Solution} \\ 98.8 \pm 7.8, \text{ Solution} \\ 98.8 \pm 7.8, \text{ Solution} \\ 152.6 \pm 14.2, \text{ Solution} \\ 152.6 \pm 14.2, \text{ Solution} \end{cases}$$

$$B_{1} \equiv \frac{\Gamma_{\phi_{R} \to K^{+}(1460)K^{-}}}{\Gamma_{\phi_{R} \to K^{+}(1400)K^{-}}} = \frac{\beta r [\phi_{R} \to K^{+}(1460)K^{-}]}{\beta r [\phi_{R} \to K^{+}(1400)K^{-}]} [\Gamma_{R}^{e^{+}e^{-}} = 3.0 \pm 3.8,$$

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$$B_{3} \equiv \frac{\Gamma_{\phi_{R} \to K^{+}_{1}(1270)K^{-}}}{\Gamma_{\phi_{R} \to K^{+}_{1}(1400)K^{-}}} = \frac{\beta r [\phi_{R} \to K^{+}_{1}(1270)K^{-}]}{\beta r [\phi_{R} \to K^{+}_{1}(1400)K^{-}]} [\Gamma_{R}^{e^{+}e^{-}} = \begin{cases} 7.6 \pm 3.7, \text{ Solution} \\ 152.6 \pm 14.2, \text{ Solution} \end{cases}$$

 $\Gamma_R^{e^+e^-} \equiv$ Partial decay width of $\phi(2170) \rightarrow e^+e^-$



RESULTS

- BESIII collaboration: $\mathscr{B}r\Gamma_R^{e^+e^-}$
 - $\mathscr{B}r \equiv \text{Branching fraction } \phi(2170) \rightarrow RK^{-}$ $\Gamma_R^{e^+e^-} \equiv$ Partial decay width of $\phi(2170) \rightarrow e^+e^-$

$$B_1 \equiv \frac{\Gamma_{\phi_R \to K^+(1460)K^-}}{\Gamma_{\phi_R \to K_1^+(1400)K^-}} =$$

$$B_2 \equiv \frac{\Gamma_{\phi_R \to K^+(1460)K^-}}{\Gamma_{\phi_R \to K_1^+(1270)K^-}} =$$

$$B_3 \equiv \frac{\Gamma_{\phi_R \to K_1^+(1270)K^-}}{\Gamma_{\phi_R \to K_1^+(1400)K^-}}$$

$$\frac{\mathcal{B}r[\phi_R \to K^+(1460)K^-]}{\mathcal{B}r[\phi_R \to K_1^+(1400)K^-]}$$

$$\frac{\mathcal{B}r[\phi_R \to K^+(1460)K^-]}{\mathcal{B}r[\phi_R \to K_1^+(1270)K^-]}$$

 $\mathcal{B}r[\phi_R \to K_1^+(1270)K^-]$ $\overline{\mathcal{B}r[\phi_R \to K_1^+(1400)K^-]}$

$$B_1^{\exp} = \begin{cases} 0.64 \pm 0.92, \text{ Solution 1,} \\ 0.03 \pm 0.04, \text{ Solution 2,} \end{cases}$$
$$B_2^{\exp} = \begin{cases} 0.40 \pm 0.54, \text{ Solution 1,} \\ 0.02 \pm 0.03, \text{ Solution 2,} \end{cases}$$
$$B_3^{\exp} = \begin{cases} 1.62 \pm 1.38, \text{ Solution 1,} \\ 1.55 \pm 0.19, \text{ Solution 2.} \end{cases}$$

RESULTS

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$$B_1 = \frac{\Gamma_{\phi_R \to K^+(1460)K^-}}{\Gamma_{\phi_R \to K_1^+(1400)K^-}} \quad B_2 = \frac{\Gamma_{\phi_R \to K^+(1460)K^-}}{\Gamma_{\phi_R \to K_1^+(1270)K^-}}$$

$$B_{3} = \frac{\Gamma_{\phi_{R} \to K^{+}(1270)K^{-}}}{\Gamma_{\phi_{R} \to K_{1}^{+}(1400)K^{-}}}$$

