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Minimal complete sets for two-pseudoscalar-meson photoproduction

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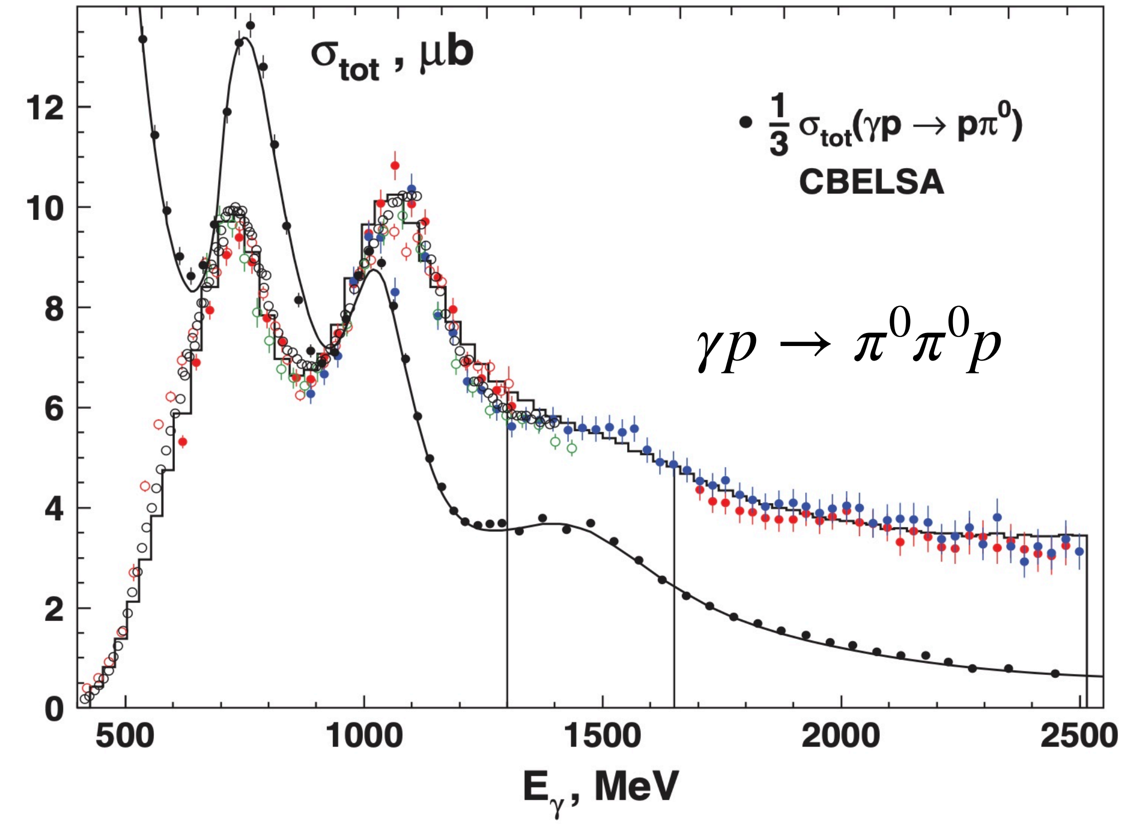
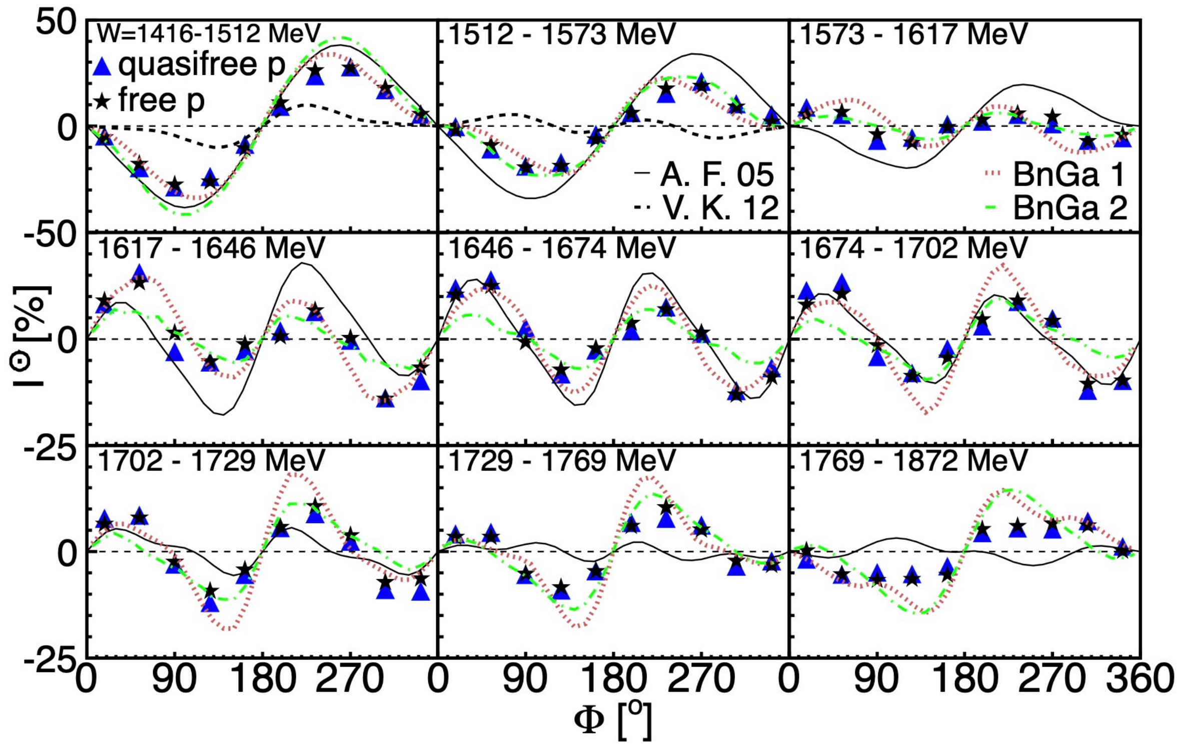
PHYSICAL REVIEW C ***103**, 014607 (2021)*

