

Leading Isospin Breaking effects in nucleon and Δ masses

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Strategy

We consider a background of pure QCD in isospin symmetry:

$$S_0 = \int d^4x \mathcal{L} = \int d^4x \mathcal{L}_{QCD}(u = d = \ell)$$

Isospin Breaking (IB)

IB splits the degenerate doublet. At LO we have 2 types of effects:

- strong IB : $\hat{m}_u \neq \hat{m}_d \rightarrow O(\frac{\hat{m}_d - \hat{m}_u}{\Lambda_{QCD}}) \sim O(1\%)$
- QED : $q_f = ee_f \neq 0 \rightarrow O(\alpha_{EM}) \sim O(1\%)$

RM123 method

At LO, the slopes can be calculated in the isosymmetric theory (isoQCD).
→ Their linear combinations with appropriate counterterms give the observables in the full theory QCD+QED

Preliminary results

	our prediction	experiment
M_N	$0.957(20)$ GeV	$\approx 0.938 - 0.939$ GeV (PDG)
M_Δ	$1.267(38)$ GeV	$1.230 - 1.234$ GeV (PDG)
$M_{\pi^+}^2 - M_{\pi^0}^2$	$1120(110)$ MeV 2	$1261.2(1)$ MeV 2 (PDG)
$M_n - M_p$	$1.69(71)$ MeV	$1.29333205(51)$ MeV (PDG)
$M_{\Delta^{++}} - M_{\Delta^0}$	$-2.2(1.1)$ MeV	$-2.86(30)$ MeV (PDG)
$[M_{\Delta^{++}} + M_{\Delta^-} - M_{\Delta^+} - M_{\Delta^0}]$	$2.41(51)$ MeV	$2.84 - 3.5$ MeV (Phys. Rev. C 47, 367)

Outline

1 isoQCD background

- Strategy and renormalization scheme
- Extrapolations of M_N and M_Δ

2 QCD+QED at LO

- Mass corrections at LO
- Tuning of counterterms
- Pion mass difference
- $M_n - M_p$
- Mass splittings of Δ resonances

3 Backup

Steps of the analysis

- Extraction of masses from the leading exponentials in the correlators ($t/a \gg 1$)

Extrapolation among 3 values of am_s for each ensemble:

$$r_s = \frac{2M_K^2 - M_\pi^2}{M_\Omega^2} \rightarrow r_s^{(\text{phys})}$$

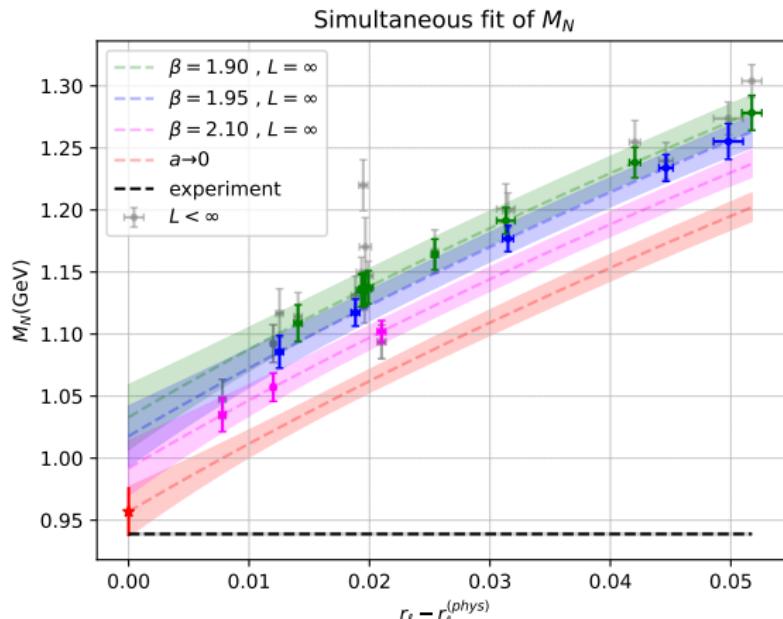
Extrapolation among the am_ℓ : $r_\ell = \frac{M_\pi^2}{M_\Omega^2} \rightarrow r_\ell^{(\text{phys})}$

- Correction of FVEs from asymptotic formula of ChPT at finite volume
- Scale setting: $a = (aM_\Omega)/M_\Omega^{(\text{phys})}$
- Final extrapolation in physical units:
 $a \rightarrow 0$, $V \rightarrow \infty$ and $r_\ell \rightarrow r_\ell^{(\text{phys})}$.

Global extrapolations (M_N)

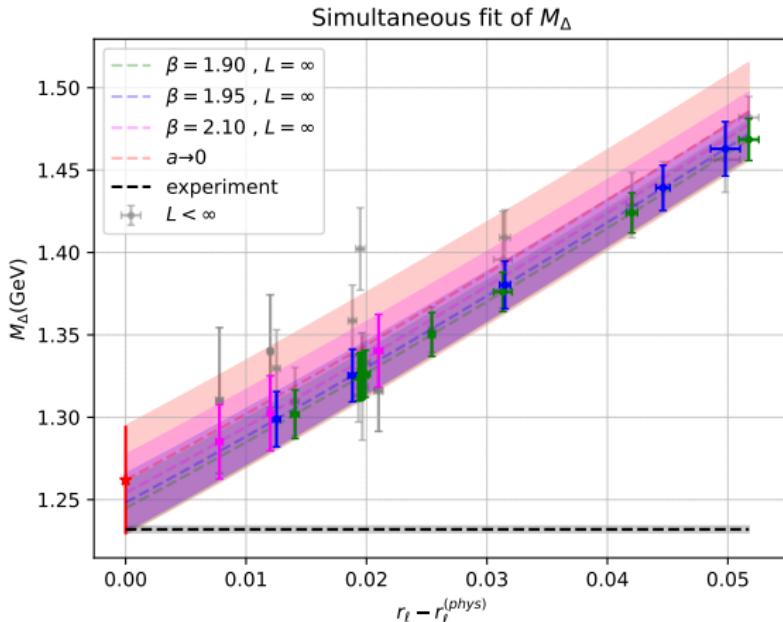
Inspired by ChPT results we fit according to:

$$M_N(L, a, r_\ell) = M_0^{(N)} \left[1 + c_L^{(N)} M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)} + c_a^{(N)} (a \Lambda_{QCD})^2 + c_\ell^{(N)} r_\ell + c_{3/2}^{(N)} r_\ell^{3/2} \right]$$



Global extrapolation ($M_\Delta(1232)$)

$$M_\Delta(L, a, r_\ell) = M_0^{(\Delta)} \left[1 + c_L^{(\Delta)} M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)} + c_a^{(\Delta)} (a \Lambda_{QCD})^2 + c_\ell^{(\Delta)} r_\ell + c_{3/2}^{(\Delta)} r_\ell^{3/2} \right]$$



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QCD+QED at LO

$$\mathcal{L} = \mathcal{L}_0 - \Delta m_{ud} \bar{q} \tau_3 q + e A_\mu \bar{q} \gamma_\mu \left(\frac{\tau_3}{2} + \frac{1}{6} \right) q$$

QED variation (lattice)

$$\begin{aligned} \Delta[T \langle O(x_i) \rangle] &= \int d^4x_1 d^4x_2 D_{\mu\nu}(x_1|x_2) T \langle O(x_i) J_\mu(x_1) J_\nu(x_2) \rangle \\ &\quad + \int d^4x D_{\mu\mu}(x|x) T \langle O(x_i) T_\mu(x) \rangle \quad . \end{aligned}$$

