Systematic Treatment of Hypernuclear Data and Application to the Hypertriton

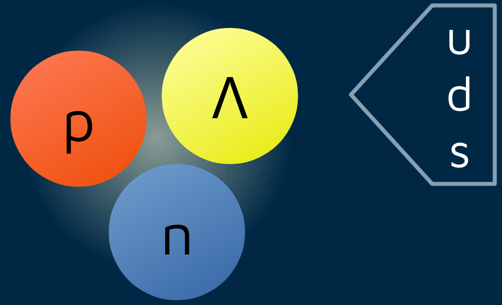
Philipp Eckert

HADRON 21

ABOUT HYPERTRITON [$^{3}_{\Lambda}H$]

- Simplest known hypernucleus with one Λ baryon
- Λ loosely bound, order of 100 keV

→ Expected $^{3}_{\Lambda}H$ lifetime close to free Λ



However:

- Many experiments with lifetimes shorter than expected

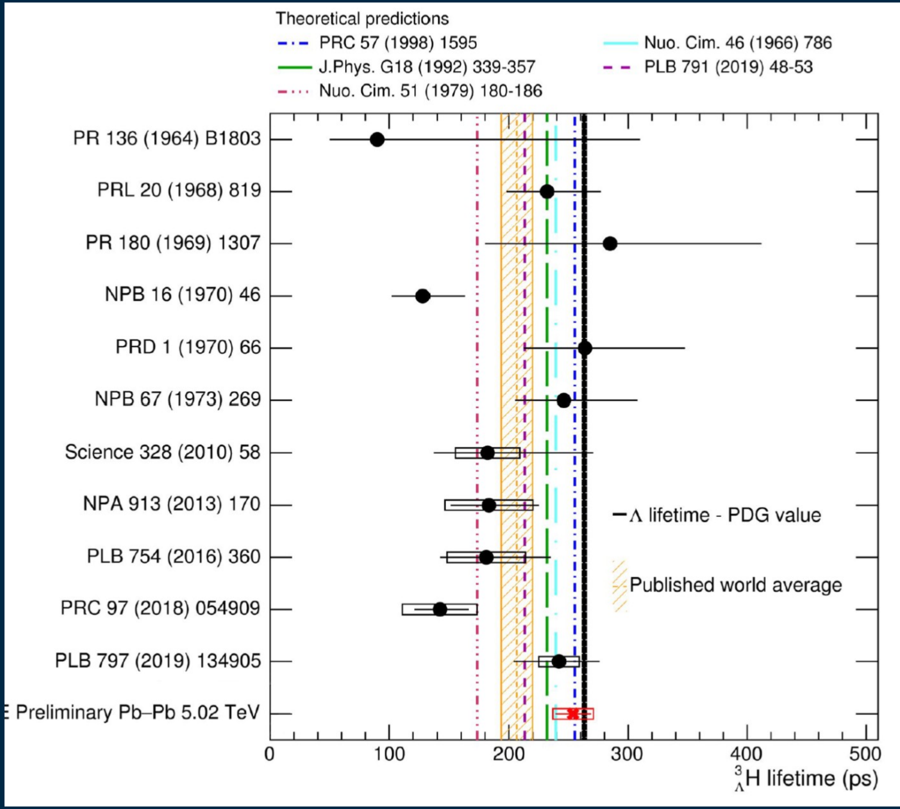
  → Hard to explain by theory
- Contradictory data for binding energy?