Motivation

- Search for missing resonances
- More measurements, observables
- Increase in precision
- Interest in complete experiment analysis



Thiel et al. [Phys. Rev. Lett. **114**, 091803 (2015)]



Oberle et al. [Phys. Lett. B **721**, 237 (2013)]

Pool of Measurements

Observable	Energy range E_γ^{lab}	Facility	Reference	Year of publication
		$\gamma p \to p\pi^0\pi^0$		
I_0	309–792 MeV	TAPS at MAMI	Härter *et al.* [20]	1997
I_0	309–820 MeV	TAPS at MAMI	Wolf *et al.* [21]	2000
I_0	200–820 MeV	TAPS at MAMI	Kleber *et al.* [22]	2000
I_0	300–425 MeV	TAPS at MAMI	Kotulla *et al.* [23]	2004
I_0	309–800 MeV	CB/TAPS at MAMI	Zehr *et al.* [24]	2012
I_0	309–1400 MeV	CB/TAPS at MAMI	Kashevarov *et al.* [25]	2012
I_0	432–1374 MeV	CB/TAPS at MAMI	Dieterle *et al.* [26]	2015
I_0	400–800 MeV	DAPHNE at MAMI	Braghieri *et al.* [27]	1995
I_0	400–800 MeV	DAPHNE at MAMI	Ahrens *et al.* [28]	2005
I_0	309–820 MeV	TAPS at MAMI, CB at ELSA	Sarantsev *et al.* [29]	2008
I_0	400–1300 MeV	CB at ELSA	Thoma *et al.* [30]	2008
I_0	≈750–2500 MeV	CBELSA/TAPS at ELSA	Thiel *et al.* [31]	2015
I_0, Σ	600–2500 MeV	CB/TAPS at ELSA	Sokhoyan *et al.* [1]	2015
I_0, Σ	650–1450 MeV	GRAAL	Assafiri *et al.* [32]	2003
Σ	650–1450 MeV	CB at ELSA	Thoma *et al.* [30]	2008
$I^\odot$	560–810 MeV	CB/TAPS at MAMI	Krambrich *et al.* [33]	2009
$I^\odot$	≈600–1400 MeV	CB/TAPS at MAMI	Oberle *et al.* [34]	2013
$I^\odot$	550–820 MeV	CB/TAPS at MAMI	Zehr *et al.* [24]	2012
E, $\sigma_{1/2}$, $\sigma_{3/2}$	≈431–1455 MeV	CB/TAPS at MAMI	Dieterle *et al.* [35]	2020
P_x, P_y, T, H, P	650–2600 MeV	CBELSA/TAPS at ELSA	Seifen *et al.* [14]	2020
I^c, I^s	970–1650 MeV	CB/TAPS at ELSA	Sokhoyan *et al.* [1]	2015
		$\gamma p \to p\pi^+\pi^-$		
I_0	400–800 MeV	DAPHNE at MAMI	Braghieri *et al.* [27]	1995
I_0	400–800 MeV	DAPHNE at MAMI	Ahrens *et al.* [36]	2007
I_0	370–940 MeV	LNF	Carbonara *et al.* [37]	1976
I_0	800–1100 MeV	NKS at LNS	Hirose *et al.* [38]	2009
I_0	500–4800 MeV	CEA	Crouch *et al.* [39]	1964
I_0	≈560–2560 MeV	SAPHIR at ELSA	Wu *et al.* [40]	2005
I_0	≈895–1663 MeV	CLAS at JLAB	Golovatch *et al.* [41]	2019
$I^\odot$	575–815 MeV	CB/TAPS at MAMI	Krambrich *et al.* [33]	2009
$I^\odot$	502–2350 MeV	CLAS at JLAB	Strauch *et al.* [42]	2005
$I^\odot$	1100–5400 MeV	CLAS at JLAB	Badui *et al.* [43]	2016
		$\gamma p \to p\pi^0\eta$		
I_0	≈930–2500 MeV	CB/TAPS at ELSA	Gutz *et al.* [15]	2014
I_0	≈1070–2860 MeV	CB at ELSA	Horn *et al.* [44]	2008
I_0	950–1400 MeV	CB/TAPS at MAMI	Kashevarov *et al.* [45]	2009
I_0	1000–1150 MeV	GeV-γ at LNS	Nakabayashi *et al.* [46]	2006
I_0, Σ	≈930–1500 MeV	GRAAL	Ajaka *et al.* [47]	2008
Σ	970–1650 MeV	CBELSA/TAPS at ELSA	Gutz *et al.* [48]	2008
Σ	≈1070–1550 MeV	CB/TAPS at ELSA	Gutz *et al.* [15]	2014
I^c, I^s	970–1650 MeV	CBELSA/TAPS at ELSA	Gutz *et al.* [49]	2010
I^c, I^s	≈1081–1550 MeV	CB/TAPS at ELSA	Gutz *et al.* [15]	2014
		$\gamma p \to n\pi^+\pi^0$		
I_0	300–820 MeV	TAPS at MAMI	Langgärtner *et al.* [50]	2001
I_0	≈325–800 MeV	CB/TAPS at MAMI	Zehr *et al.* [24]	2012
I_0	400–800 MeV	DAPHNE at MAMI	Braghieri *et al.* [27]	1995
I_0	400–800 MeV	DAPHNE at MAMI	Ahrens *et al.* [51]	2003
$I^\odot$	520–820 MeV	CB/TAPS at MAMI	Krambrich *et al.* [33]	2009
$I^\odot$	≈550–820 MeV	CB/TAPS at MAMI	Zehr *et al.* [24]	2012
		$\gamma n \to n\pi^0\pi^0$		
$I^\odot$	≈600–1400 MeV	CB/TAPS at MAMI	Oberle *et al.* [34]	2013
I_0, Σ	≈600–1500 MeV	GRAAL	Ajaka *et al.* [52]	2007
I_0	≈430–1371 MeV	CB/TAPS at MAMI	Dieterle *et al.* [26]	2015
		$\gamma n \to p\pi^-\pi^0$		
I_0	≈370–940 MeV	LNF	Carbonara *et al.* [37]	1976
I_0	≈450–800 MeV	DAPHNE at MAMI	Zabrodin *et al.* [53]	1997
I_0	≈500–800 MeV	DAPHNE at MAMI	Zabrodin *et al.* [54]	1999
		$\gamma n \to n\pi^+\pi^-$		
I_0	370–940 MeV	LNF	Carbonara *et al.* [37]	1976
		$\gamma p \to pK^+K^-$		
I_0	3000–3800 MeV	CLAS at JLAB	Lombardo *et al.* [55]	2018
$I^\odot$	1100–5400 MeV	CLAS at JLAB	Badui *et al.* [43]	2016