Renormalization in tmQCD+QED

$$m_f \rightarrow \Delta m_f [\bar{\psi}_f \psi_f]$$

$$m_f^{(crit)} \rightarrow \Delta m_f^{(crit)} [\bar{\psi}_f i\gamma^5 \tau^3 \psi_f]$$

$$g_s \rightarrow \Delta g_s G_{\mu\nu} G_{\mu\nu}$$

Quark propagator in QCD+QED

$$\begin{aligned}
 \Delta \xrightarrow{\quad\quad\quad}^{\pm} &= -(m_f - m_f^{(0)}) \xrightarrow{\quad\quad\quad} \otimes \mp (m_f - m_f^{(0)})^{(crit)} \xrightarrow{\quad\quad\quad} \textcolor{red}{\otimes} \\
 \\
 + (e_f e)^2 \xrightarrow{\text{wavy}} &+ (e_f e)^2 \xrightarrow{\text{star}} - e_f e^2 \sum_{f_1} \textcolor{blue}{e}_{f_1} \xrightarrow{\text{wavy}} \textcolor{blue}{\textcirclearrowleft} \\
 \\
 - e^2 \sum_{f_1} \textcolor{blue}{e}_{f_1}^2 \xrightarrow{\text{double wavy}} &- e^2 \sum_{f_1} \textcolor{blue}{e}_{f_1}^2 \xrightarrow{\text{star wavy}} + e^2, \sum_{f_1, f_2} \textcolor{blue}{e}_{f_1} \textcolor{red}{e}_{f_2} \xrightarrow{\text{wavy}} \textcolor{blue}{\textcirclearrowleft} \textcolor{red}{\textcirclearrowright} \\
 \\
 + \sum_{f_1} (m_{f_1} - m_{f_1}^{(0)}) \xrightarrow{\text{wavy}} &(m_{f_1} - m_{f_1}^{(0)})^{(crit)} \xrightarrow{\text{wavy}} \textcolor{blue}{\textcirclearrowleft} \textcolor{red}{\otimes} \\
 + (g_s^2 - g_s^{(0)2}) \xrightarrow{\boxed{G_{\mu\nu}G^{\mu\nu}}} &
 \end{aligned}$$

Example: ΔM_{K^+}

$$\begin{aligned}
 \Delta M_{K^+} = & + (m_u - m_{ud}^{(0)}) \partial_t \frac{\text{Diagram 1}}{\text{Diagram 2}} + (m_s - m_s^{(0)}) \partial_t \frac{\text{Diagram 3}}{\text{Diagram 4}} \\
 & - (m_u - m_{ud}^{(0)})^{(\text{crit})} \partial_t \frac{\text{Diagram 5}}{\text{Diagram 6}} + (m_s - m_s^{(0)})^{(\text{crit})} \partial_t \frac{\text{Diagram 7}}{\text{Diagram 8}} \\
 & - (e_s e)^2 \partial_t \frac{\text{Diagram 9}}{\text{Diagram 10}} - (e_u e)^2 \partial_t \frac{\text{Diagram 11}}{\text{Diagram 12}} - (e_s e)^2 \partial_t \frac{\text{Diagram 13}}{\text{Diagram 14}} - (e_u e)^2 \partial_t \frac{\text{Diagram 15}}{\text{Diagram 16}} \\
 & - e_u e_s e^2 \partial_t \frac{\text{Diagram 17}}{\text{Diagram 18}} - e_s e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t \frac{\text{Diagram 19}}{\text{Diagram 20}} - e_u e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t \frac{\text{Diagram 21}}{\text{Diagram 22}} \\
 & + [\text{isosymm. vac. pol. diag.}]
 \end{aligned}$$

The equations show the expression for ΔM_{K^+} as a sum of terms involving partial derivatives of ratios of loop diagrams. The diagrams are represented by semi-circular arcs with arrows indicating direction. Red arcs represent the numerator and black arcs represent the denominator. Various symbols are placed on the arcs, such as crossed-out symbols (\otimes , \times , \star), wavy lines, and gears, which likely indicate different contributions or cancellations in the loop diagrams.

Recap and details of the method

Expansion over isosymmetric background

We stop at $O(\alpha_{EM}) \sim O\left(\frac{m_d - m_u}{\Lambda_{QCD}}\right)$

- recycle gauge configurations in QCD with isospin
- Linear corrections in the counterterms

Steps of the analysis

- Tuning of $\Delta m_f^{(crit)}$ at fixed ensemble from the PCAC Ward Identity
(keep maximal twist at $O(\alpha_{EM}) \implies O(a)$ improvement)
- Correction of universal FVEs from QED_L .
- Tuning of Δm_f at fixed ensemble from hadronic ratios
- Extrapolation to the physical point, $a \rightarrow 0, V \rightarrow \infty$.

QED at Finite volume

The lattice photon propagator $D_{\mu\nu}(k)$ is divergent at $k = 0$:

$$D_{\mu\nu}(k) = \frac{g_{\mu\nu}}{\hat{k}^2} = \frac{g_{\mu\nu}}{\sin^2(k_\mu)}$$

QED_L prescription

We remove the 0-mode manually \implies Finite volume effects.

$$M(L) = M(\infty) \left[1 + Q^2 \alpha \frac{\kappa}{2ML} \left(1 + \frac{2}{ML} \right) \right] + O\left(\frac{1}{L^3}\right)$$

$(\kappa \approx 2.837297) \sim L^{-3}, \sim L^{-4}$ come from the structure of the hadron.

We implement the correction on the slopes \implies counterterms don't contain the universal QED_L effects.

QCD+QED : Tuning of mass counterterms

- Tuning of $a\Delta m_u$, $a\Delta m_d$, $a\Delta m_s$ from the ratios

$$r_s = \frac{2(M_{K^+}^2 + M_{K^0}^2) - (M_{\pi^+}^2 + M_{\pi^0}^2)}{2M_{\Omega^-}^2} = r_s^{(0)} + \sum_f a\Delta m_f \bar{\Delta}r(f) + \Delta r_s^{(EMC)}$$

$$r_\ell = \frac{M_{\pi^+}^2 + M_{\pi^0}^2}{2M_{\Omega^-}^2} = r_\ell^{(0)} + \sum_f a\Delta m_f \bar{\Delta}r_\ell(f) + \Delta r_\ell^{(EMC)}$$

$$r_p = \frac{M_{K^+}^2}{M_{\Omega^-}^2} = r_p^{(0)} + \sum_f a\Delta m_f \bar{\Delta}r_p(f) + \Delta r_p^{(EMC)}$$

We define the physical point of isoQCD such that the total IB correction vanishes.

QCD+QED : Tuning of mass counterterms

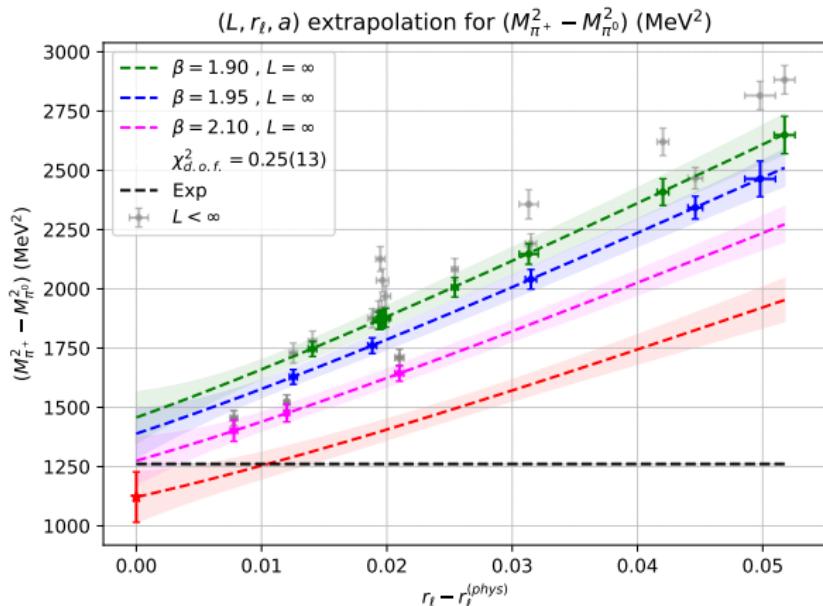
We require the ratios r_s , r_ℓ , r_p to be at the physical point when we reach the physical m_s and m_ℓ found in isoQCD:

$$\begin{bmatrix} \bar{\Delta}r_s(u) & \bar{\Delta}r_s(d) & \bar{\Delta}r_s(s) \\ \bar{\Delta}r_\ell(u) & \bar{\Delta}r_\ell(d) & \bar{\Delta}r_\ell(s) \\ \bar{\Delta}r_p(u) & \bar{\Delta}r_p(d) & \bar{\Delta}r_p(s) \end{bmatrix} \begin{bmatrix} a\Delta m_u \\ a\Delta m_d \\ a\Delta m_s \end{bmatrix} = - \begin{bmatrix} \Delta r_s^{(EMC)} \\ \Delta r_\ell^{(EMC)} \\ \Delta r_p^{(EMC)} \end{bmatrix}$$

