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A new theoretical determination of

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$$ (P = π,K)

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1. Motivation

- ✓ Lepton Universality (LU) as a basic tenet of the Standard Model (SM).
  - ✓ A few anomalies observed in semileptonic B meson decays*.
- ✓ We aim to test muon-tau lepton universality through the ratio (P = π, K)**:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1 + \delta R_{\tau/P}\right)$$

  - ✓ $g_\tau = g_\mu$ according to LU.
  - ✓ $R_{\tau/P}^{(0)}$ is the LO result $R^{(0)}_{\tau/P} = \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}$.
  - ✓ $\delta R_{\tau/P}$ encodes the radiative corrections.
- ✓ $\delta R_{\tau/P}$ was calculated by Decker & Finkemeier (DF'95)***:
  - ✓ $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$.
- ✓ Important phenomenological and theoretical reasons to address the analysis again.

* Albrecht et al.'21
** Marciano & Sirlin'93
*** Decker & Finkemeier'95

1. Motivation

- ✓ Phenomenological disagreement in LU tests:
  - ✓ Using $\frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$ and DF'95*, HFLAV** reported:
    - ✓ $|g_\tau/g_\mu|_\pi$ = 0.9958 ± 0.0026 (at 1.6σ of LU)
    - ✓ $|g_\tau/g_\mu|_K$ = 0.9879 ± 0.0063 (at 1.9σ of LU)
  - ✓ Using $\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau[\gamma])}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu[\gamma])}$, HFLAV** reported:
    - ✓ $|g_\tau/g_\mu|$ = 1.0010 ± 0.0014 (at 0.7σ of LU)
  - ✓ Using $\frac{\Gamma(W \to \tau\nu_\tau)}{\Gamma(W \to \mu\nu_\mu)}$, CMS and ATLAS*** and reported:
    - ✓ $|g_\tau/g_\mu|$ = 0.995 ± 0.006 (at 0.8σ of LU)

- ✓ Theoretical issues within DF'95*:
  - ✓ Hadronic form factors do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.
  - ✓ A cutoff to regulate the loop integrals (separating long- and short-distance corrections)
  - ✓ Underestimated uncertainties (purely ChPT size).

- ✓ By-products of the project:
  - ✓ Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.
  - ✓ CKM unitarity test via $\Gamma(\tau \to K\nu_\tau[\gamma])$ or via the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$.
  - ✓ Constraints on possible non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$^.

* Decker & Finkemeier'95
** HFLAV'21
*** CMS'21, ATLAS'21

^ Cirigliano et al.'10 '19
^ González-Alonso & Martín Calamich '16
^ Gonzàlez-Solís et al. '20

2. $P \to \mu \nu_\mu [\gamma]$ (P=π,K)

- ✓ Calculated unambigously within the Standard Model (Chiral Perturbation Theory, ChPT*).
- ✓ Notation by Marciano & Sirlin** and numbers by Cirigliano & IR*** (D=d,s for π,K):

LO result — short-distance EW correction — structure independent (SI) contributions (point-like approximation)^

$$\Gamma(P \to \mu\nu_\mu[\gamma]) = \frac{G_F^2 |V_{uD}|^2 F_P^2}{4\pi} m_P m_\mu^2 \left(1 - \frac{m_\mu^2}{m_P^2}\right)^2 \left\{1 + \frac{2\alpha}{\pi} \log\frac{m_Z}{m_\rho}\right\} \left\{1 + \frac{\alpha}{\pi} F(m_\mu^2/m_P^2)\right\} \times$$

$$\left\{1 - \frac{\alpha}{\pi}\left[\frac{3}{2}\log\frac{m_\rho}{m_P} + c_1^{(P)} + \frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)} \log\frac{m_\rho^2}{m_\mu^2} + c_3^{(P)} + c_4^{(P)}(m_\mu/m_P)\right) - \frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)} \log\frac{m_\rho^2}{m_\mu^2}\right]\right\}$$

structure-dependent (SD) contributions [coefficents reported in Cirigliano & IR'07]

- ✓ The only model-dependence is the determination of the counterterms in $c_1^{(P)}$ and $c_3^{(P)}$:
  - ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies†.

