



THE OHIO STATE UNIVERSITY

# Triangle singularities in production of $X(3872)$

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# Outline

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- **Brief review of X(3872)**
- **Charm-meson triangle singularity**
- **Production of X(3872):**
  - ◆  **$e^+e^-$  annihilation** [PRD100, 031501(2019), PRD101, 014021(2020), PRD 101, 096020(2020)]
  - ◆ **B meson decays** [PRD100, 074028(2019)]
  - ◆ **hadron colliders** [PRD100, 094006(2019)]
- **Summary**
  - ◆ Triangle singularity produces **peaks in reaction rates**
  - ◆ The observation of the peaks would definitely **resolve the nature of X(3872)**

# Brief review of X(3872) ( $\equiv \chi_{c1}(3872)$ )

- ✓ **discovery at  $e^+e^-$  collider [Belle (2003)]:**

$$B^+ \rightarrow K^+ + X$$

$$X \rightarrow J/\psi \pi^+ \pi^-$$

- ✓ **confirmation at  $p\bar{p}$  collider [CDF (2003)]:**

$$p\bar{p} \rightarrow X + \text{anything}$$

- **quantum numbers [LHCb (2013)]:**

$$J^{PC} = 1^{++}$$

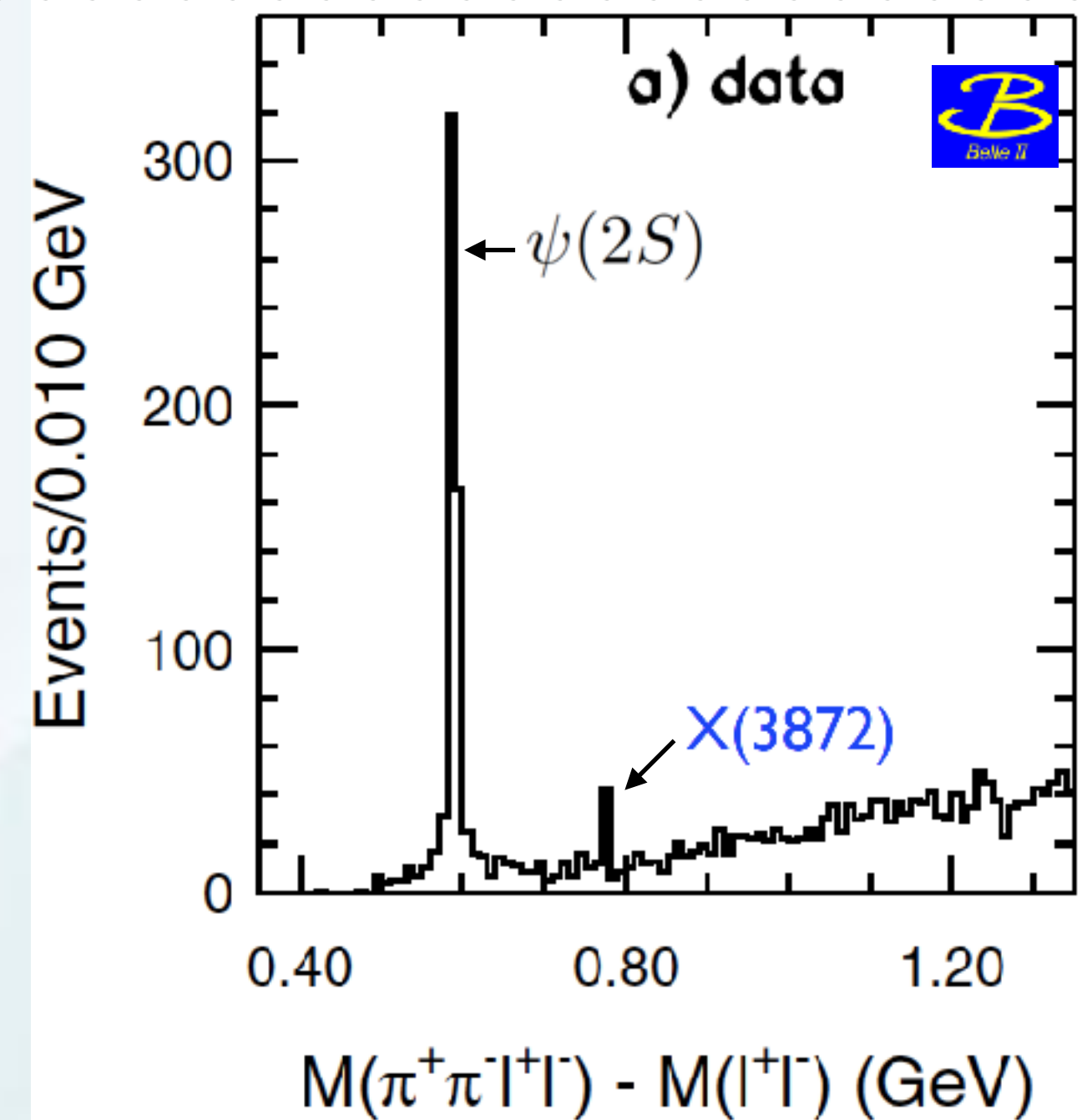
- **mass [LHCb (2020)]:**

$$E_X = M_X - (M_{D^{*0}} + M_{D^0}) = (-0.07 \pm 0.12) \text{ MeV} \quad |E_X| < 0.22 \text{ MeV at 90\% CL}$$

- **first measurement of width (Breit-Wigner) [LHCb (2020) average]:**

$$\Gamma_X = (1.19 \pm 0.19) \text{ MeV}$$

- **7 observed decay modes:  $J/\psi \pi^+ \pi^-$ ,  $J/\psi \pi^+ \pi^- \pi^0$ ,  $J/\psi \gamma$ ,  $\psi(2S) \gamma$ ,  $D^0 \bar{D}^0 \pi^0$ ,  $D^0 \bar{D}^0 \gamma$ ,  $\chi_{c1} \pi^0$**



# Brief review of $X(3872)$ ( $\equiv \chi_{c1}(3872)$ )

What is the  $X(3872)$ ?

$J^{PC} = 1^{++}$   $\rightarrow$  S-wave coupling to  $D^{*0}\bar{D}^0/\bar{D}^{*0}D^0$

$|E_X| < 0.22 \text{ MeV}$   $\rightarrow$  resonant coupling

S-wave loosely bound **charm-meson molecule!!**

$$X = \frac{1}{\sqrt{2}}(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0})$$

other components of wave functions have small probabilities:

- at long distances:  $D^0\bar{D}^0\pi^0$
- at short distances:
  - ✦  $\chi_{c1}(2P)$ ?
  - ✦ charged charm mesons?
  - ✦ compact tetraquark  $[cq][\bar{c}\bar{q}]$ ?



# Brief review of $X(3872)$ ( $\equiv \chi_{c1}(3872)$ )

## What is the $X(3872)$ ?

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S-wave loosely bound **charm-meson molecule!!**

$$X = \frac{1}{\sqrt{2}}(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0})$$

## Universal properties determined by the binding energy $|E_X|$

- \* large scattering length:  $|a| = \pm 1/\sqrt{2\mu|E_X|}$ ,  $|a| \gg \text{range}$
- \* large mean separation:  $\langle r \rangle = a/2$ ,  $|E_X| < 0.22$  MeV implies  $\langle r \rangle > 5$  fm
- \* scattering amplitude at  $E \ll 1/(2\mu \text{range}^2)$ :  $f(E) = 1/(-1/a + i\sqrt{2\mu E})$
- \* wavefunction:  $\psi(r) = e^{-r/a}/r$

## XEFT

effective field theory for charm mesons and pions

Fleming, Kusunoki, Mehen & van Kolck [PRD 76, 034006(2007)]

## Galilean-invariant XEFT

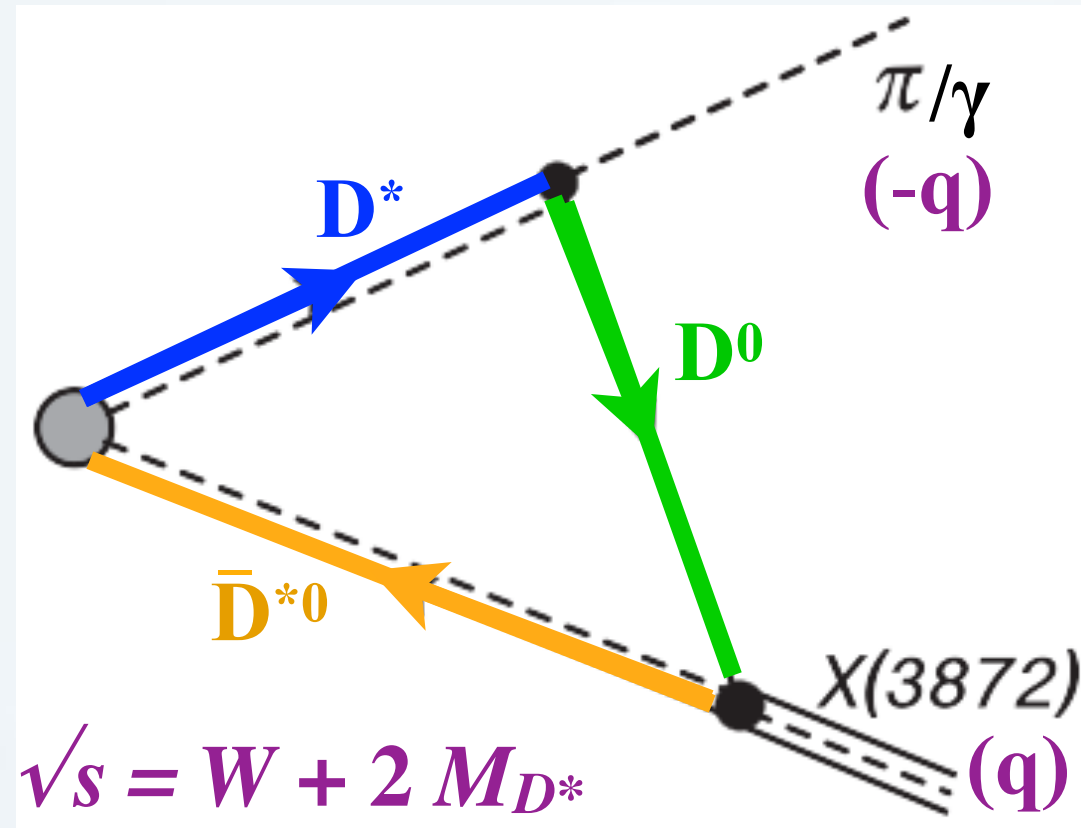
Braaten [PRD 91, 114007(2015)]

Braaten, He & Jiang [PRD 103, 036014(2021)]

# Charm-meson triangle singularity

three charm mesons can be on shell simultaneously →

$\log^2(s/s_\Delta)$  divergence in reaction rate at  $s_\Delta$   
determined by masses



loop amplitude near singularity:

$$F(W) \propto \log \frac{\sqrt{M_* W} + (M_*/M_X)q}{\sqrt{M_* W} - (M_*/M_X)q}$$

( $M_* = M_{D^*}$ )

divergence at energy W above  $D^*\bar{D}^*$  threshold:

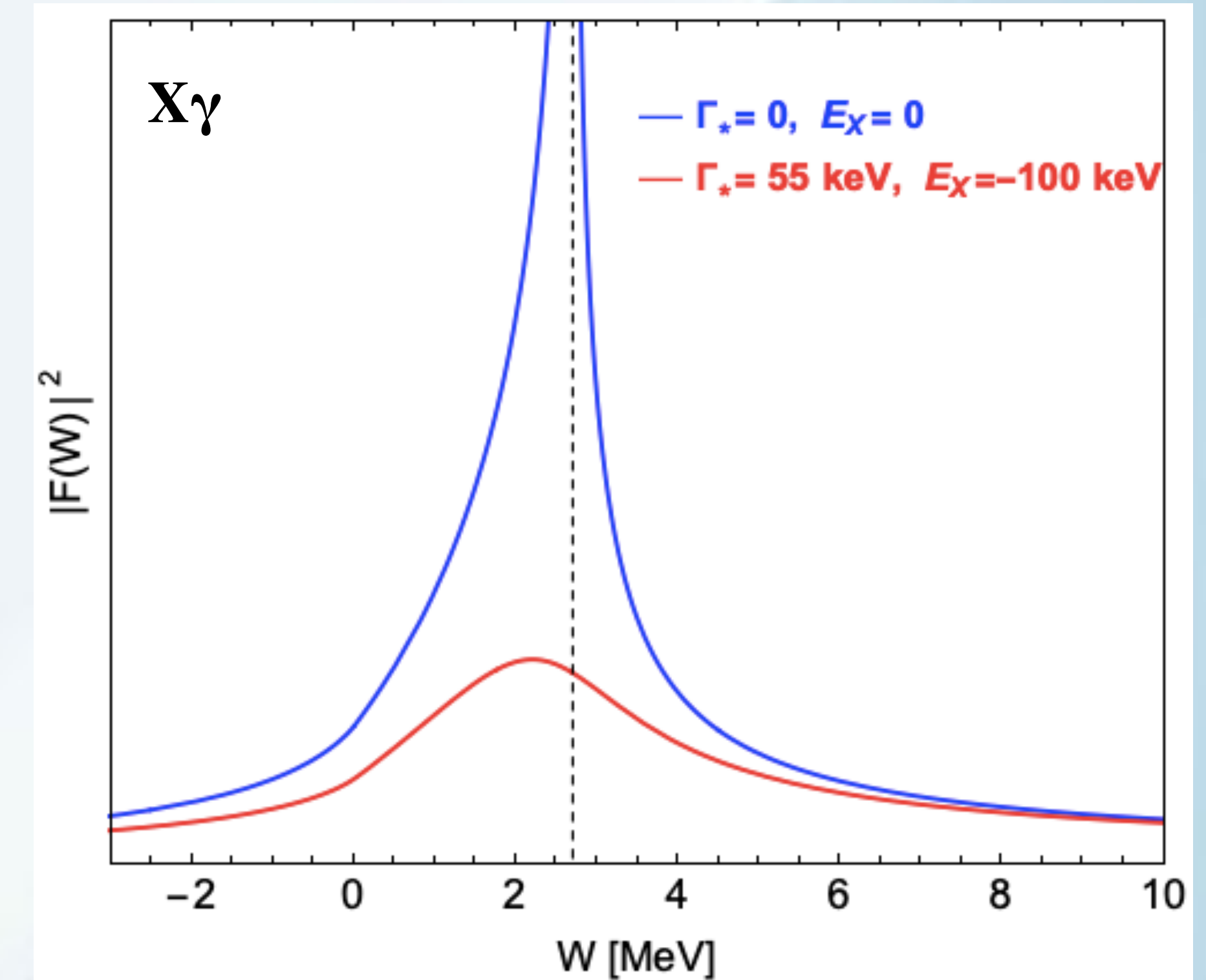
- ❖  $X\gamma$ :  $(M_{D^{*0}}/M_X^2)(M_{D^{*0}} - M_{D^0})^2 = 2.7 \text{ MeV}$
- ❖  $X\pi^0$ :  $(m_{\pi^0}/2M_{D^0})(M_{D^{*0}} - M_{D^0} - m_{\pi^0}) = 0.3 \text{ MeV}$
- ❖  $X\pi^\pm$ :  $(m_{\pi^0}/2M_{D^0})(M_{D^{*+}} - M_{D^0} - m_{\pi^+}) = 0.2 \text{ MeV}$

**BUT**

- \* nonzero decay width for  $D^*$
- \* nonzero binding energy ( $-E_X$ ) for X

$$F(W) \propto \log \frac{\sqrt{2\mu E_X + i\mu\Gamma_*} + \sqrt{M_*(W + i\Gamma_*)} + (M_*/M_X)q}{\sqrt{2\mu E_X + i\mu\Gamma_*} + \sqrt{M_*(W + i\Gamma_*)} - (M_*/M_X)q}$$

→ narrow peak in reaction rate



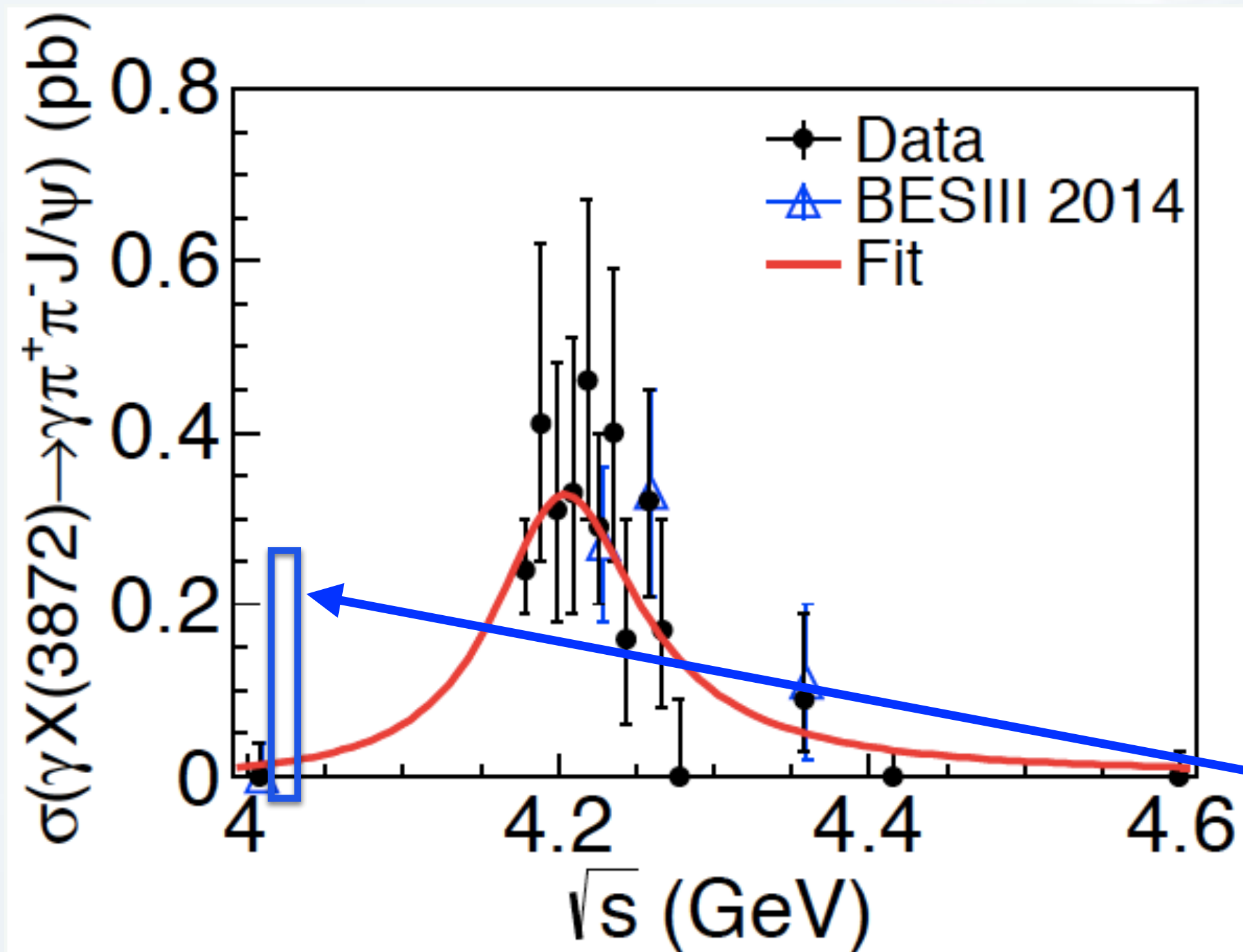
review on TS: Guo, Liu, Sakai [Prog. Part. Nucl. Phys. 112, 103757 (2020)]



# production of $X(3872) + \gamma$ in $e^+e^-$ annihilation

## Experimental observation:

BESIII:  $e^+e^- \rightarrow X\gamma$ ,  $X \rightarrow J/\psi \pi^+\pi^-$ ,  $J/\psi \omega$   
[PRL122,232002 (2019)]



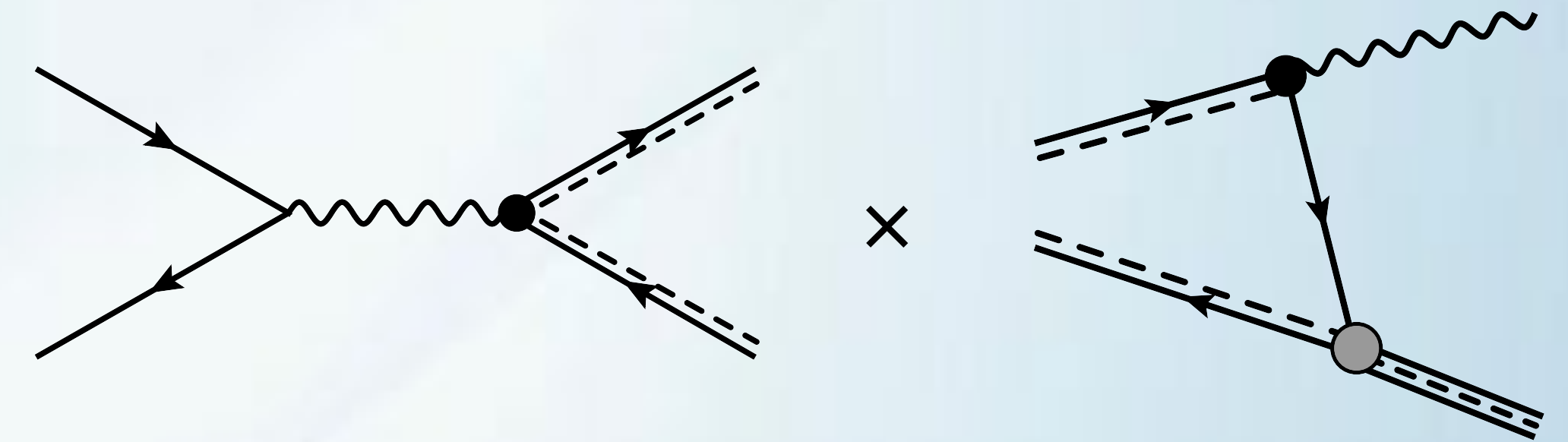
## First theoretical calculation:

Dubynskiy & Voloshin [PRD 74, 094017 (2006)]