Note: using r_s , r_ℓ and r_p we loose predictivity on π , K , Ω masses, but we can predict the other corrections (e.g. M_n , M_p , $M_{\Delta^{++}}$, etc.)

$$M_{\pi^+} - M_{\pi^0}$$

$$\Delta M_\pi = \frac{(e_u - e_d)^2}{2} e^2 \left[\partial_t \frac{\text{diagram}}{\text{diagram}} - \partial_t \frac{\text{diagram}}{\text{diagram}} \right] \approx \frac{(e_u - e_d)^2}{2} e^2 \partial_t \frac{\text{diagram}}{\text{diagram}} + O(e^2 \hat{m}_\ell)$$



$$M_n - M_p$$

$$\begin{aligned}
 M_n - M_p &= 2\Delta m_{ud} \left[\partial_t \frac{\text{Diagram A}}{\text{Diagram B}} - \partial_t \frac{\text{Diagram C}}{\text{Diagram D}} - \partial_t \frac{\text{Diagram E}}{\text{Diagram F}} \right] \\
 &\quad - 2\Delta m_{ud}^{(crit)} \left[\partial_t \frac{\text{Diagram G}}{\text{Diagram H}} - \partial_t \frac{\text{Diagram I}}{\text{Diagram J}} - \partial_t \frac{\text{Diagram K}}{\text{Diagram L}} \right] \\
 &\quad - (q_u^2 - q_d^2) \left[\partial_t \frac{\text{Diagram M}}{\text{Diagram N}} - \partial_t \frac{\text{Diagram O}}{\text{Diagram P}} - \partial_t \frac{\text{Diagram Q}}{\text{Diagram R}} + (\text{tadpole diagrams}) \right] \\
 &\quad + (q_u^2 - q_d^2) \partial_t \frac{\text{Diagram S}}{\text{Diagram T}} - [\text{exchange diagrams}]
 \end{aligned}$$

Separation of strong IB and QED

The separation is scheme dependent:

$$\Delta m_{ud} = \frac{m_d - m_u}{2} = \Delta m_{ud}^{(QCD)} + \Delta m_{ud}^{(QED)} = Z_P^{(0)} \Delta \hat{m}_{ud} + \mathcal{Z}_{ud}^{-1} \hat{m}_{ud}$$

$$\mathcal{Z}_{ud}^{-1}(\mu) = Z_P^{(0)} \frac{q_d^2 - q_u^2}{32\pi^2} [6 \log(a\mu) - 22.595 + \dots]$$

Physical interpretation

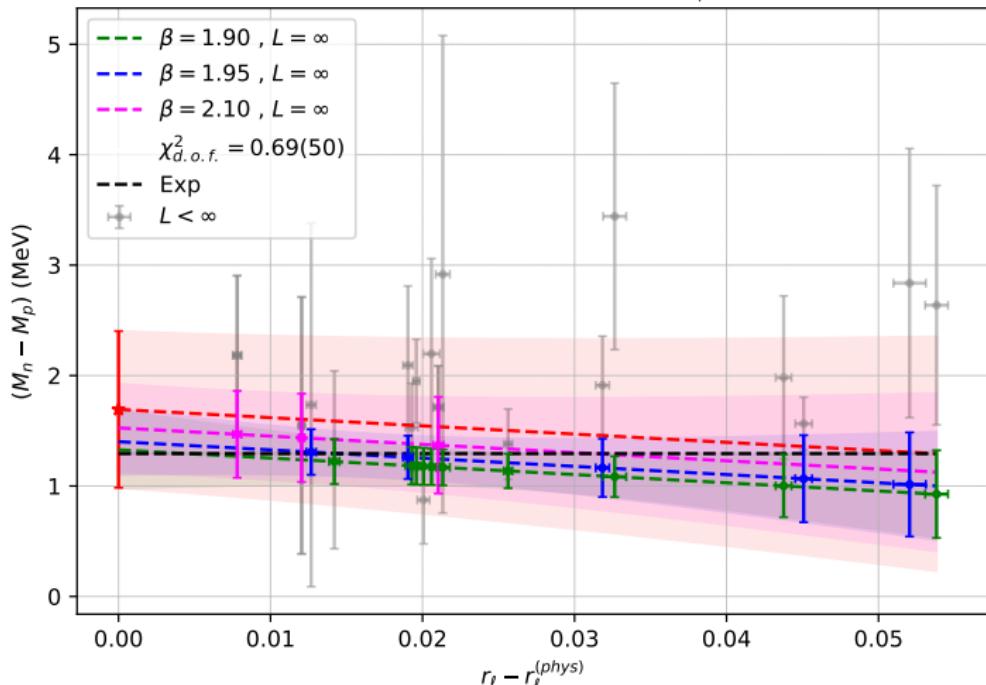
- QCD : $m_d > m_u \implies M_n > M_p$
- QED : $|Q_p| > |Q_n| \implies M_p > M_n$

The combination of these 2 effects cancel almost exactly:

$$M_n - M_p \approx 1.3 \text{ MeV} = O(10^{-3}) M_N$$

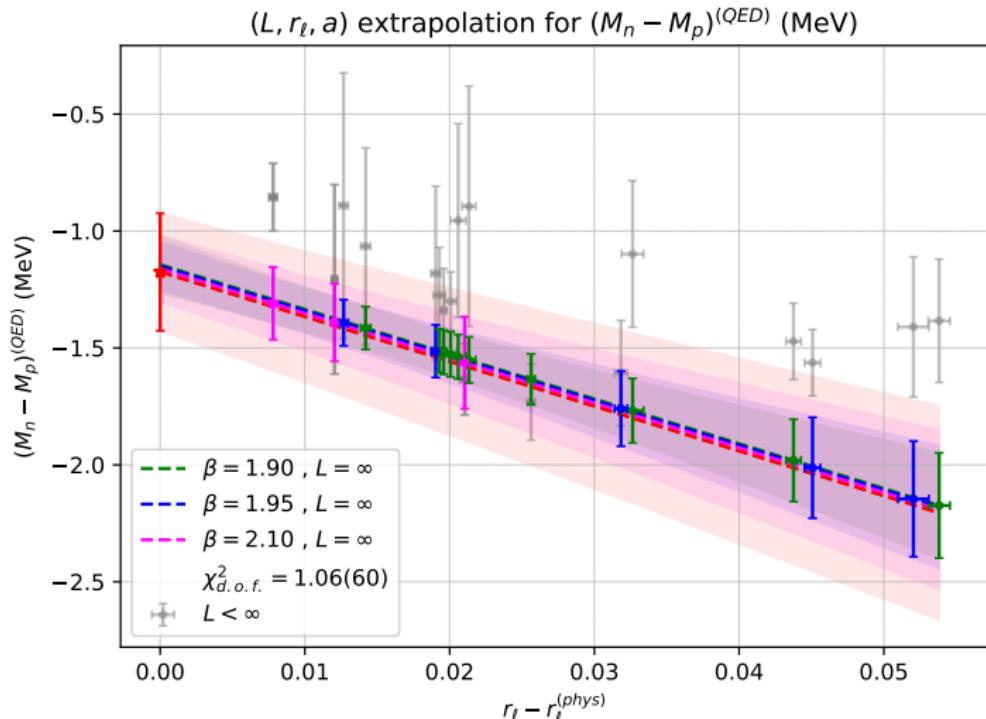
$$M_n - M_p = 1.69(71) \text{ MeV}$$

(L, r_ℓ, a) extrapolation for ($M_n - M_p$) (MeV)



