



Hadron 2021

Is $P_{cs}(4459)$ one state or
two?

Chu-Wen Xiao

Central South University

Collaborators: Juan Nieves, Eulogio Oset,
Jia-Jun Wu, Bing-Song Zou

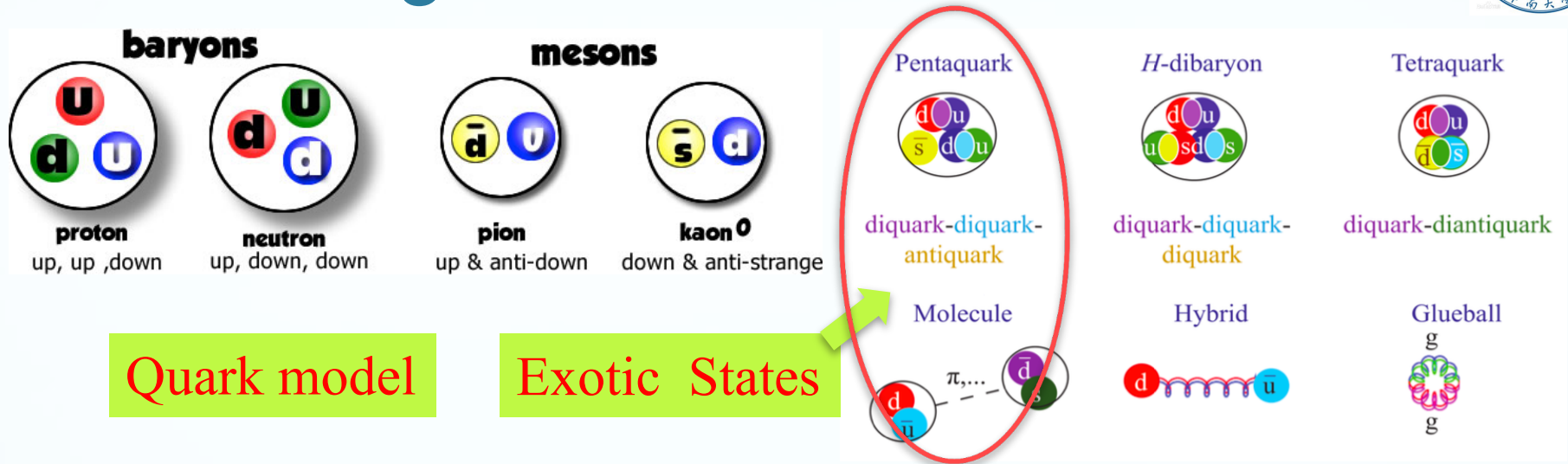
2021. Mexico



Outline

1. Introduction
2. Formalism
3. Results
4. Summary

§1. Introduction



H.-X. Chen, W. Chen, X. Liu and S.-L. Zhu, **Phys. Rept.** **639** (2016) 1.

A. Esposito, A. Pilloni and A. D. Polosa, **Phys. Rept.** **668** (2016) 1.

F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao and B.-S. Zou, **Rev. Mod. Phys.** **90** (2018) 0115004.

S. L. Olsen, T. Skwarnicki and D. Zieminska, **Rev. Mod. Phys.** **90** (2018) 0115003.

C. Z. Yuan, **Int. J. Mod. Phys. A** **33**,1830018 (2018).

Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, **Prog. Part. Nucl. Phys.** **107**, 237 (2019) .

N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, **Phys. Rept.** **873** (2020) 1.

In hidden charm strangeness sector

PRL **105**, 232001 (2010)

PHYSICAL REVIEW LETTERS

week ending
3 DECEMBER 2010

Prediction of Narrow N^* and Λ^* Resonances with Hidden Charm above 4 GeV

Jia-Jun Wu,^{1,2} R. Molina,^{2,3} E. Oset,^{2,3} and B. S. Zou^{1,3}

J. J. Wu, R. Molina, E. Oset and B. S. Zou, **Phys. Rev. Lett.** **105** (2010) 232001.

J. J. Wu, R. Molina, E. Oset and B. S. Zou, **Phys. Rev. C** **84** (2011) 015202.

H. X. Chen, L. S. Geng, W. H. Liang, E. Oset, E. Wang and J. J. Xie, **Phys. Rev. C** **93** (2016) 065203.

R. Chen, J. He and X. Liu, **Chin. Phys. C** **41** (2017) 103105.

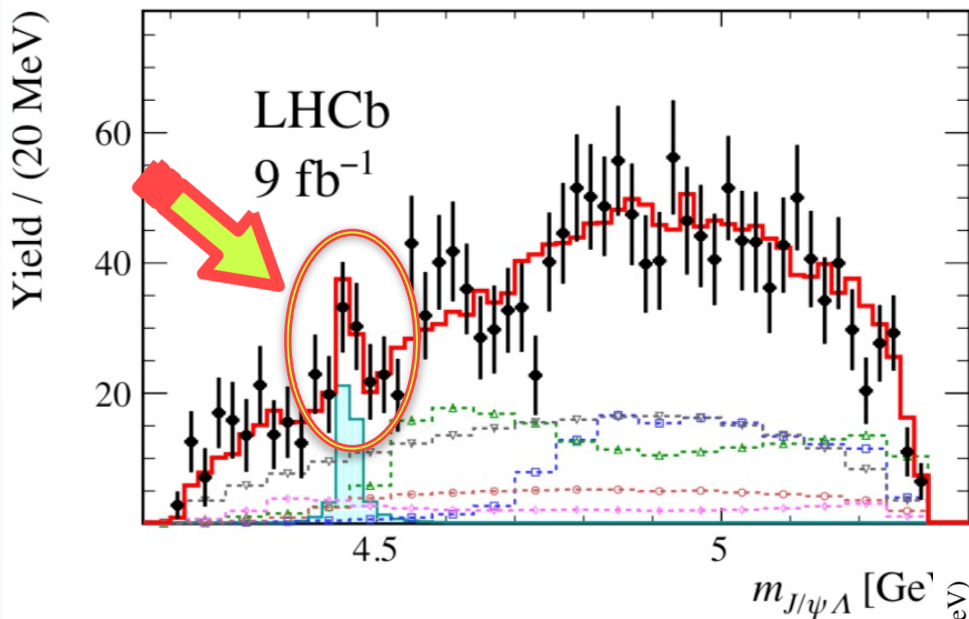
E. Santopinto and A. Giachino, **Phys. Rev. D** **96** (2017) 014014.

C. W. Shen, J. J. Wu and B. S. Zou, **Phys. Rev. D** **100** (2019) 056006.

CWX, J. Nieves and E. Oset, **Phys. Lett. B** **799** (2019) 135051.

