Heavy Quark Hybrid Decays

19th International Conference on hadron Spectroscopy and Structure
(Hadron 2021)
July 28, 2021

Abhishek Mohapatra (TU Munich)

Nora Brambilla (TU Munich)
Wai Kin Lai (South China Normal university)
Antonio Vairo (TU Munich)

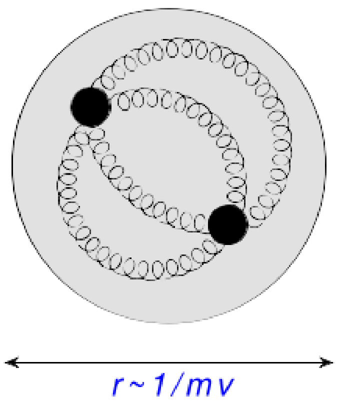
Technische Universität München

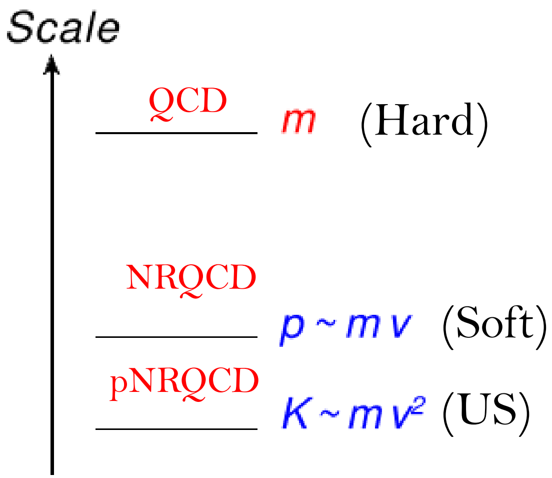
Outline

- Introduction to X Y Z mesons
- EFT for Quarkonium Hybrids
  - BO-EFT effective theory
  - Quarkonium Hybrid Spectrum
- Inclusive Decay Rates for hybrid to quarkonium
- Summary and Outlook

Introduction

- Quarkonium: Color singlet bound state of $Q\bar{Q}$ (Q = c, b).



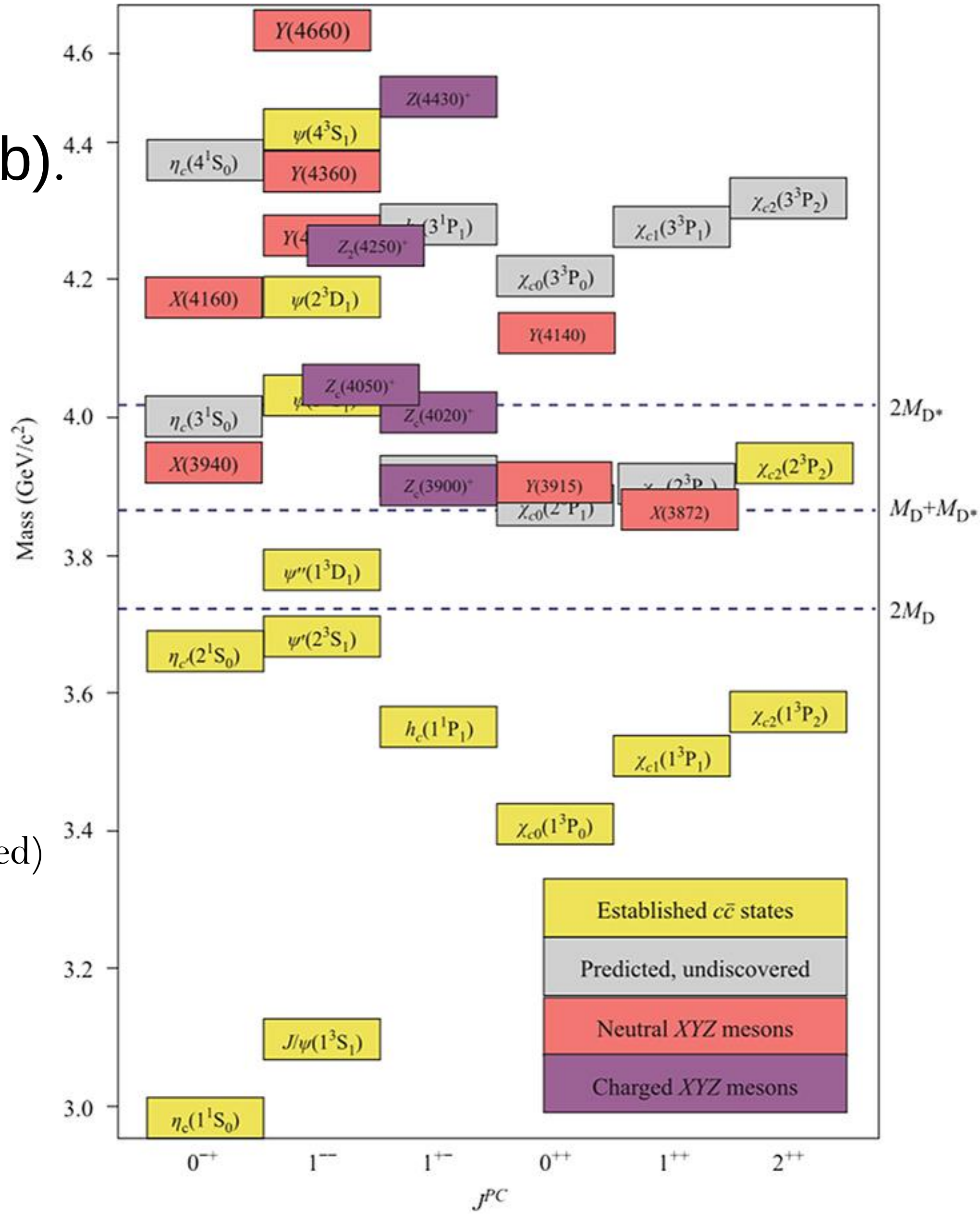


- Hierarchy of Energy Scales in $Q\bar{Q}$:

$m \gg mv \gg mv^2, \Lambda_{\text{QCD}}$ (perturbative dynamics: Weakly Coupled)

$m \gg mv, \Lambda_{\text{QCD}} \gg mv^2$ (nonperturbative dynamics: Strongly Coupled)

- pNRQCD: Relevant EFT for Quarkonium.



Front. Phys. 10 101401 (2015)

Bodwin, Braaten & Lepage (1994), Mehen and Fleming, Phys. Rev. D73, 034502 (2005)

Brambilla, Vairo, and Rosch, Phys. Rev. D72, 034021 (2005) Luke, Manohar & Rothstein (2000)

Introduction

- Quark Model:

  Mesons: quark-antiquark states Baryons: 3-quark states

- QCD spectrum also allows for more complex structures called as Exotics.

- Exotic states: XYZ mesons

  ✓ Quarkonium-like states that don't fit traditional $Q\bar{Q}$ spectrum.

  ✓ In some cases exotic quantum numbers (charged Z_c and Z_b states)

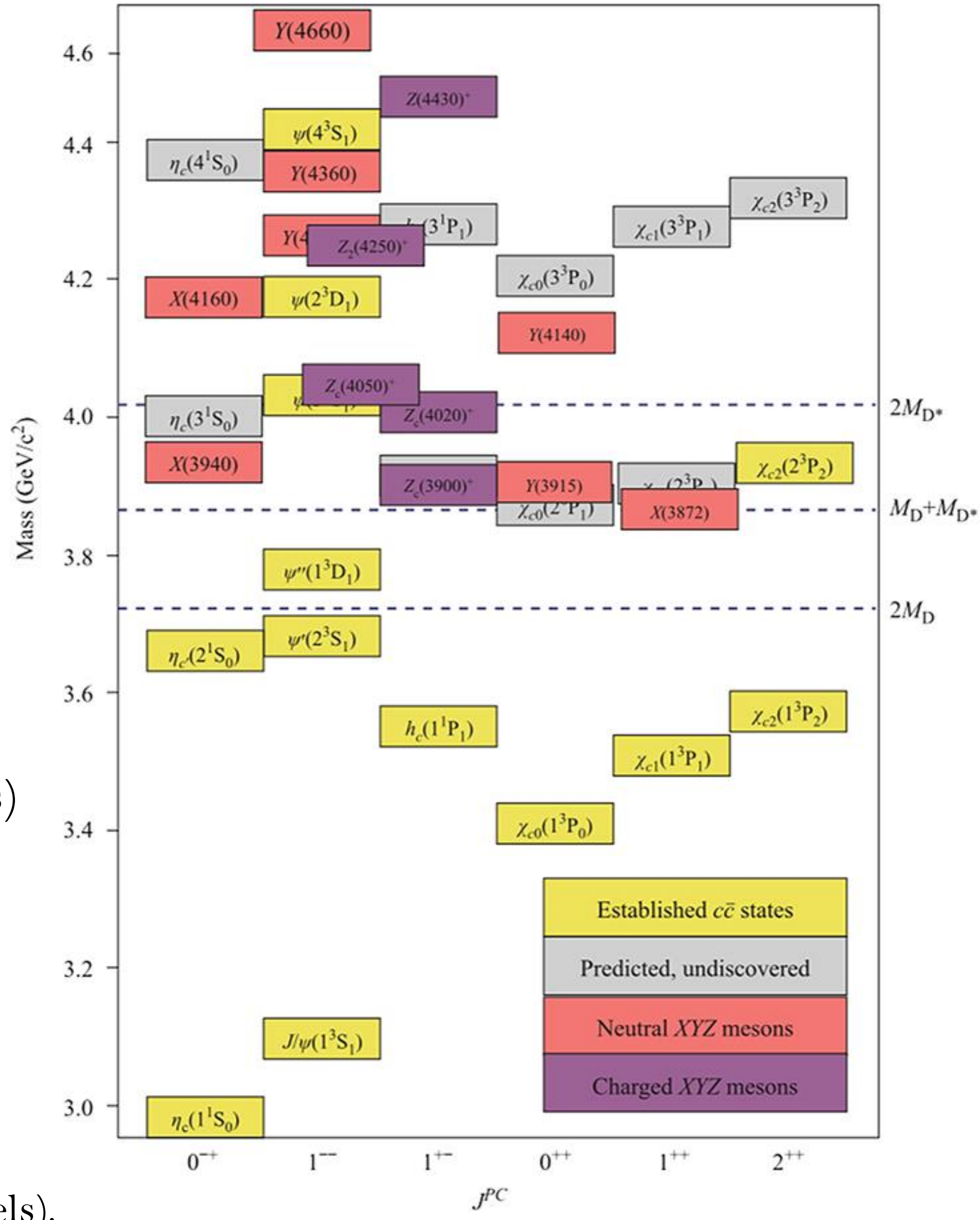
  For review see Brambilla et al. Phys. Reports. 873 (2020)