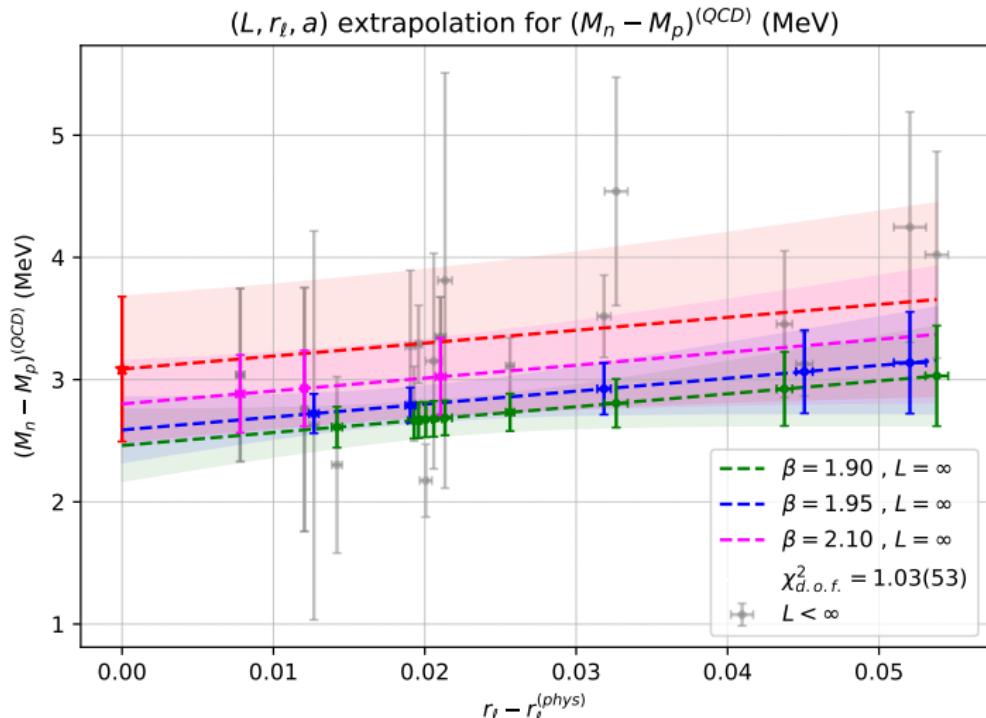
$$(M_n - M_p)^{(QED)}$$

$$(M_n - M_p)^{(QED)} = -1.17(25) \text{ MeV}$$



$$(M_n - M_p)^{(QCD)}$$

$$(M_n - M_p)^{(QCD)} = 3.09(59) \text{ MeV}$$



$\Delta(1232)$ mass splittings

$$(M_{++} + M_-) - (M_+ + M_0) = -\frac{2}{3}(q_u - q_d)^2$$

$$\left\{ \partial_t \frac{\text{Diagram 1}}{\text{Diagram 2}} + \partial_t \frac{\text{Diagram 3}}{\text{Diagram 2}} + \partial_t \frac{\text{Diagram 4}}{\text{Diagram 2}} - 2[\text{exchange diagrams}] \right\}$$

The equation shows a set of three diagrams (Diagram 1, Diagram 3, Diagram 4) each divided by a common denominator, followed by a bracketed term for exchange diagrams.

- Diagram 1: Two vertical rectangles representing quarks. The top quark has a diagonal line with arrows pointing from left to right, and a vertical dashed line with arrows pointing down. The bottom quark has two horizontal lines with arrows pointing right.
- Diagram 2: Two vertical rectangles representing quarks. The top quark has two horizontal lines with arrows pointing right. The bottom quark has two horizontal lines with arrows pointing right.
- Diagram 3: Two vertical rectangles representing quarks. The top quark has a diagonal line with arrows pointing from left to right, and a vertical dashed line with arrows pointing down. The bottom quark has two horizontal lines with arrows pointing right.
- Diagram 4: Two vertical rectangles representing quarks. The top quark has two horizontal lines with arrows pointing right. The bottom quark has a diagonal line with arrows pointing from left to right, and a vertical dashed line with arrows pointing down.

This is a purely electromagnetic effect (scheme independent).

4 masses \implies 3 independent mass splittings, but at LO:

$$M_{++} - M_- = 3(M_+ - M_0)$$

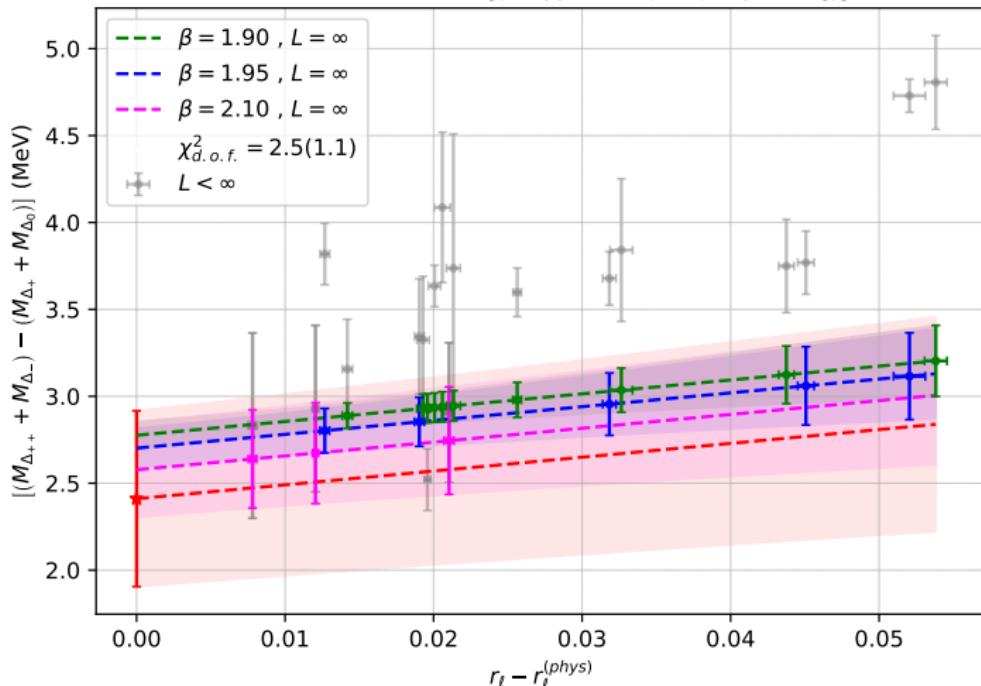
\implies 2 independent mass splittings.

\implies from $M_{++} - M_0$ and $[(M_{++} + M_-) - (M_+ + M_0)]$ we find them all.

$$(M_{++} + M_-) - (M_+ + M_0)$$

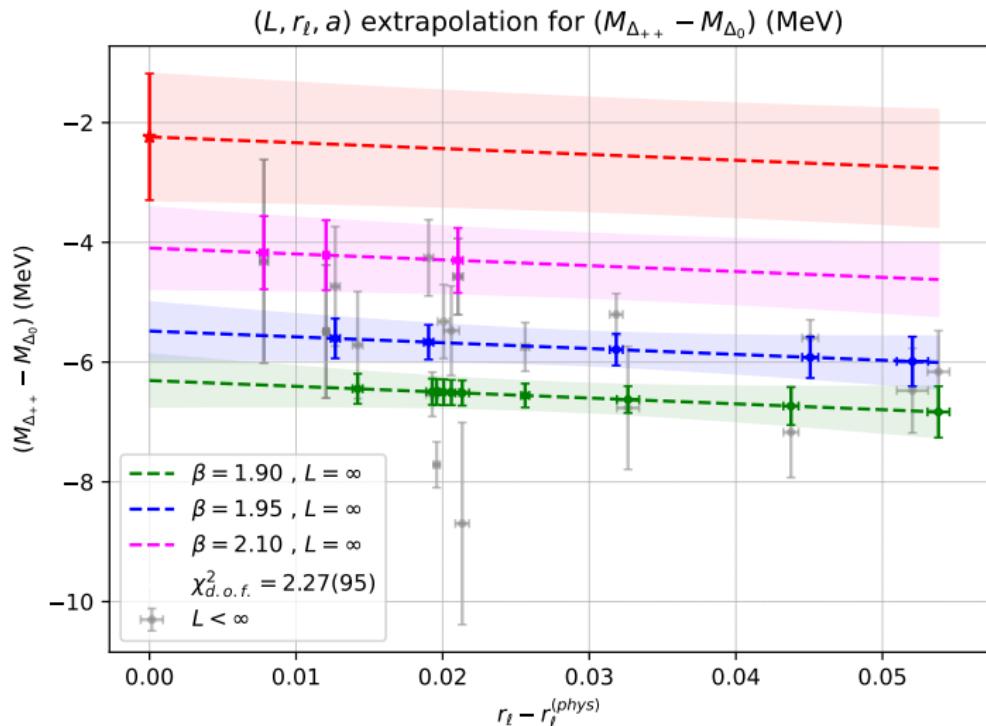
$$(M_{++} + M_-) - (M_+ + M_0) = 2.41(51) \text{ MeV}$$

