Anatomy of the internal structures of the exotic hadrons in a quantitative manner

Zhi-Hui Guo (郭志辉)

Southeast University（东南大学）

Outline:

1. Background & Introduction
2. Effective range expansion & near-threshold state
3. Zc(3900)、Zcs(3985) and X(4020)
4. X(6900) and X(6825)
5. Summary

Background & Introduction

Renaissance of hadron physics:

[X(3872), Zc(3900), Zcs(3985), X(4020), X(6900), Pc(4312) ...]

Many of them lie close to some underlying thresholds.

❖ **Important question that follows:**

Kinematical effects ? Or **Hadron molecules ?** Or **Elementary/Compact states ?**

❖ **Theoretcial methods to probe the composition of hadrons:**

- ✓ QCD sum rules
- ✓ Pole counting rule
- ✓ Nc trajecotries of resonance poles
- ✓ Weinberg's compositeness relation

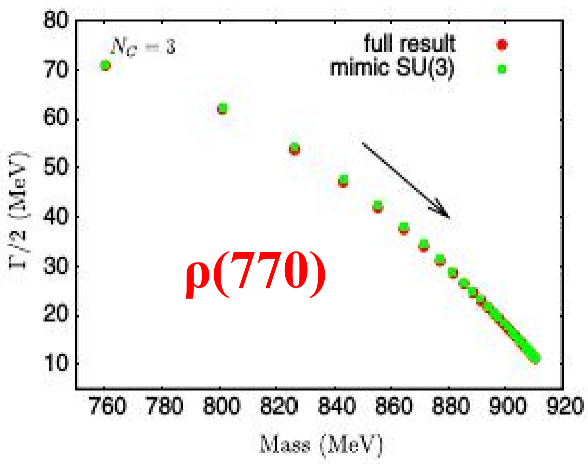
❑ Pole counting rule [Morgan, NPA'92]

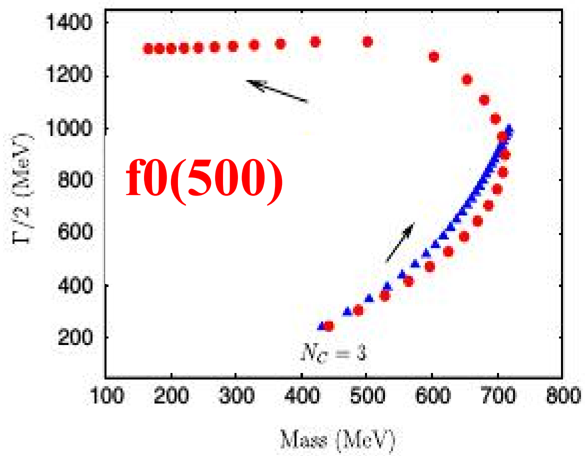
Criteria: Number of nearby poles in S-wave scattering amplitudes

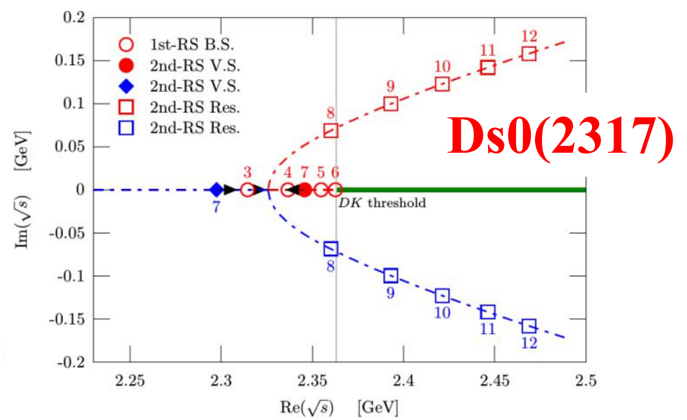
- **Elementary particle: A pair of poles close to threshold**
- **Molecular type: Single pole close to threshold**

Zc(3900): molecule of Dbar-D* [Zheng et al., PRD '16]

❑ Nc trajecotries of resonance poles [Guo et al., PRD '12 '15; Guo, EPJST '21]







❑ Pole counting rule and Nc trajectories only give qualitative conclusions !

❑ Compositeness and elementariness relation

[Weinberg, PR '63 ' 65]

$$|\psi_B\rangle = \int d\alpha \langle\varphi_\alpha|\psi_B\rangle|\varphi_\alpha\rangle + \sum_n \langle\varphi_n|\psi_B\rangle|\varphi_n\rangle$$

Compositeness — **X + Z = 1** — **Elementariness**

$$\int d\alpha |\langle\varphi_\alpha|\psi_B\rangle|^2 \qquad \sum_n |\langle\varphi_n|\psi_B\rangle|^2$$

In the non-relativistic situation, Quantum Mechanics gives

$$X = 1 - Z = \int d\alpha \frac{|\langle\varphi_\alpha|V|\psi_B\rangle|^2}{(E_\alpha - E_B)^2}$$

[Hyodo et al., PRC'12]

[Aceti, Oset, PRD'12]

$$X = \frac{2\mu_m^2}{\pi^2}\int_0^{+\infty} dk k^2 \frac{g^2(k^2)}{(k^2-\kappa^2)^2}, \qquad \kappa^2 = 2\mu_m E_B,$$

[Oller, AnnP'18]

$$X = -g^2(\kappa)^2 \frac{\partial G(E_B)}{\partial E_B} - \frac{\partial g^2(\kappa^2)}{\partial \kappa^2}\frac{\mu^2|\kappa|}{\pi} \qquad \mathcal{O}(\kappa^2/\Lambda^2)$$

$$G(E) = \int_0^\infty \frac{dk k^2}{2\pi^2}\frac{1}{E - k^2/2\mu_m}$$

[ZHG, Oller, PRD'21]

$$X = -g^2(\kappa)^2 \frac{\partial G(E_B)}{\partial E_B}$$

Coupling/Residue

$$G(E) = \int_0^\infty \frac{dk k^2}{2\pi^2} \frac{1}{E - k^2/2\mu_m}$$

- **Bound state √: Z and X are positive real numbers, allowing probabilistic interpretations**
- **Resonance ×: Z and X are usually complex, meaningless to be interpreted as probabilities**

❑ We propose an alternative way to generalize Weinberg compositeness relation for resonances.

