Leading Isospin Breaking effects in nucleon and Δ masses

S. Romiti, V. Lubicz, S. Simula, F. Sanfilippo

Roma Tre University

July 2021

Strategy

We consider a background of pure QCD in isospin symmetry:

$$S_0 = \int d^4x \mathcal{L} = \int d^4x \mathcal{L}_{QCD}(u = d = \ell)$$

Isospin Breaking (IB)

IB splits the degenerate doublet. Al LO we have 2 types of effects:

- strong IB : $\hat{m}_u \neq \hat{m}_d \to O(\frac{\hat{m}_d - \hat{m}_u}{\Lambda_{QCD}}) \sim O(1\%)$
- QED : $q_f = ee_f \neq 0 \to O(\alpha_{EM}) \sim O(1\%)$

RM123 method

At LO, the slopes can be calculated in the isosymmetric theory (isoQCD).
$\to$ Their linear combinations with appropriate counterterms give the observables in the full theory QCD+QED

Preliminary results

	our prediction	experiment
M_N	$0.957(20)$ GeV	$\approx 0.938 - 0.939$ GeV (PDG)
M_Δ	$1.267(38)$ GeV	$1.230 - 1.234$ GeV (PDG)
$M^2_{\pi^+} - M^2_{\pi^0}$	$1120(110)$ MeV^2	$1261.2(1)$ MeV^2 (PDG)
$M_n - M_p$	$1.69(71)$ MeV	$1.29333205(51)$ MeV (PDG)
$M_{\Delta^{++}} - M_{\Delta^0}$	$-2.2(1.1)$ MeV	$-2.86(30)$ MeV (PDG)
$[M_{\Delta^{++}} + M_{\Delta^-} - M_{\Delta^+} - M_{\Delta^0}]$	$2.41(51)$ MeV	2.84–3.5 MeV (Phys. Rev. C 47, 367)

Outline

1. isoQCD background
   - Strategy and renormalization scheme
   - Extrapolations of M_N and M_Δ
2. QCD+QED at LO
   - Mass corrections at LO
   - Tuning of counterterms
   - Pion mass difference
   - $M_n - M_p$
   - Mass splittings of Δ resonances
3. Backup

Steps of the analysis

- Extraction of masses from the leading exponentials in the correlators $(t/a \gg 1)$

Extrapolation among 3 values of am_s for each ensemble:
$r_s = \frac{2M_K^2 - M_\pi^2}{M_\Omega^2} \to r_s^{\text{(phys)}}$

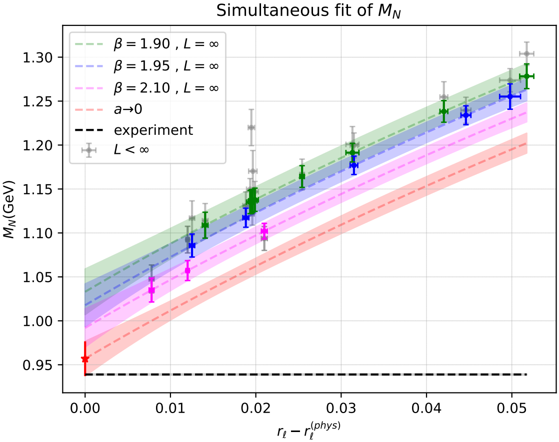
Extrapolation among the am_ℓ: $r_\ell = \frac{M_\pi^2}{M_\Omega^2} \to r_\ell^{\text{(phys)}}$

- Correction of FVEs from asymptotic formula of ChPT at finite volume
- Scale setting: $a = (aM_\Omega)/M_\Omega^{\text{(phys)}}$
- Final extrapolation in physical units:
  $a \to 0$, $V \to \infty$ and $r_\ell \to r_\ell^{\text{(phys)}}$.

Global extrapolations (M_N)

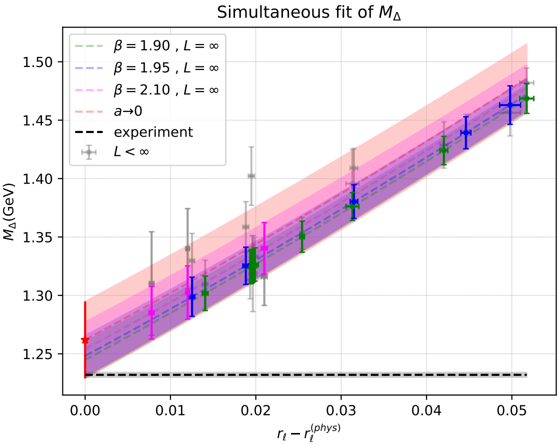
Inspired by ChPT results we fit according to:

$$M_N(L,a,r_\ell) = M_0^{(N)} \left[1 + c_L^{(N)} M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)} + c_a^{(N)} (a\Lambda_{QCD})^2 + c_\ell^{(N)} r_\ell + c_{3/2}^{(N)} r_\ell^{3/2}\right]$$



Global extrapolation ($M_\Delta(1232)$)

$$M_\Delta(L, a, r_\ell) = M_0^{(\Delta)} \left[1 + c_L^{(\Delta)} M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)} + c_a^{(\Delta)} (a\Lambda_{QCD})^2 + c_\ell^{(\Delta)} r_\ell + c_{3/2}^{(\Delta)} r_\ell^{3/2}\right]$$



QCD+QED at LO

$$\mathcal{L} = \mathcal{L}_0 - \Delta m_{ud}\, \bar{q}\tau_3 q + eA_\mu \bar{q}\gamma_\mu \left(\frac{\tau_3}{2} + \frac{1}{6}\right) q$$

QED variation (lattice)

$$\Delta[T\langle O(x_i)\rangle] = \int d^4x_1 d^4x_2 D_{\mu\nu}(x_1|x_2)\, T\langle O(x_i) J_\mu(x_1) J_\nu(x_2)\rangle + \int d^4x D_{\mu\mu}(x|x)\, T\langle O(x_i) T_\mu(x)\rangle \quad .$$

Renormalization in tmQCD+QED

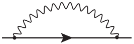
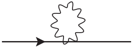
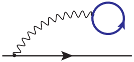
$$m_f \rightarrow \Delta m_f\, [\bar{\psi}_f \psi_f]$$

