A theoretical analysis of the semileptonic decays $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$

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Outline

- Motivation
- SM Calculations
- SM Results
- BSM *CP* Violation

Motivation

- A number of low-energy precision measurements are sensitive to BSM Physics*
 - SM prediction for the measured quantity is precisely known
 - SM background is small
- Experiments currently underway
 - Muon g 2
 - EDMs
 - Neutron decays
 - Etc.

Motivation

- The η and η' mesons are special*:
 - The η is a pseudo-Goldstone boson
 - The η' is largely influenced by the U(1)_A anomaly
 - The η and η' are eigenstates of the *C*, *P*, *CP* and *G* operators: $I^GJ^{PC} = 0^+0^{-+}$
 - All their additive quantum numbers are zero: flavour conserving decays
 - All their strong and EM decays are forbidden at lowest order
 - Decays are mostly free from SM background



Perfect laboratory to stress-test the SM in search for physics BSM

* https://redtop.fnal.gov/the-physics/

Motivation

- The semileptonic decays $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta^\prime \to \eta \ell^+ \ell^-$ ($\ell = e$ or μ) can be used as fine probes to assess physics BSM.
 - SM contributes through the *C*-conserving exchange of two photons that is highly suppressed (no contribution at tree-level, only corrections at one-loop and higher orders)
- Latest theoretical estimations for $\eta \to \pi^0 \ell^+ \ell^-$ date back to the 90s
- No theoretical studies for $\eta' \to \pi^0 \ell^+ \ell^-$ or $\eta' \to \eta \ell^+ \ell^-$ to the best of our knowledge

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- Analysis of the *C*-conserving semileptonic decays $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$
 - Assess accurately SM background
- Calculations performed within the Vector Meson Dominance (VMD) framework
 - Decay processes dominated by the exchange of vector resonances
- VMD coupling constants parametrised using an existing phenomenological model
 - Numerical values obtained from an optimisation fit to $V \to P\gamma$ and $P \to V\gamma$ radiative decays ($V = \rho^0, \omega, \phi$ and $P = \pi^0, \eta, \eta'$)
 - See *Phys. Lett. B* 807 (2020) 135534, <u>arXiv:2003.08379</u> for details

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(b) *u*-channel Feynman diagram

QED vertex

VMD vertex

Contributing Feynman diagrams

(a) t-channel Feynman diagram

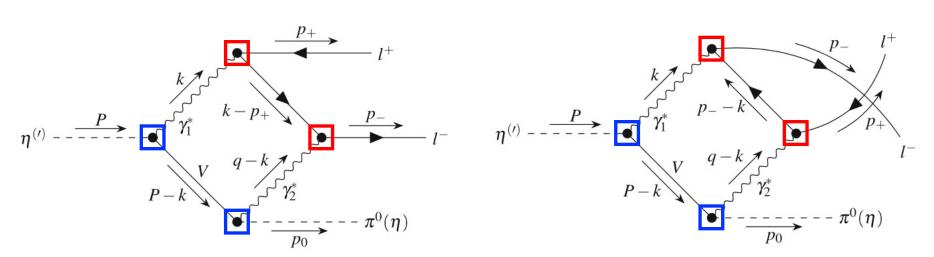
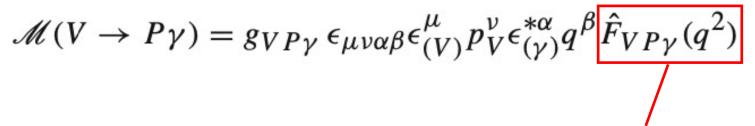


Fig. 1 Feynman diagrams contributing to the C-conserving semileptonic decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta^\prime \to \eta l^+ l^-$ (l = e or μ). Note that $q = p_+ + p_-$ and $V = \rho^0$, ω , ϕ

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• $VP\gamma$ interaction amplitude consistent with Lorentz, P, C and EM gauge invariance can be written as