### absorptive contribution only:

$e^+e^- \rightarrow D^{*0}\bar{D}^{*0} \text{ (P-wave)} \rightarrow X\gamma$

- $e^+e^-$  annihilation creates  $D^{*0}\bar{D}^{*0} \text{ (P-wave)}$
- rescattering of **real**  $D^{*0}\bar{D}^{*0}$  into  $X\gamma$



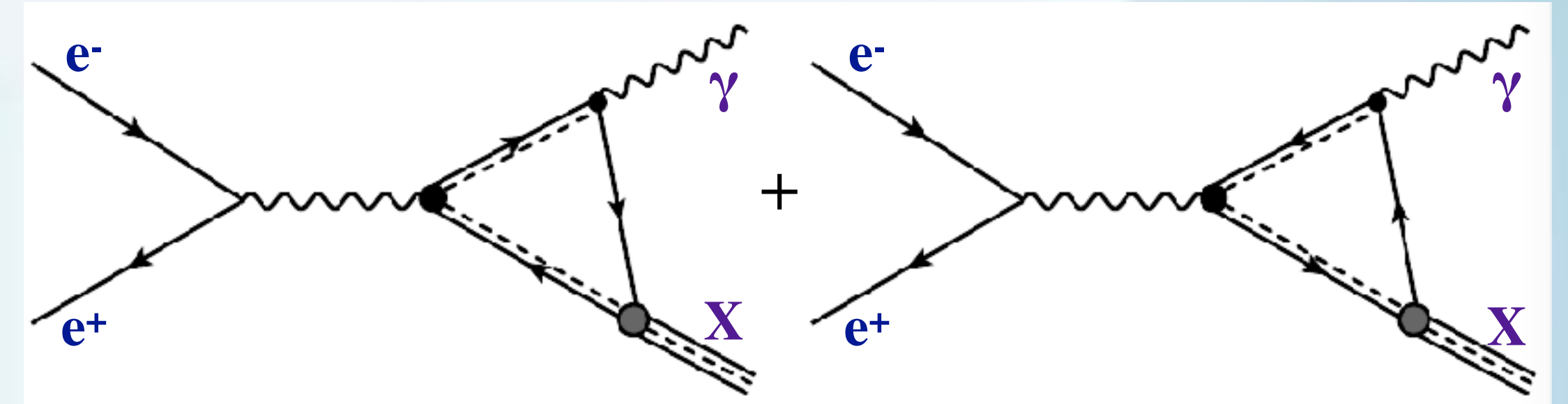
- ❖ Line shape of  $X\gamma$  has narrow peak a few MeV above  $D^{*0}\bar{D}^{*0}$  threshold
- ❖  $\sigma[X\gamma]$ : of order **1pb** near the **peak**

# production of $X(3872) + \gamma$ in $e^+e^-$ annihilation

Braaten, He & Ingles [PRD 100, 031501(2019), PRD 101, 014021(2020)]

$$e^+e^- \rightarrow D^{*0}\bar{D}^{*0} (\text{P-wave}) \rightarrow X\gamma$$

- $e^+e^-$  annihilation creates  $D^{*0}\bar{D}^{*0}$  (P-wave)
- rescattering of virtual  $D^{*0}\bar{D}^{*0}$  into  $X\gamma$



\* **improvements** over Dubynskiy & Voloshin:

- ❖ include  $\text{Re}[M]$  as well as  $\text{Im}[M]$
- ❖ include **decay width of  $D^{*0}$**
- ❖ **normalize** cross section using  $\sigma[D^{*+}D^{*-}]$   
Uglov *et al.* (JETP Lett. 105,1 (2017))

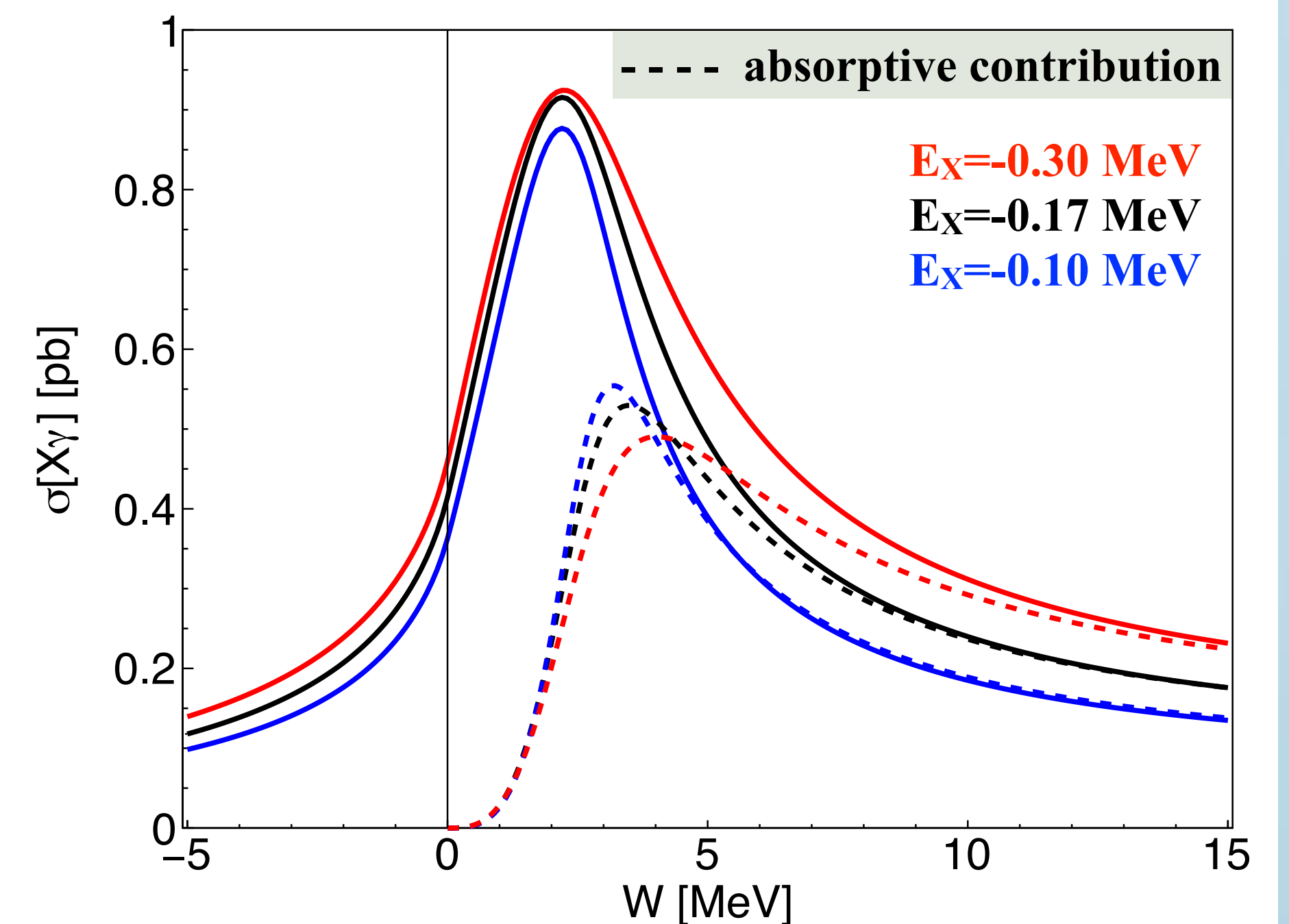
\* **cross section:**

- ❖ **triangle singularity** gives narrow **peak at 2.2 MeV**  
above  $D^{*0}\bar{D}^{*0}$  threshold at 4013.7 MeV
- ❖ **position of peak** insensitive to **binding energy**
- ❖ may be observable by **BESIII detector**!

absorptive contribution only is not a good approximation!