* Weinberg'79
* Gasser & Leutwyler'84 '85
** Marciano & Sirlin'93
*** Cirigliano & IR'07
^ Kinoshita'59
† Ecker et al.'89
† Cirigliano et al.'06

3. $\tau \to P\, \nu_\tau\, [\gamma]$ ($P=\pi,K$)

- ✓ Calculated within an effective approach encoding the hadronization:
  - ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies*.
- ✓ We follow a similar notation to $P\to\mu\nu_\mu[\gamma]$ ($D=d,s$ for π,K):

LO result — short-distance EW correction — structure independent (SI) contributions (point-like approximation)**

$$\Gamma(\tau\to P\nu_\tau[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{8\pi}M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2\left\{1+\frac{2\,\alpha}{\pi}\log\frac{m_Z}{m_\rho}\right\}\left\{1+\frac{\alpha}{\pi}\,G(m_P^2/M_\tau^2)\right\}\times$$
$$\left\{1-\frac{3\alpha}{2\pi}\log\frac{m_\rho}{M_\tau}+\delta_{\tau P}\big|_{\mathrm{rSD}}+\delta_{\tau P}\big|_{\mathrm{vSD}}\right\}$$

real-photon structure-dependent (rSD) contributions — virtual-photon structure-dependent (vSD) contributions

- ✓ Real-photon structure-dependent (rSD) contributions from Guo & Roig'10***.
- ✓ Virtual-photon structure-dependent (vSD) contributions not calculated in the literature.

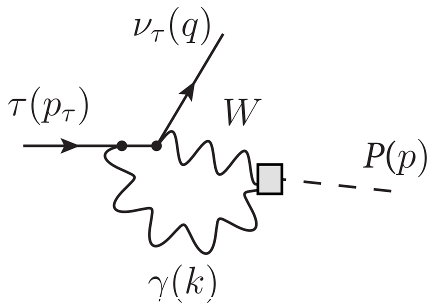
* Ecker et al.'89
* Cirigliano et al.'06
** Kinoshita'59
*** Guo & Roig'10

3. τ → P ν_τ [γ]   (P=π,K)

✓ Virtual-photon structure-dependent contribution (vSD):

$$i\mathcal{M}[\tau\to P\nu_\tau]|_{\rm SD}=iG_F V_{uD} e^2\int\frac{{\rm d}^d k}{(2\pi)^d}\frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2-M_\tau^2]}\left[i\epsilon_{\mu\nu\lambda\rho}k^\lambda p^\rho F_V^P(W^2,k^2)+F_A^P(W^2,k^2)\lambda_{1\mu\nu}+2B(k^2)\lambda_{2\mu\nu}\right]$$

$$\begin{aligned}\ell^{\mu\nu} &= \bar u(q)\gamma^\mu(1-\gamma_5)[(\not p_\tau+\not k)+M_\tau]\gamma^\nu u(p_\tau)\\ \lambda_{1\mu\nu} &= \left[(p+k)^2+k^2-m_P^2\right]g_{\mu\nu}-2k_\mu p_\nu\\ \lambda_{2\mu\nu} &= k^2 g_{\mu\nu}-\frac{k^2(p+k)_\mu p_\nu}{(p+k)^2-m_P^2}\end{aligned}$$



✓ Form factors from Guo & Roig'10 and Guevara et al.'13*:

$$\begin{aligned}F_V^P(W^2,k^2) &= \frac{-N_C M_V^4}{24\pi^2 F_P(k^2-M_V^2)(W^2-M_V^2)}\\ F_A^P(W^2,k^2) &= \frac{F_P}{2}\frac{M_A^2-2M_V^2-k^2}{(M_V^2-k^2)(M_A^2-W^2)}\\ B(k^2) &= \frac{F_P}{M_V^2-k^2}\end{aligned}$$

✓ Well-behaved two- and three-point Green functions.

✓ Chiral and U(3) limits.

✓ M_V and M_A large-N_C vector- and axial-vector resonance mass.

* Guo & Roig'10
* Guevara et al.'13

4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tau P} - \delta_{P\mu})$

1. Structure-independent contribution (point-like approximation): SI.

✓ We confirm the results by DF'95*.

$$\delta R_{\tau/P}\big|_{\rm SI} = \frac{\alpha}{2\pi}\left\{\frac{3}{2}\log\frac{M_\tau^2 m_P^2}{m_\mu^4} + \frac{3}{2} + g\left(\frac{m_P^2}{M_\tau^2}\right) - f\left(\frac{m_\mu^2}{m_P^2}\right)\right\}$$

$$f(x) = 2\left(\frac{1+x}{1-x}\log x - 2\right)\log(1-x) - \frac{x(8-5x)}{2(1-x)^2}\log x + 4\frac{1+x}{1-x}\mathrm{Li}_2(x) - \frac{x}{1-x}\left(\frac{3}{2} + \frac{4}{3}\pi^2\right)$$

$$g(x) = 2\left(\frac{1+x}{1-x}\log x - 2\right)\log(1-x) - \frac{x(2-5x)}{2(1-x)^2}\log x + 4\frac{1+x}{1-x}\mathrm{Li}_2(x) + \frac{x}{1-x}\left(\frac{3}{2} - \frac{4}{3}\pi^2\right)$$

$\delta R_{\tau/\pi}|_{SI}$ = 1.05% and $\delta R_{\tau/K}|_{SI}$ = 1.67%

2. Real-photon structure-dependent contribution: rSD.

✓ $\delta_{P\mu}|_{rSD}$ from Cirigliano & IR'07**: $\delta_{\pi\mu}|_{rSD} = -1.3\cdot 10^{-8}$ and $\delta_{K\mu}|_{rSD} = -1.7\cdot 10^{-5}$.

