Exclusive production of $f_1(1285)$ meson in proton-(anti)proton collisions

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Introduction | Motivation

In this talk we will be concerned with central exclusive production (CEP) of axial-vector $f_1(1285)$ meson in proton-(anti)proton collisions at c.m. energies:

- **low**: HADES ($pp$) and PANDA ($pp$) at FAIR ← Lebiedowicz, Nachtmann, Salabura, Szczurek, arXiv:2105.07192
- **intermediate**: WA102, COMPASS
- **high**: RHIC, LHC ← Lebiedowicz, Leutgeb, Nachtmann, Rebhan, Szczurek, PRD 102 (2020) 114003

The $f_1(1285)$ meson was measured:

- in two-photon interactions in $e^+e^-$ reactions (MARKII, TPC/Two-Gamma, L3) see: A. Szczurek, PRD 102 (2020) 113015 ← on production of $f_1$ mesons at $e^+e^-$ collisions with double-tagging as a way to constrain the axial meson light-by-light contribution to the muon g-2 and hyperfine splitting of muonic hydrogen
- in photoproduction process $\gamma p \rightarrow f_1 p$ (CLAS Collaboration)
- in CEP $pp$ collisions for c.m. energies 12.7 and 29.1 GeV (WA102) and for 13 TeV at the LHC (ATLAS-ALFA) [R. Sikora, CERN-THESIS-2020-235]

Why is it interesting to study the $pp \rightarrow ppf_1(1285)$ process?

- **What is underlying production mechanism for studies of $f_1$ CEP at near threshold and at LHC?**
  - Poorly known $VVf_1$ and pomeron-pomeron-$f_1$ coupling strengths and the vertex form factors
  - Can it be described in holographic QCD?
- **What is underlying decay mechanism?**
  - e.g., $f_1(1285) \rightarrow 4\pi$ decay via $\rho \rho$ or/and $\pi f_1(1260)(\rightarrow \rho \pi)$
  - transition form factors e.g., $\gamma^*\gamma^* \rightarrow f_1$, $f_1 \rightarrow \gamma\gamma^* \rightarrow \gamma e^+e^-$ [see, e.g., Zanke, Hoferichter, Kubis, JHEP 07 (2021) 106]
  - What is the nature of the $f_1(1285)$? For instance, is it a normal $q\bar{q}$ state or $KK^*$ molecule? see: Aceti, Dias, Oset, EPJA 51 (2015) 48; Aceti, Xie, Oset, PLB 750 (2015) 609
- **What is optimal observation channel of the $f_1(1285)$?**
VV-fusion mechanism

\[ p(p_a, \lambda_a) + p(p_b, \lambda_b) \rightarrow p(p_1, \lambda_1) + f_1(k, \lambda_{f_1}) + p(p_2, \lambda_2) \]

\[ p_{a,b}, p_{1,2} \text{ and } \lambda_{a,b}, \lambda_{1,2} = \pm \frac{1}{2} \text{: the four-momenta and helicities of protons } k \text{ and } \lambda_{f_1} = 0, \pm 1 \text{: the four-momentum and helicity of the } f_1 \text{ meson} \]

\[ q_1 = p_a - p_1, \quad q_2 = p_b - p_2, \quad k = q_1 + q_2 \]
\[ t_1 = q_1^2, \quad t_2 = q_2^2, \quad m_{f_1}^2 = k^2 \]
\[ s = (p_a + p_b)^2 = (p_1 + p_2 + k)^2, \text{ c.m. energy squared} \]
\[ s_1 = (p_1 + k)^2, \quad s_2 = (p_2 + k)^2 \]

**VV-fusion amplitude:**

\[ \mathcal{M}^{(VV \text{ fusion})}_{pp \rightarrow ppf_1} = \mathcal{M}^{(p\rho \text{ fusion})}_{pp \rightarrow ppf_1} + \mathcal{M}^{(\omega\omega \text{ fusion})}_{pp \rightarrow ppf_1} \]

\[ \mathcal{M}^{(VV \text{ fusion})}_{\lambda_a \lambda_b \rightarrow \lambda_1 \lambda_{f_1}} = (-i) (e^\alpha(\lambda_{f_1}))^* \bar{u}(p_1, \lambda_1) i \Gamma_{\mu_1}^{(pp)}(p_1, p_2) u(p_a, \lambda_a) \times i \Delta^{(V)}(s_1, t_1) i \Gamma_{\nu_1}^{(V \omega f_1)}(q_1, q_2) i \Delta^{(V)}(s_2, t_2) \times \bar{u}(p_2, \lambda_2) i \Gamma_{\mu_2}^{(pp)}(p_2, p_b) u(p_b, \lambda_b) \]

\[ i \Gamma_{\mu}^{(pp)}(p', p) = -i \Gamma_{\mu}^{(pp)}(p', p) = -i g_{VNN} F_{VNN}(t) \left[ \gamma_\mu - i \frac{\kappa_V}{2m_p} \sigma_{\mu\nu}(p' - p)^\nu \right] \]

\[ g_{ppp} = 3.0, \quad \kappa_\rho = 6.1, \quad \kappa_\omega = 9.0, \quad \kappa_V = 0 \]