cross section for  $X\gamma$

$$\sqrt{s} = W + 2 M_{D^*}$$

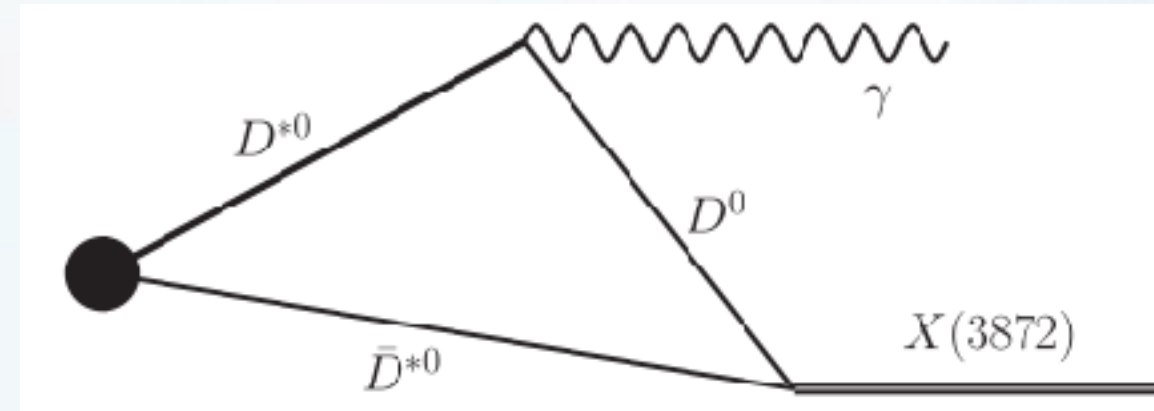




# production of $X(3872) + \gamma$ in $e^+e^-$ annihilation

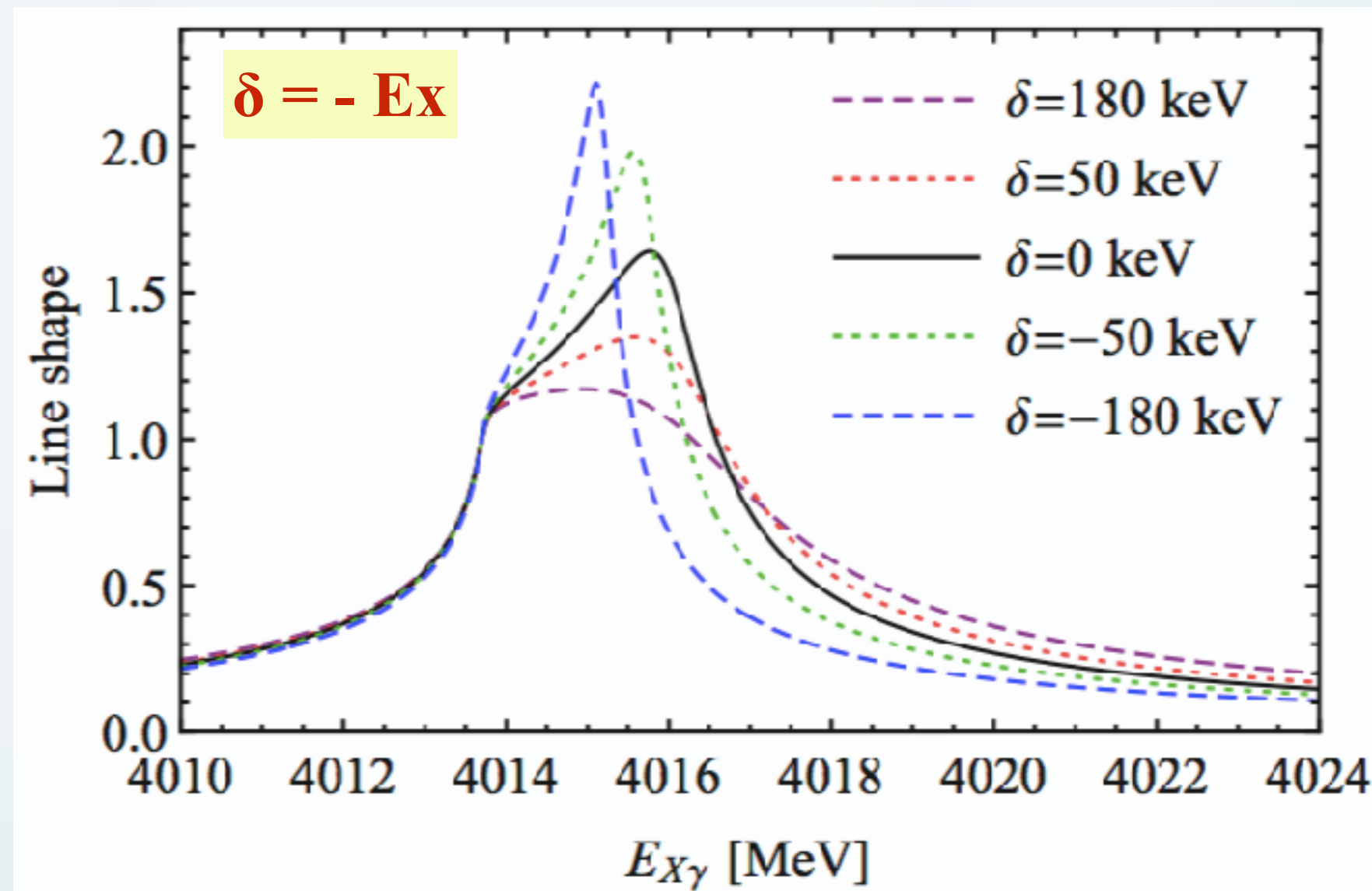
## • Guo [PRL 112, 202002 (2019)]

- creation of  $D^{*0}\bar{D}^{*0}$ (S-wave) at short distance
- rescattering of **virtual**  $D^{*0}\bar{D}^{*0}$  into  $X\gamma$



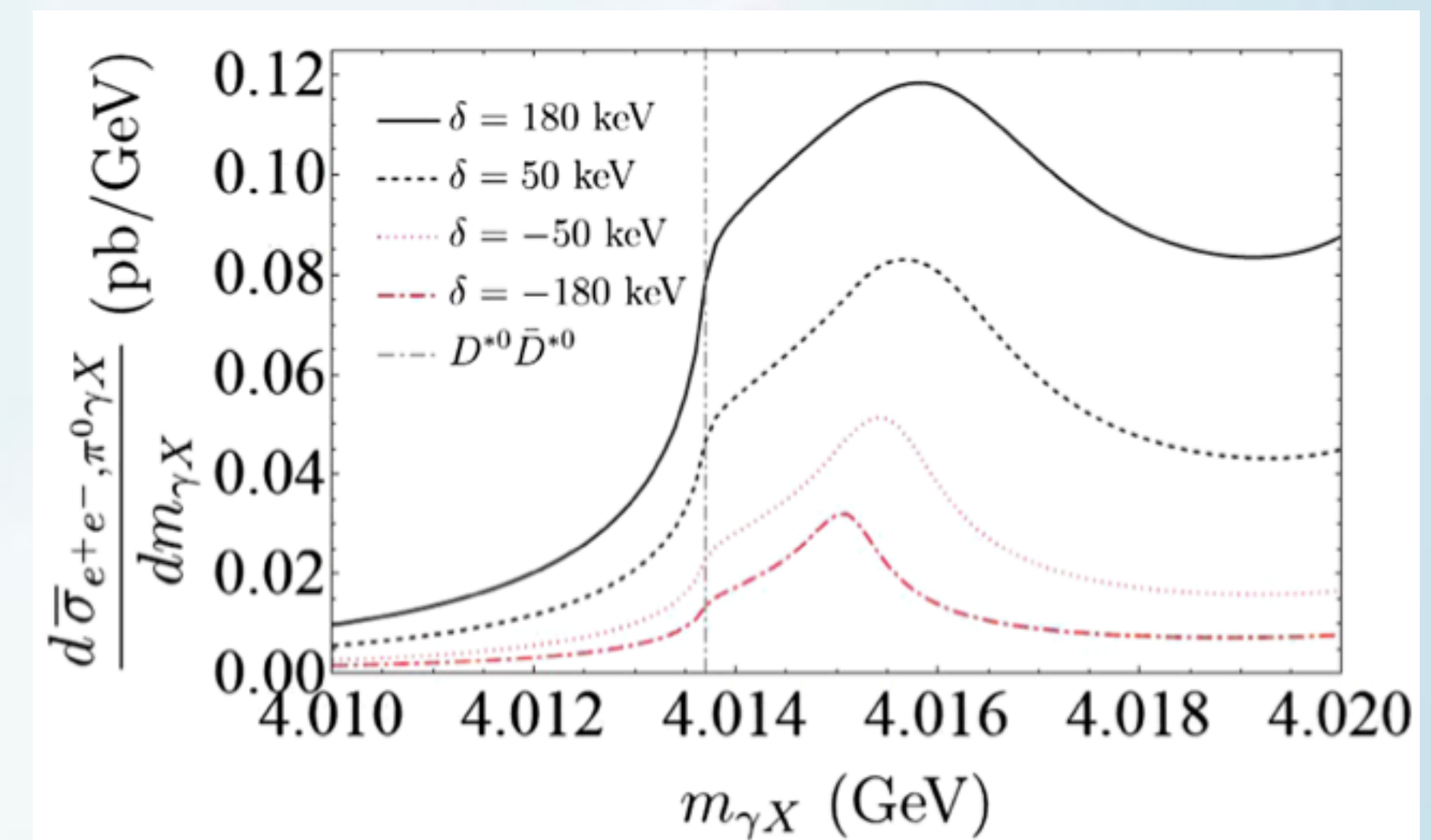
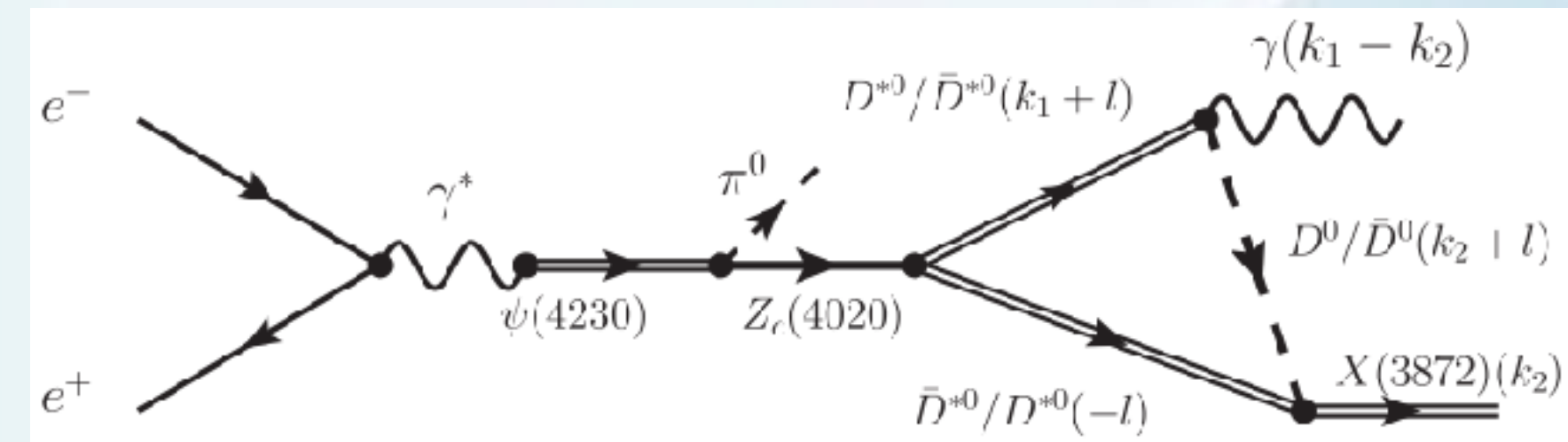
### Line shape in $X\gamma$ :

- ❖ peak a few MeV above  $D^{*0}\bar{D}^{*0}$  threshold
- ❖ can be used to measure  $E_X$



## • Sakai, Jing & Guo [PRD 102, 114041(2020)]

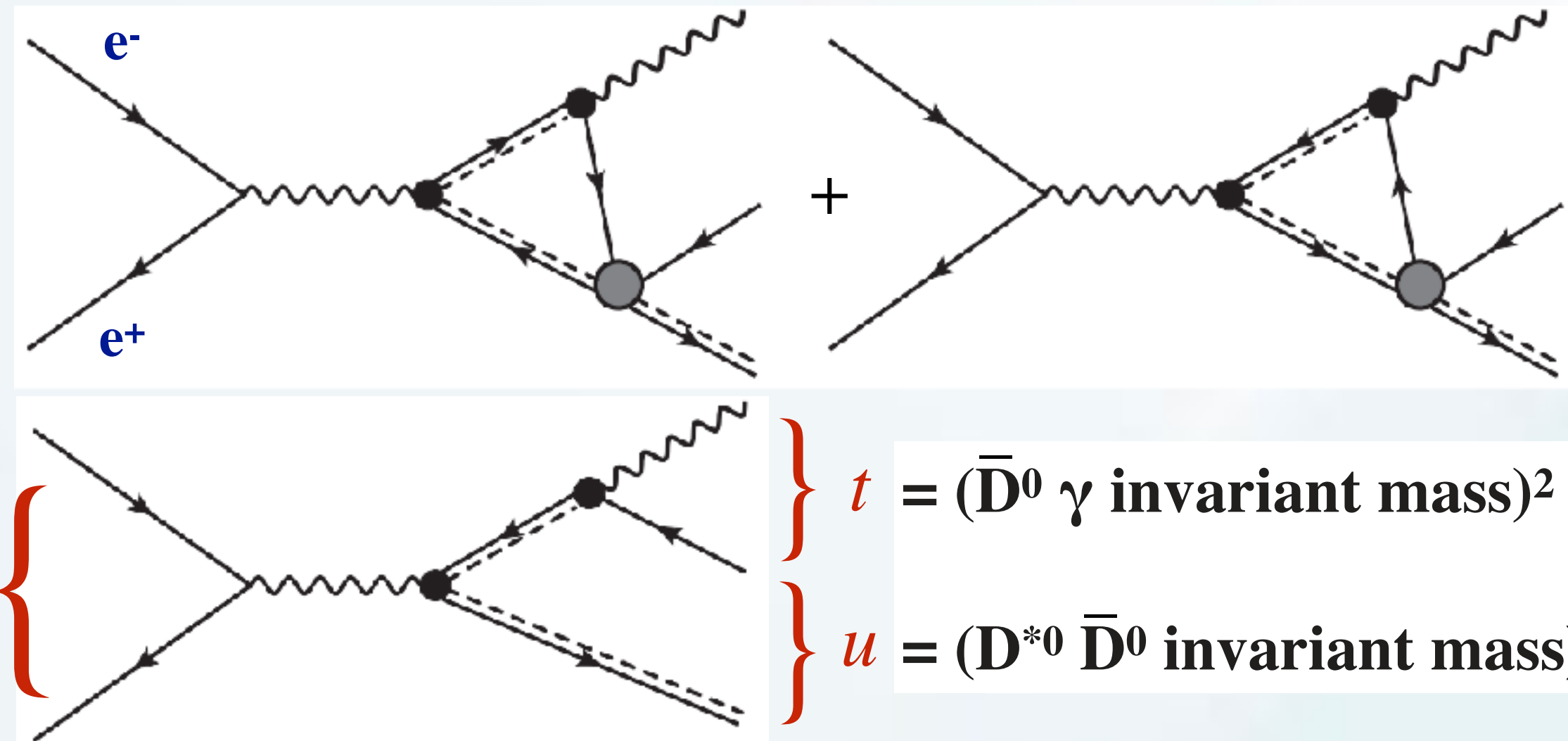
$$e^+e^- \rightarrow Z_c(4020) \pi^0, Z_c(4020) \rightarrow D^{*0}\bar{D}^{*0}(\text{S-wave}) \rightarrow X\gamma$$