✓ $\delta_{\tau P}|_{rSD}$ from Guo & Roig'10***: $\delta_{\tau\pi}|_{rSD}$ = 0.15% and $\delta_{\tau K}|_{rSD}$ = (0.18 ± 0.05)%.

$\delta R_{\tau/\pi}|_{rSD}$ = 0.15% and $\delta R_{\tau/K}|_{rSD}$ = (0.18 ± 0.15)%

* Decker & Finkemeier'95
** Cirigliano & IR'07
*** Guo & Roig'10

4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tau P} - \delta_{P\mu})$

3. Virtual-photon structure-dependent contribution: vSD.

- ✓ $\delta_{P\mu}|_{vSD}$ from Cirigliano & IR'07*: $\delta_{\pi\mu}|_{vSD} = (0.54 \pm 0.12)\%$ and $\delta_{K\mu}|_{vSD} = (0.43 \pm 0.12)\%$.
- ✓ $\delta_{\tau P}|_{vSD}$, **new calculation**: $\delta_{\tau\pi}|_{vSD} = (-0.48 \pm 0.56)\%$ and $\delta_{\tau K}|_{vSD} = (-0.45 \pm 0.57)\%$.

$$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$$

- ✓ Uncertainties dominated by $\delta_{\tau P}|_{vSD}$:
  - ✓ P decays within ChPT [counterterms can be determined by matching ChPT with the resonance effective approach at higher energies], whereas τ decays within resonance effective approach [no matching to determine the counterterms].
  - ✓ Estimation of the model-dependence by comparing our results with a less general scenario where only well-behaved two-point Green functions and a reduced resonance Lagrangian is used: ±0.22% and ±0.24% for the pion and the kaon case.
  - ✓ Estimation of the counterterms by considering the running between 0.5 and 1.0 GeV: ±0.52%.

* Cirigliano & IR'07

5. Results

Contribution	$\delta R_{\tau/\pi}$	$\delta R_{\tau/K}$	Ref.
SI	$+1.05\%$	$+1.67\%$	*
rSD	$+0.15\%$	$+(0.18 \pm 0.05)\%$	**
vSD	$-(1.02 \pm 0.57)\%$	$-(0.88 \pm 0.58)\%$	new
Total	$+(0.18 \pm 0.57)\%$	$+(0.97 \pm 0.58)\%$	new

✓ Central values agree remarkably with DF'95: $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$, **but** in that work:

- ✓ problematic hadronization: form factors do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.
- ✓ a cutoff to regulate the loop integrals.
- ✓ underestimated uncertainties (purely ChPT size).

* Decker & Finkemeier'95
** Cirigliano & IR'07
** Guo & Roig'10

6. Application I: lepton universality test

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}(1+\delta R_{\tau/P})$$

PDG

$\delta R_{\tau/\pi} = (0.18 \pm 0.57)\%$
$\delta R_{\tau/K} = (0.97 \pm 0.58)\%$

$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\rm th} \pm 0.0025_{\rm exp} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\rm th} \pm 0.0072_{\rm exp} = 0.9857 \pm 0.0078$$

- ✓ π case: at 0.9σ of LU vs. 1.6σ of LU in HFLAV'21* using DF'95**
- ✓ K case: at 1.8σ of LU vs. 1.9σ of LU in HFLAV'21* using DF'95**

* HFLAV'21
** Decker & Finkemeier'95

6. Application II: Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$

short-distance EW correction

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |V_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{ew} (1 + \delta_{\tau P})$$

✓ $\delta_{\tau P}$ includes SI and SD radiative corrections.

$$\delta_{\tau P} = \frac{\alpha}{2\pi}\left(g\left(\frac{m_P^2}{M_\tau^2}\right) + \frac{19}{4} - \frac{2\pi^2}{3} - 3\log\frac{m_\rho}{M_\tau}\right) + \delta_{\tau P}\big|_{\mathrm{rSD}} + \delta_{\tau P}\big|_{\mathrm{vSD}} = \begin{cases} \delta_{\tau\pi} = (-0.24 \pm 0.56)\% \\ \delta_{\tau K} = (-0.15 \pm 0.57)\% \end{cases}$$