**K: tensor-to-vector coupling ratio, \( \kappa_V = f_{VNN}/g_{VNN} \)**

\[ F_{VNN}(t) = \frac{\Delta^2_{VNN} - m_V^2}{\Delta^2_{VNN} - t} \]

For the proton-antiproton collisions we have

\[ \bar{u}(p_2, \lambda_2) i \Gamma_{\mu_2}^{(pp)}(p_2, p_b) u(p_b, \lambda_b) \rightarrow \bar{u}(p_b, \lambda_b) i \Gamma_{\mu_2}^{(pp)}(p_2, p_b) u(p_2, \lambda_2) \]

\[ = - i \Gamma_{\mu_2}^{(pp)}(p_2, p_b) u(p_b, \lambda_b) \]

\[ \mathcal{M}^{(VV \text{ fusion})}_{pp \rightarrow ppf_1} = - \mathcal{M}^{(VV \text{ fusion})}_{pp \rightarrow ppf_1} \]

The standard form of the vector-meson propagator:

\[ i \Delta^{(V)}_{\mu \nu}(q) = i \left( -g_{\mu \nu} + \frac{q_\mu q_\nu}{q^2 + i\epsilon} \right) \Delta^{(V)}_{\mu \nu}(q^2) - i \frac{q_\mu q_\nu}{q^2 + i\epsilon} \Delta^{(V)}_{\mu \nu}(q^2) \]

\[ \Delta^{(V)}_{\mu}(t) = (t - m_V^2)^{-1} \]

For higher values of \( s_1 \) and \( s_2 \) we must take into account reggeization:

\[ \Delta^{(V)}_{\mu}(t) \rightarrow \tilde{\Delta}^{(V)}_{\mu}(s_1, t_1) = \Delta^{(V)}_{\mu}(t_1) \left( \exp(i\phi(s_1)) \frac{s_1}{s_{thr}} \right) \]

\[ \phi(s_1) = \frac{\pi}{2} \exp \left( \frac{s_{thr} - s_1}{s_{thr}} \right) - \frac{\pi}{2} \]

where \( s_{thr} \) is the lowest value of \( s_i \), possible here: \( s_{thr} = (m_p + m_{f_1})^2 \)

We use the linear form for the vector meson Regge trajectories:

\[ \alpha_V(t) = \alpha_V(0) + \alpha'_V t, \quad \alpha_V(0) = 0.5, \quad \alpha'_V = 0.9 \text{ GeV}^{-2} \]

**VVf_1 coupling**

\[ \mathcal{L}_{VVf_1}(x) = \frac{1}{M_0^4} g_{VVf_1} \left( V_{\lambda a}(x) \partial_\mu \partial_\nu V_{\rho a}(x) \partial_\omega (U_{\beta a}(x) - \partial_\beta U_{\alpha a}(x)) \right) g^{\rho \omega} g^{\mu \nu} \]

\[ V_{\lambda a}(x) = \partial_\lambda V_a(x) - \partial_\lambda U_{\alpha a}(x), \text{ U}_a(x) \text{ and } V_a(x) \text{ are the fields of the } f_1 \text{ and the vector meson } V, \text{ } M_0 \equiv 1 \text{ GeV and } g_{VVf_1} \text{ is a dimensionless coupling constant} \]

\[ i \Gamma_{\mu \nu \alpha}(q_1, q_2) = \frac{2 g_{VVf_1}}{M_0^4} \left[ (q_1 - q_2)^\alpha (q_1 - q_2)^\nu \right] \Delta^{(V a)}(q_1, q_2) \]

\[ \times (q_{1\alpha} \delta_{\mu \nu} - q_{1}^\mu g_{\mu \nu} - q_{2}^\nu g_{\mu \nu} - q_{1}^\nu g_{\mu \nu}) + (q_1 \leftrightarrow q_2, \mu \leftrightarrow \nu) \]

\[ F^{(VVf_1)}(q_1^2, q_2^2, m_{f_1}^2) = \tilde{F}_V(q_1^2) \tilde{F}_V(q_2^2) F(m_{f_1}^2) = \frac{4}{V} \frac{\Lambda^4_V}{t_1 + (t_2 - m_{f_1}^2)^2} \]

with \( F(m_{f_1}^2) = 1 \)
Results

\[\gamma p \rightarrow f_1(1285) p \rightarrow (\eta \pi^+ \pi^-) p\]
\[W_Tp = 2.35 \text{ GeV}\]