- X(3872): First exotic state discovered in 2003 by Belle.

  Phys. Rev. Lett. 91, 262001 (2003)

- Several **new** heavy quark exotic states have been discovered since 2003 (masses & decay rates measured in various channels).

  PDG 2021



Front. Phys. 10 101401 (2015)

Introduction

- Exotics broadly classified as
  - ❖ Structures with active gluons ❖ Multiquark states
- Several interpretations of Exotics:

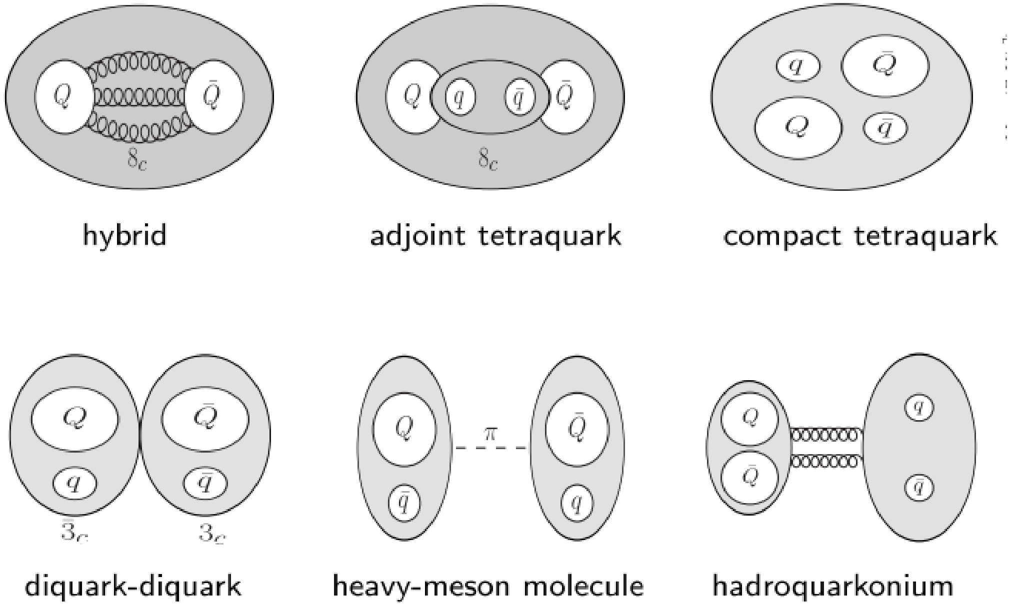
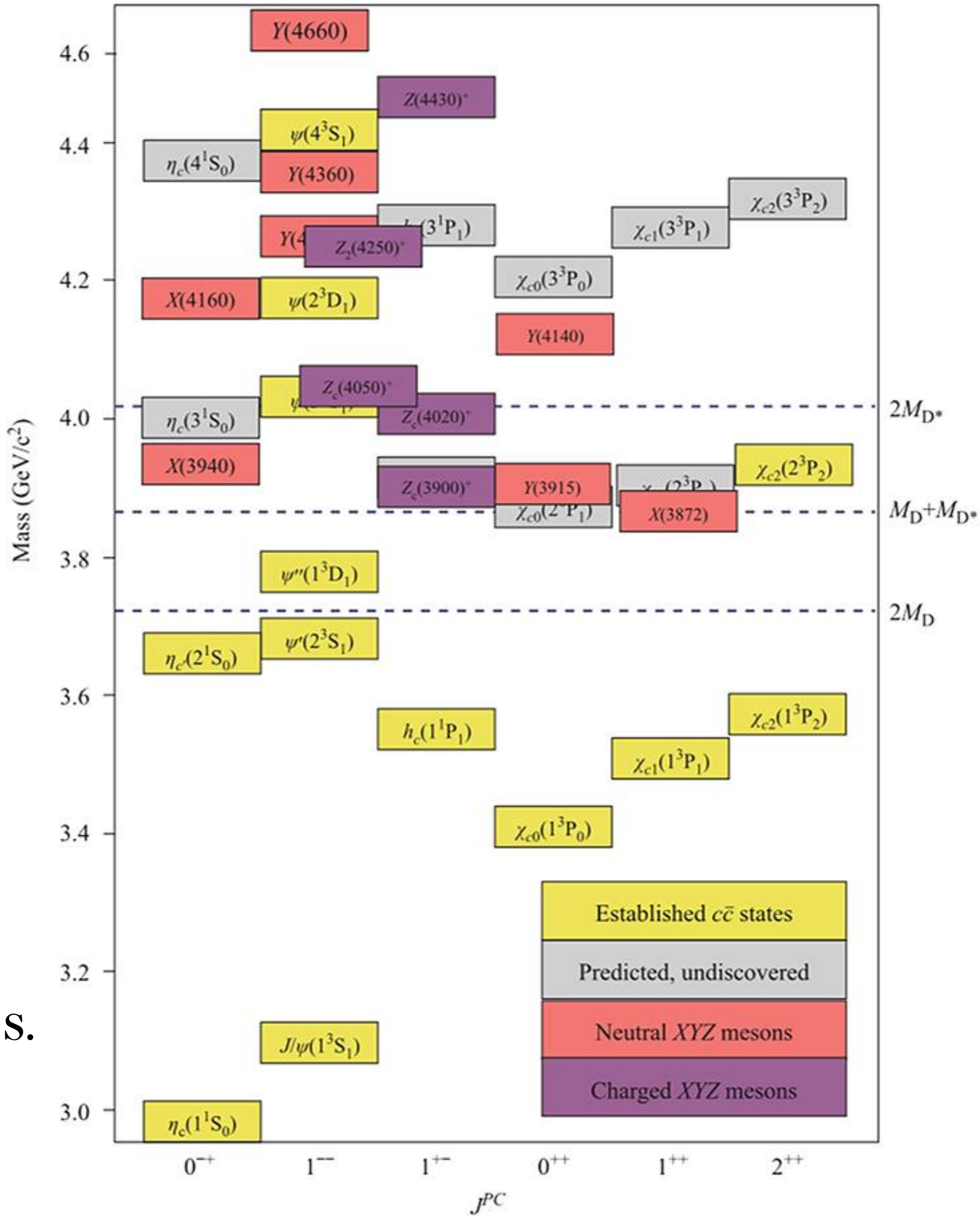


Figure from W.K.Lai talk



Front. Phys. 10 101401 (2015)

- No single model completely describes all the XYZ states.
- Hybrids ($Q\bar{Q}g$): Focus of this talk. Use EFT + lattice to have model independent description.