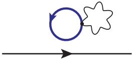
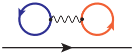
$$m_f^{(crit)} \rightarrow \Delta m_f^{(crit)}\, [\bar{\psi}_f\, i\gamma^5 \tau^3 \psi_f]$$

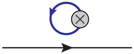
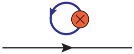
$$g_s \rightarrow \Delta g_s\, G_{\mu\nu} G_{\mu\nu}$$

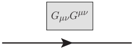
Quark propagator in QCD+QED

Δ  $^{\pm} = -(m_f - m_f^{(0)})$  $\mp (m_f - m_f^{(0)})^{(crit)}$ 

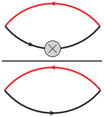
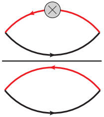
$+ (e_f e)^2$  $+ (e_f e)^2$  $- e_f e^2 \sum_{f_1} e_{f_1}$ 

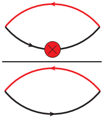
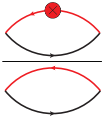
$- e^2 \sum_{f_1} e_{f_1}^2$  $- e^2 \sum_{f_1} e_{f_1}^2$  $+ e^2, \sum_{f_1, f_2} e_{f_1} e_{f_2}$ 

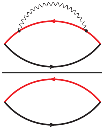
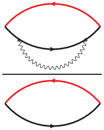
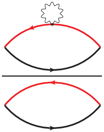
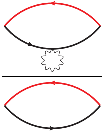
$+ \sum_{f_1} (m_{f_1} - m_{f_1}^{(0)})$  $(m_{f_1} - m_{f_1}^{(0)})^{(crit)}$ 

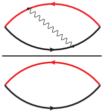
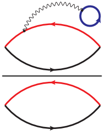
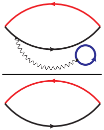
$+ (g_s^2 - {g_s^{(0)}}^2)$ 

Example: ΔM_{K^+}

$\Delta M_{K^+} = +(m_u - m_{ud}^{(0)})\partial_t$  $+ (m_s - m_s^{(0)})\partial_t$ 

$- (m_u - m_{ud}^{(0)})^{(crit)}\partial_t$  $+ (m_s - m_s^{(0)})^{(crit)}\partial_t$ 

$- (e_s e)^2 \partial_t$  $- (e_u e)^2 \partial_t$  $- (e_s e)^2 \partial_t$  $- (e_u e)^2 \partial_t$ 

$- e_u e_s e^2 \partial_t$  $- e_s e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t$  $- e_u e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t$ 

$+$ [isosymm. vac. pol. diag.]

Recap and details of the method

Expansion over isosymmetric background

We stop at $O(\alpha_{EM}) \sim O\left(\frac{m_d - m_u}{\Lambda_{QCD}}\right)$

- recycle gauge configurations in QCD with isospin
- Linear corrections in the counterterms

Steps of the analysis

- Tuning of $\Delta m_f^{(crit)}$ at fixed ensemble from the PCAC Ward Identity (keep maximal twist at $O(\alpha_{EM}) \implies O(a)$ improvement)
- Correction of universal FVEs from QED_L.
- Tuning of Δm_f at fixed ensemble from hadronic ratios
- Extrapolaton to the physical point, $a \to 0$, $V \to \infty$.

QED at Finite volume

The lattice photon propagator $D_{\mu\nu}(k)$ is divergent at $k = 0$:

$$D_{\mu\nu}(k) = \frac{g_{\mu\nu}}{\hat{k}^2} = \frac{g_{\mu\nu}}{\sin^2(k_\mu)}$$

QED_L prescription

We remove the 0-mode manually $\implies$ Finite volume effects.

$$M(L) = M(\infty)\left[1 + Q^2\alpha\frac{\kappa}{2ML}\left(1 + \frac{2}{ML}\right)\right] + O\left(\frac{1}{L^3}\right)$$

$(\kappa \approx 2.837297) \sim L^{-3}, \sim L^{-4}$ come from the structure of the hadron.

We implement the correction on the slopes $\implies$ counterterms don't contain the universal QED_L effects.

QCD+QED : Tuning of mass conterterms

- Tuning of $a\Delta m_u$, $a\Delta m_d$, $a\Delta m_s$ from the ratios

$$r_s = \frac{2(M^2_{K^+} + M^2_{K^0}) - (M^2_{\pi^+} + M^2_{\pi^0})}{2M^2_{\Omega^-}} = r_s^{(0)} + \sum_f a\Delta m_f \bar{\Delta} r(f) + \Delta r_s^{(EMC)}$$

$$r_\ell = \frac{M^2_{\pi^+} + M^2_{\pi^0}}{2M^2_{\Omega^-}} = r_\ell^{(0)} + \sum_f a\Delta m_f \bar{\Delta} r_\ell(f) + \Delta r_\ell^{(EMC)}$$

$$r_p = \frac{M^2_{K^+}}{M^2_{\Omega^-}} = r_p^{(0)} + \sum_f a\Delta m_f \bar{\Delta} r_p(f) + \Delta r_p^{(EMC)}$$

We define the physical point of isoQCD such that the total IB correction vanishes.

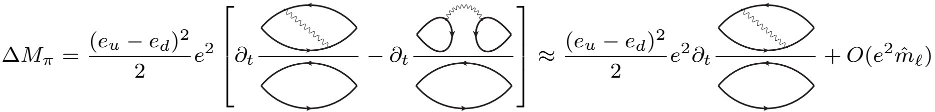
QCD+QED : Tuning of mass counterterms

We require the ratios r_s , r_ℓ , r_p to be at the physical point when we reach the physical m_s and m_ℓ found in isoQCD:

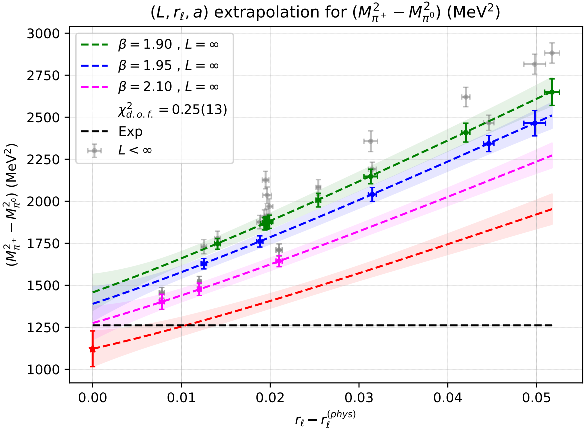
$$\begin{bmatrix} \bar{\Delta} r_s(u) & \bar{\Delta} r_s(d) & \bar{\Delta} r_s(d) \\ \bar{\Delta} r_\ell(u) & \bar{\Delta} r_\ell(d) & \bar{\Delta} r_\ell(s) \\ \bar{\Delta} r_p(u) & \bar{\Delta} r_p(d) & \bar{\Delta} r_p(s) \end{bmatrix} \begin{bmatrix} a\Delta m_u \\ a\Delta m_d \\ a\Delta m_s \end{bmatrix} = - \begin{bmatrix} \Delta r_s^{(EMC)} \\ \Delta r_\ell^{(EMC)} \\ \Delta r_p^{(EMC)} \end{bmatrix}$$

Note: using r_s, r_ℓ and r_p we loose predictivity on π, K, Ω masses, but we can predict the other corrections (e.g. M_n, M_p, $M_{\Delta^{++}}$, etc.)

$M_{\pi^+} - M_{\pi^0}$

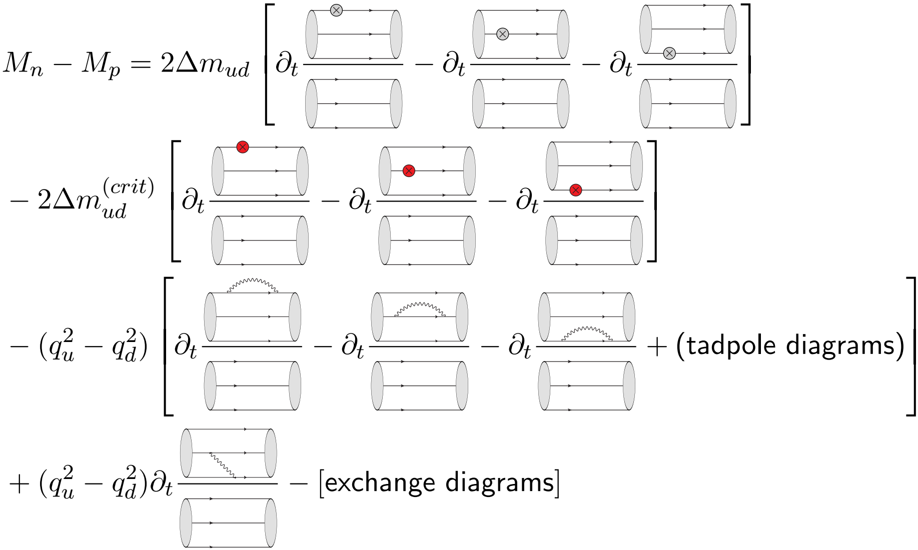


$\Delta M_\pi = \frac{(e_u - e_d)^2}{2} e^2 \Big[\partial_t \; - \partial_t \Big] \approx \frac{(e_u - e_d)^2}{2} e^2 \partial_t \; + O(e^2 \hat{m}_\ell)$



(L, r_ℓ, a) extrapolation for $(M^2_{\pi^+} - M^2_{\pi^0})$ (MeV^2)
$\beta = 1.90$, $L = \infty$
$\beta = 1.95$, $L = \infty$
$\beta = 2.10$, $L = \infty$
$\chi^2_{d.o.f.} = 0.25(13)$
Exp
$L < \infty$
$(M^2_{\pi^+} - M^2_{\pi^0})$ (MeV^2)
$r_\ell - r_\ell^{(phys)}$
3000
2750
2500
2250
2000
1750
1500
1250
1000
0.00
0.01
0.02
0.03
0.04
0.05

$M_n - M_p$


$M_n - M_p = 2\Delta m_{ud}$ [∂_t − ∂_t − ∂_t]
$- 2\Delta m_{ud}^{(crit)}$ [∂_t − ∂_t − ∂_t]
$-(q_u^2 - q_d^2)$ [∂_t − ∂_t − ∂_t + (tadpole diagrams)]
$+(q_u^2 - q_d^2)\partial_t$ − [exchange diagrams]

Separation of strong IB and QED

The separation is scheme dependent:

$$\Delta m_{ud} = \frac{m_d - m_u}{2} = \Delta m_{ud}^{(QCD)} + \Delta m_{ud}^{(QED)} = Z_P^{(0)} \Delta \hat{m}_{ud} + \mathcal{Z}_{ud}^{-1} \hat{m}_{ud}$$

$$\mathcal{Z}_{ud}^{-1}(\mu) = Z_P^{(0)} \frac{q_d^2 - q_u^2}{32\pi^2} \left[6 \log(a\mu) - 22.595 + ...\right]$$

Physical interpretation

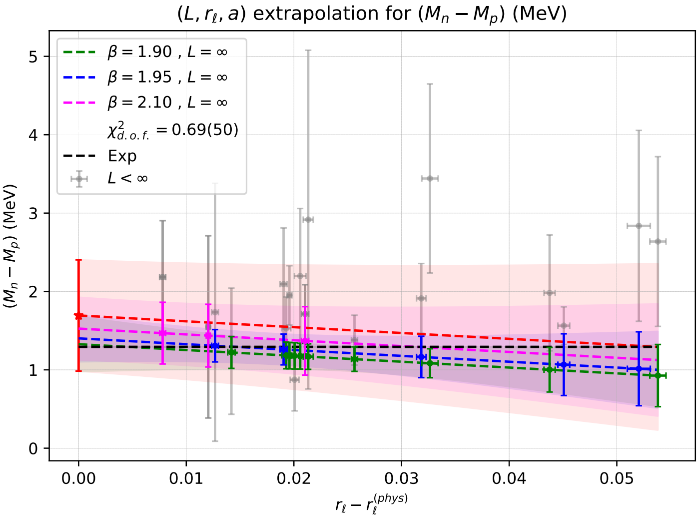
- QCD : $m_d > m_u \implies M_n > M_p$
- QED : $|Q_p| > |Q_n| \implies M_p > M_n$

