



武汉大学
WUHAN UNIVERSITY



Amplitude analyses of multibody hadronic D decays at BESIII

Panting Ge

Wuhan University

on behalf of the BESIII collaboration

19th International Conference on Hadron Spectroscopy and Structure

26th - 31st of July, 2021, Universidad Nacional Autónoma de México

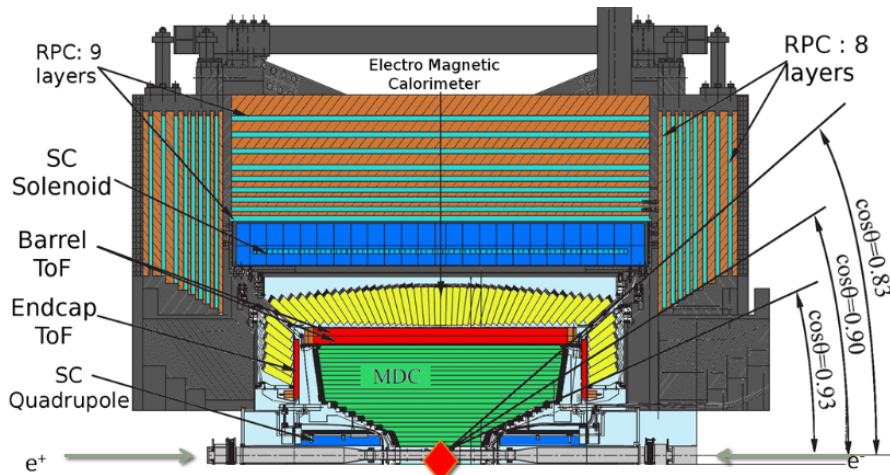
Outline

- Introduction
- Strategy
- Amplitude analyses and BF measurements
- Summary

Introduction

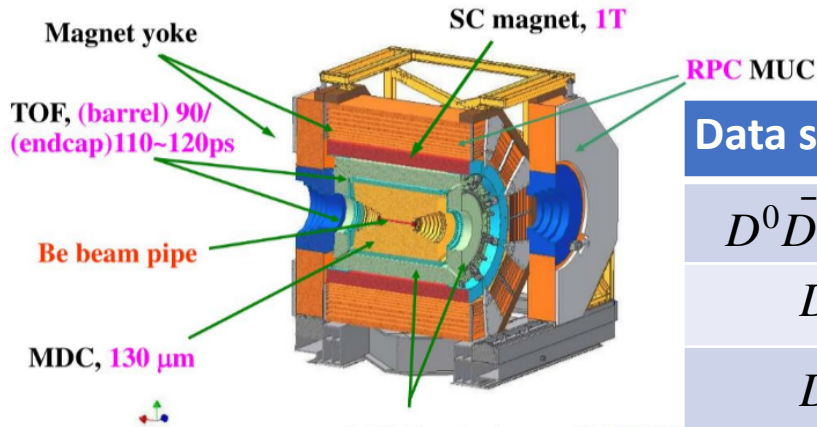
- Amplitude (Dalitz plot) analyses provide a method to study the hadronic decays of D_s^+ / D^+ .
 - Hadron spectroscopy & Structure of the resonance
 - Determine the phase of the intermediate mode
- Understand the dynamics of 2-body decays of D_s^+ / D^+ .
 - $D_s^+ \rightarrow SP, D_s^+ \rightarrow AP, D_s^+ / D^+ \rightarrow VP...$ (test $SU(3)_F$ symmetry)
- Reduce the systematic uncertainties related to the substructures in branching fraction measurement
- Analyses of
 - $D_s^+ \rightarrow K^- K^+ \pi^+$ [arXiv:2011.08041]
 - $D_s^+ \rightarrow K_s^0 \pi^+ \pi^0$ [JHEP06(2021)181]
 - $D_s^+ \rightarrow K_s^0 K^- \pi^+ \pi^+$ [Phys. Rev. D **103**, 092006 (2021)]
 - $D_s^+ \rightarrow K^- K^+ \pi^+ \pi^0$ [arXiv:2103.02482]
 - $D_s^+ \rightarrow \eta \pi^+ \pi^+ \pi^-$ [arXiv:2106.13536]
 - $D^+ \rightarrow K_s^0 K^+ \pi^0$ [Phys. Rev. D **104**, 012006 (2021)]

Data sets



- pair production at threshold
- Fully reconstructed event
- Almost free of background

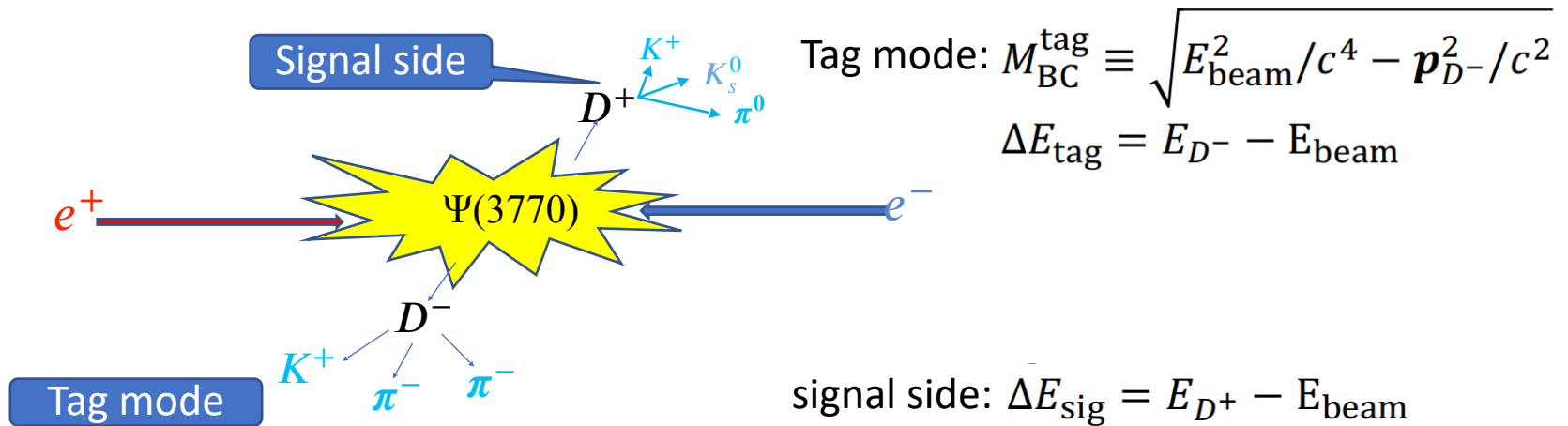
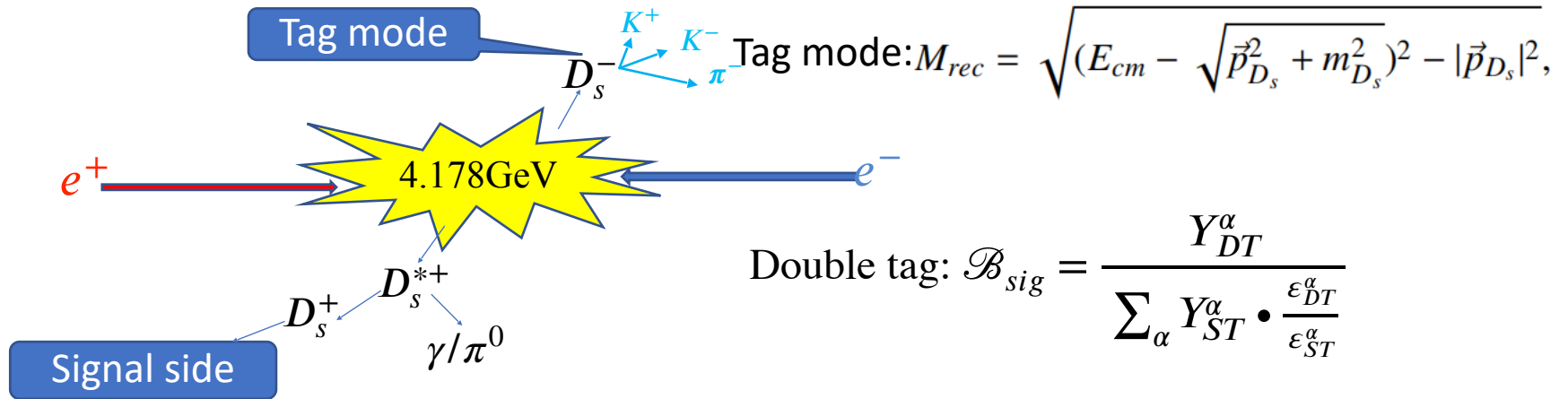
BESIII detector



CsI(Tl) calorimeter, 2.5% @ 1GeV

Data samples	(GeV)	Lum. ()	xCLEO-c
$D^0\bar{D}^0/D^-D^+$	3.773	2.93	3.6x
$D_s D_s^*$	4.178	3.19	5.3x
$D_s D_s^*$	4.189-4.226	3.13	-

