Photo- and hadron-production of mesons

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on behalf of the JPAC collaboration

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I will focus on two topics recently studied by the JPAC collaboration:

1. High energy $\pi^- p \rightarrow \pi^- \eta(\prime) p$ production
2. Photoproduction of resonances in the $\pi^+\pi^-$ system
Motivation of the $\pi^-\eta^{(i)}$ channel study

- $\pi^-\eta^{(i)}$ pairs constitute a golden channel for the searches of hybrid exotics (odd partial waves are exotic)
- COMPASS experiment at CERN (Adolph et al. PLB 740, 2015) observed a strong forward-backward asymmetry in the $\pi^-\eta^{(i)}$ channels (stronger in the $\pi\eta'$ channel)
- The asymmetry is due to odd-even partial wave interference
- The strongest odd wave is the $P-$wave, which in the resonance region can be attributed to the $\pi_1$ hybrid
- Relation between the high and low invariant mass region can be described in terms of Finite Energy Sum Rules (special kind of dispersion relations)
Experimental motivation

Objectives:

- Describe the $\pi^- \eta'(\ ')$ production above the resonance region.
- Identify dominant Regge exchanges and amplitude strengths.
**Kinematics**

- The $2 \rightarrow 3$ reaction (upolarized) is determined by 5 kinematical variables.

- In Regge analysis it is customary to choose 5 Lorentz invariants, eg:
  
  \[ s = (q + p_1)^2, \quad t_1 = (q - k_\eta)^2, \quad t_2 = (p_1 - p_2)^2, \quad s_1(k_\eta + k_\pi)^2, \quad s_2 = (k_\pi + p_2)^2. \]

- COMPASS analysis was performed in the Gottfried-Jackson frame, with $\theta$ and $\phi$ defining the direction of outgoing $\eta^{(')}$.

- In this frame $\cos \theta$ is related to $t_1$ and $\phi$ is related to $s_2$ (the other variables being $s$, $s_1$ and $t_2$).
Model vs. experimental data

- The COMPASS experiment operated with a fixed beam momentum of 191 GeV and $t_2$ was integrated in the region $t_2 \in [-1.0, -0.1]$ GeV$^2$.

- The invariant mass ($m = \sqrt{s_1}$) and angular dependent intensity was parametrized as:

$$I(m, \Omega)_{\text{COMPASS}} = \left| \sum_{\epsilon = \pm 1} \sum_{L, M} f^{\epsilon}_{LM}(m) \Psi^{\epsilon}_{LM}(\Omega) \right|^2,$$

where $\epsilon$ is the reflectivity ($\approx$ naturality in high energy limit), kept $\epsilon = +1$ and

$$\Psi^{\epsilon = +1}_{LM}(\theta, \phi) = \sqrt{2} Y_{LM}(\theta, 0) \sin(M\phi)$$

$$f^{\epsilon = +1}_{LM}(m) = \sqrt{I_{LM}} \ e^{i\phi_{LM}}$$

where the $I_{LM}$ and $\phi_{LM}$ are the COMPASS experimental partial wave intensities and phaseshifts (determined relative to $\phi_{21}$).
Fitting experimental data at large invariant masses

- The easiest approach: fit the model partial waves to experimental ones

But:

- COMPASS analysis was based on partial wave expansion truncated at \( L = 6 \) and \( M = 1 \), even though it converges slowly for the \( \pi \eta^{(')} \) invariant masses above the resonance region,

- The intensity was normalized to the number of events in the mass bin.

Therefore:

- The angular distribution constructed from the truncated partial wave expansion represents total yield.
Fitting experimental data at large invariant masses

On the model side

- Model intensities for full amplitudes and partial wave amplitudes truncated at \( L = 6 \) differ considerably.

