Determination of complete experiments using graphs

Talk based on work done in collaboration with:

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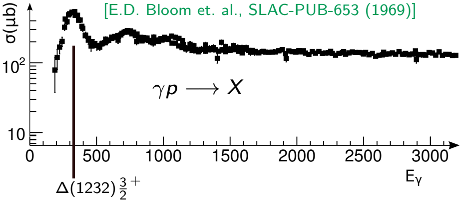
HISKP, University of Bonn

July 28, 2021

Introduction: why spin-amplitudes?

∗) Photoproduction is a generic reaction used to study baryon resonances:



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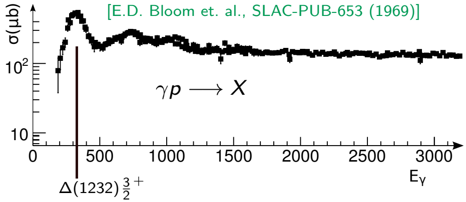

∗) Baryon resonances $\left(\Delta(1232)\frac{3}{2}^+, N(1440)\frac{1}{2}^+, \ldots\right)$ are *Fermions* (or 'fermionic modes')

- ↔ Resonances have half-integer spin
- ∗) <u>However:</u> a pure orbital angular-momentum among scattered particles can only generate integer spins:

$$\ell = 0, 1, 2, \ldots; \ \left|\vec{L}\right|^2 = \ell(\ell+1)\hbar^2.$$

- ⇒ Need to study reactions involving particles with spin in the initial- and/or final state!

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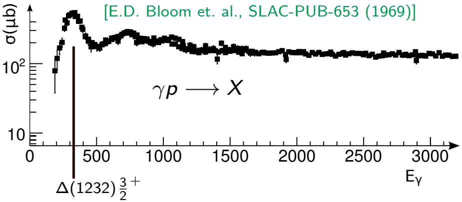

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∗) The full amplitude $\mathcal{T}_{fi}$ ('T-matrix') for reactions among particles with spin can be decomposed into N spin-amplitudes $\{b_i, i = 1, \ldots, N\}$

∗) The usual reactions under study are:

- Pion-Nucleon (πN-) scattering: $\pi N \longrightarrow \pi N$ (2 spin-amplitudes)
- Pion photoproduction: $\gamma N \longrightarrow \pi N$ (4 spin-amplitudes)
- Pion electroproduction: $eN \longrightarrow e'\pi N$ (6 spin-amplitudes)
- 2-Pion photoproduction: $\gamma N \longrightarrow \pi\pi N$ (8 spin-amplitudes)
- ...

Algebraic starting point I

$*)$ Generic problem with N amplitudes $\{b_i, i = 1, \ldots, N\}$: the N^2 (polarization-) observables are bilinear hermitean forms (def. via orthogonal matrices $\tilde{\Gamma}^{\alpha}$):

$$\mathcal{O}^{\alpha} = \boldsymbol{c}^{\alpha} \sum_{i,j=1}^{N} b_i^{*} \tilde{\Gamma}^{\alpha}_{ij} b_j, \text{ for } \alpha = 1, \ldots, N^2.$$

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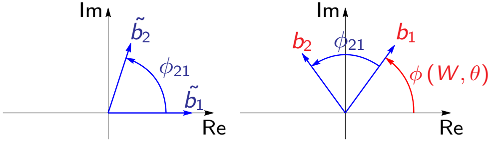
↪ Complete-experiment problem:
What are the minimal subsets of the observables $\mathcal{O}^{\alpha}$, which allow for the unique extraction of the amplitudes b_i up to one unknown overall phase $\phi(W, \theta)$?

- ∗) Analysis operates on each bin in (W, θ) individually.
- ∗) Consider idealized (academic) case without measurement uncertainty!



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$*)$ Expression $\mathcal{O}^{\alpha} = \boldsymbol{c}^{\alpha} \sum_{i,j=1}^{N} b_i^* \tilde{\Gamma}_{ij}^{\alpha} b_j$ can be 'inverted' (using the *completeness* of the $\tilde{\Gamma}$-matrices):

$$b_i^* b_j = \frac{1}{\tilde{N}} \sum_{\alpha=1}^{N^2} \left(\tilde{\Gamma}_{ij}^{\alpha} \right)^* \left(\frac{\mathcal{O}^{\alpha}}{\boldsymbol{c}^{\alpha}} \right) .$$

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$\ast$) Finding a generic solution for such problems, for arbitrary N, can be quite tough in the $\mathcal{O}^{\alpha}$-basis.
However: In the $b_i^{\ast} b_j$-basis, a general solution exists:

Moravcsik's Theorem!

Discrete ambiguities

'Cosine-type' ambiguities:

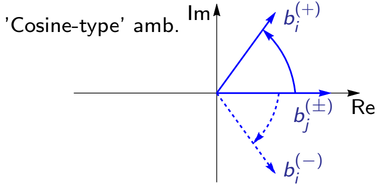
The real part

$$\mathrm{Re}\left[b_j^* b_i\right] = |b_i|\,|b_j|\,\mathrm{Re}\left[e^{i\phi_{ij}}\right] = |b_i|\,|b_j|\cos\phi_{ij},$$

fixes the relative phase ϕ_{ij} up to the discrete ambiguity:

$$\phi_{ij} \longrightarrow \phi_{ij}^{\pm} = \begin{cases} +\alpha_{ij}, \\ -\alpha_{ij}, \end{cases}$$

with a unique $\alpha_{ij} \in [0, \pi]$.



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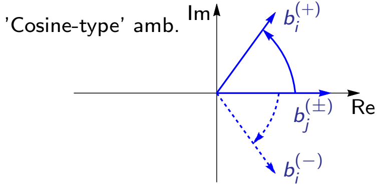

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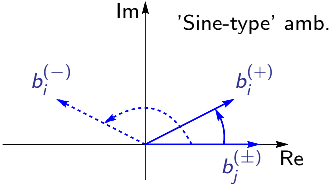

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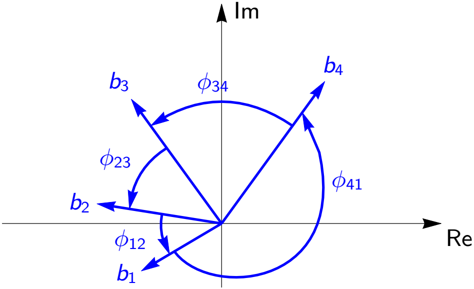
$\hookrightarrow$ Discrete ambiguities for a subset of real-and imaginary parts of bilinear products $b_j^* b_i$, defined by N amplitudes $\{b_i, i = 1, \ldots, N\}$, are 'direct (or Kronecker-) products' of these fundamental discrete ambiguities.