Double tag technique



DT method

6

- $M(D_s)$
- **Background analysis**
 - MC simulation
 - sideband
 - multivariate analysis
- **Description of signal spectrum ($M(D_s)$)**
 - MC shape \otimes Gaussian
 - MC shape or polynomial for background

Tag mode	M_{tag} (GeV/ c^2)
$D_s^- \rightarrow K_S^0 K^-$	[1.948, 1.991]
$D_s^- \rightarrow K^+ K^- \pi^-$	[1.950, 1.986]
$D_s^- \rightarrow K_S^0 K^- \pi^0$	[1.946, 1.987]
$D_s^- \rightarrow K^+ K^- \pi^- \pi^0$	[1.947, 1.982]
$D_s^- \rightarrow K_S^0 K^- \pi^- \pi^+$	[1.958, 1.980]
$D_s^- \rightarrow K_S^0 K^+ \pi^- \pi^-$	[1.953, 1.983]
$D_s^- \rightarrow \pi^- \eta_{\gamma\gamma}$	[1.930, 2.000]
$D_s^- \rightarrow \pi^- \eta'_{\pi^- \pi^+ \eta_{\gamma\gamma}}$	[1.940, 1.996]

- ΔE_{tag}
- M_{BC}^{tag} : within (1.863, 1.879) GeV/ c^2

Tag mode	ΔE_{tag} (GeV)
$D^- \rightarrow K^+ \pi^- \pi^-$	[-0.022, 0.021]
$D^- \rightarrow K^+ \pi^- \pi^- \pi^0$	[-0.060, 0.034]
$D^- \rightarrow K_s^0 \pi^-$	[-0.019, 0.021]
$D^- \rightarrow K_s^0 \pi^- \pi^0$	[-0.071, 0.041]
$D^- \rightarrow K_s^0 \pi^+ \pi^- \pi^-$	[-0.025, 0.023]
$D^- \rightarrow K^+ K^- \pi^-$	[-0.019, 0.018]

Formalism

- **Amplitude analysis:**

- **Unbinned maximum likelihood Fit:**

- PDF = $f_s S + (1 - f_s) B = \epsilon(p_j) R_4(p_j) \left[f_s \frac{|A_{D_s}(a_i, p_j)|^2}{\int \epsilon(p_j) |A_{D_s}(a_i, p_j)|^2 R_4(p_j) dp_j} + (1 - f_s) \frac{B_\epsilon(p_j)}{\int B(p_j) R_4(p_j) dp_j} \right]$

Acceptance function

MC integration

Bkg function
(RooNDKeysPdf)

$$B_\epsilon = B/\epsilon$$

For $D^+ \rightarrow K_s^0 K^+ \pi^0$, $f_s = 1$

- $A_{D_s \rightarrow \text{sig}} = \sum_n c_n A_n$, $c_n = \rho_n e^{i\phi_n}$

- **Covariant tensor formalism**

- $A_n = P_n^1 P_n^2 S_n F_n^1 F_n^2 F_n^{D_s}$, P_n^i propagator, F_n^i barrier, S_n angular distribution.

- **Log-likelihood:**

- $\ln \mathcal{L}_{\text{total}} = \sum_k^{N_{\text{data}}} \ln(f_s S(p_k) + (1 - f_s) B(p_k))$. RooNDKeysPdf

- For $D_s^+ \rightarrow \pi^+ \pi^0 \eta$:

- $\ln \mathcal{L}_{\text{total}} = \sum_k^{N_{\text{data}}} \ln(S(p_k)) - \sum_k^{N_{\text{data}}} w_{\text{bkg}}^k \ln(S(p_k))$ adding the negative weight

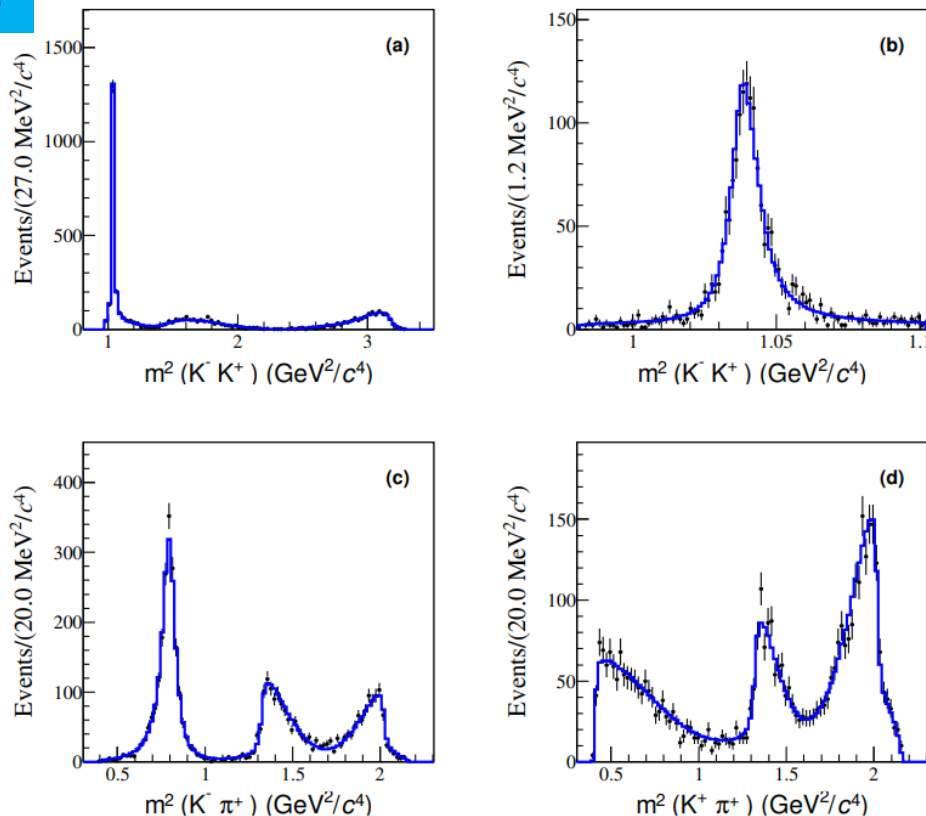
- **BF Measurement**

- Update the MC samples with the results of amplitude analysis.
- Looser selection criteria, more statistics.

Amplitude analysis of $D_s^+ \rightarrow K^- K^+ \pi^+$

arXiv:2011.08041

8



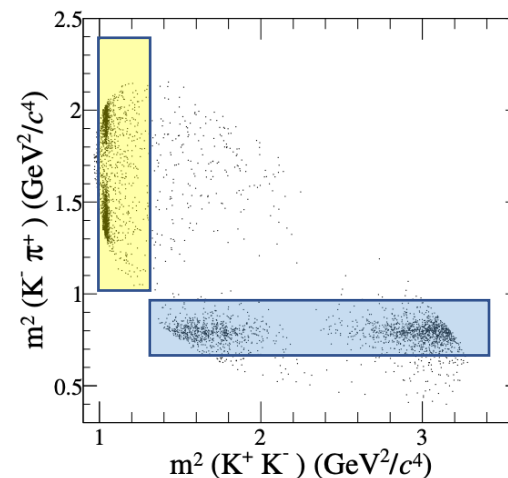
4397 DT events with a purity of 99.6% @ $\sqrt{s} = 4.178$ GeV

- $D_s^+ \rightarrow K^- K^+ \pi^+$ is a golden channel which is often used as the normalization mode.
- BF of $D_s^+ \rightarrow f_0(980)\pi^+$
PRD 79, 072008 (CLEO-c)
PRD 83, 052001 (BABAR)
- Do not distinguish between $f_0(980)/a_0(980)$
- Background free