B. Wang, L. Meng and S. L. Zhu, **Phys. Rev. D** **101** (2020) 034018.

Experimental Findings for Pcs

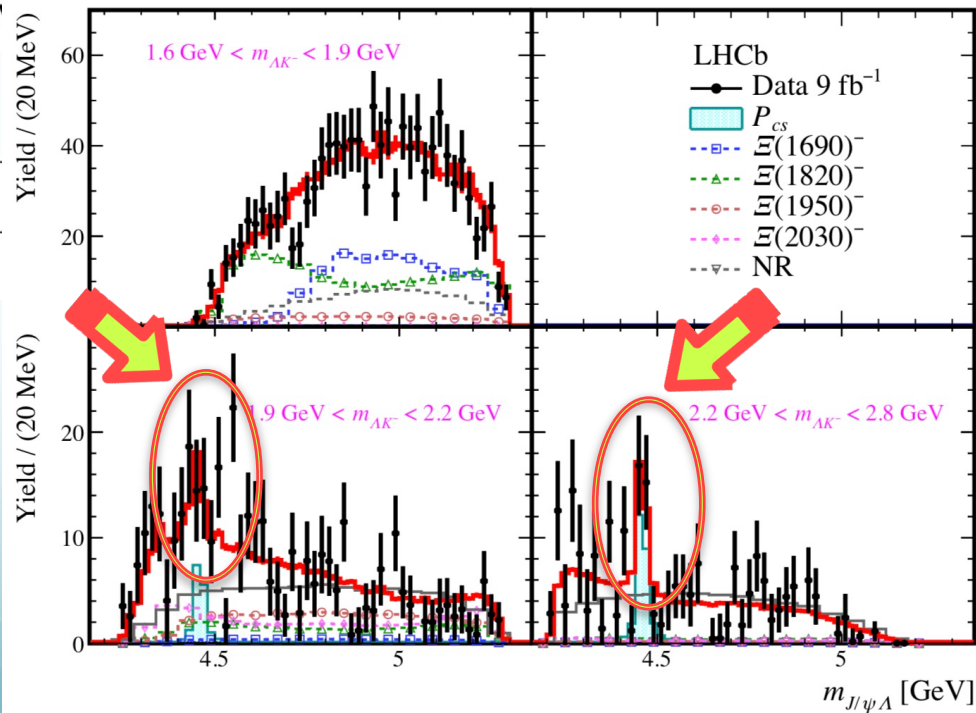


R. Aaij et al. (LHCb Collaboration),
arXiv:2012.10380

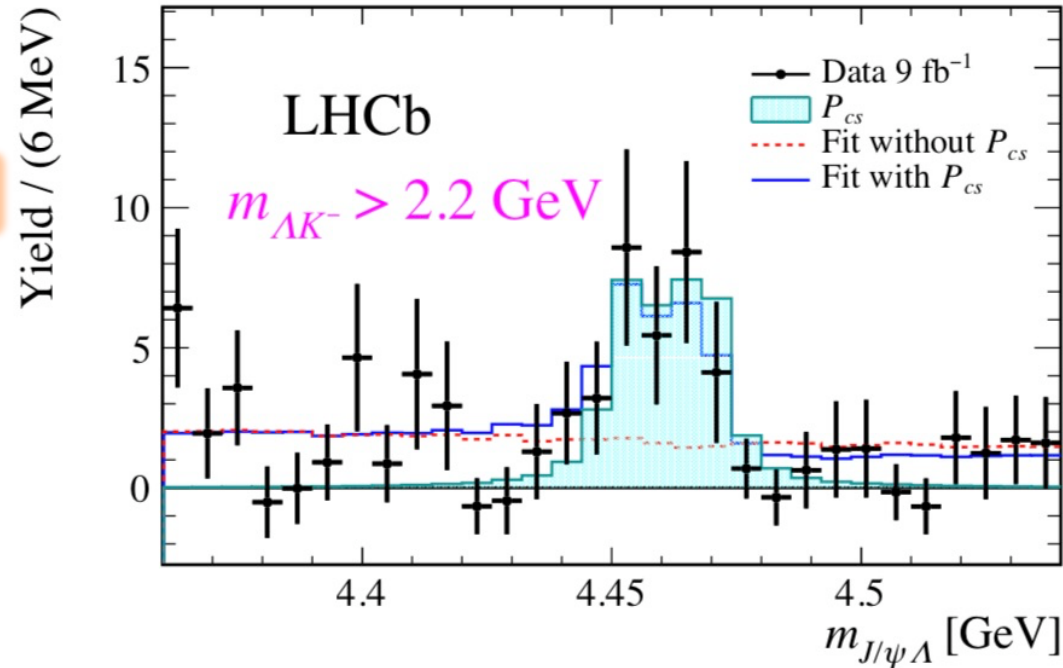
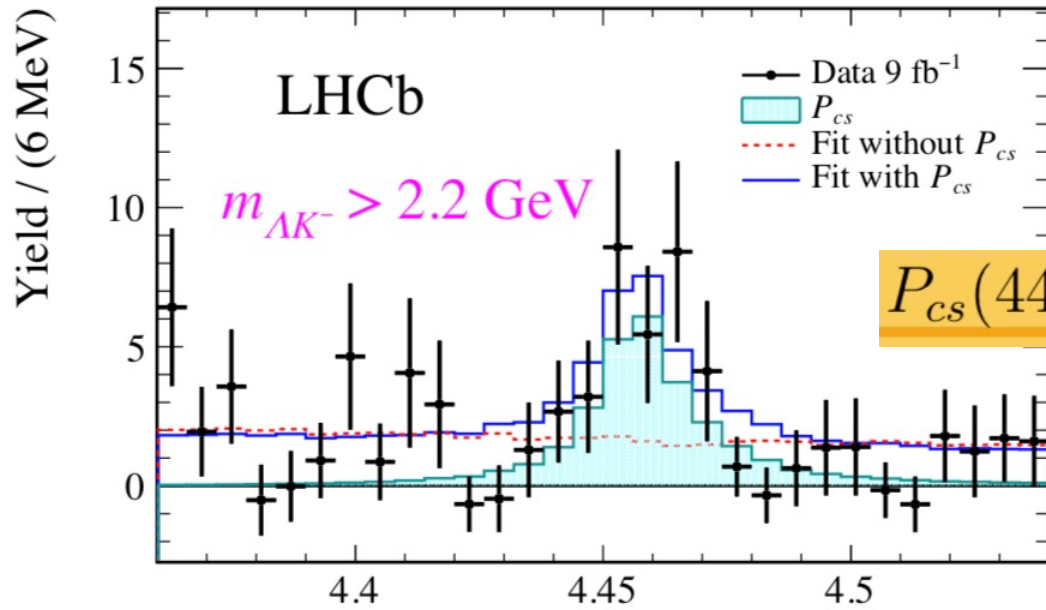
$$\Xi_b^- \rightarrow J/\psi \Lambda K^-$$

State	M_0 [MeV]	Γ_0 [MeV]
$P_{cs}(4459)^0$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$

$J^P ?$



$m_{J/\psi \Lambda}$ [GeV]



mass difference of about 6 MeV

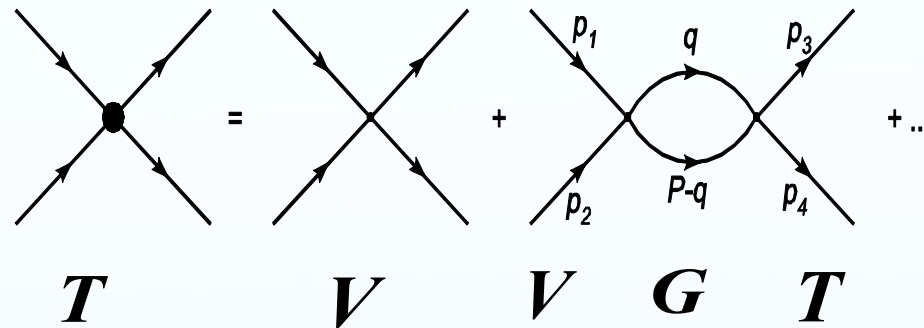
B. Wang, L. Meng and S. L. Zhu, **Phys. Rev. D** 101 (2020) 034018.