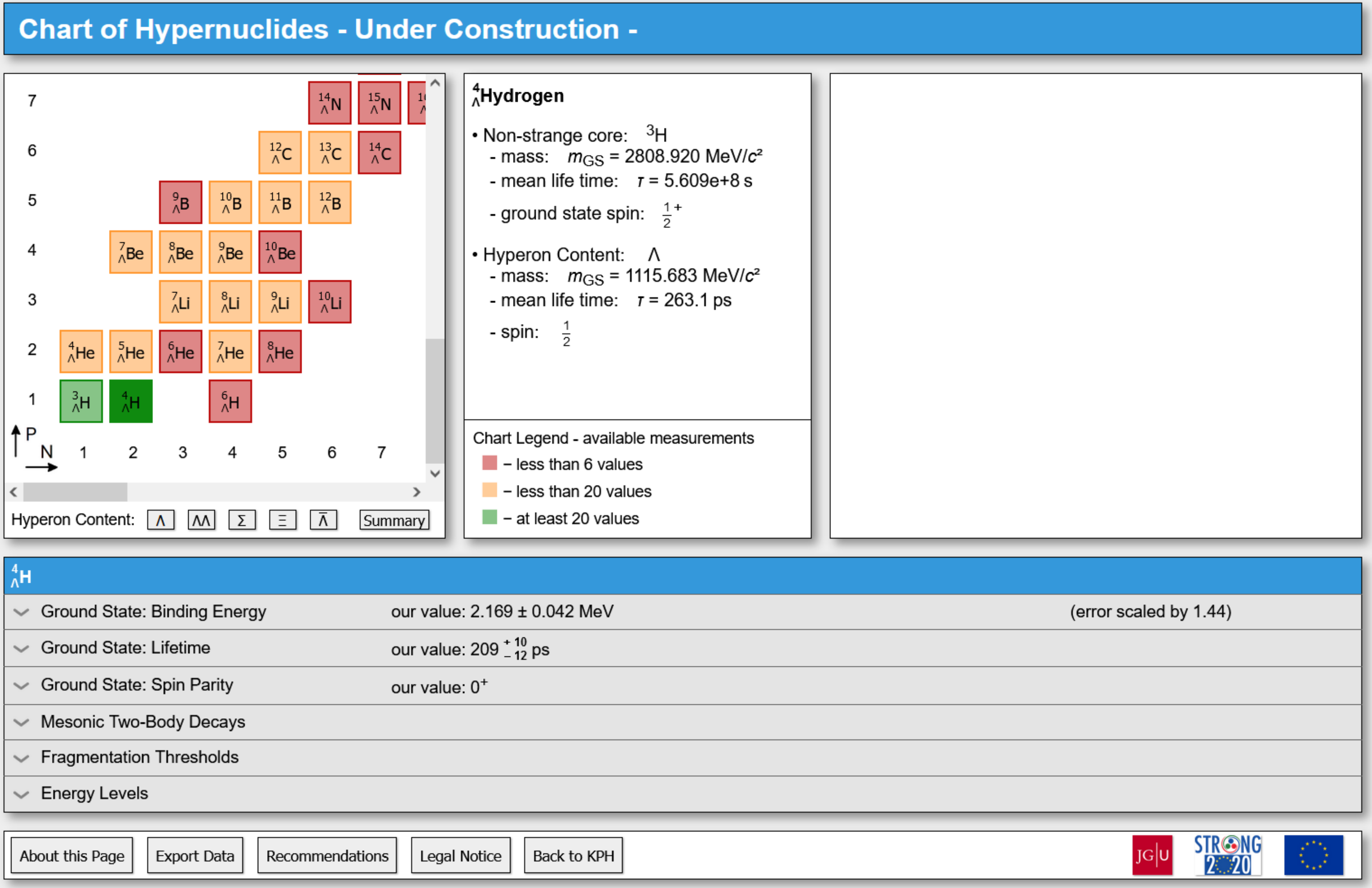
  → Hypertriton lifetime puzzle

HYPERTRITON LIFETIME –
Why to collect and treat hypernuclear data?

- History of almost 60 years
- Different exp. approaches (Emulsion, Bubble Ch., Heavy Ion)
- Asymmetric errors
- Missing systematics before 2010
- Large progress in last years
- Conflicts within the data?

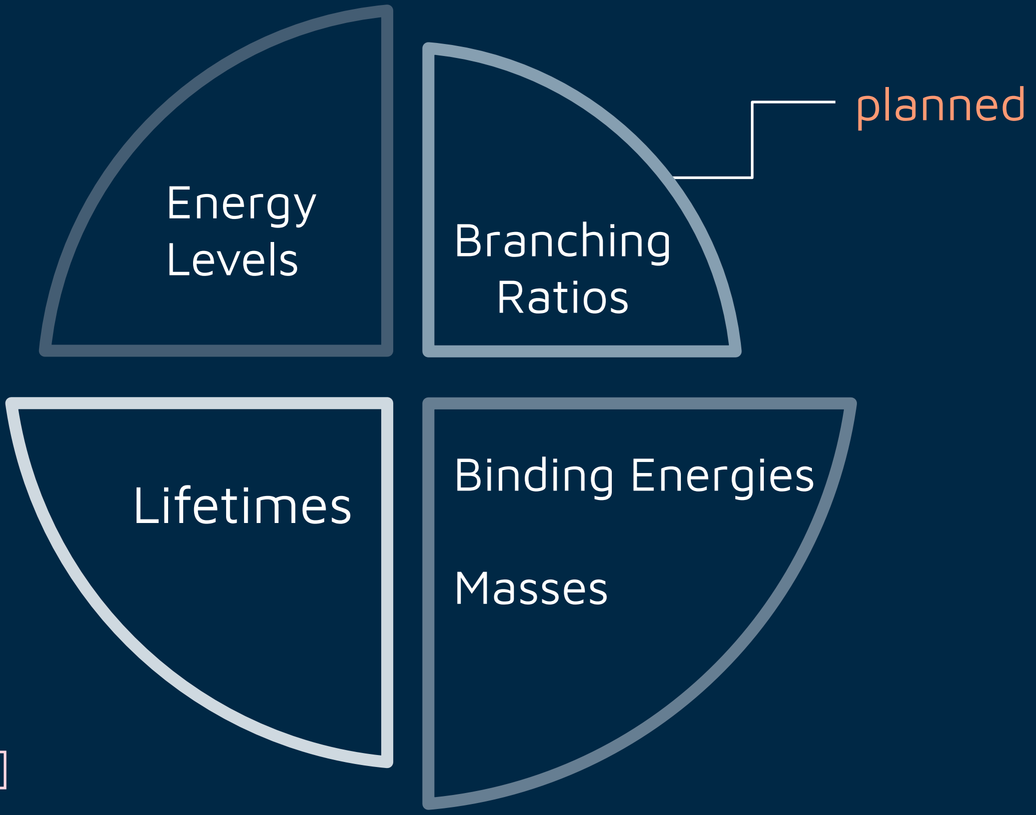
⇨ Consistent way to collect and combine data needed





Current status of the database interface – to be published at *hypernuclei.kph.uni-mainz.de*

CONTENT OF THE DATABASE



Further Information about:

- Author
- Year
- Collaboration
- Significance, etc.
- Exp. Method, etc.
- Reference

...and more!

