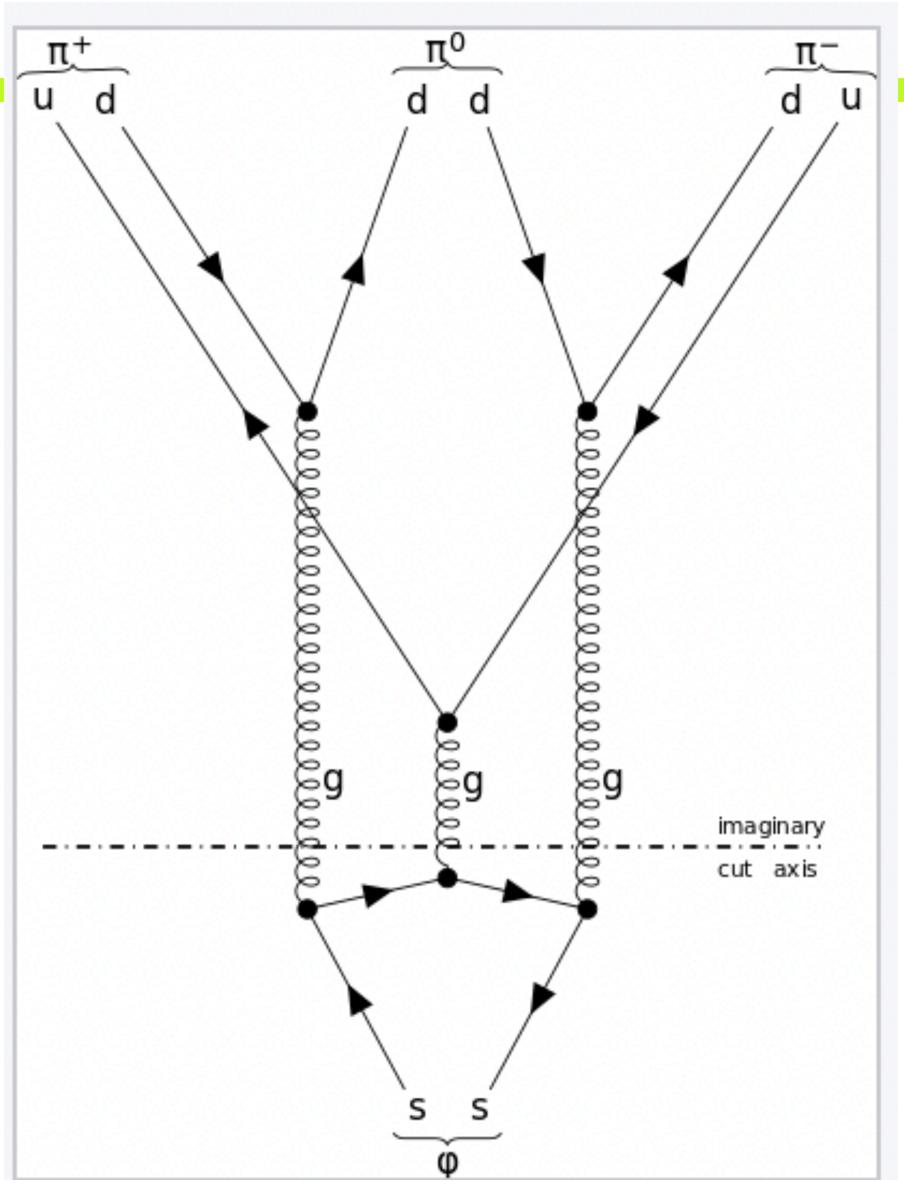
19th International Conference on Hadron Spectroscopy and Structure in memoriam Simon Eidelman

WILLIAM & MARY
CHARTERED 1693

Excited J^{--} resonances from meson-meson scattering at the SU(3) flavor point in lattice QCD.