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- More than 55 measurements
- Publications from 1976 - 2020

Introduction

<u>Overall goal:</u>

determine matrix elements of transition matrix $\mathcal{T}$
for certain reaction, i.e. $\gamma N \to \pi\pi N$

- Polarisation observable: $\mathcal{O}_i = < t \,|\, \hat{A}_i \,|\, t > = \sum_{k,j} t_k^* \, (\hat{A}_i)_{kj} \, t_j$
- Bilinear product $\longrightarrow$ mathematical ambiguities arise
- Perform complete experiment analysis
- Analytical approach very hard for $N \geq 4$ ($N = 8$ for $\gamma N \to \pi\pi N$)

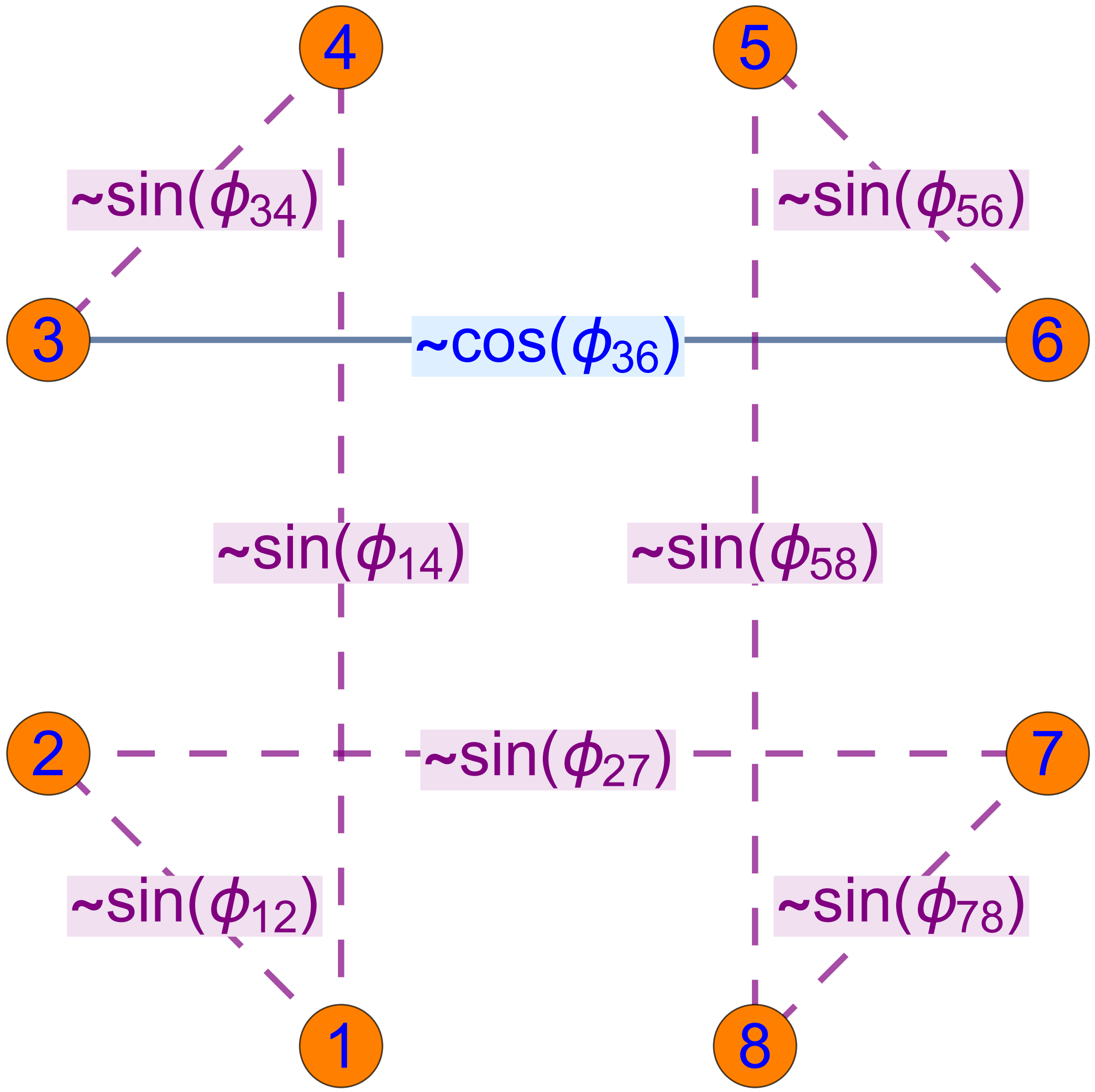
Moravcsik's Theorem

Explanation:

- Node: complex amplitude t_i
- Edge: Re/Im of bilinear product $t_i t_j^*$
- $\mathrm{Re}(t_i t_j^*) \sim \cos(\phi_{ij})$
- $\mathrm{Im}(t_i t_j^*) \sim \sin(\phi_{ij})$

Complete Set of Observables if:

- Connected Graph
- Odd number of „sine-type“ ambiguities



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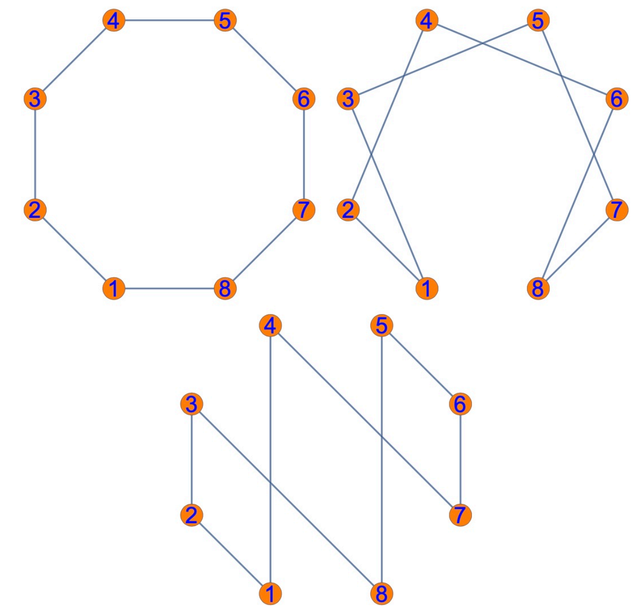
Approach

- Construct all unique graph topologies



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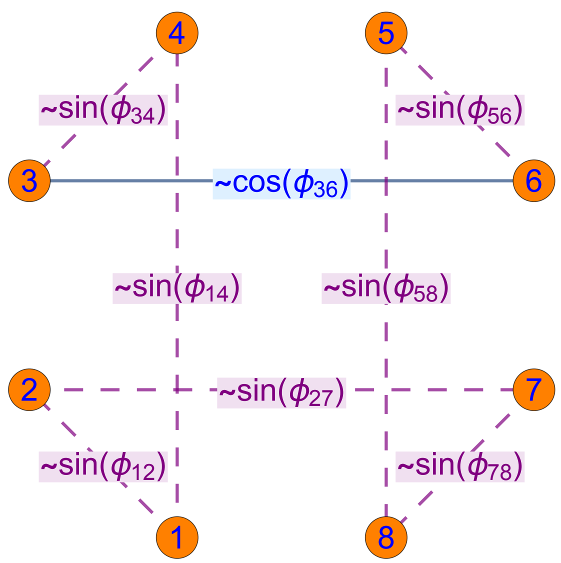
in total $\dfrac{(N-1)!}{2} = 2520$



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- Construct all possible edge configurations

$\sum_{k=1}^{N} \binom{N}{k}$ for all odd $k \leq N = 128$

- Map bilinear forms to observables

$$t_j^* t_i = \frac{1}{8} \sum_{\alpha=1}^{64} \Gamma_{ij}^{\alpha} \mathcal{O}^{\alpha}$$

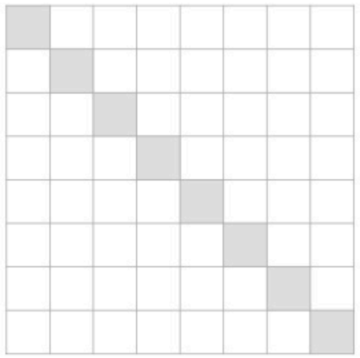
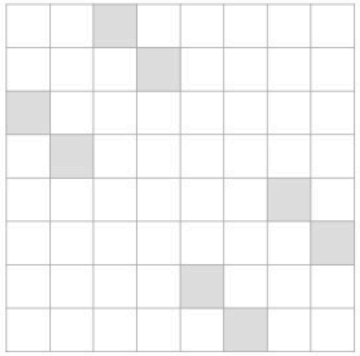
Polarisation Observables for Two-Pion Photoproduction off the Nucleon

First published by Roberts and Oed [Phys. Rev. C **71**, 055201 (2005)]

