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Study of *N(1440)* structure via $\gamma p \to$*N(1440)* transition

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- Results and discussions
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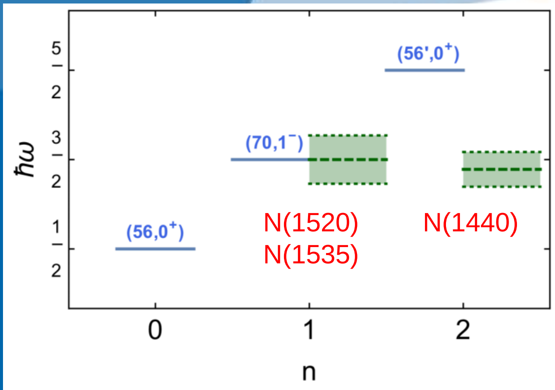
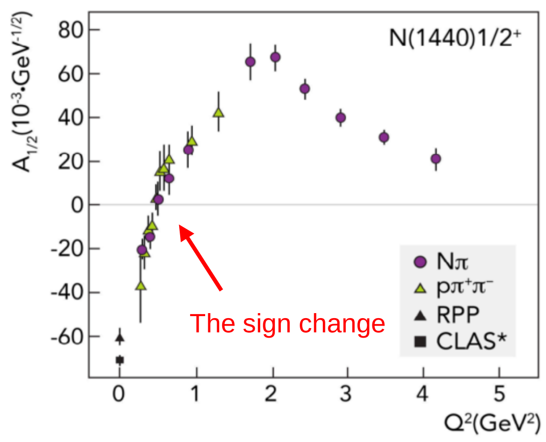
Introduction