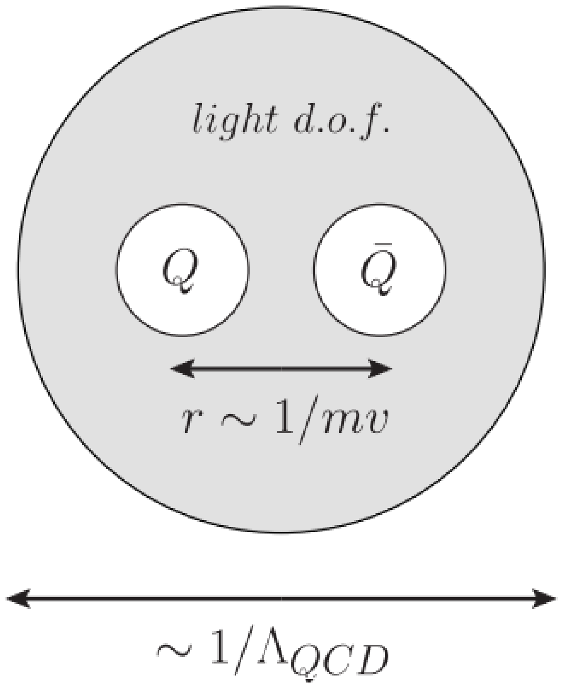
Quarkonium hybrids: EFT

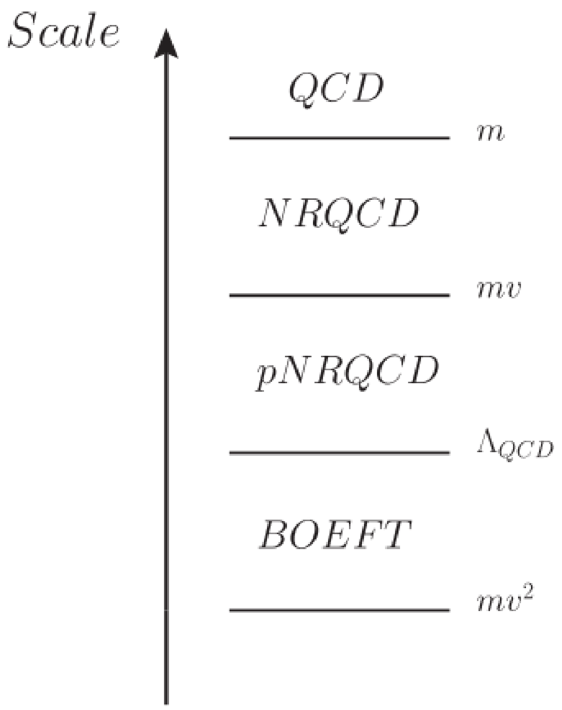
- Hybrids ($Q\bar{Q}g$): Color singlet combination of color octet $Q\bar{Q}$ + gluonic excitations.

- Separation of scales in hybrids:

$$\boxed{m \gg mv \gg \Lambda_{QCD} \gg mv^2}$$

  - ❖ light d.o.f: $\Lambda_{\mathbf{QCD}}$
  - ❖ Relative separation between heavy quarks: $r \sim 1/mv$
  - ❖ Heavy Quark K.E scale: mv^2





- Time-scale for dynamics of $Q\bar{Q}$: $\sim \frac{1}{mv^2} \gg \frac{1}{\Lambda_{QCD}}$

Born-Oppenheimer Approximation

Braaten, Langmack, Smith
Phys. Rev. D. 90, 014044 (2014)

- Appropriate EFT framework for Hybrids: **Born-Oppenheimer EFT (BOEFT)**

$$\text{QCD} \rightarrow \text{NRQCD} \rightarrow \text{pNRQCD} \rightarrow \text{BOEFT}$$

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)
Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)
R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Quarkonium hybrids: BOEFT

- Static limit (m→ ∞): Quantum #'s for hybrid

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g),\ -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

- Static Energies (Σ, Π, Δ): Eigenvalue of NRQCD Hamiltonian in the static limit.

- For r→ 0: static energies are degenerate. Characterized by O(3)×C symmetry group.

Labelled by: $(K^{PC}, \Lambda^{\sigma}_{\eta})$

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Gluonic static energies

M. Foster and C. Michael, Phys. Rev. D59 (1999)

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

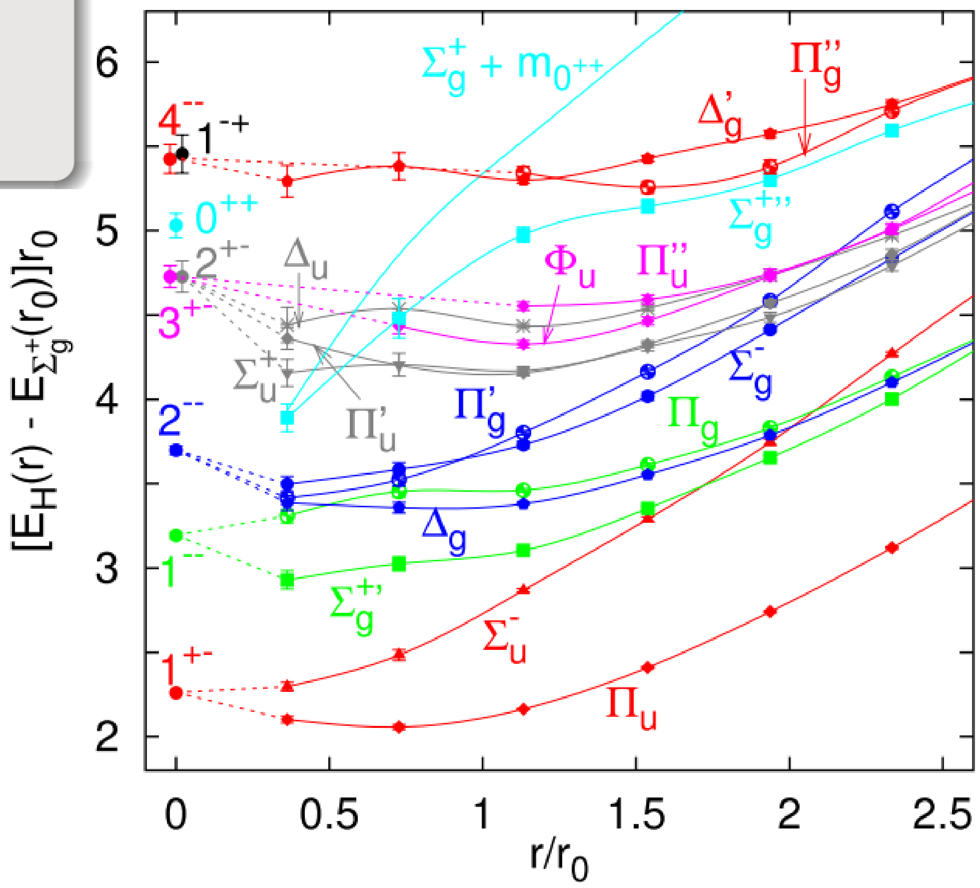


Fig from G. S. Bali and A. Pineda, Phys. Rev. D69 (2004)

Quarkonium hybrids: BOEFT

- Static limit (m$\rightarrow \infty$): Quantum #'s for hybrid

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g),\ -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

Gluonic operators characterizing Hybrids in Wilson loop

- Static Energies (Σ, Π, Δ): Eigenvalue of NRQCD Hamiltonian in the static limit.

- For r$\rightarrow$ 0: static energies are degenerate. Characterized by O(3)×C symmetry group.

Labelled by: $(K^{PC}, \Lambda_\eta^\sigma)$

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Λ_η^σ	K^{PC}	O_n
Σ_u^-	1^{+-}	$\hat{\boldsymbol{r}} \cdot \boldsymbol{B},\ \hat{\boldsymbol{r}} \cdot (\boldsymbol{D} \times \boldsymbol{E})$
Π_u	1^{+-}	$\hat{\boldsymbol{r}} \times \boldsymbol{B},\ \hat{\boldsymbol{r}} \times (\boldsymbol{D} \times \boldsymbol{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\boldsymbol{r}} \cdot \boldsymbol{E},\ \hat{\boldsymbol{r}} \cdot (\boldsymbol{D} \times \boldsymbol{B})$
Π_g	1^{--}	$\hat{\boldsymbol{r}} \times \boldsymbol{E},\ \hat{\boldsymbol{r}} \times (\boldsymbol{D} \times \boldsymbol{B})$
Σ_g^-	2^{--}	$(\hat{\boldsymbol{r}} \cdot \boldsymbol{D})(\hat{\boldsymbol{r}} \cdot \boldsymbol{B})$
Π_g'	2^{--}	$\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} \cdot \boldsymbol{D})\boldsymbol{B} + \boldsymbol{D}(\hat{\boldsymbol{r}} \cdot \boldsymbol{B}))$
Δ_g	2^{--}	$(\hat{\boldsymbol{r}} \times \boldsymbol{D})^i(\hat{\boldsymbol{r}} \times \boldsymbol{B})^j + (\hat{\boldsymbol{r}} \times \boldsymbol{D})^j(\hat{\boldsymbol{r}} \times \boldsymbol{B})^i$
Σ_u^+	2^{+-}	$(\hat{\boldsymbol{r}} \cdot \boldsymbol{D})(\hat{\boldsymbol{r}} \cdot \boldsymbol{E})$
Π_u'	2^{+-}	$\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} \cdot \boldsymbol{D})\boldsymbol{E} + \boldsymbol{D}(\hat{\boldsymbol{r}} \cdot \boldsymbol{E}))$
Δ_u	2^{+-}	$(\hat{\boldsymbol{r}} \times \boldsymbol{D})^i(\hat{\boldsymbol{r}} \times \boldsymbol{E})^j + (\hat{\boldsymbol{r}} \times \boldsymbol{D})^j(\hat{\boldsymbol{r}} \times \boldsymbol{E})^i$

Focus on these two for low lying hybrids

Quarkonium hybrids: BOEFT

- BOEFT d.o.f involve color singlet fields $\hat{\Psi}_{\kappa\lambda}(\boldsymbol{r},\boldsymbol{R},t) \propto P^i_{\kappa\lambda} O^{a\dagger}(\boldsymbol{r},\boldsymbol{R},t) G^{ia}_{\kappa}(\boldsymbol{R},t)$
  - $O^{a\dagger}(\boldsymbol{r},\boldsymbol{R},t)G^{ia}_{\kappa}(\boldsymbol{R},t)$: Gluelump operator. Eigenvector of NRQCD Hamiltonian in (m⟶ ∞):

$$\boxed{H^{(0)} O^{a\dagger}(\boldsymbol{r},\boldsymbol{R},\boldsymbol{t}) G^{ia}_{\kappa}(\boldsymbol{R},t)|0\rangle = (V_0(r) + \Lambda_\kappa)\, O^{a\dagger}(\boldsymbol{r},\boldsymbol{R},t) G^{ia}_{\kappa}(\boldsymbol{R},t)|0\rangle}$$

Λ_κ : Gluelump energy

  - $P^i_{\kappa\lambda}$: Projection operators of light d.o.f along heavy quark-antiquark axis.