Recipe:

- Fit the mass and angular dependent intensity.
Fitting experimental data at large invariant masses

- Practically we fit the extended log likelihood (Barlow, Nucl.Instr.Meth. A 297, 1990)

\[ L_{\text{ext}} = \sum_{m_i} \int d\Omega [I_{\text{JPAC}}(m_i, \Omega) - I_{\text{COMPASS}}(m_i, \Omega) \log I_{\text{JPAC}}(m_i, \Omega)] \]

with model intensity defined as:

\[ I_{\text{JPAC}}(m, z = \cos \theta, \phi) = q |T|^2 = q \left| \sum_{i=1}^{6} a_i T^i(m_{\pi\eta}, z, \phi) \right|^2 \]

where \[ q = \lambda^{1/2}(m^2, m_{\pi}^2, m_{\eta}^2)/m. \]
Double Regge exchange model

- Leading natural exchanges

<table>
<thead>
<tr>
<th>Type of diagram</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
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<td>I (fast $\eta$)</td>
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<td>$\alpha_{P}$</td>
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<td>$\alpha_{a_2}$</td>
<td>$\alpha_{f_2}$</td>
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<tr>
<td>II (fast $\pi$)</td>
<td>$\alpha_{f_2}$</td>
<td>$\alpha_{P}$</td>
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<td>$\alpha_{f_2}$</td>
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<tr>
<td></td>
<td>$\alpha_{P}$</td>
<td>$\alpha_{f_2}$</td>
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</tbody>
</table>

Type I - fast $\eta$       Type II - fast $\pi$

- Individual diagram strengths depend on the 6 coupling triples:
  - $G_{a_2\pi\eta} G_{a_2\pi P} G_{PNN}$,
  - $G_{a_2\pi\eta} G_{a_2\pi f_2} G_{f_2NN}$,
  - $G_{f_2\pi\pi} G_{f_2\eta f_2} G_{f_2NN}$,
  - $G_{P\pi\pi} G_{\eta P P} G_{PNN}$,
  - $G_{P\pi\pi} G_{f_2\eta P} G_{f_2NN}$,

  wherein at least one coupling is unknown.

- Therefore we treat diagram strengths as parameters to be fitted.
General form of the amplitude

- Double reggeon exchange (we follow Shimada et al. NPB 142, 1978)

\[ T = -K \Gamma(1 - \alpha_1)\Gamma(1 - \alpha_2) \]

\[ \left[ (\alpha's)^{\alpha_1-1}(\alpha's_2)^{\alpha_2-\alpha_1}\xi_1\xi_21 \hat{V}_1 + (\alpha's)^{\alpha_2-1}(\alpha's_1)^{\alpha_1-\alpha_2}\xi_2\xi_12 \hat{V}_2 \right] \]

where \( \xi_i \) and \( \xi_{ij} \) signature factors, \( \hat{V}_1 \)

\[ \hat{V}_1(\alpha_1, \alpha_2, \eta) = \frac{\Gamma(\alpha_1 - \alpha_2)}{\Gamma(1 - \alpha_2)} \,_1F_1 \left(1 - \alpha_1, 1 - \alpha_1 + \alpha_2, -\kappa\right) \]

\[ \hat{V}_2 = \hat{V}_1(\alpha_1 \leftrightarrow \alpha_2) \quad \text{with} \quad \kappa^{-1} = s/(\alpha's_1s_2) \]

and \( K = -4\sqrt{s_1}|q||k_\eta||p_2| \sin \theta_2 \sin \theta \sin \phi \)

- Both \( \alpha_1 \) and \( \alpha_2 \) are of \( 2^{++} \) type so only positive signature and naturality.

- Regge trajectories:

\[ \alpha_{f_2}(t) = 0.47 + 0.89t \quad \alpha_{a_2}(t) = 0.53 + 0.90t \quad \alpha_P(t) = 1.08 + 0.25t \]
Fitting procedure

1. Full amplitude ($a_i$ to be fitted):

$$T(s, s_1, t_2, \cos \theta, \phi; a_i) = K \sum_{i \in I, II} a_i \tilde{T}_i(\alpha_1(t_1), \alpha_2(t_2); s, s_1, t_2, \cos \theta, \phi)$$

2. Model intensity:

$$I_{JPAC}(m, z = \cos \theta, \phi) = q |T|^2 = q \left| \sum_{i=1}^{6} a_i T^i(m_{\pi \eta}, z, \phi) \right|^2$$

3. Extended log likelihood fit:

$$\mathcal{L}_{ext} = \sum_{m_i} \int d\Omega \left[ I_{JPAC}(m_i, \Omega) - I_{COMPASS}(m_i, \Omega) \log I_{JPAC}(m_i, \Omega) \right]$$
Fit results
Minimal set of amplitudes

Fitting the full set of 6 amplitudes is statistically inconclusive.