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$

FLAG'20*: $F_K/F_\pi = 1.1932 \pm 0.0019$

PDG

$$\delta = \frac{\alpha}{2\pi}\left\{g\left(\frac{m_K^2}{M_\tau^2}\right) - g\left(\frac{m_\pi^2}{M_\tau^2}\right)\right\} + \delta_{\tau K}\big|_{\rm rSD} - \delta_{\tau\pi}\big|_{\rm rSD} + \delta_{\tau K}\big|_{\rm vSD} - \delta_{\tau\pi}\big|_{\rm vSD} = +(0.10 \pm 0.80)\%$$

$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

- ✓ 2.1σ away from CKM unitarity, considering $|V_{ud}| = 0.97373 \pm 0.00031$**.

* FLAG'20
** Hardy & Towner'20

6. Application IV: CKM unitarity test in $\Gamma(\tau \to K\nu_\tau[\gamma])$

FLAG'20*: $\sqrt{2}F_K = (155.7 \pm 0.3)$ MeV

short-distance EW correction Sew=1.0232**

$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

PDG

$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi} |V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{ew} (1 + \delta_{\tau K})$$

$$|V_{us}| = 0.2220 \pm 0.0008_{\text{th}} \pm 0.0016_{\text{exp}} = 0.2220 \pm 0.0018$$

✓ 2.6σ away from CKM unitarity, considering $|V_{ud}|=0.97373\pm0.00031$***.

* FLAG'20
** Marciano & Sirlin'93
*** Hardy & Towner'20

6. Application V: constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$

$|V_{ud}|$ = 0.97373 ± 0.00031*
$|V_{us}/V_{ud}|$ = 0.2288 ± 0.0020

FLAG'20*:
$\sqrt{2}F_\pi$ = (130.2 ± 0.8) MeV
$\sqrt{2}F_K$ = (155.7 ± 0.3) MeV

short-distance EW correction
Sew=1.0232***

$$\delta_{\tau\pi} = (-0.24 \pm 0.56)\%$$
$$\delta_{\tau K} = (-0.15 \pm 0.57)\%$$

PDG

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |\widetilde{V}_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{ew} \left(1 + \delta_{\tau P} + 2\Delta^{\tau P}\right)$$

Values of $\Delta^{\tau P}$ reported in the MS-scheme and at a scale of μ=2 GeV.

$$\Delta^{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{M_\tau(m_u + m_D)}\epsilon_P^\tau = \begin{cases} \Delta^{\tau\pi} = -(0.15 \pm 0.72)\cdot 10^{-2} \\ \Delta^{\tau K} = -(0.36 \pm 1.18)\cdot 10^{-2} \end{cases}$$

- ✓ To be compared with $\Delta^{\tau\pi} = -(0.15 \pm 0.67)\cdot 10^{-2}$ of Cirigliano et al.'19^.
- ✓ To be compared with $\Delta^{\tau\pi} = -(0.12 \pm 0.68)\cdot 10^{-2}$ and $\Delta^{\tau K} = (-0.41 \pm 0.93)\cdot 10^{-2}$ of González-Solís et al.'20†.

* Hardy & Towner'20
** FLAG'20
*** Marciano & Sirlin'93

^ Cirigliano et al.'19
† Gonzàlez-Solís et al. '20

7. Conclusions

✓ The observable and **our result**:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R_{\tau/P}^{(0)} \left(1 + \delta R_{\tau/P}\right) \longrightarrow \begin{cases} \delta R_{\tau/\pi} = (0.18 \pm 0.57)\% \\ \delta R_{\tau/K} = (0.97 \pm 0.58)\% \end{cases}$$

✓ Framework: ChPT for π decays and a resonance extension of ChPT for τ decays.

✓ Consistent with DF'95*, but with more robust assumptions and yielding a reliable uncertainty.

✓ Applications:

- ✓ $|g_\tau/g_\mu|_P$ at 0.9σ (π) and 1.8σ (K) of LU, reducing HFLAV'21** disagreement with LU.
- ✓ Theoretical determination of radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.
- ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])/\Gamma(\tau\to\pi\nu_\tau[\gamma])$: $|V_{us}/V_{ud}| = 0.2288 \pm 0.0020$, at 2.1σ from unitarity.
- ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])$: $|V_{us}| = 0.2220 \pm 0.0018$, at 2.6σ from unitarity.
- ✓ Constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$: update of $\Delta^{\tau P}$.

* Decker & Finkemeier'95
** HFLAV'21