The combination of these 2 effects cancel almost exactly:

$$M_n - M_p \approx 1.3\,\text{MeV} = O(10^{-3}) M_N$$

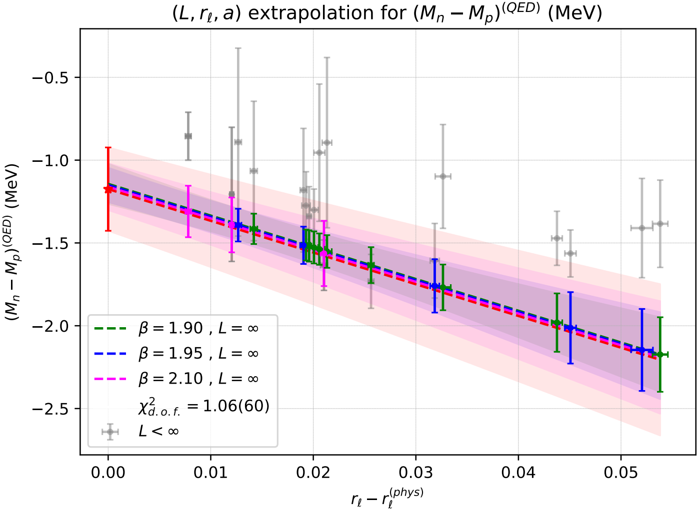
$M_n - M_p$

$$M_n - M_p = 1.69(71)\,\text{MeV}$$



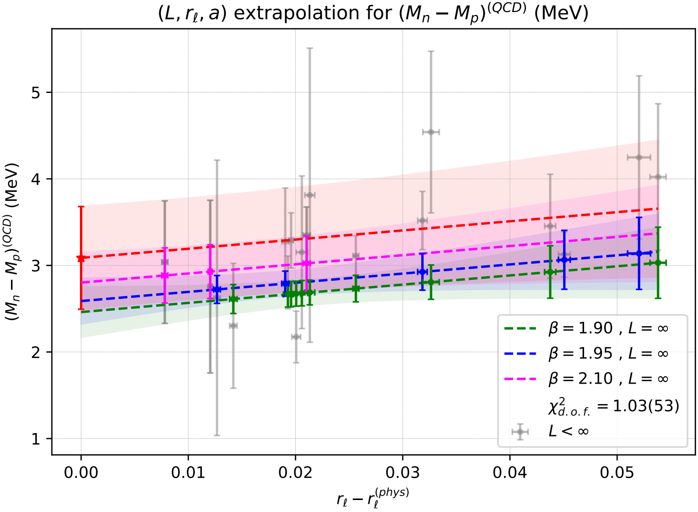
$(M_n - M_p)^{(QED)}$

$$(M_n - M_p)^{(QED)} = -1.17(25)\,\text{MeV}$$



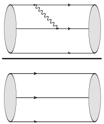
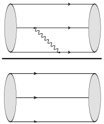
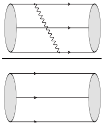
$(M_n - M_p)^{(QCD)}$

$$(M_n - M_p)^{(QCD)} = 3.09(59)\,\text{MeV}$$



$\Delta(1232)$ mass splittings

$$(M_{++} + M_{-}) - (M_{+} + M_{0}) = -\frac{2}{3}(q_u - q_d)^2$$

$\Big\{ \partial_t$  $+ \partial_t$  $+ \partial_t$  $- 2$[exchange diagrams] $\Big\}$

This is a purely electromagnetic effect (scheme independent).

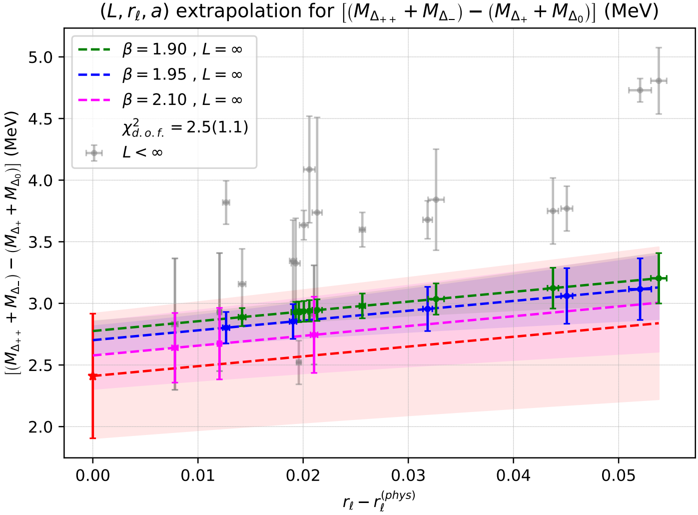
4 masses $\implies$ 3 independent mass splittings, but at LO:

$$M_{++} - M_{-} = 3(M_{+} - M_{0})$$

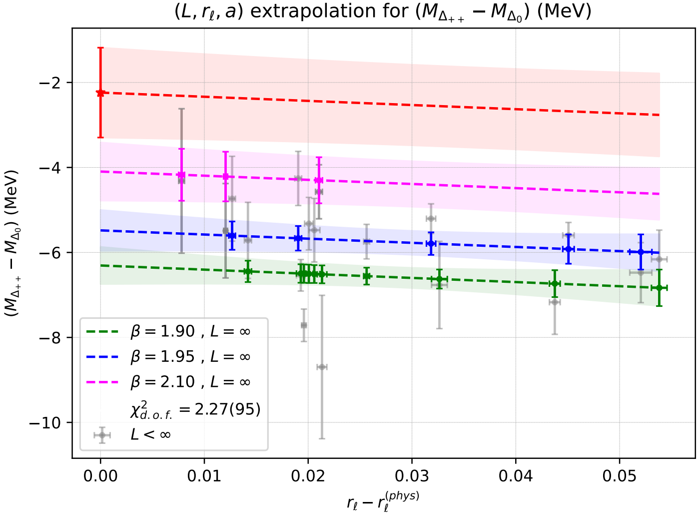
$\implies$ 2 independent mass splittings.
$\implies$ from $M_{++} - M_0$ and $[(M_{++} + M_{-}) - (M_{+} + M_{0})]$ we find them all.