From the analysis of the forward and backward mass and $\phi$ distributions we infer that the minimal set of amplitudes should include $a_2/\mathcal{P}$, $a_2/f_2$ and $f_2/f_2$. 
Parameter values from the fits

- To make the parameter evaluation stable we can fit at most 4 amplitudes.
- We exclude $P/f_2$ from all fits as it may disrupt the $\phi$ distribution.

<table>
<thead>
<tr>
<th>Channel</th>
<th>MIN $\times 10^{-4}$</th>
<th>MVR</th>
<th>MCR</th>
<th>MIN $+f/P$</th>
<th>MVR</th>
<th>MCR</th>
<th>MIN $+P/P$</th>
<th>MVR</th>
<th>MCR</th>
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<tr>
<td>$\eta\pi$</td>
<td>$L \times 10^{-4}$</td>
<td>-22.8</td>
<td>-21.9 ± 0.9</td>
<td>-22.7</td>
<td>-22.0 ± 0.9</td>
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<td>-22.1 ± 0.8</td>
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<td></td>
<td>$c_{a2}P$</td>
<td>0.29</td>
<td>0.42 ± 0.03</td>
<td>0.28</td>
<td>0.40 ± 0.04</td>
<td>0.29</td>
<td>0.36 ± 0.04</td>
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<td>$c_{a2}f_2$</td>
<td>3.67</td>
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<td>-0.30 ± 0.05</td>
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<td>$\eta'\pi$</td>
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<td>-11.4 ± 1.0</td>
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<td>$c_{a2}f_2$</td>
<td>1.50</td>
<td>0.4 ± 0.6</td>
<td>1.51</td>
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<tr>
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<td>$c_{f2}P$</td>
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<td>—</td>
<td>-0.21</td>
<td>-0.29 ± 0.03</td>
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<tr>
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<td>$c_{f2}f_2$</td>
<td>-11.42</td>
<td>-11.0 ± 0.5</td>
<td>-7.73</td>
<td>-5.5 ± 0.7</td>
<td>-9.01</td>
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<td>$c_{PP}$</td>
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<td>0.012</td>
<td>0.018 ± 0.002</td>
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Introduction

High energy $\pi^- p \rightarrow \pi^- \eta(\prime) p$ production

Summary

Forward and backward peaks for $\pi\eta$
Introduction

High energy $\pi^- p \rightarrow \pi^- \eta'$ production

Summary

Forward and backward peaks for $\pi\eta'$

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Photo- and hadron-production of mesons
Forward-backward asymmetry

Define the forward-backward asymmetry as:

\[ A(m) \equiv \frac{F(m) - B(m)}{F(m) + B(m)}, \]

with

\[ F(m) \equiv \int_{0}^{1} d\cos\theta \, I_{\theta}(m, \cos\theta), \quad B(m) \equiv \int_{-1}^{0} d\cos\theta \, I_{\theta}(m, \cos\theta) \]
Motivation for the $\pi^+\pi^-$ photoproduction study

- Basic interpretation of the $\gamma p \rightarrow \pi^+\pi^- p$ reaction was formulated in classical SLAC papers (Ballam at al, 197-)

- Diffractive peak in the $d\sigma/dt$ distribution,

- Line shape distorted due to interference with Drell background,

- These properties are grasped by Söding model (assumed diffractive $\pi p$ scattering)
Motivation for the $\pi^+\pi^-$ photoproduction study

- Decade ago the topic was revisited by the CLAS collaboration (Battaglieri et al. PRD 80, 2009)

New results:
- Comprehensive analysis of the angular distribution with moments
- First observation of the $f_0(980)$ in the photoproduction
Motivation for the $\pi^+\pi^-$ photoproduction study

Is there anything left to study? It is!

