# Cluster reducibility of multiquark operators and tetraquark – adequate QCD sum rules

**Dmitri Melikhov**

*SINP, M. V. Lomonosov Moscow State University & BLTP, JINR, Dubna & University of Vienna*

- **Interpolating currents for tetraquark states**
- **Four-quark singularities of Feynman diagrams, $T$-phile diagrams**
- **Adequate formulation of QCD sum rules for $T$-states**
- **Conclusions**

**Based on W.Lucha, D.M., H.Sazdjian,**

*Cluster reducibility of multiquark operators*, **PRD100, 094017, 2019**

*OPE and quark-hadron duality for two-point functions of tetraquark currents in 1/Nc expansion*, **PRD103, 014012, 2021**

*Tetraquarks in large-$N_c$ QCD*, **PPNP, 2021**

**QCD: Green functions of interpolating currents - (local) gauge-invariant operators built out of quark and gluon fields and** $\langle 0|j|H\rangle \neq 0$

## 1. INTERPOLATING CURRENTS

• *Color structure of interpolating currents*

**For hadrons, globally color-singlet currents**

**mesons:** $\bar{q}^i q^i$

**baryons:** $q^i q^j q^k \epsilon_{ijk}$

**(mesons and baryons - no colorless clusters inside the interpolating currents)**

**tetraquarks:** $(\bar{q}^i q^i)(\bar{q}^j q^j)$ **- can isloate colorless clusters (reducibility)**

$\epsilon_{ijk} q^i q^j \epsilon^{ij'k'} \bar{q}_{j'} \bar{q}_{k'}$

$(\bar{q}\lambda^A q)(\bar{q}\lambda^A q)$

**All forms for the tetraquarks are equivalent (reduced to color singlets by Fierz transforms)**

**Sufficient to consider singlet-singlet** $(\bar{q}q)(\bar{q}q)$ **color structure of interpolating current.**

• *Flavour structure*: **Consider** *flavour-exotic* **currents of four different flavours** $a, \bar{b}, c, \bar{d}$
**Colorless clusters of two different flavor structures emerge in QCD** $\theta_{\bar{a}b\bar{c}d} = j_{\bar{a}b} j_{\bar{c}d}$ **and** $\theta_{\bar{a}d\bar{c}b} = j_{\bar{a}d} j_{\bar{c}b}$.

• *Lorentz structure* **of the currents is irrelevant for our discussion and will be suppressed**