Observable	Definition in terms of polar coordinates / 2	Bilinear form	Roberts, Oed
$\mathcal{O}^{\mathrm{I}}_{1}$	$\frac{1}{2}(\lvert t_1\rvert^2+\lvert t_2\rvert^2+\lvert t_3\rvert^2+\lvert t_4\rvert^2-\lvert t_5\rvert^2-\lvert t_6\rvert^2-\lvert t_7\rvert^2-\lvert t_8\rvert^2)$	$\langle t\lvert\Gamma^{\mathrm{I}}_{1}\rvert t\rangle$	$\mathrm{I}^{\odot}$
$\mathcal{O}^{\mathrm{I}}_{2}$	$\frac{1}{2}(\lvert t_1\rvert^2+\lvert t_2\rvert^2-\lvert t_3\rvert^2-\lvert t_4\rvert^2+\lvert t_5\rvert^2+\lvert t_6\rvert^2-\lvert t_7\rvert^2-\lvert t_8\rvert^2)$	$\langle t\lvert\Gamma^{\mathrm{I}}_{2}\rvert t\rangle$	$\mathrm{P_y}$
$\mathcal{O}^{\mathrm{I}}_{3}$	$\frac{1}{2}(\lvert t_1\rvert^2-\lvert t_2\rvert^2+\lvert t_3\rvert^2-\lvert t_4\rvert^2+\lvert t_5\rvert^2-\lvert t_6\rvert^2+\lvert t_7\rvert^2-\lvert t_8\rvert^2)$	$\langle t\lvert\Gamma^{\mathrm{I}}_{3}\rvert t\rangle$	$\mathrm{P_{y'}}$
$\mathcal{O}^{\mathrm{I}}_{4}$	$\frac{1}{2}(\lvert t_1\rvert^2-\lvert t_2\rvert^2-\lvert t_3\rvert^2+\lvert t_4\rvert^2-\lvert t_5\rvert^2+\lvert t_6\rvert^2+\lvert t_7\rvert^2-\lvert t_8\rvert^2)$	$\langle t\lvert\Gamma^{\mathrm{I}}_{4}\rvert t\rangle$	$\mathcal{O}^{\odot}_{\mathrm{yy'}}$
$\mathcal{O}^{\mathrm{I}}_{5}$	$\frac{1}{2}(\lvert t_1\rvert^2-\lvert t_2\rvert^2-\lvert t_3\rvert^2+\lvert t_4\rvert^2+\lvert t_5\rvert^2-\lvert t_6\rvert^2-\lvert t_7\rvert^2+\lvert t_8\rvert^2)$	$\langle t\lvert\Gamma^{\mathrm{I}}_{5}\rvert t\rangle$	$\mathcal{O}_{\mathrm{yy'}}$
$\mathcal{O}^{\mathrm{I}}_{6}$	$\frac{1}{2}(\lvert t_1\rvert^2-\lvert t_2\rvert^2+\lvert t_3\rvert^2-\lvert t_4\rvert^2-\lvert t_5\rvert^2+\lvert t_6\rvert^2-\lvert t_7\rvert^2+\lvert t_8\rvert^2)$	$\langle t\lvert\Gamma^{\mathrm{I}}_{6}\rvert t\rangle$	$\mathrm{P}^{\odot}_{\mathrm{y'}}$
$\mathcal{O}^{\mathrm{I}}_{7}$	$\frac{1}{2}(\lvert t_1\rvert^2+\lvert t_2\rvert^2-\lvert t_3\rvert^2-\lvert t_4\rvert^2-\lvert t_5\rvert^2-\lvert t_6\rvert^2+\lvert t_7\rvert^2+\lvert t_8\rvert^2)$	$\langle t\lvert\Gamma^{\mathrm{I}}_{7}\rvert t\rangle$	$\mathrm{P}^{\odot}_{\mathrm{y}}$
$\mathcal{O}^{\mathrm{I}}_{8}$	$\frac{1}{2}(\lvert t_1\rvert^2+\lvert t_2\rvert^2+\lvert t_3\rvert^2+\lvert t_4\rvert^2+\lvert t_5\rvert^2+\lvert t_6\rvert^2+\lvert t_7\rvert^2+\lvert t_8\rvert^2)$	$\langle t\lvert\Gamma^{\mathrm{I}}_{8}\rvert t\rangle$	I_0
$\mathcal{O}^{\mathrm{II}}_{c1}$	$\lvert t_1\rvert\lvert t_3\rvert\cos(\phi_{13})+\lvert t_2\rvert\lvert t_4\rvert\cos(\phi_{24})+\lvert t_5\rvert\lvert t_7\rvert\cos(\phi_{57})+\lvert t_6\rvert\lvert t_8\rvert\cos(\phi_{68})$	$\langle t\lvert\Gamma^{\mathrm{II}}_{c1}\rvert t\rangle$	$-\mathrm{P_z}$
$\mathcal{O}^{\mathrm{II}}_{c2}$	$\lvert t_1\rvert\lvert t_3\rvert\cos(\phi_{13})+\lvert t_2\rvert\lvert t_4\rvert\cos(\phi_{24})-\lvert t_5\rvert\lvert t_7\rvert\cos(\phi_{57})-\lvert t_6\rvert\lvert t_8\rvert\cos(\phi_{68})$	$\langle t\lvert\Gamma^{\mathrm{II}}_{c2}\rvert t\rangle$	$-\mathrm{P}^{\odot}_{\mathrm{z}}$
$\mathcal{O}^{\mathrm{II}}_{c3}$	$\lvert t_1\rvert\lvert t_3\rvert\cos(\phi_{13})-\lvert t_2\rvert\lvert t_4\rvert\cos(\phi_{24})+\lvert t_5\rvert\lvert t_7\rvert\cos(\phi_{57})-\lvert t_6\rvert\lvert t_8\rvert\cos(\phi_{68})$	$\langle t\lvert\Gamma^{\mathrm{II}}_{c3}\rvert t\rangle$	
$\mathcal{O}^{\mathrm{II}}_{c4}$	$\lvert t_1\rvert\lvert t_3\rvert\cos(\phi_{13})-\lvert t_2\rvert\lvert t_4\rvert\cos(\phi_{24})-\lvert t_5\rvert\lvert t_7\rvert\cos(\phi_{57})+\lvert t_6\rvert\lvert t_8\rvert\cos(\phi_{68})$	$\langle t\lvert\Gamma^{\mathrm{II}}_{c4}\rvert t\rangle$	
$\mathcal{O}^{\mathrm{II}}_{s1}$	$\lvert t_1\rvert\lvert t_3\rvert\sin(\phi_{13})+\lvert t_2\rvert\lvert t_4\rvert\sin(\phi_{24})+\lvert t_5\rvert\lvert t_7\rvert\sin(\phi_{57})+\lvert t_6\rvert\lvert t_8\rvert\sin(\phi_{68})$	$\langle t\lvert\Gamma^{\mathrm{II}}_{s1}\rvert t\rangle$	
$\mathcal{O}^{\mathrm{II}}_{s2}$	$\lvert t_1\rvert\lvert t_3\rvert\sin(\phi_{13})+\lvert t_2\rvert\lvert t_4\rvert\sin(\phi_{24})-\lvert t_5\rvert\lvert t_7\rvert\sin(\phi_{57})-\lvert t_6\rvert\lvert t_8\rvert\sin(\phi_{68})$	$\langle t\lvert\Gamma^{\mathrm{II}}_{s2}\rvert t\rangle$	
$\mathcal{O}^{\mathrm{II}}_{s3}$	$\lvert t_1\rvert\lvert t_3\rvert\sin(\phi_{13})-\lvert t_2\rvert\lvert t_4\rvert\sin(\phi_{24})+\lvert t_5\rvert\lvert t_7\rvert\sin(\phi_{57})-\lvert t_6\rvert\lvert t_8\rvert\sin(\phi_{68})$	$\langle t\lvert\Gamma^{\mathrm{II}}_{s3}\rvert t\rangle$	
$\mathcal{O}^{\mathrm{II}}_{s4}$	$\lvert t_1\rvert\lvert t_3\rvert\sin(\phi_{13})-\lvert t_2\rvert\lvert t_4\rvert\sin(\phi_{24})-\lvert t_5\rvert\lvert t_7\rvert\sin(\phi_{57})+\lvert t_6\rvert\lvert t_8\rvert\sin(\phi_{68})$	$\langle t\lvert\Gamma^{\mathrm{II}}_{s4}\rvert t\rangle$	

⋮

PHYSICAL REVIEW C **103**, 014607 (2021)