Fit model:

$$\chi^2/\text{ndf} = 290/280$$

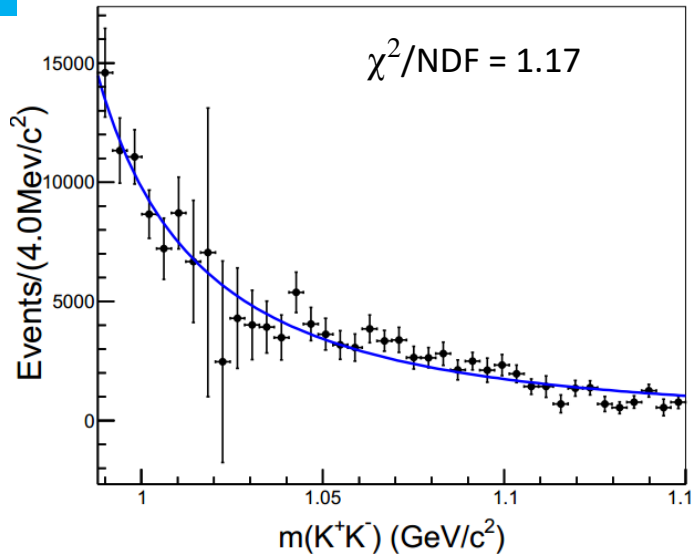
Amplitude	BABAR	CLEO	BESIII (this analysis)
$D_s^+ \rightarrow \bar{K}^*(892)^0 K^+$	$47.9 \pm 0.5 \pm 0.5$	$47.4 \pm 1.5 \pm 0.4$	$48.3 \pm 0.9 \pm 0.6$
$D_s^+ \rightarrow \phi(1020)\pi^+$	$41.4 \pm 0.8 \pm 0.5$	$42.2 \pm 1.6 \pm 0.3$	$40.5 \pm 0.7 \pm 0.9$
$D_s^+ \rightarrow S(980)\pi^+$	$16.4 \pm 0.7 \pm 2.0$	$28.2 \pm 1.9 \pm 1.8$	$19.3 \pm 1.7 \pm 2.0$
$D_s^+ \rightarrow K_0^*(1430)^0 K^+$	$2.4 \pm 0.3 \pm 1.0$	$3.9 \pm 0.5 \pm 0.5$	$3.0 \pm 0.6 \pm 0.5$
$D_s^+ \rightarrow f_0(1710)\pi^+$	$1.1 \pm 0.1 \pm 0.1$	$3.4 \pm 0.5 \pm 0.3$	$1.9 \pm 0.4 \pm 0.6$
$D_s^+ \rightarrow f_0(1370)\pi^+$	$1.1 \pm 0.1 \pm 0.2$	$4.3 \pm 0.6 \pm 0.5$	$1.2 \pm 0.4 \pm 0.2$
$\sum FF(\%)$	$110.2 \pm 0.6 \pm 2.0$	$129.5 \pm 4.4 \pm 2.0$	$114.2 \pm 1.7 \pm 2.3$
χ^2/NDF	$2843/2291=1.2$	$170/117=1.5$	$290/280=1.04$
Events	96307 ± 369 (purity 95%)	14400 (purity 85%)	4397 (purity 99.6%)



Amplitude analysis of $D_s^+ \rightarrow K^- K^+ \pi^+$

arXiv:2011.08041

9



The parameterization of $S(980)$:

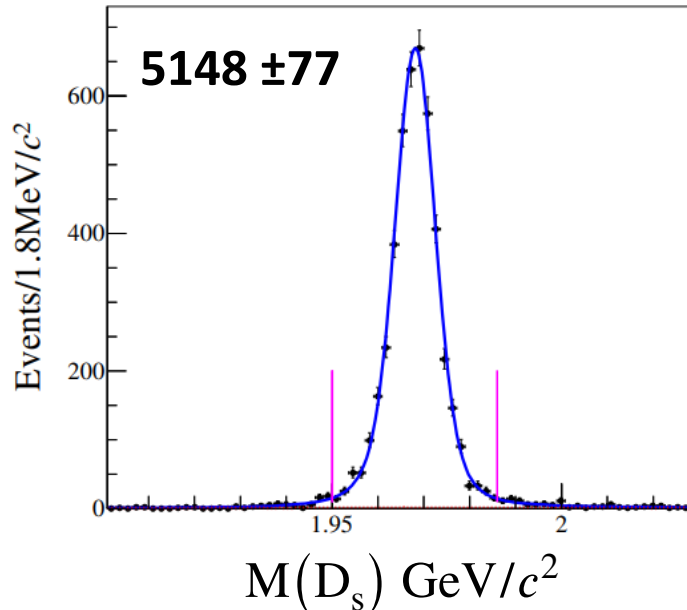
$$A_{S(980)} = \frac{1}{m_0^2 - m^2(K^+K^-) - im_0\Gamma_0\rho_{K^+K^-}}$$

$$\rho_{K^+K^-} = \frac{1}{\sqrt{(1 - 4m(K)^2)/m(K^+K^-)^2}}$$

$$m_0 = (0.919 \pm 0.006_{stat} \pm 0.030_{sys}) GeV/c^2$$

$$\Gamma_0 = (0.272 \pm 0.040 \pm 0.024) GeV$$

$K^+ K^-$ S-wave ($S(980)$) is extracted from model-independent partial wave analysis



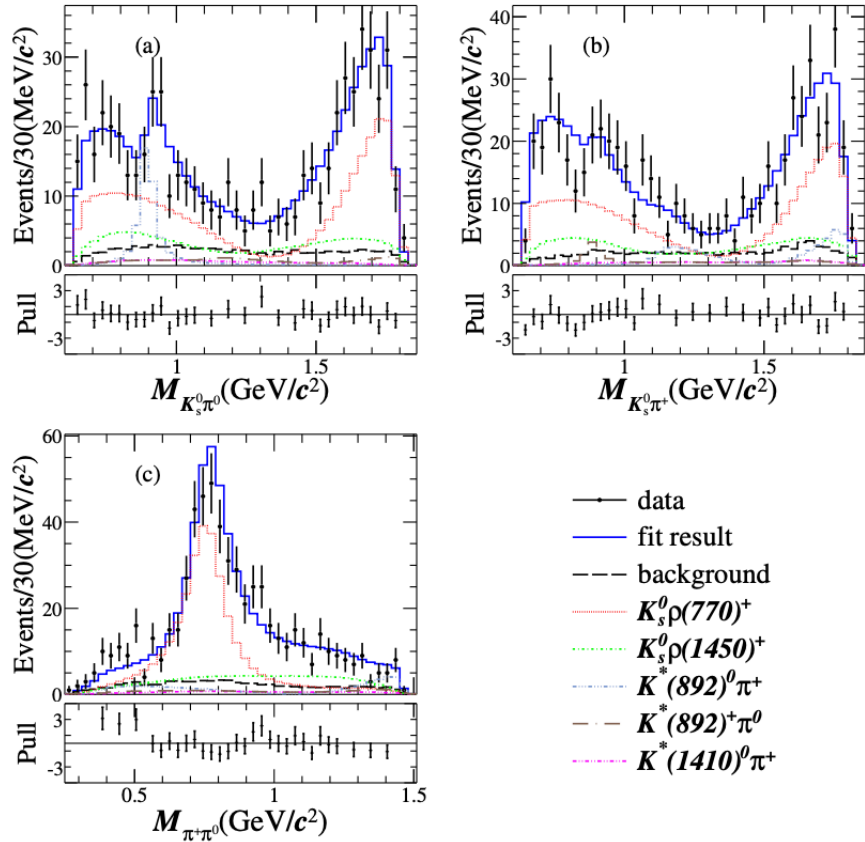
Branching Fraction:

- $\mathcal{B}(D_s^+ \rightarrow K^- K^+ \pi^+) = (5.47 \pm 0.08 \pm 0.13) \%$
- $\mathcal{B}(D_s^+ \rightarrow \phi(1020)\pi^+) = (4.60 \pm 0.17) \%$
- $\mathcal{B}(D_s^+ \rightarrow k^*(892)^0 K^+) = (3.94 \pm 0.12) \%$

Consistent with theoretical predictions

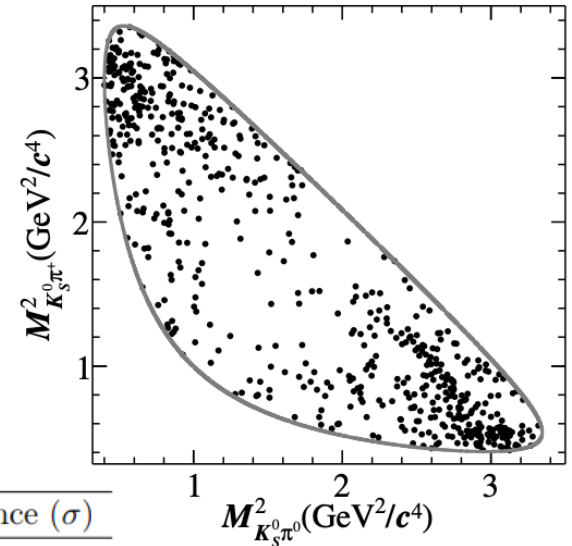
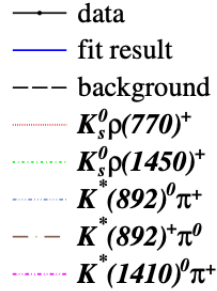
[Phys. Rev. D **93**, 114010 \(2016\)](#)

Best precision



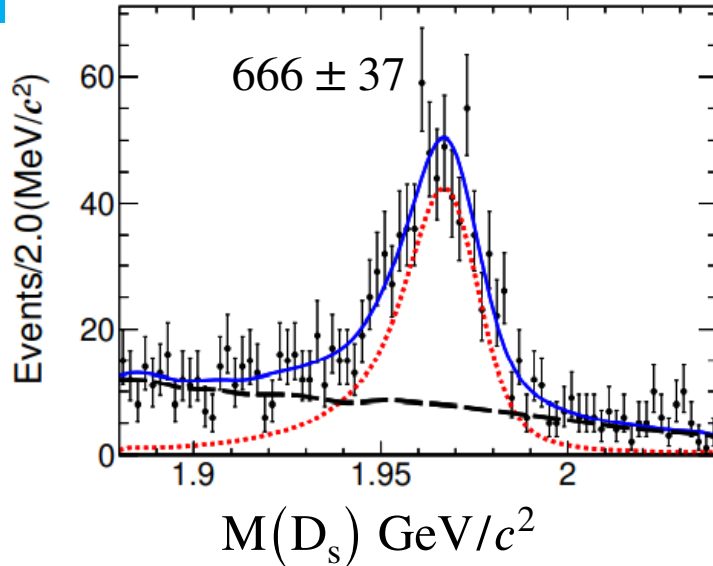
352 DT events with purity of 88.9% @ $\sqrt{s} = 4.178$ GeV

$D_s^+ \rightarrow K^{*+} \pi^0, K^{*0} \pi^+$ are good modes to search for CPV and study the $SU(3)_F$ symmetry and its breaking effect.