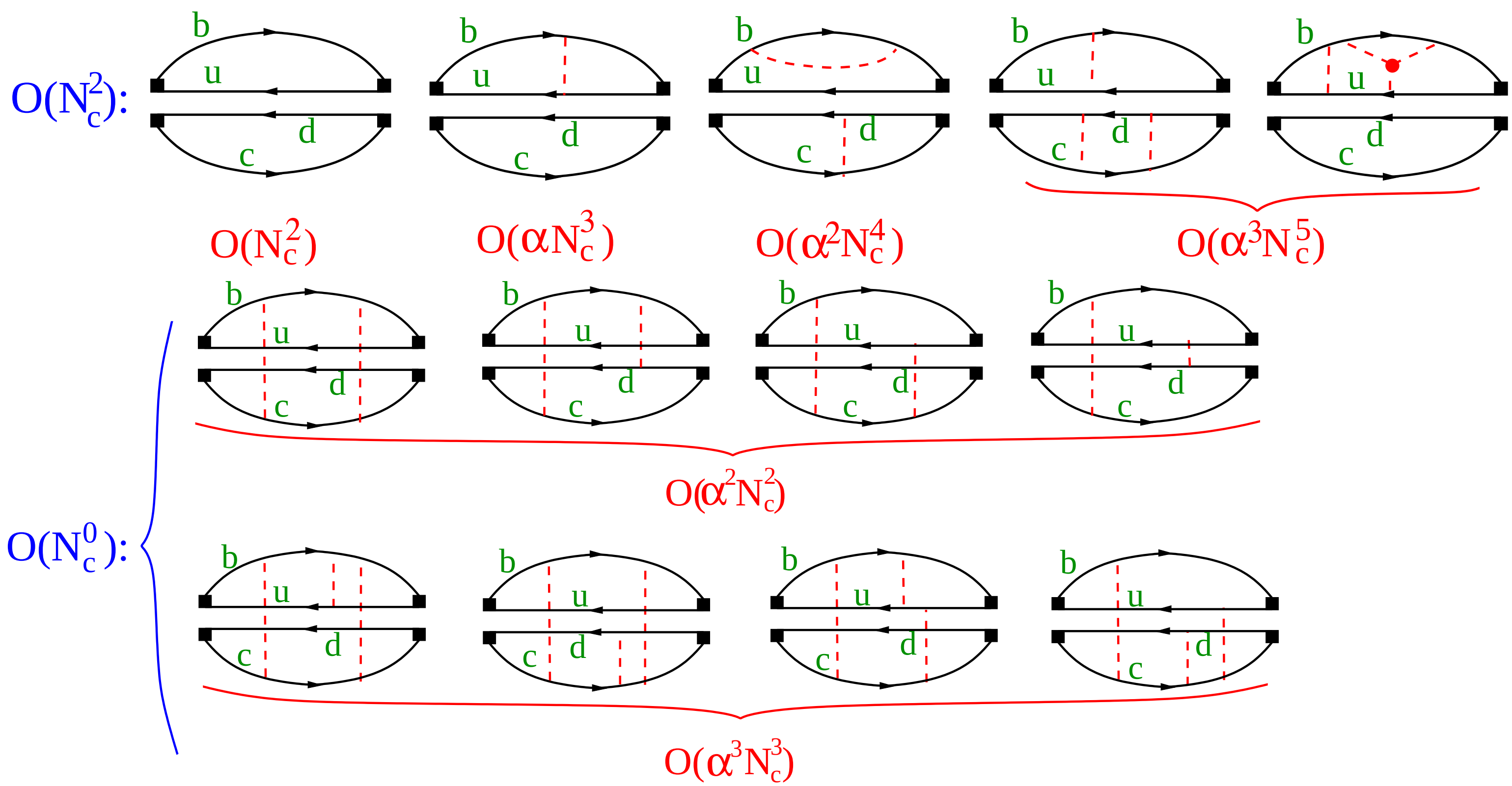
• *Singularities of QCD Feynman diagrams*: **For tetraquark bound states four-quark singularities are crucial**

## 2. GREEN FUNCTIONS OF INTERPOLATING CURRENTS

**Two different color/flavor contractions emerge:** $\theta^{(1)}_{\bar a b \bar c d} = j_{\bar a b} j_{\bar c d}$ **and** $\theta^{(2)}_{\bar a d \bar c b} = j_{\bar a d} j_{\bar c b}$**.**

$$\Pi^{\rm dir}(x) \equiv \langle T\{\theta_{\bar b u \bar c d}(x)\theta^\dagger_{\bar b u \bar c d}(0)\}\rangle = \Pi_{\bar b u}(x)\Pi_{\bar c d}(x) + \Pi^{\rm dir}_{\rm NF}(x).$$

$$\Pi_{\bar b u}(x) \equiv \langle T\{j_{\bar b u}(x) j^\dagger_{\bar b u}(0)\}\rangle = \sum_{h_{\bar b u}} R_{\bar b u}(x), \quad \Pi_{\bar c d}(x) \equiv \langle T\{j_{\bar c d}(x) j^\dagger_{\bar c d}(0)\}\rangle = \sum_{h_{\bar c d}} R_{\bar c d}(x).$$




**The factorizabe part contains only ordinary hadrons as intermediate states**
**— no relationship to tetraquarks**

# Four – point Green functions of bilinear currents and tetraquark poles

• **Study four-point Green functions of bilinear color-singlet quark currents of the form $j_{ab} = \bar{q}_a q_b$. Depend on 6 variables $p_1^2$, $p_2^2$, $p_1'^2$, $p_2'^2$, $p = p_1 + p_2 = p_1' + p_2'$, and the two Mandelstam variables $s = p^2$ and $t = (p_1 - p_1')^2$.**

**Criteria for selecting diagrams which potentially contribute to the tetraquark pole at $s = M_T^2$:**

1. **The diagram should depend on the variable $s$.**
2. **The diagram should have a four-particle cut (i.e. threshold at $s = (m_a + m_b + m_c + m_d)^2$), where $m_i$ are the masses of the quarks forming the tetraquark bound state. The presence or absence of this cut is established by solving the Landau equations for the corresponding diagram.**