Normalised form factor to account for off-shell photons mediating the transition

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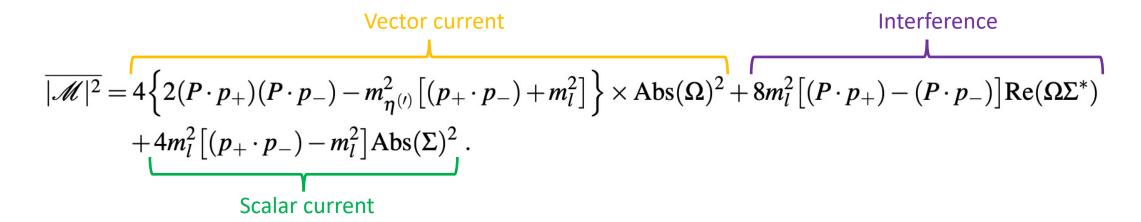
Decay amplitude

$$\mathcal{M} = ie^2 \sum_{V = \rho^0, \omega, \phi} g_{V \eta^{(\prime)} \gamma} g_{V \pi^0(\eta) \gamma} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \frac{1}{(k - q)^2 + i\epsilon} \epsilon_{\mu\nu\alpha\beta} \left[\frac{k^{\mu} (P - k)^{\alpha} (k - q)^{\rho} (P - k)^{\delta}}{(P - k)^2 - m_V^2 + i\epsilon} \right] \epsilon_{\rho\sigma\delta}^{\beta}$$

$$\bar{u}(p_-) \left[\gamma^{\sigma} \frac{\not k - \not p_+ + m_l}{(k - p_+)^2 - m_l^2 + i\epsilon} \gamma^{\nu} + \gamma^{\nu} \frac{\not p_- - \not k + m_l}{(k - p_-)^2 - m_l^2 + i\epsilon} \gamma^{\sigma} \right] v(p_+) ,$$

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Unpolarised squared amplitude



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with

$$\Omega = \sum_{V=\rho^{0},\omega,\phi} \alpha_{V} + \sigma_{V} , \qquad \alpha_{V} = e^{2\frac{g_{V\eta^{(\prime)}\gamma}g_{V\pi^{0}(\eta)\gamma}}{16\pi^{2}}} \int dxdydz \left[\frac{2A_{1}}{\Delta_{1V}+i\epsilon} - \frac{B_{1}}{(\Delta_{1V}+i\epsilon)^{2}} \right] ,
\Sigma = \sum_{V=\rho^{0},\omega,\phi} \beta_{V} + \tau_{V} , \qquad \beta_{V} = e^{2\frac{g_{V\eta^{(\prime)}\gamma}g_{V\pi^{0}(\eta)\gamma}}{16\pi^{2}}} \int dxdydz \left[\frac{2C_{1}}{\Delta_{1V}+i\epsilon} - \frac{D_{1}}{(\Delta_{1V}+i\epsilon)^{2}} \right] ,
\sigma_{V} = e^{2\frac{g_{V\eta^{(\prime)}\gamma}g_{V\pi^{0}(\eta)\gamma}}{16\pi^{2}}} \int dxdydz \left[\frac{2A_{2}}{\Delta_{2V}+i\epsilon} - \frac{B_{2}}{(\Delta_{2V}+i\epsilon)^{2}} \right] ,
\tau_{V} = e^{2\frac{g_{V\eta^{(\prime)}\gamma}g_{V\pi^{0}(\eta)\gamma}}{16\pi^{2}}} \int dxdydz \left[\frac{2C_{2}}{\Delta_{2V}+i\epsilon} - \frac{D_{2}}{(\Delta_{2V}+i\epsilon)^{2}} \right] ,$$