$\Rightarrow$ Such ambiguities turn up time and again in the discussion of complete experiments! Is there help? Yes! $\rightarrow$ Consistency Relations

Consistency relations

∗) Consider amplitude-arrangement in the complex plane (e.g.: $N = 4$):



∗) Natural constraint satisfied by this constellation: consistency relation

$$\phi_{12} + \phi_{23} + \phi_{34} + \phi_{41} = 0 \text{ (up to add. of multiples of } 2\pi\text{).}$$

Consistency relations

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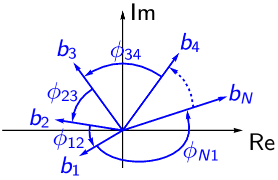

$*)$ Fundamental consistency relation for a problem with N amplitudes:

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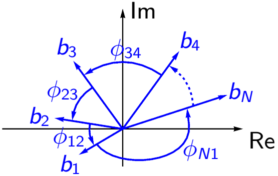

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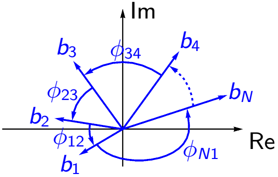

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↪ Moravcsik's Theorem is a systematic study of all cases where such non-degeneracies are obtained, in the $b_j^* b_i$-basis.

Moravcsik's Theorem (modified form)

From [YW, P. Kroenert, F. Afzal, A. Thiel, Phys. Rev. C **102**, no.3, 034605 (2020)], based on [Moravcsik, J. Math. Phys. **26**, 211 (1985).]:

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'Geometrical (graphical) analog': Represent every amplitude from $b_1, \ldots, b_N$ by a *point* and every relative-phase ϕ_{ij} by a *line connecting points* 'i' and 'j'.

Furthermore: $\hookrightarrow$ Represent every $\mathrm{Re}\left[b_i^* b_j\right] \propto \cos\phi_{ij}$ by a *solid line*,

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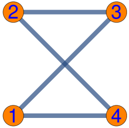
(i) the graph is fully *connected* and all points (i.e. vertices) have order to have *order two* (i.e. are attached to two lines):

- all continuous ambiguities are resolved,
- existence of consistency relation is ensured.

Example:



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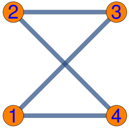

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(ii) the graph has to have an *odd* number of dashed lines, as well as *any* number of solid lines:

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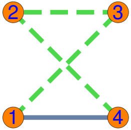
E.g.:



is complete ✓

Proof: see App. A of [Phys. Rev. C **102**, no.3, 034605 (2020)].

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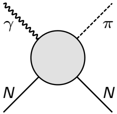

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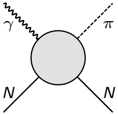

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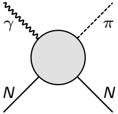

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$\hookrightarrow$ The $N^2 = 16$ polarization observables (or polarization asymmetries)

$$\mathcal{O} = \left[\left(\frac{d\sigma}{d\Omega}\right)^{(B_1,T_1,R_1)} - \left(\frac{d\sigma}{d\Omega}\right)^{(B_2,T_2,R_2)} \right],$$

take a particularly convenient form in the transversity basis.

Example 1: photoproduction (observables)