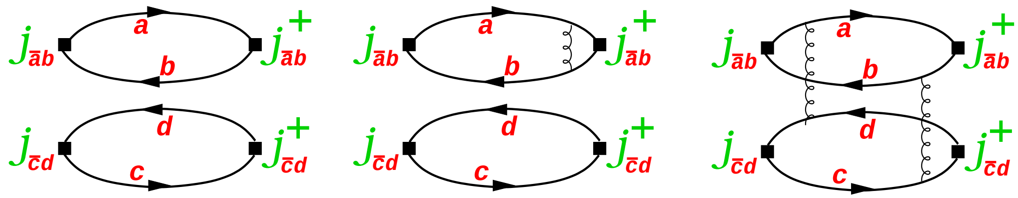
**Diagrams satisfying these criteria are "T-phile" diagrams.**

• **We consider "direct" and "recombination" Green functions:**

$$(dir) : \langle j_{ab} j_{cd} j^\dagger_{ab} j^\dagger_{cd} \rangle, \langle j_{ad} j_{cb} j^\dagger_{ad} j^\dagger_{cb} \rangle \qquad (rec) : \langle j_{ab} j_{cd} j^\dagger_{ad} j^\dagger_{cb} \rangle$$

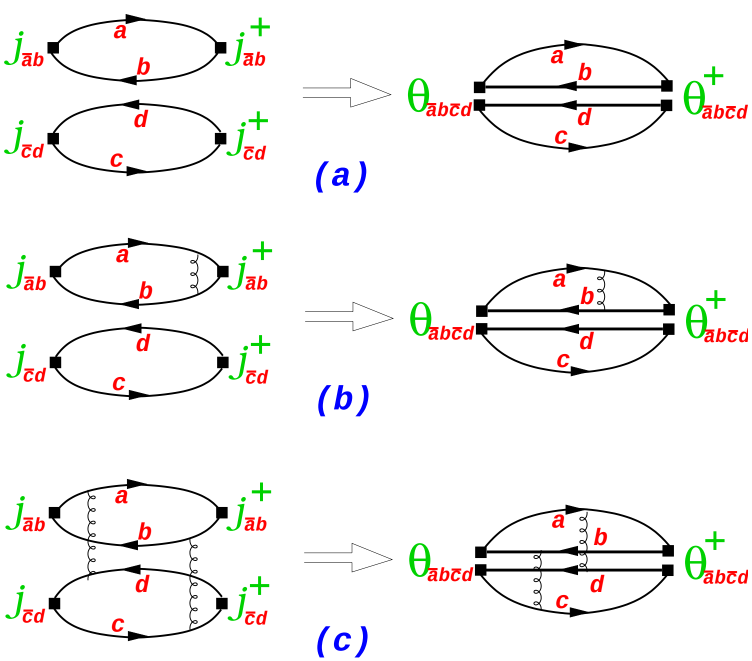
## Direct Green functions

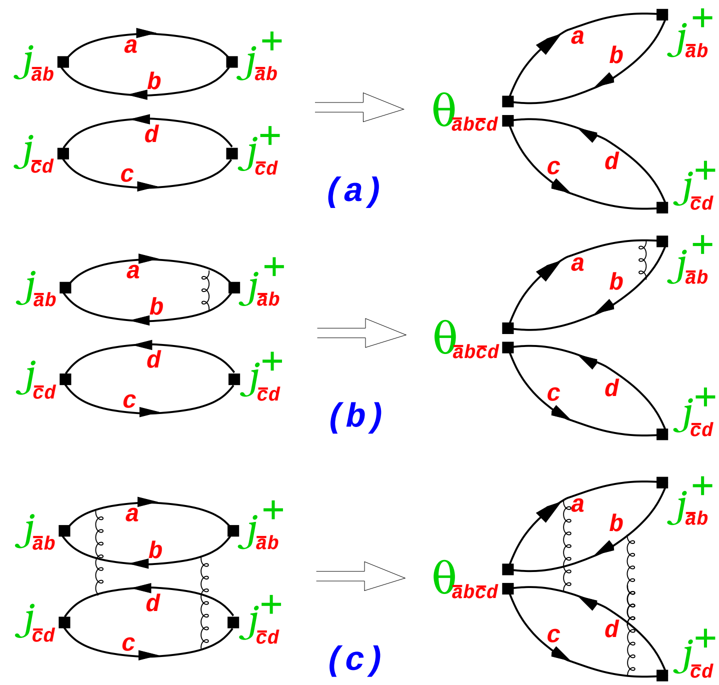
**$\langle j_{ab} j_{cd} j^\dagger_{ab} j^\dagger_{cd}\rangle$ (similar diagrams for $\langle j_{ad} j_{cb} j^\dagger_{ad} j^\dagger_{cb}\rangle$):**




