Decays of an exotic 1^{-+} hybrid meson resonance in QCD

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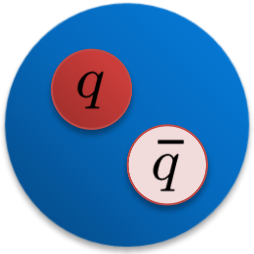
Exotic mesons



Experiments have observed a number of structures that don’t fit into the quark-model picture of mesons.

Exotic quantum numbers are particularly interesting, e.g. $J^{PC} = 0^{--}, 0^{+-}, \mathbf{1^{-+}}, 2^{+-}$

One proposed type of exotic is a **hybrid meson**.

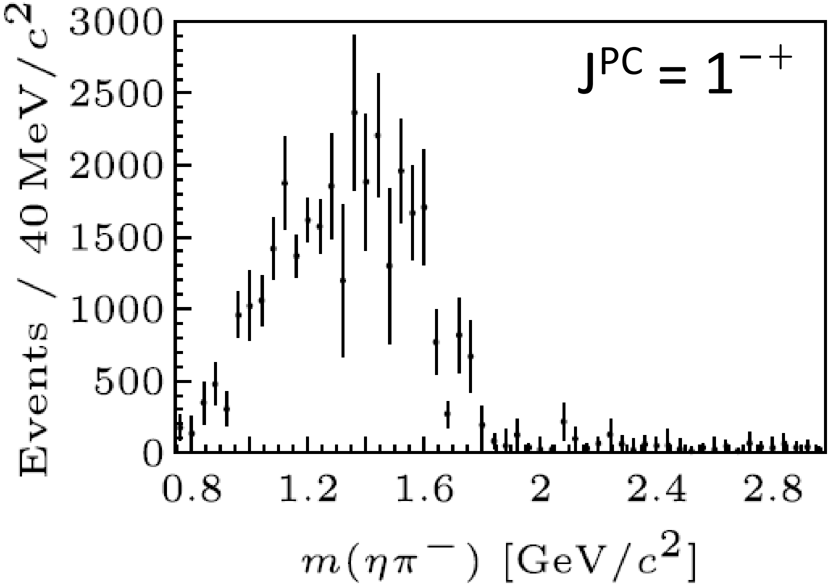
Exotic mesons

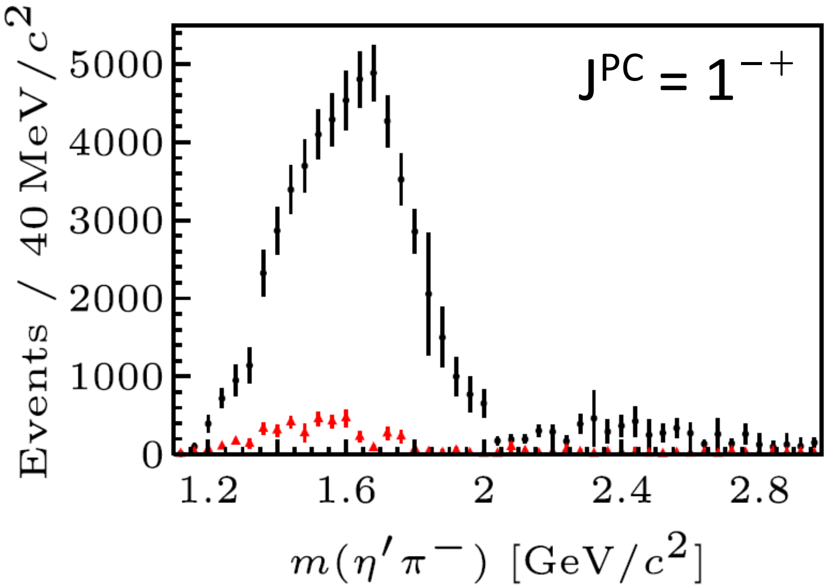
[PL B 740, 303 (2015)]

$\eta^{(\prime)}\pi^-$ in $\pi^-\ p \rightarrow \eta^{(\prime)}\pi^-\ p$

$\pi_1(1400)/\pi_1(1600)$?

COMPASS





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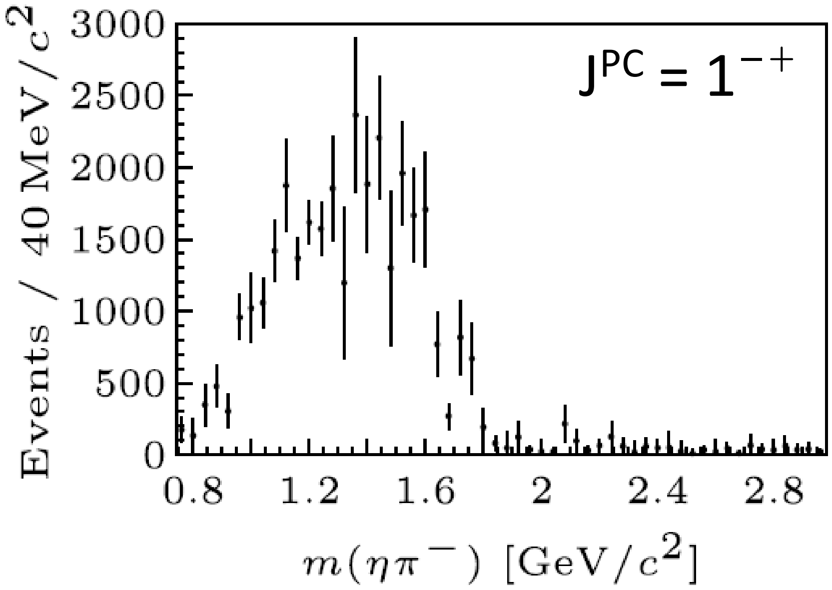

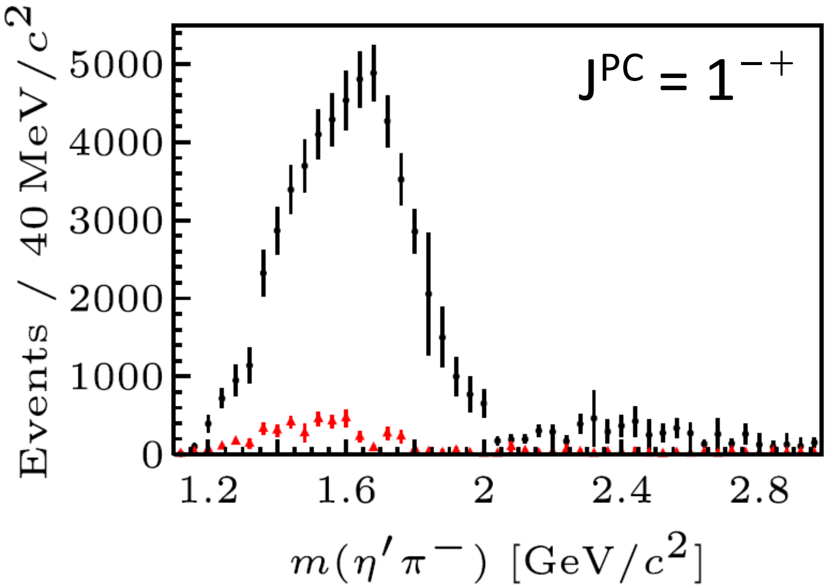

Rodas *et al* (JPAC) [PRL 122, 042002 (2019)]: single resonance, m = 1564(24)(86) MeV, Γ = 492(54)(102) MeV

Kopf *et al* [2008.11566] CB and COMPASS data: single resonance, $m = (1561.6 \pm 3.0\,^{+6.6}_{-2.6})\,\mathrm{MeV}$, $\Gamma = (388.1 \pm 5.4\,^{+0.2}_{-14.1})\,\mathrm{MeV}$

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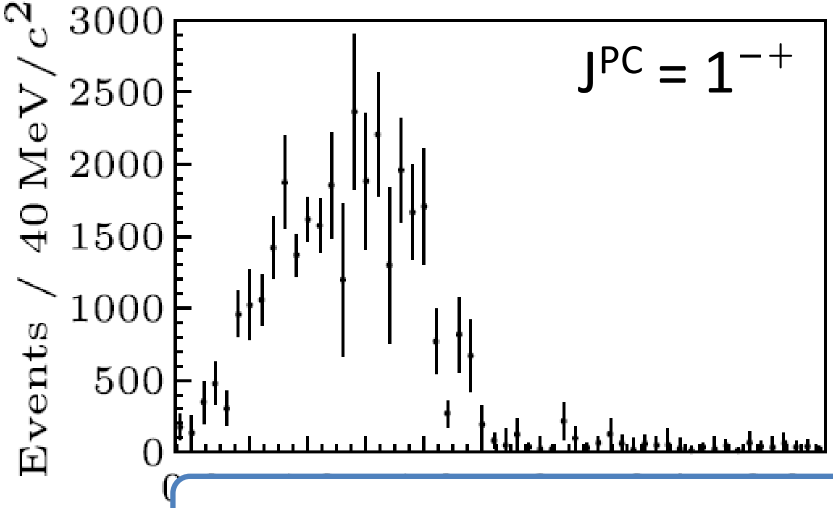