Observable	Bilinear form	Shape-class
$\sigma_0 = \frac{1}{2}\left(\lvert b_1\rvert^2 + \lvert b_2\rvert^2 + \lvert b_3\rvert^2 + \lvert b_4\rvert^2\right)$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^1\,\lvert b\rangle$	
$-\check{\Sigma} = \frac{1}{2}\left(\lvert b_1\rvert^2 + \lvert b_2\rvert^2 - \lvert b_3\rvert^2 - \lvert b_4\rvert^2\right)$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^4\,\lvert b\rangle$	$\mathcal{S} = \mathrm{D}$
$-\check{T} = \frac{1}{2}\left(-\lvert b_1\rvert^2 + \lvert b_2\rvert^2 + \lvert b_3\rvert^2 - \lvert b_4\rvert^2\right)$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^{10}\,\lvert b\rangle$	
$\check{P} = \frac{1}{2}\left(-\lvert b_1\rvert^2 + \lvert b_2\rvert^2 - \lvert b_3\rvert^2 + \lvert b_4\rvert^2\right)$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^{12}\,\lvert b\rangle$	
$\mathcal{O}^a_{1+} = \lvert b_1\rvert\lvert b_3\rvert \sin\phi_{13} + \lvert b_2\rvert\lvert b_4\rvert \sin\phi_{24} = \mathrm{Im}\left[b_3^* b_1 + b_4^* b_2\right] = -\check{G}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^3\,\lvert b\rangle$	
$\mathcal{O}^a_{1-} = \lvert b_1\rvert\lvert b_3\rvert \sin\phi_{13} - \lvert b_2\rvert\lvert b_4\rvert \sin\phi_{24} = \mathrm{Im}\left[b_3^* b_1 - b_4^* b_2\right] = \check{F}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^{11}\,\lvert b\rangle$	$a = \mathcal{BT} = \mathrm{PR}$
$\mathcal{O}^a_{2+} = \lvert b_1\rvert\lvert b_3\rvert \cos\phi_{13} + \lvert b_2\rvert\lvert b_4\rvert \cos\phi_{24} = \mathrm{Re}\left[b_3^* b_1 + b_4^* b_2\right] = -\check{E}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^9\,\lvert b\rangle$	
$\mathcal{O}^a_{2-} = \lvert b_1\rvert\lvert b_3\rvert \cos\phi_{13} - \lvert b_2\rvert\lvert b_4\rvert \cos\phi_{24} = \mathrm{Re}\left[b_3^* b_1 - b_4^* b_2\right] = \check{H}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^5\,\lvert b\rangle$	
$\mathcal{O}^b_{1+} = \lvert b_1\rvert\lvert b_4\rvert \sin\phi_{14} + \lvert b_2\rvert\lvert b_3\rvert \sin\phi_{23} = \mathrm{Im}\left[b_4^* b_1 + b_3^* b_2\right] = \check{O}_{z'}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^7\,\lvert b\rangle$	
$\mathcal{O}^b_{1-} = \lvert b_1\rvert\lvert b_4\rvert \sin\phi_{14} - \lvert b_2\rvert\lvert b_3\rvert \sin\phi_{23} = \mathrm{Im}\left[b_4^* b_1 - b_3^* b_2\right] = -\check{C}_{x'}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^{16}\,\lvert b\rangle$	$b = \mathcal{BR} = \mathrm{AD}$
$\mathcal{O}^b_{2+} = \lvert b_1\rvert\lvert b_4\rvert \cos\phi_{14} + \lvert b_2\rvert\lvert b_3\rvert \cos\phi_{23} = \mathrm{Re}\left[b_4^* b_1 + b_3^* b_2\right] = -\check{C}_{z'}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^2\,\lvert b\rangle$	
$\mathcal{O}^b_{2-} = \lvert b_1\rvert\lvert b_4\rvert \cos\phi_{14} - \lvert b_2\rvert\lvert b_3\rvert \cos\phi_{23} = \mathrm{Re}\left[b_4^* b_1 - b_3^* b_2\right] = -\check{O}_{x'}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^{14}\,\lvert b\rangle$	
$\mathcal{O}^c_{1+} = \lvert b_1\rvert\lvert b_2\rvert \sin\phi_{12} + \lvert b_3\rvert\lvert b_4\rvert \sin\phi_{34} = \mathrm{Im}\left[b_2^* b_1 + b_4^* b_3\right] = -\check{L}_{x'}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^8\,\lvert b\rangle$	
$\mathcal{O}^c_{1-} = \lvert b_1\rvert\lvert b_2\rvert \sin\phi_{12} - \lvert b_3\rvert\lvert b_4\rvert \sin\phi_{34} = \mathrm{Im}\left[b_2^* b_1 - b_4^* b_3\right] = -\check{T}_{z'}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^{13}\,\lvert b\rangle$	$c = \mathcal{TR} = \mathrm{PL}$
$\mathcal{O}^c_{2+} = \lvert b_1\rvert\lvert b_2\rvert \cos\phi_{12} + \lvert b_3\rvert\lvert b_4\rvert \cos\phi_{34} = \mathrm{Re}\left[b_2^* b_1 + b_4^* b_3\right] = -\check{L}_{z'}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^{15}\,\lvert b\rangle$	
$\mathcal{O}^c_{2-} = \lvert b_1\rvert\lvert b_2\rvert \cos\phi_{12} - \lvert b_3\rvert\lvert b_4\rvert \cos\phi_{34} = \mathrm{Re}\left[b_2^* b_1 - b_4^* b_3\right] = \check{T}_{x'}$	$\frac{1}{2}\langle b\rvert\,\tilde{\Gamma}^6\,\lvert b\rangle$	

Example 1: photoproduction (further preliminaries)

$*$) Standard assumption: moduli are known from group $\mathcal{S}$ observables:

$$|b_1| = \frac{1}{2} \left(\sigma_0 - \check{\Sigma} + \check{T} - \check{P} \right), |b_2| = \frac{1}{2} \left(\sigma_0 - \check{\Sigma} - \check{T} + \check{P} \right),$$

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$*)$ Define a basis of 'decoupled' observables $\tilde{\mathcal{O}}^n_{\nu\pm}$, which isolate the real- and imaginary parts of the bilinear products $b_j^* b_i$:

$$\tilde{\mathcal{O}}^n_{1\pm} := \frac{1}{2}\left(\mathcal{O}^n_{1+} \pm \mathcal{O}^n_{1-}\right), \; n = a, b, c,$$
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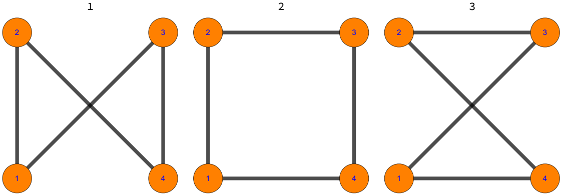
- Example:

$$\mathrm{Im}\left[b_4^* b_2\right] = |b_2|\,|b_4| \sin\phi_{24} = \tilde{\mathcal{O}}^a_{1-} = \tfrac{1}{2}\left(\mathcal{O}^a_{1+} - \mathcal{O}^a_{1-}\right) = \tfrac{1}{2}\left(-\check{G} - \check{F}\right).$$

Example 1: photoproduction (à la Moravcsik) I

$*$) For $N = 4$ amplitudes, one gets $\frac{(N-1)!}{2} = \frac{3!}{2} = 3$ possible graph-topologies :



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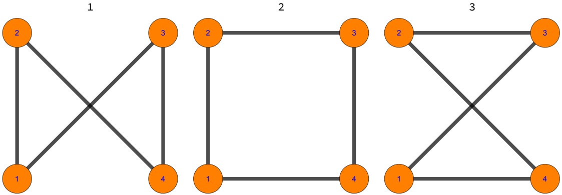
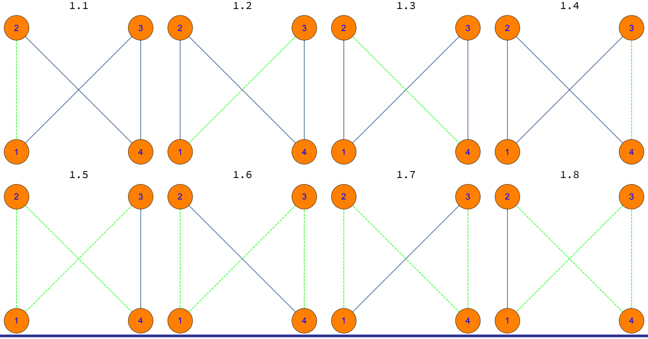
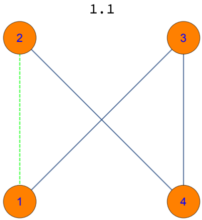


$*$) <u>Example:</u> (fully) complete graphs coming from topology 1:



Example 1: photoproduction (à la Moravcsik) II

*) Example (1.1) (fully complete):



$\longrightarrow \{\sin \phi_{12}, \cos \phi_{24}, \cos \phi_{34}, \cos \phi_{13}\}$