- BOEFT Lagrangian:

$$L_{\text{BOEFT}} = \int d^3R\, d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \hat{\Psi}^{\dagger}_{\kappa\lambda}(\boldsymbol{r},\boldsymbol{R},t) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m} P^i_{\kappa\lambda'} \right\} \hat{\Psi}_{\kappa\lambda'}(\boldsymbol{r},\boldsymbol{R},t) + \ldots$$

- Schrödinger Eq: Dynamics of $Q\bar{Q}$ at scale $mv^2 \ll \Lambda_{QCD}$

Schrödinger equation

$$\left[-P^{i\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m} P^i_{\kappa\lambda'} + V_{\kappa\lambda\lambda'}(r) \right] \Psi^n_{\kappa\lambda'}(\boldsymbol{r}) = E_n \Psi^n_{\kappa\lambda}(\boldsymbol{r})$$

Hybrid wf

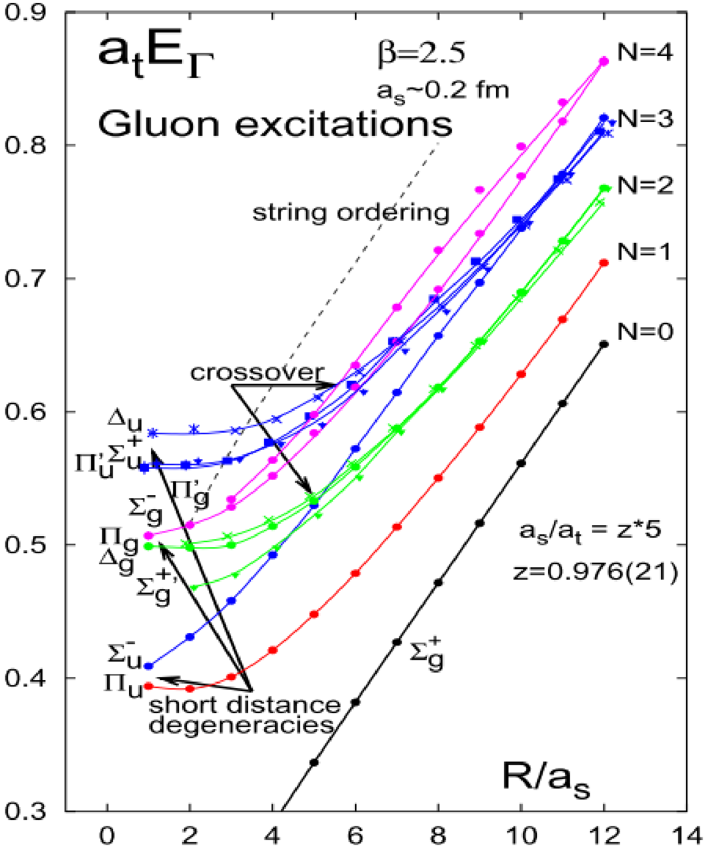
- Coupled Eq. due to projection operators. Mixes Σ_u and Π_u states.

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)
Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Quarkonium hybrids: Spectrum

- Lattice potentials for solving the Schrödinger Eq:

Gluonic Static energies from lattice:



K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

Potential after fitting lattice data:

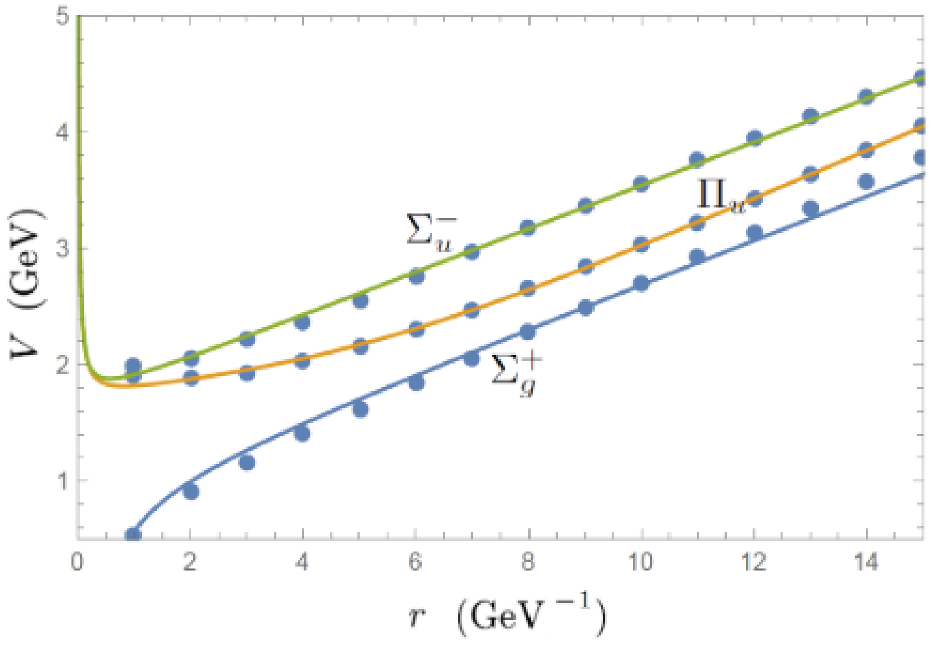


Fig from R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Quarkonium hybrids: Spectrum

- Using potentials for V_Π and V_Σ from R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Charmonium hybrids

NL_J	w-f	$M_{c\bar{c}}$	$M_{c\bar{c}g}$	$S=0$ $\mathcal{J}^{PC}$	$S=1$ $\mathcal{J}^{PC}$	Λ_η^ϵ
$1s$	S	3068		0^{-+}	1^{--}	Σ_g^+
$2s$	S	3678		0^{-+}	1^{--}	Σ_g^+
$3s$	S	4131		0^{-+}	1^{--}	Σ_g^+
$1p_0$	P^+		4486	0^{++}	1^{+-}	Σ_u^-
$4s$	S	4512		0^{-+}	1^{--}	Σ_g^+
$2p_0$	P^+		4920	0^{++}	1^{+-}	Σ_u^-
$3p_0$	P^+		5299	0^{++}	1^{+-}	Σ_u^-
$4p_0$	P^+		5642	0^{++}	1^{+-}	Σ_u^-
$1p$	S	3494		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p$	S	3968		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$	P^{+-}		4011	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$	P^0		4145	1^{++}	$(0,1,2)^{+-}$	Π_u
$2(s/d)_1$	P^{+-}		4355	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p$	S	4369		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p_1$	P^0		4511	1^{++}	$(0,1,2)^{+-}$	Π_u
$3(s/d)_1$	P^{+-}		4692	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4(s/d)_1$	P^{+-}		4718	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4p$	S	4727		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3p_1$	P^0		4863	1^{++}	$(0,1,2)^{+-}$	Π_u
$5(s/d)_1$	P^{+-}		5043	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$5p$	S	5055		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1d$	S	3793		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d$	S	4210		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$	P^{+-}		4231	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$1d_2$	P^0		4334	2^{--}	$(1,2,3)^{-+}$	Π_u
$2(p/f)_2$	P^{+-}		4563	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$3d$	S	4579		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d_2$	P^0		4693	2^{--}	$(1,2,3)^{-+}$	Π_u
$3(p/f)_2$	P^{+-}		4886	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$4d$	S	4916		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$4(p/f)_2$	P^{+-}		4923	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$3d_2$	P^0		5036	2^{--}	$(1,2,3)^{-+}$	Π_u