- CLAS data
- Eq. (C7)
- Eq. (C10)
- Eq. (C11)

\[\gamma p \rightarrow f_1(1285) p \rightarrow (\eta \pi^+ \pi^-) p\]
\[W_Tp = 2.75 \text{ GeV}\]

- CLAS data
- Eq. (C7)
- Eq. (C9)
- Eq. (C10)
- Eq. (C11)

**N(2300) (?)**

\[\text{we use}\]

The \(\rho \rho f\) coupling constant is extracted from the radiative decay rate \(f_1 \rightarrow \rho^0 \gamma\) using the VMD approach.

\[\Gamma(f_1(1285) \rightarrow \gamma \rho^0) = 1384.7^{+305.1}_{-283.1} \text{ keV}\]

from PDG:

\[\Gamma(f_1(1285) \rightarrow \gamma \rho^0) = (453 \pm 177) \text{ keV}\]

from CLAS:

We consider decay \(f_1 \rightarrow \rho^0 \gamma \rightarrow \pi^+ \pi^- \gamma\) taking \(\rho^0\) mass distribution. We estimate the cutoff parameter \(\Lambda_\rho\) in the \(f_1 \rho \rho\) form factor:

\[F_{\rho \rho f}(k_\rho^2, k_\gamma^2, k^2) = F_{\rho \rho f}(k_\rho^2, 0, m_{f_1}^2) \tilde{F}_\rho(k_\rho^2) \tilde{F}_\rho(0) F(m_{f_1}^2) = \tilde{F}_\rho(k_\rho^2) \tilde{F}_\rho(0)\]

**Photoproduction process:**

- We assume \(g_{\omega f_1} = g_{\rho f_1}\) based on arguments from the quark model and VMD. We assume \(\Lambda_\rho = \Lambda_\omega = \Lambda_V\) and \(\Lambda_{\rho NN} = \Lambda_{\omega NN} = \Lambda_{VNN}\).
- Reggeization effect included
- The t-channel V-exchange mechanism play a crucial role in reproducing the forward-peaked angular distributions, especially at higher energies. From the comparison of differential cross sections to the CLAS data we estimate:

  (C7) : \(\Lambda_{VNN} = 1.35 \text{ GeV for } \Lambda_V = 0.65 \text{ GeV , } |g_{VVf_1}| = 20.03\)
  (C9) : \(\Lambda_{VNN} = 1.01 \text{ GeV for } \Lambda_V = 0.8 \text{ GeV , } |g_{VVf_1}| = 12.0\)
  (C10) : \(\Lambda_{VNN} = 0.9 \text{ GeV for } \Lambda_V = 1.0 \text{ GeV , } |g_{VVf_1}| = 8.49\)
  (C11) : \(\Lambda_{VNN} = 0.834 \text{ GeV for } \Lambda_V = 1.5 \text{ GeV , } |g_{VVf_1}| = 6.59\)

(C11) is excluded due to small \(\Lambda_{VNN}\), we stay with (C7) - (C10)

- Missing N* resonances and s/u-channel proton exchange
  Possible N(2300) contribution
  → postulated in Y.-Y. Wang et al., PRD 95 (2017) 096015

CLAS data:

R. Dickson et al. (CLAS Collaboration), PRC 93 (2016) 065202
**Results**

- No data for the $pp \rightarrow pp f_1$ and $p\bar{p} \rightarrow p\bar{p} f_1$ reactions
- High sensitivity of the VV-fusion cross section to the different sets of parameters.

In our procedure of extracting the coupling constants and the form-factor cutoff parameters from the CLAS data the dominant sensitivity is on coupling constants not on the form factors

- Reggeization effect included
- The NN FSI effect can be neglected
- We predict for the VV-fusion mechanism:
  - Diffractive contribution (IPIP fusion) is very small for the HADES and PANDA energy range
  - $\sqrt{s} = 3.46$ GeV: $\sigma(pp \rightarrow pp f_1) = 0.04 - 0.15 \mu b$
  - $\sqrt{s} = 5.0$ GeV: $\sigma(p\bar{p} \rightarrow p\bar{p} f_1) = 0.41 - 2.07 \mu b$

$\sqrt{s} = 2.25 - 5.47$ GeV

- IPIP-fusion contribution should be considered as upper limit of the cross section.