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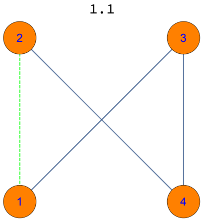

$\longrightarrow \{\sin\phi_{12}, \cos\phi_{24}, \cos\phi_{34}, \cos\phi_{13}\}$

$\hookrightarrow$ Map this result to observables (in $\tilde{\mathcal{O}}$- and $\mathcal{O}$-basis):

$$|b_1|\,|b_2|\sin\phi_{12} = \tilde{\mathcal{O}}^c_{1+} = (1/2)\left[\mathcal{O}^c_{1+} + \mathcal{O}^c_{1-}\right] = (1/2)\left[-\check{L}_{x'} - \check{T}_{z'}\right],$$
$$|b_2|\,|b_4|\cos\phi_{24} = \tilde{\mathcal{O}}^a_{2-} = (1/2)\left[\mathcal{O}^a_{2+} - \mathcal{O}^a_{2-}\right] = (1/2)\left[-\check{E} - \check{H}\right],$$
$$|b_3|\,|b_4|\cos\phi_{34} = \tilde{\mathcal{O}}^c_{2-} = (1/2)\left[\mathcal{O}^c_{2+} - \mathcal{O}^c_{2-}\right] = (1/2)\left[-\check{L}_{z'} - \check{T}_{x'}\right],$$
$$|b_1|\,|b_4|\cos\phi_{13} = \tilde{\mathcal{O}}^a_{2+} = (1/2)\left[\mathcal{O}^a_{2+} + \mathcal{O}^a_{2-}\right] = (1/2)\left[-\check{E} + \check{H}\right].$$

Example 1: photoproduction (à la Moravcsik) II

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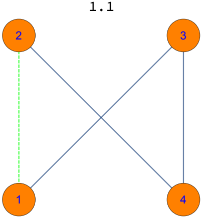

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$$|b_1|\,|b_4| \cos \phi_{13} = \tilde{\mathcal{O}}^a_{2+} = (1/2)\left[\mathcal{O}^a_{2+} + \mathcal{O}^a_{2-}\right] = (1/2)\left[-\check{E} + \check{H}\right].$$

$\Rightarrow$ Extract the 'Moravcsik-complete' set (combined with $\{\sigma_0, \check{\Sigma}, \check{T}, \check{P}\}$):

$$\left\{\mathcal{O}^a_{2+}, \mathcal{O}^a_{2-}, \mathcal{O}^c_{1+}, \mathcal{O}^c_{1-}, \mathcal{O}^c_{2+}, \mathcal{O}^c_{2-}\right\} \equiv \left\{\check{E}, \check{H}, \check{L}_{x'}, \check{T}_{z'}, \check{L}_{z'}, \check{T}_{x'}\right\}.$$

Example 1: photoproduction (à la Moravcsik) III

∗) Similar procedure, applied to all the remaining relevant graphs, leads to 12 non-redundant 'Moravcsik-complete' sets for photoproduction (always in combination with $\left\{ \sigma_0, \check{\Sigma}, \check{T}, \check{P} \right\}$):

Set-Nr.	Observables			Set-Nr.	Observables		
1	$\mathcal{O}^a_{2\pm}$	$\mathcal{O}^c_{1\pm}$	$\mathcal{O}^c_{2\pm}$	7	$\mathcal{O}^b_{1\pm}$	$\mathcal{O}^c_{1\pm}$	$\mathcal{O}^c_{2\pm}$
2	$\mathcal{O}^a_{1\pm}$	$\mathcal{O}^a_{2\pm}$	$\mathcal{O}^c_{2\pm}$	8	$\mathcal{O}^b_{1\pm}$	$\mathcal{O}^b_{2\pm}$	$\mathcal{O}^c_{1\pm}$
3	$\mathcal{O}^a_{1\pm}$	$\mathcal{O}^c_{1\pm}$	$\mathcal{O}^c_{2\pm}$	9	$\mathcal{O}^a_{1\pm}$	$\mathcal{O}^a_{2\pm}$	$\mathcal{O}^b_{2\pm}$
4	$\mathcal{O}^a_{1\pm}$	$\mathcal{O}^a_{2\pm}$	$\mathcal{O}^c_{1\pm}$	10	$\mathcal{O}^a_{2\pm}$	$\mathcal{O}^b_{1\pm}$	$\mathcal{O}^b_{2\pm}$
5	$\mathcal{O}^b_{2\pm}$	$\mathcal{O}^c_{1\pm}$	$\mathcal{O}^c_{2\pm}$	11	$\mathcal{O}^a_{1\pm}$	$\mathcal{O}^a_{2\pm}$	$\mathcal{O}^b_{1\pm}$
6	$\mathcal{O}^b_{1\pm}$	$\mathcal{O}^b_{2\pm}$	$\mathcal{O}^c_{2\pm}$	12	$\mathcal{O}^a_{1\pm}$	$\mathcal{O}^b_{1\pm}$	$\mathcal{O}^b_{2\pm}$

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Observation: Moravcsik-complete sets contain 2 observables more than complete sets with an absolutely minimal amount of observables, i.e. with $2N = 8$ observables [Chiang & Tabakin (1997), Nakayama (2018)].

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$\leftrightarrow$ What happened?!

- Not fully clear yet. Possible method to reduce this mismatch $\rightarrow$ later

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Now: consider next more complicated example of *electroproduction* ...

Example 2: electroproduction (formalism)

$*)$ Reaction: $eN \longrightarrow e'\pi N \Rightarrow$ no. of amplitudes $= \underbrace{3}_{\gamma^*} * \underbrace{2}_{N} * \underbrace{2}_{N} / \underbrace{2}_{\text{Parity}} = 6$.

One has: 6 amplitudes $b_1, \ldots, b_6$ vs. 36 observables.