CHRIS JOHNSON

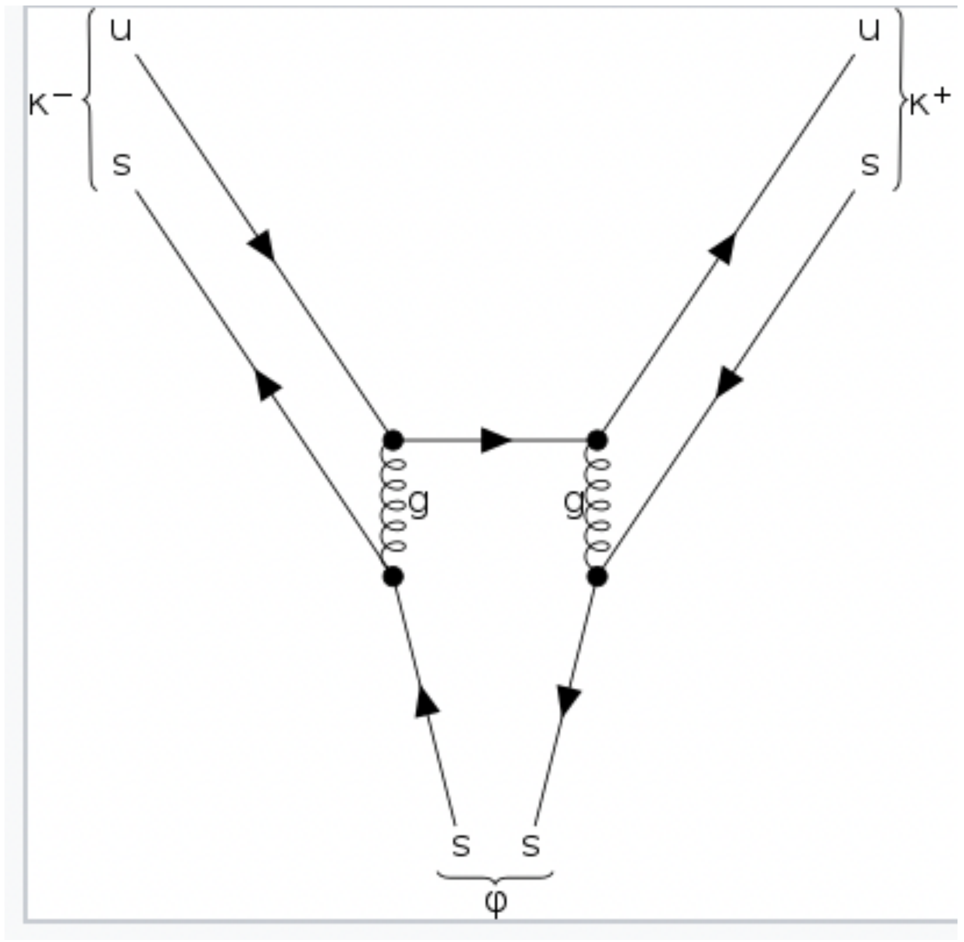
Experimental Status



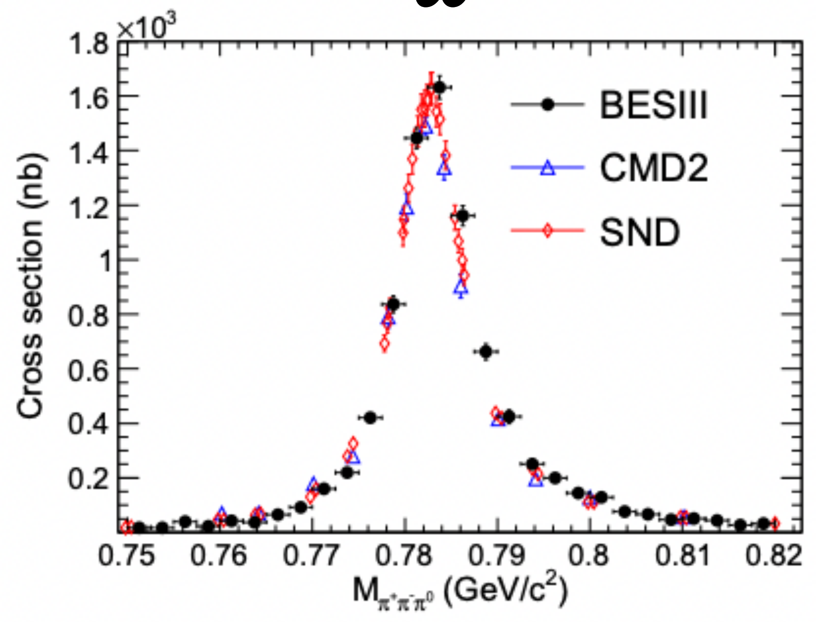
The lightest vector ($J^{PC} = 1^{--}$) mesons are the $\rho(770)$, $\omega(782)$, $\phi(1020)$

States are well understood in e⁺e⁻ annihilation due to their narrow widths and little background into decay into simple states like $\pi\pi$, $\pi\pi\pi$, $K\bar{K}$.

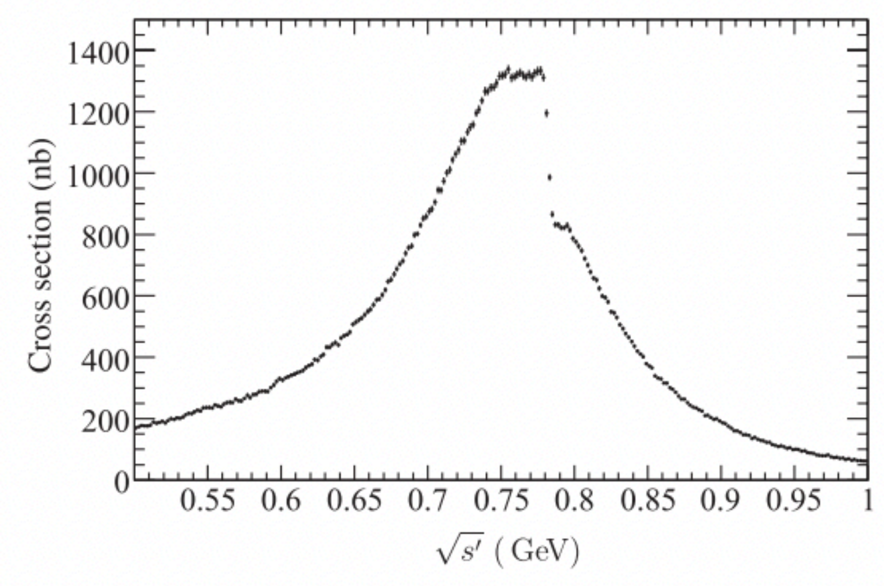
ω and ϕ states separated via decay channels $\pi\pi\pi$ vs $K\bar{K}$ (OZI)