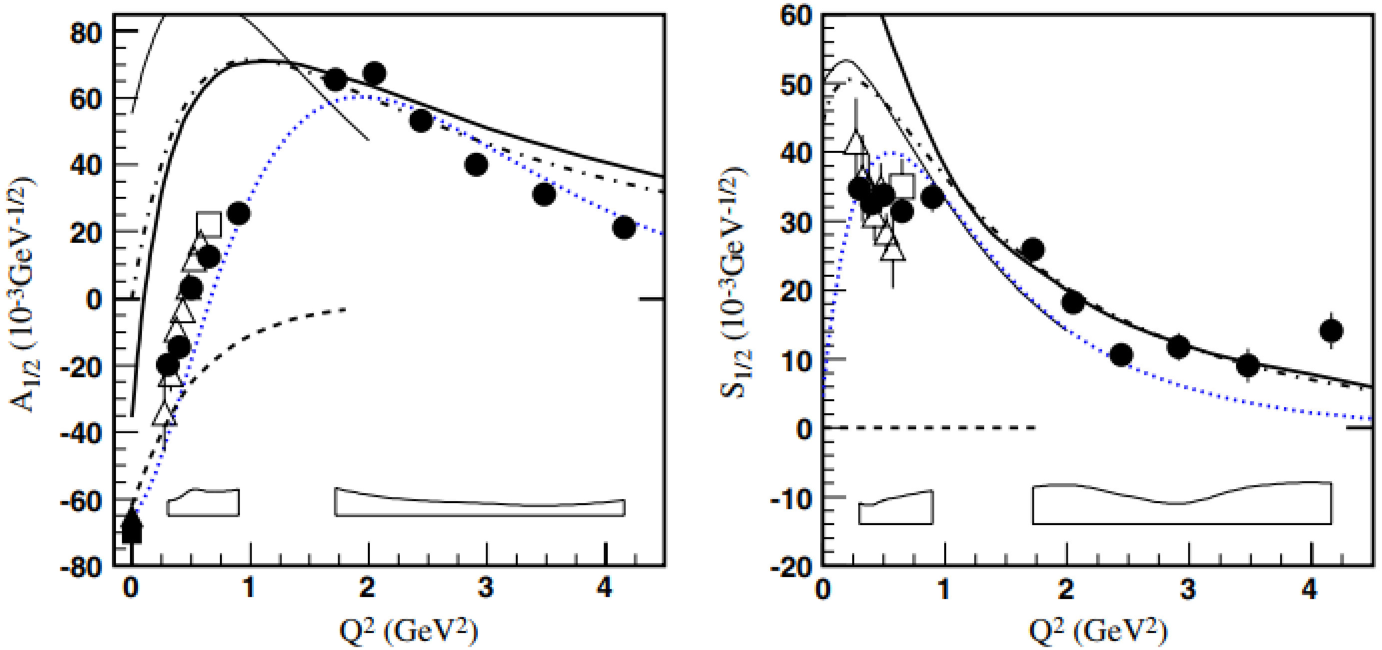




- In the traditional q^3 picture, the Roper $N_{1/2+}(1440)$ usually gets a mass $\sim$ 100 MeV above the $N_{3/2-}(1520)$ and $N_{1/2-}(1535)$, but not 100 MeV below it.
- Roper resonance is usually blamed sitting at a wrong place or intruding the q^3 spectrum. It could be $q^4\bar{q}$ pentaquark, q^3g hybrid, $q^3(q\bar{q})$ resonance...
- The sign change in the helicity amplitude of the Roper resonance as a function of Q^2 suggests a node in the wave function and thus a radially excited state.
- In the $N_{3/2-}(1520)$ decay modes, branching ratio of $\Gamma_{\eta N}/\Gamma_{tot}$ is almost zero.
- In the $N_{1/2-}(1535)$ decay modes, the branching ratio of $N(1535) \to N\eta$ process is as large as $N(1535) \to N\pi$.
- $N_{1/2+}(1440)$ may mainly be a q^3 first radial excitation
- $N_{3/2-}(1520)$ may contain a large ground-state ($l = 0$) non-strange pentaquark component.
- $N_{1/2-}(1535)$ may contain a large ground-state ($l = 0$) $uuds\bar{s}$ component.

Helicity amplitudes of N(1440) resonance

I.G. Aznauryan, V.D. Burkert / Progress in Particle and Nuclear Physics 67 (2012) 1–54



- The thick curves correspond to quark models assuming that N(1440) is a three-quark first radial excitation: dash dotted (Capstick and Keister, 1995), solid (Aznauryan, 2007).
- The thin dashed curves are obtained assuming that N(1440) is a three-quark hybrid state (Li et al., 1992).
- The thin solid curves are non-relativistic quark model (Li et al., 1992)
- The blue dotted curves are the MAID2007 fitting (global analysis).

The solid circles are from CLAS (Aznauryan, 2008).
The solid triangle is the RPP estimate (Nagamura et al.,2010).
The open triangles are from Mokeev, et al., 2009.

Formalism

Wave function of proton and *N(1440)* in the harmonic oscillator basis

The proton and *N(1440)* spatial wave function in a three-quark picture:

$$\psi^O = \sum_{N=0,2,4,\ldots} A_{N00}\Psi^O_{N00}.$$

The corresponding algebraic structure consists of the usual spin, flavor, and color algebras :

$$SU_f(3) \otimes SU_s(2) \otimes SU_c(3)$$

Harmonic oscillator basis for 3q system

$$H^{q^3} = \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + \frac{1}{2}C\left(\lambda^2 + \rho^2\right)$$

Harmonic oscillator basis

$$\Psi^{\mathrm{O}}_{NLM} = \sum_{n_\lambda, n_\rho, l_\lambda, l_\rho} A(n_\lambda, n_\rho, l_\lambda, l_\rho)\psi_{n_\lambda l_\lambda m_\lambda}(\boldsymbol{\lambda}) C(l_\lambda, m_\lambda; l_\rho, m_\rho; l_{\lambda\rho}, m_{\lambda\rho})\psi_{n_\lambda l_\lambda m_\lambda}(\boldsymbol{\lambda})\psi_{n_\rho l_\rho m_\rho}(\boldsymbol{\rho})$$