If at the WA102 c.m. energy (29.1 GeV) there are important contributions from subleading reggeon exchanges, the IPIP contribution could be smaller (by a factor of up to 4)
Results

$\sqrt{s} = 3.46$ GeV (top) and 5.0 GeV (bottom)

- At near threshold energy (HADES) the values of small $|t_1|$ and $|t_2|$ are not accessible kinematically.
- HADES and PANDA experiments have a good opportunity to study physics of large four-momentum transfer squared $\rightarrow$ probes corresponding form factors at relatively large values of $|t_{1,2}|$.
- $\rho^0\rho^0$- and $\omega\omega$-fusion processes have different kinematic dependences. Both terms play similar role. But with increasing c.m. energy the averages of $|t_{1,2}|$ decrease (damping by form factors), hence the $\omega\omega$ term becomes more important.
- We predict a strong preference for the outgoing nucleons to be produced with their transverse momenta being back-to-back,
  \[ \frac{d\sigma}{d\phi_{pp}} \text{ at } \phi_{pp} = \pi \]
  \[ \left( 0 \leq \phi_{pp} \leq \pi \right) \]
Simulations for HADES experiment for $\sqrt{s} = 3.46$ GeV using PLUTO MC generator:

$$pp \rightarrow pp\pi^+\pi^-\pi^+\pi^-$$

$BR(f_1(1285) \rightarrow \pi^+\pi^-\pi^+\pi^-) = (10.9 \pm 0.6)\%$

$\sigma_{\text{back}}^{4\pi} \sim 227 \mu b$ [1], $\sigma_{f_1}^{4\pi} = 16 \mu b$

Difficult to see $f_1$ peak on the $4\pi$ background without additional cuts

$$pp \rightarrow pp\pi^+\pi^-\pi^+\pi^0$$

$BR(f_1(1285) \rightarrow \pi^+\pi^-\eta) = (35 \pm 15)\%$

$BR(\eta \rightarrow \pi^+\pi^-\pi^0) = (22.92 \pm 0.28)\%$

The narrow width of the $\eta$ meson allows to set a mass cut on the $\pi^+\pi^-\pi^0$ invariant mass and suppresses the multi-pion background efficiently

The study of $f_1(1285)$ production at HADES should be feasible!
Pomeron-Pomeron fusion mechanism

At high energies double pomeron (IP) exchange is dominant production mechanism of the $f_1(1285)$

see: Lebiedowicz, Leutgeb, Nachtmann, Rebhan, Szczurek, PRD 102 (2020) 114003

We treat our reaction in the tensor-pomeron approach


The pomeron and the charge conjugation C=+1 reggeons are described as effective rank 2 symmetric tensor exchanges. The odderon and the C= -1 reggeons are described as effective vector exchanges.

This approach has a good basis from nonperturbative QCD considerations. The IP exchange can be understood as a coherent sum of exchanges of spin 2+4+6+ ...  


A tensor character of the pomeron is also preferred in holographic QCD, see e.g.,

Brower, Polchinski, Strassler, Tan, JHEP 12 (2007) 005

Domokos, Harvey, Mann, PRD 80 (2009) 126015

Iatrakis, Ramamurti, Shuryak, PRD 94 (2016) 045005
Applications of the tensor-pomeron and vector-odderon model

- \( \gamma p \rightarrow \pi^+ \pi^- p \) Bolz, Ewerz, Maniatis, Nachtmann, Sauter, Schöning, JHEP 01 (2015) 151
  \( \leftarrow \) interference between \( \gamma p \rightarrow (\rho^0 \rightarrow \pi^+ \pi^-) p \) (IP exchange) and \( \gamma p \rightarrow (f_2(1270) \rightarrow \pi^+ \pi^-) p \) (O exchange) processes and as a consequence \( \pi^+ \pi^- \) charge asymmetries

- **Photoproduction and low x DIS** Britzger, Ewerz, Glazov, Nachtmann, Schmitt, PRD100 (2019) 114007
  \( \leftarrow \) a “vector pomeron” decouples completely in the total photoabsorption cross section and in the structure functions of DIS

- **Helicity in proton-proton elastic scattering and the spin structure of the pomeron**
  Ewerz, P.L., Nachtmann, Szczurek, PLB 763 (2016) 382
  \( \leftarrow \) studying the ratio \( r_5 \) of single-helicity-flip to non-flip amplitudes we found that the STAR data [L. Adamczyk et al., PLB 719 (2013) 62] are compatible with the tensor pomeron ansatz while they clearly exclude a scalar character of the pomeron

- **Central Exclusive Production (CEP)**, \( pp \rightarrow pp X \), P.L., Nachtmann, Szczurek:

  \[
  X: \eta, \eta', f_0 \\
  \rho^0 \quad \text{PRD91 (2015) 074023} \\
  \pi^+ \pi^-, f_0, f_2 \rightarrow (\pi^+ \pi^-) \quad \text{PRD93 (2016) 054015, PRD101 (2020) 034008} \\
  \pi^+ \pi^- \pi^+ \pi^- \quad \rho^0 \rho^0 \quad \text{PRD94 (2016) 034017} \\
  \rho^0 \text{ with proton diss.} \quad \text{PRD95 (2017) 034036} \\
  \rho \phi \quad \text{PRD97 (2018) 094027} \\
  K^+ K^- \quad \text{PRD98 (2018) 014001} \\
  \phi \rightarrow K^+ K^-, \mu^+ \mu^- \quad \text{PRD101 (2020) 094012} \\
  \phi \phi \rightarrow K^+ K^- K^+ K^- \quad \text{PRD99 (2019) 094034} \\
  \{ \text{odderon exchange} \}
  \\
  f_1(1285) \quad \text{P.L., Leutgeb, Nachtmann, Rebhan, Szczurek, PRD102 (2020) 114003} \\
  K^0 \bar{K}^0, \text{continuum vs } f_2(1950) \quad \text{P.L., PRD103 (2021) 054039}
The Born-level amplitude within the tensor-pomeron approach:

\[
M_{\lambda_a \lambda_b \rightarrow \lambda_1 \lambda_2 \lambda_{f_1}}^{\text{Born}} = (-i) \left( \epsilon^\mu (\lambda_{f_1}) \right)^* \bar{u}(p_1, \lambda_1) i \Gamma^{(IPpp)}_{\mu_1 \nu_1}(p_1, p_a) u(p_a, \lambda_a) \\
\times i \Delta^{(IP)}_{\mu_1 \nu_1, \alpha_1 \beta_1}(s_1, t_1) i \Gamma^{(IPPf_1)}_{\alpha_1 \beta_1, \alpha_2 \beta_2, \mu}(q_1, q_2) i \Delta^{(IP)}_{\alpha_2 \beta_2, \mu \nu_2}(s_2, t_2) \\
\times \bar{u}(p_2, \lambda_2) i \Gamma^{(IPpp)}_{\mu_2 \nu_2}(p_2, p_b) u(p_b, \lambda_b)
\]

with terms of the effective pomeron propagator and the pomeron-proton vertex

\[
i \Delta^{(IP)}_{\mu \nu, \kappa \lambda}(s, t) = \frac{1}{4s} \left( g_{\mu \kappa} g_{\nu \lambda} + g_{\mu \lambda} g_{\nu \kappa} - \frac{1}{2} g_{\mu \nu} g_{\kappa \lambda} \right) (-is \alpha'_{IP}(t))^{-1}
\]

\[
i \Gamma^{(IPpp)}_{\mu \nu}(p', p) = -i 3 \beta_{IPNN} F_1 \left((p' - p)^2\right) \left\{ \frac{1}{2} \left[ \gamma_\mu (p' + p)_\nu + \gamma_\nu (p' + p)_\mu \right] - \frac{1}{4} g_{\mu \nu} (p' + p) \right\}
\]

\[
\alpha_{IP}(t) = \alpha_{IP}(0) + \alpha'_{IP} t, \quad \alpha_{IP}(0) = 1.0808, \quad \alpha'_{IP} = 0.25 \text{ GeV}^{-2}
\]

\[
\beta_{IPNN} = 1.87 \text{ GeV}^{-1}, \quad F_1(t): \text{ Dirac form factor of the proton}
\]

Absorption effects:

\[
M_{pp \rightarrow ppf_1} = M_{pp \rightarrow ppf_1}^{\text{Born}} + M_{pp \rightarrow ppf_1}^{\text{pp-recattering}}
\]

\[
M_{pp \rightarrow ppf_1}^{\text{pp-recattering}}(s, \vec{p}_{1\perp}, \vec{p}_{2\perp}) = \frac{i}{8\pi^2 s} \int d^2 \vec{k}_\perp M_{pp \rightarrow ppf_1}^{\text{Born}}(s, \vec{p}_{1\perp} - \vec{k}_\perp, \vec{p}_{2\perp} + \vec{k}_\perp) M_{pp \rightarrow pp}^{\text{pp-exchange}}(s, -\vec{k}_\perp^2)
\]

where \( \vec{k}_\perp \) is the transverse momentum carried around the loop
IP IP f1 coupling

\[ \mathcal{L}^{(\text{IP IP } f_1)} \]

"bare" vertex function
\[ i \Gamma^{(\text{IP IP } f_1)}_{\kappa \lambda, \rho \sigma, \alpha} (q_1, q_2) \big|_{\text{bare}} \]

CEP reaction

vertex function supplemented by suitable form factor
\[ i \Gamma^{(\text{IP IP } f_1)}_{\kappa \lambda, \rho \sigma, \alpha} (q_1, q_2) = i \Gamma^{(\text{IP IP } f_1)}_{\kappa \lambda, \rho \sigma, \alpha} (q_1, q_2) \big|_{\text{bare}} \tilde{F}^{(\text{IP IP } f_1)} (q_1^2, q_2^2, k^2) \]

For the on-shell meson we have set \( k^2 = m_{f_1}^2 \). 