ω

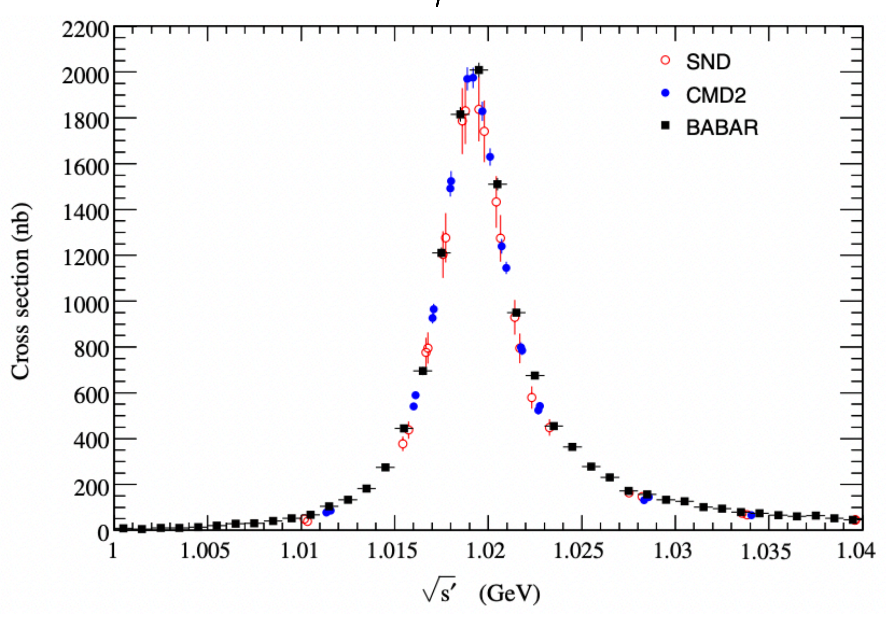


ρ



BaBar

ϕ



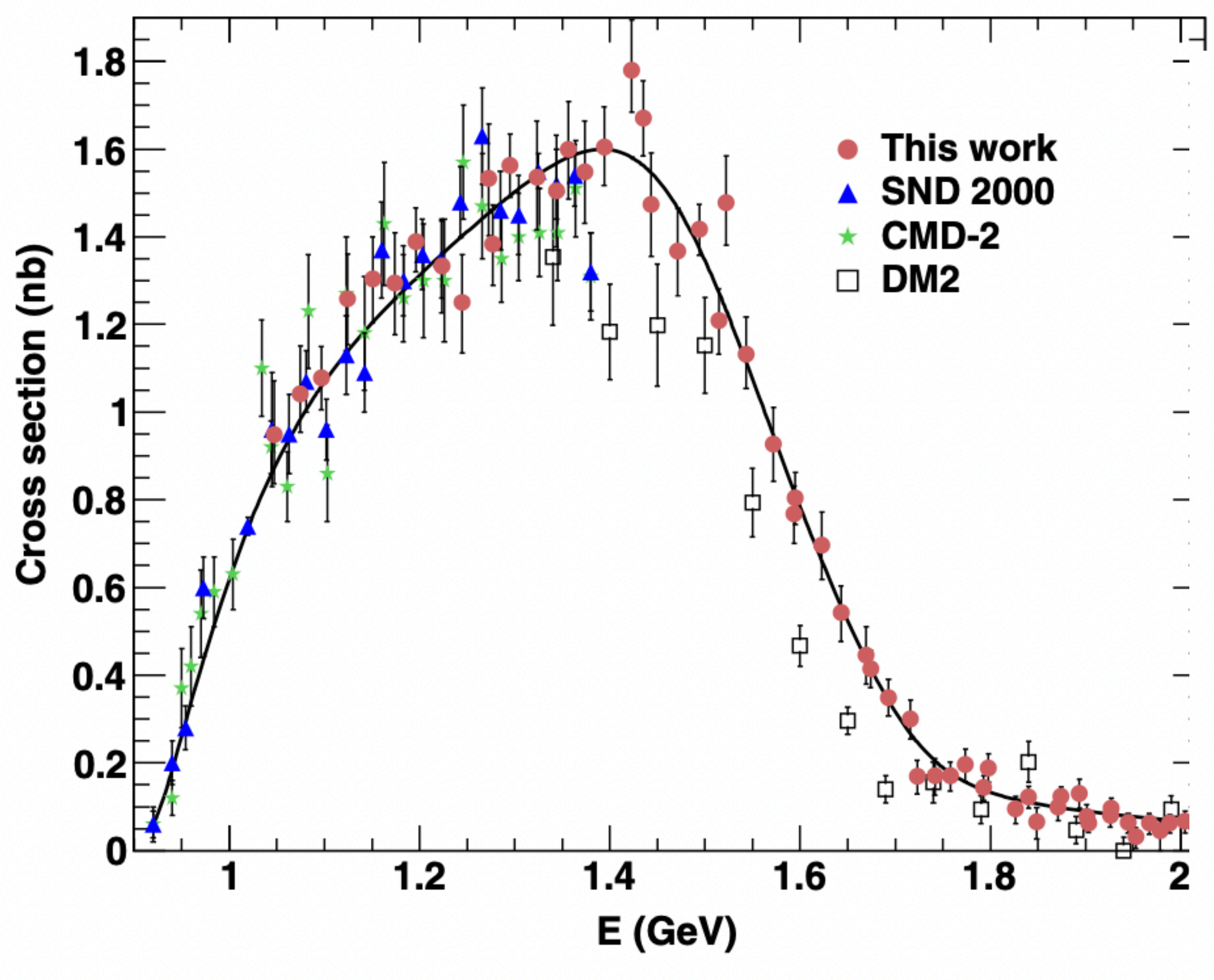
arXiv:1912.11208v1 BES III

PR D86 032013 J.P. Lees et al.

PR D88, 032013 J.P. Lees et al.