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\boldsymbol{r}_1 - \boldsymbol{r}_2),$$

$$\boldsymbol{\lambda} = \frac{1}{\sqrt{6}}(\boldsymbol{r}_1 + \boldsymbol{r}_2 - 2\boldsymbol{r}_3)$$

$$\boldsymbol{R} = \frac{1}{\sqrt{3}}(\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3)$$

$$N = 2(n_\lambda + n_\rho) + l_\lambda + l_\rho$$

$$\psi_{nlm}(\boldsymbol{r}) = \left[\frac{2\alpha^3 n!}{(\frac{1}{2} + n + l)!}\right]^{\frac{1}{2}} (\alpha r)^l e^{-\frac{1}{2}\alpha^2 r^2} L_n^{l+\frac{1}{2}}(\alpha^2 r^2) Y_{lm}(\hat{\boldsymbol{r}})$$

$$\left\langle \Psi^{\mathrm{O}}_{NLM} \,\middle|\, \Psi^{\mathrm{O}}_{N'L'M'} \right\rangle = \delta_{NN'}\delta_{LL'}\delta_{MM'}$$

$$\left\langle \psi_{nlm} \,\middle|\, \psi_{n'l'm'} \right\rangle = \delta_{nn'}\delta_{ll'}\delta_{mm'}$$

A relation between momentum space and position space

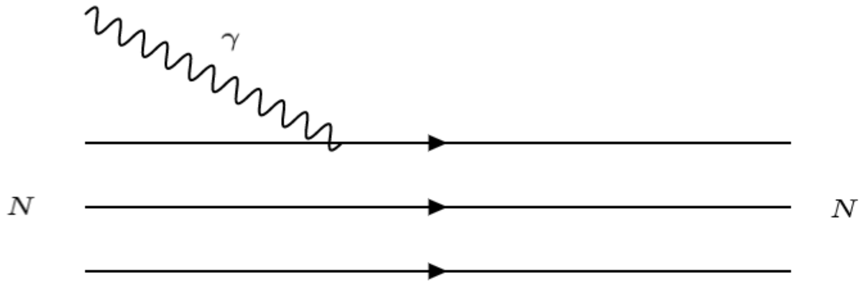
$$\Psi^{\mathrm{O}}_{NLM}(\{\boldsymbol{r}\}) = (-i)^N \Psi^{\mathrm{O}}_{NLM}(\{\boldsymbol{p}\})$$

Explicit form of harmonic oscillator basis

NLM	Wave function
0 00	$\Psi^{O}_{000} = \psi_{000}(\boldsymbol{\rho})\psi_{000}(\boldsymbol{\lambda})$
2 00	$\Psi^{O}_{200} = \frac{1}{\sqrt{2}}[\psi_{100}(\boldsymbol{\rho})\psi_{000}(\boldsymbol{\lambda}) + \psi_{000}(\boldsymbol{\rho})\psi_{100}(\boldsymbol{\lambda})]$
4 00	$\Psi^{O}_{400} = \frac{\sqrt{5}}{4}[\psi_{200}(\boldsymbol{\rho})\psi_{000}(\boldsymbol{\lambda}) + \sqrt{\frac{6}{5}}\psi_{100}(\boldsymbol{\rho})\psi_{100}(\boldsymbol{\lambda}) + \psi_{000}(\boldsymbol{\rho})\psi_{200}(\boldsymbol{\lambda})]$
6 00	$\Psi^{O}_{600} = \frac{\sqrt{14}}{8}[\psi_{300}(\boldsymbol{\rho})\psi_{000}(\boldsymbol{\lambda}) + \frac{3}{\sqrt{7}}\psi_{200}(\boldsymbol{\rho})\psi_{100}(\boldsymbol{\lambda}) + \frac{3}{\sqrt{7}}\psi_{100}(\boldsymbol{\rho})\psi_{200}(\boldsymbol{\lambda})$ $+\psi_{000}(\boldsymbol{\rho})\psi_{300}(\boldsymbol{\lambda})]$
8 00	$\Psi^{O}_{800} = \frac{\sqrt{42}}{16}[\psi_{400}(\boldsymbol{\rho})\psi_{000}(\boldsymbol{\lambda}) + \frac{2}{\sqrt{3}}\psi_{300}(\boldsymbol{\rho})\psi_{100}(\boldsymbol{\lambda}) + \sqrt{\frac{10}{7}}\psi_{200}(\boldsymbol{\rho})\psi_{200}(\boldsymbol{\lambda})]$ $+\frac{2}{\sqrt{3}}\psi_{100}(\boldsymbol{\rho})\psi_{300}(\boldsymbol{\lambda}) + \psi_{000}(\boldsymbol{\rho})\psi_{400}(\boldsymbol{\lambda})]$
10 00	$\Psi^{O}_{1000} = \frac{\sqrt{33}}{16}[\psi_{500}(\boldsymbol{\rho})\psi_{000}(\boldsymbol{\lambda}) + \sqrt{\frac{15}{11}}\psi_{400}(\boldsymbol{\rho})\psi_{100}(\boldsymbol{\lambda}) + 5\sqrt{\frac{2}{33}}\psi_{300}(\boldsymbol{\rho})\psi_{200}(\boldsymbol{\lambda})]$ $+5\sqrt{\frac{2}{33}}\psi_{200}(\boldsymbol{\rho})\psi_{300}(\boldsymbol{\lambda}) + \sqrt{\frac{15}{11}}\psi_{100}(\boldsymbol{\rho})\psi_{400}(\boldsymbol{\lambda}) + \psi_{000}(\boldsymbol{\rho})\psi_{500}(\boldsymbol{\lambda})]$
12 00	$\Psi^{O}_{1200} = \frac{\sqrt{429}}{64}[\psi_{600}(\boldsymbol{\rho})\psi_{000}(\boldsymbol{\lambda}) + 3\sqrt{\frac{2}{13}}\psi_{500}(\boldsymbol{\rho})\psi_{100}(\boldsymbol{\lambda}) + \frac{15}{\sqrt{143}}\psi_{400}(\boldsymbol{\rho})\psi_{200}(\boldsymbol{\lambda})]$ $+10\sqrt{\frac{7}{429}}\psi_{300}(\boldsymbol{\rho})\psi_{300}(\boldsymbol{\lambda}) + \frac{15}{\sqrt{143}}\psi_{200}(\boldsymbol{\rho})\psi_{400}(\boldsymbol{\lambda}) + 3\sqrt{\frac{2}{13}}\psi_{100}(\boldsymbol{\rho})\psi_{500}(\boldsymbol{\lambda})]$ $+\psi_{000}(\boldsymbol{\rho})\psi_{600}(\boldsymbol{\lambda})]$