B_1 _		B_3				
$\frac{1}{1}$		Model B	0.04 ± 0.01			
Model D 0.02 ± 0.20			0.09 ± 0.02	(Solution \mathbb{S}_1)		
Model C 0.11 ± 0.04		Model C	0.96 ± 0.16	(Solution \mathbb{S}_2)		
			2.40 ± 0.40	(Solution \mathbb{S}_3)		
B_2						
1.3 ± 0.4	(Poles z_1, z_2))				
Model A 3.6 ± 1.2	2 (Pole z_1)					
8.8 ± 2.8	8 (Pole z_2)					
Model B 16 ± 6						
1.2 ± 0.4	$($ Solution $\mathbb{S}_1)$					
Model C 0.12 ± 0.0	04 (Solution \mathbb{S}_2)					
0.05 ± 0.0	02 (Solution \mathbb{S}_3)					

odel A: Molecular model for $K_1(1270)$ odel B: Mixing scheme odel C: Phenomenological (radiative decays)

RESULTS K*

More details in the talk of Brenda B. Malabarba, Thursday 12:40 (Exotic hadrons and candidates)



21) Xiu-Lei Ren, Brenda B. Malabarba, Li-Sheng Geng, K. P. Khemchandani, A. Martínez Torres, Phys. Lett. B785, 112-117 (2018); Xiu-Lei Ren, K. P. Khemchandani, A. Martínez Torres, JHEP 05, 103 (2019) and Phys. Rev. D102, 016005 (2020).

1680)	1/2(1-)
1770)	1/2(2-)
1780)	1/2(3-)
1820)	1/2(2-)
830)	1/2(0-)
1950)	1/2(0+)
1980)	1/2(2+)
2045)	1/2(4+)
2250)	1/2(2-)
2320)	1/2(3+)
2380)	1/2(5 ⁻)
2500)	1/2(4-)
100)	$?^{?}(?^{??})$

RESULTS *K**



21) Xiu-Lei Ren, Brenda B. Malabarba, Li-Sheng Geng, K. P. Khemchandani, A. Martínez Torres, Phys. Lett. B785, 112-117 (2018); Xiu-Lei Ren, K. P. Khemchandani, A. Martínez Torres, JHEP 05, 103 (2019) and Phys. Rev. D102, 016005 (2020).









RESULTS K*



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CONCLUSIONS

- $\phi(2170)$ as a $\phi f_0(980)$ state:
 - Explains its suppressed decay to $K^*(892)\bar{K}^*(892)$
 - K(1460) as a state which couples to Kf_0 , $K_1(1400)$ mixing angle scheme/

phenomenological approach: compatible results for

- $K_1(1270)$ as state related to two poles arising from PV dynamics/ phenomenological approach for $K_1(1270) \& K_1(1400)$. Compatible results for

 $B_2 = \frac{\Gamma_{\phi_R \to K^+(1460)K^-}}{\Gamma_{\phi_R \to K^+_1(1270)K^-}}.$ The mixing scheme considered for $K_1(1270)$ & $K_1(1400)$ is not compatible.

- The ratio $B_3 = \frac{\Gamma_{\phi_R \to K^+(1270)K^-}}{\Gamma_{\phi_R \to K_1^+(1400)K^-}}$ is compatible with the mixing scheme/phenomenological approach.

• K^* resonance with hidden charm is predicted around 4307 MeV (also found by Ma, Wang, Meissner in Chin. Phys. C43, 014102 (2019) and by Tian-Wei Wu, Ming-Zhu Liu, Li-Sheng Geng in Phys. Rev. D103,3 (2021))

$$r B_1 = \frac{\Gamma_{\phi_R \to K^+(1460)K^-}}{\Gamma_{\phi_R \to K_1^+(1400)K^-}}.$$

• Triangular loops:



18) Palomar, Roca, Oset, Vacas, Nucl. Phys. A729, 743 (2003)

 $K_1(1270) \& K_1(1400)$ as mixed states: K_{1A}, K_{1B}

- Tensor meson formalism for the vector mesons (rank 2) - Couplings of $K_1(1270)$ & $K_1(1400)$ to ϕK are obtained

$${}_{\to\phi K^{+}} = \frac{|g_{K_{1}^{+}\to\phi K^{+}}^{T}|^{2}}{2\pi} \frac{1}{\mathcal{N}} \int_{M_{K_{1}}-a\Gamma_{K_{1}}}^{M_{K_{1}}+a\Gamma_{K_{1}}} d\tilde{M}_{K_{1}} (2\tilde{M}_{K_{1}}) \frac{|\vec{p}|}{\tilde{M}_{K_{1}}^{2}} \left[1 + \frac{2}{3} \frac{|\vec{p}|}{M_{K_{1}}}\right] \\ = \frac{1}{\tilde{M}_{K_{1}}^{2} - M_{K_{1}}^{2} + iM_{K_{1}}\Gamma_{K_{1}}} \left[\theta(\tilde{M}_{K_{1}} - M_{\phi} - M_{K})\theta(\tilde{M}_{K_{1}} - M_{\pi} - M_{K^{*}})\right]$$