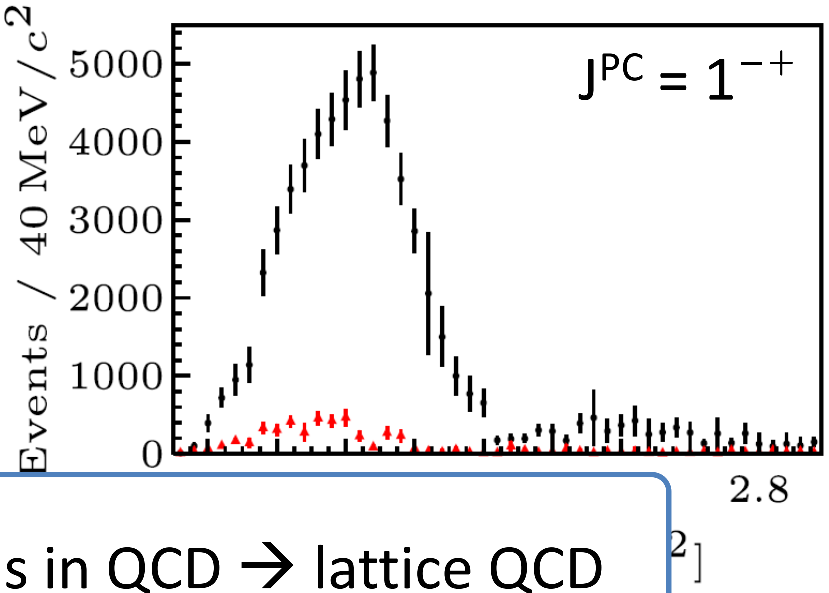

First-principles calculations in QCD → lattice QCD

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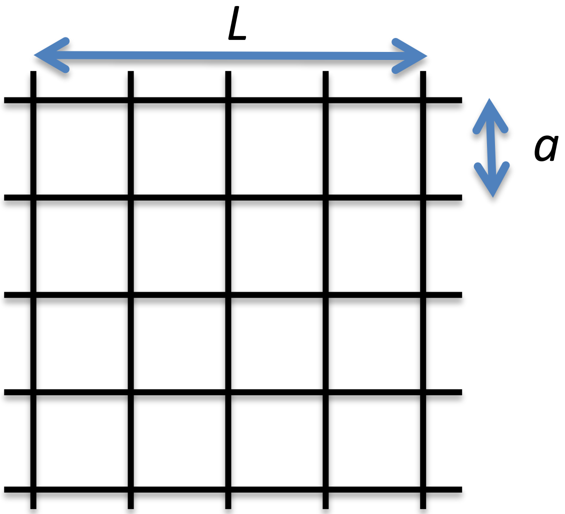
Lattice QCD spectroscopy

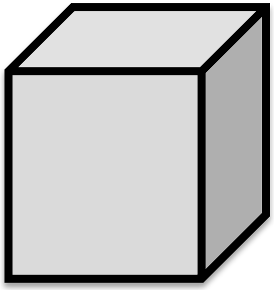
See Dave Wilson’s talk on Thursday for more of an overview



Systematically-improvable first-principles calculations

Finite-volume energy eigenstates from:

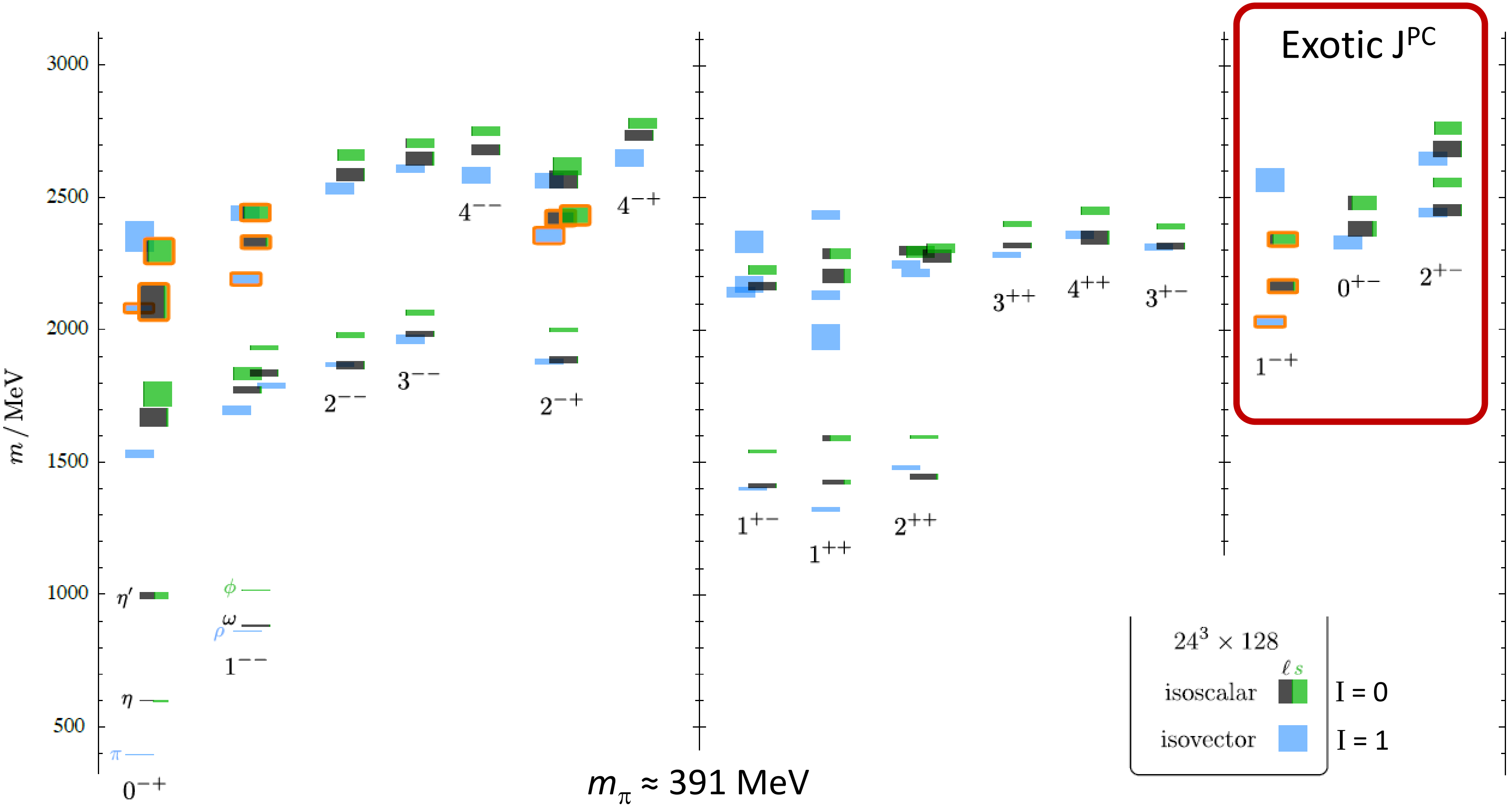
$$C_{ij}(t) = \left\langle 0 \left| \underline{\mathcal{O}_i(t)}\,\underline{\mathcal{O}_j^{\dagger}(0)} \right| 0 \right\rangle$$



Large bases of interpolating operators (with appropriate structures)

Light mesons (isospin = 0 and 1)

[Dudek, Edwards, Guo, CT, PR D88, 094505 (2013)]



$m_\pi \approx 391$ MeV

Large bases of only fermion-bilinear ops $\sim \bar{\psi}\Gamma D \ldots \psi$

(also other m_π and volumes)

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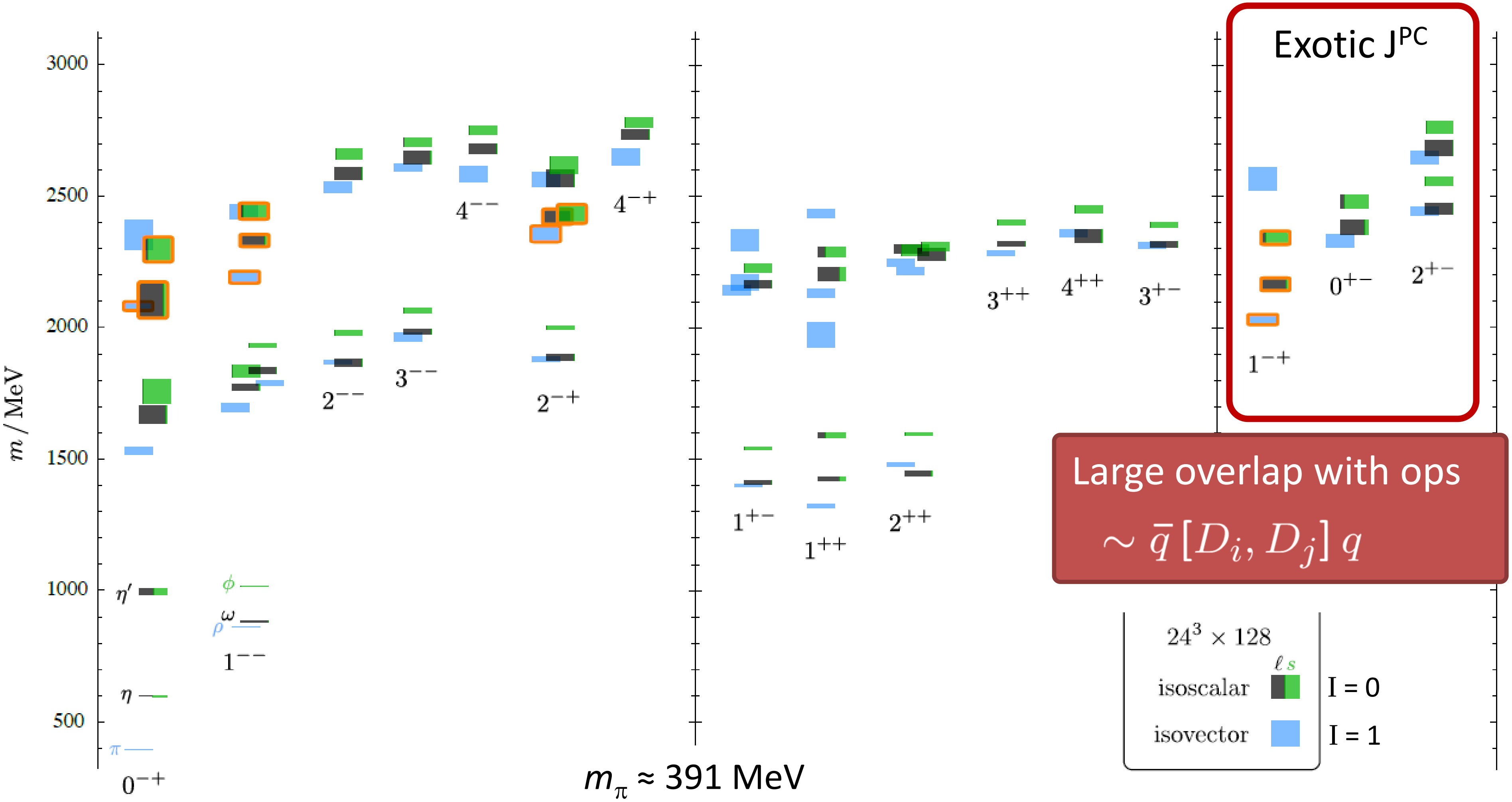