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VMD couplings parametrisation

$$g_{
ho^0\pi^0\gamma} = rac{1}{3}g \; ,$$
 $g_{
ho^0\eta\gamma} = gz_{
m NS}\cos\phi_P \; ,$ $g_{
ho^0\eta'\gamma} = gz_{
m NS}\sin\phi_P \; ,$ $g_{\omega\pi^0\gamma} = g\cos\phi_V \; ,$

$$g_{\omega\eta\gamma} = \frac{1}{3}g\left(z_{\rm NS}\cos\phi_P\cos\phi_V - 2\frac{\overline{m}}{m_S}z_{\rm S}\sin\phi_P\sin\phi_V\right),$$

$$g_{\omega\eta'\gamma} = \frac{1}{3}g\left(z_{\rm NS}\sin\phi_P\cos\phi_V + 2\frac{\overline{m}}{m_S}z_{\rm S}\cos\phi_P\sin\phi_V\right),$$

$$g_{\phi\pi^0\gamma} = g\sin\phi_V,$$

$$g_{\phi\eta\gamma} = \frac{1}{3}g\left(z_{\rm NS}\cos\phi_P\sin\phi_V + 2\frac{\overline{m}}{m_S}z_{\rm S}\sin\phi_P\cos\phi_V\right),$$

$$g_{\phi\eta'\gamma} = \frac{1}{3}g\left(z_{\rm NS}\sin\phi_P\sin\phi_V - 2\frac{\overline{m}}{m_S}z_{\rm S}\cos\phi_P\cos\phi_V\right),$$

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• Numerical values from optimisation fit to $VP\gamma$ radiative decays

$$g = 0.70 \pm 0.01 \text{ GeV}^{-1}, \quad z_{\text{S}}\overline{m}/m_s = 0.65 \pm 0.01,$$

 $\phi_P = (41.4 \pm 0.5)^{\circ}, \quad \phi_V = (3.3 \pm 0.1)^{\circ},$
 $z_{\text{NS}} = 0.83 \pm 0.02.$

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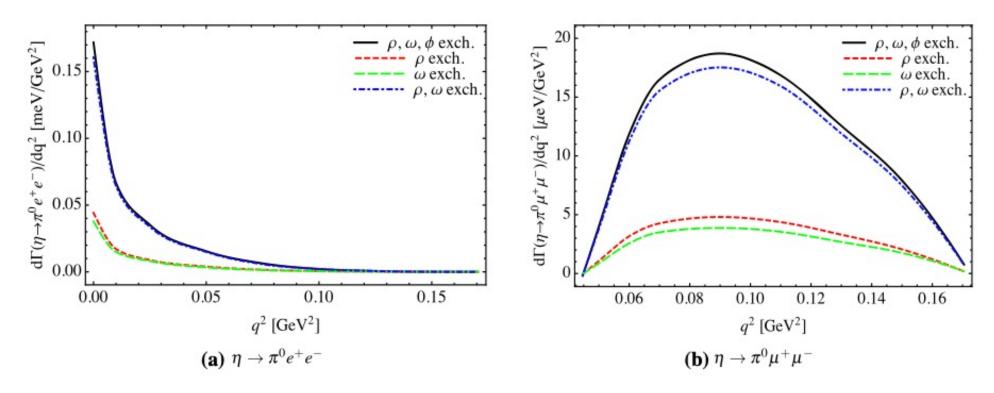
Decay widths and branching ratios

Table 1 Decay widths and branching ratios for the six C-conserving decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta^\prime \to \eta l^+ l^-$ (l = e or μ). First error is experimental, second is down to numerical integration and third is due to model dependency

Decay	$arGamma_{ ext{th}}$	BR_{th}	BR_{exp}
$ \eta \to \pi^0 e^+ e^- \eta \to \pi^0 \mu^+ \mu^- \eta' \to \pi^0 e^+ e^- \eta' \to \pi^0 \mu^+ \mu^- \eta' \to \eta^0 e^+ e^- $	$2.8(2)(3)(5) \times 10^{-6} \text{ eV}$ $1.6(1)(2)(2) \times 10^{-6} \text{ eV}$ $8.7(0.5)(0.9)(1.0) \times 10^{-4} \text{ eV}$ $3.5(2)(4)(5) \times 10^{-4} \text{ eV}$ $7.6(0.4)(0.8)(1.3) \times 10^{-5} \text{ eV}$	$2.1(1)(2)(4) \times 10^{-9}$ $1.2(1)(1)(2) \times 10^{-9}$ $4.5(3)(5)(6) \times 10^{-9}$ $1.8(1)(2)(3) \times 10^{-9}$ $3.9(3)(4)(7) \times 10^{-10}$	$< 7.5 \times 10^{-6} \text{ (CL=90\%) WASA-at-COSY}$ $< 5 \times 10^{-6} \text{ (CL=90\%)}$ $< 1.4 \times 10^{-3} \text{ (CL=90\%)}$ $< 6.0 \times 10^{-5} \text{ (CL=90\%)}$ $< 2.4 \times 10^{-3} \text{ (CL=90\%)}$
$\eta' \rightarrow \eta^0 \mu^+ \mu^-$	$3.1(2)(3)(2) \times 10^{-5} \text{ eV}$	$1.6(1)(2)(1) \times 10^{-10}$	$< 1.5 \times 10^{-5} \text{ (CL=90\%)}$