AVERAGING PROCEDURES – Error Weighted Mean

- Set of N measurements: $\mu_i \pm \sigma_i\,,\quad \sigma_i^2 = \sigma_{stat,i}^2 + \sigma_{syst,i}^2$
- Weight for i-th measurement: $w_i = \frac{1}{\sigma_i^2}$
- Weights normalized to 1: $w'_i = \frac{w_i}{\sum_j w_j}$
- Total mean: $\bar{x} = \sum_{i=1}^{N} w'_i \mu_i$
- Total variance: $\sigma_{\bar{x}}^2 = \sum_{i=1}^{N} w'^2_i \sigma_i^2$

Simple, but:

Works only for symmetric errors

The resulting error is always smaller than the smallest contributing error

AVERAGING PROCEDURES – Asymmetric Errors

- Parametrise pdf via asymmetric Gaussian curve

$$L(x) = e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma(x)}\right)^2}$$

- With a linear function

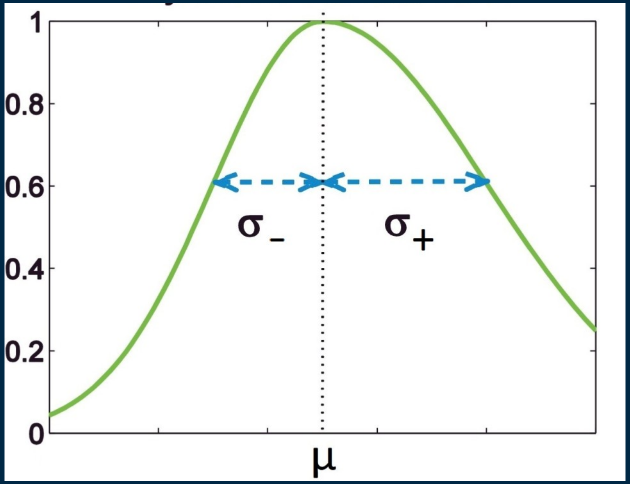
$$\sigma(x) = \sigma_1 + \sigma_2 \cdot (x-\mu)$$

$$\sigma_1 = \frac{2\sigma_+\sigma_-}{\sigma_+ + \sigma_-} \quad \sigma_2 = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

- Weights given by

$$w_i = \frac{\sigma_{1i}}{\left(\sigma_{1i} + \sigma_{2i}(\bar{x} - \mu_i)\right)^3}$$

depend on mean
→ iterative



Method by: R. Barlow: "Asymmetric Statistical Errors" (2004) https://arxiv.org/abs/physics/0406120v1

First application in hypernuclear physics: C. Rappold PLB728, 543 (2014)