$(M_{++} + M_{-}) - (M_{+} + M_0)$

$$(M_{++} + M_{-}) - (M_{+} + M_0) = 2.41(51)\,\text{MeV}$$



$$M_{++} - M_0 = -2.2(1.1)\,\text{MeV}$$



Conclusions

We are able to include IB at 1st order in the spectrum of mesons and baryons.

- No need for QED in the Lattice Lagrangian at LO.
  (same isoQCD gauge configurations)
- Hadronic scheme to reach the physical point and tune mass counterterms.
  (independence from renormalization contants)
- 1σ compatibility with experimental values and consistent separation of strong IB and QED.
- Prediction for mass splittings involving M_{Δ^-}
  (no experimental value available yet)

Thank you for the attention

Backup

1. isoQCD background
   - Strategy and renormalization scheme
   - Extrapolations of M_N and M_Δ
2. QCD+QED at LO
   - Mass corrections at LO
   - Tuning of counterterms
   - Pion mass difference
   - $M_n - M_p$
   - Mass splittings of Δ resonances
3. Backup

Ensembles

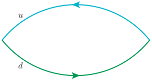
Ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	κ	N_{cfg}
$A30.32$	1.90	$32^3 \times 64$	0.0030	0.15	0.19	0.163272	150
$A40.32$			0.0040			0.163270	150
$A50.32$			0.0050			0.163267	150
$A40.20$	1.90	$20^3 \times 48$	0.0040	0.15	0.19	0.163270	150
$A40.24$	1.90	$24^3 \times 48$	0.0040	0.15	0.19	0.163270	150
$A60.24$			0.0060			0.163265	150
$A80.24$			0.0080			0.163255	150
$A100.24$			0.0100			0.163260	150
$A40.48$	1.90	$48^3 \times 96$	0.0040	0.15	0.19	0.163270	90
$A40.40$	1.90	$40^3 \times 80$	0.0040	0.15	0.19	0.163270	150

Ensembles

Ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	κ	N_{cfg}
$B25.32$	1.95	$32^3 \times 64$	0.0025	0.135	0.170	0.1612420	150
$B35.32$			0.0035			0.1612400	150
$B55.32$			0.0055			0.1612360	150
$B75.32$			0.0075			0.1612320	75
$B85.24$	1.95	$24^3 \times 48$	0.0085	0.135	0.170	0.1612312	150
$D15.48$	2.10	$48^3 \times 96$	0.0015	0.12	0.1385	0.156361	90
$D20.48$			0.0020			0.156357	90
$D30.48$			0.0030			0.156355	90

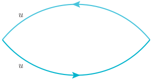
- Partial quenching in the strange sector
- A40.XX ensembles differ only for the volume
- 3 values of the lattice spacing ($a^{-1} \sim 2 - 3$ GeV)
- $M_\pi \simeq 200 - 450$ MeV

$$C_{\pi^+\pi^-}(x) = -\sum_{\vec{x}} \langle [\bar{u}\gamma_5 d](x)\,[\bar{d}\gamma_5 u](0)\rangle = -$$

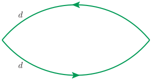


$$C_{\pi^0\pi^0}(x) = -\frac{1}{2}\sum_{\vec{x}} \langle [\bar{u}\gamma_5 u - \bar{d}\gamma_5 d](x)[\bar{u}\gamma_5 u - \bar{d}\gamma_5 d](0)\rangle$$

$$= -\frac{1}{2}\Big[$$

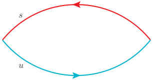


$$+$$

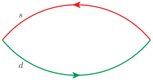


$$\Big]$$

$$C_{K^+K^-}(x) = -\sum_{\vec{x}} \langle [\bar{s}\gamma_5 u](x)\,[\bar{u}\gamma_5 s](0)\rangle =$$



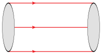
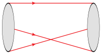
$$C_{K^0\bar{K}^0}(x) = -\sum_{\vec{x}} \langle [\bar{s}\gamma_5 d](x)\,[\bar{d}\gamma_5 s](0)\rangle =$$



Baryonic Correlators ($\vec{p} = \vec{0}$, $J^P = 3/2^+$)

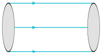
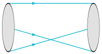
Ω^-

$$\begin{aligned} C(t) \propto\ & \epsilon_{a_1 b_1 c_1} \epsilon_{a_2 b_2 c_2} \\ & + \mathrm{Tr}[S_s^{T\,a_1\,a_2}(x|0) C\gamma_\nu S_s{}^{b_1\,b_2}(x|0) C\gamma_\mu]\,\mathrm{Tr}[P_+ P^{3/2}_{\mu\nu} S_s{}^{c_1\,c_2}(x|0)] \\ & - 2\mathrm{Tr}[S_s{}^{a_1\,b_2}(x|0) P_+ P^{3/2}_{\mu\nu} S_s{}^{b_1\,a_2}(x|0) C\gamma_\nu S_s^{T\,c_1\,c_2}(x|0) C\gamma_\mu] \end{aligned}$$

$=$  $-\,2$ 

Δ^{++}

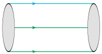
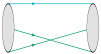
$$\begin{aligned} C(t) \propto\ & \epsilon_{a_1 b_1 c_1} \epsilon_{a_2 b_2 c_2} \\ & + \mathrm{Tr}[S_u^{T\,a_1\,a_2}(x|0) C\gamma_\nu S_u{}^{b_1\,b_2}(x|0) C\gamma_\mu]\,\mathrm{Tr}[P_+ P^{3/2}_{\mu\nu} S_u{}^{c_1\,c_2}(x|0)] \\ & - 2\mathrm{Tr}[S_u{}^{a_1\,b_2}(x|0) P_+ P^{3/2}_{\mu\nu} S_u{}^{b_1\,a_2}(x|0) C\gamma_\nu S_u^{T\,c_1\,c_2}(x|0) C\gamma_\mu] \end{aligned}$$

$=$  $-\,2$ 

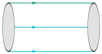
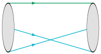
$\ldots(\Delta^+,\ \Delta^0,\ \Delta^-)$