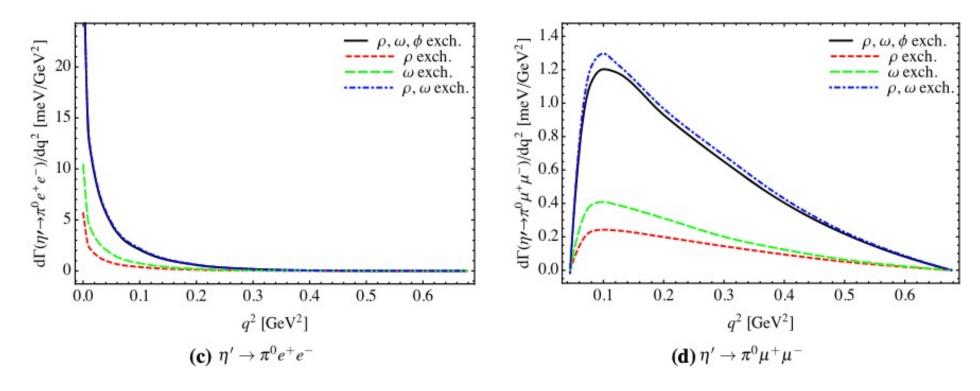
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• $\eta \to \pi^0 \ell^+ \ell^-$ dilepton spectrum



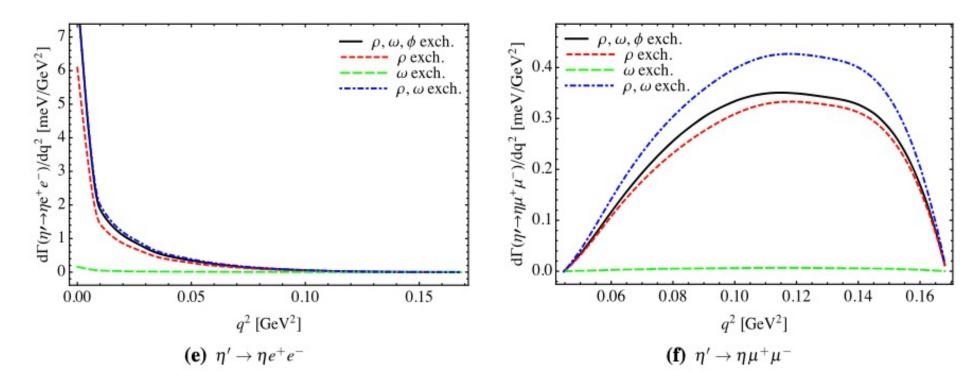
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• $\eta' \to \pi^0 \ell^+ \ell^-$ dilepton spectrum



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• $\eta' \rightarrow \eta \ell^+ \ell^-$ dilepton spectrum



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- REDTOP is a new Fermilab project that belongs to the high intensity class of experiments
 - It aims at detecting small variations from the SM by looking at a large number of events produced with very intense beams
- 1.8 GeV continuous proton beam impinging on a target made with 10 foils of beryllium to produce about $2.5 \times 10^{13} \frac{\eta}{year}$ and $2.5 \times 10^{11} \frac{\eta'}{year}$
- More information about REDTOP can be found in https://redtop.fnal.gov
- REDTOP may be able measure these BRs with significantly improved accuracy!