Fit model:

Amplitude	Magnitude (ρ_n)	Phase (ϕ_n)	FF (%)	Significance (σ)
$D_s^+ \rightarrow K_S^0 \rho^+$	1.0(fixed)	0.0(fixed)	$50.2 \pm 7.2 \pm 3.9$	>10
$D_s^+ \rightarrow K_S^0 \rho(1450)^+$	2.7 ± 0.5	$2.2 \pm 0.2 \pm 0.1$	$20.4 \pm 4.3 \pm 4.4$	>10
$D_s^+ \rightarrow K^*(892)^0 \pi^+$	0.4 ± 0.1	$3.2 \pm 0.2 \pm 0.1$	$8.4 \pm 2.2 \pm 0.9$	5.0
$D_s^+ \rightarrow K^*(892)^+ \pi^0$	0.3 ± 0.1	$0.2 \pm 0.2 \pm 0.2$	$4.6 \pm 1.4 \pm 0.4$	4.0
$D_s^+ \rightarrow K^*(1410)^0 \pi^+$	0.8 ± 0.2	$0.2 \pm 0.3 \pm 0.1$	$3.3 \pm 1.6 \pm 0.5$	3.7



Showing the measured BFs of $D_s \rightarrow VP$ and theoretical predictions from various models ($\times 10^{-3}$)

Channel	PDG [1]	Y.L. Wu et al. [7]	H.Y. Cheng et al. [8]	F.S. Yu et al. [4]
$K^0 \rho^+$	—	9.1 ± 7.7	11.47 ± 0.48	7.5 ± 2.1
$K^*(892)^0 \pi^+$	2.13 ± 0.36	3.3 ± 3.5	3.65 ± 0.24	1.5 ± 0.7
$K^*(892)^+ \pi^0$	—	1.3 ± 1.3	1.02 ± 0.07	0.1 ± 0.1

[4] PRD 84 (2011) 074019

[7] EPJC 42, (2005) 391

[8] PRD 100, (2019) 093002

- Signal shape: MC simulated shape \otimes Gaussian
- Background shape: MC simulated shape
- $\mathcal{B}(D_s^+ \rightarrow K_s^0 \pi^+ \pi^0) = (5.43 \pm 0.30 \pm 0.15) \times 10^{-3}$
- $\mathcal{B}(D_s^+ \rightarrow K^0 \rho^+) = (5.46 \pm 0.84 \pm 0.44) \times 10^{-3}$
- $\mathcal{B}(D_s^+ \rightarrow K^{*0} \pi^+) = (2.71 \pm 0.72 \pm 0.30) \times 10^{-3}$
- $\mathcal{B}(D_s^+ \rightarrow K^{*+} \pi^0) = (0.75 \pm 0.24 \pm 0.06) \times 10^{-3}$

Test the CP conservation:

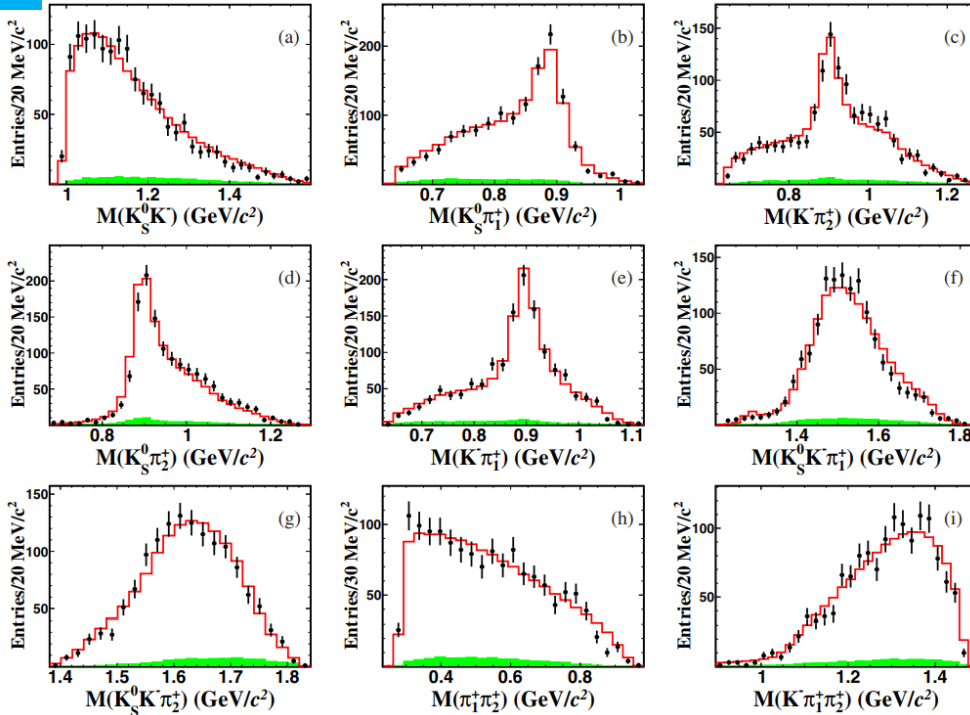
$$A_{CP} = \frac{\mathcal{B}(D_s^+) - \mathcal{B}(D_s^-)}{\mathcal{B}(D_s^+) + \mathcal{B}(D_s^-)} = (2.7 \pm 5.5 \pm 0.9)\%$$

Statistical error dominant.

No CPV is observed.

Most precise measurements

Amplitude analysis of $D_s^+ \rightarrow K_s^0 K^- \pi^+ \pi^+$



Argus Collaboration (1992):

$$B(D_s^+ \rightarrow K^{*+} \bar{K}^{*0}) = (7.2 \pm 2.6) \%$$

Z.Phys.C 53 (1992) 361

1308 DT events with a purity of 94.9% @ $\sqrt{s} = 4.178 \sim 4.226$ GeV

- $\mathcal{B}(D_s^+ \rightarrow K_s^0 K^- \pi^+ \pi^+) = (1.46 \pm 0.05 \pm 0.05) \%$
- $\mathcal{B}(D_s \rightarrow K^{*+} \bar{K}^{*0}) = (5.34 \pm 0.39 \pm 0.64) \%$

Much more precise

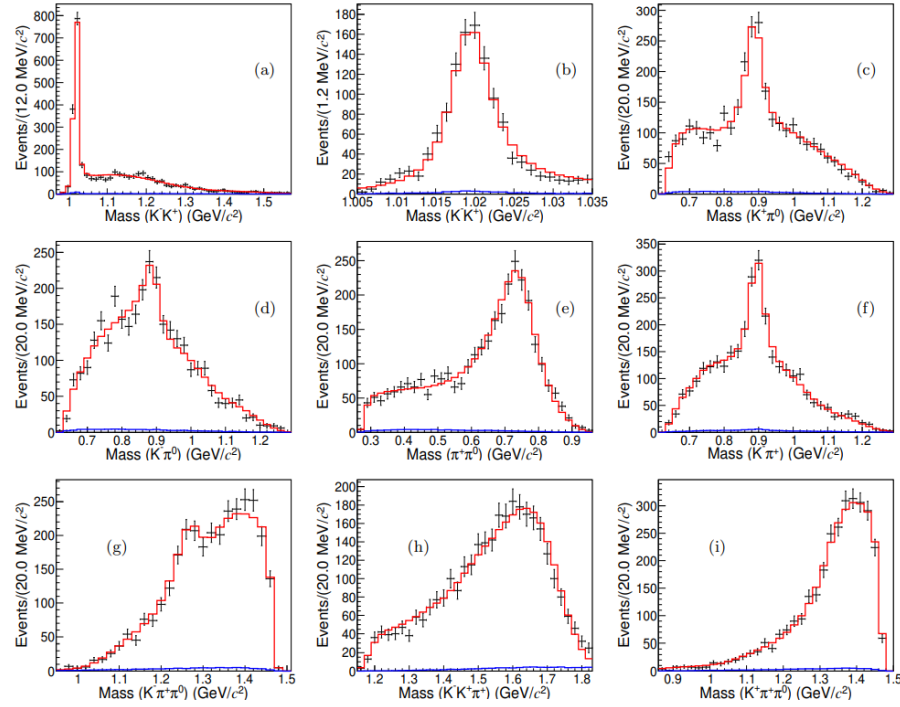
FIT model:

Label	Component	ϕ	FF(%)	Significance (σ)
I	$D_s^+[S] \rightarrow K^*(892)^+ \bar{K}^*(892)^0$	0 (fixed)	$34.3 \pm 3.1 \pm 5.2$	>10.0
II	$D_s^+[P] \rightarrow K^*(892)^+ \bar{K}^*(892)^0 - 1.61$	$0.08 \pm 0.03 \pm 7.5$	$1.1 \pm 0.1 \pm 8.3$	
III	$D_s^+[D] \rightarrow K^*(892)^+ \bar{K}^*(892)^0$	$-0.16 \pm 0.14 \pm 0.04$	$4.5 \pm 0.8 \pm 0.3$	8.2
IV	$D_s^+ \rightarrow K^*(892)^+ \bar{K}^*(892)^0$		$40.6 \pm 2.9 \pm 4.9$	
V	$D_s^+ \rightarrow K^*(892)^+ (K^- \pi^+)_{S\text{-wave}}$	$1.85 \pm 0.15 \pm 0.09$	$5.0 \pm 1.2 \pm 1.0$	6.2
VI	$D_s^+ \rightarrow \bar{K}^*(892)^0 (K_S^0 \pi^+)_{S\text{-wave}}$	$-1.57 \pm 0.12 \pm 0.13$	$7.3 \pm 1.1 \pm 0.9$	9.1
VII	$D_s^+ \rightarrow \eta(1475) \pi^+, \eta(1475) \rightarrow a_0(980)^- \pi^+$	$-1.95 \pm 0.15 \pm 0.07$	$10.8 \pm 2.6 \pm 5.2$	4.4
VIII	$D_s^+ \rightarrow \eta(1475) \pi^+, \eta(1475) \rightarrow \bar{K}^*(892)^0 K_S^0$	$0.05 \pm 0.15 \pm 0.11$	$2.2 \pm 0.6 \pm 0.2$	4.5
IX	$D_s^+ \rightarrow \eta(1475) \pi^+, \eta(1475) \rightarrow K^*(892)^+ K^-$	$0.05 \pm 0.15 \pm 0.11$	$2.2 \pm 0.6 \pm 0.2$	4.5
IXX	$D_s^+ \rightarrow \eta(1475) \pi^+, \eta(1475) \rightarrow K^*(892) K$		$4.9 \pm 1.4 \pm 1.0$	
IIIX	$D_s^+ \rightarrow \eta(1475) \pi^+, \eta(1475) \rightarrow (K_S^0 \pi^+)_{S\text{-wave}} K^-$	$2.30 \pm 0.11 \pm 0.07$	$23.6 \pm 3.6 \pm 7.5$	6.7
X	$D_s^+ \rightarrow f_1(1285) \pi^+, f_1(1285) \rightarrow a_0(980)^- \pi^+$	$-0.89 \pm 0.26 \pm 0.14$	$2.2 \pm 0.5 \pm 0.2$	6.0
XI	$D_s^+ \rightarrow (K^*(892)^+ K^-)_P \pi^+, (K^*(892)^+ K^-)_P \rightarrow K^*(892)^+ K^-$	$-1.07 \pm 0.11 \pm 0.03$	$10.8 \pm 1.9 \pm 1.7$	9.2

Dominant

Amplitude analysis of $D_s^+ \rightarrow K^- K^+ \pi^+ \pi^0$

13

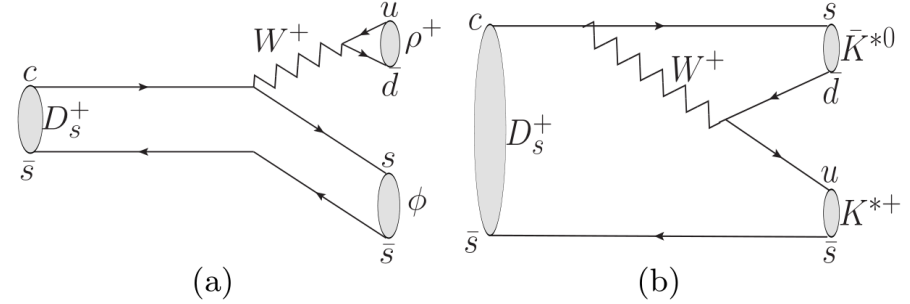


$$\chi^2/\text{ndf} = 288.6/273$$

3088 DT events with purity of 97.5% @
 $\sqrt{s} = 4.178 \sim 4.226$ GeV

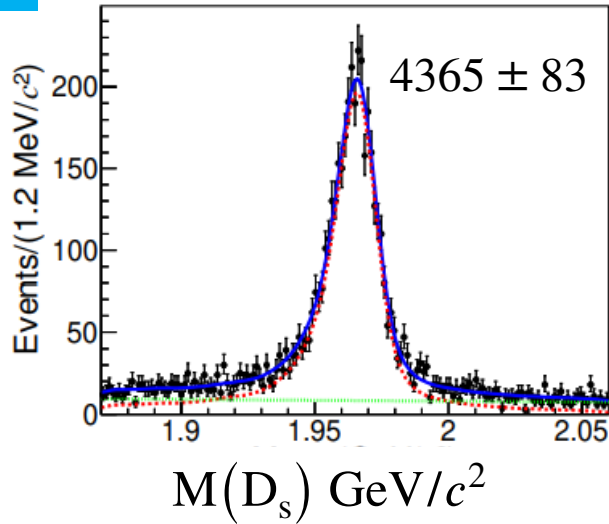
The amplitudes of VIII, IX, X and XII are fixed by Clebsch Gordan coefficients and charge conjugation relations

Dominated by $D_s^+ \rightarrow VV$ processes:



Label	Amplitude	Phase (ϕ_n)	FF (%)	SS (σ)
I	$D_s^+[S] \rightarrow \phi\rho^+$	0.0 (fixed)	$42.64 \pm 1.30 \pm 0.77$	>20
II	$D_s^+[P] \rightarrow \phi\rho^+$	$1.64 \pm 0.05 \pm 0.02$	$8.58 \pm 0.69 \pm 0.37$	15.2
III	$D_s^+[D] \rightarrow \phi\rho^+$	$1.58 \pm 0.06 \pm 0.02$	$4.89 \pm 0.79 \pm 0.47$	8.4
	$D_s^+ \rightarrow \phi\rho^+$...	$56.17 \pm 1.05 \pm 1.24$...
IV	$D_s^+[S] \rightarrow \bar{K}^{*0}K^{*+}$	$1.13 \pm 0.06 \pm 0.03$	$15.49 \pm 0.81 \pm 0.36$	>20
V	$D_s^+[P] \rightarrow \bar{K}^{*0}K^{*+}$	$2.82 \pm 0.07 \pm 0.03$	$6.13 \pm 0.50 \pm 0.19$	16.2
VI	$D_s^+[D] \rightarrow \bar{K}^{*0}K^{*+}$	$1.76 \pm 0.07 \pm 0.03$	$4.00 \pm 0.47 \pm 0.34$	12.5
	$D_s^+ \rightarrow \bar{K}^{*0}K^{*+}$...	$22.44 \pm 0.81 \pm 0.32$...
VII	$D_s^+ \rightarrow \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270) \rightarrow K^-\rho^+$	$5.36 \pm 0.06 \pm 0.10$	$9.81 \pm 0.80 \pm 0.46$	>20
	$D_s^+ \rightarrow \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[S] \rightarrow \bar{K}^{*0}\pi^0$...	$0.69 \pm 0.13 \pm 0.12$...
	$D_s^+ \rightarrow \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[S] \rightarrow K^{*-}\pi^+$...	$1.27 \pm 0.27 \pm 0.25$...
VIII	$D_s^+ \rightarrow \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[S] \rightarrow K^*\pi$	$0.09 \pm 0.14 \pm 0.12$	$1.87 \pm 0.39 \pm 0.36$	7.2
	$D_s^+ \rightarrow \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[D] \rightarrow \bar{K}^{*0}\pi^0$...	$0.22 \pm 0.05 \pm 0.03$...
	$D_s^+ \rightarrow \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[D] \rightarrow K^{*-}\pi^+$...	$0.41 \pm 0.10 \pm 0.05$...
IX	$D_s^+ \rightarrow \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[D] \rightarrow K^*\pi$	$1.62 \pm 0.15 \pm 0.12$	$0.64 \pm 0.16 \pm 0.08$	5.5
	$D_s^+ \rightarrow \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270) \rightarrow K^*\pi$...	$2.57 \pm 0.42 \pm 0.42$...
	$D_s^+ \rightarrow \bar{K}_1^0(1400)K^+, \bar{K}_1^0(1400)[S] \rightarrow \bar{K}^{*0}\pi^0$...	$2.67 \pm 0.36 \pm 0.17$...
	$D_s^+ \rightarrow \bar{K}_1^0(1400)K^+, \bar{K}_1^0(1400)[S] \rightarrow K^{*-}\pi^+$...	$4.90 \pm 0.65 \pm 0.29$...
X	$D_s^+ \rightarrow \bar{K}_1^0(1400)K^+, \bar{K}_1^0(1400)[S] \rightarrow K^*\pi$	$5.66 \pm 0.08 \pm 0.05$	$7.23 \pm 0.95 \pm 0.41$	12.0
XI	$D_s^+ \rightarrow a_0^0(980)\rho^+$	$2.33 \pm 0.10 \pm 0.09$	$1.61 \pm 0.29 \pm 0.21$	6.0
	$D_s^+ \rightarrow f_1(1420)\pi^+, f_1(1420) \rightarrow K^{*-}K^+$...	$0.87 \pm 0.17 \pm 0.07$...
	$D_s^+ \rightarrow f_1(1420)\pi^+, f_1(1420) \rightarrow K^{*+}K^-$...	$0.87 \pm 0.17 \pm 0.07$...
XII	$D_s^+ \rightarrow f_1(1420)\pi^+, f_1(1420) \rightarrow K^{*\mp}K^\pm$	$5.14 \pm 0.10 \pm 0.05$	$1.35 \pm 0.28 \pm 0.11$	6.5
XIII	$D_s^+ \rightarrow f_1(1420)\pi^+, f_1(1420) \rightarrow a_0^0(980)\pi^0$	$5.77 \pm 0.14 \pm 0.07$	$0.65 \pm 0.24 \pm 0.12$	3.6
XIV	$D_s^+ \rightarrow \eta(1475)\pi^+, \eta(1475) \rightarrow a_0^0(980)\pi^0$	$0.98 \pm 0.08 \pm 0.06$	$3.28 \pm 0.38 \pm 0.25$	9.7