[ZHG, Oller, PRD '16]

Transformation of S matrix and Probability interpretation of X

$$\hat{S}_u(s) = \hat{\mathcal{U}}_m \hat{S}_m(s) \hat{\mathcal{U}}_m^T$$

$$\hat{\gamma}_u = \hat{\rho}(s_P)^{-\frac{1}{2}} \hat{\mathcal{U}}_m \hat{\rho}(s_P)^{\frac{1}{2}} \hat{\gamma}$$

$$\hat{\mathcal{U}}_m = \text{diag}(e^{i\phi_1}, \ldots, e^{i\phi_m})$$

$$1 = \hat{\gamma}^T \left[-\frac{d\hat{G}_m(s_P)}{ds} + \hat{G}_m(s_P) \frac{d\hat{\mathcal{K}}_m(s_P)}{ds} \hat{G}_m(s_P) \right] \hat{\gamma}$$

$$1 = -\hat{\gamma}_u^T \frac{d\hat{G}_m(s_P)}{ds} \hat{\gamma}_u + \hat{\gamma}_u^T \hat{G}_m \frac{d\hat{\mathcal{K}}_u(s_P)}{ds} \hat{G}_m \hat{\gamma}_u$$

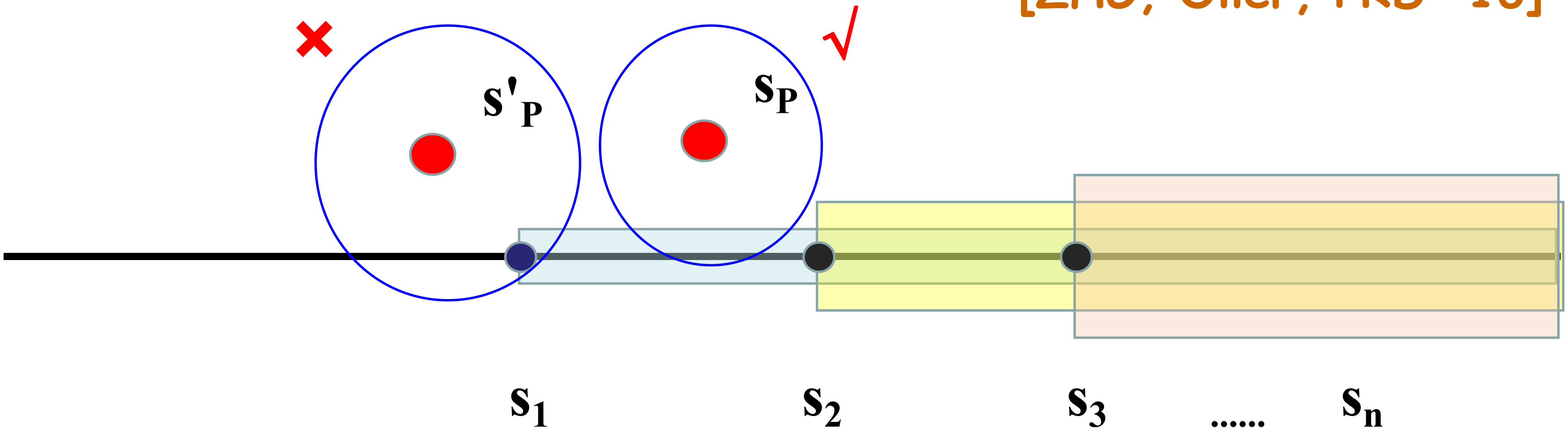
This procedure gives a positive real value for X

$$X_i^R = -e^{2i\phi_i} \eta_i^2 |\gamma_i|^2 \frac{dG(s_P)_i}{ds} = |\gamma_i|^2 \left| \frac{dG(s_P)_i}{ds} \right|$$

Caveats: only valid for resonance poles when $M_R > m_{threshold}$

Working assumption / Condition

[ZHG, Oller, PRD '16]



Summary: Once *S* matrix is known in a region (no matter how small) of physical axis around M_R, we can perfrom analytic extrapolation from the real axis to the complex plane, due to the convergence of the Laurent series for *s* around M_R.

- **Then, compositeness is model independently determined by the properties of resonance pole (pole position and residues)**

$$X_i^R = |\gamma_i|^2 \left| \frac{dG(s_P)_i}{ds} \right|$$

Hidden-channel effects are identified as "Elementariness" !

Applications to some well-established hadrons

[Guo, Oller, PRD '16]