Baryonic Correlators ($\vec{p}=\vec{0},\ J^P=1/2^+$)

$$
\begin{aligned}
p \quad C(t) = {} & \epsilon_{a_1 b_1 c_1}\epsilon_{a_2 b_2 c_2} \\
& + \mathrm{Tr}[S_u^{T\,a_1\,a_2}(x|0)C\gamma_5 S_u{}^{b_1\,b_2}(x|0)C\gamma_5]\,\mathrm{Tr}[P_+ S_d{}^{c_1\,c_2}(x|0)] \\
& - \mathrm{Tr}[S_u{}^{a_1\,b_2}(x|0)P_+ S_u{}^{b_1\,a_2}(x|0)C\gamma_5 S_d^{T\,c_1\,c_2}(x|0)C\gamma_5]
\end{aligned}
$$

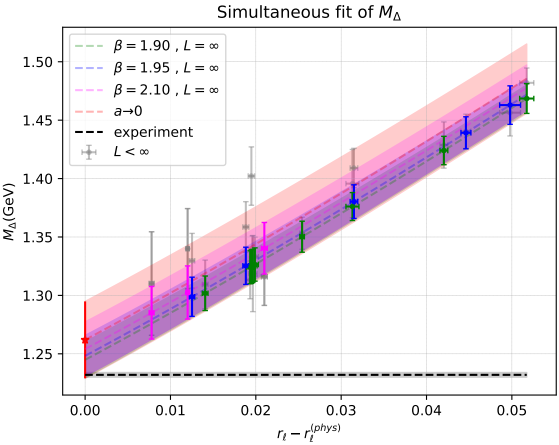
$=$  $-$ 

$$
\begin{aligned}
n \quad C(t) = {} & \epsilon_{a_1 b_1 c_1}\epsilon_{a_2 b_2 c_2} \\
& + \mathrm{Tr}[S_d^{T\,a_1\,a_2}(x|0)C\gamma_5 S_d{}^{b_1\,b_2}(x|0)C\gamma_5]\,\mathrm{Tr}[P_+ S_u{}^{c_1\,c_2}(x|0)] \\
& - \mathrm{Tr}[S_d{}^{a_1\,b_2}(x|0)P_+ S_d{}^{b_1\,a_2}(x|0)C\gamma_5 S_u^{T\,c_1\,c_2}(x|0)C\gamma_5]
\end{aligned}
$$

$=$  $-$ 

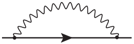
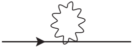
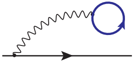
Global extrapolation ($M_\Delta(1232)$)

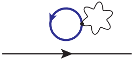
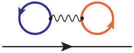
$$M_\Delta(L,a,r_\ell) = M_0^{(\Delta)}\left[1 + c_L^{(\Delta)} M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)} + c_a^{(\Delta)} (a\Lambda_{QCD})^2 + c_\ell^{(\Delta)} r_\ell + c_{3/2}^{(\Delta)} r_\ell^{3/2}\right]$$

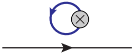
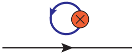


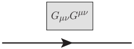
Quark propagator in QCD+QED

Δ  $^{\pm} = -(m_f - m_f^{(0)})$  $\mp (m_f - m_f^{(0)})^{(crit)}$ 

$+ (e_f e)^2$  $+ (e_f e)^2$  $- e_f e^2 \sum_{f_1} e_{f_1}$ 

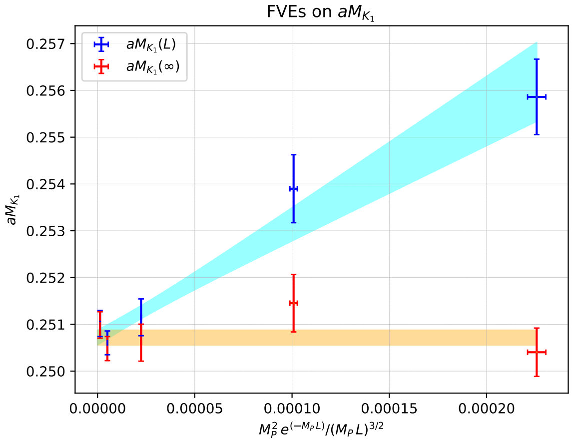
$- e^2 \sum_{f_1} e_{f_1}^2$  $- e^2 \sum_{f_1} e_{f_1}^2$  $+ e^2, \sum_{f_1, f_2} e_{f_1} e_{f_2}$ 

$+ \sum_{f_1} (m_{f_1} - m_{f_1}^{(0)})$  $(m_{f_1} - m_{f_1}^{(0)})^{(crit)}$ 

$+ (g_s^2 - g_s^{(0)^2})$ 

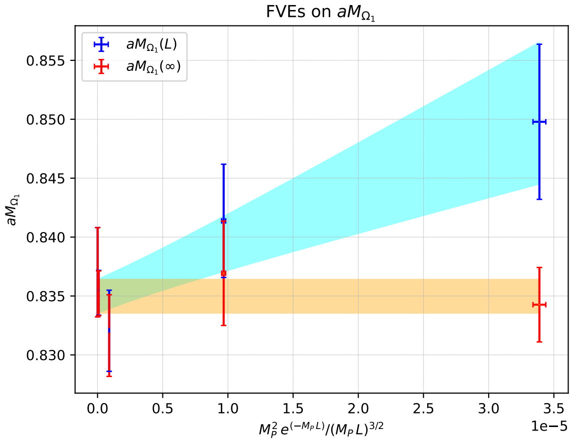
Correction of FVEs (mesons)

$$M_P(L) = M\left(1 + C_P\, M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}}\right)$$



FVEs (M_Ω)

$$M_\Omega(L) = M_\Omega(\infty)\left(1 + C_B\, M_K^2 \frac{e^{-M_K L}}{(M_K L)^{3/2}}\right)$$



Chiral perturbation theory (ChPT)

ChPT approximates low-energy QCD $\rightarrow$ base for fit ansatz.