(L, r_ℓ, a) extrapolation for $[(M_{\Delta_{++}} + M_{\Delta_-}) - (M_{\Delta_+} + M_{\Delta_0})]$ (MeV)



$$M_{++} - M_0$$

$$M_{++} - M_0 = -2.2(1.1) \text{ MeV}$$



Conclusions

We are able to include IB at 1st order in the spectrum of mesons and baryons.

- No need for QED in the Lattice Lagrangian at LO.
(same isoQCD gauge configurations)
- Hadronic scheme to reach the physical point and tune mass counterterms.
(independence from renormalization constants)
- 1σ compatibility with experimental values and consistent separation of strong IB and *QED*.
- Prediction for mass splittings involving M_{Δ^-}
(no experimental value available yet)

Thank you for the attention

Backup

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Ensembles

Ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	κ	N_{cfg}
$A30.32$	1.90	$32^3 \times 64$	0.0030	0.15	0.19	0.163272	150
$A40.32$			0.0040			0.163270	150
$A50.32$			0.0050			0.163267	150
$A40.20$	1.90	$20^3 \times 48$	0.0040	0.15	0.19	0.163270	150
$A40.24$	1.90	$24^3 \times 48$	0.0040	0.15	0.19	0.163270	150
$A60.24$			0.0060			0.163265	150
$A80.24$			0.0080			0.163255	150
$A100.24$			0.0100			0.163260	150
$A40.48$	1.90	$48^3 \times 96$	0.0040	0.15	0.19	0.163270	90
$A40.40$	1.90	$40^3 \times 80$	0.0040	0.15	0.19	0.163270	150

Ensembles

Ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	κ	N_{cfg}
$B25.32$	1.95	$32^3 \times 64$	0.0025	0.135	0.170	0.1612420	150
$B35.32$			0.0035			0.1612400	150
$B55.32$			0.0055			0.1612360	150
$B75.32$			0.0075			0.1612320	75
$B85.24$	1.95	$24^3 \times 48$	0.0085	0.135	0.170	0.1612312	150
$D15.48$	2.10	$48^3 \times 96$	0.0015	0.12	0.1385	0.156361	90
$D20.48$			0.0020			0.156357	90
$D30.48$			0.0030			0.156355	90

- Partial quenching in the strange sector
- A40.XX ensembles differ only for the volume
- 3 values of the lattice spacing ($a^{-1} \sim 2 - 3 \text{ GeV}$)
- $M_\pi \simeq 200 - 450 \text{ MeV}$

Mesonic correlators ($\vec{p} = \vec{0}$)

$$C_{\pi^+\pi^-}(x) = - \sum_{\vec{x}} \langle [\bar{u}\gamma_5 d](x) [\bar{d}\gamma_5 u](0) \rangle = - \begin{array}{c} \text{Diagram of a loop with two external gluons (blue and red), each with a quark (u or d) and an antiquark (d or u). Arrows indicate flow from left to right.} \end{array}$$

$$C_{\pi^0\pi^0}(x) = -\frac{1}{2} \sum_{\vec{x}} \langle [\bar{u}\gamma_5 u - \bar{d}\gamma_5 d](x) [\bar{u}\gamma_5 u - \bar{d}\gamma_5 d](0) \rangle$$

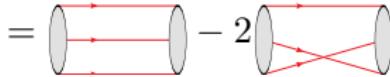
$$= -\frac{1}{2} \left[\begin{array}{c} \text{Diagram of a loop with two external gluons (blue and red), each with a quark (u or d). Arrows indicate flow from left to right.} \\ + \begin{array}{c} \text{Diagram of a loop with two external gluons (blue and red), each with a quark (d or u). Arrows indicate flow from left to right.} \end{array} \end{array} \right]$$

$$C_{K^+K^-}(x) = - \sum_{\vec{x}} \langle [\bar{s}\gamma_5 u](x) [\bar{u}\gamma_5 s](0) \rangle = \begin{array}{c} \text{Diagram of a loop with two external gluons (red and blue), each with a quark (s or u). Arrows indicate flow from left to right.} \end{array}$$

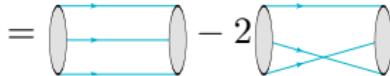
$$C_{K^0\bar{K}^0}(x) = - \sum_{\vec{x}} \langle [\bar{s}\gamma_5 d](x) [\bar{d}\gamma_5 s](0) \rangle = \begin{array}{c} \text{Diagram of a loop with two external gluons (red and green), each with a quark (s or d). Arrows indicate flow from left to right.} \end{array}$$

Baryonic Correlators ($\vec{p} = \vec{0}$, $J^P = 3/2^+$)

$$\Omega^- \quad C(t) \propto \epsilon_{a_1 b_1 c_1} \epsilon_{a_2 b_2 c_2}$$
$$+ \text{Tr}[S_s^{T^{a_1 a_2}}(x|0) C \gamma_\nu S_s^{b_1 b_2}(x|0) C \gamma_\mu] \text{Tr}[P_+ P_{\mu\nu}^{3/2} S_s^{c_1 c_2}(x|0)]$$
$$- 2 \text{Tr}[S_s^{a_1 b_2}(x|0) P_+ P_{\mu\nu}^{3/2} S_s^{b_1 a_2}(x|0) C \gamma_\nu S_s^{T^{c_1 c_2}}(x|0) C \gamma_\mu]$$

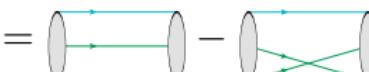


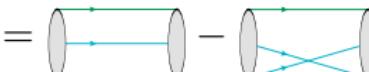
$$\Delta^{++} \quad C(t) \propto \epsilon_{a_1 b_1 c_1} \epsilon_{a_2 b_2 c_2}$$
$$+ \text{Tr}[S_u^{T^{a_1 a_2}}(x|0) C \gamma_\nu S_u^{b_1 b_2}(x|0) C \gamma_\mu] \text{Tr}[P_+ P_{\mu\nu}^{3/2} S_u^{c_1 c_2}(x|0)]$$
$$- 2 \text{Tr}[S_u^{a_1 b_2}(x|0) P_+ P_{\mu\nu}^{3/2} S_u^{b_1 a_2}(x|0) C \gamma_\nu S_u^{T^{c_1 c_2}}(x|0) C \gamma_\mu]$$



... (Δ^+ , Δ^0 , Δ^-)