FURTHER DATA TREATMENT

EXCLUDE MEASUREMENTS

By low weight or
low weight to Chi^2 ratio

TREAT SHARED SYSTEMATICS

Within values from
the same experiment

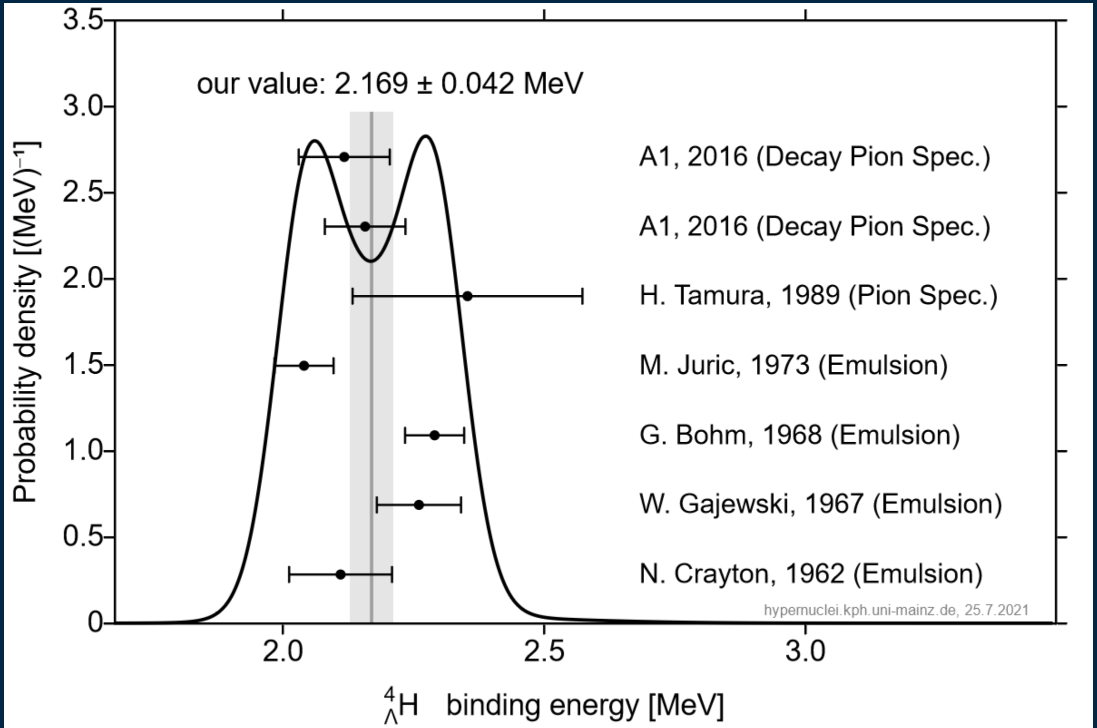
ERROR SCALING

For resulting average
if Chi^2 too large

ADD MISSING ERRORS

Collect and apply
suggestions by experts

ERROR SCALING – ${}^{4}_{\Lambda}H$ binding energy



- Reported errors small
- Two tendencies in prob. curve

$$\chi^2 = \sum_i w_i (\mu_i - \bar{x})^2$$

$$S = [\chi^2/(N-1)]^{1/2}$$

X^2 sum:	12.5
DoF:	$N - 1 = 6$
Scaling:	$S = 1.44$
⇨ 29 keV	→ 42 keV

SYSTEMATIC ERROR FOR HISTORICAL BINDING ENERGIES

Suggestion by Davis[1]: $\sigma_{syst} = 40\,\mathrm{keV}$

new STAR value more than 3 times larger than Juric`s

Hypertriton binding energy:

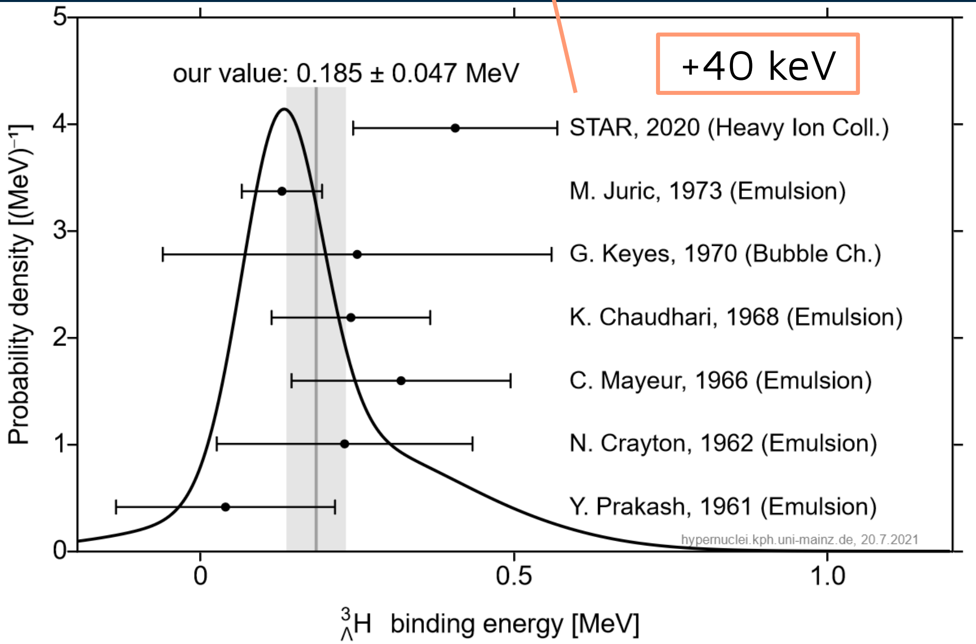
- Dominated by Juric`s value

$$B_\Lambda = 130 \pm 50\,\mathrm{keV} \rightarrow \pm 64\,\mathrm{keV}$$

- Change of ~8% in average:

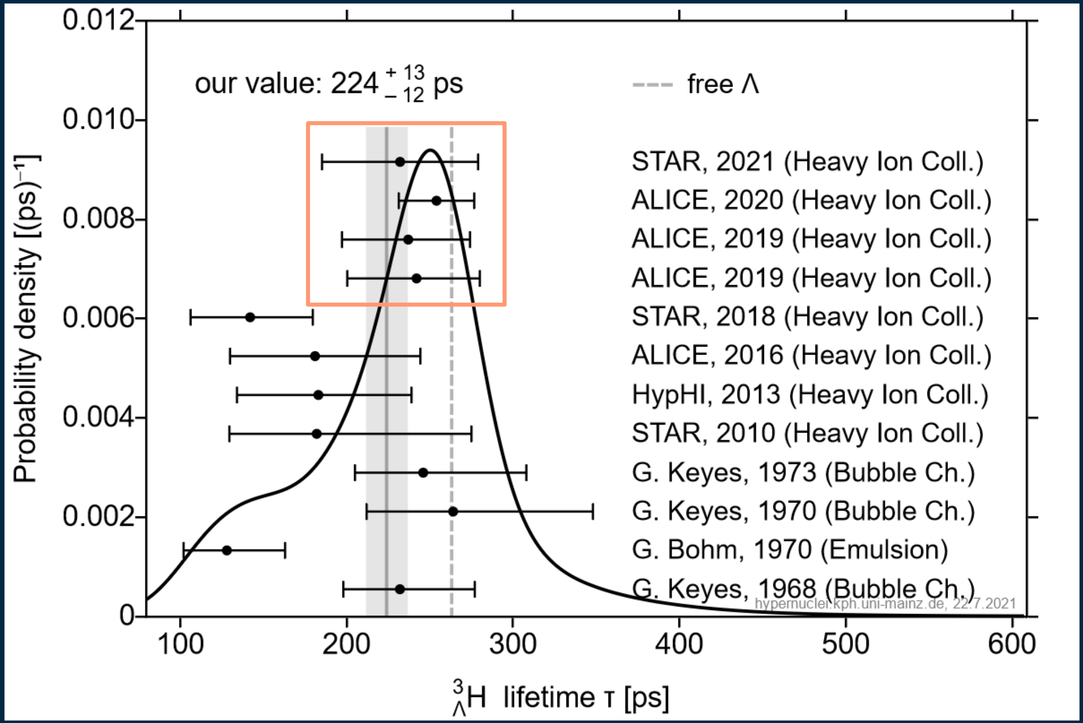
$$\bar{B}_\Lambda = 171 \pm 40\,\mathrm{keV} \rightarrow 185 \pm 47\,\mathrm{keV}$$

→ How accurate is old data?



[1]: D.H. Davis [NPA 547 (1992) 369c-378c]

RESULTS – HYPERTRITON LIFETIME



- Error scaling of 1.03
  → Data almost consistent
- Downward trend not confirmed by recent experiments
- Even free Λ lifetime seems possible

Hypertriton Lifetime puzzle solved?