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Observable	Bilinear form	Shape-class
$R_T^{00} = \frac{1}{2}\left(\lvert b_1\rvert^2 + \lvert b_2\rvert^2 + \lvert b_3\rvert^2 + \lvert b_4\rvert^2\right)$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^1 \,\lvert b\rangle$	
$-\,{}^cR_{TT}^{00} = \frac{1}{2}\left(\lvert b_1\rvert^2 + \lvert b_2\rvert^2 - \lvert b_3\rvert^2 - \lvert b_4\rvert^2\right)$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^4 \,\lvert b\rangle$	D1
$-R_T^{0y} = \frac{1}{2}\left(-\lvert b_1\rvert^2 + \lvert b_2\rvert^2 + \lvert b_3\rvert^2 - \lvert b_4\rvert^2\right)$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^{10} \,\lvert b\rangle$	
$-R_T^{y'0} = \frac{1}{2}\left(-\lvert b_1\rvert^2 + \lvert b_2\rvert^2 - \lvert b_3\rvert^2 + \lvert b_4\rvert^2\right)$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^{12} \,\lvert b\rangle$	
$\mathcal{O}_{1+}^a = \lvert b_1\rvert\,\lvert b_3\rvert \sin\phi_{13} + \lvert b_2\rvert\,\lvert b_4\rvert \sin\phi_{24} = \mathrm{Im}\left[b_3^* b_1 + b_4^* b_2\right] = -\,{}^sR_{TT}^{0z}$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^3 \,\lvert b\rangle$	
$\mathcal{O}_{1-}^a = \lvert b_1\rvert\,\lvert b_3\rvert \sin\phi_{13} - \lvert b_2\rvert\,\lvert b_4\rvert \sin\phi_{24} = \mathrm{Im}\left[b_3^* b_1 - b_4^* b_2\right] = R_{TT'}^{0x}$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^{11} \,\lvert b\rangle$	$a = \mathrm{PR1}$
$\mathcal{O}_{2+}^a = \lvert b_1\rvert\,\lvert b_3\rvert \cos\phi_{13} + \lvert b_2\rvert\,\lvert b_4\rvert \cos\phi_{24} = \mathrm{Re}\left[b_3^* b_1 + b_4^* b_2\right] = R_{TT'}^{0z}$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^9 \,\lvert b\rangle$	
$\mathcal{O}_{2-}^a = \lvert b_1\rvert\,\lvert b_3\rvert \cos\phi_{13} - \lvert b_2\rvert\,\lvert b_4\rvert \cos\phi_{24} = \mathrm{Re}\left[b_3^* b_1 - b_4^* b_2\right] = {}^sR_{TT}^{0x}$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^5 \,\lvert b\rangle$	
$\mathcal{O}_{1+}^b = \lvert b_1\rvert\,\lvert b_4\rvert \sin\phi_{14} + \lvert b_2\rvert\,\lvert b_3\rvert \sin\phi_{23} = \mathrm{Im}\left[b_4^* b_1 + b_3^* b_2\right] = -\,{}^sR_{TT}^{z'0}$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^7 \,\lvert b\rangle$	
$\mathcal{O}_{1-}^b = \lvert b_1\rvert\,\lvert b_4\rvert \sin\phi_{14} - \lvert b_2\rvert\,\lvert b_3\rvert \sin\phi_{23} = \mathrm{Im}\left[b_4^* b_1 - b_3^* b_2\right] = -R_{TT'}^{x'0}$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^{16} \,\lvert b\rangle$	$b = \mathrm{AD1}$
$\mathcal{O}_{2+}^b = \lvert b_1\rvert\,\lvert b_4\rvert \cos\phi_{14} + \lvert b_2\rvert\,\lvert b_3\rvert \cos\phi_{23} = \mathrm{Re}\left[b_4^* b_1 + b_3^* b_2\right] = R_{TT'}^{z'0}$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^2 \,\lvert b\rangle$	
$\mathcal{O}_{2-}^b = \lvert b_1\rvert\,\lvert b_4\rvert \cos\phi_{14} - \lvert b_2\rvert\,\lvert b_3\rvert \cos\phi_{23} = \mathrm{Re}\left[b_4^* b_1 - b_3^* b_2\right] = -\,{}^sR_{TT}^{x'0}$	$\frac{1}{2}\langle b\rvert\, \tilde{\Gamma}^{14} \,\lvert b\rangle$	

Example 2: electroproduction (formalism)