(K. Xu et at., 2020)

Electric Form Factor



Breit frame $P_i = (E_N, 0, 0, -|\boldsymbol{k}/2|), \quad P_f = (E_N, 0, 0, |\boldsymbol{k}/2|), \quad k = (0, 0, 0, |\boldsymbol{k}|)$

$$Q^2 = -k^2 = |\boldsymbol{k}|^2, \qquad \boldsymbol{P}_i + \boldsymbol{P}_f = 0$$

Lorentz invariance

$$G_E = \left\langle N, S'_z = \frac{1}{2} \middle| q'_1 q'_2 q'_3 \right\rangle T_B(q_1 q_2 q_3 \to q'_1 q'_2 q'_3) \left\langle q_1 q_2 q_3 \middle| N, S_z = \frac{1}{2} \right\rangle \Big|_{\mathrm{Breit}}$$

$$\begin{aligned} & T_B(q_1 q_2 q_3 \to q'_1 q'_2 q'_3) \\ &= e_3 \bar{u}_{s'}(p') \gamma^\mu u_s(p) \epsilon_\mu(k) \langle q'_1 q'_2 \,|\, q_1 q_2 \rangle \\ &= e_3 T^B_{s's} \langle q'_1 q'_2 \,|\, q_1 q_2 \rangle \,. \end{aligned}$$

$$\epsilon = (1, 0, 0, 0)$$

$$p_\pm = \frac{1}{\sqrt{2}}(p_x \pm i p_y)$$

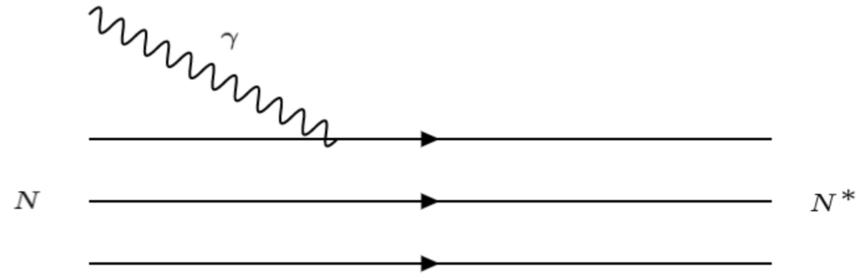
$$T^B_{\uparrow\uparrow} = \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left(1 + \frac{p'_z p_z + 2p'_- p_+}{(E'+m)(E+m)}\right),$$