BF measurement of $D_s^+ \rightarrow K^- K^+ \pi^+ \pi^0$



- $\mathcal{B}(D_s^+ \rightarrow K^- K^+ \pi^+ \pi^0) = (5.42 \pm 0.10 \pm 0.17) \%$

- $\mathcal{B}(D_s^+ \rightarrow \phi \rho^+) = (6.22 \pm 0.17 \pm 0.24) \%$

Consistent with theoretical prediction
PRD 49, 269(1994)

- $\mathcal{B}(D_s^+ \rightarrow K^{*+} K^{*-0}) = (5.46 \pm 0.23 \pm 0.18) \%$

$$R_{K_{1(1270)}} \equiv \frac{\mathcal{B}(K_1^0 \rightarrow K^* \pi)}{\mathcal{B}(K_1^0 \rightarrow K \rho)} = (0.99 \pm 0.15 \pm 0.18) \%$$

Our result is consistent with the results measured by LHCb [JHEP02(2019)126] and CLEO [PRD **85**, 122002]

$R_{K_{1(1270)}}$	Process	Experiment
0.81 ± 0.10	$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$	LHCb [19]
1.18 ± 0.43	$D^0 \rightarrow K^- K_1^+(1270)$	CLEO [20]
0.11 ± 0.06	$D^0 \rightarrow K^+ K_1^-(1270)$	CLEO [20]
0.19 ± 0.10	$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	BESIII [21]
0.24 ± 0.04	$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	LHCb [22]
0.45 ± 0.05	$B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$	Belle [23] (Fit 1)
0.30 ± 0.04	$B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$	Belle [23] (Fit 2)
0.38 ± 0.13	$K^- p \rightarrow K^- \pi^- \pi^+ p$	ACCMOR [24]
0.45 ± 0.14	$D^0 \rightarrow K^- K_1^+(1270)$	CLEO [25]

[19] PRD85, 122002 (2012)

[20] PRD95, 072010 (2017)

[21] EPJC78, 443 (2018)

[22] PRD83, 032005 (2011)

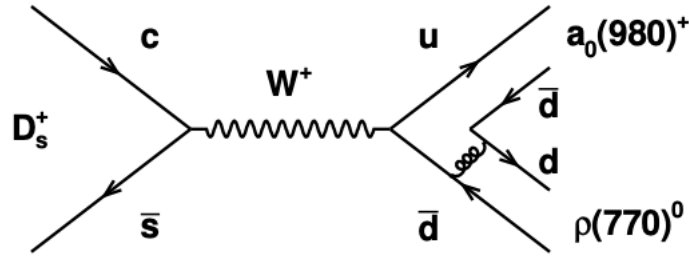
[23] NPB187, 1 (1981)

[24] JHEP05, 143 (2017)

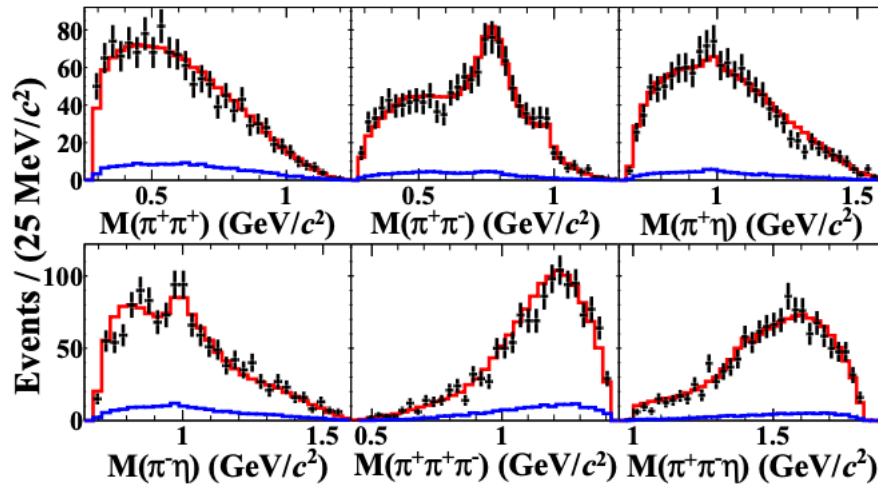
Amplitude analysis of $D_s^+ \rightarrow \eta\pi^+\pi^+\pi^-$

15

Observe W-annihilation for $D_s^+ \rightarrow a_0(980)^+\rho(770)^0$



1306 DT candidates with larger than 85% purity
@ $\sqrt{s} = 4.178\sim 4.226$ GeV



Amplitude	Phase	FF(%)
$a_1(1260)^+(\rho(770)^0\pi^+)\eta$	0.0(fixed)	$55.4 \pm 3.9 \pm 2.0$
$a_1(1260)^+(f_0(500)\pi^+)\eta$	$5.0 \pm 0.1 \pm 0.1$	$8.1 \pm 1.9 \pm 2.1$
$a_0(980)^+\rho(770)^0$	$2.5 \pm 0.1 \pm 0.1$	$6.7 \pm 2.5 \pm 1.5$
$\eta(1405)(a_0(980)^-\pi^+)\pi^+$	$0.2 \pm 0.2 \pm 0.1$	$0.7 \pm 0.2 \pm 0.1$
$\eta(1405)(a_0(980)^+\pi^-\pi^+)\pi^+$	$0.2 \pm 0.2 \pm 0.1$	$0.7 \pm 0.2 \pm 0.1$
$f_1(1420)(a_0(980)^-\pi^+)\pi^+$	$4.3 \pm 0.2 \pm 0.4$	$1.9 \pm 0.5 \pm 0.3$
$f_1(1420)(a_0(980)^+\pi^-\pi^+)\pi^+$	$4.3 \pm 0.2 \pm 0.4$	$1.7 \pm 0.5 \pm 0.3$
$[a_0(980)^-\pi^+]_S\pi^+$	$0.1 \pm 0.2 \pm 0.2$	$5.1 \pm 1.2 \pm 0.9$
$[a_0(980)^+\pi^+]_S\pi^+$	$0.1 \pm 0.2 \pm 0.2$	$3.4 \pm 0.8 \pm 0.6$
$[f_0(980)\eta]_S\pi^+$	$1.4 \pm 0.2 \pm 0.3$	$6.2 \pm 1.7 \pm 0.9$
$[f_0(500)\eta]_S\pi^+$	$2.5 \pm 0.2 \pm 0.3$	$12.7 \pm 2.6 \pm 2.0$

First measurement:

- $\mathcal{B}(D_s^+ \rightarrow \eta\pi^+\pi^+\pi^-) = (3.12 \pm 0.13 \pm 0.09) \%$

Dominant process:

- $\mathcal{B}(D_s^+ \rightarrow a_1(1260)^+\eta, a_1(1260)^+ \rightarrow \rho^0\pi^+) = (1.73 \pm 0.14 \pm 0.08) \%$

W-annihilation contribution:

- $\mathcal{B}(D_s^+ \rightarrow a_0(980)^+\rho^+, a_0(980)^+ \rightarrow \eta\pi^+) = (0.21 \pm 0.08 \pm 0.05) \%$

Larger than the fractions of most other measured pure W-annihilation decays

Amplitude analysis of $D^+ \rightarrow K_S^0 K^+ \pi^0$

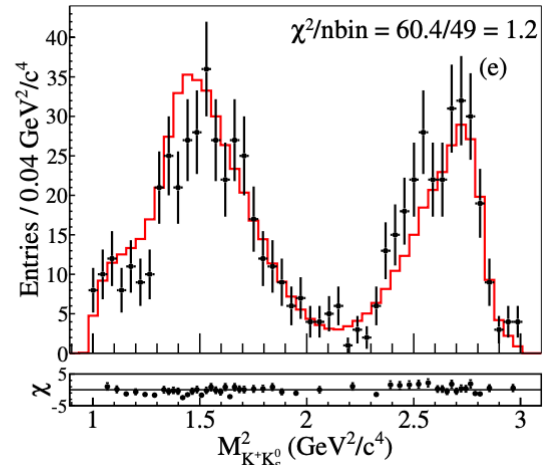
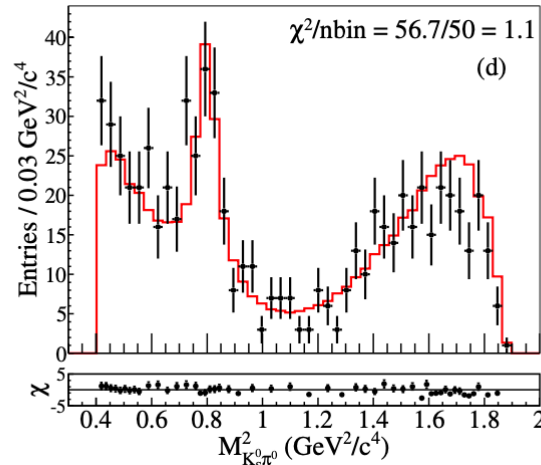
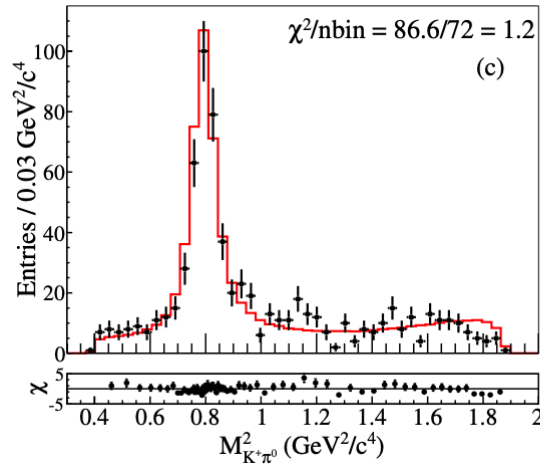
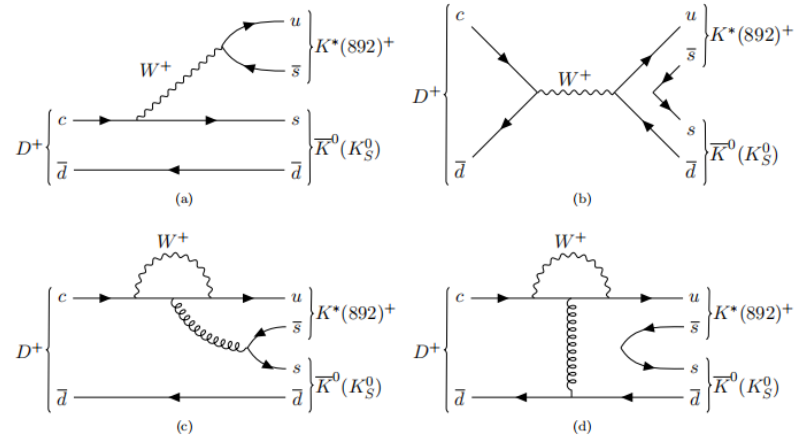
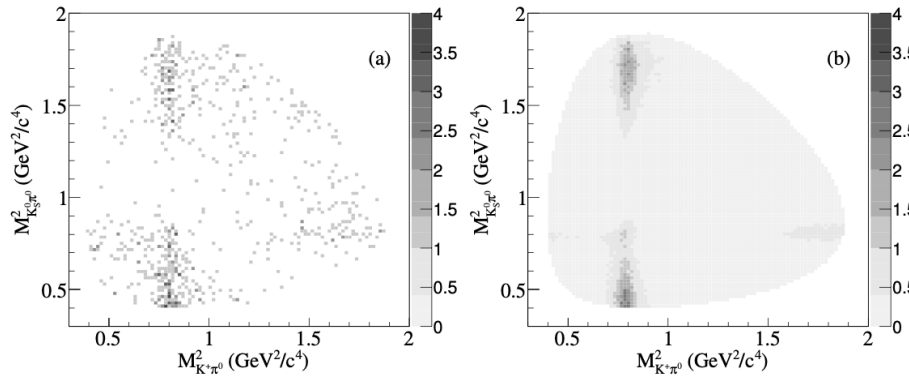
Phys. Rev. D **104**, 012006 (2021)

- First amplitude analysis
- 692 DT candidates with 97.4% @ $\sqrt{s} = 3.773$ GeV

Topological diagrams for $D^+ \rightarrow K^*(892)^+ K_S^0$

Data

Fit model



Amplitude analysis of $D^+ \rightarrow K_S^0 K^+ \pi^0$

Phys. Rev. D **104**, 012006 (2021)

17

A factor of 4.6 improvement for $\mathcal{B}_{K^*(892)^+ K_S^0}$

Amplitude	Magnitude	Phase ϕ ($^\circ$)	FF (%)	Significance
$D^+ \rightarrow K^*(892)^+ K_S^0$	1.0 (fixed)	0.0 (fixed)	57.1 ± 2.6	29.6σ
$D^+ \rightarrow \bar{K}^*(892)^0 K^+$	0.41 ± 0.04	162 ± 10	10.2 ± 1.5	11.6σ
$D^+ \rightarrow (K^+ \pi^0)_{\mathcal{S}\text{-wave}} K_S^0$	2.02 ± 0.37	140 ± 14	3.9 ± 1.5	5.2σ
$D^+ \rightarrow (K_S^0 \pi^0)_{\mathcal{S}\text{-wave}} K^+$	3.14 ± 0.46	-173.7 ± 9.7	9.7 ± 2.6	7.4σ

BF	This work	PDG
$\frac{\mathcal{B}(D^+ \rightarrow K^*(892)^+ (K^+ \pi^0) K_S^0)}{\mathcal{B}(D^+ \rightarrow K^+ K_S^0 \pi^0)}$	$(57.1 \pm 2.6_{\text{stat.}} \pm 4.2_{\text{syst.}})\%$	—
$\frac{\mathcal{B}(D^+ \rightarrow \bar{K}^*(892)^0 (K_S^0 \pi^0) K^+)}{\mathcal{B}(D^+ \rightarrow K^+ K_S^0 \pi^0)}$	$(10.2 \pm 1.5_{\text{stat.}} \pm 2.2_{\text{syst.}})\%$	—
$\mathcal{B}(D^+ \rightarrow K^*(892)^+ K_S^0)$	$(8.69 \pm 0.40_{\text{stat.}} \pm 0.64_{\text{syst.}} \pm 0.51_{\text{Br.}}) \times 10^{-3}$	$(17 \pm 8) \times 10^{-3}$
$\mathcal{B}(D^+ \rightarrow \bar{K}^*(892)^0 K^+)$	$(3.10 \pm 0.46_{\text{stat.}} \pm 0.68_{\text{syst.}} \pm 0.18_{\text{Br.}}) \times 10^{-3}$	$(3.74_{-0.20}^{+0.12}) \times 10^{-3}$

Before release of this analysis:
4.0 σ variation

After this release: accord well
Latest calculated: arXiv: 2014.13548

Model	$\mathcal{B}(D^+ \rightarrow K^*(892)^+ K_S^0) (\times 10^{-3})$	Mode	$\mathcal{B}_{\text{theory}}$	\mathcal{B}_{exp}
Pole	6.2 ± 1.2	$D^+ \rightarrow K^+ \bar{K}^{*0}$	$5.92_{-0.18}$	$3.71_{-0.16}$
FAT[mix]	5.5			
TDA[tree]	5.02 ± 1.31	$D^+ \rightarrow \bar{K}^0 K^{*+}$	$16.28_{-0.61}$	$17.6_{-1.8}$
TDA[QCD-penguin]	4.90 ± 0.21			
PDG	17 ± 8			

Our understanding of the charmed dynamics improved

Amplitude analyses of other $D^+/D^0/D_s^+$ Decays

Amplitude analysis of $D^+ \rightarrow K_s^0 \pi^+ \pi^+ \pi^-$ PRD 100, 072008(2019)

Amplitude analysis of $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ PRD 99, 092008(2019)

Amplitude analysis of $D_s^+ \rightarrow \pi^+ \pi^0 \eta$ PRL 123, 112001 (2019)

Amplitude analysis of $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ PRD 95, 072010(2017)

Amplitude analysis of $D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$ BESIII Preliminary

.....