The data cannot confirm or refute the two-peak hypothesis.

§2. Formalism

- Coupled Channel Unitary Approach (CCUA): solving Bethe-Salpeter (BS) equations, which take on-shell approximation to loops.

$$T = V + V G T, \quad T = [1 - V G]^{-1} V$$



where **V matrix (potentials)** can be evaluated from the interaction Lagrangians.

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438

E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99

J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

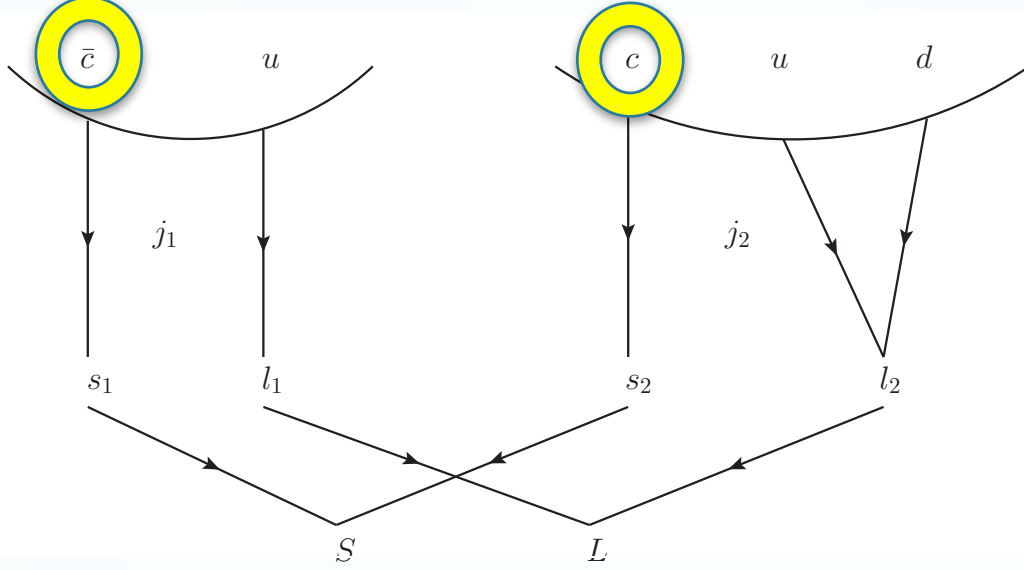
The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

$$\text{Im } T_{ij} = T_{in} \sigma_{nn} T_{nj}^*$$

$$\sigma_{nn} \equiv \text{Im } G_{nn} = - \frac{q_{cm}}{8\pi\sqrt{s}} \theta(s - (m_1 + m_2)^2)$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^I(s) + i \frac{q_{cm}}{4\pi\sqrt{s}}$$



Considering the heavy quark spin symmetry

$$\bar{D} \longrightarrow \bar{D}^* \quad \Xi_c \longrightarrow \Xi_c^*$$

$$\langle \ell'_M \ell'_B S' | S'_{c\bar{c}} \mathcal{L}'; J' | H^{QCD} | S_{c\bar{c}} \mathcal{L}; J, \rangle_{(\ell_M \ell_B S)} = \delta_{JJ'} \delta_{S'_{c\bar{c}} S_{c\bar{c}}} \delta_{\mathcal{L}\mathcal{L}'} \langle \ell'_M \ell'_B S' | | H^{QCD} | | \ell_M \ell_B S \rangle_{\mathcal{L}}$$

$$\begin{aligned} |\ell_M s_M j_M S; \ell_B s_B j_B; J\rangle &= \sum_{\mathcal{L}, S_{c\bar{c}}} [(2S_{c\bar{c}} + 1)(2\mathcal{L} + 1)(2j_M + 1)(2j_B + 1)]^{\frac{1}{2}} \\ &\times \begin{Bmatrix} \ell_M & \ell_B & \mathcal{L} \\ s_M & s_B & S_{c\bar{c}} \\ j_M & j_B & J \end{Bmatrix} |\mathcal{L} S_{c\bar{c}}; J\rangle_{(\ell_M \ell_B S)} \end{aligned}$$