Bottomonium hybrids

NL_J	w-f	$M_{b\bar{b}}$	$M_{b\bar{b}g}$	$S=0$ $\mathcal{J}^{PC}$	$S=1$ $\mathcal{J}^{PC}$	Λ_η^ϵ
$1s$	S	9442		0^{-+}	1^{--}	Σ_g^+
$2s$	S	10009		0^{-+}	1^{--}	Σ_g^+
$3s$	S	10356		0^{-+}	1^{--}	Σ_g^+
$4s$	S	10638		0^{-+}	1^{--}	Σ_g^+
$1p_0$	P^+		11011	0^{++}	1^{+-}	Σ_u^-
$2p_0$	P^+		11299	0^{++}	1^{+-}	Σ_u^-
$3p_0$	P^+		11551	0^{++}	1^{+-}	Σ_u^-
$4p_0$	P^+		11779	0^{++}	1^{+-}	Σ_u^-
$1p$	S	9908		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p$	S	10265		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3p$	S	10553		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$	P^{+-}		10690	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$	P^0		10761	1^{++}	$(0,1,2)^{+-}$	Π_u
$4p$	S	10806		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2(s/d)_1$	P^{+-}		10885	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$2p_1$	P^0		10970	1^{++}	$(0,1,2)^{+-}$	Π_u
$5p$	S	11035		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3(s/d)_1$	P^{+-}		11084	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4(s/d)_1$	P^{+-}		11156	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p_1$	P^0		11175	1^{++}	$(0,1,2)^{+-}$	Π_u
$6p$	S	11247		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$5(s/d)_1$	P^{+-}		11284	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1d$	S	10155		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d$	S	10454		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$3d$	S	10712		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$	P^{+-}		10819	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$1d_2$	P^0		10870	2^{--}	$(1,2,3)^{-+}$	Π_u
$4d$	S	10947		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2(p/f)_2$	P^{+-}		11005	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$2d_2$	P^0		11074	2^{--}	$(1,2,3)^{-+}$	Π_u
$5d$	S	11163		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$3(p/f)_2$	P^{+-}		11197	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$3d_2$	P^0		11275	2^{--}	$(1,2,3)^{-+}$	Π_u
$4(p/f)_2$	P^{+-}		11291	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$

$m_c = 1.47$ GeV $\qquad m_b = 4.88$ GeV

Other notation of hybrid states

		l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
$N(s/d)_1 \to$	H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
$Np_1 \to$	H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
$Np_0 \to$	H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
$N(p/f)_2 \to$	H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
$Nd_2 \to$	H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

Table taken from R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Inclusive Decays

- Dozens of XYZ states have been discovered (mass and decay rates measured) but physics still unknown.

- Several theoretical models for exotic states but no general consensus.

- Most of the exotic states discovered from decays to quarkonium. So, decays might provide information on the structure of XYZ.

- Consider the process: $\boldsymbol{H_m \to Q_n + X}$; H_m: low-lying hybrid, Q_n: low-lying quarkonium.

  ✓ $\Delta \equiv \boldsymbol{m_H - m_Q \gtrsim 1}$ **GeV**. For low-lying states, we observe that $\Delta \gg \Lambda_{\text{QCD}} \gg mv^2$

  ✓ Hierarchy of Scales: $\mathbf{mv \gg \Delta \gg \Lambda_{QCD} \gg mv^2}$

- Start with **pNRQCD** effective theory and **obtain BOEFT** by **matching**: Integrate out modes of scale $\sim \Delta$ and $\sim \Lambda_{\text{QCD}}$.

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

Inclusive Decays

- pNRQCD Lagrangian:

Weakly-coupled pNRQCD Lagrangian

$$L_{\mathrm{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger \left(i\partial_0 - h_s\right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o\right) \mathrm{O} \right]$$

$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{S} + \frac{1}{2}\mathrm{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\mathrm{O}\}\right] + \frac{g}{4m}\mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\mathrm{O}]\right]$$

$$+ \frac{gc_F}{m}\mathrm{Tr}\left[\mathrm{S}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{O} + \mathrm{O}^\dagger(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{S} + \mathrm{O}^\dagger\boldsymbol{S}_1\cdot\boldsymbol{B}\,\mathrm{O} - \mathrm{O}^\dagger\boldsymbol{S}_2\mathrm{O}\cdot\boldsymbol{B}\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}\Big\}$$

- BOEFT:

Potential term in BOEFT

BOEFT Hamiltonian

$$H_{\mathrm{BOEFT}} = \int d^3x \int d^3R\,\mathrm{Tr}\left[H^{i\dagger}\left(h_o\delta^{ij} + \boxed{V^{ij}_{soft}}\right)H^j\right]$$

- Decays are computed from local imaginary terms in the BOEFT Lagrangian.

- Imaginary term in V^{ij}_{soft} from 1-loop diagram in pNRQCD and then **matching** to BOEFT .

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

Inclusive Decays

- pNRQCD Lagrangian:

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

Weakly-coupled pNRQCD Lagrangian

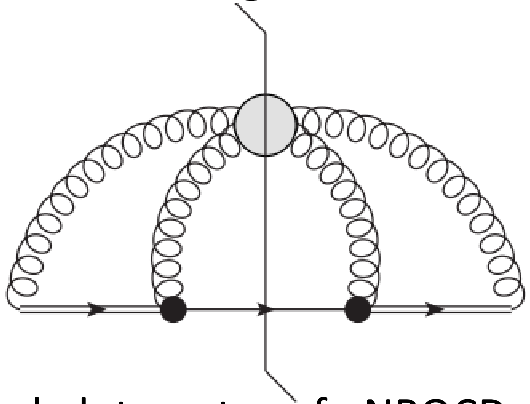
$$L_{\mathrm{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\mathrm{Tr}\left[\mathrm{S}^\dagger \left(i\partial_0 - h_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o \right) \mathrm{O} \right]$$

$$+ g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O} + \mathrm{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{S} + \frac{1}{2}\mathrm{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\mathrm{O}\} \right] + \frac{g}{4m}\mathrm{Tr}\left[\mathrm{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\mathrm{O}] \right]$$

$$+ \frac{gc_F}{m}\mathrm{Tr}\left[\mathrm{S}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{O} + \mathrm{O}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\mathrm{S} + \mathrm{O}^\dagger \boldsymbol{S}_1\cdot\boldsymbol{B}\,\mathrm{O} - \mathrm{O}^\dagger \boldsymbol{S}_2 \mathrm{O}\cdot\boldsymbol{B} \right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} \Big\}$$

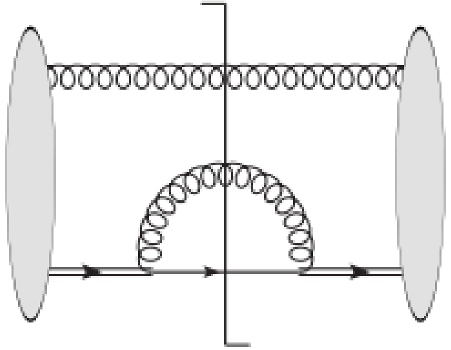
- **Spin preserving decays**
- **Spin flipping decays**

- 1-loop diagram in pNRQCD contributing to Im V^{ij}_{soft} in BOEFT:



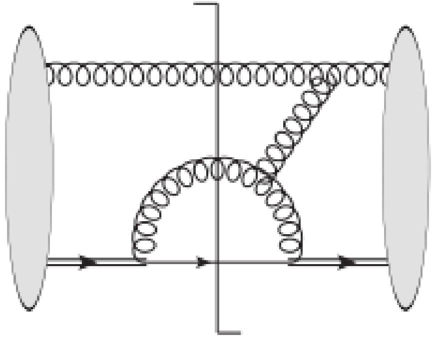
Black dot: vertex of pNRQCD

=



Spectator gluon approximation

+



example of α_s correction

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

Spin-preserving inclusive decay rate for $H_m \to Q_n + X$

$$\Gamma(H_m \to Q_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{n'} |h_{nn'}|^2 \sum_{q,q'} \int dE \int dE' \, f^i_{mq}(E) g^j_{qn}(E) \times g^{j\dagger}_{q'n}(E') f^{i\dagger}_{mq'}(E') (\Lambda + E/2 + E'/2 - E^s_n)^3$$

Overlap functions:

$$\boxed{f^i_{mq}(E) = \left[\int d^3r\, \Psi^{i\dagger}_m(\boldsymbol{r}) \Phi^o_{E,q}(\boldsymbol{r})\right]}$$

$$\boxed{g^j_{qn}(E) = \left[\int d^3r\, \Phi^{o\dagger}_{E,q}(\boldsymbol{r}) r^j \Phi^s_n(\boldsymbol{r})\right]}$$

$$\boxed{h_{nn'} = \int d^3r\, \Phi^{s\dagger}_{n'}(\boldsymbol{r}) \Phi^Q_n(\boldsymbol{r})}$$

- Double integral over the energies E, E' of the octet wave function $\Phi^{o}_{E,q}$.
- Depends on overlap function of singlet w.f Φ^{S}_{n} and quarkonium w.f Φ^{Q}_{n}.
- Last factor $\left(\Lambda + {}^{E}/_{2} + {}^{E'}/_{2} - E^S_n\right)^3 \sim \Delta^3$ parametrically. $(\Delta \equiv \boldsymbol{m_H} - \boldsymbol{m_Q})$
- Above result different from the one in R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017).