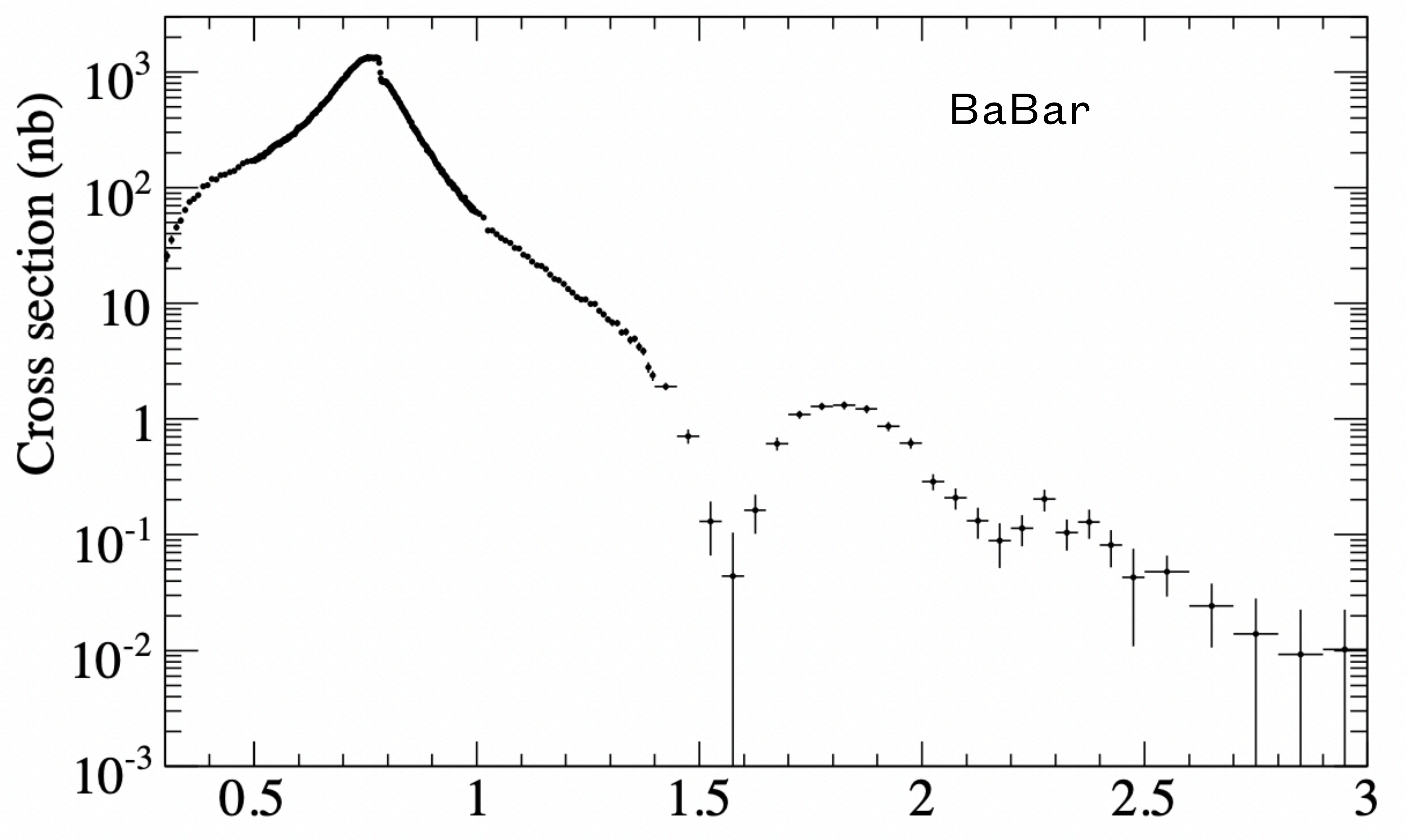
Excited light vector mesons (I=1)

Phys. Rev. D 94, 11201 (2016)



$e^+e^- \to \omega\pi^0$

Phys. Rev. D 86, 032013 (2012)