$$J = 1/2, I = 0$$

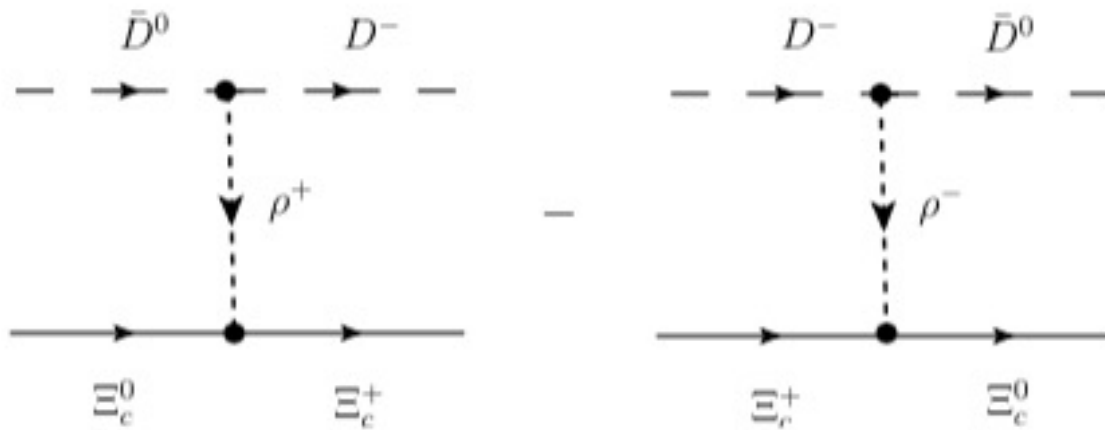
$\eta_c \Lambda$	$J/\psi \Lambda$	$\bar{D} \Xi_c$	$\bar{D}_s \Lambda_c$	$\bar{D} \Xi'_c$	$\bar{D}^* \Xi_c$	$\bar{D}_s^* \Lambda_c$	$\bar{D}^* \Xi'_c$	$\bar{D}^* \Xi_c^*$
$\hat{\mu}_1$	0	$-\frac{\hat{\mu}_{12}}{2}$	$-\frac{\hat{\mu}_{13}}{2}$	$\frac{\hat{\mu}_{14}}{2}$	$\frac{\sqrt{3}\hat{\mu}_{12}}{2}$	$\frac{\sqrt{3}\hat{\mu}_{13}}{2}$	$\frac{\hat{\mu}_{14}}{2\sqrt{3}}$	$\sqrt{\frac{2}{3}}\hat{\mu}_{14}$
0	$\hat{\mu}_1$	$\frac{\sqrt{3}\hat{\mu}_{12}}{2}$	$\frac{\sqrt{3}\hat{\mu}_{13}}{2}$	$\frac{\hat{\mu}_{14}}{2\sqrt{3}}$	$\frac{\hat{\mu}_{12}}{2}$	$\frac{\hat{\mu}_{13}}{2}$	$\frac{5\hat{\mu}_{14}}{6}$	$-\frac{\sqrt{2}\hat{\mu}_{14}}{3}$
$-\frac{\hat{\mu}_{12}}{2}$	$\frac{\sqrt{3}\hat{\mu}_{12}}{2}$	$\hat{\mu}_2$	$\hat{\mu}_{23}$	0	0	0	$\frac{\hat{\mu}_{24}}{\sqrt{3}}$	$-\sqrt{\frac{2}{3}}\hat{\mu}_{24}$
$-\frac{\hat{\mu}_{13}}{2}$	$\frac{\sqrt{3}\hat{\mu}_{13}}{2}$	$\hat{\mu}_{23}$	$\hat{\mu}_3$	0	0	0	$\frac{\hat{\mu}_{34}}{\sqrt{3}}$	$-\sqrt{\frac{2}{3}}\hat{\mu}_{34}$
$\frac{\hat{\mu}_{14}}{2}$	$\frac{\hat{\mu}_{14}}{2\sqrt{3}}$	0	0	$\frac{1}{3}(2\hat{\lambda} + \hat{\mu}_4)$	$\frac{\hat{\mu}_{24}}{\sqrt{3}}$	$\frac{\hat{\mu}_{34}}{\sqrt{3}}$	$-\frac{2(\hat{\lambda} - \hat{\mu}_4)}{3\sqrt{3}}$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\hat{\mu}_4 - \hat{\lambda})$
$\frac{\sqrt{3}\hat{\mu}_{12}}{2}$	$\frac{\hat{\mu}_{12}}{2}$	0	0	$\frac{\hat{\mu}_{24}}{\sqrt{3}}$	$\hat{\mu}_2$	$\hat{\mu}_{23}$	$\frac{2\hat{\mu}_{24}}{3}$	$\frac{\sqrt{2}\hat{\mu}_{24}}{3}$
$\frac{\sqrt{3}\hat{\mu}_{13}}{2}$	$\frac{\hat{\mu}_{13}}{2}$	0	0	$\frac{\hat{\mu}_{34}}{\sqrt{3}}$	$\hat{\mu}_{23}$	$\hat{\mu}_3$	$\frac{2\hat{\mu}_{34}}{3}$	$\frac{\sqrt{2}\hat{\mu}_{34}}{3}$
$\frac{\hat{\mu}_{14}}{2\sqrt{3}}$	$\frac{5\hat{\mu}_{14}}{6}$	$\frac{\hat{\mu}_{24}}{\sqrt{3}}$	$\frac{\hat{\mu}_{34}}{\sqrt{3}}$	$-\frac{2(\hat{\lambda} - \hat{\mu}_4)}{3\sqrt{3}}$	$\frac{2\hat{\mu}_{24}}{3}$	$\frac{2\hat{\mu}_{34}}{3}$	$\frac{1}{9}(2\hat{\lambda} + 7\hat{\mu}_4)$	$\frac{1}{9}\sqrt{2}(\hat{\lambda} - \hat{\mu}_4)$
$\sqrt{\frac{2}{3}}\hat{\mu}_{14}$	$-\frac{\sqrt{2}\hat{\mu}_{14}}{3}$	$-\sqrt{\frac{2}{3}}\hat{\mu}_{24}$	$-\sqrt{\frac{2}{3}}\hat{\mu}_{34}$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\hat{\mu}_4 - \hat{\lambda})$	$\frac{\sqrt{2}\hat{\mu}_{24}}{3}$	$\frac{\sqrt{2}\hat{\mu}_{34}}{3}$	$\frac{1}{9}\sqrt{2}(\hat{\lambda} - \hat{\mu}_4)$	$\frac{1}{9}(\hat{\lambda} + 8\hat{\mu}_4)$

LECs

$$\mathcal{L}_{VVV} = ig \langle [V_\nu, \partial_\mu V_\nu] V^\mu \rangle,$$

$$\mathcal{L}_{PPV} = -ig \langle [P, \partial_\mu P] V^\mu \rangle,$$

$$\mathcal{L}_{BBV} = g (\langle \bar{B} \gamma_\mu [V^\mu, B] \rangle + \langle \bar{B} \gamma_\mu B \rangle \langle V^\mu \rangle)$$



$$\hat{\mu}_1 = \hat{\mu}_3 = \hat{\mu}_{24} = \hat{\mu}_{34} = 0$$

$$\hat{\mu}_2 = \hat{\mu}_{23}/\sqrt{2} = \hat{\mu}_4 = \hat{\lambda} = -F, \quad F = \frac{1}{4f^2} (p^0 + p')$$

$$\hat{\mu}_{12} = -\hat{\mu}_{13}/\sqrt{2} = \hat{\mu}_{14}/\sqrt{3} = -\sqrt{\frac{2}{3}} \frac{m_V^2}{m_{D^*}^2} F,$$