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

Spin-preserving inclusive decay rate for $H_m \to Q_n + X$

$$\Gamma(H_m \to Q_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{n'} |h_{nn'}|^2 \sum_{q,q'} \int dE \int dE' \, f^i_{mq}(E) g^j_{qn}(E) \times g^{j\dagger}_{q'n}(E') f^{i\dagger}_{mq'}(E') (\Lambda + E/2 + E'/2 - E^s_n)^3$$

Assumption:
$f^i_{mq}(E) \neq 0$ only for $E_m \approx E + \Lambda$

$h_{nn'} \approx 1$ and $E^Q_m \approx E^s_m$

Spin-preserving inclusive decay rate for $H_m \to Q_n + X$

$$\Gamma(H_m \to Q_n + X) = \frac{4\alpha_s T_F}{3N_c} (E_m - E^Q_n)^3 \, T^{ij} \, (T^{ij})^*$$

$$\boxed{T^{ij} \equiv \int d^3r \, \Psi^{i\dagger}_m(\boldsymbol{r}) r^j \, \Phi^Q_n(\boldsymbol{r})}$$

- Above result looks similar to the one in R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017). In general has **tensor structure T^{ij}** that agrees with J. Castellà, E. Passemar, arXiv:2104.03975.

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

Spin-preserving inclusive decay rate for $H_m \to Q_n + X$

$$\Gamma(H_m \to Q_n + X) = \frac{4\alpha_s T_F}{3N_c}\,(E_m - E_n^Q)^3\, T^{ij}\,(T^{ij})^*$$

$$\boxed{T^{ij} \equiv \int d^3r\, \Psi_m^{i\dagger}(\boldsymbol{r}) r^j\, \Phi_n^Q(\boldsymbol{r})}$$

- R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017): only **diagonal elements** T^{ii} are considered

Inclusive decay rate for $H_m \to Q_n + X$ computed by Oncala and Soto

$$\Gamma^{\text{Oncala}}(H_m \to Q_n + X) = \frac{4\alpha_s T_F}{3N_c}\,(E_m - E_n^Q)^3\, T^{ii}\,(T^{jj})^*$$

- T^{ii} leads to **selection rule**: Hybrids such as Np_1 (H_2) where L=J **don't decay to quarkonium**.
- T^{ij}: allows for the decay of Np_1 (H_2) hybrid decays.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- **Spin conserving** decay rates for **Charm hybrids**:

$NL_J \to N'L'$	ΔE (GeV)	$\alpha_s\,(\Delta E)$	$\sqrt{T^{ij}(T^{ij})^*}$ (GeV^{-1})	Γ (MeV)
$1p_0 \to 1s$	1.418	0.307	1.187	274.48
$1p_0 \to 2s$	0.808	0.408	1.748	146.23
$2p_0 \to 1s$	1.852	0.276	0.312	38.0
$2p_0 \to 2s$	1.242	0.326	1.633	370.8
$3p_0 \to 1s$	2.231	0.259	0.144	13.23
$3p_0 \to 2s$	1.620	0.290	0.388	41.2
$1p_1 \to 1s$	1.078	0.349	1.886	346.0
$2p_1 \to 1s$	1.444	0.305	0.862	151.48
$2p_1 \to 2s$	0.834	0.401	1.908	187.90
$3p_1 \to 1s$	1.796	0.279	0.447	71.73
$3p_1 \to 2s$	1.186	0.334	0.942	109.62
$2(s/d)_1 \to 1p$	0.861	0.394	2.322	301.56
$4(s/d)_1 \to 1p$	1.224	0.329	3.667	1802.13

Decays not allowed in R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017)

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- **Spin conserving** decay rates for **Bottom hybrids**:

$NL_J \to N'L'$	ΔE (GeV)	$\alpha_s(\Delta E)$	$\sqrt{T^{ij}(T^{ij})^*}$ (GeV^{-1})	Γ (MeV)
$1p_0 \to 1s$	1.569	0.294	0.520	68.20
$1p_0 \to 2s$	1.002	0.363	1.270	130.86
$2p_0 \to 1s$	1.857	0.276	0.264	27.26
$2p_0 \to 2s$	1.290	0.321	0.583	51.96
$2p_0 \to 3s$	0.943	0.375	1.910	254.63
$3p_0 \to 1s$	2.109	0.264	0.162	14.37
$3p_0 \to 2s$	1.542	0.296	0.290	20.35
$4p_0 \to 1s$	2.338	0.254	0.111	8.88
$4p_0 \to 2s$	1.770	0.281	0.170	10.06
$4p_0 \to 3s$	1.423	0.307	0.329	21.31
$1p_1 \to 1s$	1.320	0.317	0.721	84.29
$2p_1 \to 1s$	1.528	0.297	0.546	70.35
$2p_1 \to 2s$	0.961	0.371	0.059	0.25
$3p_1 \to 1s$	1.732	0.283	0.395	51.00
$3p_1 \to 2s$	1.165	0.336	0.417	20.54
$2(s/d)_1 \to 1p$	0.978	0.367	0.890	60.51
$3(s/d)_1 \to 1p$	1.176	0.335	1.073	141.05
$3(s/d)_1 \to 2p$	0.819	0.405	0.131	0.85
$4(s/d)_1 \to 1p$	1.248	0.326	3.056	1312.72
$4(s/d)_1 \to 2p$	0.890	0.386	4.484	1218.65
$5(s/d)_1 \to 1p$	1.376	0.311	0.467	39.28
$5(s/d)_1 \to 2p$	1.019	0.360	1.011	86.36

Decays not allowed in R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017)

Inclusive Decays

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- Result for **spin-flipping** decays due to $\boldsymbol{S}.\boldsymbol{B}$ term:

Spin-flipping inclusive decay rate for $H_m \to Q_n + X$

$$\Gamma(H_m \to Q_n + X) = \frac{4\alpha_s T_F c_F^2}{3N_c m_Q^2} T^{ij} \left(T^{ij}\right)^* \left(E_m - E_n^Q\right)^3$$

$$T^{ij} \equiv \int d^3r\, \Psi_m^{i\dagger}(\boldsymbol{r}) S^j \Phi_n^Q(\boldsymbol{r})$$

- Spin-flipping decays implies $|S_H = 1 > \dashrightarrow |S_Q = 0 >$ & $|S_H = 0 > \dashrightarrow |S_Q = 1 >$
- Above result agrees with J. Castellà, E. Passemar, arXiv:2104.03975.

- $Q_m \to Q_n + X$ spin-flipping decays: Decay rate suppressed by additional $(\boldsymbol{r} \cdot \boldsymbol{E})^2 \sim v^2$ vertex factor.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- **Spin flipping** decay rates for **Charm hybrids**:

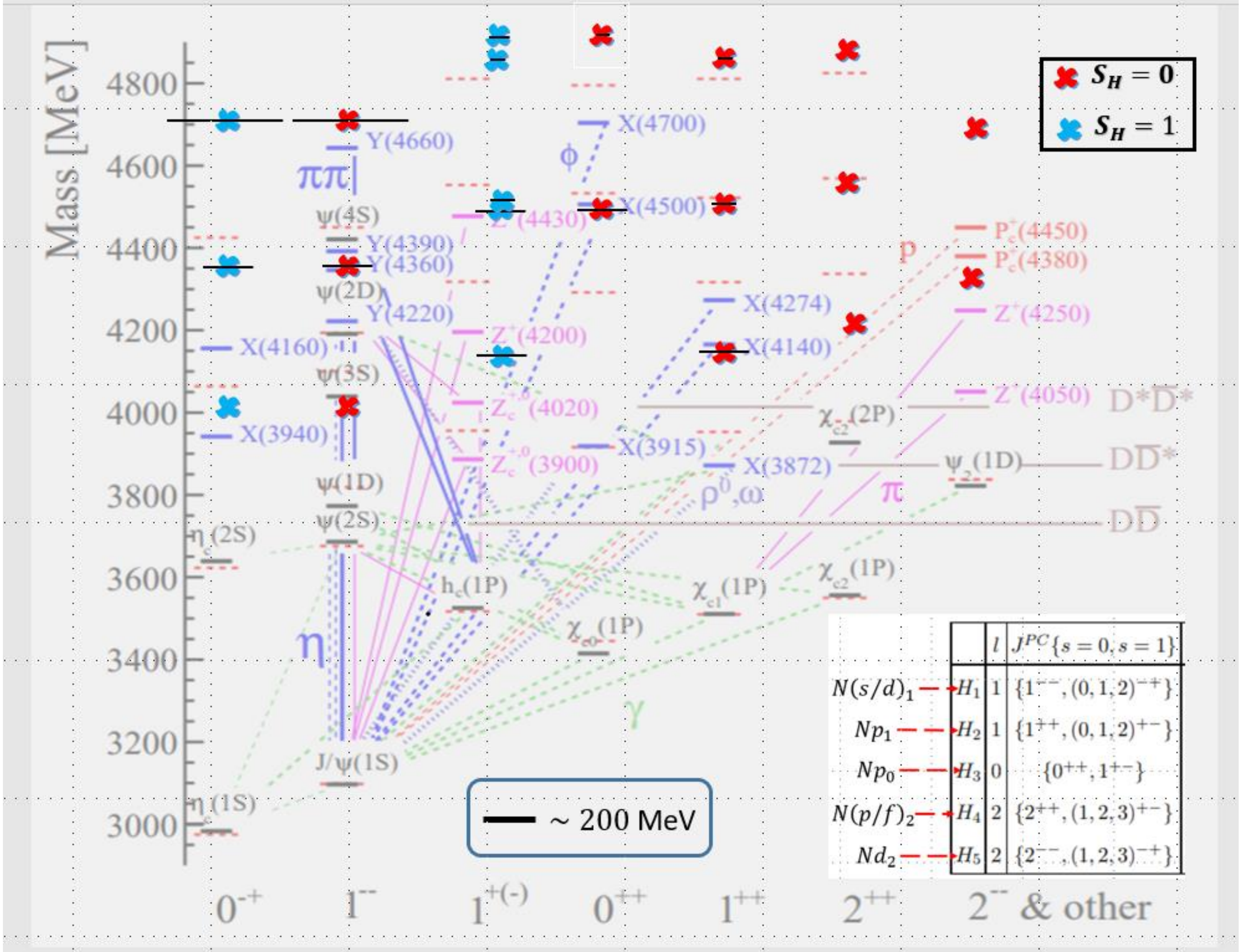
$NL_J \to N'L'$	ΔE (GeV)	$\alpha_S\,(\Delta E)$	Γ(MeV) $(1 \to 0)$	Γ(MeV) $(0 \to 1)$
$1p_0 \to 1p$	0.993	0.365	35.60	106.81
$2p_0 \to 1p$	1.426	0.307	1.44	4.31
$2p_0 \to 2p$	0.952	0.373	31.34	94.03
$3p_0 \to 1p$	1.804	0.279	0.41	1.22
$3p_0 \to 2p$	1.330	0.316	1.52	4.56
$1p_1 \to 1p$	0.653	0.467	46.57	139.72
$2p_1 \to 1p$	1.018	0.360	14.51	43.53
$2p_1 \to 2p$	0.544	0.531	26.13	78.39
$3p_1 \to 1p$	1.369	0.312	5.69	17.07
$3p_1 \to 2p$	0.895	0.385	10.97	32.90
$1(s/d)_1 \to 1s$	0.944	0.374	74.98	224.95
$2(s/d)_1 \to 1s$	1.288	0.321	32.51	97.53
$2(s/d)_1 \to 2s$	0.678	0.455	18.86	56.57
$3(s/d)_1 \to 1s$	1.624	0.290	7.73	23.18
$3(s/d)_1 \to 2s$	1.014	0.360	1.04	3.11
$4(s/d)_1 \to 1s$	1.650	0.288	7.35	22.06
$4(s/d)_1 \to 2s$	1.040	0.356	38.86	116.57