Exotic J^{PC}

Large overlap with ops

$$\sim \bar{q}\,[D_i, D_j]\,q$$

isoscalar ■🟩 I = 0

isovector 🟦 I = 1

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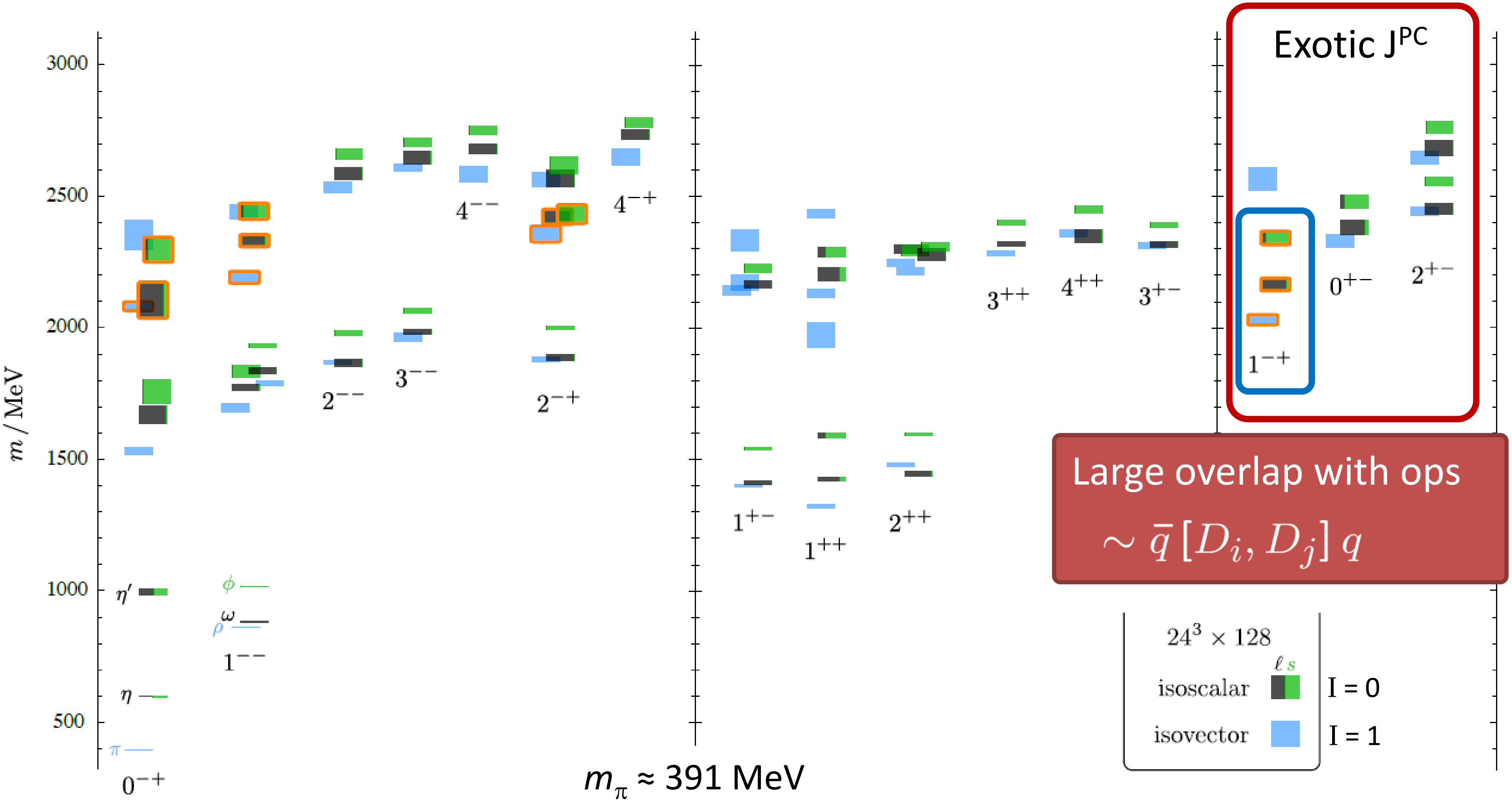

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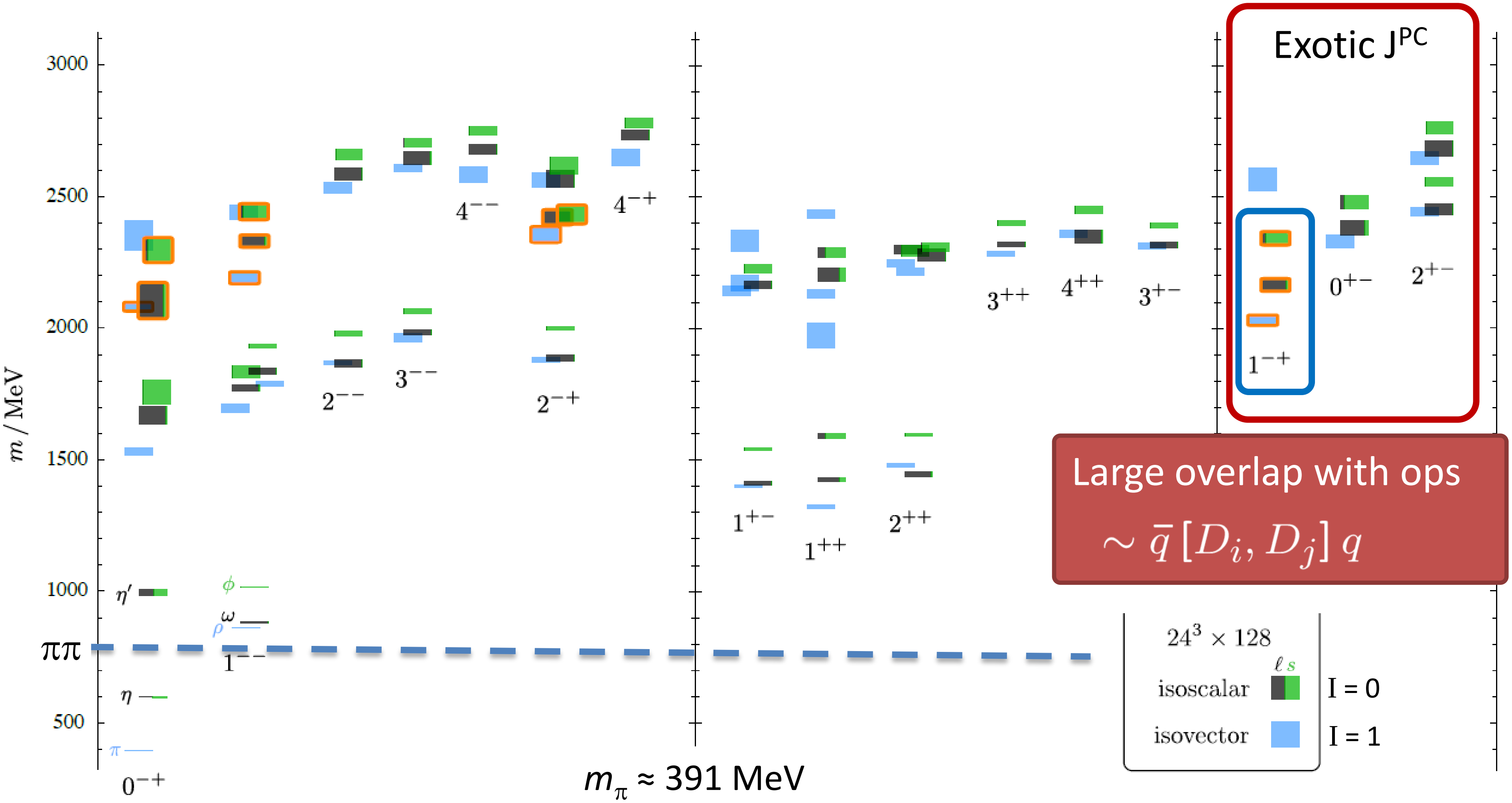

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Most hadrons appear as resonances in scattering of lighter hadrons

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Lüscher method (and extensions): relate **finite-volume energy levels** $\{E_{\mathrm{cm}}\}$ to **infinite-volume scattering *t*-matrix**.