(a) $\sim N_c^2$ (b) $\sim N_c^3 \alpha_s$ (c) $\sim N_c^2 \alpha_s^2$

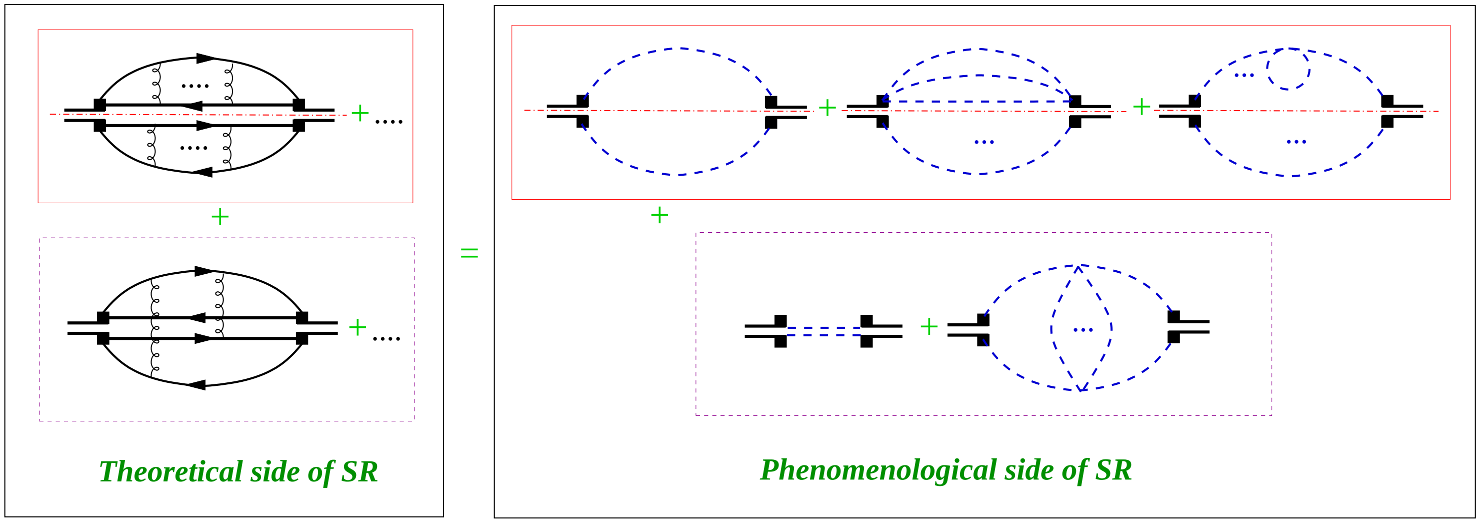
**T-phile diagrams appear at order $O(\alpha_s^2)$ and higher and behave as $O(N_c^0)$.**