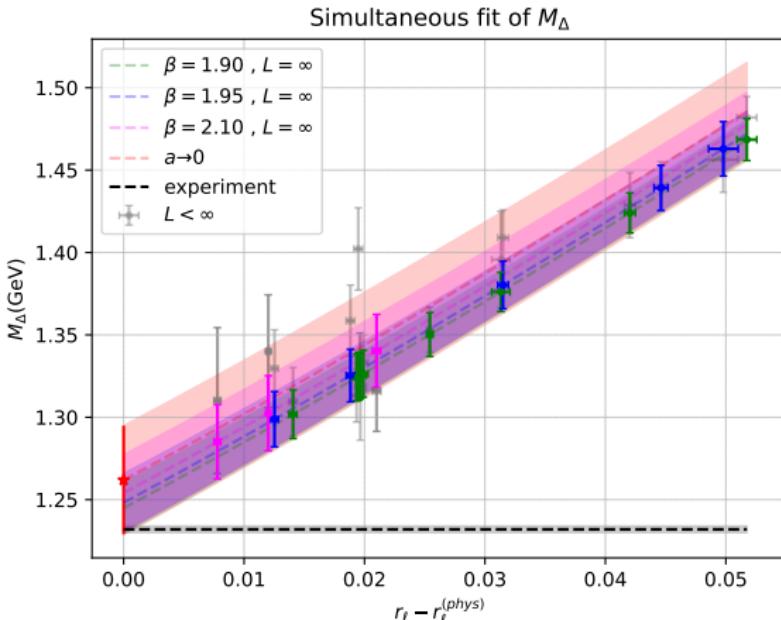
Baryonic Correlators ($\vec{p} = \vec{0}$, $J^P = 1/2^+$)

$$\begin{aligned} p \quad C(t) &= \epsilon_{a_1 b_1 c_1} \epsilon_{a_2 b_2 c_2} \\ &+ \text{Tr}[S_u^{T a_1 a_2}(x|0) C \gamma_5 S_u^{b_1 b_2}(x|0) C \gamma_5] \text{Tr}[P_+ S_d^{c_1 c_2}(x|0)] \\ &- \text{Tr}[S_u^{a_1 b_2}(x|0) P_+ S_u^{b_1 a_2}(x|0) C \gamma_5 S_d^{T c_1 c_2}(x|0) C \gamma_5] \\ &= \text{Diagram } 1 - \text{Diagram } 2 \end{aligned}$$


$$\begin{aligned} n \quad C(t) &= \epsilon_{a_1 b_1 c_1} \epsilon_{a_2 b_2 c_2} \\ &+ \text{Tr}[S_d^{T a_1 a_2}(x|0) C \gamma_5 S_d^{b_1 b_2}(x|0) C \gamma_5] \text{Tr}[P_+ S_u^{c_1 c_2}(x|0)] \\ &- \text{Tr}[S_d^{a_1 b_2}(x|0) P_+ S_d^{b_1 a_2}(x|0) C \gamma_5 S_u^{T c_1 c_2}(x|0) C \gamma_5] \\ &= \text{Diagram } 1 - \text{Diagram } 2 \end{aligned}$$


Global extrapolation ($M_\Delta(1232)$)

$$M_\Delta(L, a, r_\ell) = M_0^{(\Delta)} \left[1 + c_L^{(\Delta)} M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)} + c_a^{(\Delta)} (a \Lambda_{QCD})^2 + c_\ell^{(\Delta)} r_\ell + c_{3/2}^{(\Delta)} r_\ell^{3/2} \right]$$

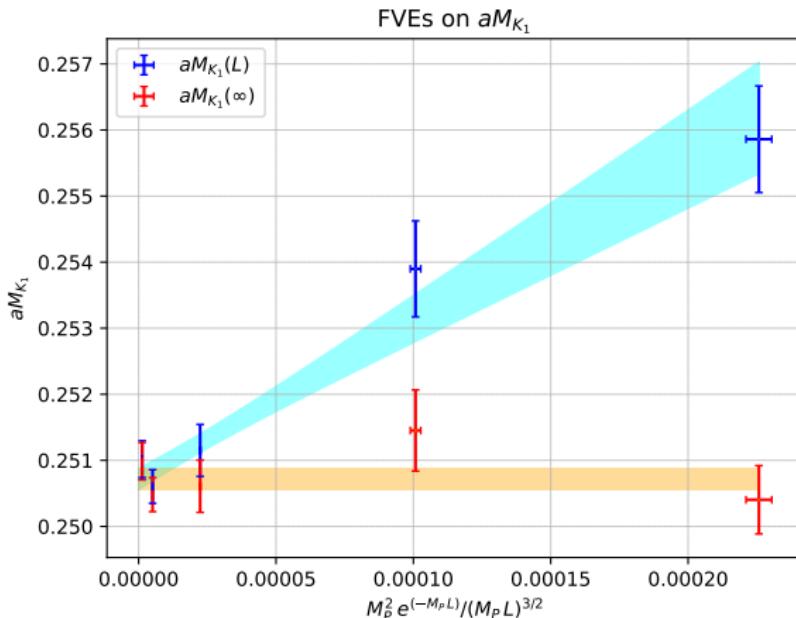


Quark propagator in QCD+QED

$$\Delta \xrightarrow{\quad \pm = -(m_f - m_f^{(0)}) \xrightarrow{\otimes} \mp (m_f - m_f^{(0)})^{(crit)} \xrightarrow{\quad \otimes \quad} } \text{red circle}$$
$$+ (e_f e)^2 \xrightarrow{\text{wavy line}} + (e_f e)^2 \xrightarrow{\text{star}} - e_f e^2 \sum_{f_1} \textcolor{blue}{e}_{f_1} \xrightarrow{\text{wavy line}} \text{blue circle}$$
$$- e^2 \sum_{f_1} \textcolor{blue}{e}_{f_1}^2 \xrightarrow{\text{double wavy line}} - e^2 \sum_{f_1} \textcolor{blue}{e}_{f_1}^2 \xrightarrow{\text{star}} + e^2, \sum_{f_1, f_2} \textcolor{blue}{e}_{f_1} \textcolor{red}{e}_{f_2} \xrightarrow{\text{wavy line}} \text{blue circle} \xrightarrow{\text{wavy line}} \text{red circle}$$
$$+ \sum_{f_1} (m_{f_1} - m_{f_1}^{(0)}) \xrightarrow{\text{circle with cross}} (m_{f_1} - m_{f_1}^{(0)})^{(crit)} \xrightarrow{\text{circle with cross}} \text{red circle}$$
$$+ (g_s^2 - g_s^{(0)2}) \xrightarrow{\boxed{G_{\mu\nu} G^{\mu\nu}}} \text{black arrow}$$

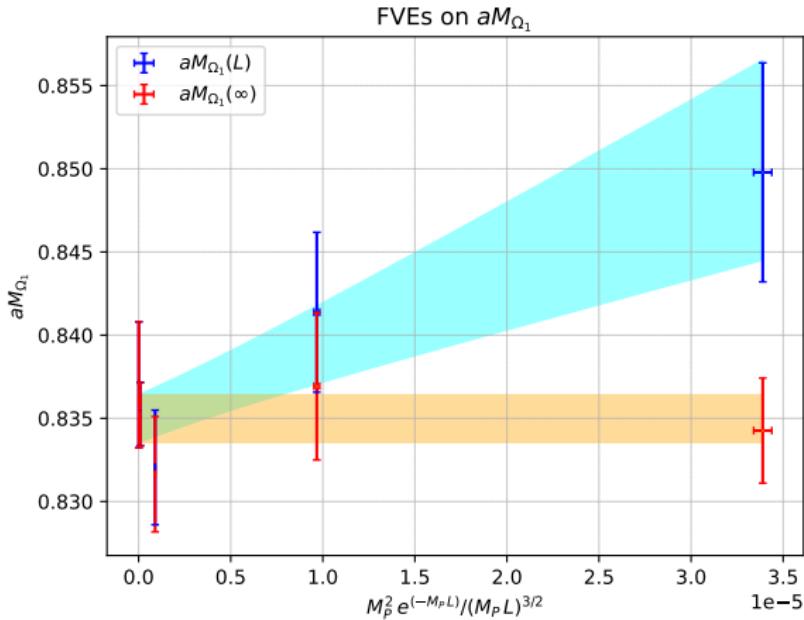
Correction of FVEs (mesons)

$$M_P(L) = M \left(1 + C_P M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}} \right)$$



FVEs (M_Ω)

$$M_\Omega(L) = M_\Omega(\infty) \left(1 + C_B M_K^2 \frac{e^{-M_K L}}{(M_K L)^{3/2}} \right)$$



Chiral perturbation theory (ChPT)

ChPT approximates low-energy QCD → base for fit ansatz.