- **Amplitude analyses and BF measurements of:**

- $D_s^+ \rightarrow K^- K^+ \pi^+$ $\mathcal{B} = (5.47 \pm 0.08 \pm 0.13)\%$
- $D_s^+ \rightarrow K_s^0 \pi^+ \pi^0$ $\mathcal{B} = (5.43 \pm 0.30 \pm 0.15) \times 10^{-3}$ (No evidence of CPV)
- $D_s^+ \rightarrow K_s^0 K^- \pi^+ \pi^+$ $\mathcal{B} = (1.46 \pm 0.05 \pm 0.05)$
- $D_s^+ \rightarrow K^- K^+ \pi^+ \pi^0$ $\mathcal{B} = (5.42 \pm 0.10 \pm 0.17)\%$
- $D_s^+ \rightarrow \eta \pi^+ \pi^+ \pi^-$ $\mathcal{B} = (3.12 \pm 0.13 \pm 0.09)\%$
- $D^+ \rightarrow K_s^0 K^+ \pi^0$

- **Precise measurements of two-body decays:**

- $\mathcal{B}(D_s^+ \rightarrow \phi \pi^+) = (4.60 \pm 0.17)\%$
- $\mathcal{B}(D_s^+ \rightarrow \bar{K}^*(892)^0 K^+) = (3.94 \pm 0.12)\%$
- $\mathcal{B}(D_s^+ \rightarrow \phi \rho) = (6.22 \pm 0.17 \pm 0.24)\%$
- $\mathcal{B}(D_s^+ \rightarrow K_s^0 \rho^+) = (5.46 \pm 0.84 \pm 0.44) \times 10^{-3}$
- $\mathcal{B}(D_s^+ \rightarrow K^{*0} \pi^+) = (2.71 \pm 0.72 \pm 0.30) \times 10^{-3}$
- $\mathcal{B}(D_s^+ \rightarrow K^{*+} \pi^0) = (0.75 \pm 0.24 \pm 0.06) \times 10^{-3}$
- $\mathcal{B}(D_s^+ \rightarrow K^{*+} \bar{K}^{*0}) = (5.46 \pm 0.23 \pm 0.18)\%$
- $\mathcal{B}(D^+ \rightarrow K^{*0} K^+) = (2.71 \pm 0.72 \pm 0.30) \times 10^{-3}$
- $\mathcal{B}(D_s^+ \rightarrow a_0(980)^+ \rho(770)^0) = (0.21 \pm 0.08 \pm 0.05) \times 10^{-3}$ Anomaly
- $\mathcal{B}(D^+ \rightarrow K^{*+} K_s^0) = (8.69 \pm 0.64 \pm 0.51) \times 10^{-3}$

Confirm theoretical predictions

Anomaly

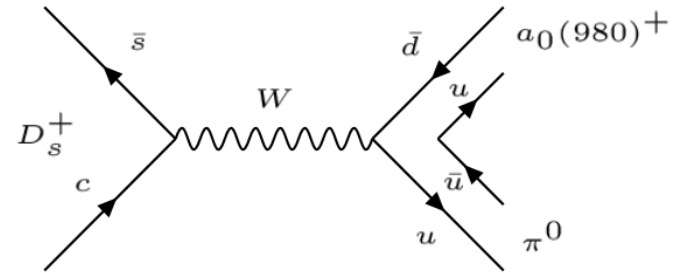
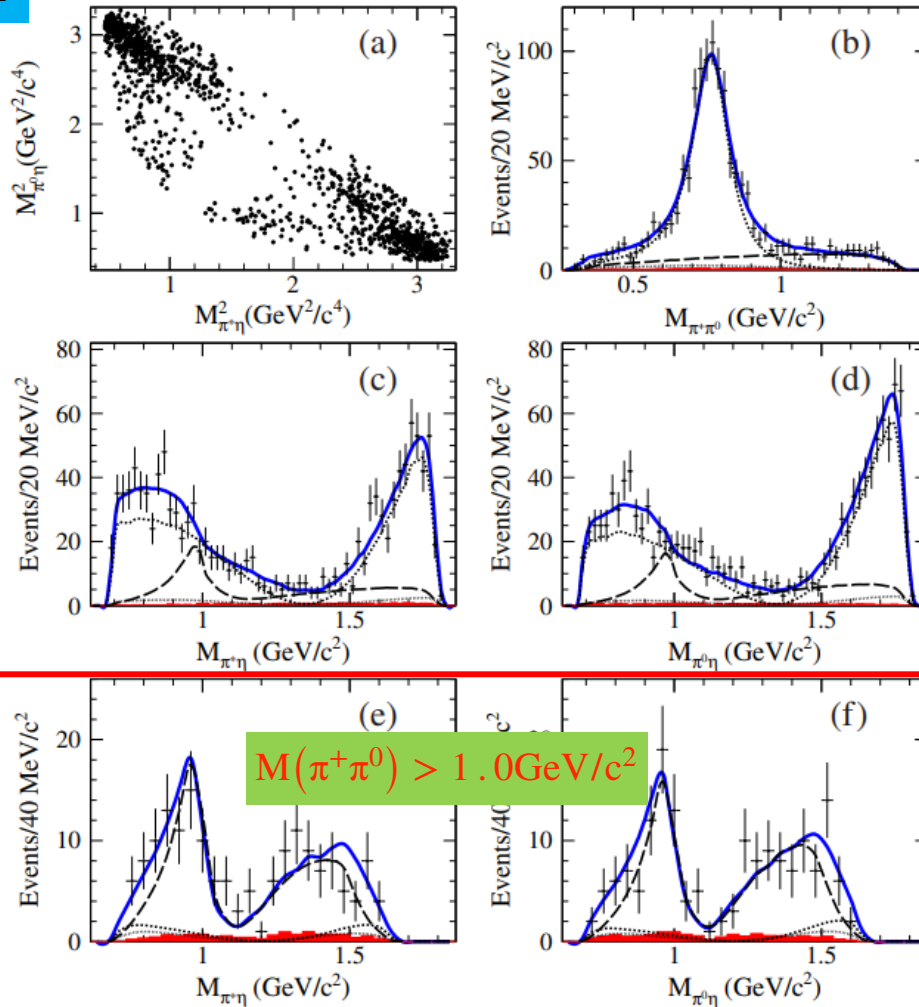
Thank you!

back up

Amplitude analysis of $D_s^+ \rightarrow \pi^+ \pi^0 \eta$

PRL 123, 112001 (2019)

21



1239 DT events with purity of 97.7% @ $\sqrt{s} = 4.178$ GeV

- $\mathcal{B}(D_s^+ \rightarrow \pi^+ \pi^0) = 3.4 \times 10^{-4}$
- $\mathcal{B}(D_s^+ \rightarrow \pi^+ \rho^0) < (1.9 \pm 1.2) \times 10^{-4}$
- $\mathcal{B}(D_s^+ \rightarrow \pi^+ \pi^0 \eta) = (9.50 \pm 0.28 \pm 0.41) \%$
- $\mathcal{B}(a_0(980)^{+(0)} \pi^{0(+)}) = (2.20 \pm 0.22 \pm 0.34) \%$

The BFs of $D_s^+ \rightarrow a_0(980)^{+(0)} \pi^{0(+)}$ are significantly larger than BFs of pure annihilation processes by 2 orders of magnitude.

Amplitude	ϕ_n (rad)	FF_n
$D_s^+ \rightarrow \rho^+ \eta$	0.0 (fixed)	$0.783 \pm 0.050 \pm 0.021$
$D_s^+ \rightarrow (\pi^+ \pi^0)_{\nu} \eta$	$0.612 \pm 0.172 \pm 0.342$	$0.054 \pm 0.021 \pm 0.025$
$D_s^+ \rightarrow a_0(980) \pi$	$2.794 \pm 0.087 \pm 0.044$	$0.232 \pm 0.023 \pm 0.033$

$\chi^2/\text{ndf} = 82.8/77$

Significance $> 5\sigma$