Elastic scattering: one-to-one mapping $E_{\mathrm{cm}} \leftrightarrow t(E_{\mathrm{cm}})$

[Complication: reduced sym. of lattice vol. → mixing of partial waves]

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Coupled channels: under-constrained problem
(each E_{cm} constrains t-matrix at that E_{cm})
Parameterise $t(E_{\mathrm{cm}})$ using various K-matrix forms, …

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Demonstrated in calcs. of ρ, light scalars, b_1, charm mesons, …

[Complication: reduced sym. of lattice vol. → mixing of partial waves]

1^{-+} channel with $SU(3)_F$ flavour sym

$SU(3)_F$ symmetry (m_u=m_d=m_s)

$m_\pi \approx$ 700 MeV, $m_\rho \approx$ 1000 MeV, $m_{\eta'} \approx$ 940 MeV

[**Woss**, Dudek, Edwards, Thomas, Wilson,
PR D103, 054502 (2021), 2009.10034]

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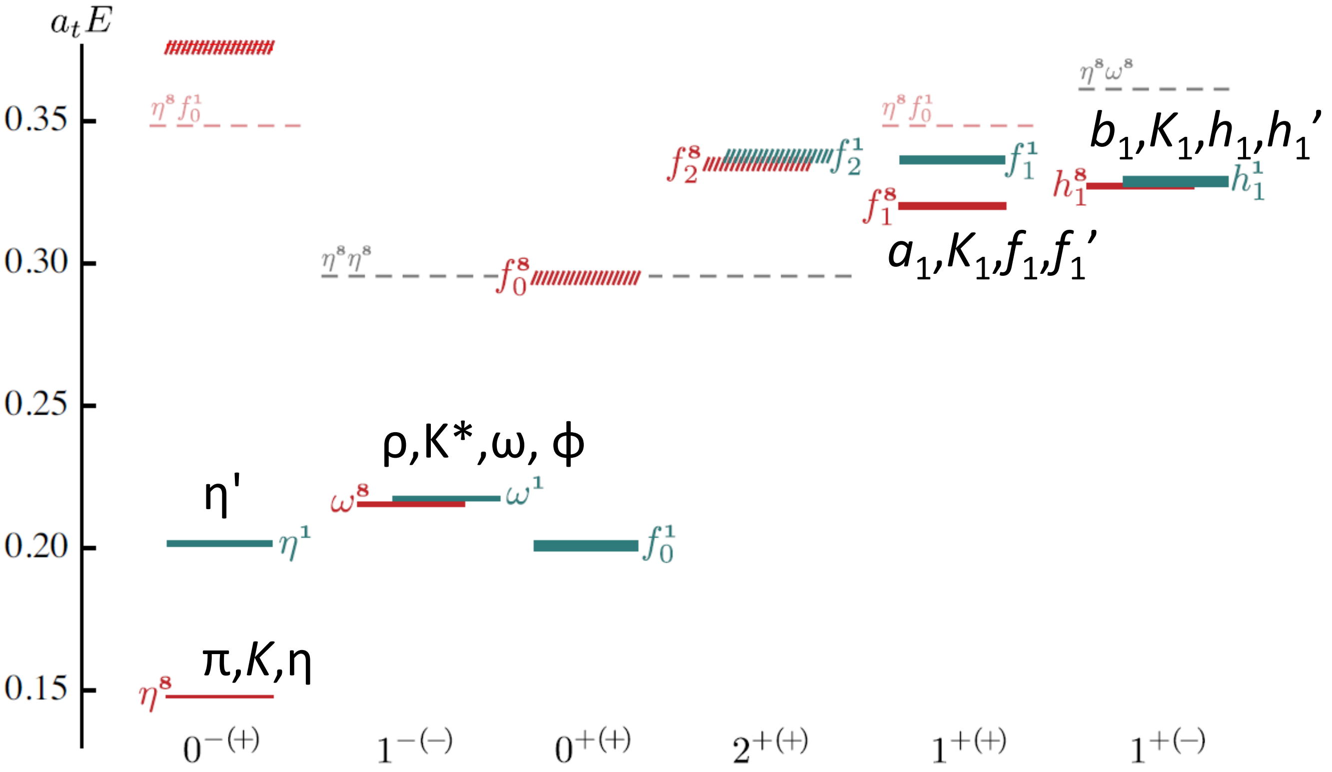

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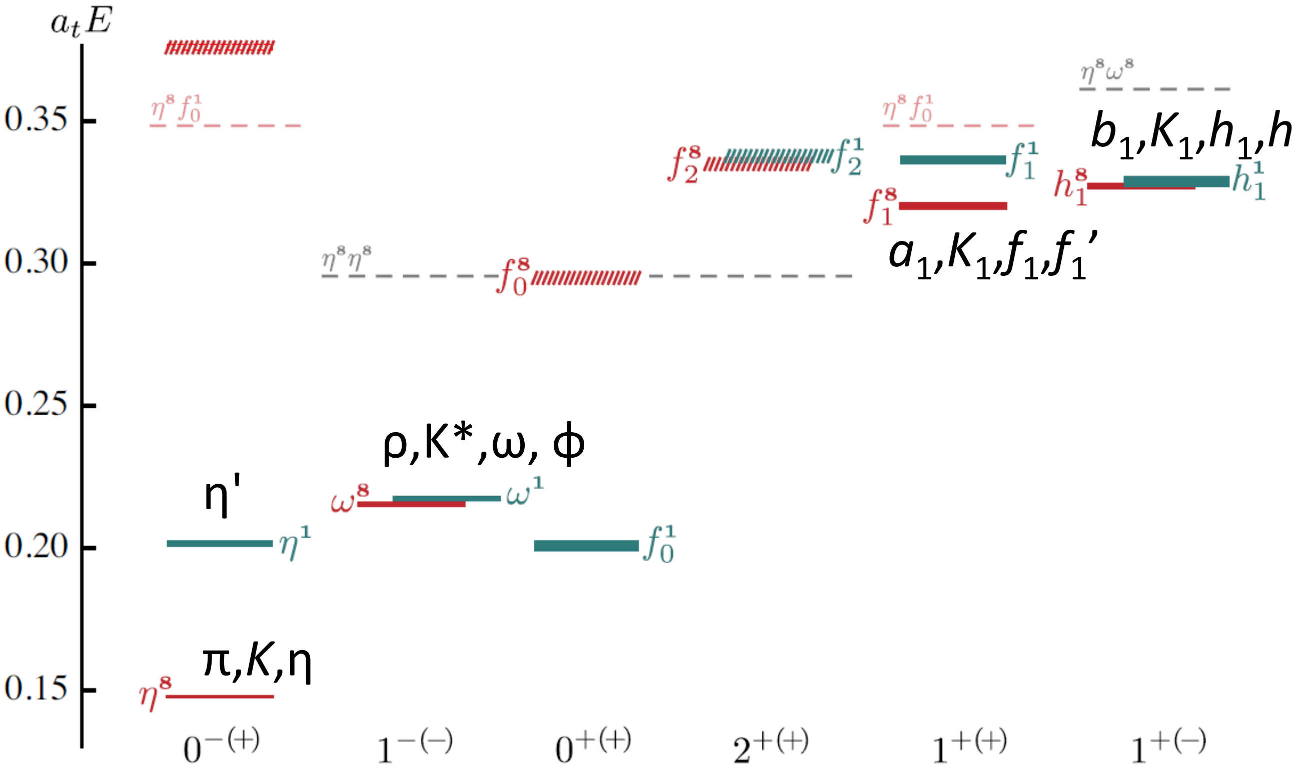


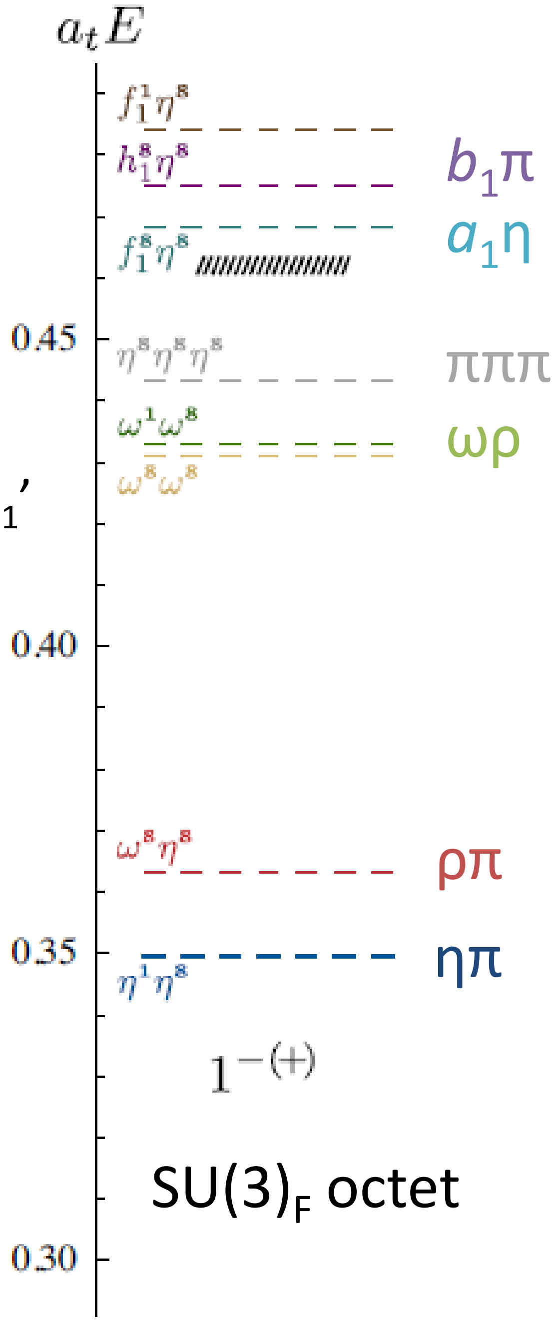
η'