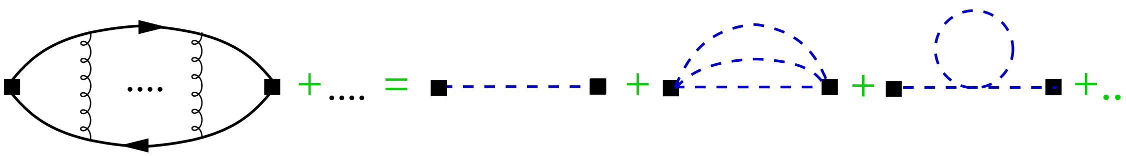


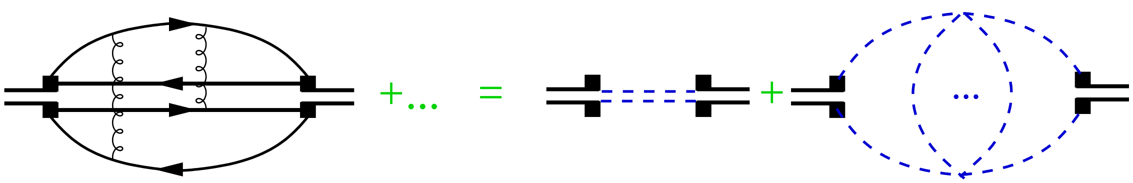



# QCD SUM RULES FOR EXOTIC STATES (DIRECT GREEN FUNCTION)

*W.L., D.M., H.S., Phys.Rev. D100, 014010 (2019)*

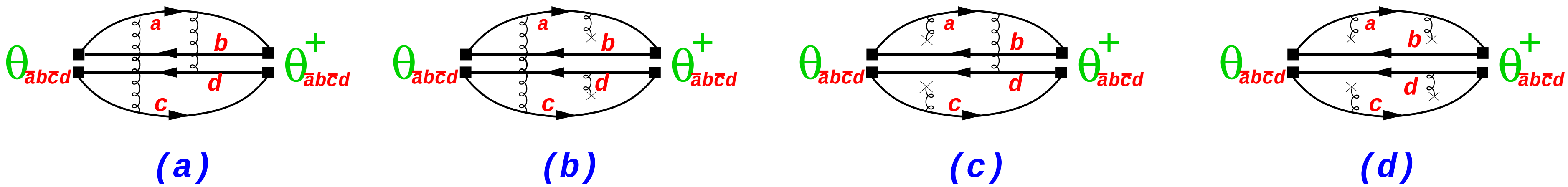


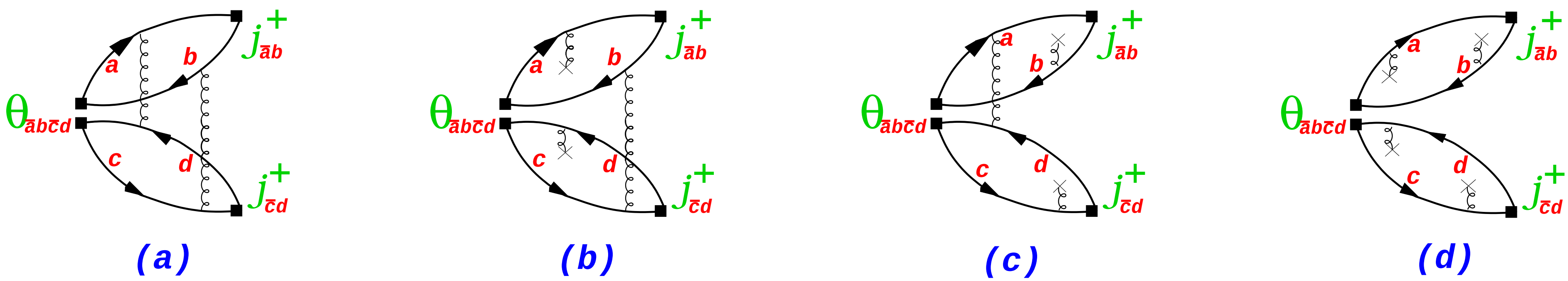


**The OPE side in the tetraquark-adequate QCD sum rule starts with diagrams of order $\alpha_s^2$.**

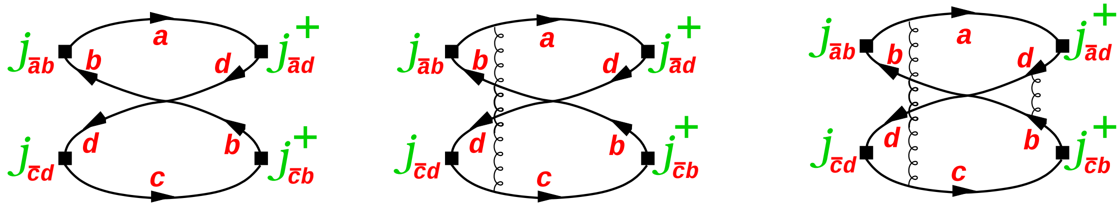
## *T*-phile diagrams on the OPE side of *T*-adequate QCD sum rule for 2-point direct Green function