- BESIII [arXiv:2101.00644]: no significant signal
- $e^+e^- \rightarrow Z_c(4020) \pi^0, Z_c(4020) \rightarrow D^{*0}\bar{D}^{*0}(\text{S-wave}) \rightarrow X\gamma$

# production of $D^{*0}\bar{D}^0 + \gamma$ in $e^+e^-$ annihilation

Braaten, He, Ingles & Jiang [PRD 101, 096020(2020)]



## \* Schmid cancellation:

Schmid [PR154, 1363(1967)]

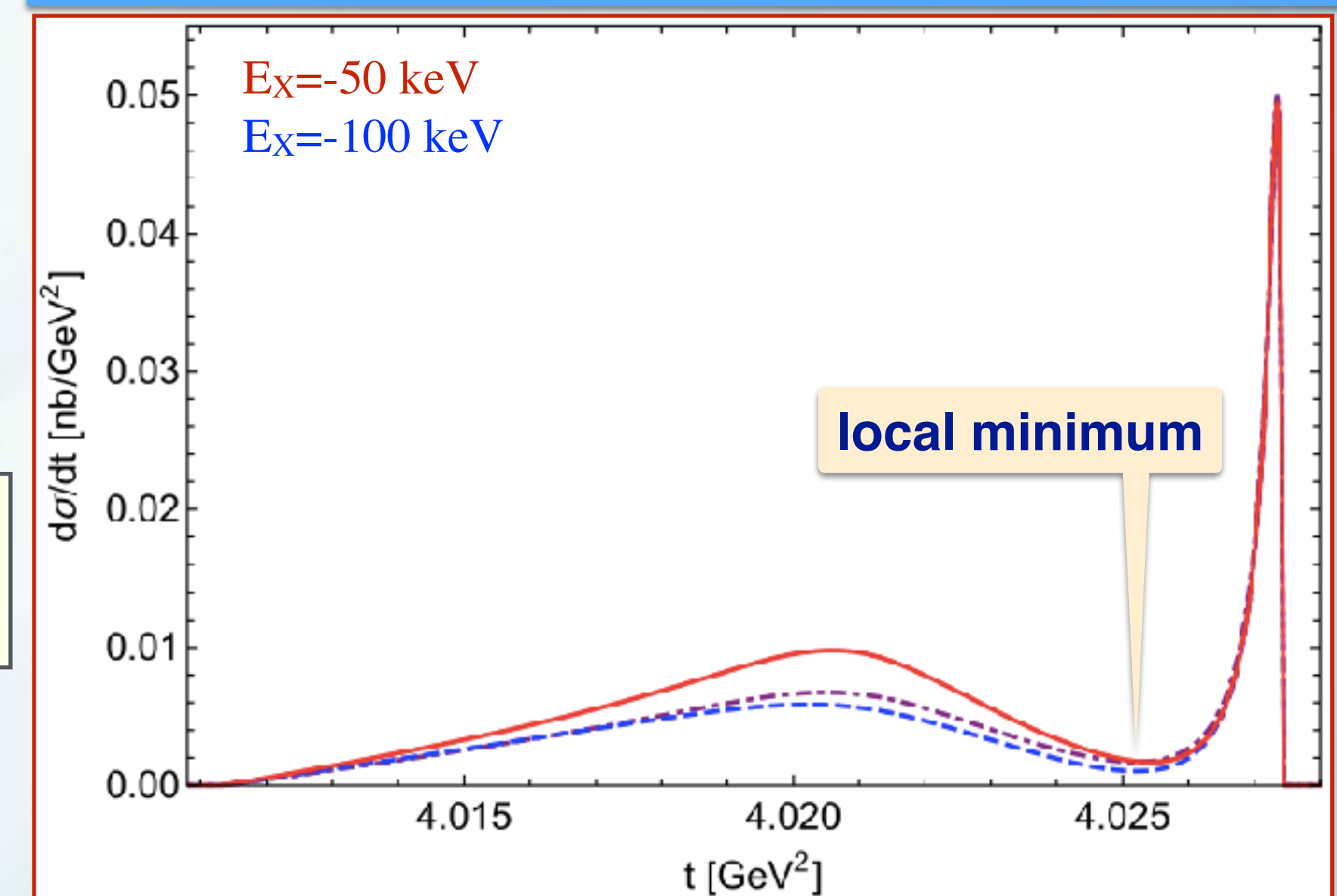
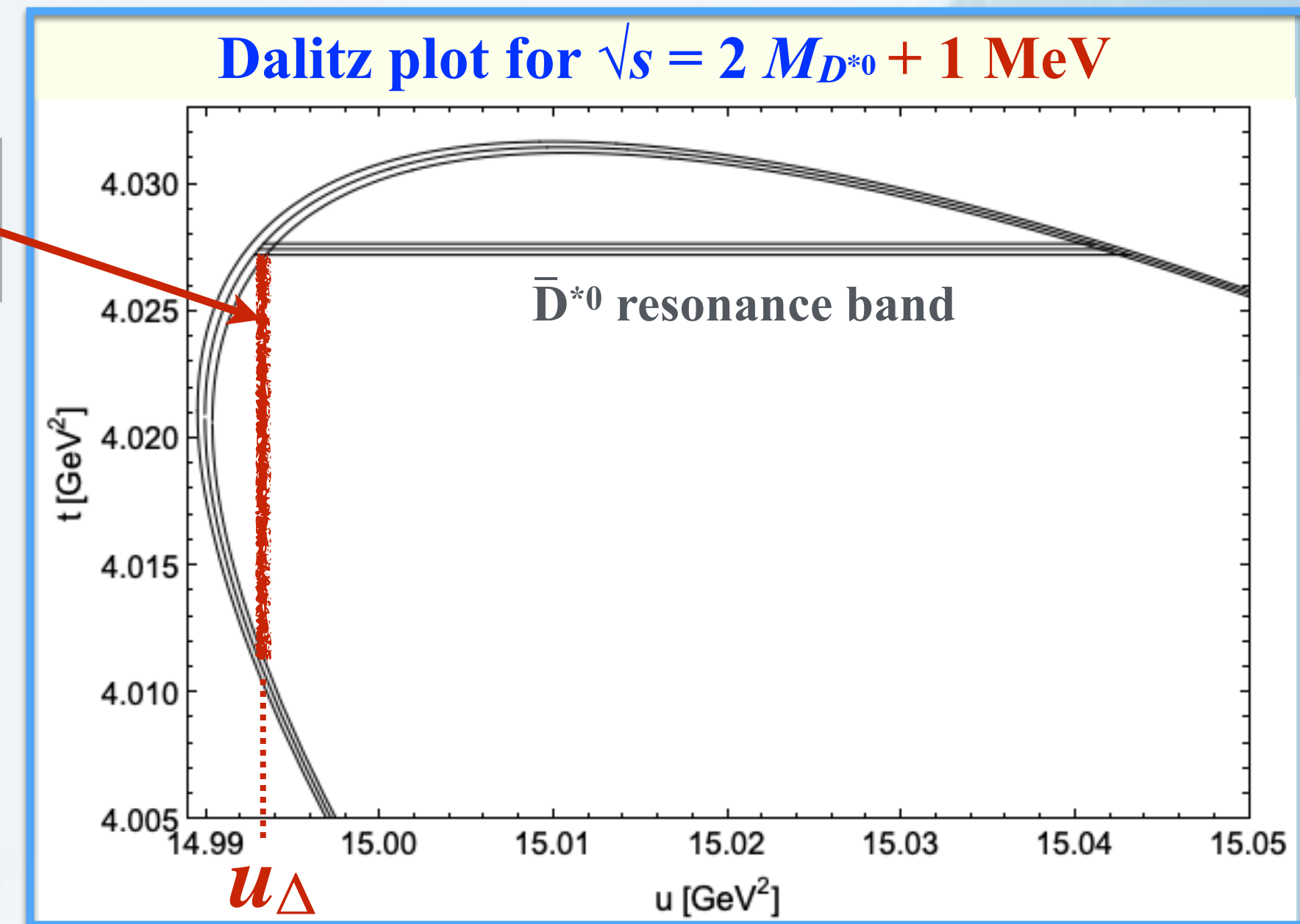
Anisovich & Anisovich [PLB345, 321(1995)]

- $d\sigma/(du dt)$  at fixed  $t$ : **log<sup>2</sup> divergence**
- $d\sigma/du$  integrated over  $t$ : **log divergence**

## \* indirect way to observe triangle singularity:

$d\sigma/dt$  integrated over  $u < u_\Delta$  has **local minimum in  $t$**

triangle singularity  
at  $u = u_\Delta$





# production of $X(3872) + \pi$ from B meson decay

Belle [PRD 91, 051101 (2015)]