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo
(in progress)

- **Spin flipping** decay rates for **Bottom hybrids**:

$NL_J \to N'L'$	ΔE (GeV)	$\alpha_S\,(\Delta E)$	$\Gamma(\mathrm{MeV})$ $(1 \to 0)$	$\Gamma(\mathrm{MeV})$ $(0 \to 1)$
$1p_0 \to 1p$	1.103	0.346	3.94	11.81
$2p_0 \to 1p$	1.391	0.310	0.54	1.61
$2p_0 \to 2p$	1.033	0.357	2.95	8.84
$3p_0 \to 1p$	1.643	0.289	0.19	0.56
$3p_0 \to 2p$	1.286	0.321	0.50	1.49
$4p_0 \to 1p$	1.872	0.275	0.09	0.28
$4p_0 \to 2p$	1.515	0.299	0.19	0.57
$4p_0 \to 3p$	1.226	0.328	0.45	1.35
$1p_1 \to 1p$	0.853	0.396	6.32	18.96
$2p_1 \to 1p$	1.062	0.352	3.22	9.67
$2p_1 \to 2p$	0.704	0.444	1.99	5.97
$3p_1 \to 1p$	1.266	0.323	1.62	4.85
$3p_1 \to 2p$	0.909	0.382	2.16	6.47
$1(s/d)_1 \to 1s$	1.247	0.326	8.6	25.86
$2(s/d)_1 \to 1s$	1.288	0.305	5.95	17.84
$2(s/d)_1 \to 2s$	0.876	0.390	0.56	1.68
$3(s/d)_1 \to 1s$	1.642	0.289	3.56	10.67
$3(s/d)_1 \to 2s$	1.075	0.350	0.95	2.84
$4(s/d)_1 \to 1s$	1.713	0.284	0.50	1.51
$4(s/d)_1 \to 2s$	1.146	0.339	2.56	7.68
$4(s/d)_1 \to 3s$	0.799	0.411	0.37	1.12



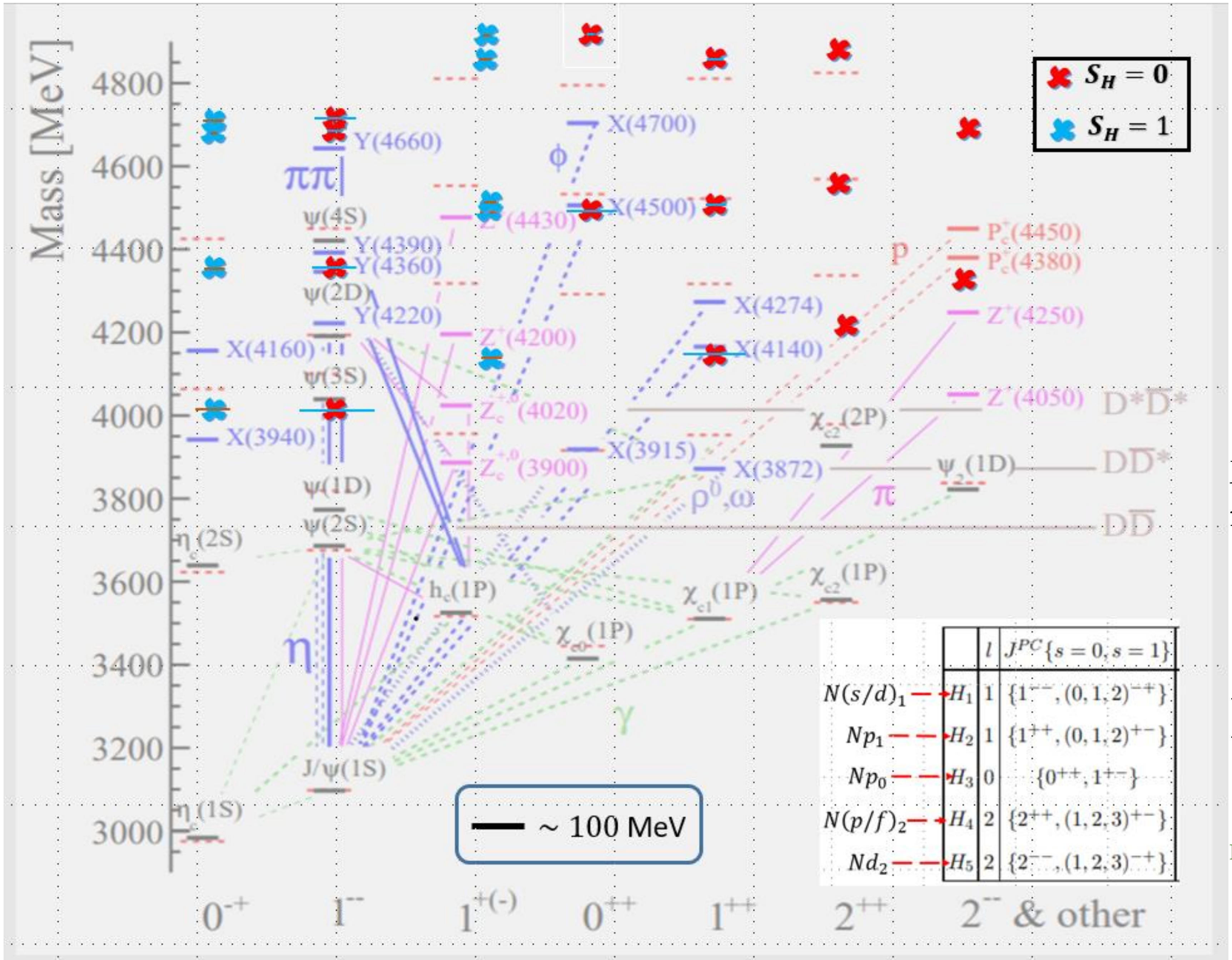
Charm Hybrids: Spin conserving Decays

Background Fig from
S.L. Olsen, T. Skwarnicki, D. Zieminska
Rev. Mod. Phys. 90, 015003 (2018)

Decay rates not computed till now for $J^{PC} = 2^{++}$ and 2^{--} states.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)



		l	$J^{PC}\{s=0, s=1\}$
$N(s/d)_1$	H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$
Np_1	H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$
Np_0	H_3	0	$\{0^{++}, 1^{+-}\}$
$N(p/f)_2$	H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$
Nd_2	H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$