## *T*-phile diagrams on the OPE side of *T*-adequate QCD sum rule for 3-point direct Green function



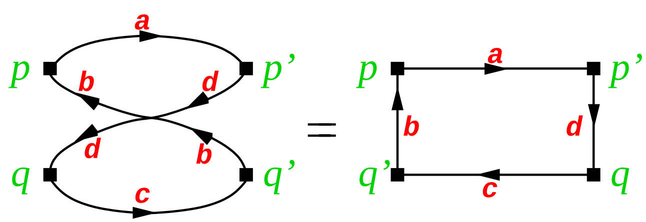
## Recombination Green functions



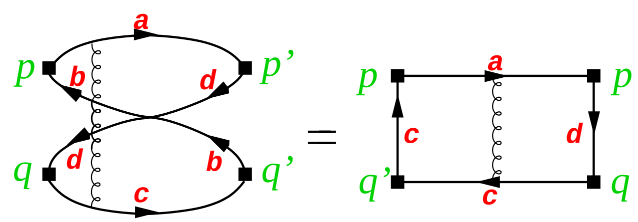

(a) ~ $N_c$ (b) ~ $N_c^2\alpha_s$ (c) ~ $N_c\,\alpha_s^2$

**Which QCD diagrams contain four-quark *s*-channel cut?**

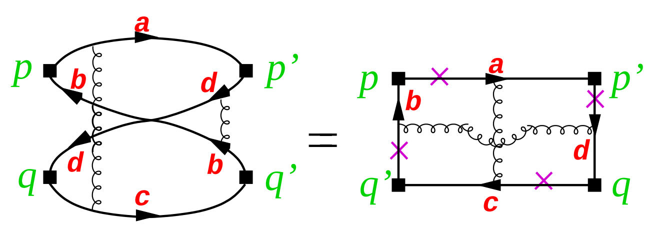
**To answer this question: (i) Unfold diagrams (ii) solve Landau equations and find *u*-channel singularities of unfolded diagrams.**




(a)