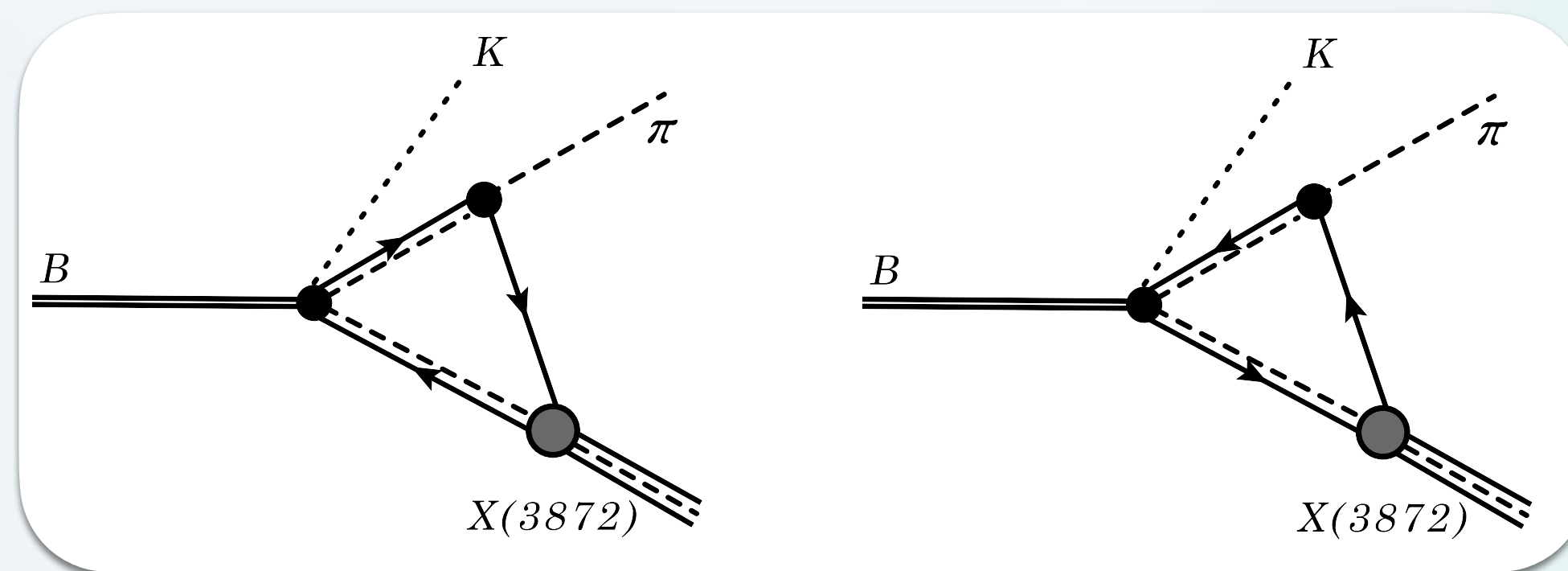
first observation of  $B^0 \rightarrow K^+ \pi^- X$ ,  $B^+ \rightarrow K^0 \pi^+ X$

34% of  $B^0 \rightarrow K^+ \pi^- X$  from  $B^0 \rightarrow K^{*0}(892) X$

Braaten, He, Ingles [PRD 100, 074028(2019)]

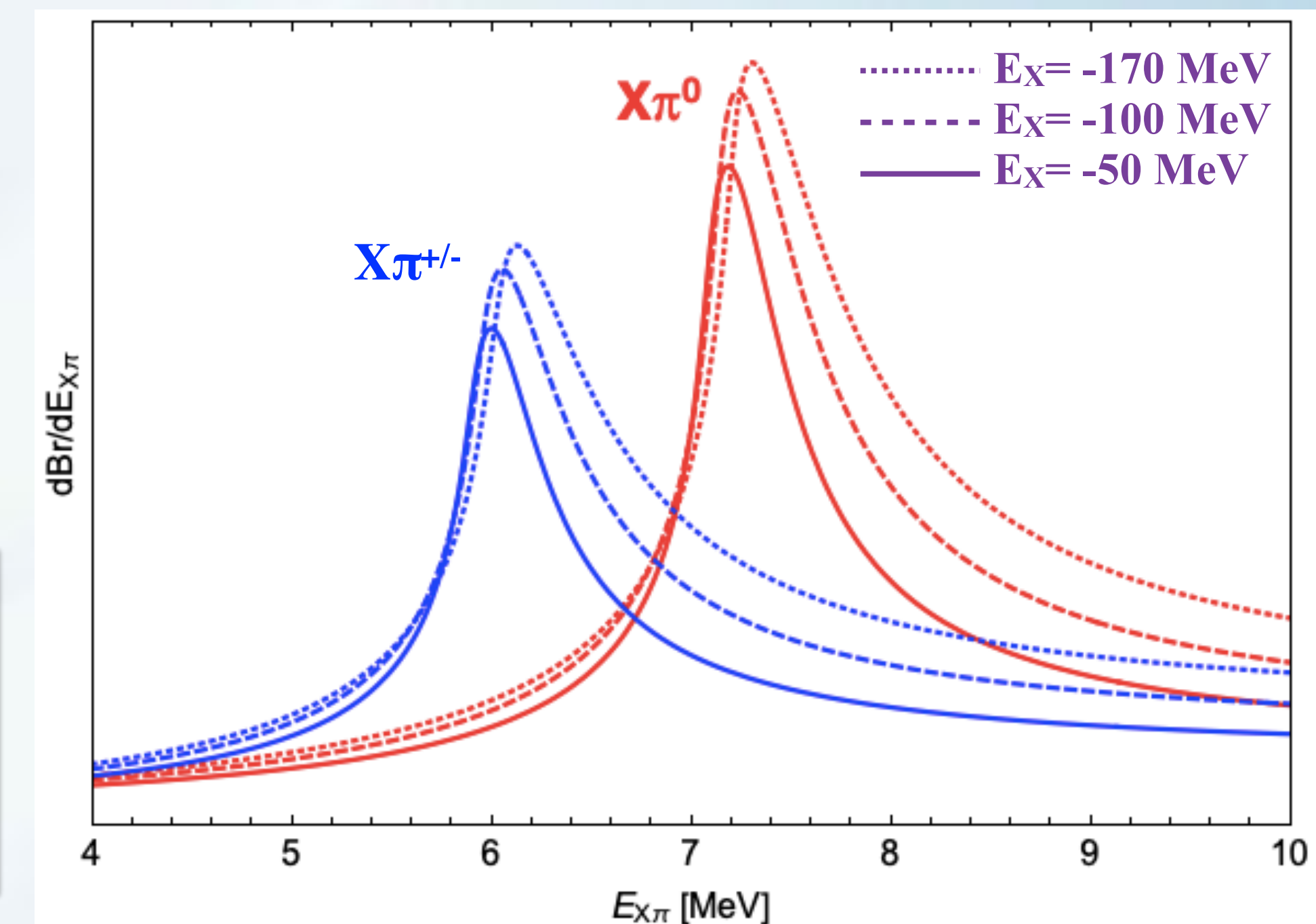
$$B \rightarrow K D^* \bar{D}^* \rightarrow K X \pi$$

decay of B meson into  $K + D^* \bar{D}^*$ , rescattering of virtual  $D^* \bar{D}^*$  into  $X \pi$



triangle singularity produces narrow peaks in  $d\text{Br}[B \rightarrow K X \pi]$

- ❖  $X\pi^\pm$ : near 6.1 MeV above  $X\pi^+$  threshold
- ❖  $X\pi^0$ : near 7.3 MeV above  $X\pi^0$  threshold





# production of $X(3872) + \pi$ from B meson decay

branching fractions for  $X\pi^\pm$  from the peak:

integrated over

$$E_{X\pi} \leq 2\delta_1 = 11.8 \text{ MeV}, \quad \delta_1 = M_{D^{*+}} - M_{D^0} - m_{\pi^+}$$

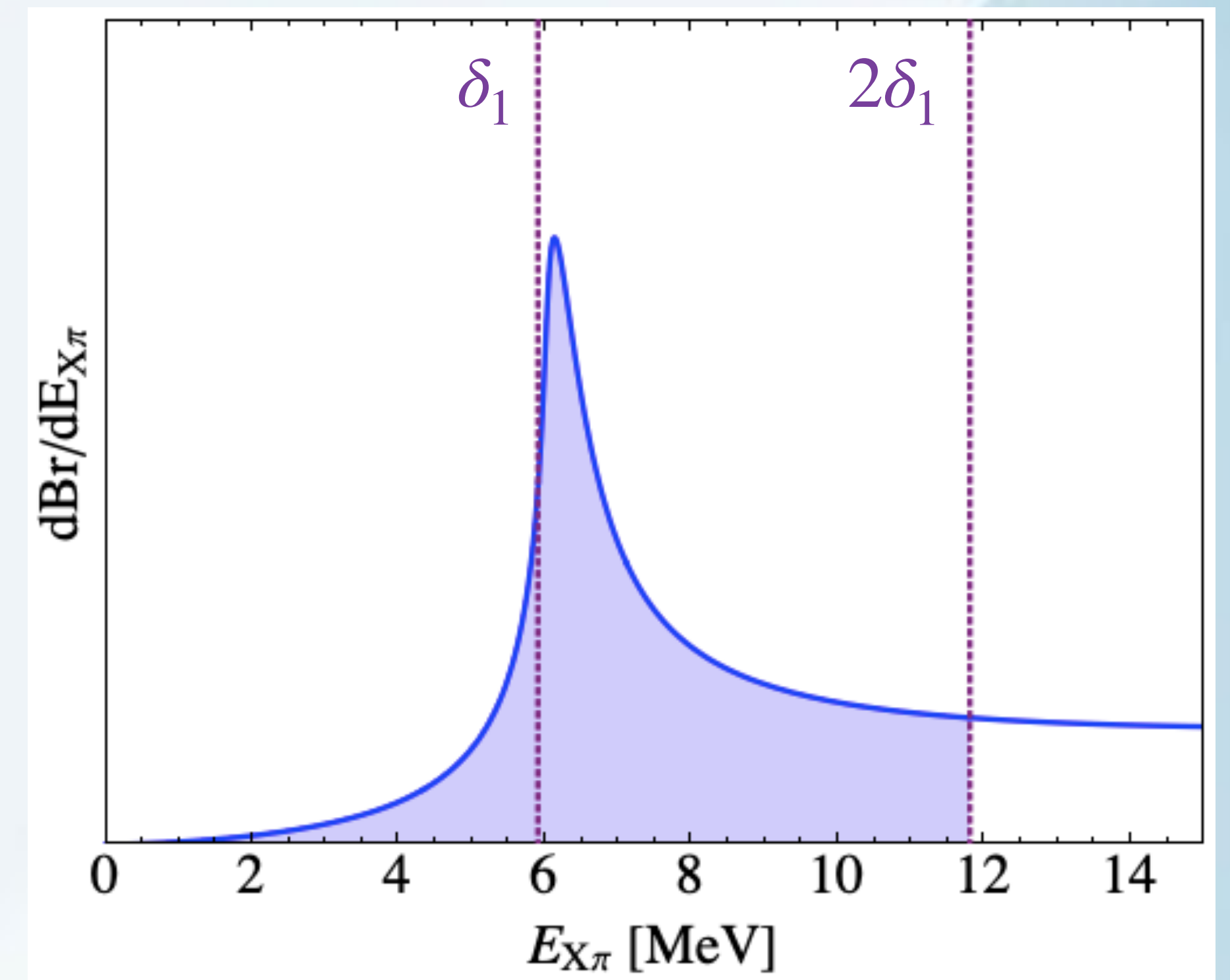
$$\frac{d\Gamma}{d^3q}[B^+ \rightarrow K^0 X \pi^+] = \frac{d\Gamma}{d^3q}[B^0 \rightarrow K^+ X \pi^-]$$

$$\text{Br}[B^0 \rightarrow K^+(X\pi^-)_\Delta] \approx (2.4 \times 10^{-7}) \left( \frac{|E_X|}{0.17 \text{ MeV}} \right)^{1/2} \left[ 2.64 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right]$$

$$|E_X| = 0.17 \text{ MeV}$$

$$6 \times 10^{-7}$$

could contribute an observable fraction of the decay of  $B^0 \rightarrow K^+ X \pi^-$