- Joined forces with Pablo Sánchez-Puertas for this study.
- Use results from previous section to fix SM background
- Some of Pablo's previous work on *CP* violation:
 - JHEP 01, 031 (2019), <u>arXiv:1810.13228</u>
 - arXiv:1909.07491



- The SMEFT assumes that new physics states are heavy
- The SMEFT is a consistent EFT generalization of the SM constructed out of a series of $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant higher dimensional operators, built out of SM fields*

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_{i} \frac{c_i}{\Lambda} \mathcal{O}_i^{d=5} + \sum_{i} \frac{c_i}{\Lambda^2} \mathcal{O}_i^{d=6} + \mathcal{O}(\Lambda^{-3})$$

BSM effects

Relevant operators for our study

$$\mathcal{O}^{prst}_{\ell edg} = (ar{\ell}^i_p e_r)(ar{d}_s q^i_t), \qquad \mathcal{O}^{(1)prst}_{\ell equ} = (ar{\ell}^i_p e_r)(ar{q}^j_s u_t)\epsilon_{ij}$$

^{*} Phys. Rept. 793 (2019) 1-98, <u>arXiv:1706.08945</u>



• The most general form factor decomposition for the $\eta^{(\prime)} \to \pi^0(\eta) \mu^+ \mu^-$ processes

$$\langle \ell^+ \ell^- | \eta \pi^0 \rangle = i \mathcal{M}(2\pi)^4 \delta(p_{\ell^+} + p_{\ell^-} - p_{\eta} - p_{\pi})$$

is

$$\mathcal{M} = m_{\ell}(\bar{u}v)F_1 + (\bar{u}i\gamma^5v)F_2 + (\bar{u}kv)F_3 + i(\bar{u}k\gamma^5v)F_4,$$

where

$$k = p_{\eta} + p_{\pi}$$



- Noting that
 - Within the SM, only $F_{1,3}$ terms contribute if one neglects electroweak effects
 - Possible effects to F_4 within the SM arise via electroweak loops
 - Within the SMEFT, the Fermi operators involving vector currents are irrelevant

one arrives at

$$F_1 = \Sigma \qquad \text{(cf. slide 15)}$$

$$F_2 = \left[\operatorname{Im} c_{\ell edq}^{2211} \left< 0 \right| \bar{d}d \left| \eta \pi^0 \right> + \operatorname{Im} c_{\ell edq}^{2222} \left< 0 \right| \bar{s}s \left| \eta \pi^0 \right> - \operatorname{Im} c_{\ell equ}^{(1)2211} \left< 0 \right| \bar{u}u \left| \eta \pi^0 \right> \right] v^{-2}$$

$$F_3 = \frac{1}{2}\Omega \qquad \text{(cf. slide 15)}$$

$$F_4 = 0$$



• Strong upper bounds for the Wilson coefficients come from nEDMs and charm decays, such as $D_s^- \to \mu^- \bar{\nu}_\mu$

$$Im \ c_{lequ}^{(1)2211} < 0.001$$
 $Im \ c_{ledq}^{2211} < 0.002$ $Im \ c_{ledq}^{2222} < 0.02$



• At LO in $LN_c \chi$ PT, the above matrix elements can be expressed as

$$\langle \pi^{0} | \bar{u}u | \eta \rangle = B_{0}(c\phi_{23} + \epsilon_{13}s\phi_{23}), \qquad \langle \pi^{0} | \bar{u}u | \eta' \rangle = -B_{0}(\epsilon_{13}c\phi_{23} - s\phi_{23})$$

$$\langle \pi^{0} | \bar{d}d | \eta \rangle = -B_{0}(c\phi_{23} - \epsilon_{13}s\phi_{23}), \qquad \langle \pi^{0} | \bar{d}d | \eta' \rangle = -B_{0}(\epsilon_{13}c\phi_{23} + s\phi_{23})$$