π,*K*,η

ρ,K*,ω, ϕ

a_1,K_1,f_1,f_1'

b_1,K_1,h_1,h_1'



$b_1\pi$

$a_1\eta$

πππ

ωρ

ρπ

ηπ

$SU(3)_F$ octet

[**Woss**, Dudek, Edwards, Thomas, Wilson, PR D103, 054502 (2021), 2009.10034]

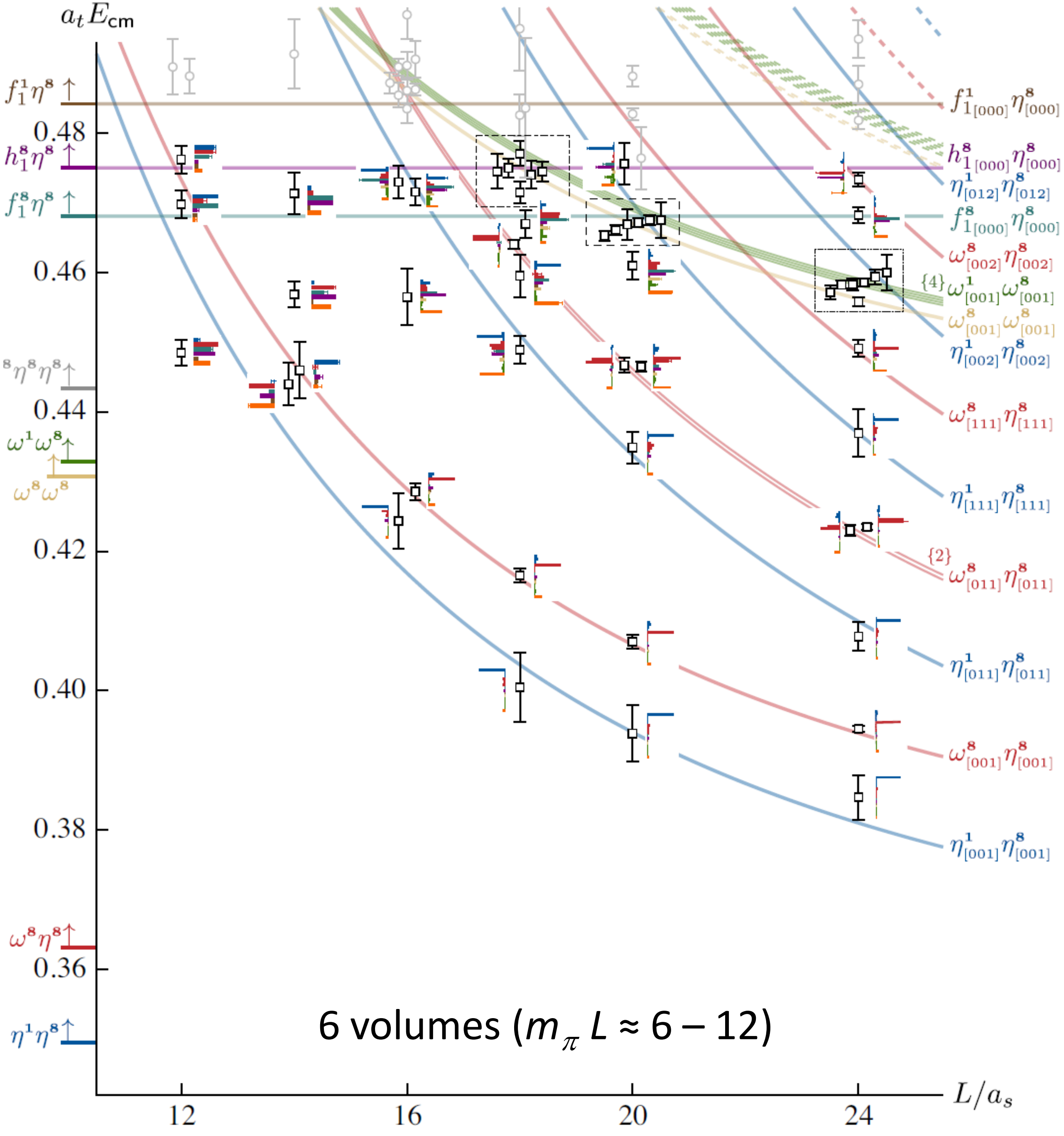
Spectra

1^{-+} (, 3^{-+}, …)

Use many ops,

$$\sim \bar{\psi}\Gamma D \ldots \psi$$

and $\eta^1\eta^8$, $\omega^8\eta^8$, $\omega^8\omega^8$, $\omega^1\omega^8$, $f_1^8\eta^8$, $h_1^8\eta^8$, $f_1^1\eta^8$



6 volumes ($m_\pi L \approx 6 - 12$)

[PR D103, 054502 (2021)]

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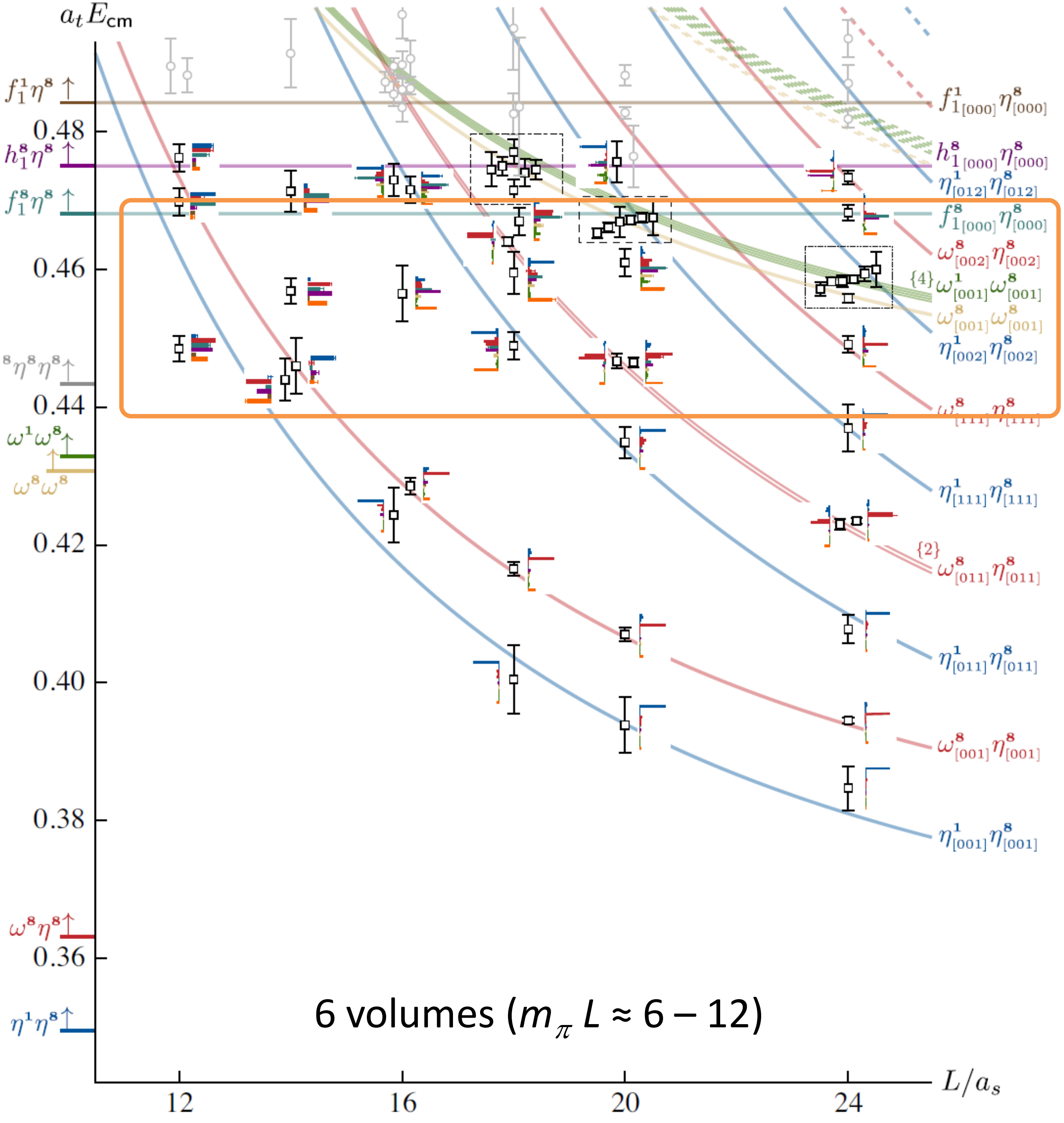

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[PR D103, 054502 (2021)]

Scattering amps

[PR D103, 054502 (2021)]

Constrain eight 1^{-+} coupled partial waves with 53 energy levels

$\eta^{\mathbf{1}}\eta^{\mathbf{8}}\{{}^1P_1\}$
$\omega^{\mathbf{8}}\eta^{\mathbf{8}}\{{}^3P_1\}$
$\omega^{\mathbf{8}}\omega^{\mathbf{8}}\{{}^3P_1\}$, $\omega^{\mathbf{1}}\omega^{\mathbf{8}}\{{}^1P_1, {}^3P_1, {}^5P_1\}$
$f_1^{\mathbf{8}}\eta^{\mathbf{8}}\{{}^3S_1\}$, $h_1^{\mathbf{8}}\eta^{\mathbf{8}}\{{}^3S_1\}$

(Another 8 energy levels constrain three 3^{-+} partial waves.)