$\mathcal{O}^c_{1+} = \lvert b_1\rvert \lvert b_2\rvert \sin\phi_{12} + \lvert b_3\rvert \lvert b_4\rvert \sin\phi_{34} = \mathrm{Im}\left[b_2^* b_1 + b_4^* b_3\right] = -R_T^{x'z}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^8 \lvert b\rangle$	
$\mathcal{O}^c_{1-} = \lvert b_1\rvert \lvert b_2\rvert \sin\phi_{12} - \lvert b_3\rvert \lvert b_4\rvert \sin\phi_{34} = \mathrm{Im}\left[b_2^* b_1 - b_4^* b_3\right] = R_T^{z'x}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{13} \lvert b\rangle$	$c = \mathrm{PL1}$
$\mathcal{O}^c_{2+} = \lvert b_1\rvert \lvert b_2\rvert \cos\phi_{12} + \lvert b_3\rvert \lvert b_4\rvert \cos\phi_{34} = \mathrm{Re}\left[b_2^* b_1 + b_4^* b_3\right] = R_T^{z'z}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{15} \lvert b\rangle$	
$\mathcal{O}^c_{2-} = \lvert b_1\rvert \lvert b_2\rvert \cos\phi_{12} - \lvert b_3\rvert \lvert b_4\rvert \cos\phi_{34} = \mathrm{Re}\left[b_2^* b_1 - b_4^* b_3\right] = R_T^{x'x}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^6 \lvert b\rangle$	
$R_L^{00} = \lvert b_5\rvert^2 + \lvert b_6\rvert^2$	$\frac{1}{\sqrt{2}}\langle b\rvert \tilde{\Gamma}^{17} \lvert b\rangle$	D2
$R_L^{0y} = \lvert b_5\rvert^2 - \lvert b_6\rvert^2$	$\frac{1}{\sqrt{2}}\langle b\rvert \tilde{\Gamma}^{18} \lvert b\rangle$	
$\mathcal{O}^d_1 = 2\lvert b_5\rvert \lvert b_6\rvert \sin\phi_{56} = 2\mathrm{Im}\left[b_6^* b_5\right] = R_L^{z'x}$	$\frac{1}{\sqrt{2}}\langle b\rvert \tilde{\Gamma}^{20} \lvert b\rangle$	$d = \mathrm{AD2}$
$\mathcal{O}^d_2 = 2\lvert b_5\rvert \lvert b_6\rvert \cos\phi_{56} = 2\mathrm{Re}\left[b_6^* b_5\right] = -R_L^{x'x}$	$\frac{1}{\sqrt{2}}\langle b\rvert \tilde{\Gamma}^{19} \lvert b\rangle$	
$\mathcal{O}^e_{1+} = \lvert b_3\rvert \lvert b_6\rvert \sin\phi_{36} + \lvert b_4\rvert \lvert b_5\rvert \sin\phi_{45} = \mathrm{Im}\left[b_6^* b_3 + b_5^* b_4\right] = -{}^sR_{LT'}^{00}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{31} \lvert b\rangle$	
$\mathcal{O}^e_{1-} = \lvert b_3\rvert \lvert b_6\rvert \sin\phi_{36} - \lvert b_4\rvert \lvert b_5\rvert \sin\phi_{45} = \mathrm{Im}\left[b_6^* b_3 - b_5^* b_4\right] = {}^sR_{LT'}^{0y}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{29} \lvert b\rangle$	$e = \mathrm{AD3}$
$\mathcal{O}^e_{2+} = \lvert b_3\rvert \lvert b_6\rvert \cos\phi_{36} + \lvert b_4\rvert \lvert b_5\rvert \cos\phi_{45} = \mathrm{Re}\left[b_6^* b_3 + b_5^* b_4\right] = {}^cR_{LT}^{00}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{21} \lvert b\rangle$	
$\mathcal{O}^e_{2-} = \lvert b_3\rvert \lvert b_6\rvert \cos\phi_{36} - \lvert b_4\rvert \lvert b_5\rvert \cos\phi_{45} = \mathrm{Re}\left[b_6^* b_3 - b_5^* b_4\right] = -{}^cR_{LT}^{0y}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{23} \lvert b\rangle$	
$\mathcal{O}^f_{1+} = \lvert b_1\rvert \lvert b_6\rvert \sin\phi_{16} + \lvert b_2\rvert \lvert b_5\rvert \sin\phi_{25} = \mathrm{Im}\left[b_6^* b_1 + b_5^* b_2\right] = -{}^sR_{LT}^{0z}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{30} \lvert b\rangle$	
$\mathcal{O}^f_{1-} = \lvert b_1\rvert \lvert b_6\rvert \sin\phi_{16} - \lvert b_2\rvert \lvert b_5\rvert \sin\phi_{25} = \mathrm{Im}\left[b_6^* b_1 - b_5^* b_2\right] = {}^cR_{LT'}^{0x}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{24} \lvert b\rangle$	$f = \mathrm{AD4}$
$\mathcal{O}^f_{2+} = \lvert b_1\rvert \lvert b_6\rvert \cos\phi_{16} + \lvert b_2\rvert \lvert b_5\rvert \cos\phi_{25} = \mathrm{Re}\left[b_6^* b_1 + b_5^* b_2\right] = {}^cR_{LT'}^{0z}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{32} \lvert b\rangle$	
$\mathcal{O}^f_{2-} = \lvert b_1\rvert \lvert b_6\rvert \cos\phi_{16} - \lvert b_2\rvert \lvert b_5\rvert \cos\phi_{25} = \mathrm{Re}\left[b_6^* b_1 - b_5^* b_2\right] = {}^sR_{LT}^{0x}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{22} \lvert b\rangle$	

Example 2: electroproduction (formalism)

$\mathcal{O}^{g}_{1+} = \lvert b_1 \rvert \, \lvert b_5 \rvert \sin\phi_{15} + \lvert b_2 \rvert \, \lvert b_6 \rvert \sin\phi_{26} = \mathrm{Im}\left[b_5^{*} b_1 + b_6^{*} b_2 \right] = - {}^{s}R^{z'0}_{LT}$	$\frac{1}{2} \langle b \rvert \tilde{\Gamma}^{33} \lvert b \rangle$	
$\mathcal{O}^{g}_{1-} = \lvert b_1 \rvert \, \lvert b_5 \rvert \sin\phi_{15} - \lvert b_2 \rvert \, \lvert b_6 \rvert \sin\phi_{26} = \mathrm{Im}\left[b_5^{*} b_1 - b_6^{*} b_2 \right] = - {}^{c}R^{x'0}_{LT'}$	$\frac{1}{2} \langle b \rvert \tilde{\Gamma}^{26} \lvert b \rangle$	$g = \mathrm{PR2}$
$\mathcal{O}^{g}_{2+} = \lvert b_1 \rvert \, \lvert b_5 \rvert \cos\phi_{15} + \lvert b_2 \rvert \, \lvert b_6 \rvert \cos\phi_{26} = \mathrm{Re}\left[b_5^{*} b_1 + b_6^{*} b_2 \right] = {}^{c}R^{z'0}_{LT'}$	$\frac{1}{2} \langle b \rvert \tilde{\Gamma}^{34} \lvert b \rangle$	
$\mathcal{O}^{g}_{2-} = \lvert b_1 \rvert \, \lvert b_5 \rvert \cos\phi_{15} - \lvert b_2 \rvert \, \lvert b_6 \rvert \cos\phi_{26} = \mathrm{Re}\left[b_5^{*} b_1 - b_6^{*} b_2 \right] = - {}^{s}R^{x'0}_{LT}$	$\frac{1}{2} \langle b \rvert \tilde{\Gamma}^{25} \lvert b \rangle$	
$\mathcal{O}^{h}_{1+} = \lvert b_3 \rvert \, \lvert b_5 \rvert \sin\phi_{35} + \lvert b_4 \rvert \, \lvert b_6 \rvert \sin\phi_{46} = \mathrm{Im}\left[b_5^{*} b_3 + b_6^{*} b_4 \right] = {}^{s}R^{x'x}_{LT'}$	$\frac{1}{2} \langle b \rvert \tilde{\Gamma}^{35} \lvert b \rangle$	
$\mathcal{O}^{h}_{1-} = \lvert b_3 \rvert \, \lvert b_5 \rvert \sin\phi_{35} - \lvert b_4 \rvert \, \lvert b_6 \rvert \sin\phi_{46} = \mathrm{Im}\left[b_5^{*} b_3 - b_6^{*} b_4 \right] = - {}^{c}R^{z'x}_{LT}$	$\frac{1}{2} \langle b \rvert \tilde{\Gamma}^{28} \lvert b \rangle$	$h = \mathrm{PR3}$
$\mathcal{O}^{h}_{2+} = \lvert b_3 \rvert \, \lvert b_5 \rvert \cos\phi_{35} + \lvert b_4 \rvert \, \lvert b_6 \rvert \cos\phi_{46} = \mathrm{Re}\left[b_5^{*} b_3 + b_6^{*} b_4 \right] = - {}^{c}R^{x'x}_{LT}$	$\frac{1}{2} \langle b \rvert \tilde{\Gamma}^{36} \lvert b \rangle$	
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Example 2: electroproduction (formalism)