$$_{\to\phi K^{+}} = \frac{|g_{K_{1}^{+}\to\phi K^{+}}|^{2}}{24\pi} \frac{1}{\mathcal{N}} \int_{M_{K_{1}}-a\Gamma_{K_{1}}}^{M_{K_{1}}+a\Gamma_{K_{1}}} d\tilde{M}_{K_{1}}(2\tilde{M}_{K_{1}}) \frac{|\vec{p}|}{\tilde{M}_{K_{1}}^{2}} \left[3 + \frac{|\vec{p}|}{M_{K_{1}}}\right]$$

 $\times \operatorname{Im} \left| \frac{1}{\tilde{M}_{K_1}^2 - M_{K_1}^2 + iM_{K_1}\Gamma_{K_1}} \right| \theta(\tilde{M}_{K_1} - M_{\phi} - M_K)\theta(\tilde{M}_{K_1} - M_{\pi} - M_{K^*(892)})$



• Triangular loops:

- Phenomenological approach for $K_1(1270)$ & $K_1(1400)$:
 - $K_1(1270) \& K_1(1400)$ to ϕK are obtained
 - Available data on the radiative decays - Vector meson dominance: Couplings of



 $t_{K_1^+ \to \phi K^+} = g_{K_1^+ \to \phi K} \epsilon_{K_1^+} \cdot \epsilon_{\phi}$

18) Palomar, Roca, Oset, Vacas, Nucl. Phys. A729, 743 (2003)



We use the tensor formalism for vector mesons: Tree level term is gauge invariant

$$t_{K_{1}^{0}\to\gamma K^{0}}^{T} = -\frac{2eF_{V}}{M_{K_{1}^{0}}} \Big[\frac{g_{K_{1}^{0}\to\rho^{0}K^{0}}^{T}}{M_{\rho^{0}}^{2}} + \frac{g_{K_{1}^{0}\to\omega K^{0}}^{T}}{3M_{\omega}^{2}} - \frac{\sqrt{2}g_{K_{1}^{0}\to\phi K^{0}}^{T}}{3M_{\phi}^{2}} \Big] \\ \times \Big[(P \cdot p)(\epsilon_{K_{1}^{0}}(P) \cdot \epsilon_{\gamma}(p)) - (P \cdot \epsilon_{\gamma}(p))(p \cdot \epsilon_{K_{1}^{0}}(P) \cdot \epsilon_{\gamma}(p)) - (P \cdot \epsilon_{\gamma}(p))(p \cdot \epsilon_{K_{1}^{0}}(P) \cdot \epsilon_{\gamma}(p)) \Big] \Big]$$



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 $I_{K_1^+ \to \phi K^+} = g_{K_1^+ \to \phi K} \epsilon_{K_1^+} \cdot \epsilon_{\phi}$

 K^{-}

(q)

18) Palomar, Roca, Oset, Vacas, Nucl. Phys. A729, 743 (2003)

We use the tensor formalism for vector mesons: Tree level term is gauge invariant

Fixed to reproduce the experimental data on the decay width



We use the experimental data

We get the coupling in the Tensor formalism

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 $\Gamma'_{K_1 \to \phi K} \sim \Gamma_{K_1 \to \phi K}$: we can determine $|g_{K_1 \to \phi K}|$

(q)

18) Palomar, Roca, Oset, Vacas, Nucl. Phys. A729, 743 (2003)

We use the tensor formalism for vector mesons: Tree level term is gauge invariant

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• Triangular loops:



 $|g_{K_1^+(1270)}|$

Phenomenological approach for $K_1(1270)$ & $K_1(1400)$:

$$|_{0) \to \phi K^+}| = \begin{cases} 3967 \pm 419 \text{ MeV}, \text{ Solution } \mathbb{S}_1, \\ 12577 \pm 763 \text{ MeV}, \text{ Solution } \mathbb{S}_2, \\ 19841 \pm 1177 \text{ MeV}, \text{ Solution } \mathbb{S}_3. \end{cases}$$

 $|g_{K_1^+(1400)\to\phi K^+}| = 8480 \pm 1333 \text{ MeV}$

No direct measurement of $K_1 \rightarrow K\gamma$