\[ \tilde{F}^{(\text{IP IP } f_1)} (t_1, t_2, m_{f_1}^2) = F_M (t_1) F_M (t_2), \quad F_M (t) = \frac{1}{1 - t / \Lambda_0^2}, \quad \Lambda_0^2 = 0.5 \text{ GeV}^2 \]

or

\[ \tilde{F}^{(\text{IP IP } f_1)} (t_1, t_2, m_{f_1}^2) = \exp \left( \frac{t_1 + t_2}{\Lambda_E^2} \right) \]

where the cutoff constant \( \Lambda_E \) should be adjusted to experimental data.
We follow two strategies for constructing coupling Lagrangian:

(1) **Phenomenological approach.** First we consider a fictitious process: the fusion of two “real spin 2 pomerons” (or tensor glueballs) of mass $m$ giving an $f_1$ meson of $J^{PC} = 1^{++}$.

$$ IP(m, \epsilon_1) + IP(m, \epsilon_2) \rightarrow f_1(m_{f_1}, \epsilon) $$

$\epsilon_{1,2}$: polarisation tensors, $\epsilon$: polarisation vector

We work in the rest system of the $f_1$ meson:

The spin 2 of these “real pomerons” can be combined to a total spin $S$ ($0 \leq S \leq 4$) and this must be combined with the orbital angular momentum $l$ to give $J^{PC} = 1^{++}$ of the $f_1$ state.

There are exactly two possibilities: $(l,S) = (2,2)$ and $(4,4)$.

Corresponding couplings are:

$$ L_{IPf_1}^{(2,2)} = \frac{g'_{IPf_1}}{32 M_0^2} \left( IP_{\kappa\lambda} \frac{\partial_{\mu_1} \partial_{\mu_2}}{\partial_{\mu} \partial_{\nu}} iP_{\rho\sigma} \right) \left( \partial_\alpha U_\beta - \partial_\beta U_\alpha \right) \Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta} $$

$$ L_{IPf_1}^{(4,4)} = \frac{g''_{IPf_1}}{24 \times 32 M_0^4} \left( IP_{\kappa\lambda} \frac{\partial_{\mu_1} \partial_{\mu_2} \partial_{\mu_3} \partial_{\mu_4}}{\partial_{\mu} \partial_{\nu}} iP_{\rho\sigma} \right) \left( \partial_\alpha U_\beta - \partial_\beta U_\alpha \right) \Gamma^{(10)}_{\kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta} $$

Here $M_0 \equiv 1$ GeV, $g'_{IPf_1}, g''_{IPf_1}$: dimensionless coupling parameters,

$IP_{\kappa\lambda}$ effective pomeron field, $U_\alpha f_1$ field, $\partial_\mu = \partial_{\mu_1} - \partial_{\mu_2}$ asymmetric derivative, and $\Gamma^{(8)}, \Gamma^{(10)}$ are known tensor functions.
(2) Holographic QCD approach using the Sakai-Sugimoto model. There, the $IP IP f_1$ coupling can be derived from the bulk Chern-Simons (CS) term requiring consistency of supergravity and the gravitational anomaly.

$$\mathcal{L}^{\text{CS}} = \kappa' U_\alpha \varepsilon^{\alpha\beta\gamma\delta} \, \Pi_\beta \, \partial_\delta \Pi_{\gamma\mu} + \kappa'' U_\alpha \varepsilon^{\alpha\beta\gamma\delta} \left( \partial_\mu P_\beta^\mu \right) \left( \partial_\delta \partial_\mu P_{\gamma\nu} - \partial_\delta \partial_\nu P_{\gamma\mu} \right)$$

$\kappa'$: dimensionless, $\kappa''$: dimension GeV$^{-2}$


For our fictitious reaction with real pomerons there is strict equivalence $\mathcal{L}^{\text{CS}} \cong \mathcal{L}^{(2,2)} + \mathcal{L}^{(4,4)}$ if the couplings satisfy:

$$g'_{IP IP f_1} = -\kappa' \frac{M_0^2}{k^2} - \kappa'' \frac{M_0^2 (k^2 - 2m^2)}{2k^2}$$

$$g''_{IP IP f_1} = \kappa'' \frac{2M_0^2}{k^2}$$

where $k^2$ is invariant mass squared of the resonance $f_1$.

For the CEP reaction the pomerons have invariant mass squared $t_1, t_2 < 0$ instead of $m^2$ and, in general, $t_1 \neq t_2$. Replacing above $2m^2 \to t_1 + t_2$ we expect for small $|t_1|$ and $|t_2|$ still approximate equivalence to hold. This is confirmed by explicit numerical studies.
Comparison with experimental results from WA102@CERN

Data: D. Barberis et al. (WA102 Collaboration), PLB 440 (1998) 225

\[ f_1(1285) \left| \sqrt{s} = 29.1 \text{ GeV}, \ |x_{F,M}| \leq 0.2 \right. \]

\[ \sigma_{\text{exp}} = (6919 \pm 886) \text{ nb} \]

\[ |g'_{IPP} f_1| = 4.89 \]

\[ |g''_{IPP} f_1| = 10.31 \]