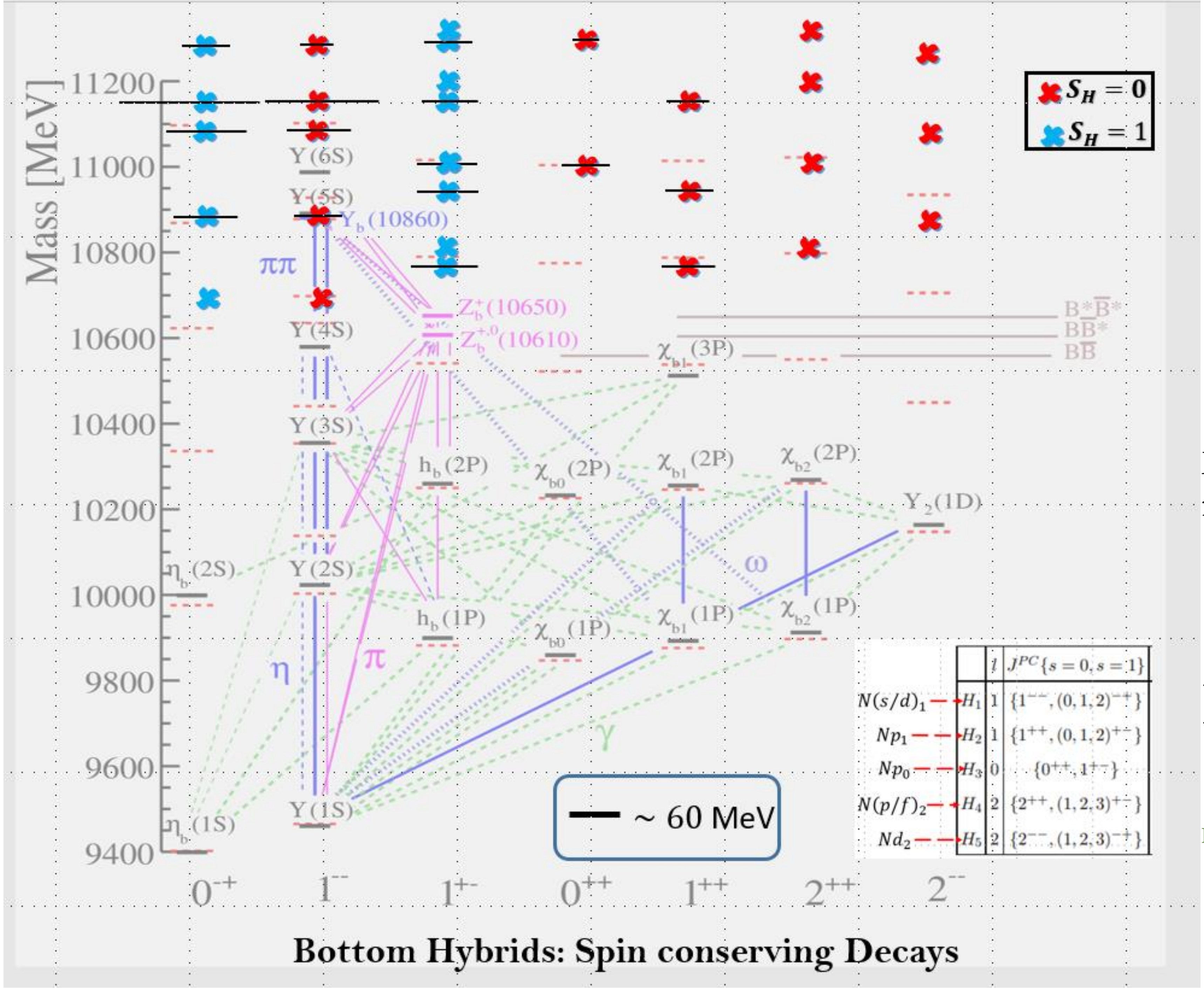
Charm Hybrids: Spin flipping Decays

Background Fig from
S.L. Olsen, T. Skwarnicki, D. Zieminska
Rev. Mod. Phys. 90, 015003 (2018)

Decay rates not computed till now for $J^{PC} = 2^{++}$ and 2^{--} states.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)



Background Fig from

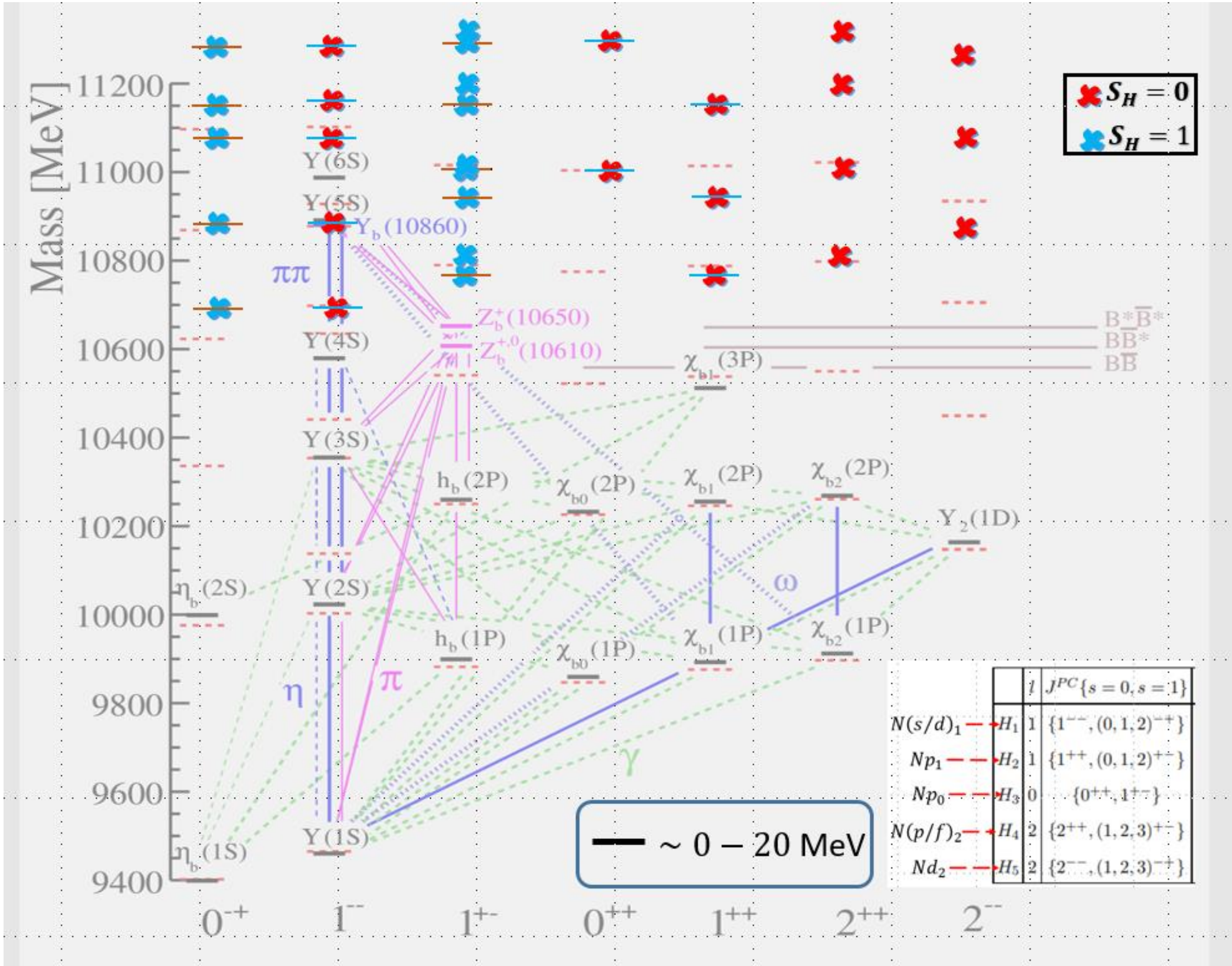
S.L. Olsen, T. Skwarnicki, D. Zieminska

Rev. Mod. Phys. 90, 015003 (2018)

Decay rates not computed till now for $J^{PC} = 2^{++}$ and 2^{--} states.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)



Bottom Hybrids: Spin flipping Decays

Background Fig from
S.L. Olsen, T. Skwarnicki, D. Zieminska
Rev. Mod. Phys. 90, 015003 (2018)

Decay rates not computed till now for $J^{PC} = 2^{++}$ and 2^{--} states.

Preliminary Results

N. Brambilla, W.K. Lai, AM, A. Vairo (in progress)

Summary/Outlook

- BOEFT provides a model-independent & systematic way to study heavy quark hybrids (exotic) and decays.
- General formula for the H_m inclusive decay based on overlap functions:

Inclusive decay rate for $H_m \to Q_n + X$

$$\Gamma(H_m \to Q_n + X) = \frac{4\alpha_s T_F}{3N_c} \sum_{n'} |h_{nn'}|^2 \sum_{q,q'} \int dE \int dE' f^i_{mq}(E) g^j_{qn}(E)$$
$$\times\, g^{j\dagger}_{q'n}(E') f^{i\dagger}_{mq'}(E') (\Lambda + E/2 + E'/2 - E^s_n)^3$$

- Computed preliminary results on decay rates for spin-flipping and spin preserving decays.

- Future work includes:
  - Computing decay rates using the formulae based on overlap functions.
  - Quantifying errors in the decay rates & comparing with the PDG data for observed exotic states.
  - Exclusive decays: $H \to Q\pi\pi$ and include effect of mixing with excited quarkonia $Q' \to Q + X$.
  - Extending this analysis to study tetraquarks.

Thank you!!