$$\langle \pi^{0} | \bar{s}s | \eta \rangle = -2B_{0}\epsilon_{13}s\phi_{23}, \qquad \langle \pi^{0} | \bar{s}s | \eta' \rangle = 2B_{0}\epsilon_{13}c\phi_{23}$$

$$\langle \eta | \bar{u}u | \eta' \rangle = -B_{0}[\epsilon_{13}(c\phi_{23}^{2} - s\phi_{23}^{2}) - (1 - 2\epsilon_{12})c\phi_{23}s\phi_{23}]$$

$$\langle \eta | \bar{d}d | \eta' \rangle = B_{0}[\epsilon_{13}(c\phi_{23}^{2} - s\phi_{23}^{2}) + (1 + 2\epsilon_{12})c\phi_{23}s\phi_{23}]$$

$$\langle \eta | \bar{s}s | \eta' \rangle = -2B_{0}c\phi_{23}s\phi_{23}$$



• Differential decays widths for $\eta^{(\prime)} \to \pi^0(\eta) \mu^+ \mu^- \to \pi^0(\eta) e^+ \nu_e \overline{\nu_\mu} e^- \overline{\nu_e} \nu_\mu$

$$d\Gamma = \sum_{\lambda\bar{\lambda}} \frac{ds dc_{\theta}}{64(2\pi)^{3} m_{\eta}} \frac{\lambda_{K}^{1/2} \beta_{\mu}}{m_{\eta}^{2}} |\mathcal{M}(\lambda \boldsymbol{n}, \bar{\lambda} \bar{\boldsymbol{n}})|^{2} \left[\frac{d\Omega}{4\pi} dx \ n(x) \left(1 - \lambda b(x) \boldsymbol{\beta} \cdot \boldsymbol{n}\right) \right] \left[\frac{d\bar{\Omega}}{4\pi} d\bar{x} \ n(\bar{x}) \left(1 + \bar{\lambda} b(\bar{x}) \bar{\boldsymbol{\beta}} \cdot \bar{\boldsymbol{n}}\right) \right]$$

$$\eta^{(\prime)} \to \pi^{0}(\eta) \mu^{+} \mu^{-} \qquad \mu^{+} \to e^{+} \nu_{e} \bar{\nu}_{\mu} \qquad \mu^{-} \to e^{-} \bar{\nu}_{e} \nu_{\mu}$$

$$d\Gamma = \frac{dsdc_{\theta}}{64(2\pi)^{3}m_{\eta}} \frac{\lambda_{K}^{1/2}\beta_{\mu}}{m_{\eta}^{2}} \left[\frac{d\Omega}{4\pi} dx \ n(x) \right] \left[\frac{d\bar{\Omega}}{4\pi} d\bar{x} \ n(\bar{x}) \right] \left[\tilde{c}_{1}|F_{1}|^{2} + \tilde{c}_{3}|F_{3}|^{2} + \tilde{c}_{13}^{R} \operatorname{Re} F_{1}F_{3}^{*} + \tilde{c}_{13}^{I} \operatorname{Im} F_{1}F_{3}^{*} + \tilde{c}_{23}^{I} \operatorname{Im} F_{1}F_{3}^{*} + \tilde{c}_{23}^{I} \operatorname{Im} F_{2}F_{3}^{*} \right] + \tilde{c}_{2}|F_{2}|^{2} + \tilde{c}_{12}^{R} \operatorname{Re} F_{1}F_{2}^{*} + \tilde{c}_{12}^{I} \operatorname{Im} F_{1}F_{2}^{*} + \tilde{c}_{23}^{R} \operatorname{Re} F_{2}F_{3}^{*} + \tilde{c}_{23}^{I} \operatorname{Im} F_{2}F_{3}^{*} \right],$$