- We get a reasonable description of WA102 data with \( \Lambda_E = 0.7 \text{ GeV} \)
- Absorption effects included

\[ <S^2> = \frac{\sigma_{\text{abs}}}{\sigma_{\text{Born}}} \approx 0.5-0.7 \]

depending on the kinematics
Comparison with experimental results from WA102@CERN


The theoretical results have been normalized to the mean value of the number of events

- An almost ‘flat’ distribution at large values of $|t_1 - t_2|$ can be observed
  → absorption effects play a significant role there,
  large damping of cross section at higher values of $\phi_{pp}$
- It seems that the $(\ell,S) = (4,4)$ term best reproduces the shape of the WA102 data
Holographic QCD approach

Fit to WA102 data using the Chern-Simons (CS) coupling.

The relation between the \((\ell,S)\) and CS forms of the couplings:

With \(\varepsilon' = -8.88\) and \(\varepsilon''/\varepsilon' = -1.0\ \text{GeV}^{-2}\)

and setting \(t_1 = t_2 = -0.1\ \text{GeV}^2\)

we get: \(g'_{PP_{f_1}} = 0.42, \quad g''_{PP_{f_1}} = 10.81\)

This CS coupling corresponds practically to a pure \((\ell,S) = (4,4)\) coupling!

The prediction for \(\varepsilon''/\varepsilon'\) obtained in the Sakai-Sugimoto model:

\[\varepsilon''/\varepsilon' = -5.631/M_{KK}^2 = -(6.25, 3.76, 2.44)\ \text{GeV}^{-2}\]

for \(M_{KK} = (949, 1224, 1519)\ \text{MeV}\)

Usually \(M_{KK}\) (Kaluza-Klein mass scale) is fixed by matching the mass of the lowest vector meson to that of the physical \(\rho\) meson, leading to \(M_{KK} = 949\ \text{MeV}\).

However, this choice leads to tensor glueball mass which is too low, \(M_T \approx 1.5\ \text{GeV}\).

The standard pomeron trajectory corresponds to \(M_T \approx 1.9\ \text{GeV}\), whereas lattice gauge theory indicates \(M_T \approx 2.4\ \text{GeV}\).
Predictions for the LHC experiments

- The contribution with $\chi''/\chi' = -6.25 \text{ GeV}^{-2}$ gives a significantly different shape.

- The absorption effects are included, $\langle S^2 \rangle \approx 0.35$. They decrease the distribution mostly at higher values of $\phi_{pp}$ and at smaller values of $p_{t,M}$ (and also $|t|$). This could be tested in ATLAS-ALFA experiment when both protons are measured.

Cross sections in $\mu$b for $pp \to ppf_1(1285)$ for $\sqrt{s} = 13 \text{ TeV}$:

| Contribution | Parameters | $|y_{f_1}| < 1.0$ | $|y_{f_1}| < 2.5$ | $|y_{f_1}| < 2.5$, $0.17 < |p_{y,p}| < 0.50 \text{ GeV}$ | $2.0 < y_{f_1} < 4.5$ |
|--------------|------------|------------------|------------------|-----------------------------------------------|------------------|
| $(l, S) = (2, 2)$ | $\Lambda_E = 0.7 \text{ GeV}$; $g'_{ppf_1} = 4.89$ | 14.8 | 37.5 | 6.46 | 18.9 |
| $(l, S) = (4, 4)$ | $g'_{ppf_1} = 10.31$ | 13.8 | 34.0 | 6.06 | 18.1 |
| $(\chi', \chi'')$ | $\chi''/\chi' = -6.25 \text{ GeV}^{-2}$ | 18.6 | 45.8 | 7.14 | 23.1 |
| $(\chi', \chi'')$ | $\chi''/\chi' = -2.44 \text{ GeV}^{-2}$ | 17.5 | 43.4 | 7.10 | 22.1 |
| $(\chi', \chi'')$ | $\chi''/\chi' = -1.0 \text{ GeV}^{-2}$ | 16.6 | 41.0 | 7.09 | 20.5 |
Predictions for the LHC experiments

- One of the most prominent decay modes of the $f_1(1285)$ is $f_1(1285) \rightarrow \pi^+\pi^-\pi^+\pi^-$

- There $f_1(1285)$ and $f_2(1270)$ are close in mass. We obtain for $\sqrt{s} = 13$ TeV and $|y_M| < 2.5$:
  \[
  \sigma_{pp\rightarrow ppf_1(1285)} \times BR(f_1(1285) \rightarrow 2\pi^+2\pi^-) = 34.0 \mu b \times 0.109 = 3.7 \mu b
  \]
  \[
  \sigma_{pp\rightarrow ppf_2(1270)} \times BR(f_2(1270) \rightarrow 2\pi^+2\pi^-) = 11.3 \mu b \times 0.028 = 0.3 \mu b
  \]