Meson ChPT

- $M_P^2 \propto (\hat{m}_1 + \hat{m}_2)$
  - $2M_K^2 - M_\pi^2 \propto \hat{m}_s$
  - $M_\pi^2 \propto \hat{m}_\ell$

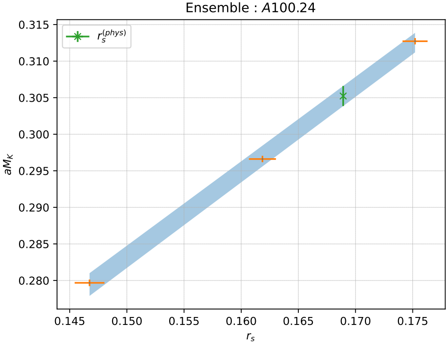
Baryon ChPT

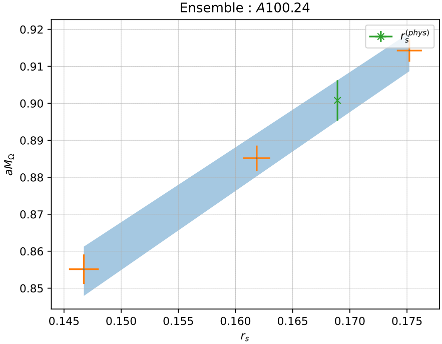
- $M_N = M_N^{(0)} - 4\,c_1\,M_\pi^2 - \frac{3g_A^2}{32\pi f_\pi} M_\pi^3 + O(M_\pi^4 \log(M_\pi^2)) + \ldots$
- $M_\Omega = M_\Omega^{(0)} - 4\,c_\Omega^{(1)}\,M_\pi^2 + O(M_\pi^4 \log(M_\pi^2)) + \ldots$

$r_s = \frac{2M_K^2 - M_\pi^2}{M_\Omega^2}$ and $r_\ell = \frac{M_\pi^2}{M_\Omega^2}$ can be used to reach the physical point of am_ℓ and am_s

r_s interpolation

For each ensemble we interpolate among the 3 values of am_s.





Perturbative expansion of QED

Consider a theory with couplings $\vec{g}^{(0)} = (g_1^{(0)}, ..., g_n^{(0)})$. When we add the perturbation $\Delta S[U, A, \psi, \bar{\psi}]$

$$\langle O \rangle^{\vec{g}} = \frac{\left\langle e^{-(\beta-\beta_0)S[U]} \prod_f \frac{det(D_f[U,\vec{g}])}{det(D_f[U,\vec{g}_0])} O[U,A,\vec{g}] \right\rangle^{A,\vec{g}_0}}{\left\langle e^{-(\beta-\beta_0)S[U]} \prod_f \frac{det(D_f[U,\vec{g}])}{det(D_f[U,\vec{g}_0])} \right\rangle^{A,\vec{g}_0}}$$

Electroquenched approximation

We consider **chargless sea quarks**. This consists in setting:

$$e^{-(\beta-\beta_0)S[U]} \prod_f \frac{det(D_f[U,\, \vec{g}])}{det(D_f[U,\, \vec{g}_0])} \to 1$$

$\to$ feasible calculation using heat-bath algorithms.

Mass corrections from lattice correlators

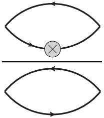
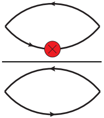
$$\Delta C = \sum_y \langle O(t)\, J(y)\, O^\dagger(0) \rangle$$

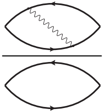
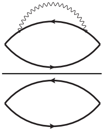
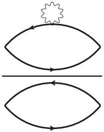
ΔM , ΔA

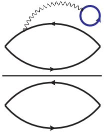
$$C(t) = A\, e^{-M\,t} = A_0(1 + \Delta A)\, e^{-(M_0 + \Delta M)\,t}$$
$$= C_0(1 + \Delta A - \Delta M\, t)$$

The mass correction is found as

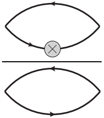
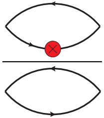
$$\Delta M = -\partial_t \frac{\Delta C}{C_0}$$

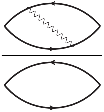
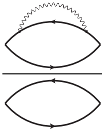
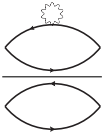
$\Delta M_{\pi^+} = +2(m_{ud} - m_{ud}^{(0)})\partial_t$  $- (m_u + m_d - 2m_{ud}^{(0)})^{(crit)}\partial_t$ 

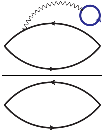
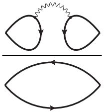
$- e_u e_d e^2 \partial_t$  $- (e_u^2 + e_d^2)e^2\partial_t$  $- (e_u^2 + e_d^2)e^2\partial_t$ 

$- (e_u + e_d)e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t$  + [isosymm. vac. pol. diag.] ,

ΔM_{π^0}

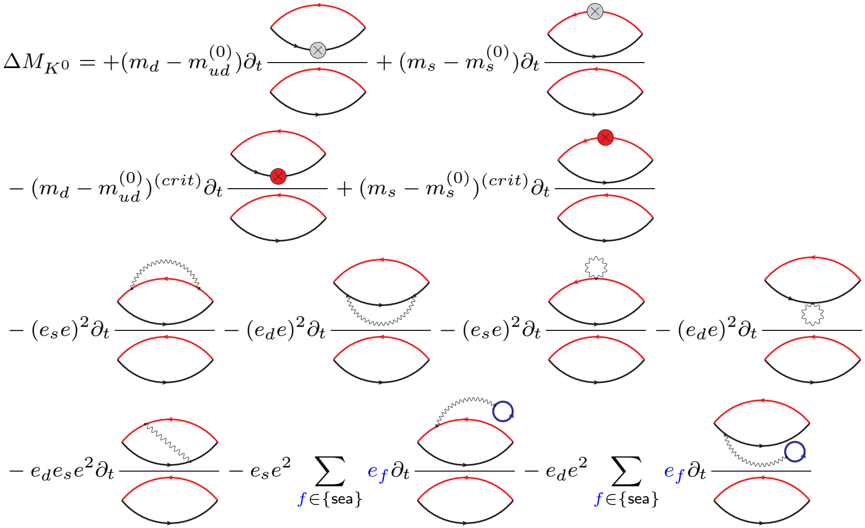
$\Delta M_{\pi^0} = +2(m_{ud} - m_{ud}^{(0)})\partial_t$  $- (m_u + m_d - m_{ud}^{(0)})^{(crit)}\partial_t$ 

$- \frac{(e_u^2 + e_d^2)}{2} e^2 \partial_t$  $- (e_u^2 + e_d^2) e^2 \partial_t$  $- (e_u^2 + e_d^2) e^2 \partial_t$ 

$- (e_u + e_d) e^2 \sum_{f \in \{\text{sea}\}} e_f \partial_t$  $+ \frac{(e_u - e_d)^2}{2} e^2 \partial_t$ 

$+$ [isosymm. vac. pol. diag.] ,

ΔM_{K^0}



+ [isosymm. vac. pol. diag.]