Scattering amps

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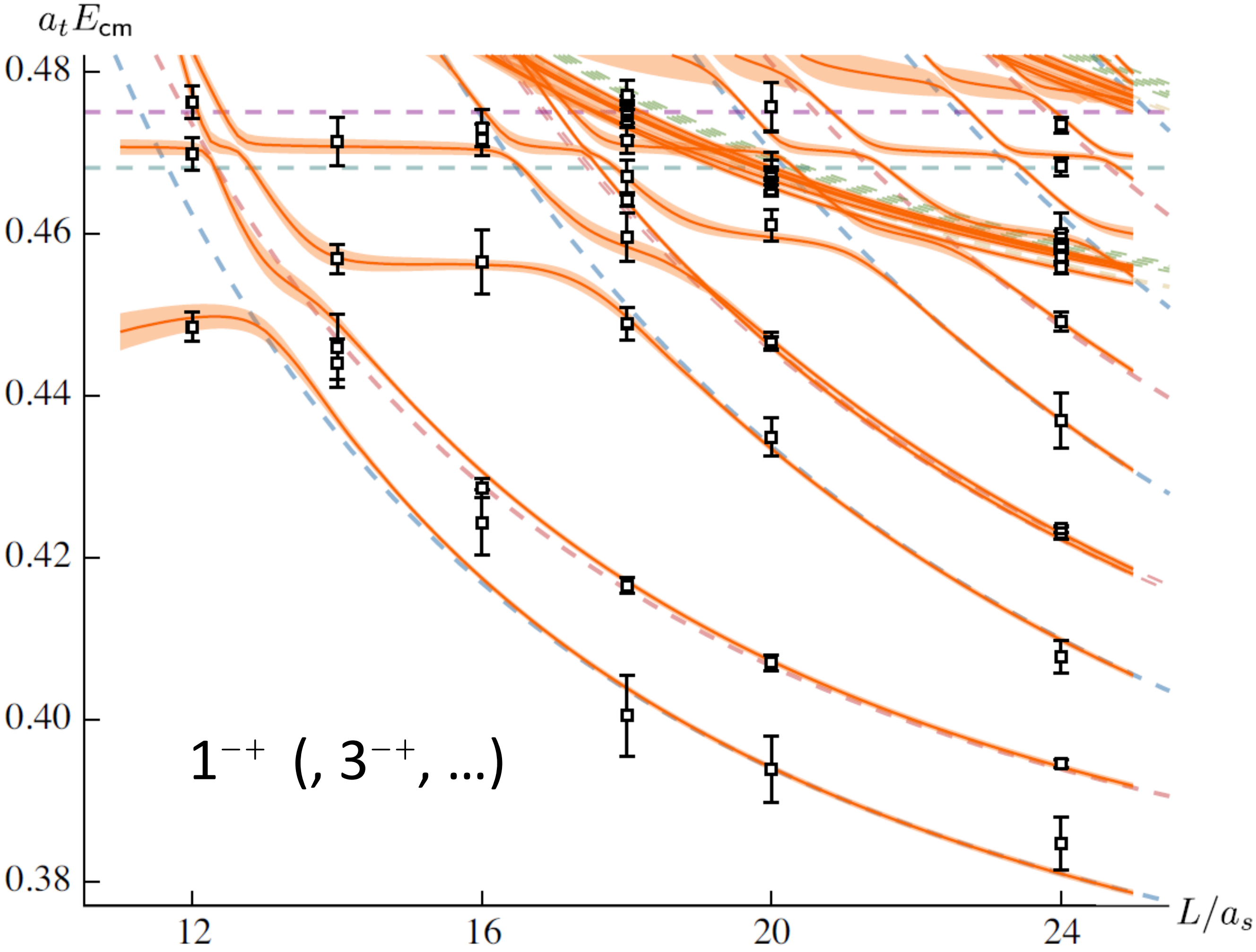
Example *K*-matrix param that gives a good description of spectra.

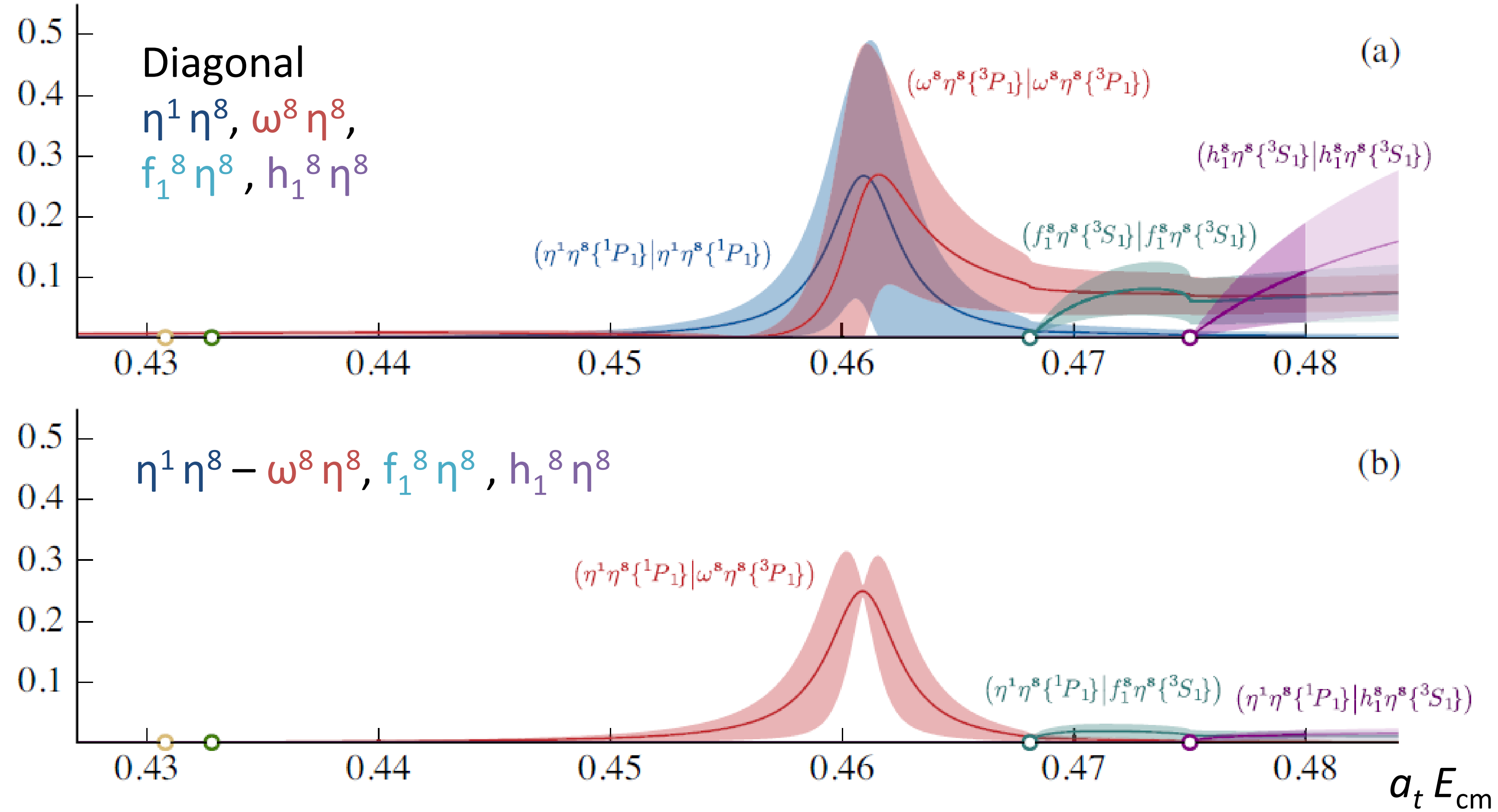
χ^2/N_{dof}
$= 43.6/(53 - 11)$
$= 1.04$



Scattering amps – example param

[PR D103, 054502 (2021)]