$$T^B_{\uparrow\downarrow} = \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left(\frac{\sqrt{2}(p'_z p_- - p'_- p_z)}{(E'+m)(E+m)}\right),$$

$$T^B_{\downarrow\uparrow} = \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left(\frac{\sqrt{2}(-p'_z p_+ + p'_+ p_z)}{(E'+m)(E+m)}\right),$$

$$T^B_{\downarrow\downarrow} = \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left(1 + \frac{p'_z p_z + 2p'_+ p_-}{(E'+m)(E+m)}\right),$$

Helicity amplitudes (1)



N* rest frame

$$P_i = (E_N, 0, 0, -|\boldsymbol{k}|), \quad P_f = (M_{N^*}, 0, 0, 0), \quad k = (\omega, 0, 0, |\boldsymbol{k}|)$$

$$\omega = \frac{M_{N^*}^2 - M_N^2 - Q^2}{2M_{N^*}} \qquad |\boldsymbol{k}| = \left[Q^2 + \left(\frac{M_{N^*}^2 - M_N^2 - Q^2}{2M_{N^*}}\right)^2\right]^{\frac{1}{2}} \qquad Q^2 = -k^2$$

$$A_{1/2} = \frac{1}{\sqrt{2K}} \left\langle N^*, S'_z = \frac{1}{2} \middle| q'_1 q'_2 q'_3 \right\rangle T^{+}_{s's}(q_1 q_2 q_3 \to q'_1 q'_2 q'_3) \left\langle q_1 q_2 q_3 \middle| N, S_z = -\frac{1}{2} \right\rangle,$$

$$A_{3/2} = \frac{1}{\sqrt{2K}} \left\langle N^*, S'_z = \frac{3}{2} \middle| q'_1 q'_2 q'_3 \right\rangle T^{+}_{s's}(q_1 q_2 q_3 \to q'_1 q'_2 q'_3) \left\langle q_1 q_2 q_3 \middle| N, S_z = \frac{1}{2} \right\rangle,$$

$$S_{1/2} = \frac{1}{\sqrt{2K}} \left\langle N^*, S'_z = \frac{1}{2} \middle| q'_1 q'_2 q'_3 \right\rangle T^{0}_{s's}(q_1 q_2 q_3 \to q'_1 q'_2 q'_3) \left\langle q_1 q_2 q_3 \middle| N, S_z = \frac{1}{2} \right\rangle \frac{|\boldsymbol{k}|}{Q}$$

$$K = \frac{M_{N^*}^2 - M_N^2}{2M_{N^*}}$$

Heliciy amplitudes (2)

$$
\begin{aligned}
T(q_1q_2q_3 \to q_1'q_2'q_3') &= e_3\bar{u}_{s'}(p')\gamma^\mu u_s(p)\epsilon_\mu^\lambda(k)\,\langle q_1'q_2' \,|\, q_1q_2\rangle \\
&= e_3 T^\lambda_{s's}\,\langle q_1'q_2' \,|\, q_1q_2\rangle
\end{aligned}
$$

$$
\epsilon_\mu^0 = \frac{1}{Q}(|\boldsymbol{k}|,0,0,\omega), \quad \epsilon_\mu^+ = -\frac{1}{\sqrt{2}}(0,1,i,0).
$$