Name of the states	Pole: $\sqrt{s_P}$ [MeV]	$X^R_{\pi\pi}$	$X^R_{\bar{K}K}$	$X^R_{\eta\eta}$	$X^R_{\eta\eta'}$	X^R	Z^R
$f_0(500)$ [17]	$442^{+4}_{-4} - i246^{+7}_{-5}$	$0.40^{+0.02}_{-0.02}$	$\cdots$	$\cdots$	$\cdots$	$0.40^{+0.02}_{-0.02}$	$0.60^{+0.02}_{-0.02}$
$f_0(980)$ [17]	$978^{+17}_{-11} - i29^{+9}_{-11}$	$0.02^{+0.01}_{-0.01}$	$0.65^{+0.27}_{-0.26}$	$\cdots$	$\cdots$	$0.67^{+0.28}_{-0.27}$	$0.33^{+0.28}_{-0.27}$
$f_0(1710)$ [14]	$1690^{+20}_{-20} - i110^{+20}_{-20}$	$0.00^{+0.00}_{-0.00}$	$0.03^{+0.01}_{-0.01}$	$0.02^{+0.01}_{-0.01}$	$0.20^{+0.07}_{-0.07}$	$0.25^{+0.10}_{-0.10}$	$0.75^{+0.10}_{-0.10}$
$\rho(770)$ [17]	$760^{+7}_{-5} - i71^{+4}_{-5}$	$0.08^{+0.01}_{-0.01}$	$\cdots$	$\cdots$	$\cdots$	$0.08^{+0.01}_{-0.01}$	$0.92^{+0.01}_{-0.01}$
		$X^R_{K\pi}$				X^R	Z^R
$K_0^*(800)$ [17]	$643^{+75}_{-30} - i303^{+25}_{-75}$	$0.94^{+0.39}_{-0.52}$				$0.94^{+0.39}_{-0.52}$	$0.06^{+0.39}_{-0.52}$
$K^*(892)$ [17]	$892^{+5}_{-7} - i25^{+2}_{-2}$	$0.05^{+0.01}_{-0.01}$				$0.05^{+0.01}_{-0.01}$	$0.95^{+0.01}_{-0.01}$
		$X^R_{\pi\eta}$	$X^R_{\bar{K}K}$	$X^R_{\pi\eta'}$		X^R	Z^R
$a_0(1450)$ [17]	$1459^{+70}_{-95} - i174^{+110}_{-100}$	$0.09^{+0.03}_{-0.07}$	$0.02^{+0.12}_{-0.02}$	$0.12^{+0.22}_{-0.09}$		$0.23^{+0.37}_{-0.18}$	$0.77^{+0.37}_{-0.18}$
		$X^R_{\rho\pi}$				X^R	Z^R
$a_1(1260)$ [18]	$1260 - i250$	0.46				0.46	0.54
Hyperon with $I = 0$		$X^R_{\pi\Sigma}$	$X^R_{\bar{K}N}$			X^R	Z^R
$\Lambda(1405)$ broad [19]	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$0.73^{+0.15}_{-0.10}$	$\cdots$			$0.73^{+0.15}_{-0.10}$	$0.27^{+0.15}_{-0.10}$
$\Lambda(1405)$ narrow [19]	$1421^{+3}_{-2} - i19^{+8}_{-5}$	$0.18^{+0.13}_{-0.08}$	$0.82^{+0.36}_{-0.17}$			$1.00^{+0.49}_{-0.25}$	$0.00^{+0.49}_{-0.25}$
		X^R_{DK}	$X^R_{D_s\eta}$	$X^R_{D_s\eta'}$		X^R	Z^R
$D_{s0}^*(2317)$ [20]	2321^{+6}_{-3}	$0.56^{+0.05}_{-0.03}$	$0.12^{+0.01}_{-0.01}$	$0.02^{+0.01}_{-0.01}$		$0.70^{+0.07}_{-0.05}$	$0.30^{+0.07}_{-0.05}$
		$X^R_{J/\psi f_0(500)}$	$X^R_{J/\psi f_0(980)}$	$X^R_{Z_c(3900)\pi}$	$X^R_{\omega\chi_{c0}}$	X^R	Z^R
$Y(4260)$ [21,22]	$4232.8 - i36.3$	0.00	0.02	0.02	0.17	0.21	0.79

Effective range expansion

Effective-range-expansion (ERE)

$$V(k) = -\frac{1}{a} + \frac{1}{2} r k^2$$

a : scattering length
r : effective range

$$k = \sqrt{2\mu(E - M_{\rm th})}$$

$$\mu = m_1 m_2/(m_1 + m_2)$$

$$M_{\rm th} = m_1 + m_2$$

$$T(k) = \frac{1}{V(k) - ik}$$

- **Crossed-channel cuts neglected**
- **Convergent radius of ERE: dictated by the nearest singularity from the crossed channel**
- **Invalid if there is a near-threshold CDD pole !**

$$\delta a = -\frac{M_{\rm th} - M_{\rm i,CDD}}{g_i} \qquad \delta r = -\frac{g_i}{\mu(M_{\rm th} - M_{\rm i,CDD})^2}$$

[Guo, Oller, PRD '16]

Single-channel case

$$t(E) = \frac{1}{-\frac{1}{a} + \frac{1}{2} r k^2 - i k} \qquad t_{II}(E) = \frac{1}{-\frac{1}{a} + \frac{1}{2} r k^2 + i k}$$

$$k = \sqrt{2\mu(E - M_{\text{th}})}$$

Determine *a* and *r* using the mass and width of resonance *R*

$$E_R = M_R - i\frac{\Gamma_R}{2}, \quad k_R = \sqrt{2\mu(E_R - M_{\text{th}})}, \quad k_R = k_r + i k_i, \quad k_i > 0.$$

Partial wave amplitude in the 2nd Riemann Sheet :

$$0 = -\frac{1}{a} + \frac{1}{2} r k_R^2 + i k_R$$

$$a = -\frac{2k_i}{|k_R|^2}, \quad r = -\frac{1}{k_i}$$

Once *a* and *r* are determined, the partial-wave amplitude is completely fixed.

Residue in the variable of three-momentum *k*

[Kang, Guo, Oller, PRD '16]

Expand the denorminator in $k - k_R$:

$$t_{II}(k) = \frac{1}{(rk_R + i)(k - k_R)} + \ldots = \frac{-k_i/k_r}{k - k_R} + \ldots$$

$$\gamma_k^2 = -\frac{k_i}{k_r} > 0$$

(Residue also fixed by the pole position)

Remember: $k_R = k_r + ik_i, \qquad k_i > 0.$

Residue in the variable of CM energy *E*:

$$t_{II}(E) \xrightarrow[{[E \to E_R]}]{} -\frac{\gamma^2}{s - E_R^2} \qquad \gamma_k^2 = -\gamma^2 \frac{dk}{ds}\bigg|_{k_R} = -\frac{\mu\gamma^2}{2E_R k_R}$$

Compositeness for a resonance within ERE

$$X = \left|\gamma^2 \frac{dG(E_R)}{ds}\right| = \left|\gamma^2 \frac{dk}{ds}\frac{dG(E_R)}{dk}\right|^2 = |\gamma_k|^2 = \left(\frac{2r}{a} - 1\right)^{-\frac{1}{2}}$$

Zcs(3985)、Zc(3900) 、X(4020)