$\mathcal{O}^{g}_{1+} = \lvert b_1\rvert \lvert b_5\rvert \sin\phi_{15} + \lvert b_2\rvert \lvert b_6\rvert \sin\phi_{26} = \mathrm{Im}\left[b_5^* b_1 + b_6^* b_2\right] = -\,{}^{s}R^{z'0}_{LT}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{33}\lvert b\rangle$	
$\mathcal{O}^{g}_{1-} = \lvert b_1\rvert \lvert b_5\rvert \sin\phi_{15} - \lvert b_2\rvert \lvert b_6\rvert \sin\phi_{26} = \mathrm{Im}\left[b_5^* b_1 - b_6^* b_2\right] = -\,{}^{c}R^{x'0}_{LT'}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{26}\lvert b\rangle$	$g = \mathrm{PR2}$
$\mathcal{O}^{g}_{2+} = \lvert b_1\rvert \lvert b_5\rvert \cos\phi_{15} + \lvert b_2\rvert \lvert b_6\rvert \cos\phi_{26} = \mathrm{Re}\left[b_5^* b_1 + b_6^* b_2\right] = {}^{c}R^{z'0}_{LT'}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{34}\lvert b\rangle$	
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$\mathcal{O}^{h}_{1-} = \lvert b_3\rvert \lvert b_5\rvert \sin\phi_{35} - \lvert b_4\rvert \lvert b_6\rvert \sin\phi_{46} = \mathrm{Im}\left[b_5^* b_3 - b_6^* b_4\right] = -\,{}^{c}R^{z'x}_{LT}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{28}\lvert b\rangle$	$h = \mathrm{PR3}$
$\mathcal{O}^{h}_{2+} = \lvert b_3\rvert \lvert b_5\rvert \cos\phi_{35} + \lvert b_4\rvert \lvert b_6\rvert \cos\phi_{46} = \mathrm{Re}\left[b_5^* b_3 + b_6^* b_4\right] = -\,{}^{c}R^{x'x}_{LT}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{36}\lvert b\rangle$	
$\mathcal{O}^{h}_{2-} = \lvert b_3\rvert \lvert b_5\rvert \cos\phi_{35} - \lvert b_4\rvert \lvert b_6\rvert \cos\phi_{46} = \mathrm{Re}\left[b_5^* b_3 - b_6^* b_4\right] = -\,{}^{s}R^{z'x}_{LT'}$	$\frac{1}{2}\langle b\rvert \tilde{\Gamma}^{27}\lvert b\rangle$	

$\hookrightarrow$ For $N = 6$: quite many observables/inteference terms. Possible algebraic dependencies among the observables are manifold and quite complicated!

$\Rightarrow$ It is quite tough to solve this problem 'by hand'.

However: Moravcsik's Theorem comes to the rescue, since its application is very systematic and quite easily automated!

Example 3: electroproduction (graph topologies)

$*$) For $N = 6$, we have $(N-1)!/2 = 5!/2 = 60$ possible graph-topologies:

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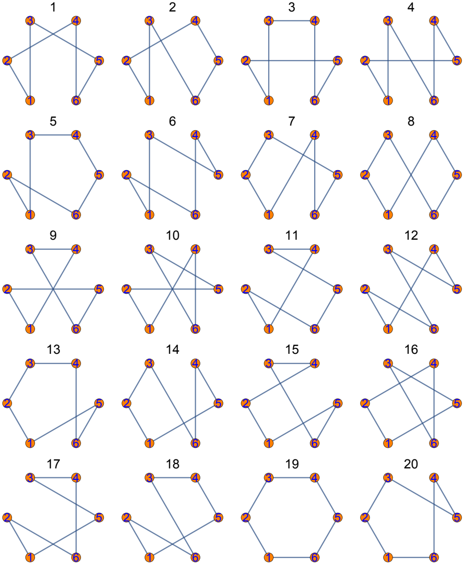

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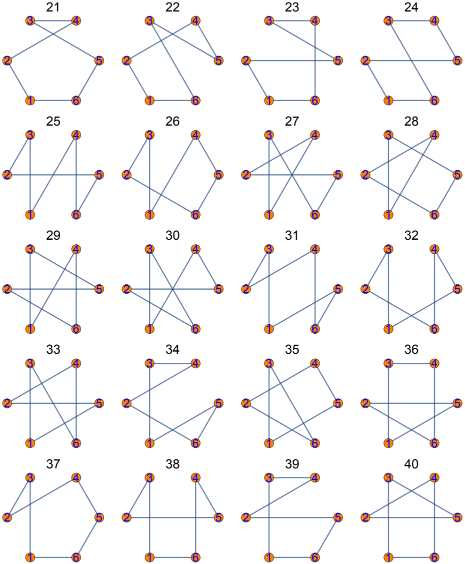

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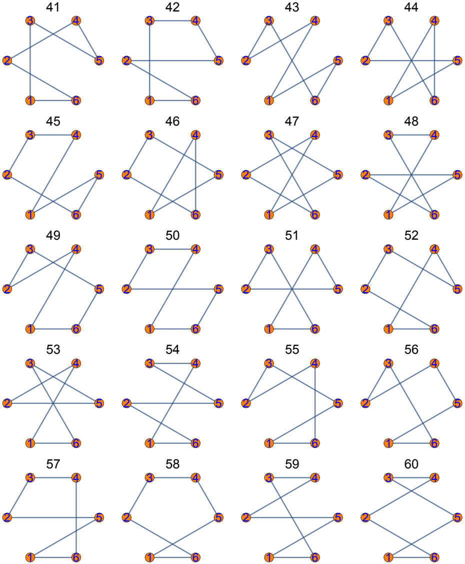
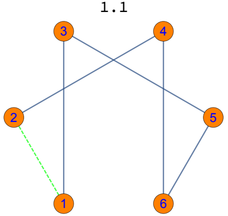