$$J = 1/2, I = 0$$

§3. Results

i) $J = 1/2, I = 0$

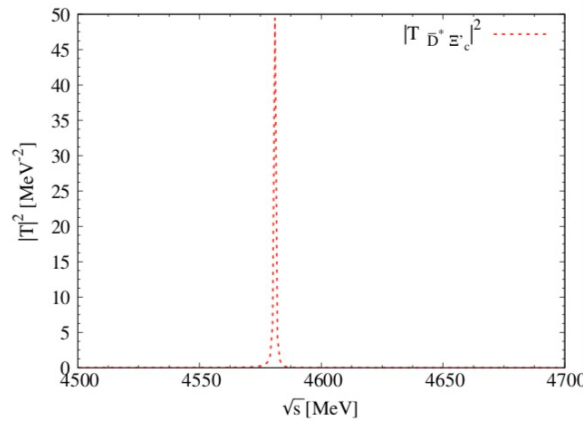
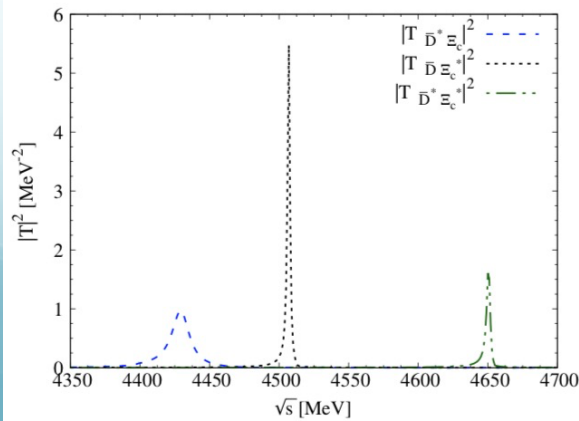
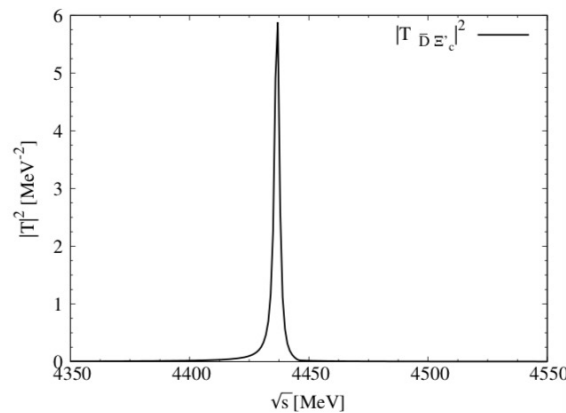
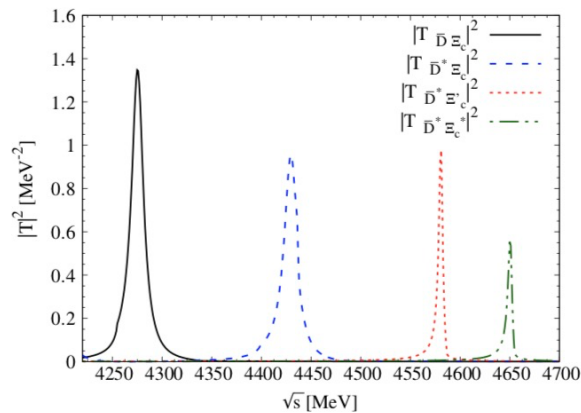
$\eta_c \Lambda, J/\psi \Lambda, \bar{D} \Xi_c, \bar{D}_s \Lambda_c, \bar{D} \Xi'_c, \bar{D}^* \Xi_c, \bar{D}_s^* \Lambda_c, \bar{D}^* \Xi'_c, \bar{D}^* \Xi_c^*$.

HQSS

ii) $J = 3/2, I = 0$

$J/\psi \Lambda, \bar{D}^* \Xi_c, \bar{D}_s \Lambda_c, \bar{D}^* \Xi'_c, \bar{D} \Xi_c^*, \bar{D}^* \Xi_c^*$.

$$a(\mu = 1 \text{ GeV}) = -2.09$$



CWX, J. Nieves and E. Oset, Phys. Rev. D 100 (2019) 014021

- $J = 1/2, I = 0$

$$a(\mu = 1 \text{ GeV}) = -2.09$$

Thres.	4099.58	4212.58	4366.61	4254.80	4445.34	4477.92	4398.66	4586.66	4654.48
	$\eta_c\Lambda$	$J/\psi\Lambda$	$\bar{D}\Xi_c$	$\bar{D}_s\Lambda_c$	$\bar{D}\Xi'_c$	$\bar{D}^*\Xi_c$	$\bar{D}_s^*\Lambda_c$	$\bar{D}^*\Xi'_c$	$\bar{D}^*\Xi_c^*$
	$4276.59 + i7.67$								
g_i	$0.17 - i0.03$	$0.29 - i0.07$	$2.93 + i0.08$	$0.76 + i0.31$	$0.00 + i0.01$	$0.01 + i0.02$	$0.01 + i0.04$	$0.01 - i0.02$	$0.01 - i0.03$
$ g_i $	0.17	0.30	2.93	0.82	0.01	0.02	0.05	0.02	0.03
	$4429.84 + i7.92$								
g_i	$0.29 - i0.11$	$0.17 - i0.07$	$0.00 - i0.00$	$0.00 - i0.00$	$0.15 - i0.26$	$2.78 + i0.01$	$0.66 + i0.32$	$0.01 + i0.05$	$0.01 + i0.03$
$ g_i $	0.31	0.18	0.00	0.00	0.30	2.78	0.73	0.05	0.04
	$4436.70 + i1.17$								
g_i	$0.24 + i0.03$	$0.14 + 0.01$	$0.00 - i0.00$	$0.00 - i0.00$	$1.72 - i0.04$	$0.22 - i0.31$	$0.06 - i0.01$	$0.01 - i0.04$	$0.01 - i0.03$
$ g_i $	0.24	0.14	0.00	0.00	1.72	0.38	0.07	0.04	0.03
	$4580.96 + i2.44$								
g_i	$0.12 - i0.00$	$0.37 - i0.04$	$0.02 - i0.01$	$0.02 - i0.01$	$0.03 - i0.00$	$0.02 - i0.02$	$0.03 - i0.02$	$1.57 - i0.17$	$0.00 + i0.02$
$ g_i $	0.12	0.37	0.02	0.02	0.03	0.03	0.03	1.58	0.02
	$4650.86 + i2.59$								
g_i	$0.32 - i0.05$	$0.19 - i0.03$	$0.02 - i0.01$	$0.03 - i0.02$	$0.02 - i0.00$	$0.01 - i0.01$	$0.02 - i0.01$	$0.01 - i0.00$	$1.41 - i0.23$
$ g_i $	0.32	0.19	0.03	0.04	0.02	0.02	0.02	0.02	1.43

- $J = 3/2, I = 0$

Thres.	4212.58	4477.92	4398.66	4586.66	4513.17	4654.48
	$J/\psi\Lambda$	$\bar{D}^*\Xi_c$	$\bar{D}_s^*\Lambda_c$	$\bar{D}^*\Xi'_c$	$\bar{D}\Xi_c^*$	$\bar{D}^*\Xi_c^*$
	$4429.52 + i7.67$					
g_i	$0.31 - i0.10$	$2.77 - i0.02$	$0.67 + i0.32$	$0.00 + i0.002$	$0.00 - i0.06$	$0.00 + i0.004$
$ g_i $	0.32	2.77	0.74	0.02	0.06	0.04
	$4506.99 + i1.03$					
g_i	$0.27 - i0.02$	$0.02 - i0.03$	$0.02 - i0.02$	$0.00 - i0.03$	$1.56 - i0.07$	$0.00 - i0.05$
$ g_i $	0.27	0.03	0.03	0.03	1.56	0.05
	$4580.96 + i0.34$					
g_i	$0.14 - i0.01$	$0.01 - i0.01$	$0.01 - i0.01$	$1.54 - i0.02$	$0.02 - i0.00$	$0.00 - i0.04$
$ g_i $	0.14	0.01	0.02	1.54	0.02	0.04
	$4650.58 + i1.48$					
g_i	$0.29 - i0.02$	$0.02 - i0.01$	$0.03 - i0.02$	$0.03 - i0.01$	$0.03 - i0.00$	$1.40 - i0.13$
$ g_i $	0.29	0.03	0.03	0.03	0.03	1.41