Elastic ERE Results for the Zcs(3985), Zc(3900) & X(4020)

[ZHG, Oller, PRD '21Mar]

Inputs

Tetraquark Resonance	Mass (MeV)	Width (MeV)
$Z_c(3900)$	3888.4 ± 2.5	28.3 ± 2.5
$X(4020)$	4024.1 ± 1.9	13 ± 5
$Z_{cs}(3985)$	3982.5 ± 3.3	12.8 ± 6.1

Outputs from ERE and compositeness relation

Tetraquark Resonance	Threshold (MeV)	a (fm)	r (fm)	X
$Z_c(3900)$	$\bar{D}D^*$ (3875.5)	-0.84 ± 0.13	-2.52 ± 0.25	0.45 ± 0.06
$X(4020)$	$\bar{D}^*D^*$ (4017.1)	-1.04 ± 0.30	-3.90 ± 1.35	0.39 ± 0.14
$Z_{cs}(3985)$	$D_s^- D^{*0}$ (3975.2)	-1.00 ± 0.47	-4.04 ± 1.82	0.38 ± 0.18
	$D_s^{*-} D^0$ (3977.0)	-1.28 ± 0.60	-3.65 ± 1.60	0.46 ± 0.19

- **Single-channel scattering is assumed for each state. X is only calculated when the working condition is satisfied.**
- **Coupled-channel analysis including J/ψ-π (competition between its coupling strength and phase space) is in order.**

Coupled-channel study: saturation of X & width

Channels included : J/ψ-π (1),   DD* (2)

$$X = X_1 + X_2 \equiv |g_1|^2 \left| \frac{\partial G_1^{\rm II}(s_R)}{\partial s} \right| + |g_2|^2 \left| \frac{\partial G_2^{\rm II}(s_R)}{\partial s} \right|$$

$$\Gamma_R = \Gamma_1 + \Gamma_2 = |g_1|^2 \frac{q_1(M_R^2)}{8\pi M_R^2} + |g_2|^2 \int_{m_{\rm th}}^{M_R + n\Gamma_R} dE \, \frac{q_2(E^2)}{16\pi^2 E^2} \frac{\Gamma_R}{(M_R - E)^2 + \frac{\Gamma_R^2}{4}}$$

- **M_R and Γ_R are known from Exp.**
- **Once g_1 and g_2 are obtained, partial decay widths and partial compositeness coefficients can then be predicted.**
- **Addtional input to get g_1 and g_2: X, branching ratio, fits to data ... ...**

Coupled-channel results for the Zc(3900)

Inputs $\Gamma_{D\bar{D}^*}/\Gamma_{J/\psi\pi} = 6.2 \pm 2.9$ [BESIII, '15PRD]

Outputs

$$|g_1| = 1.46^{+0.43}_{-0.23}, \qquad |g_2| = 7.89^{+0.18}_{-0.44}$$

$$X_1 = 0.002 \pm 0.001, \qquad X_2 = 0.436^{+0.021}_{-0.047},$$

$$X = X_1 + X_2 = 0.438^{+0.021}_{-0.047}.$$

Nice agreement with single-channel ERE $X = 0.45 \pm 0.06$

- **This inspires us to use the X from single-channel ERE for Zcs(3985) and X(4020) to solve the coupled-channel equations !**

Coupled-channel results for the Zcs(3985) & X(4020)

Channels included

[ZHG, Oller, PRD '21Mar]

X(4020): h_c-π (1), D^*D^* (2), Zcs(3985): J/ψ-K (1), D_sD^* (2)

Resonance	$\|g_1\|$ (GeV)	$\|g_2\|$ (GeV)	Γ_1 (MeV)	Γ_2 (MeV)	$X_1 \times 10^3$	X_2
$X(4020)$						
$X_{\rm ERE} = 0.39 \pm 0.14$	1.1 ± 0.2	6.5 ± 1.3	1.4 ± 0.5	11.6 ± 4.5	1 ± 1	0.39 ± 0.14
$Z_{cs}(3985)$						
Threshold $(D_s^- D^{*0})$						
$X_{\rm ERE} = 0.38 \pm 0.18$	0.8 ± 0.2	6.4 ± 1.7	1.2 ± 0.6	11.6 ± 5.3	0.8 ± 0.4	0.38 ± 0.18
Threshold $(D_s^{*-} D^0)$						
$X_{\rm ERE} = 0.46 \pm 0.19$	0.9 ± 0.2	6.8 ± 1.7	1.2 ± 0.6	11.6 ± 5.6	0.8 ± 0.4	0.46 ± 0.19

- **Updated analysis by using the LHCb reslults is ongoing! Stay tuned !**

X(6900) and prediction of a new state: X(6825)

Dynamical coupled-channel study of X(6900): 0^{++} is assumed

Channels included : J/ψ-J/ψ (1), χ_{c0}-χ_{c0} (2), χ_{c1}-χ_{c1} (3)

Scattering amplitude

$$\mathcal{T}(s) = [1 - \mathcal{V}(s)\cdot G(s)]^{-1}\cdot \mathcal{V}(s)$$

[ZHG, Oller, PRD '21Feb]

$$\mathcal{V}(s) = \begin{pmatrix} 0 & b_{12} & b_{13} \\ b_{12} & \frac{b_{22}}{M_{J/\psi}^2}(s - M_{CDD}^2) & \frac{b_{23}}{M_{J/\psi}^2}(s - M_{CDD}^2) \\ b_{13} & \frac{b_{23}}{M_{J/\psi}^2}(s - M_{CDD}^2) & \frac{b_{33}}{M_{J/\psi}^2}(s - M_{CDD}^2) \end{pmatrix}$$

Production amplitudes

$$B(s) = [1 - \mathcal{V}(s)\cdot G(s)]^{-1}\cdot \mathcal{P} \qquad \mathcal{P} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

J/ψ-J/ψ distribution

$$\frac{d\mathcal{N}(s)}{d\sqrt{s}} = |B_1(s)|^2 \frac{q_{J/\psi J/\psi}(s)}{M_{J/\psi}^2}$$