$$
\begin{aligned}
T^0_{\uparrow\uparrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{|\boldsymbol{k}|}{Q}\left(1+\frac{p_z'p_z+2p_-'p_+}{(E'+m)(E+m)}\right)\right. \\
&\quad \left. -\frac{\omega}{Q}\left(\frac{p_z}{E+m}+\frac{p_z'}{E'+m}\right)\right] \\
T^0_{\uparrow\downarrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{|\boldsymbol{k}|}{Q}\left(\frac{\sqrt{2}(p_z'p_- - p_-'p_z)}{(E'+m)(E+m)}\right)\right. \\
&\quad \left. -\frac{\omega}{Q}\left(\frac{\sqrt{2}p_-}{E+m}-\frac{\sqrt{2}p_-'}{E'+m}\right)\right] \\
T^0_{\downarrow\uparrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{|\boldsymbol{k}|}{Q}\left(\frac{\sqrt{2}(-p_z'p_+ + p_+'p_z)}{(E'+m)(E+m)}\right)\right. \\
&\quad \left. -\frac{\omega}{Q}\left(\frac{-\sqrt{2}p_+}{E+m}+\frac{\sqrt{2}p_+'}{E'+m})\right)\right] \\
T^0_{\downarrow\downarrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{|\boldsymbol{k}|}{Q}\left(1+\frac{p_z'p_z+2p_+'p_-}{(E'+m)(E+m)}\right)\right. \\
&\quad \left. -\frac{\omega}{Q}\left(\frac{p_z}{E+m}+\frac{p_z'}{E'+m}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
T^+_{\uparrow\uparrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{2p_+}{E+m}\right] \\
T^+_{\uparrow\downarrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[-\frac{\sqrt{2}p_z}{E+m}+\frac{\sqrt{2}p_z'}{E'+m}\right] \\
T^+_{\downarrow\uparrow} &= 0 \\
T^+_{\downarrow\downarrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{2p_+'}{E'+m}\right]
\end{aligned}
$$

Proton electric form factor from the fitting of low-lying baryon mass spectra

The proton wave function

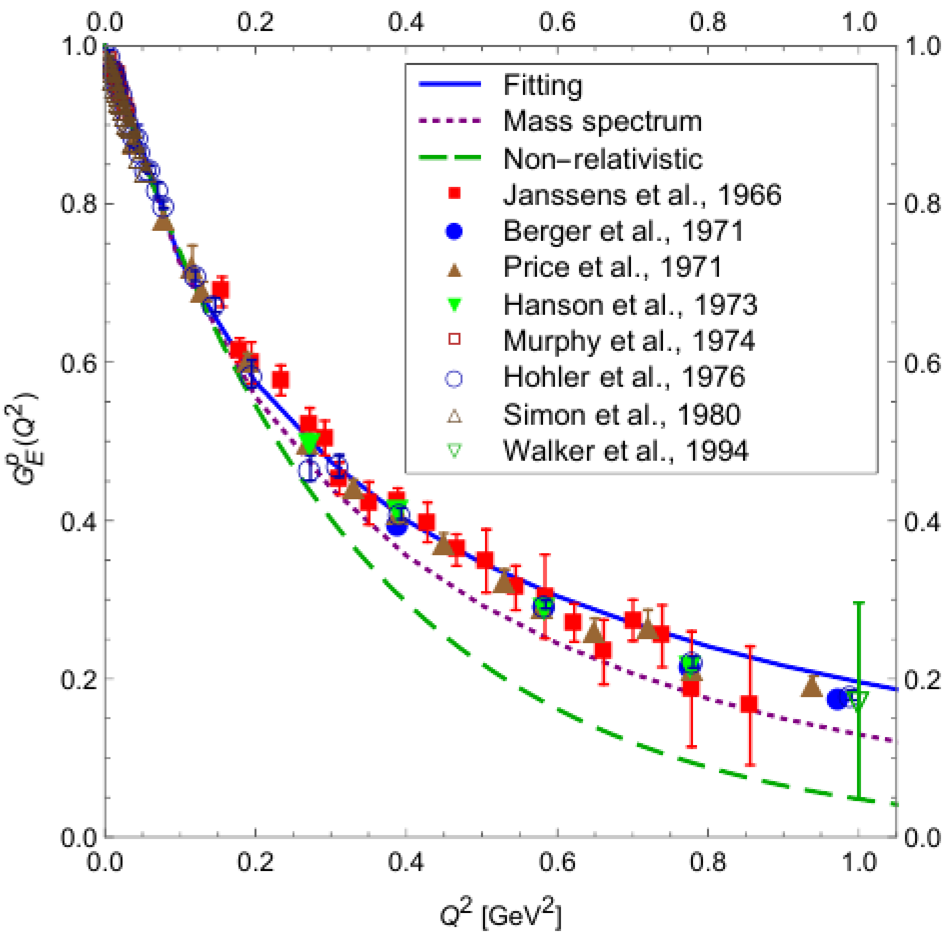
- Being the L=0 lowest state of the low-lying baryon mass spectra in the three-quark picture (K. Xu et at., 2020).