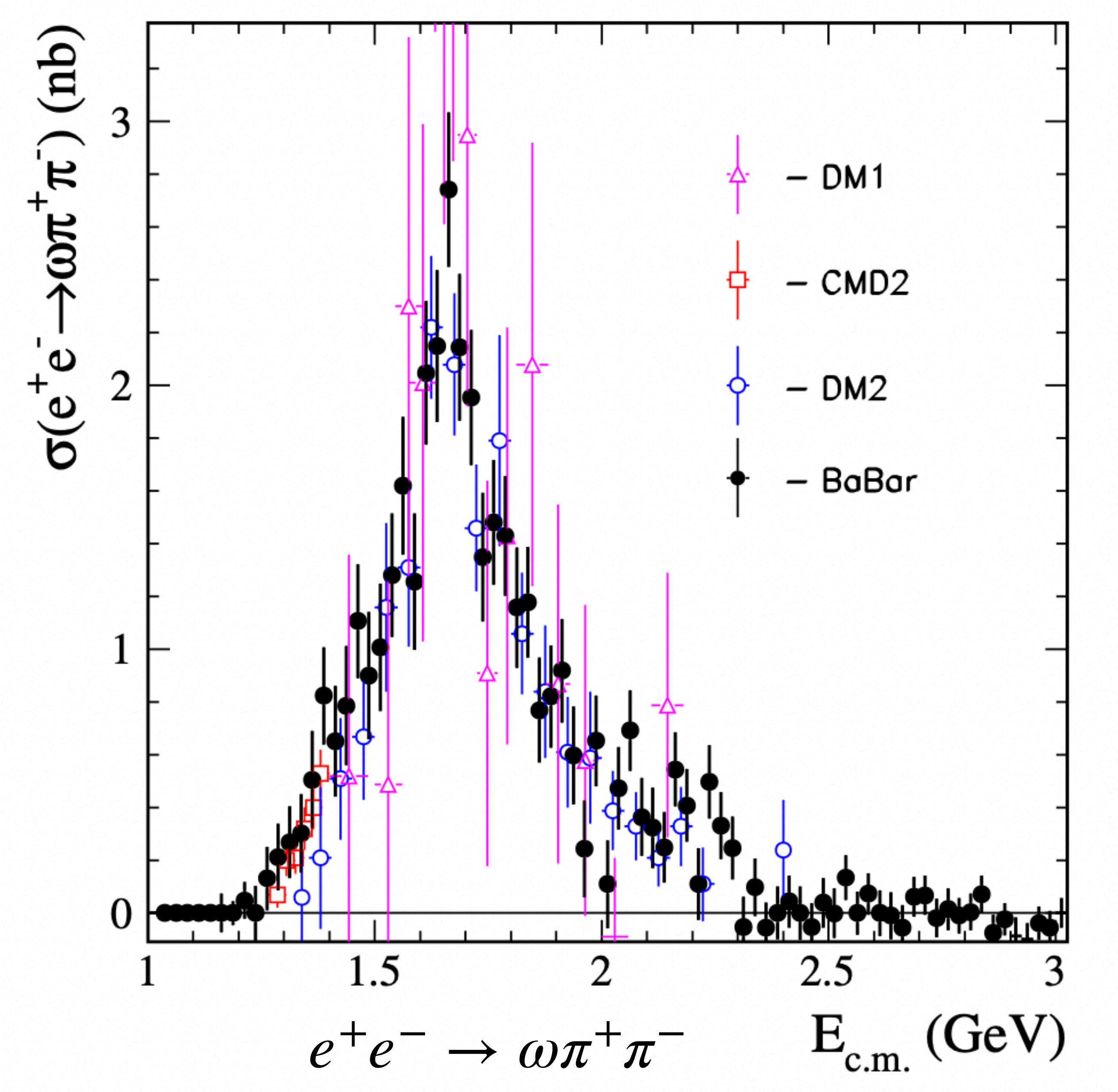


$e^+e^- \to \pi^+\pi^-(\gamma)$

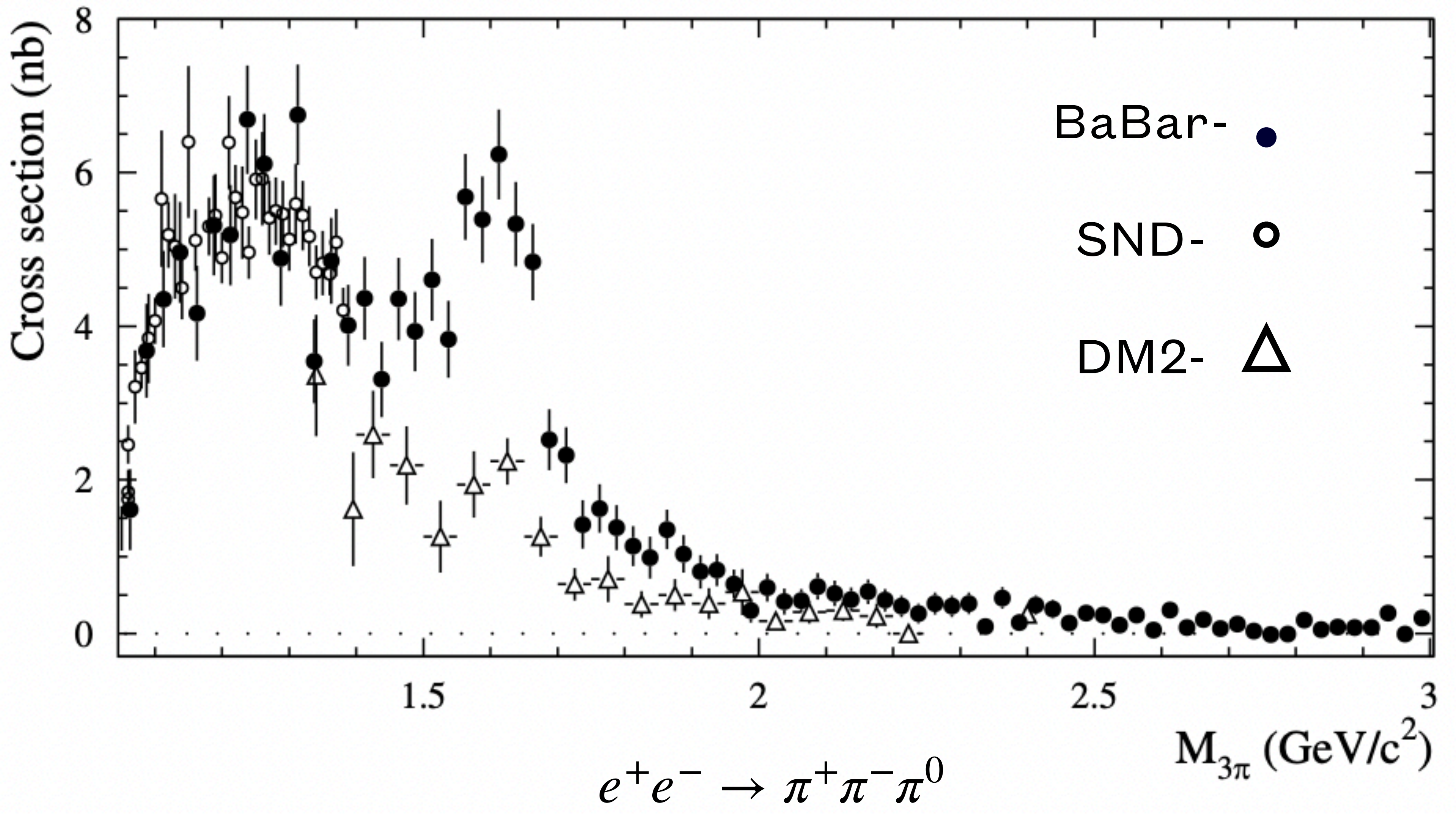
The $\rho(1450)$ and the $\rho(1700)$

Excited light vector mesons (I=0)