$\rho_a \rho_b |t_{ab}|^2 \sim |\text{amp}|^2$



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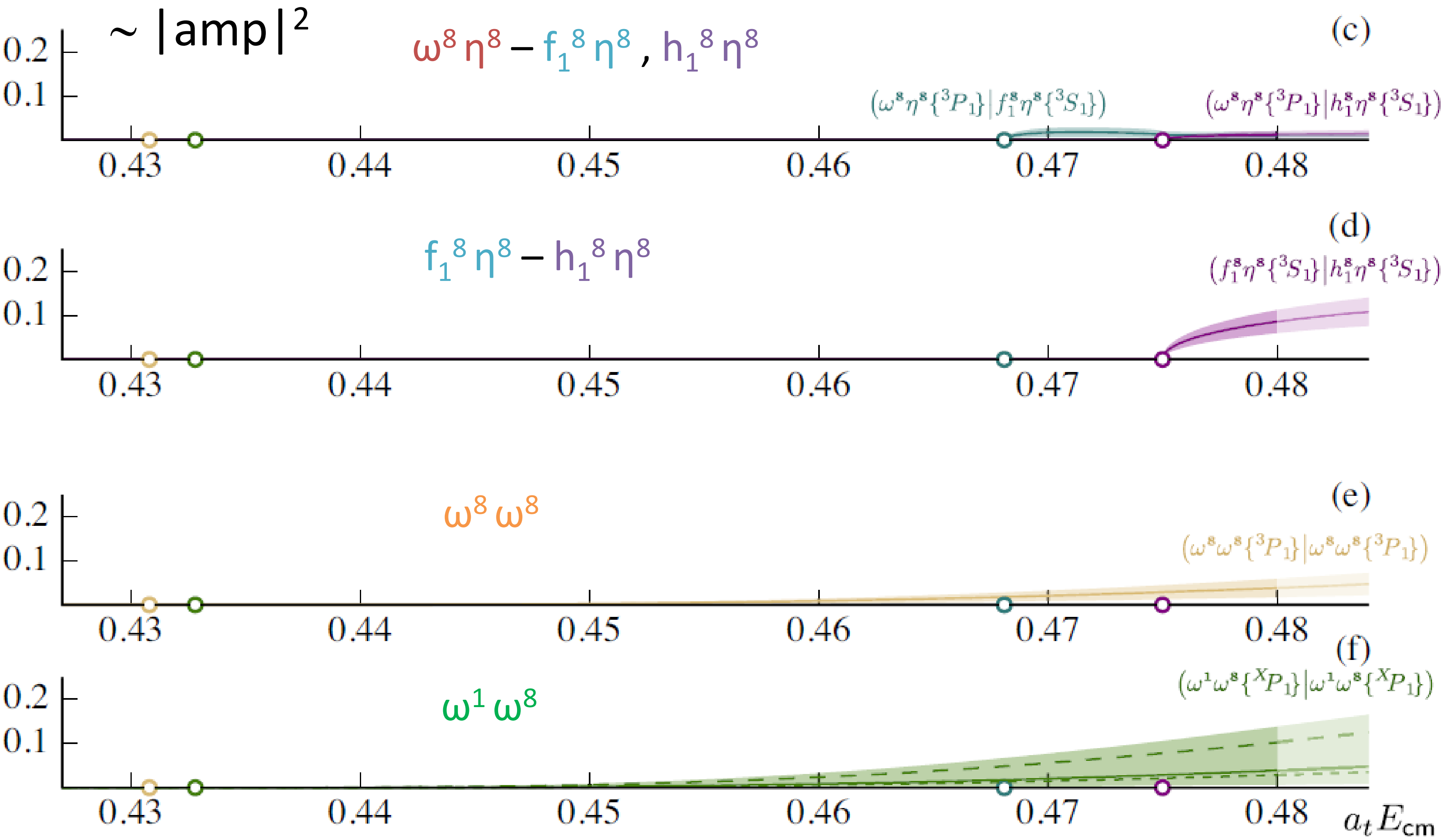
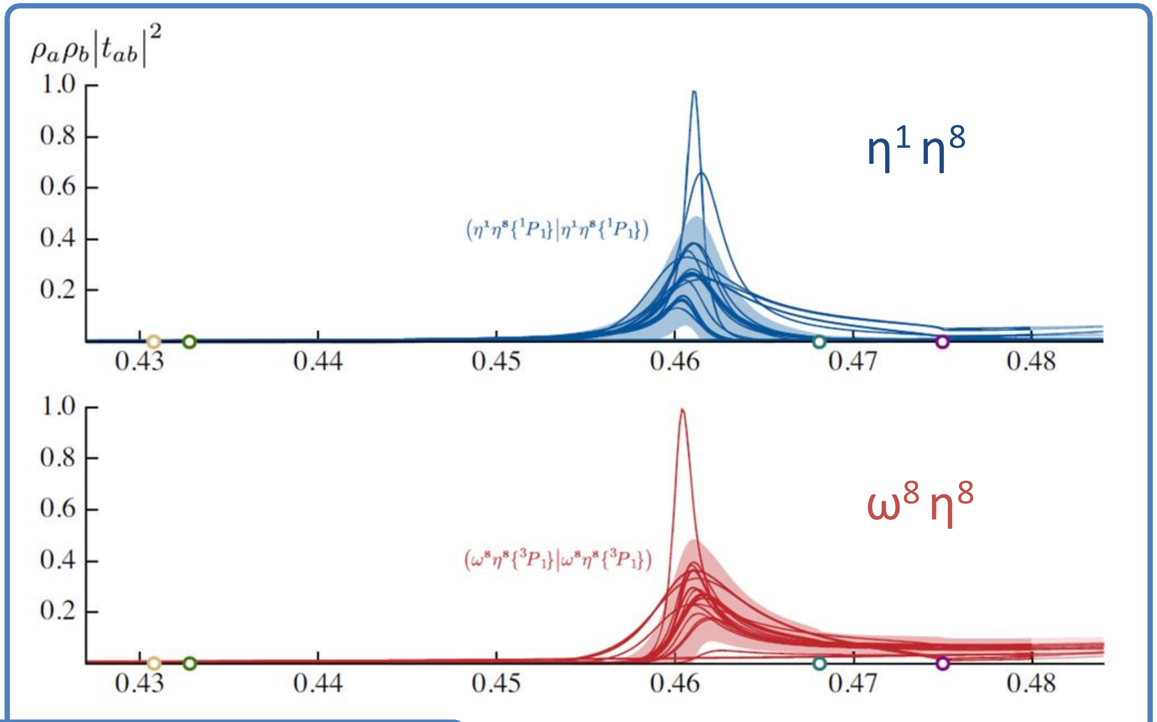
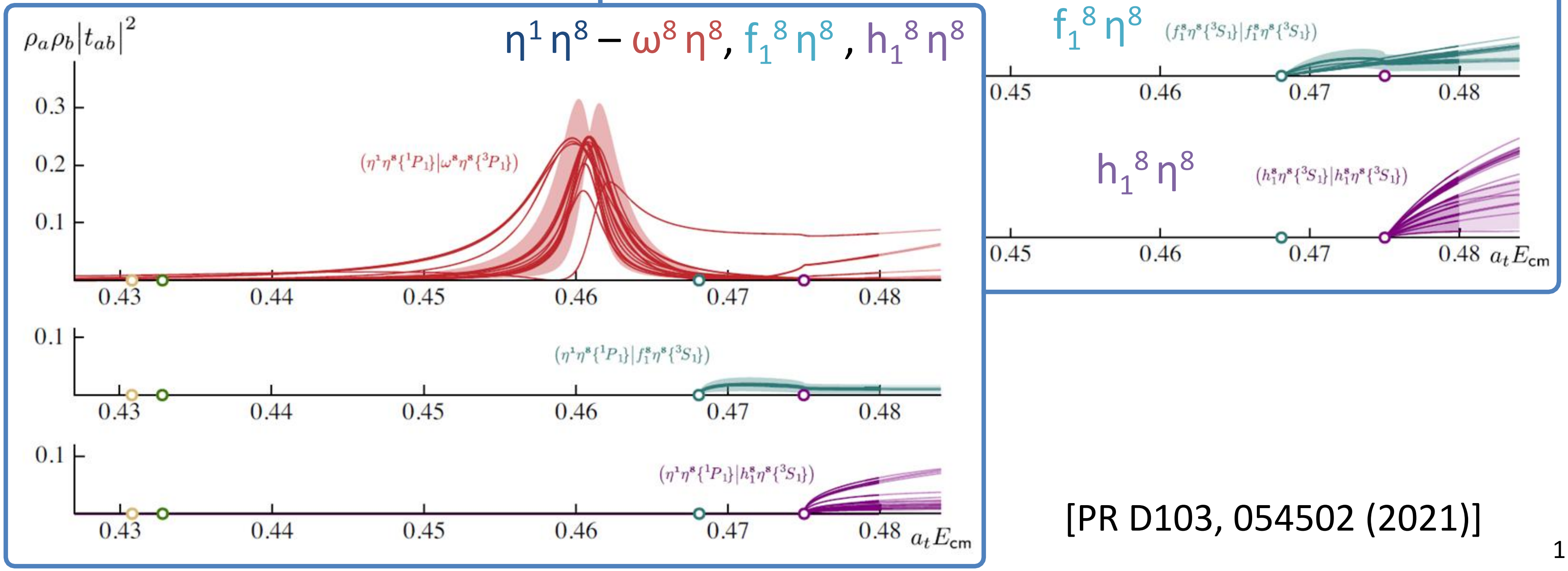


Scattering amps

27 parameterisations with $\chi^2/N_{\mathrm{dof}} \leq 1.25$





[PR D103, 054502 (2021)]