Meson ChPT

- $M_P^2 \propto (\hat{m}_1 + \hat{m}_2)$
 - $2M_K^2 - M_\pi^2 \propto \hat{m}_s$
 - $M_\pi^2 \propto \hat{m}_\ell$

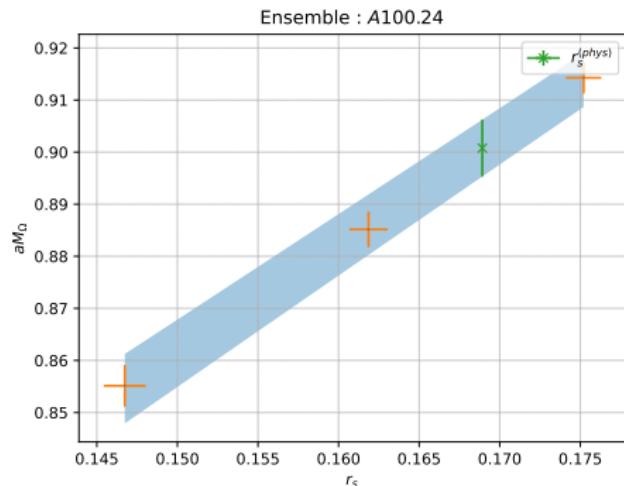
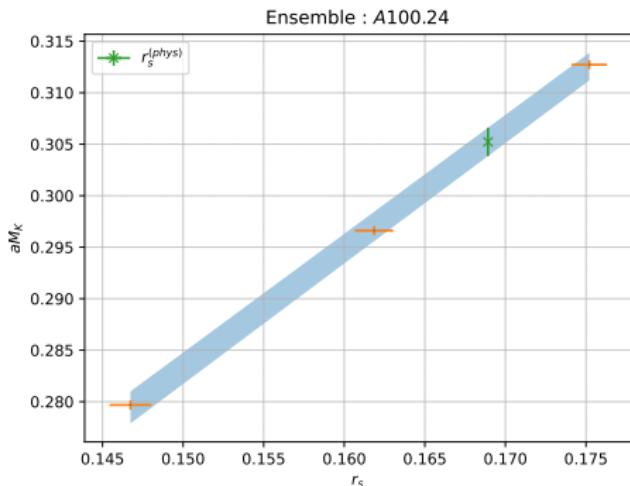
Baryon ChPT

- $M_N = M_N^{(0)} - 4c_1 M_\pi^2 - \frac{3g_A^2}{32\pi f_\pi} M_\pi^3 + O(M_\pi^4 \log(M_\pi^2)) + \dots$
- $M_\Omega = M_\Omega^{(0)} - 4c_\Omega^{(1)} M_\pi^2 + O(M_\pi^4 \log(M_\pi^2)) + \dots$

$r_s = \frac{2M_K^2 - M_\pi^2}{M_\Omega^2}$ and $r_\ell = \frac{M_\pi^2}{M_\Omega^2}$ can be used to reach the physical point of am_ℓ and am_s

r_s interpolation

For each ensemble we interpolate among the 3 values of am_s .



Perturbative expansion of QED

Consider a theory with couplings $\vec{g}^{(0)} = (g_1^{(0)}, \dots, g_n^{(0)})$. When we add the perturbation $\Delta S[U, A, \psi, \bar{\psi}]$

$$\langle O \rangle^{\vec{g}} = \frac{\langle e^{-(\beta - \beta_0)S[U]} \prod_f \frac{\det(D_f[U, \vec{g}])}{\det(D_f[U, \vec{g}_0])} O[U, A, \vec{g}] \rangle^{A, \vec{g}_0}}{\langle e^{-(\beta - \beta_0)S[U]} \prod_f \frac{\det(D_f[U, \vec{g}])}{\det(D_f[U, \vec{g}_0])} \rangle^{A, \vec{g}_0}}$$

Electroquenched approximation

We consider **chargless sea quarks**. This consists in setting:

$$e^{-(\beta - \beta_0)S[U]} \prod_f \frac{\det(D_f[U, \vec{g}])}{\det(D_f[U, \vec{g}_0])} \rightarrow 1$$

→ feasible calculation using heat-bath algorithms.

Mass corrections from lattice correlators

$$\Delta C = \sum_y \langle O(t) J(y) O^\dagger(0) \rangle$$

ΔM , ΔA

$$\begin{aligned} C(t) &= A e^{-M t} = A_0(1 + \Delta A) e^{-(M_0 + \Delta M)t} \\ &= C_0(1 + \Delta A - \Delta M t) \end{aligned}$$

The mass correction is found as

$$\Delta M = -\partial_t \frac{\Delta C}{C_0}$$

$$\Delta M_{\pi^+}$$

$$\begin{aligned}
 \Delta M_{\pi^+} = & +2(m_{ud} - m_{ud}^{(0)})\partial_t \frac{\text{Diagram with crossed lines}}{\text{Diagram with solid lines}} - (m_u + m_d - 2m_{ud}^{(0)})^{(\text{crit})}\partial_t \frac{\text{Diagram with red dot}}{\text{Diagram with solid lines}} \\
 & - e_u e_d e^2 \partial_t \frac{\text{Diagram with wavy line}}{\text{Diagram with solid lines}} - (e_u^2 + e_d^2) e^2 \partial_t \frac{\text{Diagram with dashed line}}{\text{Diagram with solid lines}} - (e_u^2 + e_d^2) e^2 \partial_t \frac{\text{Diagram with star}}{\text{Diagram with solid lines}} \\
 & - (e_u + e_d) e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t \frac{\text{Diagram with blue circle}}{\text{Diagram with solid lines}} + [\text{isosymm. vac. pol. diag.}] ,
 \end{aligned}$$

$$\Delta M_{\pi^0}$$

$$\begin{aligned}
 \Delta M_{\pi^0} = & +2(m_{ud} - m_{ud}^{(0)})\partial_t \frac{\text{Diagram with crossed gluons}}{\text{Diagram with gluon loop}} - (m_u + m_d - m_{ud}^{(0)})^{(crit)}\partial_t \frac{\text{Diagram with red dot}}{\text{Diagram with gluon loop}} \\
 & - \frac{(e_u^2 + e_d^2)}{2} e^2 \partial_t \frac{\text{Diagram with gluon loop and wavy line}}{\text{Diagram with gluon loop}} - (e_u^2 + e_d^2) e^2 \partial_t \frac{\text{Diagram with gluon loop and wavy line}}{\text{Diagram with gluon loop}} - (e_u^2 + e_d^2) e^2 \partial_t \frac{\text{Diagram with gluon loop and wavy line}}{\text{Diagram with gluon loop}} \\
 & - (e_u + e_d) e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t \frac{\text{Diagram with gluon loop and blue Q}}{\text{Diagram with gluon loop}} + \frac{(e_u - e_d)^2}{2} e^2 \partial_t \frac{\text{Diagram with two gluon loops}}{\text{Diagram with gluon loop}} \\
 & + [\text{isosymm. vac. pol. diag.}] ,
 \end{aligned}$$

ΔM_{K^0}

$$\begin{aligned}
 \Delta M_{K^0} = & + (m_d - m_{ud}^{(0)}) \partial_t \frac{\text{Diagram 1}}{\text{Diagram 2}} + (m_s - m_s^{(0)}) \partial_t \frac{\text{Diagram 3}}{\text{Diagram 4}} \\
 & - (m_d - m_{ud}^{(0)})^{(\text{crit})} \partial_t \frac{\text{Diagram 5}}{\text{Diagram 6}} + (m_s - m_s^{(0)})^{(\text{crit})} \partial_t \frac{\text{Diagram 7}}{\text{Diagram 8}} \\
 & - (e_s e)^2 \partial_t \frac{\text{Diagram 9}}{\text{Diagram 10}} - (e_d e)^2 \partial_t \frac{\text{Diagram 11}}{\text{Diagram 12}} - (e_s e)^2 \partial_t \frac{\text{Diagram 13}}{\text{Diagram 14}} - (e_d e)^2 \partial_t \frac{\text{Diagram 15}}{\text{Diagram 16}} \\
 & - e_d e_s e^2 \partial_t \frac{\text{Diagram 17}}{\text{Diagram 18}} - e_s e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t \frac{\text{Diagram 19}}{\text{Diagram 20}} - e_d e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t \frac{\text{Diagram 21}}{\text{Diagram 22}} \\
 & + [\text{isosymm. vac. pol. diag.}]
 \end{aligned}$$

Diagrams are represented by horizontal lines with arrows indicating direction. Red lines represent quarks and gluons, black lines represent gluons, and wavy lines represent photons. External gluon lines are red, while internal gluon lines are black. External photon lines are wavy, while internal photon lines are black. External quark lines are red, while internal quark lines are black.