- Assumption of diffractive $\pi p$ scattering (Deck (1964), Söding (1966), Pumplin(1970)) is usually not valid,
- No uniform description of partial waves varying slopes suggest different production mechanisms.
- Coupled channel effects may be strong particularly in the $S$– and $D$–waves.
- Description of polarisation effects still in early stage (see Mathieu, Pilloni et al. PRD 102, 2020).
Introduction

High energy $\pi^-$ $p \rightarrow \pi^- \eta(\prime) p$ production

π$^+$ π$^-$ photoproduction

Summary

Unitarity compatible form of the $\gamma p \rightarrow \pi^+ \pi^- p$ amplitude

- Aitchinson and Bowler, J. Phys. G., 3, 1978 had shown that the general form of the unitarity compatible photoproduction amplitude reads:

$$M = M_{\text{diffuse}} e^{i\delta_{\pi\pi}} \cos \delta_{\pi\pi} + M_{\text{compact}} e^{i\delta_{\pi\pi}} \sin \delta_{\pi\pi}$$

- $M_{\text{diffuse}}$ and $M_{\text{compact}}$ denote the production amplitudes where pion pair is produced from the compact or diffuse (asymptotically stretched) source.

- In hadron physics a practical infinitely long interaction is achieved with exchanging the particle of smallest mass, which is pion.
- One pion exchange brings us to the Deck model.
- Short range interaction can be modelled by flat energy dependence so we put $M_{\text{compact}} = A + Bs_{\pi\pi}$.
Deck model - brief recap

Deck amplitude (Pumplin 1970)

\[ M_{\lambda_2 \lambda_1} = -e \left[ \left( \frac{\epsilon_\lambda \cdot k_2}{q \cdot k_2} - \frac{\epsilon_\lambda \cdot (p_1 + p_2)}{q \cdot (p_1 + p_2)} \right) T_{\lambda_1 \lambda_2}^+ - \left( \frac{\epsilon_\lambda \cdot k_1}{q \cdot k_1} - \frac{\epsilon_\lambda \cdot (p_1 + p_2)}{q \cdot (p_1 + p_2)} \right) T_{\lambda_1 \lambda_2}^- \right] \]

Partial wave projection

\[ M^{lm}_{\pi^+ \pi^-} (\lambda_2 \lambda_1) = \frac{1}{\sqrt{4\pi}} \int d\Omega \ Y^*_l(m) M_{\lambda_2 \lambda_1} \]

Deck+FSI

\[ \mathcal{T}^{lm}_{\pi^+ \pi^-} (\lambda_2 \lambda_1) = (1 + i \rho \ t^l) M^{lm}_{\pi^+ \pi^-} (\lambda_2 \lambda_1) \]
Deck model - brief recap

- To describe the $\pi p$ interaction we use the SAID amplitudes (in ongoing analysis we use amplitudes from Mathieu et al. PRD 92, 2015).
- We use the model of $\pi\pi$ FSI from Bydžovský et al. PRD 94, 2016.
- The only parameters to be fitted are $A$ and $B$ coefficients in the linear approximation of the short range amplitude.
- Deck amplitude is essentially parameter free, thus enables precise determination of the short range contribution.
Model predictions vs. CLAS data

Work in progress:

- Better description of the short range component, eg. through Regge exchange Mathieu et al. PRD 102, 2020
- Prediction of polarized moments of angular distribution for CLAS12 and GlueX (talk by Nicholas Zachariou of CLAS12 earlier today).
Summary

- We found that $a_2/P$, $a_2/f_2$, $f_2/f_2$ and either $f_2P$ or $P/P$ (the data do not show clear preference for either exchange) are required to describe $\pi\eta$ intensity.

- To describe $\pi\eta'$ intensity the $a_2/P$, $a_2/f_2$, $f_2/f_2$ and $P/P$ exchanges are necessary. Glue rich exchange points toward $\pi_1$ hybrid production.

- Quite surprisingly, lower $f_2$ exchange is necessary to describe COMPASS data.

- The model which combines diffuse source (Deck) and compact source components properly describes the $\pi^+\pi^-$ mass distributions at fixed $t$ in $S$, $P$ and $D$ partial waves and reproduces the dominance of the $f_0(980)$, $\rho(770)$ and $f_2(1270)$ respectively while respecting the 2-particle unitarity in the $\pi\pi$ system.
Thank you for your attention