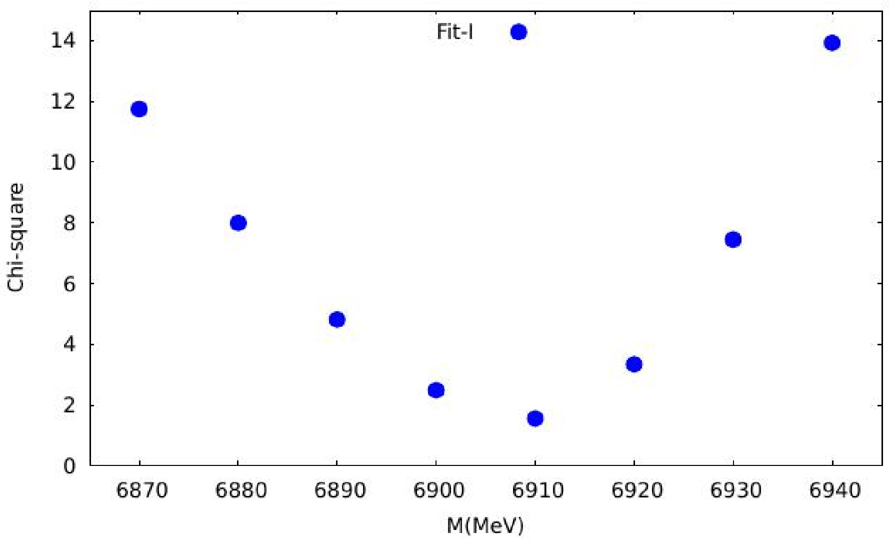
Heavy-quark symmetry

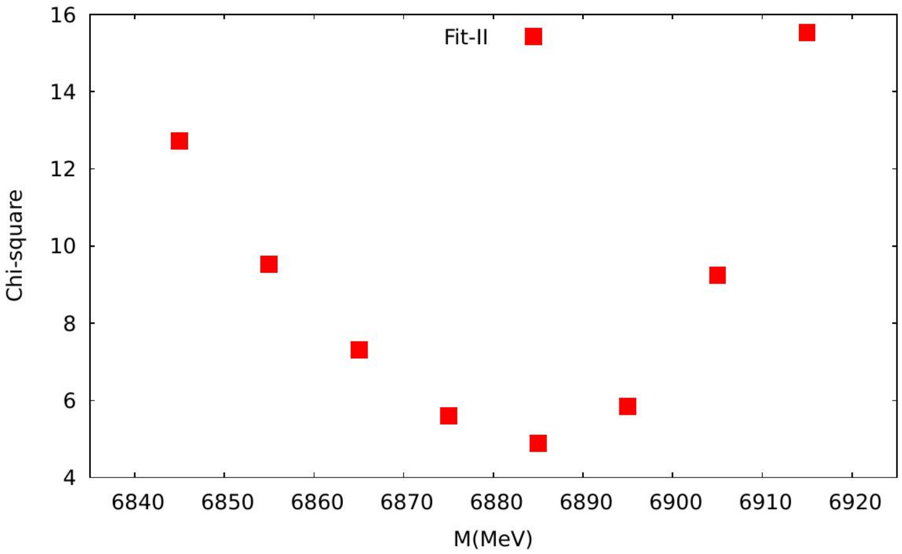
$$b_{13} = \frac{b_{12}}{\sqrt{3}}, \qquad b_{23} = \frac{b_{22}}{\sqrt{3}}, \qquad b_{33} = \frac{b_{22}}{3} \qquad d_3 = d_2/\sqrt{3}$$

Fit results

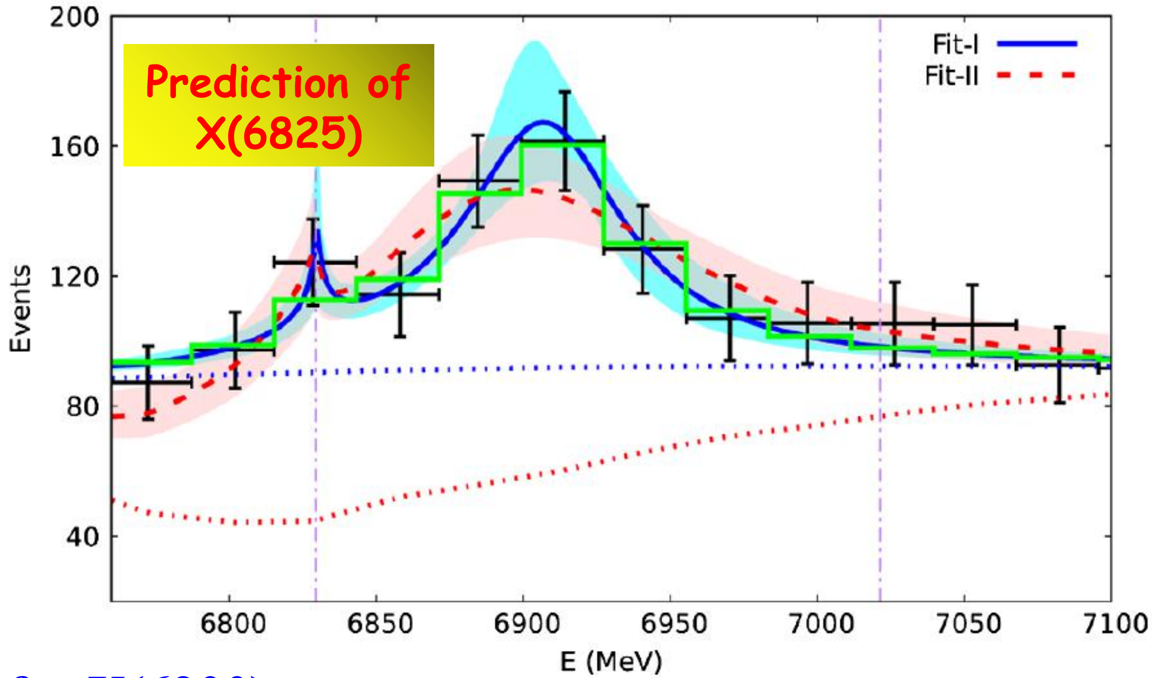
	$\chi^2/\text{d.o.f}$	$a(\mu)$	M_{CDD}	b_{22}	b_{12}	d_2
Fit I	$1.6/(12-3)$	-3.0^*	6910^*	10817^{+8378}_{-2096}	151^{+153}_{-99}	2213^{+2106}_{-316}
Fit II	$4.9/(12-3)$	-3.0^*	6885^*	21085^{+15141}_{-7359}	484^{+239}_{-112}	3646^{+1325}_{-714}

$$a(\mu) = -2\log\left(1+\sqrt{1+\frac{m^2}{q_{\max}^2}}\right) + \cdots \simeq -3.0$$