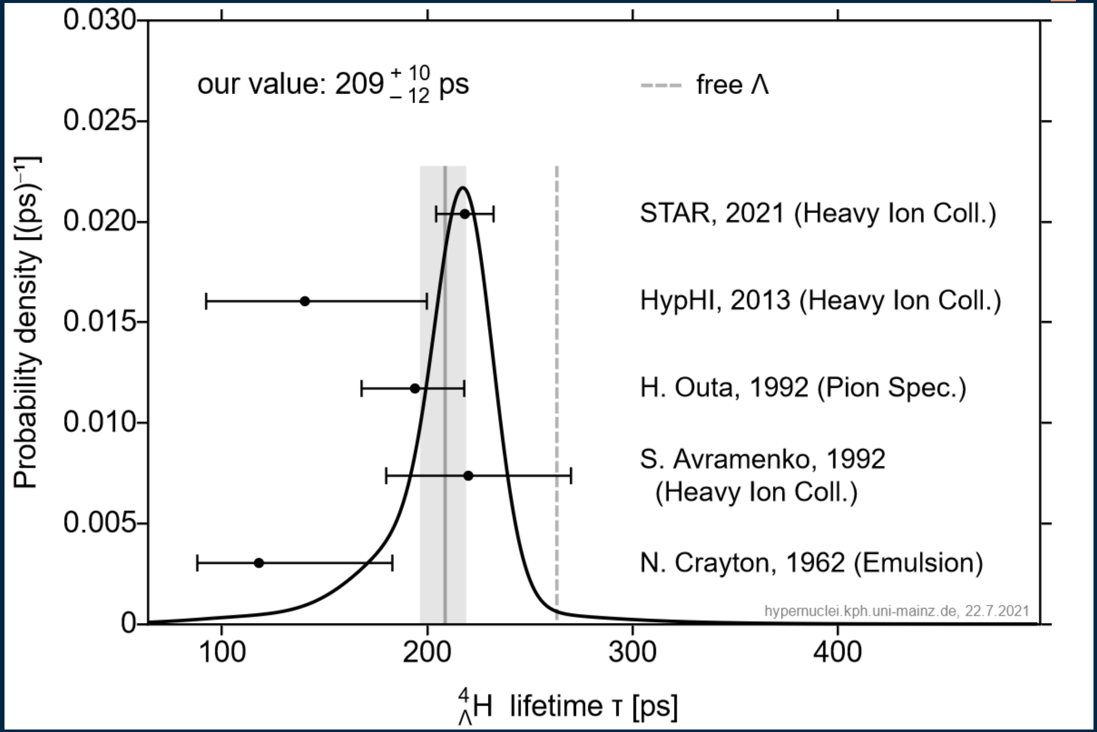
SITUATION FOR HYDROGEN-4-LAMBDA

- No scaling needed
- Lifetime and binding energy fit together

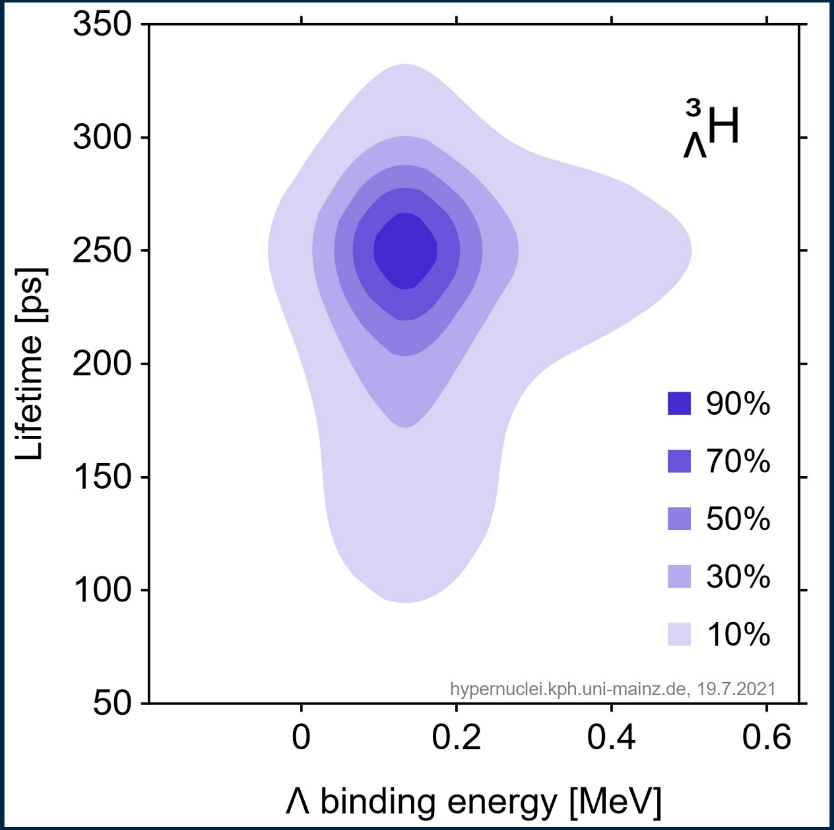
...but:

New measurement by E73: 180 ± 7 ps
(systematic error not published yet)

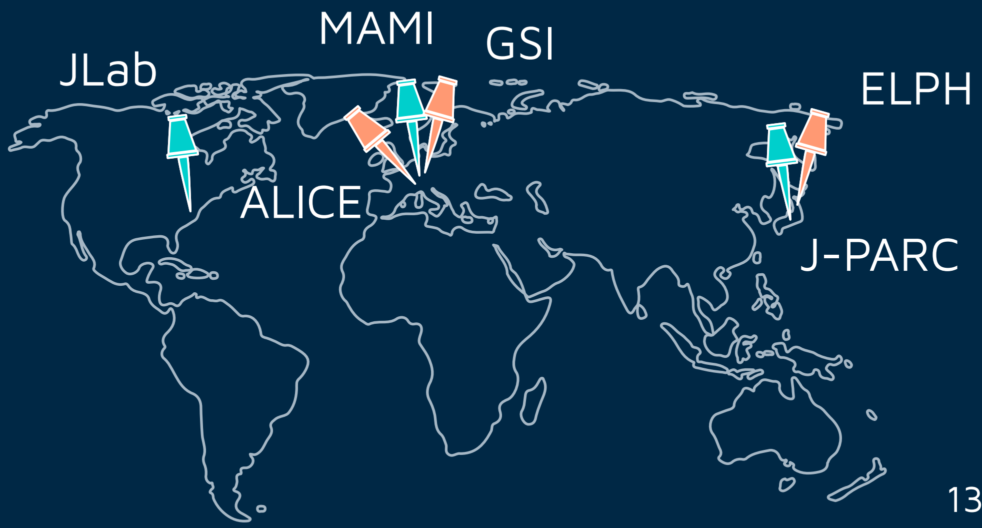
→ Again a lifetime lower than expected?