Γ-matrices	Definition	Shape-class
Γ^{I}_{1}	$\sigma^3\otimes I_2\otimes I_2$	
Γ^{I}_{2}	$I_2\otimes\sigma^3\otimes I_2$	
Γ^{I}_{3}	$I_2\otimes I_2\otimes\sigma^3$	
Γ^{I}_{4}	$\sigma^3\otimes\sigma^3\otimes\sigma^3$	
Γ^{I}_{5}	$I_2\otimes\sigma^3\otimes\sigma^3$	
Γ^{I}_{6}	$\sigma^3\otimes I_2\otimes\sigma^3$	
Γ^{I}_{7}	$\sigma^3\otimes\sigma^3\otimes I_2$	
Γ^{I}_{8}	$I_2\otimes I_2\otimes I_2$	
$\Gamma^{\mathrm{II}}_{c1}$	$I_2\otimes\sigma^1\otimes I_2$	
$\Gamma^{\mathrm{II}}_{c2}$	$\sigma^3\otimes\sigma^1\otimes I_2$	
$\Gamma^{\mathrm{II}}_{c3}$	$I_2\otimes\sigma^1\otimes\sigma^3$	
$\Gamma^{\mathrm{II}}_{c4}$	$\sigma^3\otimes\sigma^1\otimes\sigma^3$	
$\Gamma^{\mathrm{II}}_{s1}$	$-I_2\otimes\sigma^2\otimes I_2$	
$\Gamma^{\mathrm{II}}_{s2}$	$-\sigma^3\otimes\sigma^2\otimes I_2$	
$\Gamma^{\mathrm{II}}_{s3}$	$-I_2\otimes\sigma^2\otimes\sigma^3$	
$\Gamma^{\mathrm{II}}_{s4}$	$-\sigma^3\otimes\sigma^2\otimes\sigma^3$	

⋮

$$\Gamma^{\mathrm{II}}_{c3}=\begin{pmatrix}0&0&1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&1&0&0&0\\0&0&0&0&0&-1&0&0\end{pmatrix}$$

- 64 observables
- 8 shape-classes
- Single-, double- and triple polarisation observables

Intermediate Results for N=8

- 322,560 edge configurations wich yield a complete set of observables
- 5964 unique sets of observables
- 392 distinct sets of length 24 (slightly over-complete)

At least five triple polarisation observables

Constructed from four or five different shape classes

Reduction to Minimal Sets of 2N=16

Algorithm (already applied in Tiator et al., Phys. Rev. C **96**, 025210 (2017))

- System of multivariate homogenous polynomials: $\mathcal{O}_1(\vec{t}\,) = g_1, \ldots, \mathcal{O}_n(\vec{t}\,) = g_n$
- Fix overall phase of amplitudes, i.e. $\mathrm{Re}(t_1) > 0$ and $\mathrm{Im}(t_1) = 0$
- \`NSolve\` from Mathematica is used to solve the system

> "For systems of algebraic equations,
>
> NSolve computes a numerical Gröbner basis using an efficient monomial ordering,
>
> then uses eigensystem methods to extract numerical roots."

[https://reference. wolfram.com/language/tutorial/SomeNotesOnInternal Implementation.html]

- Other methods like Homotopy Continuation are also possible

Implication for Experimentalist

(1)	P_z	$P_{x'}$	$P_{z'}$	P^s_x	P^c_z	$P^\odot_z$	$P^\odot_{x'}$	$P^\odot_{z'}$
(2)	P_z	P^s_x	P^c_z	$P^\odot_z$	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$
(3)	P_z	P^s_x	P^c_z	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(4)	$P_{x'}$	$P_{z'}$	P^s_x	P^c_z	$P^\odot_z$	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{zy'}$
(5)	P^s_x	P^c_z	$P^\odot_z$	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(6)	P_x	P_z	$P_{z'}$	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_z$	$P^\odot_{z'}$
(7)	P_x	P_z	$P_{z'}$	P^s_x	P^c_z	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{zy'}$
(8)	P_x	P_z	$P_{z'}$	P^s_x	P^c_z	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(9)	$P_{z'}$	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_z$	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{zy'}$
(10)	$P_{z'}$	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_z$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(11)	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_z$	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(12)	P_z	P^c_x	P^s_x	P^c_z	P^s_z	$P^\odot_z$	$P^\odot_{z'}$	$\mathcal{O}_{yz'}$
(13)	P_z	P^c_x	P^s_x	P^c_z	P^s_z	$P^\odot_{z'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(14)	P^c_x	P^s_x	P^c_z	P^s_z	$P^\odot_z$	$P^\odot_{z'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(15)	P_x	P_z	$P_{x'}$	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_z$	$P^\odot_{x'}$
(16)	P_x	P_z	$P_{x'}$	P^s_x	P^c_z	$P^\odot_{x'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{zy'}$
(17)	P_x	P_z	$P_{x'}$	P^s_x	P^c_z	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{zy'}$
(18)	$P_{x'}$	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_z$	$P^\odot_{x'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{zy'}$
(19)	$P_{x'}$	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_z$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{zy'}$
(20)	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_z$	$P^\odot_{x'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{zy'}$
(21)	P_z	$P_{x'}$	$P_{z'}$	P^c_x	P^s_z	$P^\odot_z$	$P^\odot_{x'}$	$P^\odot_{z'}$
(22)	P_z	P^c_x	P^s_z	$P^\odot_z$	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$
(23)	P_z	$P_{x'}$	$P_{z'}$	P^c_x	P^s_z	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(24)	P_z	P^c_x	P^s_z	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(25)	$P_{x'}$	$P_{z'}$	P^c_x	P^s_z	$P^\odot_z$	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{zy'}$
(26)	P^c_x	P^s_z	$P^\odot_z$	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(27)	P_z	P^c_x	P^s_x	P^c_z	P^s_z	$P^\odot_z$	$P^\odot_{x'}$	$\mathcal{O}_{yx'}$
(28)	P_z	P^c_x	P^s_x	P^c_z	P^s_z	$P^\odot_{x'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{zy'}$
(29)	P^c_x	P^s_x	P^c_z	P^s_z	$P^\odot_z$	$P^\odot_{x'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{zy'}$
(30)	P_x	P_z	$P_{z'}$	P^c_x	$P^\odot_x$	P^s_z	$P^\odot_z$	$P^\odot_{z'}$
(31)	P_x	P_z	$P_{z'}$	P^c_x	P^s_z	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{zy'}$
(32)	P_x	P_z	$P_{z'}$	P^c_x	P^s_z	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(33)	$P_{z'}$	P^c_x	$P^\odot_x$	P^s_z	$P^\odot_z$	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{zy'}$
(34)	$P_{z'}$	P^c_x	$P^\odot_x$	P^s_z	$P^\odot_z$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(35)	P^c_x	$P^\odot_x$	P^s_z	$P^\odot_z$	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yz'}$	$\mathcal{O}_{zy'}$
(36)	P_x	$P_{x'}$	$P_{z'}$	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_{x'}$	$P^\odot_{z'}$
(37)	P_x	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$
(38)	P_x	P^s_x	P^c_z	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$
(39)	$P_{x'}$	$P_{z'}$	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$
(40)	P^s_x	$P^\odot_x$	P^c_z	$P^\odot_{x'}$	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{yz'}$
(41)	P_x	P_z	$P_{x'}$	P^c_x	$P^\odot_x$	P^s_z	$P^\odot_z$	$P^\odot_{x'}$
(42)	P_x	P_z	$P_{x'}$	P^c_x	P^s_z	$P^\odot_{x'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{zy'}$
(43)	P_x	P_z	$P_{x'}$	P^c_x	P^s_z	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{zy'}$
(44)	$P_{x'}$	P^c_x	$P^\odot_x$	P^s_z	$P^\odot_z$	$P^\odot_{x'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{zy'}$
(45)	$P_{x'}$	P^c_x	$P^\odot_x$	P^s_z	$P^\odot_z$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{zy'}$
(46)	P^c_x	$P^\odot_x$	P^s_z	$P^\odot_z$	$P^\odot_{x'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yx'}$	$\mathcal{O}_{zy'}$