• $J = 1/2, I = 0$

$a_\mu(\mu = 1 \text{ GeV}) = -1.94$

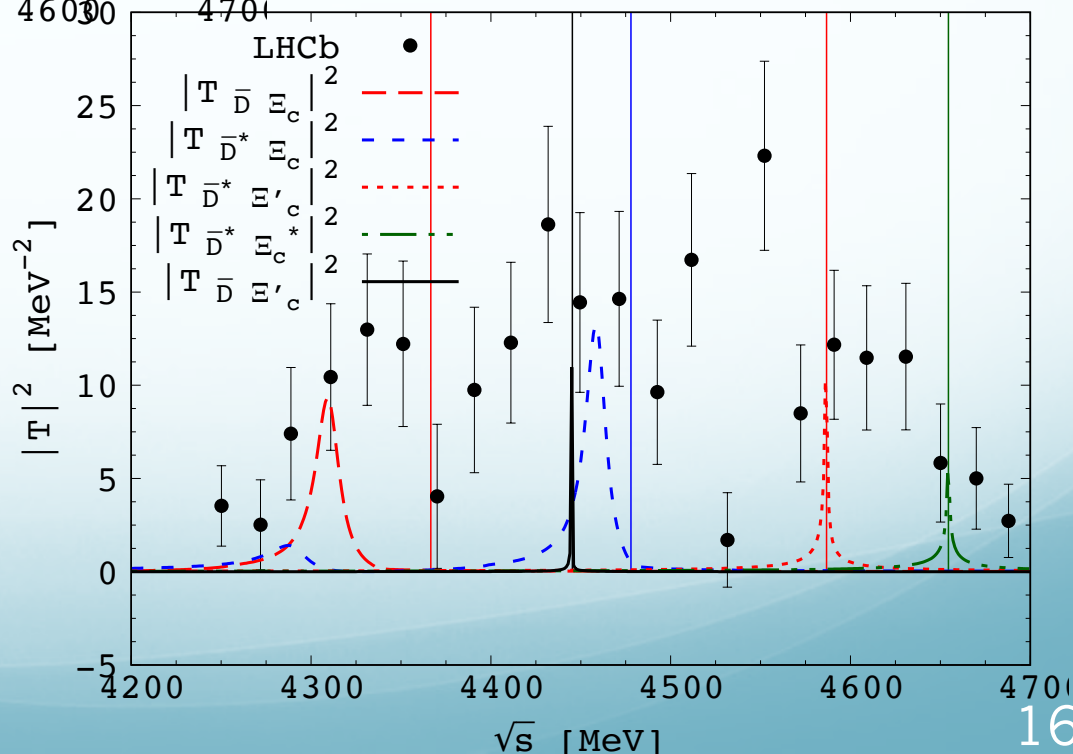
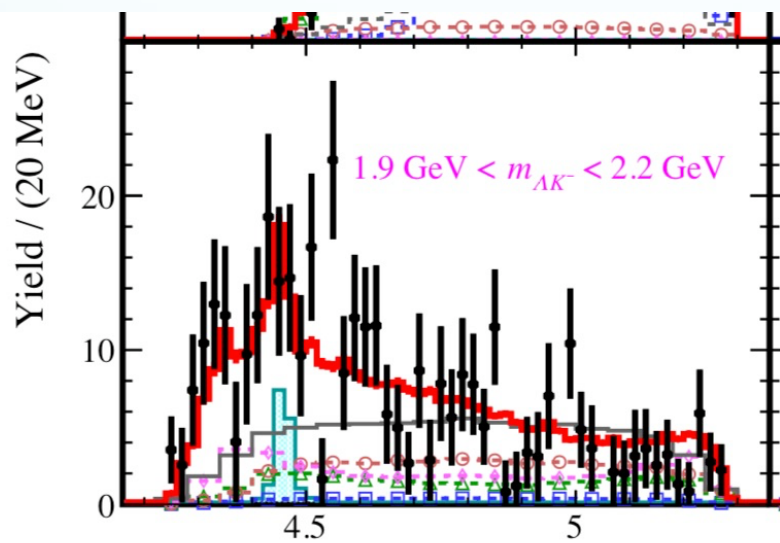
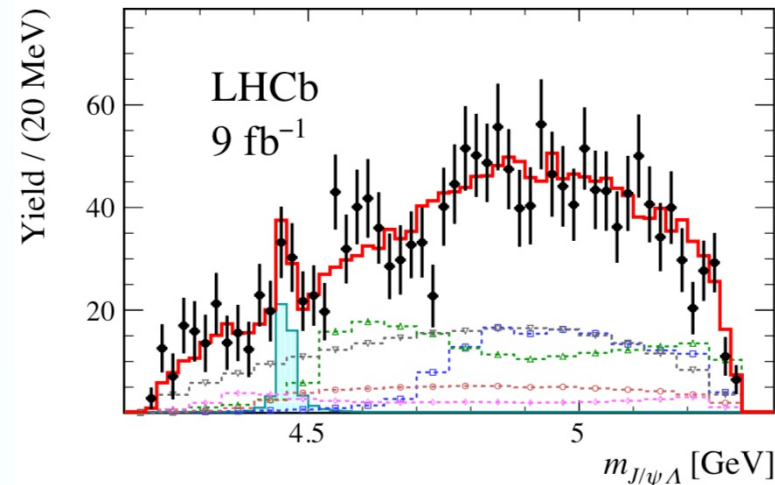
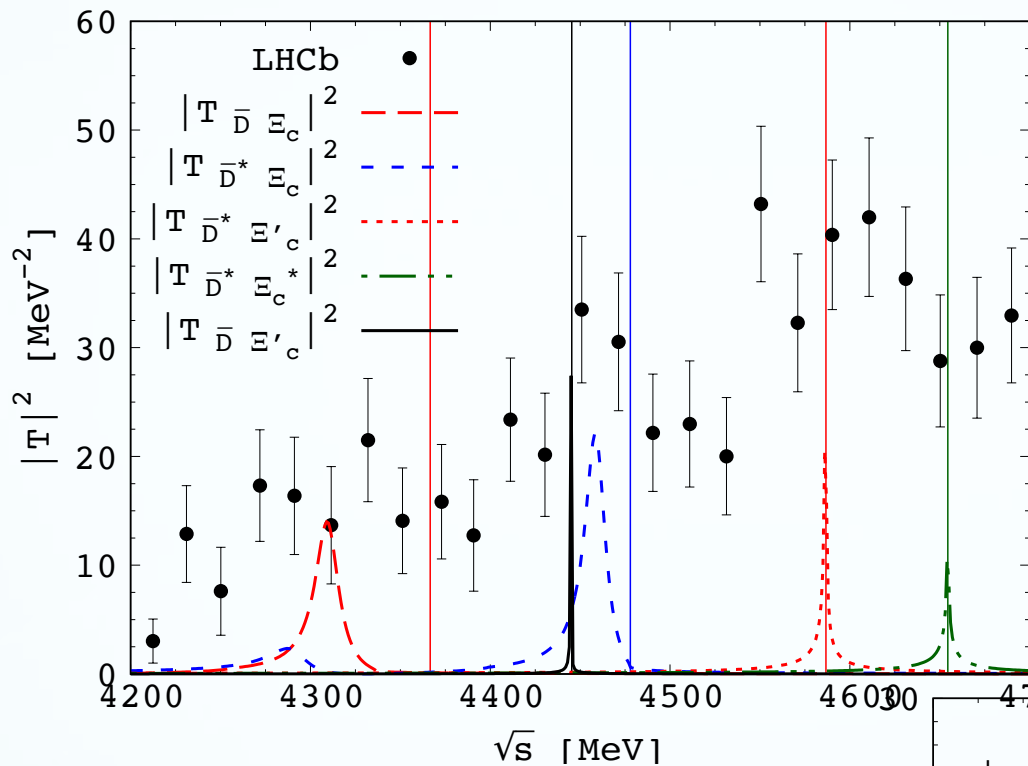