Tuning of $\Delta m_f^{(crit)}$

In the physical basis the critical mass counterterm is modified by the insertion of

$$J(x) = \sum_f \bar{\psi}_f(x) i\gamma_5 \tau_3 \psi_f(x)$$

PCAC Ward Identity

$$\partial_t \sum_{\vec{x}} \langle \bar{\psi}_f(t, \vec{x}) i\gamma_0 \psi_f(t, \vec{x}) O(y) \rangle = 2m_f^{(PCAC)} \sum_{\vec{x}} \langle \bar{\psi}_f(t, \vec{x}) i\gamma_5 \psi_f(t, \vec{x}) O(y) \rangle$$

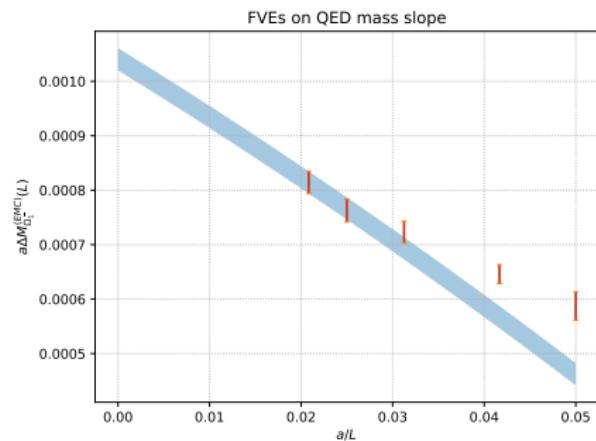
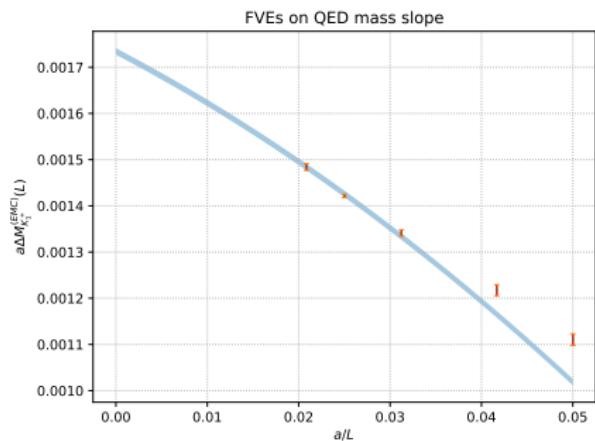
- In isoQCD the PCAC mass was tuned to 0 . \rightarrow maximal twist $\rightarrow O(a)$ improvement
- In QCD+QED we impose that maximal twist is preserved at $O(\alpha_{EM})$:

$$\Delta \left(\frac{\partial_t \langle V_0 P_5 \rangle}{\langle P_5 P_5 \rangle} \right)^{(EMC)} = 0$$

FVEs on QED mass corrections

QED_L formula ($\kappa \approx 2.837297$)

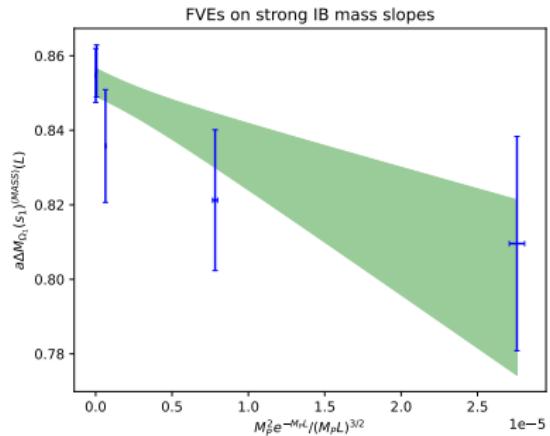
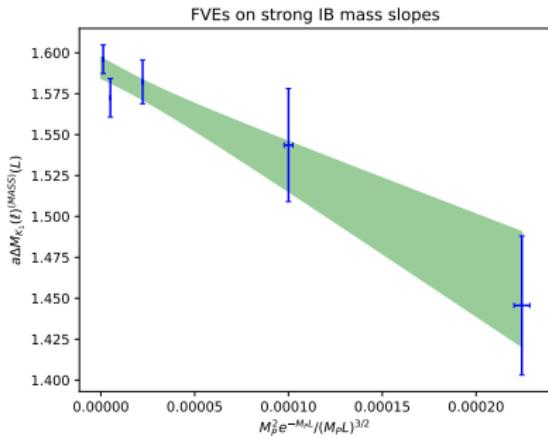
$$\Delta M(L) = \Delta M(\infty) - Q^2 \alpha \frac{\kappa}{2ML} \left(1 + \frac{2}{ML} \right) + O\left(\frac{1}{L^3}\right)$$



FVEs on strong IB mass corrections

strong IB ($\Delta m_f \neq 0$)

$$\Delta M(L) \sim \Delta M(\infty) \left(1 + C \frac{e^{-M_\pi L}}{(M_\pi L)^\alpha} \right)$$

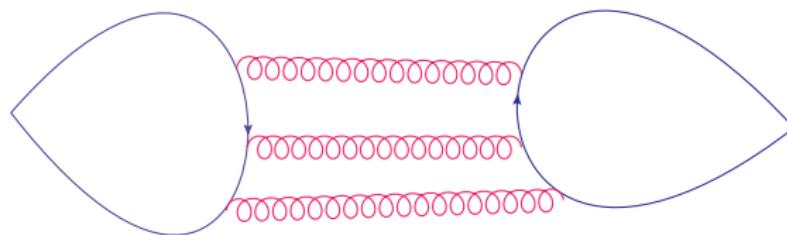


Approximations in the isoQCD analysis

- Non-unitary setup in the strange sector. Osterwalder-Seiler strange quarks.

$$m_q^{(val)} \neq m_q^{(sea)}$$

- We neglect “fermion-disconnected” contributions, e.g. :



Ansatz $M_{\pi^+} - M_{\pi^-}$

$$(M_{\pi^+} - M_{\pi^-})(L, a, r_\ell) =$$

$$\delta_0 \left[1 + c_L M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}} + c_3 L^{-3} + c_4 L^{-4} + c_a a^2 + c_\ell^{(1)} r_\ell + c_\ell^{(2)} r_\ell^2 + d_\ell r_\ell \log(r_\ell) - \right]$$

Ansatz $M_n - M_p$

$$(M_n - M_p)(L, a, r_\ell) =$$

$$\delta_0 \times \left[1 + c_L M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}} + c_3 L^{-3} + c_4 L^{-4} + c_a a^2 + c_\ell^{(1)} r_\ell + c_\ell^{(2)} r_\ell^2 \right]$$

Hadronic correlators

Mesonic correlators : $O(x) = \bar{\psi}_1(x) \Gamma \psi_2(x)$

- Integer spin (bosons)
- Coupled forward and backward signals

Baryonic correlators(fermions) : $O(x)_\alpha = \bar{\psi}_1(x) \Gamma \psi_2(x) \psi_3(x)_\alpha$

- Half-integer spin (fermions) → free Dirac index in the interpolator
- Parity projection acts on backward signals:
 - $P = +1$ propagate only forward (backward)
 - $P = -1$ propagate only backward (forward)

isoQCD background

Steps of the analysis

- Extraction of the leading exponential signal
 - Fit of the effective masse
 - n -exponential fit
 - Prony methods (ODE)
- Correction of Finite Volume Effects for each ensemble
- Extrapolation to the physical point of m_s for each ensemble
- Extrapolation to the physical point of m_ℓ and to the continuum

Strategy: We find the physical point in terms of hadronic ratios r_s and r_ℓ , and use it the tuning of counterterms in QCD+QED.