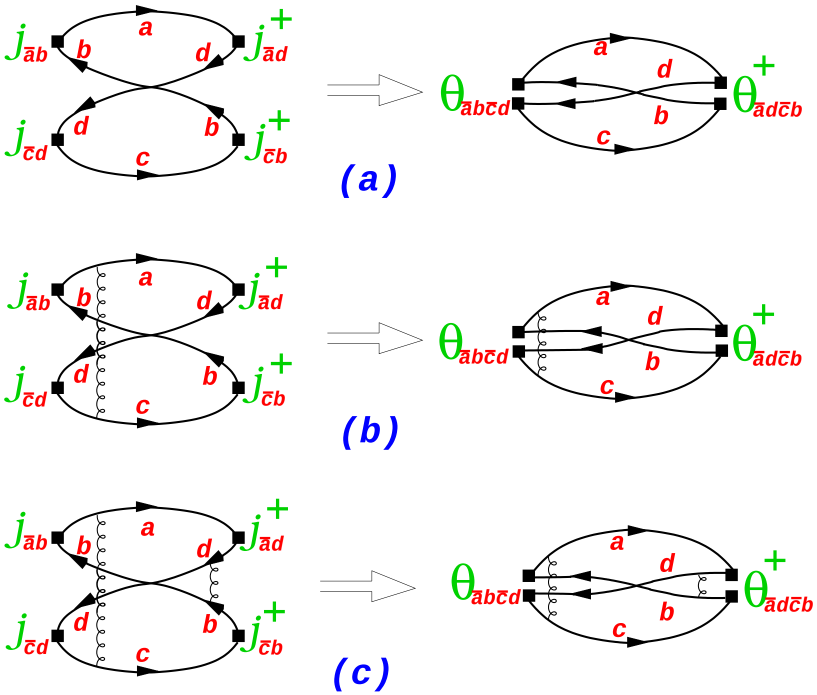


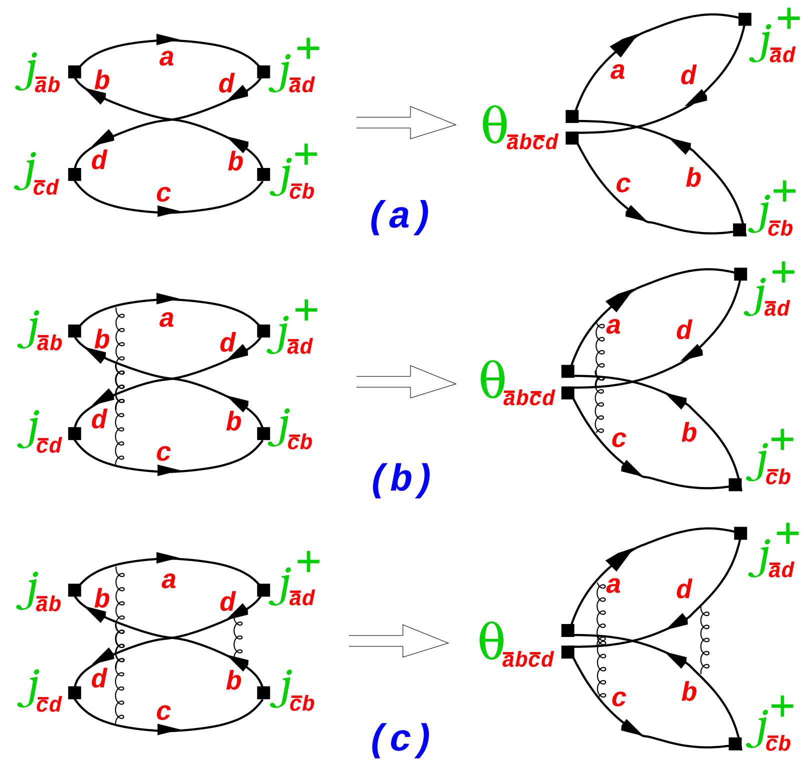

(b)




(c)

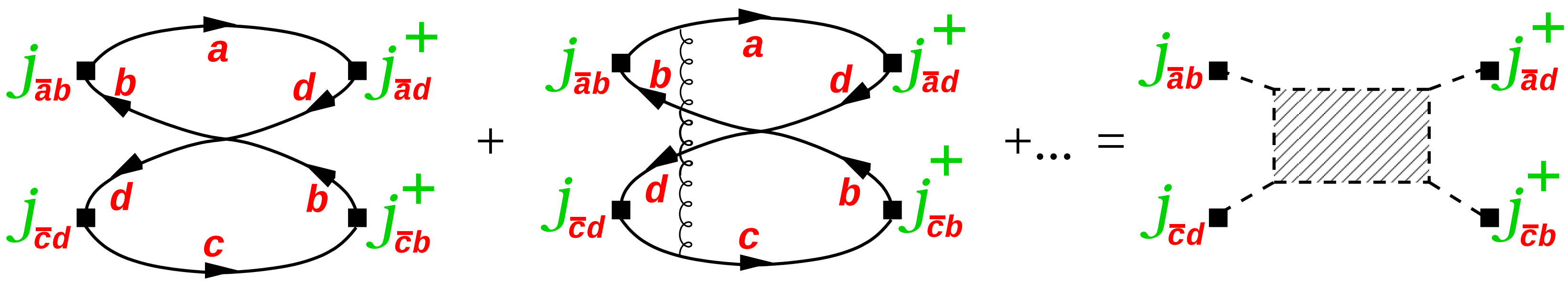
**Result: (a) and (b) do not have 4-quark cut; one needs two gluon exchanges $O(\alpha_s^2)$ (c).**


(a)
(b)
(c)

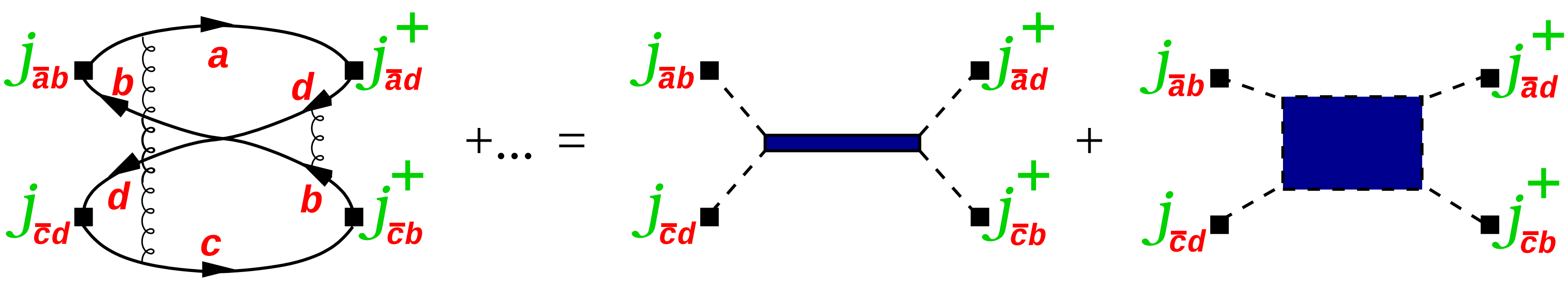

(a)
(b)
(c)