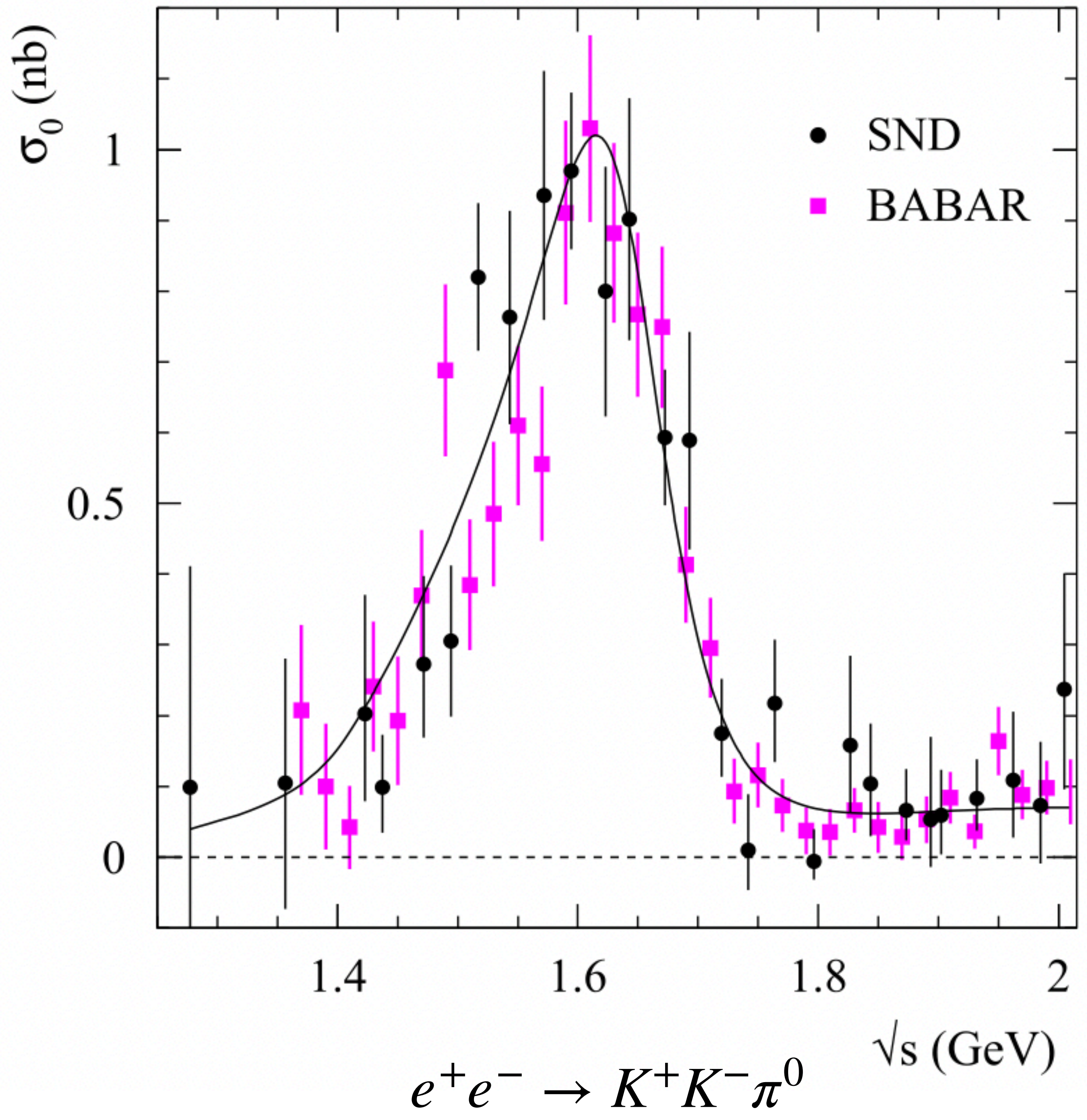
Phys. Rev. D 76, 092005 (2007)



Phys. Rev. D 70, 072004 (2004)



Eur. Phys. J. C 80, 1139 (2020)



The $\omega(1420)$, $\omega(1650)$, and the $\phi(1680)$

A Place to start

Presence of two states in 1^{--} from quark model it is natural to interpret these states as a radial excitation in S-wave $[2^3S_1]$, and an orbital excitation in D-wave $[^3D_1]$ (or some linear combination of the two).

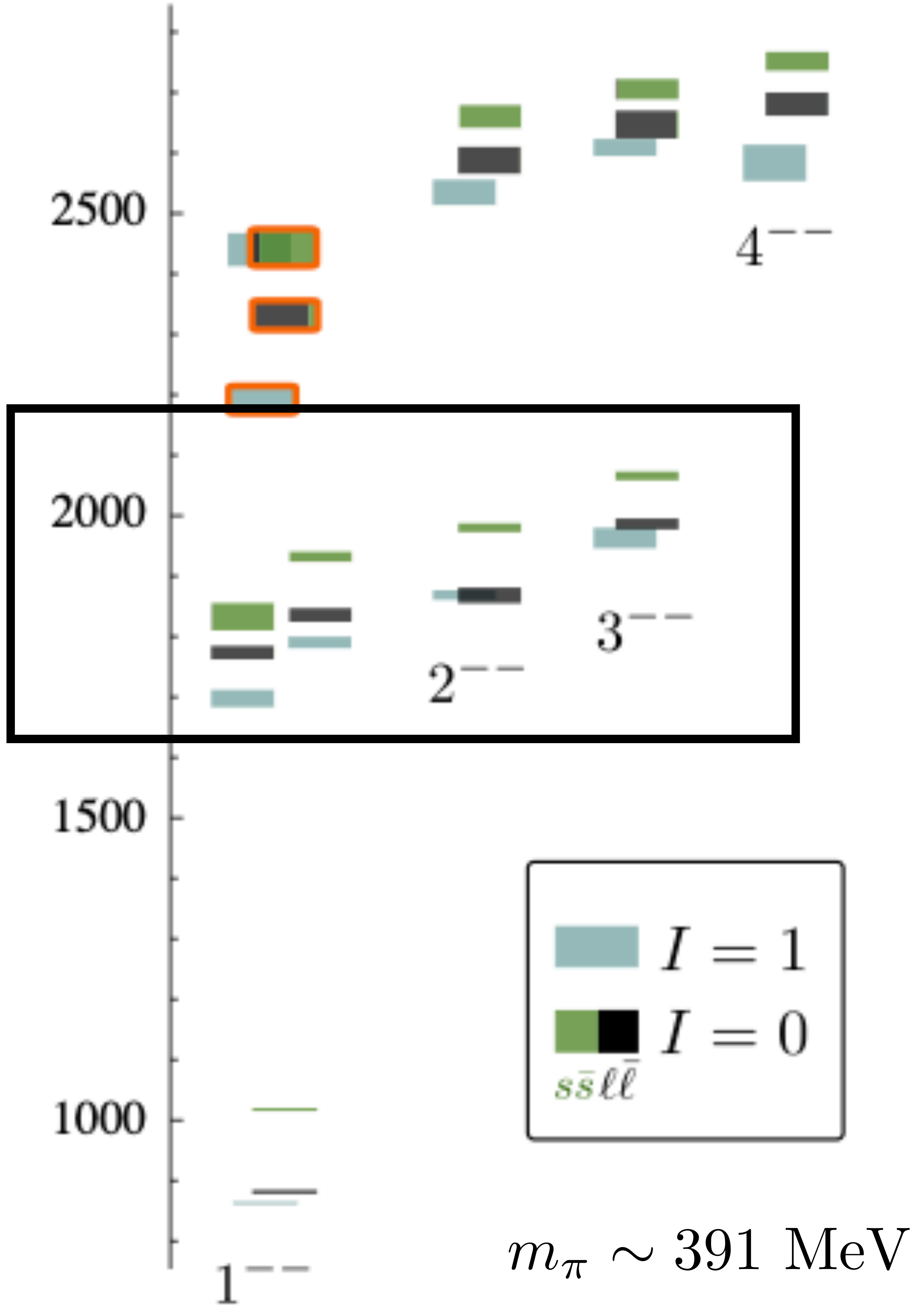
	J^P
$\ell = 0$	1^-
$\ell = 1$	$(0,1,2)^+$
$\ell = 2$	$(1,2,3)^-$
$\dots$	$\dots$