Fit results



[ZHG, Oller, PRD '21Feb]

Resonance poles for X(6900)

	Mass (MeV)	Width/2 (MeV)	$\lvert\gamma_1\rvert$ (GeV)	$\lvert\gamma_2\rvert$ (GeV)	$\lvert\gamma_3\rvert$ (GeV)	X_1	X_2	X_3	$X=\sum_{i=1}^3 X_i$
Fit I	6907^{+5}_{-3}	33^{+14}_{-10}	$4.6^{+2.5}_{-2.8}$	$9.7^{+1.4}_{-2.6}$	$5.6^{+0.8}_{-1.5}$	$0.01^{+0.01}_{-0.01}$	$0.13^{+0.04}_{-0.06}$	$0.03^{+0.01}_{-0.01}$	$0.17^{+0.04}_{-0.07}$
Fit II	6892^{+2}_{-2}	80^{+24}_{-17}	$10.3^{+1.8}_{-1.4}$	$6.9^{+1.4}_{-1.9}$	$4.0^{+0.8}_{-1.1}$	$0.05^{+0.02}_{-0.01}$	$0.06^{+0.03}_{-0.03}$	$0.01^{+0.01}_{-0.01}$	$0.13^{+0.03}_{-0.03}$

Resonance poles for X(6825) [virtual pole of χ_{c0}-χ_{c0} (6829.4)]

Values for fit I:

$E'_R = 6827.0^{+1.6}_{-4.8} - i1.1^{+1.3}_{-1.0}$, $|\gamma'_1| = 1.4^{+0.6}_{-0.9}$,

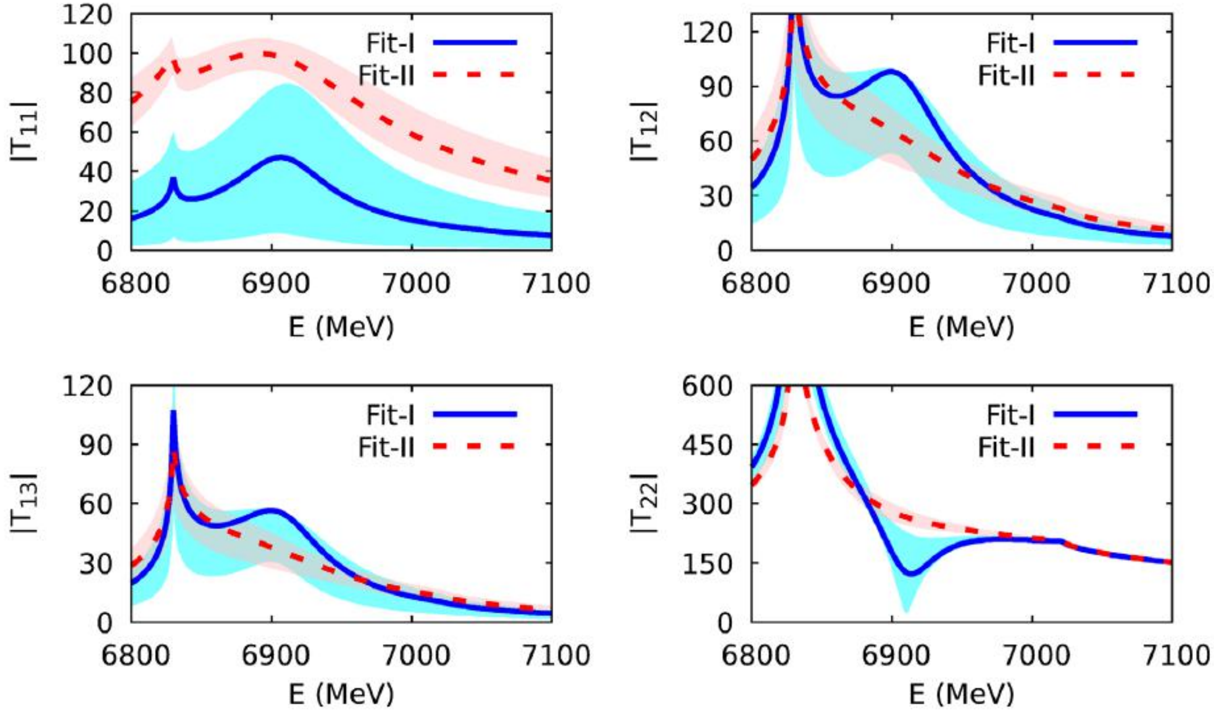
$|\gamma'_2| = 11.9^{+3.2}_{-3.1}$, $|\gamma'_3| = 6.8^{+1.8}_{-1.8}$,

Values for fit II:

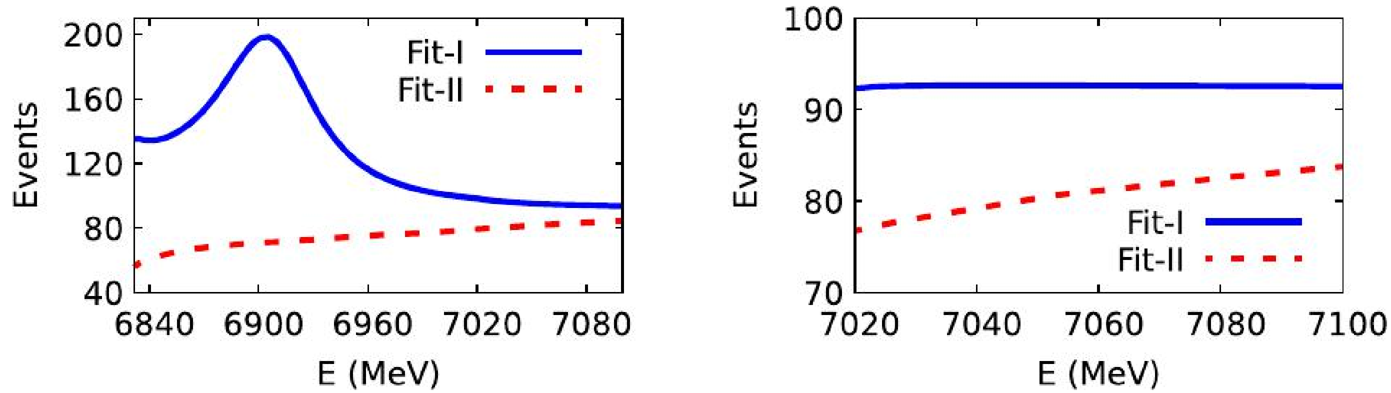
$E'_R = 6820.6^{+3.0}_{-2.7} - i4.0^{+1.7}_{-1.6}$, $|\gamma'_1| = 2.5^{+0.5}_{-0.6}$,

$|\gamma'_2| = 15.8^{+0.7}_{-0.6}$, $|\gamma'_3| = 9.1^{+0.4}_{-0.4}$,

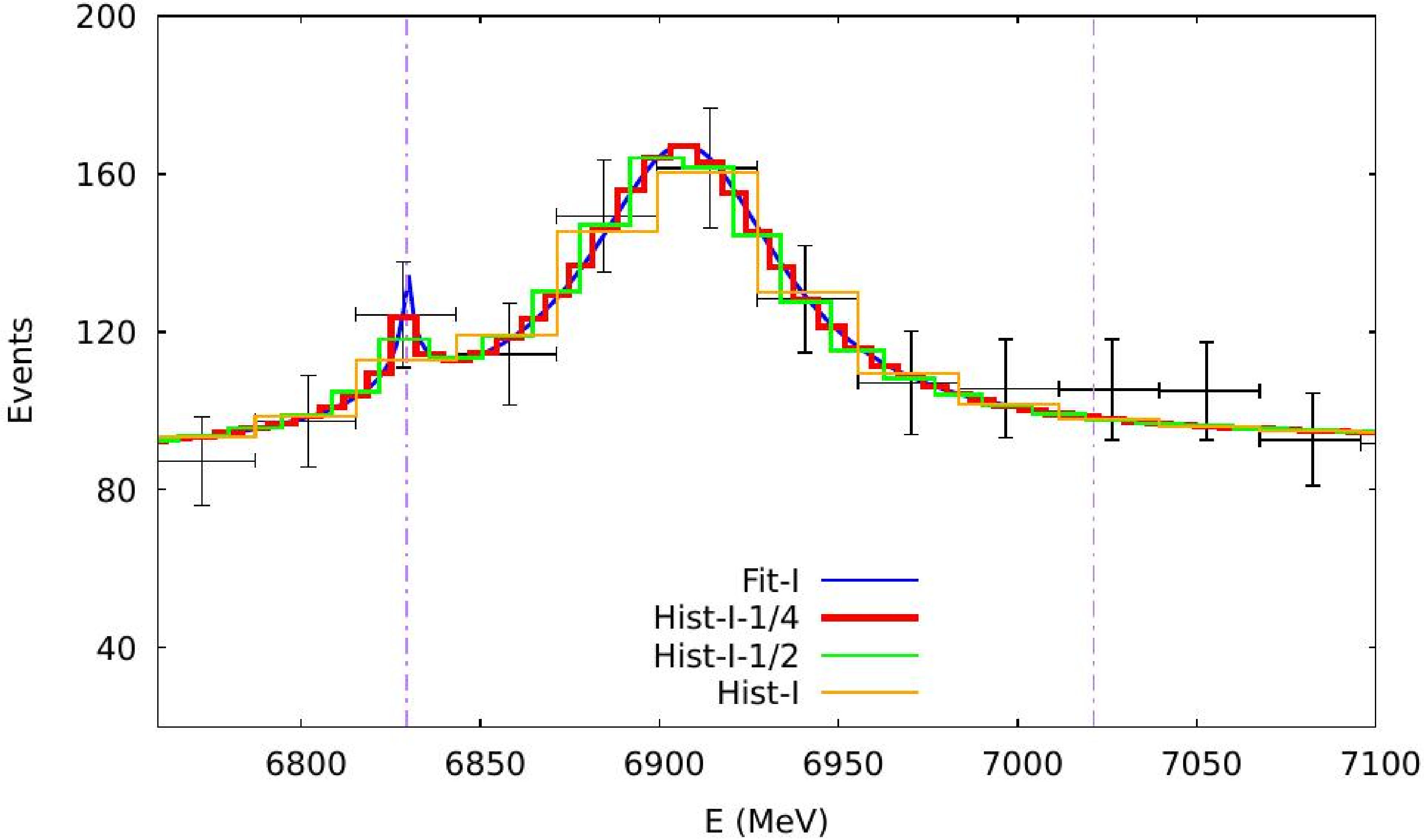
Resonance shapes in the amplitudes



Predictions of event distributions for χ_{c0}-χ_{c0} and χ_{c1}-χ_{c1}



Scrutinizing the X(6825) signal



- **Results to include J/ψ-ψ(3770) are also discussed.**

[ZHG, Oller, PRD '21Feb]

Summary

- **Combination of effective-range expansion and Weinberg's compositeness relations provides a powerful tool to analyze the near-threshold resonances.**

- **Zc(3900)、Zcs(3985) and X(4020) can be simultaneously described in our case and share similar properties.**

- **X(6900) is demonsrtated to behave like a elementary state and the two-meson components [J/ψ-J/ψ, χ_{c0}-χ_{c0}, χ_{c1}-χ_{c1}, J/ψ-ψ(3770)] are subdominant.**

- **The promising X(6825) state, just below χ_{c0}-χ_{c0} threshold, is predicted, which deserves further verification both in experiment and theory.**

Thanks very much for your attention!