Extraction of proton spatial wave fucntion from electric form factor

The proton wave function

- Being the L=0 lowest nucleon state in the three-quark picture.
- The dimension of the basis is 6: $N = 0, 2, 4, ..., 10$
- The length parameter $\alpha = 400 \ \mathrm{MeV}$
- Fit by the least square method

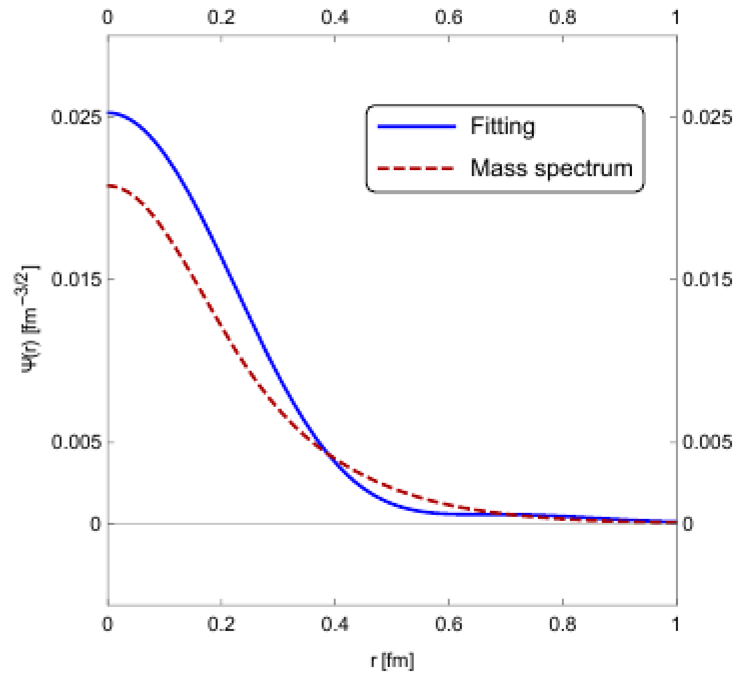
Results of the proton electric form factor



Non-relativistic
- The Result is in good agreement only in the small Q^2 region.

Mass spectrum
- The result almost repeats the data.

Fitting
- The proton is mainly the three-quark bound state.



Helicity amplitudes of N(1440) with the wave function from mass spectrum fitting

The proton wave function

- Being the L=0 lowest state of nucleon resonance of the low-lying baryon mass spectra in the three-quark picture.

The N(1440) wave function

- Being the L=0 first radial excited state of nucleon resonance of the low-lying baryon mass spectra in the three-quark picture.

Extraction of quark distribution of N(1440)

The proton wave function

- Imported from the section of the proton form factor.

The N(1440) wave function

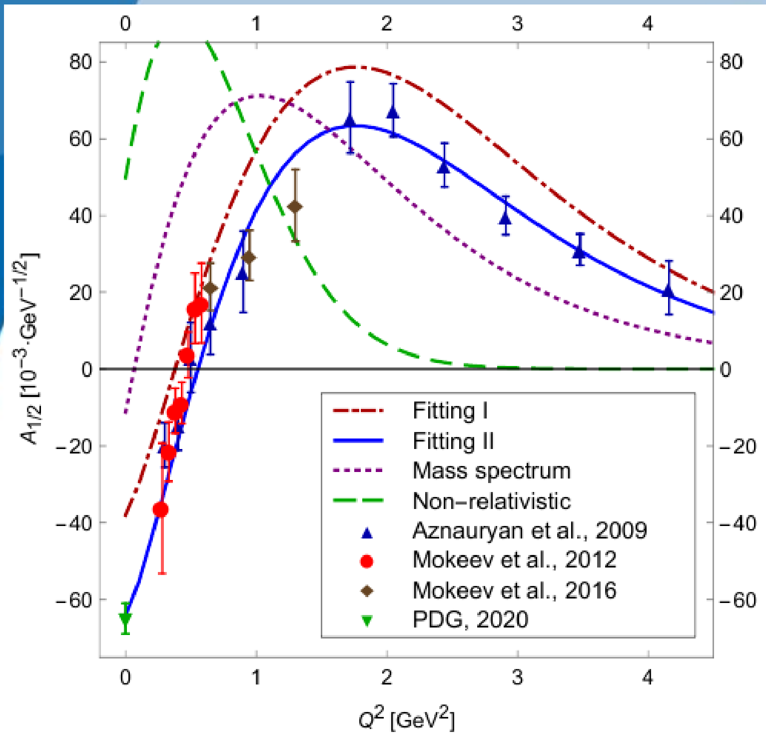
- Considered in 2 cases:

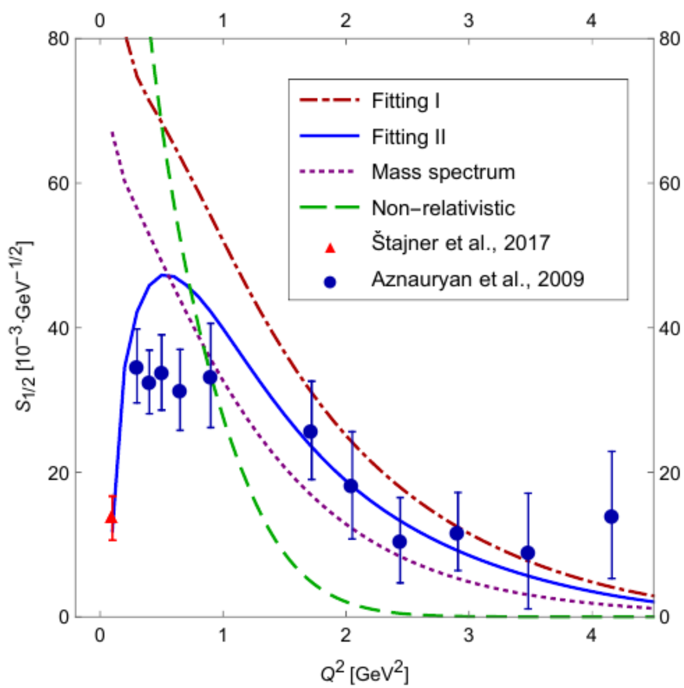
Fitting I: the proton and the N(1440) spatial wave functions are orthogonal each other: $\left\langle \psi_p^O \middle| \psi_{N(1440)}^O \right\rangle = 0.$

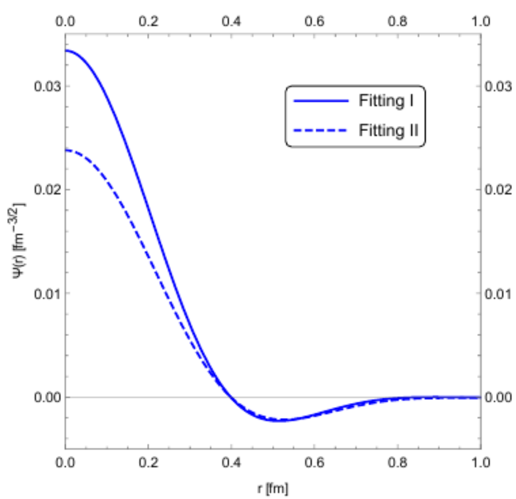
Fitting II: the proton and the N(1440) spatial wave functions may not be orthogonal each other.

- Expanded in the 3-quark harmonic oscillator basis.
- The dimension of the basis is 6: $N = 0, 2, 4, ..., 10.$
- The length parameter $\alpha = 400\ \mathrm{MeV}.$
- Fit by the least square method.

Results of the helicity amplitudes of N(1440)







Non-relativistic

- Can not give the changing sign.

Mass spectrum

- Consistent with the three-quark light front model and covariant spectator quark model.

Fitting

- The N(1440) is dominantly the radial excitation of nucleon.
- May have other components like L=2 three-quark states, pentaquarks, molecule states.

Thank you for your attention

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