What the PDG says:

Isovector : $\rho(1450), \rho(1700), \rho_3(1690)$

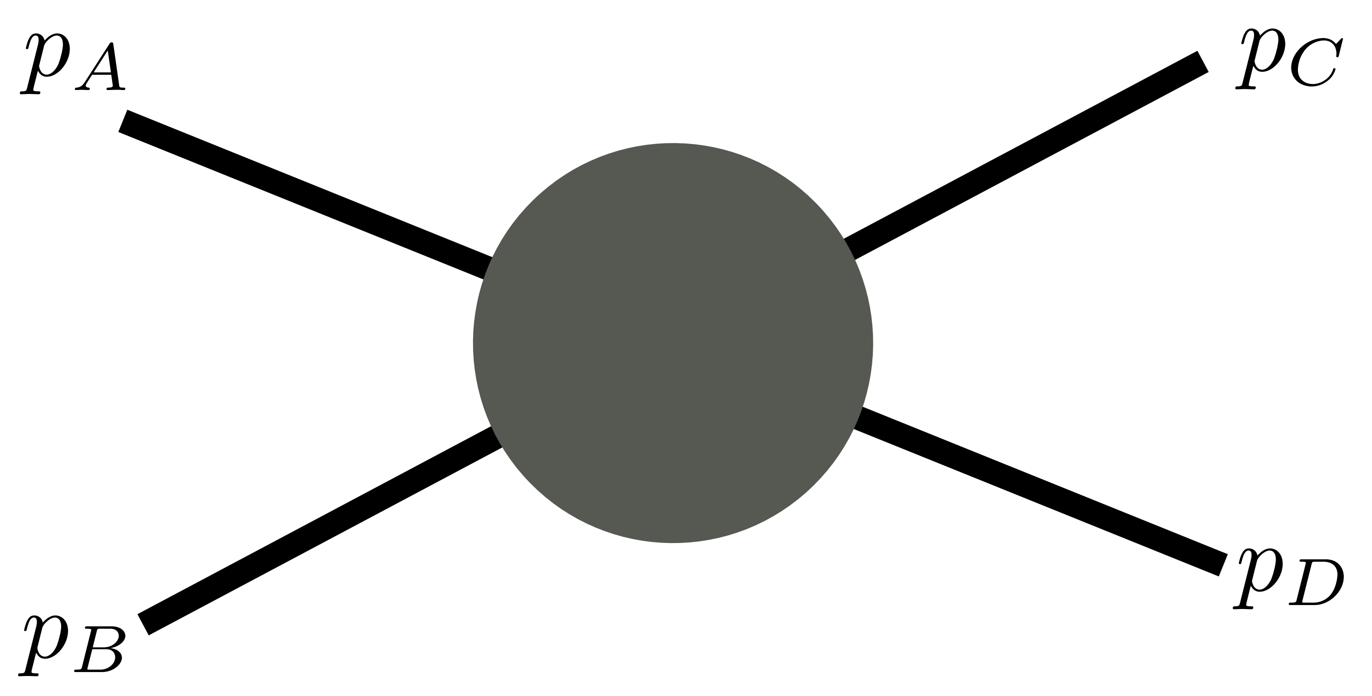
Isoscalar: $\omega(1420), \omega(1650), \omega_3(1670)$ / $\phi(1680), \phi_3(1850)$.

Lattice: $C_{ij}(t) = \sum_\alpha \langle 0|O_i|\alpha\rangle\langle\alpha|O_j|0\rangle e^{-E_\alpha t}$



J. J. Dudek, R. G. Edwards, P. Guo, and C. E. Thomas (Hadron Spectrum), Phys. Rev. D88, 094505 (2013), arXiv:1309.2608 [hep-lat].