Backup slides

Blind use of elastic ERE for X(6900)

Inputs

$$\text{Model I}: M = 6905 \pm 11 \pm 7 \text{ MeV},$$
$$\Gamma = 80 \pm 19 \pm 33 \text{ MeV},$$
$$\text{Model II}: M = 6886 \pm 11 \pm 11 \text{ MeV},$$
$$\Gamma = 168 \pm 33 \pm 69 \text{ MeV},$$

[LHCb]

Near-threshold channels tested

$\chi_{c0}\chi_{c0}$, $\chi_{c1}\chi_{c1}$

Outputs

Resonance	Threshold (MeV)	a (fm)	r (fm)	X
$X(6900)$ I	$\chi_{c0}\chi_{c0}$ (6829.4)	-0.18 ± 0.07	-1.52 ± 0.69	0.25 ± 0.11
$X(6900)$ II	$\chi_{c0}\chi_{c0}$ (6829.4)	-0.32 ± 0.06	-0.72 ± 0.26	0.53 ± 0.16

χ_{c1}-χ_{c1} (7021.3)

$$a = -0.59 \pm 0.04, \qquad r = -0.31 \pm 0.02 \text{ (case I)},$$
$$a = -0.51 \pm 0.05, \qquad r = -0.28 \pm 0.02 \text{ (case II)},$$

- **Caveats: It is probably unreliable to rely on elastic ERE to address X(6900). Coupled-channel study at least with J/ψ-J/ψ is needed !**

Coupled-channel study: saturation of X & width

Channels included : J/ψ-J/ψ (1), χ_{c0}-χ_{c0} (2), χ_{c1}-χ_{c1} (3)

Heavy-quark symmetry: $\frac{g_2}{g_3} = \sqrt{3}$, **[0^{++} is assumed for X(6900)]**

$$X = X_1 + X_2 + X_3$$

$$= |g_a|^2 \left| \frac{dG_1^{\mathrm{II}}(s_R)}{ds} \right| + |g_b|^2 \left| \frac{dG_2^{\mathrm{II}}(s_R)}{ds} \right| + \frac{|g_b|^2}{3} \left| \frac{dG_3(s_R)}{ds} \right|$$

$$\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3$$

$$= |g_a|^2 \frac{q_1(M_R^2)}{8\pi M_R^2} + |g_b|^2 \int_{m_{\mathrm{th},2}}^{M_R+2\Gamma_R} dw \frac{q_2(w^2)}{16\pi^2 w^2} \frac{\Gamma_R}{(M_R - w)^2 + \Gamma_R^2/4} + \frac{|g_b|^2}{3} \int_{m_{\mathrm{th},3}}^{M_R+2\Gamma_R} dw \frac{q_3(w^2)}{16\pi^2 w^2} \frac{\Gamma_R}{(M_R - w)^2 + \Gamma_R^2/4}$$

Heavy-quark symm. & C-G coefficient

- ***X* is needed to solve the two equations!**

Arbitrary X for testing purposes

[ZHG, Oller, PRD '21Feb]

Channel	$\lvert g_a \rvert$ (GeV)	$\lvert g_b \rvert$ (GeV)	Γ_1 (MeV) J/ψ-J/ψ	Γ_2 (MeV) χ_{c0}-χ_{c0}	Γ_3 (MeV) χ_{c1}-χ_{c1}	X_1	X_2	X_3
$X(6900)$ I								
$X = 0.1$	$7.1^{+2.2}_{-2.1}$	$6.8^{+0.6}_{-0.8}$	$64.7^{+42.5}_{-33.4}$	$15.2^{+5.3}_{-4.6}$	$0.1^{+0.3}_{-0.1}$	$0.02^{+0.02}_{-0.01}$	$0.06^{+0.01}_{-0.01}$	$0.01^{+0.00}_{-0.00}$
$X = 0.4$	$0.6^{+5.7}_{-0.5}$	$15.4^{+0.5}_{-0.5}$	$0.4^{+49.7}_{-0.4}$	$79.2^{+11.1}_{-12.5}$	$0.4^{+1.8}_{-0.4}$	$0.00^{+0.02}_{-0.01}$	$0.33^{+0.01}_{-0.01}$	$0.07^{+0.01}_{-0.01}$
$X(6900)$ II								
$X = 0.1$	$11.3^{+2.7}_{-3.5}$	$5.0^{+1.6}_{-3.1}$	$160.7^{+83.3}_{-83.6}$	$7.0^{+6.8}_{-6.1}$	$0.3^{+0.1}_{-0.3}$	$0.06^{+0.03}_{-0.03}$	$0.03^{+0.03}_{-0.03}$	$0.01^{+0.01}_{-0.01}$
$X = 0.4$	$8.9^{+3.0}_{-5.5}$	$15.0^{+0.5}_{-1.0}$	$100.9^{+79.0}_{-86.2}$	$64.3^{+13.7}_{-16.9}$	$2.8^{+3.7}_{-2.6}$	$0.04^{+0.03}_{-0.03}$	$0.31^{+0.03}_{-0.03}$	$0.06^{+0.01}_{-0.01}$
$X = 0.9$	$1.0^{+6.6}_{-1.0}$	$23.7^{+1.5}_{-1.4}$	$1.3^{+71.9}_{-1.3}$	$159.9^{+44.4}_{-38.9}$	$6.8^{+10.1}_{-4.8}$	$0.00^{+0.03}_{-0.00}$	$0.76^{+0.02}_{-0.02}$	$0.14^{+0.01}_{-0.02}$

- **Alternatively a dynamical model respecting the two-body unitarity is constructed to fit the J/ψ-J/ψ event distributions to obtain the couplings g_a and g_b!**