(47)	P_x	P^c_x	P^s_x	$P^\odot_x$	P^c_z	P^s_z	$P^\odot_{z'}$	$\mathcal{O}_{yz'}$
(48)	P_x	P^c_x	P^s_x	P^c_z	P^s_z	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yz'}$
(49)	P^c_x	P^s_x	$P^\odot_x$	P^c_z	P^s_z	$P^\odot_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{yz'}$
(50)	P_x	P_z	$P^\odot_x$	$P^\odot_z$	$P^c_{x'}$	$P^s_{z'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$
(51)	P_x	P_z	$P^c_{x'}$	$P^s_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zy'}$
(52)	$P^\odot_x$	$P^\odot_z$	$P^c_{x'}$	$P^s_{z'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zy'}$
(53)	I^c	P_x	$P^\odot_x$	P^c_y	$\mathcal{O}_{xx'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zz'}$
(54)	I^c	P_x	$P^\odot_x$	$P^c_{y'}$	$\mathcal{O}_{xx'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zz'}$
(55)	P_x	$P^\odot_x$	P^c_y	$P^c_{y'}$	$\mathcal{O}_{xx'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zz'}$
(56)	I^c	P_x	P^c_y	$\mathcal{O}_{xx'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zz'}$
(57)	I^c	P_x	$P^c_{y'}$	$\mathcal{O}_{xx'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zz'}$
(58)	P_x	P^c_y	$P^c_{y'}$	$\mathcal{O}_{xx'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zz'}$
(59)	I^c	$P^\odot_x$	P^c_y	$\mathcal{O}_{xx'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zz'}$
(60)	$P^\odot_x$	P^c_y	$P^c_{y'}$	$\mathcal{O}_{xx'}$	$\mathcal{O}_{xy'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$	$\mathcal{O}_{zz'}$
(61)	I^c	P_x	$P^\odot_x$	$P^c_{x'}$	$P^s_{x'}$	$P^c_{y'}$	$P^c_{z'}$	$P^s_{z'}$
(62)	P_x	$P^\odot_x$	P^c_y	$P^c_{x'}$	$P^s_{x'}$	$P^c_{y'}$	$P^c_{z'}$	$P^s_{z'}$
(63)	I^c	P_x	P^c_y	$P^c_{x'}$	$P^s_{x'}$	$P^c_{z'}$	$P^s_{z'}$	$\mathcal{O}_{xy'}$
(64)	I^c	$P^\odot_x$	P^c_y	$P^c_{x'}$	$P^s_{x'}$	$P^c_{z'}$	$P^s_{z'}$	$\mathcal{O}_{xy'}$
(65)	I^c	$P^\odot_x$	$P^c_{x'}$	$P^s_{x'}$	$P^c_{y'}$	$P^c_{z'}$	$P^s_{z'}$	$\mathcal{O}_{xy'}$
(66)	$P^\odot_x$	P^c_y	$P^c_{x'}$	$P^s_{x'}$	$P^c_{y'}$	$P^c_{z'}$	$P^s_{z'}$	$\mathcal{O}_{xy'}$
(67)	I^c	I^s	P^c_y	P^s_y	$P^c_{x'}$	$P^s_{z'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$
(68)	I^c	I^s	$P^c_{x'}$	$P^c_{y'}$	$P^s_{y'}$	$P^s_{z'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$
(69)	P^c_y	P^s_y	$P^c_{x'}$	$P^c_{y'}$	$P^s_{y'}$	$P^s_{z'}$	$\mathcal{O}_{xz'}$	$\mathcal{O}_{zx'}$

PHYSICAL REVIEW C **103**, 014607 (2021)

+ observable class 1

$\gamma p \rightarrow \pi^0 \pi^0 p$

$$\{I^\odot, P_y, P_{y'}, \mathcal{O}^\odot_{yy'}, \mathcal{O}_{yy'}, P^\odot_{y'}, P^\odot_y, I_0, P_x, P_z, P_{x'}, P^s_x, P^\odot_x, P^c_z, P^\odot_z, P^\odot_{x'}\}.$$

three more measurements needed for remaining 8 observables

$\mathcal{B}_{lin}\mathcal{T}_{long}$: P_z, P^c_z

$\mathcal{B}_\odot \mathcal{T}_{trans}$: $P^\odot_x, P^\odot_y$

$\mathcal{B}_\odot \mathcal{T}_{trans} \mathcal{R}$: $P_{x'}, P^\odot_{x'}, P^\odot_{y'}, \mathcal{O}^\odot_{yy'}$