# QCD SUM RULES FOR EXOTIC STATES (RECOMBINATION GREEN FUNCTION)

*W.L., D.M, H.S., Phys.Rev. D100, 074029 (2019)*



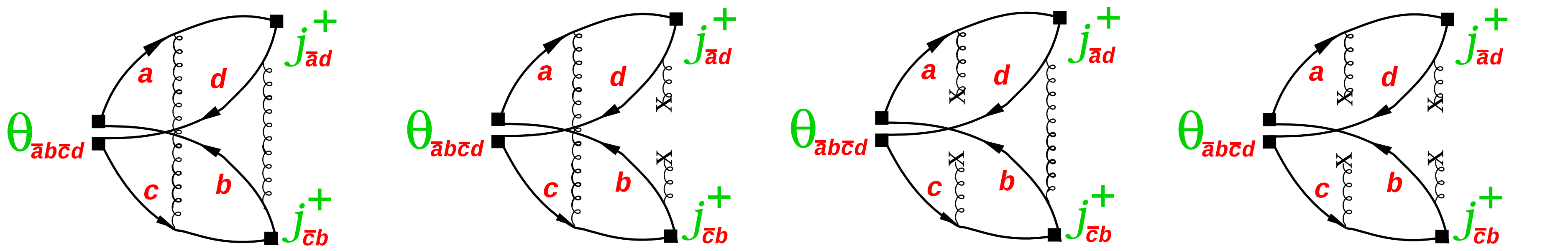

**(a)**




**(b)**

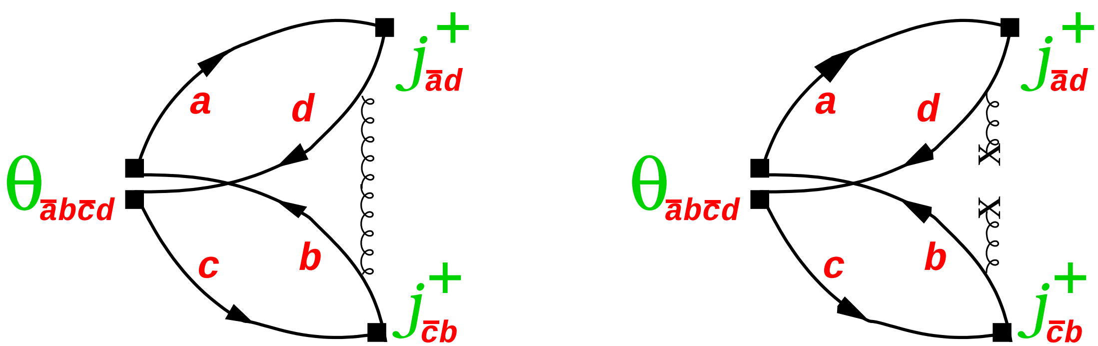
**(b) is T-adequate QCD sum rule for recombination Green functions**

**Diagrams that appear on the OPE side of the $T$-adequate QCD sum rule for recombination three-point Green function**




(a) (b) (c) (d)

**Diagrams that DO NOT appear on the OPE side of the $T$-adequate QCD sum rule for recombination three-point Green function**




(a) (b)

# Conclusions

• *Interpolating currents*

**For the analysis of exotic T it is sufficient to consider currents of a color singlet-singlet structure. Such currents describe tetraquarks of any internal structure. The choice of the color form of the interpolating current does not preselect any structure of the bound state we are studying.**

**An essential feature of any tetraquark current is that it produces from the vacuum (mainly) two-meson states and only in a relatively small fraction (at large $N_c$) an exotic tetraquark state.**

• *T-adequate QCD sum rules*

**Duality relations for correlation functions of tetraquark currents split into two non-overlapping relations. Only one of them contains the tetraquark contribution on the hadron side; its OPE side contains T-phile diagrams of order $\alpha_s^2$ and higher.**

**MANY THANKS FOR YOUR ATTENTION!**