Example 3: electroproduction (fully complete graphs)

$*)$ Example (1.1) (fully complete):



$\longrightarrow \quad \{\sin\phi_{12}, \cos\phi_{24}, \cos\phi_{46}, \cos\phi_{56}, \cos\phi_{35}, \cos\phi_{13}\}$

$\hookrightarrow$ In the same way as before, extract the 'Moravcsik-complete' set (combined with $\left\{R_{T}^{00},\ {}^{c}R_{TT}^{00}, R_{T}^{0y}, R_{T}^{y'0}, R_{L}^{00}, R_{L}^{0y}\right\}$):

$$\left\{\mathcal{O}_{2+}^{a}, \mathcal{O}_{2-}^{a}, \mathcal{O}_{1+}^{c}, \mathcal{O}_{1-}^{c}, \mathcal{O}_{2}^{d}, \mathcal{O}_{2+}^{h}, \mathcal{O}_{2-}^{h}\right\}$$
$$\equiv \left\{R_{TT'}^{0z}, {}^{s}R_{TT}^{0x}, R_{T}^{x'z}, R_{T}^{z'x}, R_{L}^{x'x}, {}^{c}R_{LT}^{x'x}, {}^{s}R_{LT'}^{z'x}\right\}.$$

Example 3: electroproduction (fully complete graphs)

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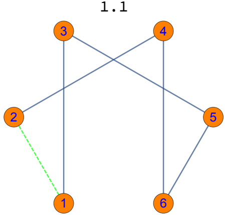

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*) In total, we obtain for the first time (!):

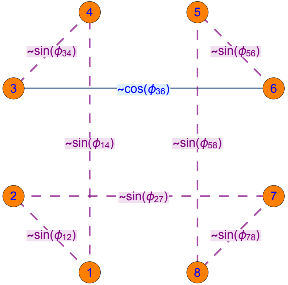
- 64 non-redundant Moravcsik-complete sets composed of 13 observables
  ↪ Only one observable more than the minimal number of $2N = 12$ observables!
- 96 non-redundant Moravcsik-complete sets composed of 14 observables

Cases with larger numbers of $N \geq 6$ amplitudes

Two-meson photoproduction

∗) 8 amplitudes vs. 64 observables

∗) Typical complete graph:



[P. Kroenert et al. (2021)]

∗) No. of start-topologies: $\frac{(N-1)!}{2} = 2520$

∗) Moravcsik-complete sets have at least:
24 observables $> 2N = 16$ observables

∗) [Phys. Rev. C **103**, 1, 014607 (2021)]
→ see also next talk by P. Kroenert

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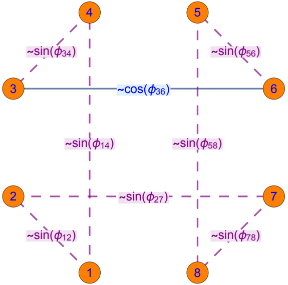

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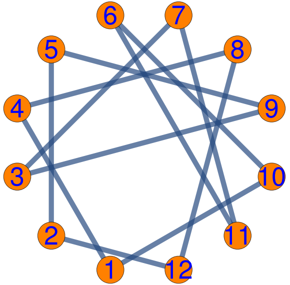
Vector-meson photoproduction

*) 12 amp.'s vs. 244 observables

*) Example for start-topology:



*) No. of start-topologies:

$$\frac{(N-1)!}{2} = 19\,958\,400$$

⇒ Numerically very demanding problem!

⇒ Needs to be implemented on a cluster

. . .

Minimal complete sets from new 'directional' graphs

Observation: Moravcsik-complete sets tend to be slightly over-complete, i.e. to contain *more than* $2N$ observables, for problems with $N \geq 4$ amplitudes

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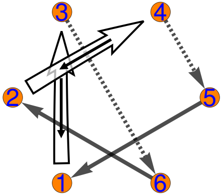

$\Leftrightarrow$ complete electroproduction-set (yields $2N = 12$ obs.'s in combination with 6 'diagonal' obs.'s):

$$\left\{ {}^{s}R^{0z}_{TT}, R^{0z}_{TT'}, {}^{s}R^{00}_{LT'}, {}^{s}R^{0y}_{LT'}, {}^{c}R^{z'0}_{LT'}, {}^{s}R^{x'0}_{LT} \right\}.$$

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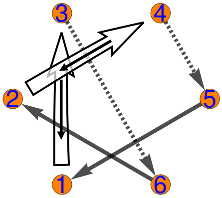

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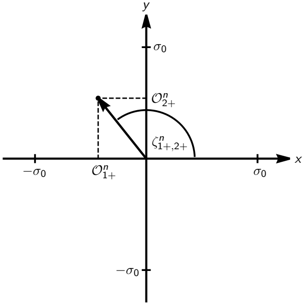
- Single-lined arrows: same as in Moravcsik's Theorem
- Double-lined arrows: 'crossed' selection $\mathcal{O}^{a}_{1\pm} \oplus \mathcal{O}^{a}_{2\pm}$
- 'Outer' direction $\Leftrightarrow$ 'directional convention' for consist. rel.: $\phi_{13} + \phi_{36} + \phi_{62} + \phi_{24} + \phi_{45} + \phi_{51} = 0$.
- Direction of 'inner' arrows: sign of 'ζ-angle' (cf. Figure on the right) in discrete-ambiguity formulas



$\Rightarrow$ More details: [arXiv:2106.00486 [nucl-th]].

Conclusion and Outlook

For a $2 \to 2$ reaction involving particles with spin:
N (transversity) amplitudes b_i vs. N^2 pol.-observables $\check{\mathcal{O}}^{\alpha} \propto \langle b| \tilde{\Gamma}^{\alpha} |b\rangle$.

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We have:

∗) (Re-) derived a modified version of Moravcsik's Theorem

∗) Treated the example of photoproduction in detail

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We have:

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∗) Further possible directions of research:

- Two-meson photoproduction [P. Kroenert et al. (2021)] $\to$ next talk!
- First ever treatment of vector-meson photoproduction ($N = 12$ amplitudes: tough!!)
- Consider mismatch between Moravcsik-complete sets and minimal complete sets of $2N$ observables
  $\Rightarrow$ new 'directional' graphs [YW, [arXiv:2106.00486 [nucl-th]] (2021)]

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Thank You!