Analytic Approach

- Use phase-fixing approach by Nakayama
  [Phys. Rev. C **100**, 035208 (2019)]

- Construct "decoupled" shape-classes
  depend on two relative phases (not four)

$$\mathrm{IIa}: \mathscr{O}^{\mathrm{II}}_{\mathrm{s1}} + \mathscr{O}^{\mathrm{II}}_{\mathrm{s2}}, \mathscr{O}^{\mathrm{II}}_{\mathrm{s3}} + \mathscr{O}^{\mathrm{II}}_{\mathrm{s4}}, \mathscr{O}^{\mathrm{II}}_{\mathrm{c1}} + \mathscr{O}^{\mathrm{II}}_{\mathrm{c2}}, \mathscr{O}^{\mathrm{II}}_{\mathrm{c3}} + \mathscr{O}^{\mathrm{II}}_{\mathrm{c4}},$$

$$\mathrm{IIb}: \mathscr{O}^{\mathrm{II}}_{\mathrm{s1}} - \mathscr{O}^{\mathrm{II}}_{\mathrm{s2}}, \mathscr{O}^{\mathrm{II}}_{\mathrm{s3}} - \mathscr{O}^{\mathrm{II}}_{\mathrm{s4}}, \mathscr{O}^{\mathrm{II}}_{\mathrm{c1}} - \mathscr{O}^{\mathrm{II}}_{\mathrm{c2}}, \mathscr{O}^{\mathrm{II}}_{\mathrm{c3}} - \mathscr{O}^{\mathrm{II}}_{\mathrm{c4}}.$$

- Choose [Xa, Xb, Y, Z], e.g. $\left[IIa, IIb, VIIIa, VIb\right]$

$$\underbrace{\phi_{13} + \phi_{24}}_{\mathrm{IIa}} + \underbrace{\phi_{57} + \phi_{68}}_{\mathrm{IIb}} = \underbrace{\phi_{18} + \phi_{27}}_{\mathrm{VIIIa}} - \underbrace{\phi_{35} - \phi_{46}}_{\mathrm{VIb}}$$

⋮

see paper

$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathrm{P^s_z}$	$\mathrm{P^c_x}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xz'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathrm{P^s_z}$	$\mathrm{P^c_x}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xx'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathrm{P^s_z}$	$\mathrm{P^c_x}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xz'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathrm{P^s_z}$	$\mathrm{P^c_x}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xx'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathrm{P^c_z}$	$\mathrm{P^s_x}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xz'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathrm{P^c_z}$	$\mathrm{P^s_x}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xx'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathrm{P^c_z}$	$\mathrm{P^s_x}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xz'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathrm{P^c_z}$	$\mathrm{P^s_x}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xx'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xz'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xx'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xz'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xx'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xz'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xx'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xz'}}$
$\mathrm{P_x}$	$\mathrm{P_z}$	$\mathrm{P^{\odot}_x}$	$\mathrm{P^{\odot}_z}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xx'}}$
$\mathrm{P^s_z}$	$\mathrm{P^c_x}$	$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xz'}}$
$\mathrm{P^s_z}$	$\mathrm{P^c_x}$	$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xx'}}$
$\mathrm{P^s_z}$	$\mathrm{P^c_x}$	$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xz'}}$
$\mathrm{P^s_z}$	$\mathrm{P^c_x}$	$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xx'}}$
$\mathrm{P^c_z}$	$\mathrm{P^s_x}$	$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xz'}}$
$\mathrm{P^c_z}$	$\mathrm{P^s_x}$	$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xx'}}$
$\mathrm{P^c_z}$	$\mathrm{P^s_x}$	$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xz'}}$
$\mathrm{P^c_z}$	$\mathrm{P^s_x}$	$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xx'}}$
$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xz'}}$
$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xx'}}$
$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xz'}}$
$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xx'}}$
$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xz'}}$
$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xx'}}$
$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zz'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xz'}}$
$\mathcal{O}_{\mathrm{xy'}}$	$\mathcal{O}_{\mathrm{zy'}}$	$\mathcal{O}^{\odot}_{\mathrm{xy'}}$	$\mathcal{O}^{\odot}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{zy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{xy'}}$	$\mathcal{O}^{\mathrm{s}}_{\mathrm{zx'}}$	$\mathcal{O}^{\mathrm{c}}_{\mathrm{xx'}}$

PHYSICAL REVIEW C **103**, 014607 (2021)

Summary

- Extensive list of two-pseudoscalar-meson photoproduction measurements
- Tackled extremely hard complete experiment analysis for N=8
- 69 minimal complete sets with 16 observables (only 1 triple polarisation observable!)
- Test of complete experiment analysis is within reach
- Most promising set for future measurements:

$$\boxed{I^{\odot}, P_{y}, P_{y'}, \mathcal{O}^{\odot}_{yy'}, \mathcal{O}_{yy'}, P^{\odot}_{y'}, P^{\odot}_{y}, I_{0}, P_{x}, P_{z}, P_{x'}, P^{s}_{x}, P^{\odot}_{x}, P^{c}_{z}, P^{\odot}_{z}, P^{\odot}_{x'}}$$

Thank you for your attention!

Backup-Slides

Backup | Homotopy Continuation

$$F(x_1, \ldots, x_n) = \begin{bmatrix} f_1(x_1, \ldots, x_n) \\ \vdots \\ f_n(x_1, \ldots, x_n) \end{bmatrix}$$

System of interest

$$G(x_1, \ldots, x_n) = \begin{bmatrix} g_1(x_1, \ldots, x_n) \\ \vdots \\ g_n(x_1, \ldots, x_n) \end{bmatrix}$$

Already know the solutions

At least as many solutions as $F(x_1, \ldots, x_n)$

Can always find homotopy H wich satisfies:

$$H(\vec{x},1) = G(\vec{x})$$
$$H(\vec{x},0) = F(\vec{x})$$

Track each solution path from $G(\vec{x})$ to $F(\vec{x})$, via the homotopy H

taken from https://www.juliahomotopycontinuation.org/guides/introduction/