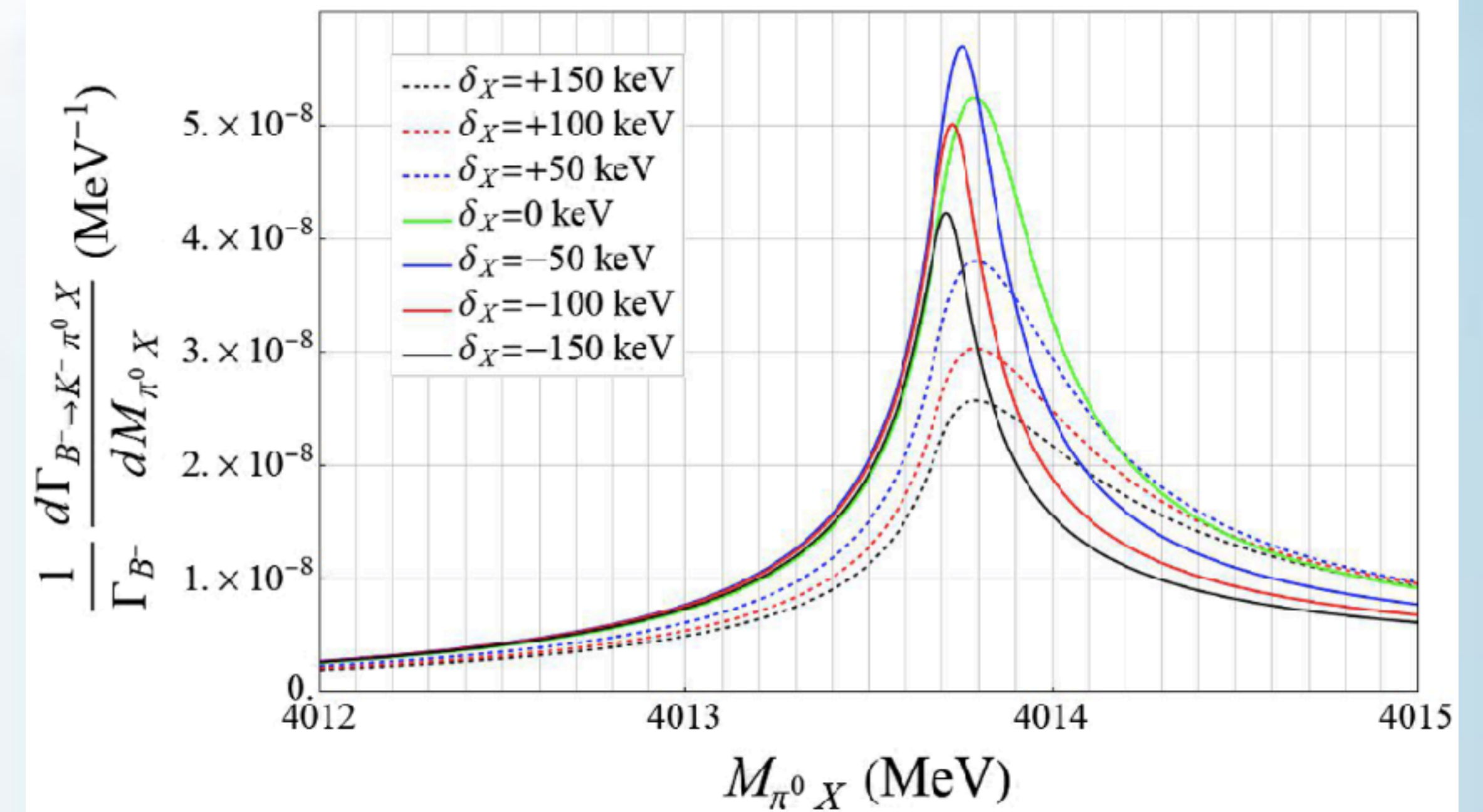


# production of $X(3872) + \pi$ from B meson decay

- Sakai, Oset & Guo [PRD 101, 054030(2020)]

$$B^- \rightarrow K^- D^{*0} \bar{D}^{*0} \rightarrow K^- X \pi^0$$

$E_X (= -\delta_X)$  may be extracted from the asymmetry of the  $X\pi$  line shape



- Nakamura [PRD 102, 074004(2020)]

$$B^0 \rightarrow K^+ D^{*0} D^{*-} \rightarrow K^+ (J/\psi \rho \pi^-)$$

triangle singularity could produce narrow peak in  $J/\psi \rho$  invariant mass near 3872 MeV even without  $X(3872)$  resonance

- Molina & Oset [EPJC 80, 451(2020)]

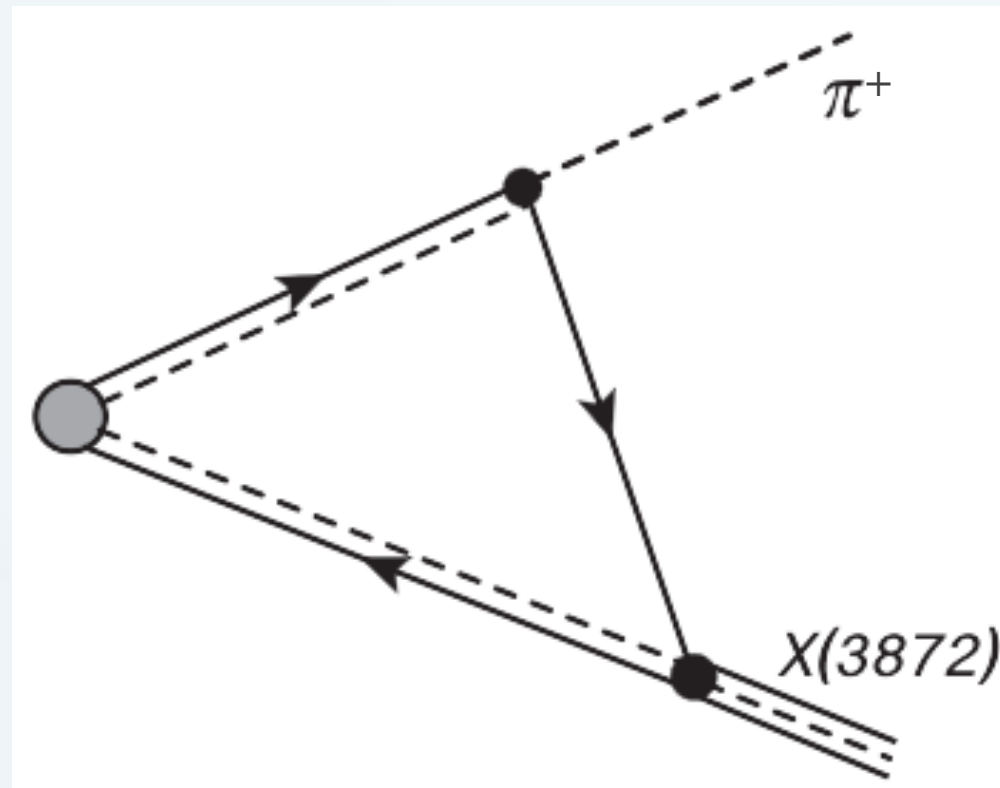
$$B^- \rightarrow K^- X, X \rightarrow (D^{*0} \bar{D}^0, \bar{D}^{*0} D^0) \rightarrow \pi^0 (\pi^+ \pi^-)$$

triangle singularity in decay of  $X$

# prompt production of $X(3872) + \pi$ at Hadron colliders

Braaten, He & Ingles [PRD 100, 094006(2019)]

$D^{*+}\bar{D}^{*0} \rightarrow X(3872)\pi^+$  from prompt production

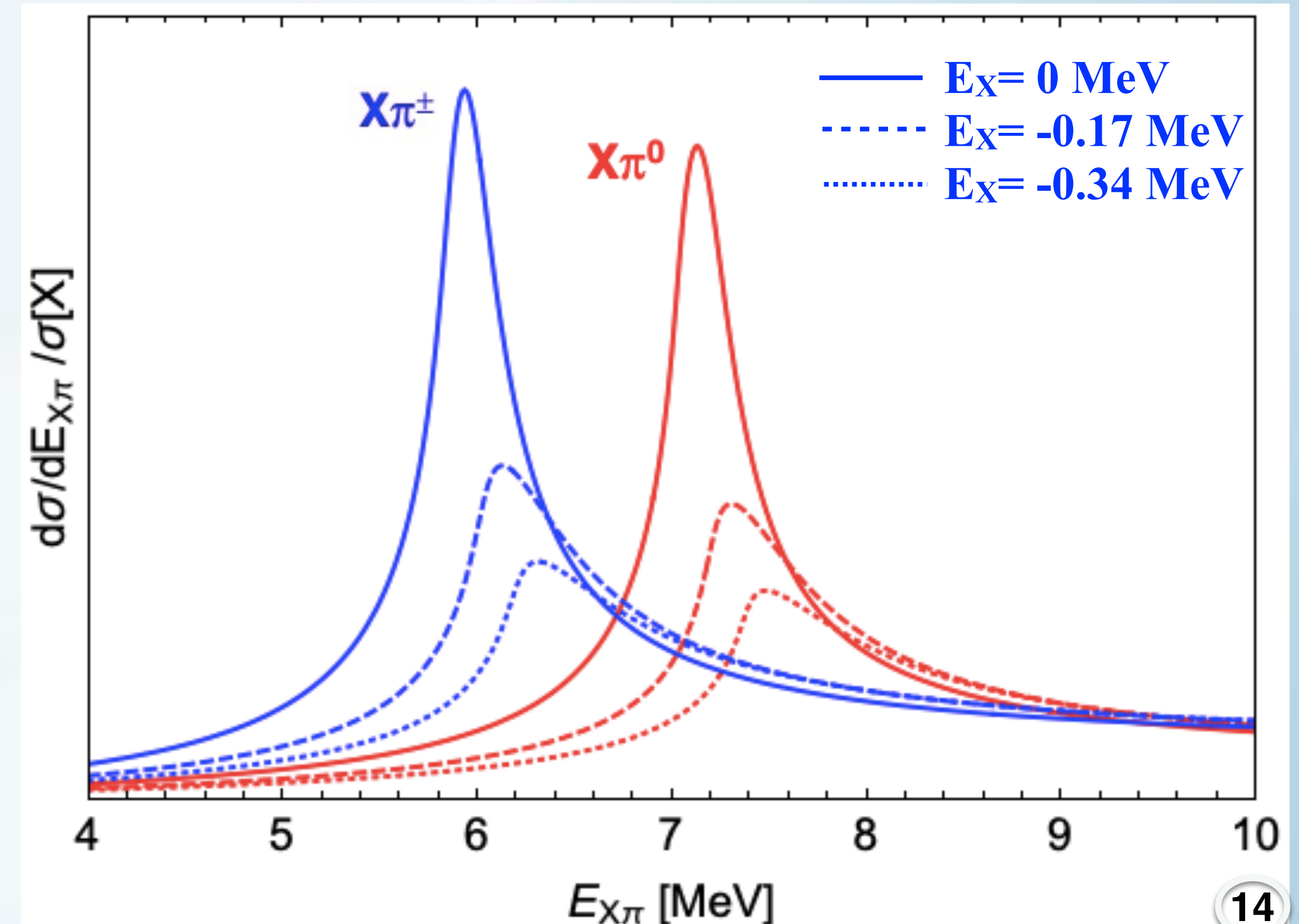


- ❖ creation of  $D^{*+}\bar{D}^{*0}$  at short distance
- ❖ rescattering of **virtual**  $D^{*+}\bar{D}^{*0}$  into  $X\pi^+$

estimated ratio of cross sections:

$$\frac{\sigma[(X\pi^+)_{\Delta}]}{\sigma[X]} \approx 0.028 \left(\frac{m_{\pi}}{\Lambda}\right)^2 \left[2.64 - \log \frac{|E_X|}{0.17 \text{ MeV}}\right]$$