Longitudinal and transverse asymmetries:

$$A_L = \frac{N(c_{\theta_{e^+}} > 0) - N(c_{\theta_{e^+}} < 0)}{N} = -\frac{2}{3} \frac{\int ds dc_{\theta} \lambda_K^{1/2} \beta_{\mu} m_{\mu} \left[\beta_{\mu} s \operatorname{Im} F_1 F_2^* + 2 \lambda_K^{1/2} c_{\theta} \operatorname{Im} F_3 F_2^* \right]}{64 (2\pi)^3 m_{\eta}^3 \int d\Gamma}$$

$$A_{T} = \frac{N(s_{\bar{\phi}-\phi} > 0) - N(s_{\bar{\phi}-\phi} < 0)}{N} = \frac{\pi}{18} \frac{\int ds dc_{\theta} \lambda_{K}^{1/2} \beta_{\mu} m_{\mu} \left[\beta_{\mu} s \operatorname{Re} F_{1} F_{2}^{*} + 2\lambda_{K}^{1/2} c_{\theta} \operatorname{Re} F_{3} F_{2}^{*} \right]}{64(2\pi)^{3} m_{\eta}^{3} \int d\Gamma}$$



• **Preliminary** results @LO in χ PT for $\eta \to \pi^0 \mu^+ \mu^-$

$$A_L = -0.179(9) \ Im \ c_{lequ}^{(1)2211} - 0.176(10) \ Im \ c_{ledq}^{2211} - 7.92(32) \cdot 10^{-3} \ Im \ c_{ledq}^{2222} < -6.9(4) \cdot 10^{-4}$$

$$A_T = 0.0800(34) \ Im \ c_{lequ}^{(1)2211} + 0.0786(26) \ Im \ c_{ledq}^{2211} + 3.52(15) \cdot 10^{-3} \ Im \ c_{ledq}^{2222} < 3.0(1) \cdot 10^{-4}$$



• **Preliminary** results @LO in χ PT for $\eta' \rightarrow \pi^0 \mu^+ \mu^-$

$$A_L = -0.0757(23) \, Im \, c_{lequ}^{(1)2211} - 0.0798(23) \, Im \, c_{ledq}^{2211} + 4.38(18) \cdot 10^{-3} \, Im \, c_{ledq}^{2222} < -1.5(1) \cdot 10^{-4}$$

$$A_T = 6.39(47) \cdot 10^{-3} \, Im \, c_{lequ}^{(1)2211} + 6.84(36) \cdot 10^{-3} \, Im \, c_{ledq}^{2211} - 3.76(17) \cdot 10^{-4} \, Im \, c_{ledq}^{2222} < 1.3(1) \cdot 10^{-5}$$



• **Preliminary** results @LO in χ PT for $\eta' \rightarrow \eta \mu^{+} \mu^{-}$

$$A_L = -0.0375(9) \ Im \ c_{lequ}^{(1)2211} + 0.0422(12) \ Im \ c_{ledq}^{2211} - 0.0794(20) \ Im \ c_{ledq}^{2222} < -1.55(4) \cdot 10^{-3}$$

$$A_T = 6.47(1.34) \cdot 10^{-4} \ Im \ c_{lequ}^{(1)2211} - 6.25(1.91) \cdot 10^{-4} \ Im \ c_{ledq}^{2211} + 1.21(9) \cdot 10^{-3} \ Im \ c_{ledq}^{2222} < 2.3(6) \cdot 10^{-5}$$



- REDTOP can perform muon polarimetry
- The expected asymmetry noise, for example, for the $\eta \to \pi^0 \mu^+ \mu^-$ is

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{2.5 \times 10^{13} \cdot 1.2 \times 10^{-9}}} \approx 6 \times 10^{-3}$$

- On the other hand, the asymmetries calculated at LO are of the order $\sim 1 \times 10^{-3}$
- Thus, it appears that the statistics of REDTOP may fall short
- Results using matrix elements at NLO with q^2 dependence still pending

STAY TUNNED

Conclusions

- The study of the η and $\eta^{(\prime)}$ phenomenology can provide a very interesting way to find physics BSM
- Theoretical predictions have been presented for the BRs of the six $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$ semileptonic processes
- Theoretical estimations for the longitudinal and transverse asymmetries of the three $\eta^{(\prime)} \to \pi^0(\eta) \mu^+ \mu^-$ processes have been presented, which will enable assessing the existence of *CP* violating effects from physics BSM. Work ongoing.

Thanks