Pole and couplings

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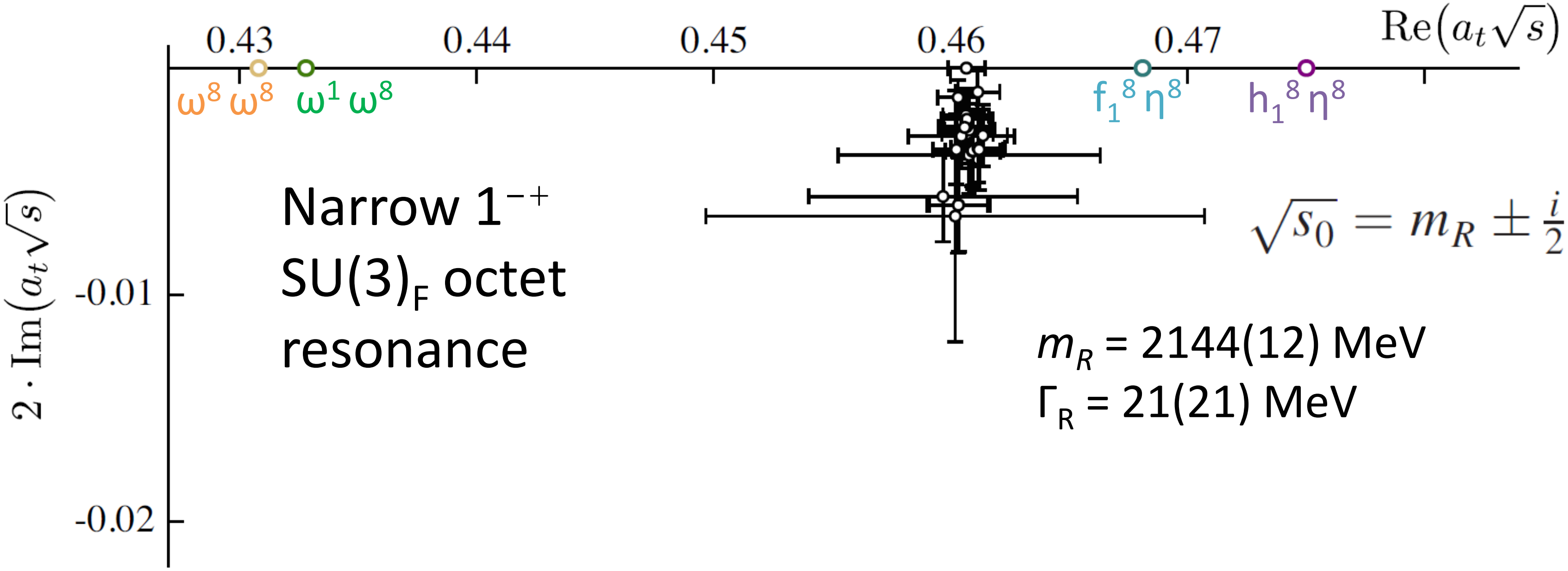

Narrow 1^{-+}
$SU(3)_F$ octet
resonance

$$\sqrt{s_0} = m_R \pm \tfrac{i}{2}\Gamma_R$$

m_R = 2144(12) MeV
Γ_R = 21(21) MeV

Pole and couplings

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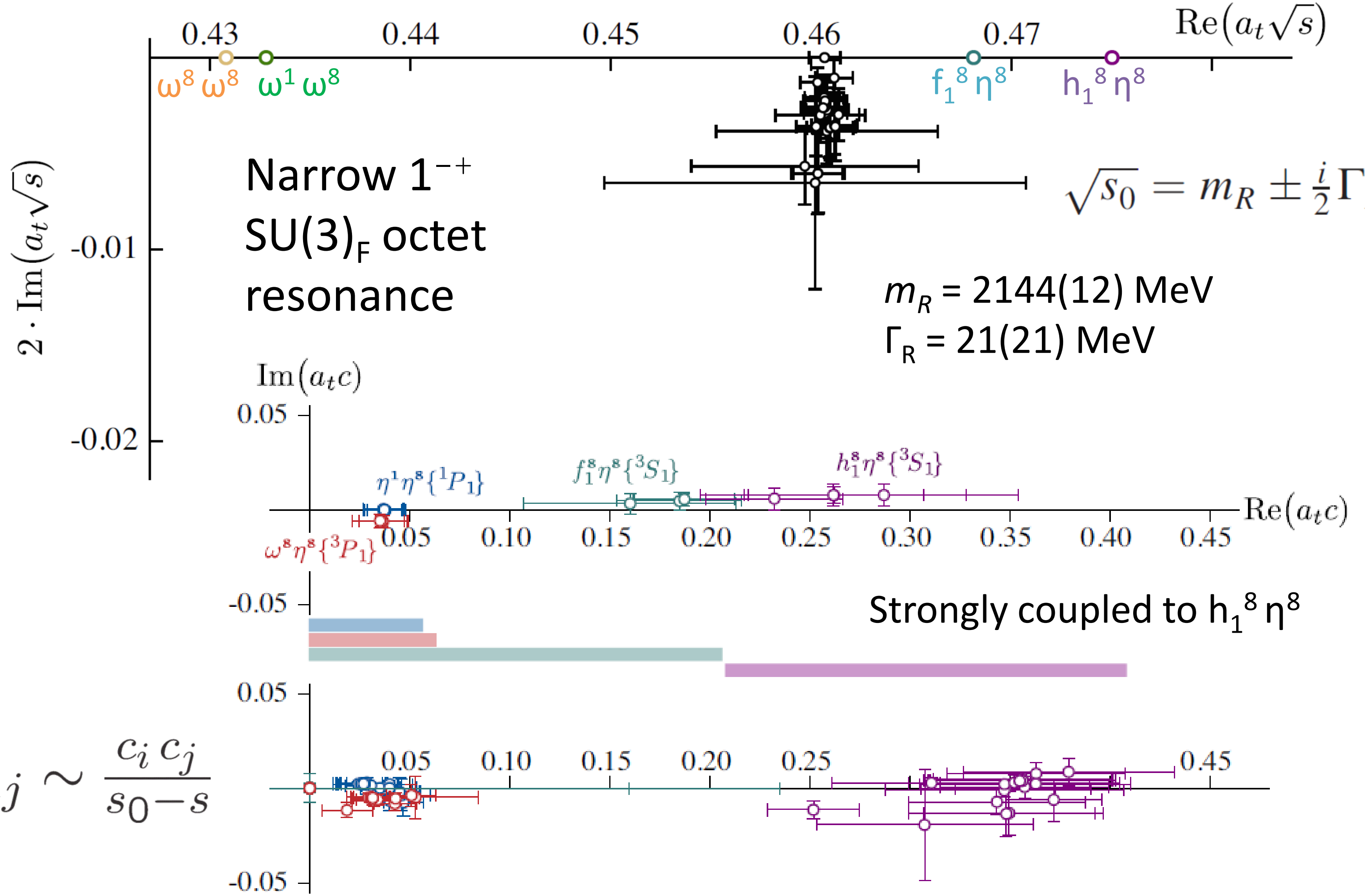

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Strongly coupled to $h_1{}^8\,\eta^8$

$$t_{ij} \sim \frac{c_i\, c_j}{s_0 - s}$$

Extrapolation of couplings

[PR D103, 054502 (2021)]

Attempt crude extrapolation to physical masses (break $SU(3)_F$ symmetry).

Assume couplings scale with appropriate barrier factor k^ℓ.

PDG masses and m_R = 1564 MeV.

$$|c|^{\text{phys}} = \left| \frac{k^{\text{phys}}(m_R^{\text{phys}})}{k(m_R)} \right|^\ell |c|$$

$$\Gamma(R \to i) = \frac{|c_i^{\text{phys}}|^2}{m_R^{\text{phys}}} \cdot \rho_i(m_R^{\text{phys}})$$

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$\Gamma = \Sigma_i \Gamma_i$ = 139 – 590 MeV

c.f. JPAC: Γ = 492(54)(102) MeV

Kopf *et al*: Γ = $(388.1 \pm 5.4\,^{+0.2}_{-14.1})$ MeV

	Γ_i/MeV
$\eta\pi$	$0 \to 1$
$\rho\pi$	$0 \to 20$
$\eta'\pi$	$0 \to 12$
$b_1\pi$	$139 \to 529$
$K^*\overline{K}$	$0 \to 2$
$f_1(1285)\pi$	$0 \to 24$
$\rho\omega\{^1P_1\}$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	$\lesssim 0.09$
$\rho\omega\{^5P_1\}$	$\lesssim 0.03$
$f_1(1420)\pi$	$0 \to 2$

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LQCD calc in McNeile & Michael [PR D73, 074506 (2006)]: consider setup with m_π ≈ 500 MeV, $m_{\pi_1} = m_{b_1} + m_\pi$

Summary

- **First determination** of the **lightest exotic $J^{PC} = 1^{-+}$ resonance** using lattice QCD.
- With $SU(3)_F$ sym and $m_\pi \approx 700$ MeV: **narrow resonance** in flavour-octet 1^{-+} channel.
- Weak coupling to open channels, strong coupling to at least one closed axial-vector–pseudoscalar channel.
- Simple extrapolation to physical masses, breaking $SU(3)_F$ → potentially **broad resonance** with large decay to $b_1\pi$ (would → $\omega\pi\pi$ → $\pi\pi\pi\pi\pi$)

- Calculation with lighter light-quarks? (three-meson decays…)
- Other exotic quantum numbers?

Acknowledgements

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UNIVERSITY OF CAMBRIDGE

Hadron Spectrum Collaboration

[*www.hadspec.org*]

hadspec

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