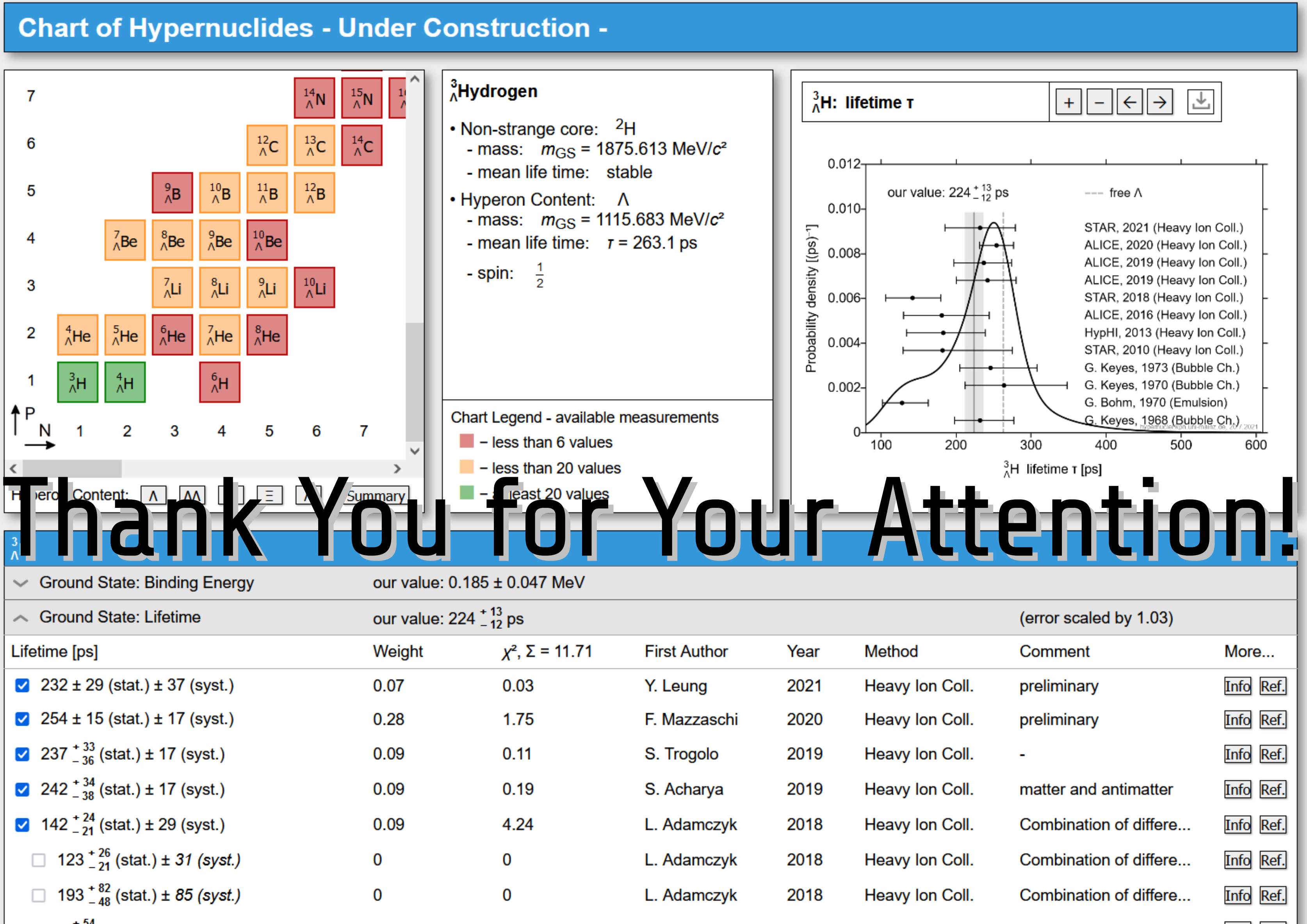
HYPERTRITON SUMMARY



- Tendency to lower lifetimes still not excluded
- STAR binding energy has to be cross checked
- New precise experiments on lifetime and binding energy planned:



Thank You for Your Attention!



Backup

AVERAGING PROCEDURES
– Asymmetric Errors

- How to define $\sigma(x) = \sigma_1 + \sigma_2 \cdot (x - \mu)$?

- Analogous to common Gaussian:

$$g(x) = e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$g(\mu-\sigma) = g(\mu+\sigma) = e^{-\frac{1}{2}} \Rightarrow L(\mu-\sigma_-) \overset{!}{=} L(\mu+\sigma_+) \overset{!}{=} e^{-\frac{1}{2}}$$

- Given by

$$\sigma_1 = \frac{2\sigma_+\sigma_-}{\sigma_+ + \sigma_-} \qquad \sigma_2 = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

- Then the $[\mu-\sigma_-,\ \mu+\sigma_+]$ interval is equivalent to a common 1σ interval,

→ both cover a probability of 68.27 %

AVERAGING PROCEDURES – Asymmetric Errors

- Set of N measurements: $\mu_i + \sigma_{+i} - \sigma_{-i}$, $\sigma_{\pm}^2 = \sigma_{stat,\pm}^2 + \sigma_{syst,\pm}^2$

- Mean value $\bar{x}$ can be found with:

$$\bar{x} \sum_i w_i = \sum_i \mu_i w_i$$

$$w_i = \frac{\sigma_{1i}}{\left(\sigma_{1i} + \sigma_{2i}(\bar{x} - \mu_i)\right)^3}$$

depends on mean

⇨ numerical solution via iterations:

- Initial value $\bar{x}_0 = \frac{1}{N} \sum_{i=0}^{N} \mu_i$
- Accuracy of 10^{-5} can be achieved in about 5 iterations

ASYMMETRIC ERRORS – Error Interval

- Log-likelihood function: $lnL(x) = -\frac{1}{2}\sum_i \left(\frac{x-\mu_i}{\sigma_i(x)}\right)^2$

- Find both $e^{-\frac{1}{2}}$ points, equivalent to $lnL(\bar{x}) - lnL(\sigma_\pm) = -\frac{1}{2}$

- Solution via iterations with initial values $\sigma_{\pm,0} = \left(\sum_i \frac{1}{\sigma^2_{\pm,i}}\right)^{-\frac{1}{2}}$

Errors with accuracy 10^{-3} already found after 3 iterations

⇨ Application in online calculator possible!