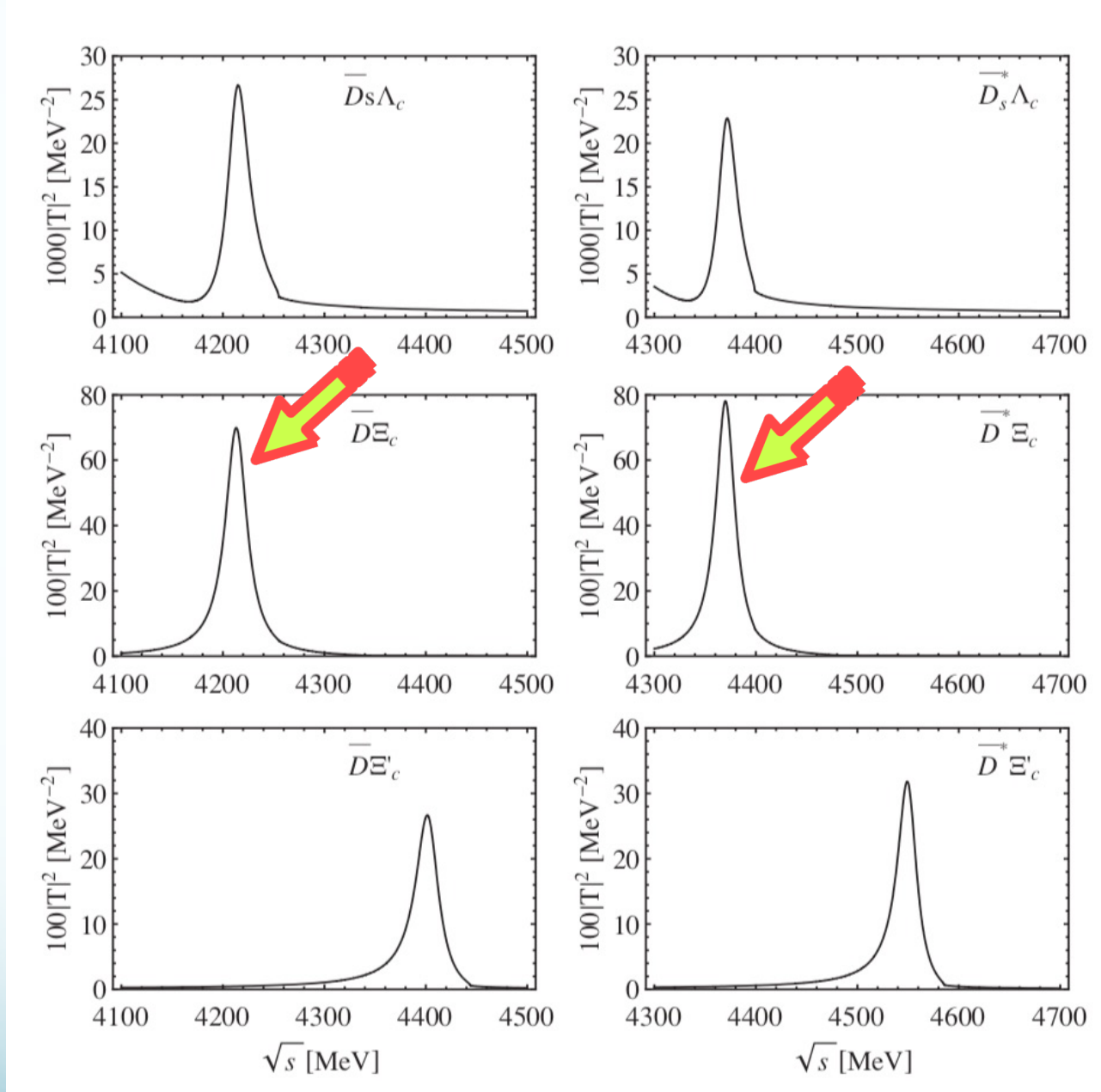
Chan.	$\eta_c \Lambda$	$J/\psi \Lambda$	$\bar{D}\Xi_c$	$\bar{D}_s \Lambda_c$	$\bar{D}\Xi'_c$	$\bar{D}^* \Xi_c$	$\bar{D}_s^* \Lambda_c$	$\bar{D}^* \Xi'_c$	$\bar{D}^* \Xi_c^*$
Thres.	4099.58	4212.58	4366.61	4254.80	4445.34	4477.92	4398.66	4586.66	4654.48
	$4310.53 + i8.23$								
$ g_i $	0.15	0.27	2.33	0.69	0.00	0.04	0.09	0.01	0.02
Γ_i	0.57	1.18	–	13.86	–	–	–	–	–
Br.	3.47%	7.16%	–	84.21%	–	–	–	–	–
	$4445.12 + i0.19$								
$ g_i $	0.10	0.06	0.00	0.00	0.72	0.08	0.04	0.01	0.01
Γ_i	0.29	0.08	0.00	0.00	–	–	0.04	–	–
Br.	74.74%	21.22%	0.01%	0.01%	–	–	10.62%	–	–
	$4459.07 + i6.89$	$P_{cs}(4459)$							
$ g_i $	0.22	0.13	0.00	0.00	0.07	2.16	0.61	0.03	0.02
Γ_i	1.59	0.46	0.00	0.00	0.01	–	11.14	–	–
Br.	11.57%	3.31%	0.00%	0.00%	0.70%	–	80.86%	–	–
	4586.66?								
$ g_i $	–	–	–	–	–	–	–	–	–
	4654.48?								
$ g_i $	–	–	–	–	–	–	–	–	–

- $J = 3/2, I = 0$

Chan.	$J/\psi\Lambda$	$\bar{D}^*\Xi_c$	$\bar{D}_s^*\Lambda_c$	$\bar{D}^*\Xi'_c$	$\bar{D}\Xi_c^*$	$\bar{D}^*\Xi_c^*$
Thres.	4212.58	4477.92	4398.66	4586.66	4513.17	4654.48
	$4459.02 + i6.83$	$P_{cs}(4459)$				
$ g_i $	0.28	2.16	0.61	0.02	0.04	0.03
Γ_i	2.00	—	11.15	—	—	—
Br.	14.68%	—	81.64%	—	—	—
	4586.66?					
$ g_i $	—	—	—	—	—	—
	4513.17?					
$ g_i $	—	—	—	—	—	—
	4654.48?					
$ g_i $	—	—	—	—	—	—









states from $P B \rightarrow P B$. The units :

(I, S)	z_R	Real axis	
		M	Γ
$(1/2, 0)$	4269	4267	34.3
$(0, -1)$	4213	4213	26.4
	4403	4402	28.2

for the states from $V B \rightarrow V B$.