Tuning of $\Delta m_f^{(crit)}$

In the physical basis the critical mass counterterm is modified by the insertion of

$$J(x) = \sum_f \bar{\psi}_f(x) i\gamma_5 \tau_3 \psi_f(x)$$

PCAC Ward Identity

$$\partial_t \sum_{\vec{x}} \langle \bar{\psi}_f(t,\vec{x}) i\gamma_0 \psi_f(t,\vec{x})\, O(y) \rangle = 2m_f^{(PCAC)} \sum_{\vec{x}} \langle \bar{\psi}_f(t,\vec{x}) i\gamma_5 \psi_f(t,\vec{x})\, O(y) \rangle$$

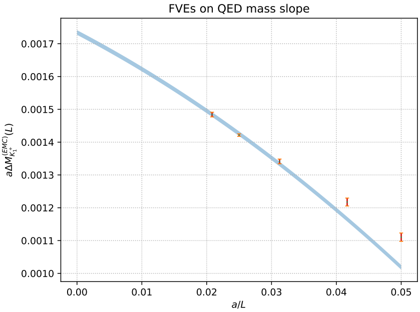
- In isoQCD the PCAC mass was tuned to 0 . $\rightarrow$ maximal twist $\rightarrow$ $O(a)$ improvement
- In QCD+QED we impose that maximal twist is preserved at $O(\alpha_{EM})$:

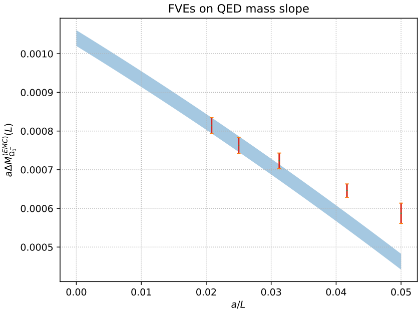
$$\Delta \left(\frac{\partial_t \langle V_0 P_5 \rangle}{\langle P_5 P_5 \rangle} \right)^{(EMC)} = 0$$

FVEs on QED mass corrections

QED_L formula($\kappa \approx 2.837297$)

$$\Delta M(L) = \Delta M(\infty) - Q^2 \alpha \frac{\kappa}{2ML}\left(1 + \frac{2}{ML}\right) + O(\frac{1}{L^3})$$

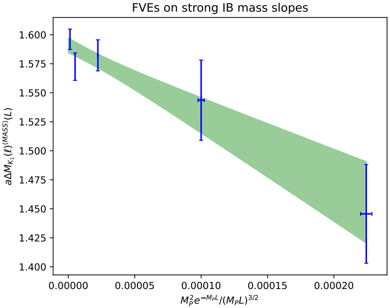


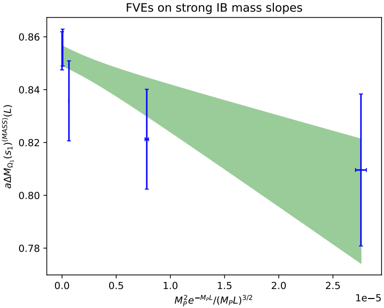


FVEs on strong IB mass corrections

strong IB ($\Delta m_f \neq 0$)

$$\Delta M(L) \sim \Delta M(\infty)\left(1 + C\frac{e^{-M_\pi L}}{(M_\pi L)^\alpha}\right)$$



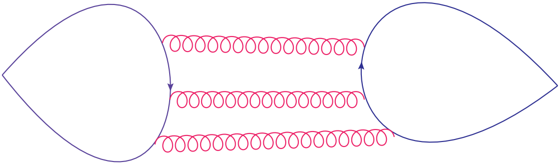


Approximations in the isoQCD analysis

- Non-unitary setup in th strange sector. Osterwalder-Seiler strange quarks.

$$m_q^{(val)} \neq m_q^{(sea)}$$

- We neglect "fermion-disconnected" contributions, e.g. :



Ansatz $M_{\pi^+} - M_{\pi^-}$

$$(M_{\pi^+} - M_{\pi^-})(L, a, r_\ell) =$$
$$\delta_0 \left[1 + c_L M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}} + c_3 L^{-3} + c_4 L^{-4} + c_a a^2 + c_\ell^{(1)} r_\ell + c_\ell^{(2)} r_\ell^2 + d_\ell r_\ell \log(r_\ell) + \right.$$

Ansatz $M_n - M_p$

$$(M_n - M_p)(L, a, r_\ell) =$$
$$\delta_0 \times \left[1 + c_L M_\pi^2 \frac{e^{-M_\pi L}}{(M_\pi L)^{3/2}} + c_3 L^{-3} + c_4 L^{-4} + c_a a^2 + c_\ell^{(1)} r_\ell + c_\ell^{(2)} r_\ell^2\right]$$

Hadronic correlators

Mesonic correlators : $O(x) = \bar{\psi}_1(x)\,\Gamma\,\psi_2(x)$

- Integer spin (bosons)
- Coupled forward and backward signals

Baryonic correlators(fermions) : $O(x)_\alpha = \bar{\psi}_1(x)\,\Gamma\,\psi_2(x)\psi_3(x)_\alpha$

- Half-integer spin (fermions) $\rightarrow$ free Dirac index in the interpolator
- Parity projection acts on backward signals:
  - $P = +1$ propagate only forward (backward)
  - $P = -1$ propagate only backward (forward)

isoQCD background

Steps of the analysis

- Extraction of the leading exponential signal
  - Fit of the effective masse
  - n-exponential fit
  - Prony methods (ODE)
- Correction of Finite Volume Effects for each ensemble
- Extrapolation to the physical point of m_s for each ensemble
- Extrapolation to the physical point of m_ℓ and to the continuum

Strategy: We find the physical point in terms of hadronic ratios r_s and r_ℓ, and use it the tuning of counterterms in QCD+QED.