- As the $f_1(1285)$ has a much narrower width than the $f_2(1270)$ it would be seen in the $M(4\pi)$ distribution as a peak on top of broader $f_2(1270)$ and of the $4\pi$ continuum background

- $f_1(1285)$ is seen in the preliminary ATLAS-ALFA results for $pp \rightarrow pp\pi^+\pi^-\pi^+\pi^-$ at $\sqrt{s} = 13$ TeV and for $|\eta_{\pi}| < 2.5, p_{t,\pi} > 0.1$ GeV, $\max(p_{t,\pi}) > 0.2$ GeV, $0.17$ GeV $< |p_{y,p}| < 0.5$ GeV
  [R. Sikora, CERN-THESIS-2020-235]

- Theoretical studies of the reaction $pp \rightarrow pp\,4\pi$ including both the resonances and continuum contributions within the tensor-pomeron approach → in progress:
  4π production via the intermediate $\sigma\sigma$ and $pp$ states: Lebiedowicz, Nachtmann, Szczurek, PRD 94 (2016) 034017
  4π continuum: Kycia, Lebiedowicz, Szczurek, Turnau, PRD 95 (2017) 094020
  $f_1(1285)$ production: Lebiedowicz, Leutgeb, Nachtmann, Rebhan, Szczurek, PRD 102 (2020) 114003

using GenEx MC generator for exclusive reactions and DECAY MC library for the decay of a particle with ROOT compatibility:
Conclusions

- We have given predictions for forthcoming experiments with HADES and PANDA at FAIR.
  
  We have performed feasibility studies and estimated that a 30-days measurement with HADES should allow to identify $f_1(1285)$ meson in the $\pi^+\pi^-\eta$ channel.
  From such experiments we will learn more on the production mechanism, in particular, about the $\rho\rho f_1$ and $\omega\omega f_1$ coupling strengths.

- We have discussed in detail CEP of $f_1(1285)$ meson in $pp$ collisions at high energies in the tensor-pomeron approach. Different forms of the $IP IP f_1$ coupling are possible. Tests of the Sakai-Sugimoto model are possible.

- We obtain a good description of the WA102 data for the $pp \rightarrow pp f_1(1285)$ reaction assuming that the reaction is dominated by IP exchange. We have given predictions for experiments at the LHC. We have included - very important - absorptive corrections.

  Experimental studies of single meson CEP reactions will give many $IP IP M$ coupling parameters. Their theoretical calculation is a challenging problem of nonperturbative QCD.

- We are looking forward to first experimental results on production of $f_1(1285)$ at HADES and at the LHC.

Thank you for your attention
Results (HADES and PANDA)

Other decay channels?

PDG: \( BR(f_1(1285) \rightarrow \rho^0\gamma) = (6.1 \pm 1.0)\% \)

CLAS: \( BR(f_1(1285) \rightarrow \rho^0\gamma) = (2.5^{+0.7}_{-0.6})\% \)

- For the 4\( \pi \) channel it may be difficult to identify the \( f_1(1285) \) due to large continuum background e.g. \( pp \rightarrow N(1440)N(1440) \rightarrow N\pi N\pi \)
  - we have found that fusion mechanisms for the \( \rho^0\rho^0 \) production: \( \pi^0-\omega-\pi^0 \) and \( \omega-\pi^0-\omega \) exchanges (treated with exact 2 \( \rightarrow \) 4 kinematics) give much smaller background cross sections

- The \( \rho^0\gamma \) channel should be much better suited. There, however, dominant background channel \( pp\pi^+\pi^-\pi^0 \) is of the order of 2 mb [1] and \( \rho^0 \) is so broad that it will not provide sufficient reductions (as it is the case in \( \eta \) decay channel)
  

\[ + (p_a \leftrightarrow p_b, p_1 \leftrightarrow p_2) \]

The \( \pi\pi\)-continuum contribution is larger than the VV-continuum term. In both cases the \( f_1 \) resonance is clearly visible, even without the reggeization (green lines) in the continuum processes. We get:

- for \( \sqrt{s} = 3.46 \) GeV: \( \sigma_{pp \rightarrow pp(f_1 \rightarrow \rho^0\gamma)} = 5.38 \) nb
- for \( \sqrt{s} = 5.0 \) GeV: \( \sigma_{pp \rightarrow pp(f_1 \rightarrow \rho^0\gamma)} = 62.86 \) nb

\( \sim 10 \times \) larger

This result makes us rather optimistic that an experimental study of the \( f_1 \) in the \( \rho\gamma \) decay channel should be possible at PANDA@FAIR.

For our exploratory study we have neglected interference effects between the background \( \rho\gamma \) and the signal \( f_1 \rightarrow \rho\gamma \) processes.

We have also neglected the background processes due to bremsstrahlung of \( \gamma \) and \( \rho^0 \) from the nucleon lines.