(I, S)	z_R	Real Axis	
		M	Γ
$(1/2, 0)$	4418	4416	28.4
$(0, -1)$	4370	4371	23.3
	4550	4549	23.7

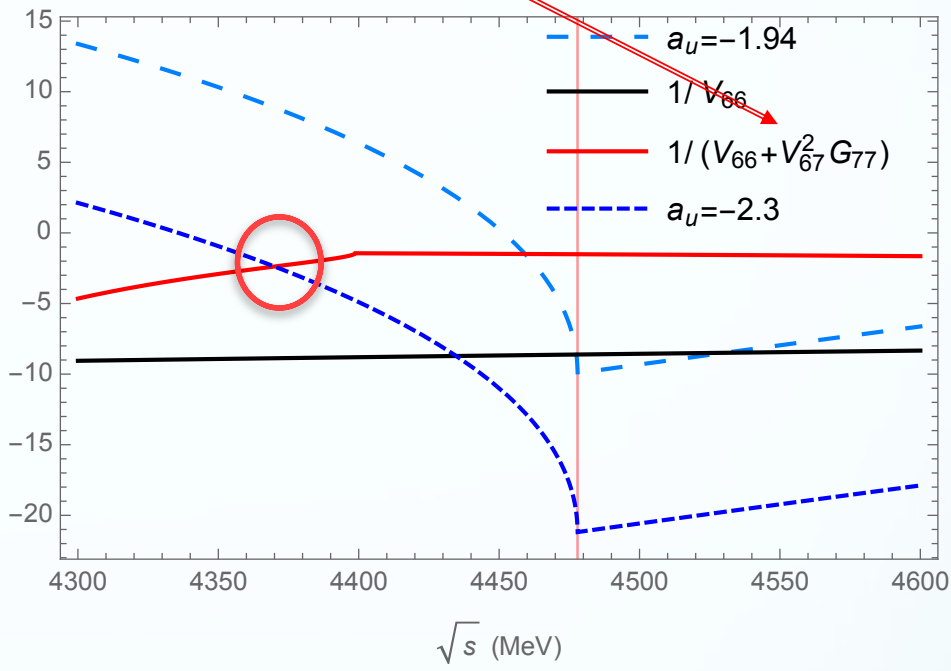
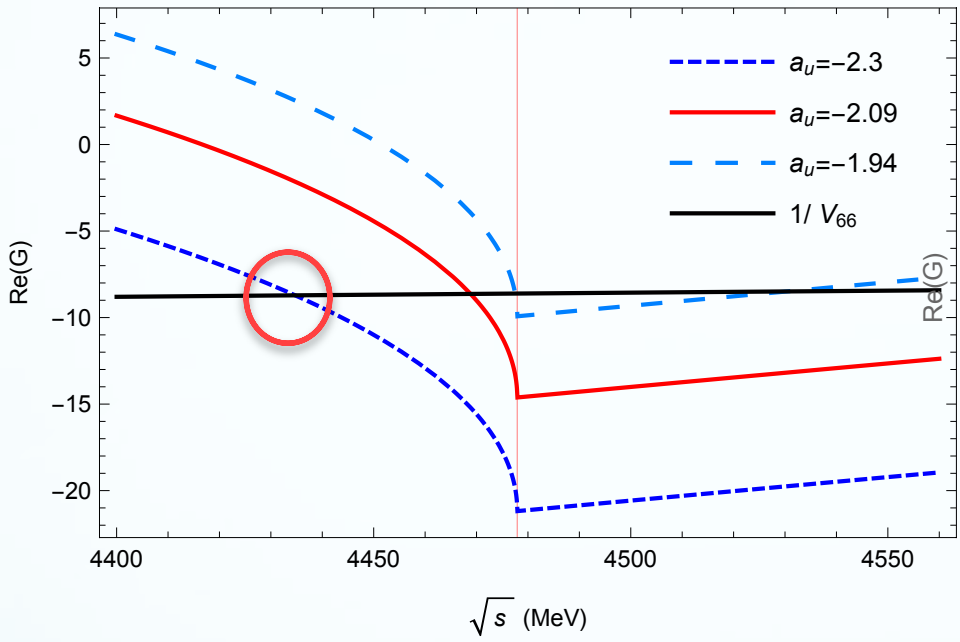
$\Lambda_{c\bar{c}}(4213)$	$\Lambda_{c\bar{c}}(4403)$	$\Lambda_{c\bar{c}}(4370)$	$\Lambda_{c\bar{c}}(4490)$	$\Lambda_{c\bar{c}}(4550)$		
$D_s^- \Lambda_c^+(4255)$	$\bar{D} \Xi_c(4337)$	$\bar{D} \Xi_c'(4445)$	$D_s^{*-} \Lambda_c^+(4399)$	$\bar{D}^* \Xi_c(4478)$	$\bar{D} \Xi_c^*(4513)$	$\bar{D}^* \Xi_c'(4587)$

$$\mu = 1000 \text{ MeV}$$

$$a(\mu) = -2.3$$

for the $\bar{D}^*\Xi_c$ channel.

$\bar{D}_s^*\Lambda_c$



CWX, J. J. Wu and B. S. Zou, Phys. Rev. D 103 (2021) 054016.

$\bar{D}\Xi_c$ $\bar{D}_s\Lambda_c$

§4. Summary

- Our results of bound states —molecular states

a $\bar{D}\Xi_c$ state 4310.53 $\bar{D}\Xi'_c$ $P_c(4312)^+$
 Having $J = 1/2$.

a $\bar{D}\Xi_c^*$ state
 With $J = 3/2$. $P_c(4450)$

a $\bar{D}^*\Xi_c$ state ? $P_{cs}(4459)$ $\bar{D}^*\Xi'_c$ $P_c(4440)^+$
 Degenerate in $J = 1/2, 3/2$. $P_c(4457)^+$

a $\bar{D}^*\Xi_c^*$ state
 Degenerate in $J = 1/2, 3/2, 5/2$.

Hope that our predictions can be found in the future experiments!



Thanks for your attention!