**triangle singularity produces narrow peak in  $X\pi^{\pm}$  invariant mass**  
peak near 6.1 MeV above  $X\pi^+$  threshold





# Experimental observation of $X(3872) + \pi^\pm$ in $p\bar{p}$ collisions

D0 Collaboration [PRD 102, 072005 (2020)]

prompt and b-hadron decay production of  $X(3872) + \text{soft } \pi^\pm$

$T(X\pi) < 11.8 \text{ MeV}$       **observed events**       **$X + \text{random } \pi$**

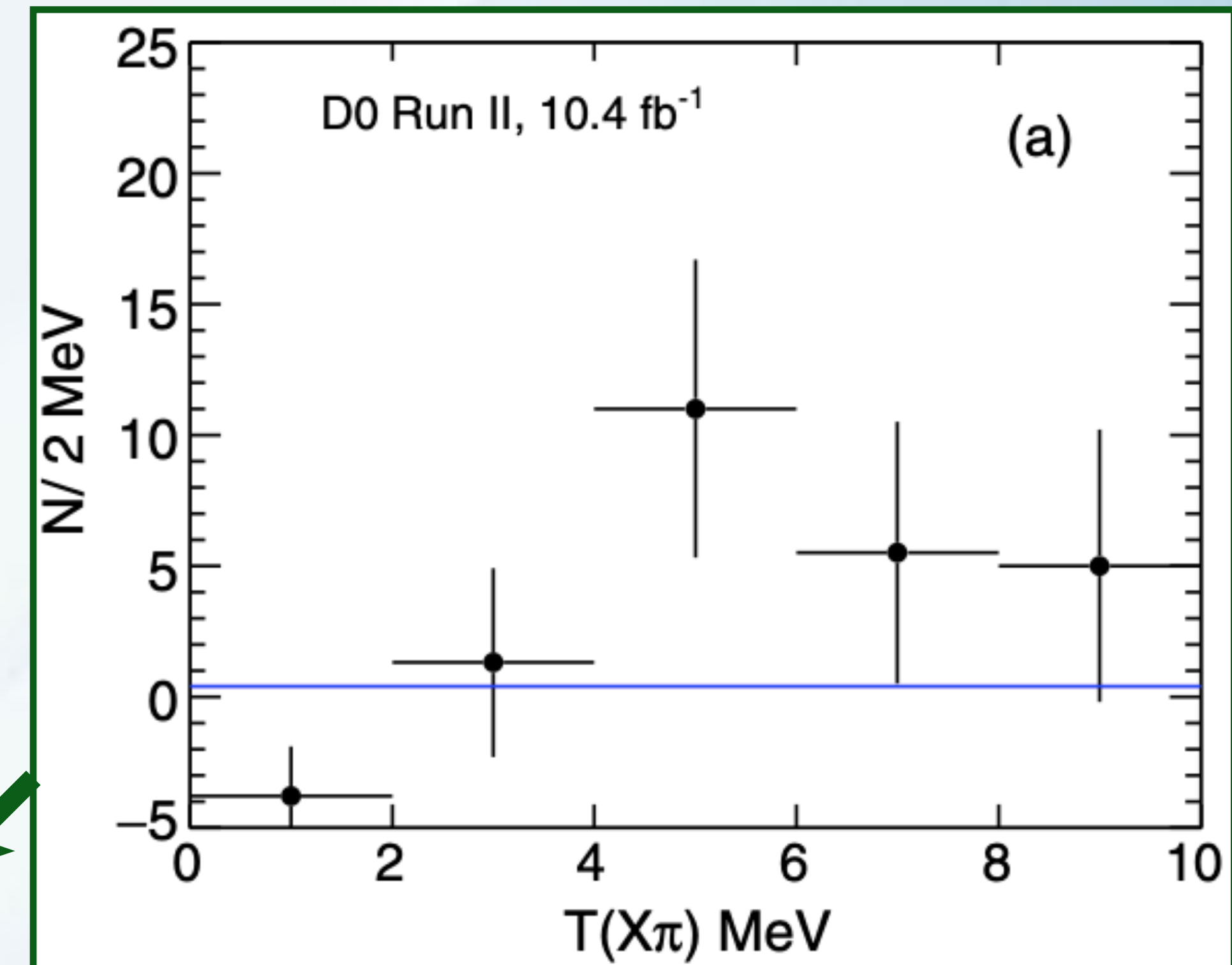
prompt production:                       **$18 \pm 16$**                       **6**

b-decay:                                       **$27 \pm 12$**                       **2**

conclusions:

- \* **prompt production:** no evidence for an enhancement as expected from the triangle singularity
- \* **b-decay:** no “significant” evidence for an enhancement as expected from the triangle singularity

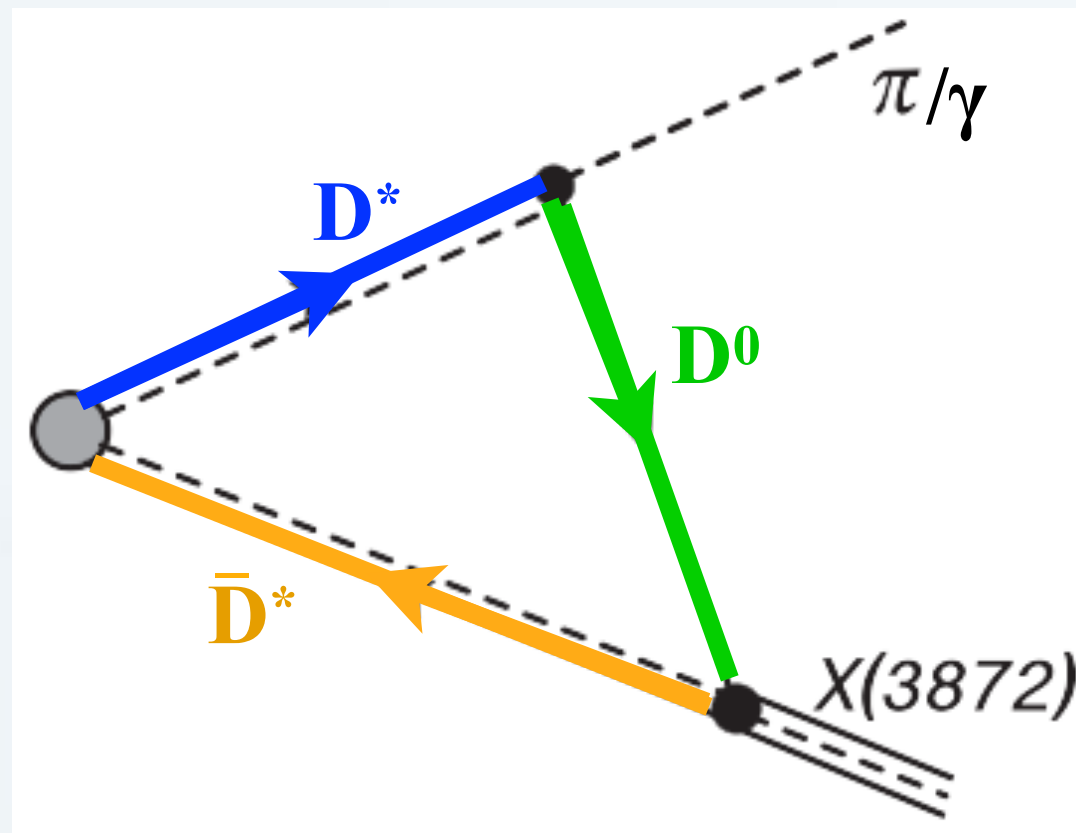
a small excess in small  $T(X\pi)$  region, significance of  $2\sigma$



# Summary

## Production of $X+\gamma$ or $X+\pi$

charm meson triangle singularity produces **narrow peaks** just above  $D^*\bar{D}^*$  threshold



Thank you!

### ■ $e^+e^-$ annihilation

- ◇  $\sigma[X\gamma]$ : narrow peak at 4015.9 MeV
- ◇ peak is in region not yet measured by BESIII

### ■ B meson decay

- ◇  $d\text{Br}[X\pi^0]/dE_{X\pi}$ : peak near 7.3 MeV above  $X\pi^0$  threshold
- ◇  $d\text{Br}[X\pi^\pm]/dE_{X\pi}$ : peak near 6.1 MeV above  $X\pi^+$  threshold
- ◇ could be observed by Belle II or LHCb

### ■ Hadron colliders

- ◇  $d\sigma[X\pi^\pm]/dE_{X\pi}$ : peak near 6.1 MeV above  $X\pi^+$  threshold
- ◇ hint of peak at  $p\bar{p}$  collider by D0
- ◇ could be observed at  $pp$  collider by LHCb, CMS, ATLAS

The observation of the peaks would definitely resolve the nature of X(3872)

# Backup

## B meson decay

$$\frac{d\Gamma}{d^3q}[B^+ \rightarrow K^+ X \pi^0] = \frac{|\mathcal{A}[K^+ X \pi^0]|^2}{4|\mathcal{A}[K^0 X \pi^0]|^2} \frac{d\Gamma}{d^3q}[B^0 \rightarrow K^0 X \pi^0], \quad (36a)$$

$$\frac{d\Gamma}{d^3q}[B^+ \rightarrow K^0 X \pi^+] = \frac{d\Gamma}{d^3q}[B^0 \rightarrow K^+ X \pi^-]. \quad (36b)$$

$$\text{Br}[B^0 \rightarrow K^+(X\pi^-)_\Delta] \approx (2.4 \times 10^{-7}) \left( \frac{|E_X|}{0.17 \text{ MeV}} \right)^{1/2} \times \left[ 2.64 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right].$$

$$\text{Br}[B^0 \rightarrow K^+ X \pi^-] \text{Br}[X \rightarrow J\psi \pi^+ \pi^-] = (7.9 \pm 1.3 \pm 0.4) \times 10^{-6}$$

## Hadron collider

$$d\sigma[D^{*0} \bar{D}^{*0}] \approx d\sigma[X(3872)] \frac{12\pi\mu}{\gamma_X \Lambda^2} \frac{d^3k}{(2\pi)^3 M_{*0}}$$

$$\frac{\sigma[(X\pi^0)_\Delta]}{\sigma[X]} \approx 0.049 \left( \frac{m_\pi}{\Lambda} \right)^2 \left[ 2.82 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right],$$

$$\frac{\sigma[(X\pi^+)_\Delta]}{\sigma[X]} \approx 0.028 \left( \frac{m_\pi}{\Lambda} \right)^2 \left[ 2.64 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right].$$

$$\text{Br}[B^0 \rightarrow K^0(X\pi^0)_\Delta] < (8 \times 10^{-8}) \left( \frac{|E_X|}{0.17 \text{ MeV}} \right)^{1/2} \times \left[ 2.82 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right].$$

- ◆ **prompt production** by **QCD mechanisms**:  
 decay products emerge from primary collision vertex
- ◆ **production by *b* hadron decay**:  
 decay products emerge from displaced secondary vertex