SHARED SYSTEMATIC ERRORS

Set of measurements with same systematic error $\mu_i \pm \sigma_{stat,i} \pm \sigma_{syst}$

- resulting error can`t be smaller than σ_{syst}

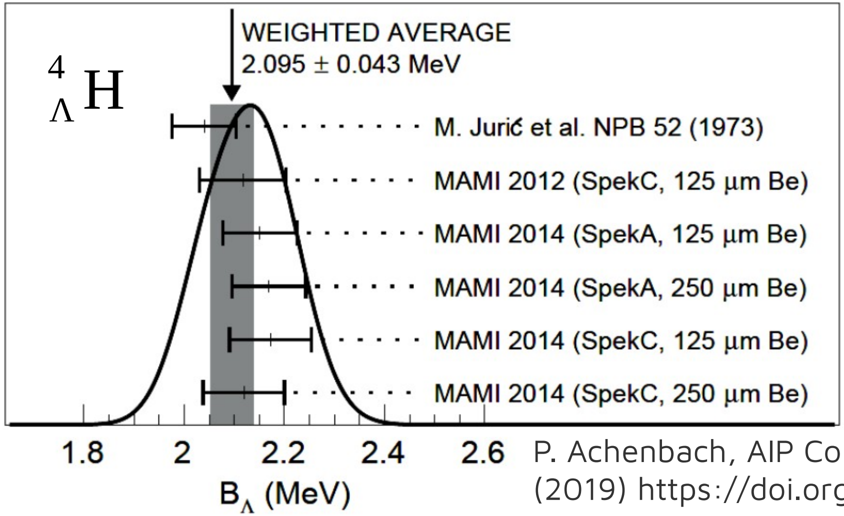
Not guaranteed by the averaging procedures!

⇨ modified systematic error:

$$\sigma_{syst,i} = \sigma_{syst} \cdot \sigma_{stat,i} \left(\sum_j \frac{1}{\sigma_{stat,j}^2} \right)^{\frac{1}{2}}$$

„This procedure has the advantage that, with the modified systematic errors [...], each measurement may be treated as independent and averaged in the usual way with other data.“ – PDG

SHARED SYSTEMATICS – Example



B_Λ (MeV)	δB (MeV)	w	χ^2
2.04	0.064	0.45	0.74
2.117	0.087	0.24	0.06
2.151	0.074	0.09	0.15
2.169	0.074	0.11	0.34
2.172	0.081	0.06	0.20
2.119	0.081	0.05	0.02
		1.00	1.50

P. Achenbach, AIP Conference Proceedings **2130**, 030007 (2019) https://doi.org/10.1063/1.5118397

- error treatment avoids overestimation of MAMI's influence:

  → weights distributed almost 1:1 instead of 1:5

- no underestimation of resulting error

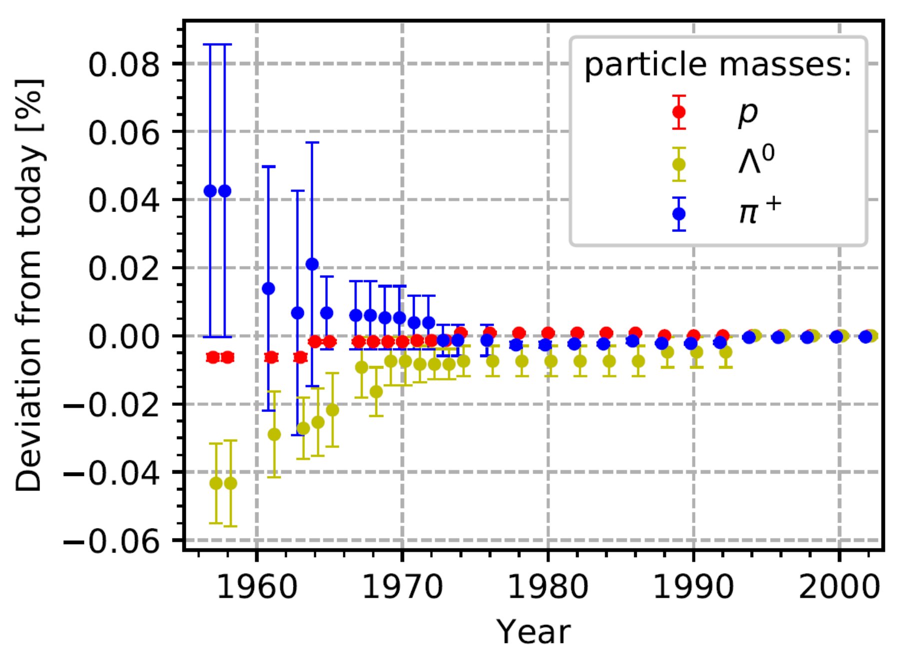
Recalibration of historical Data?

Particle masses changed over decades

- Deviation of 0.02 % in Λ mass
  → 220 keV in total!
- Influence on weakly bound nuclei?

- Correction by P. Liu et al.[1] on hypertriton binding energy:
- 150 keV → 270 keV

→ Find agreement for consistent correction



PDG values

[1]: Chinese Physics C Vol. 37, No. 1 (2019) 010201