Scattering

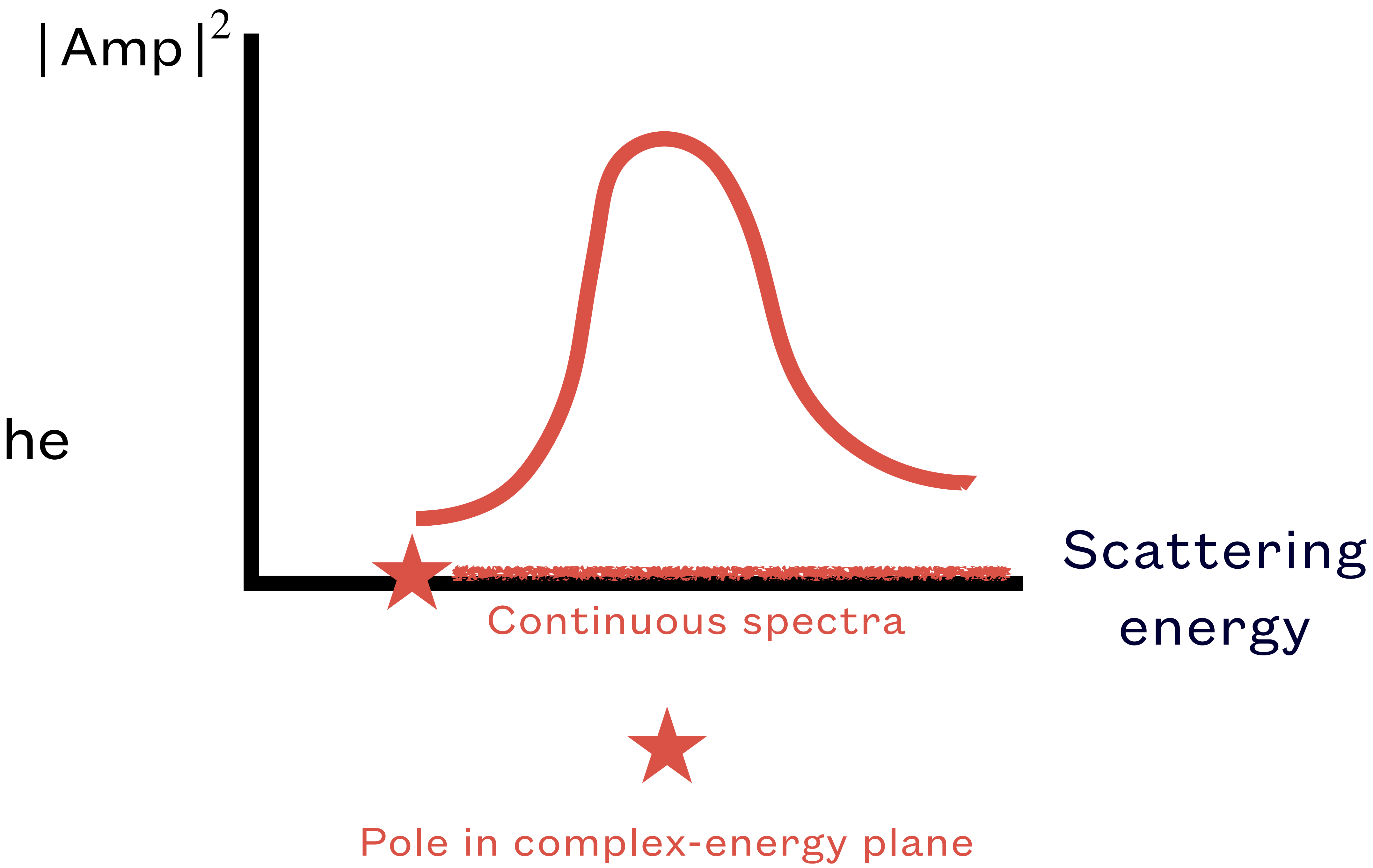


Instead of studying the J^{--} states in production, we can study them in scattering.

Example: $\rho \to \pi\pi$

Production: $e^+e^- \to \pi\pi$

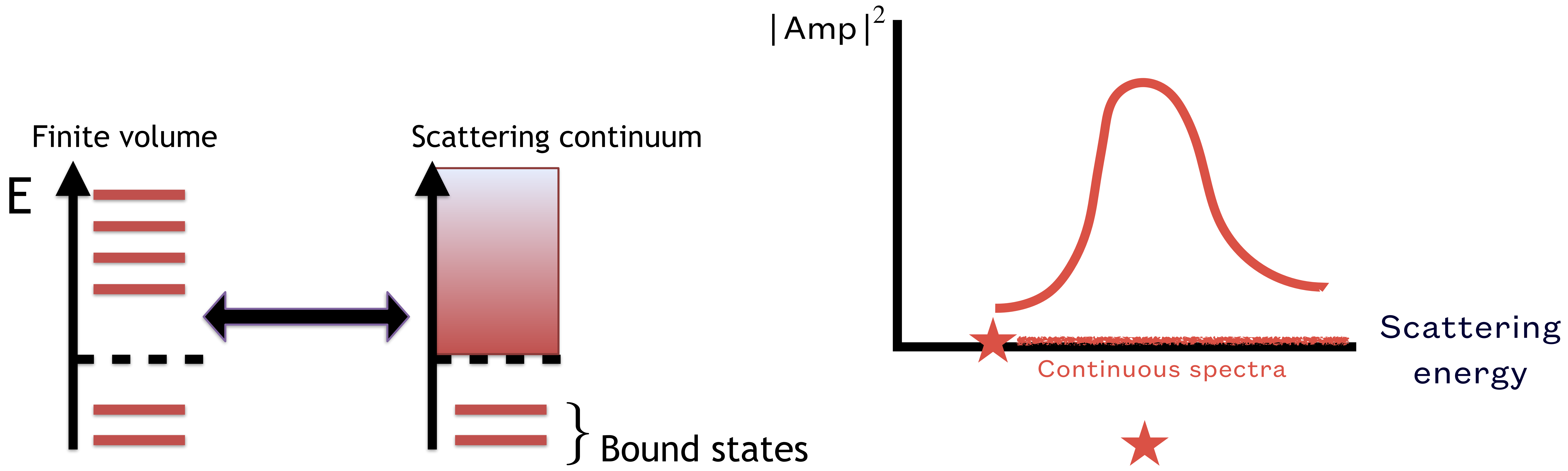
Scattering: $\pi\pi \to \pi\pi$



Resonances can be understood by the energy dependence of the scattering amplitude.

Scattering in a finite volume

Energy levels are now discrete, they feel the effects of the finite-volume (no asymptotic states).



Lattice QCD

Introduces three fundamental changes:

- Lattice spacing → does not likely play a big role
- Lattice volume → tool we need for scattering
- Quark mass → feature we make use of increasing pion mass

Compute correlation functions $C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle$ to extract the finite volume spectrum

$$\Rightarrow C_{ij}(t) = \sum_{\alpha} \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$$

Scattering in a finite volume

$2 \to 2$ scattering amplitudes are related to the finite volume spectrum via Lüscher's quantization condition: $\det\left[1 + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1} + i\mathcal{M}\right)\right] = 0$

$\rho_i(E) = \frac{2k_i}{E}$ is diagonal matrix of the phase space

$t_{ij}(E)$ is the symmetric scattering matrix satisfying unitarity $\mathrm{Im}(t_{ij}^{-1}) = -\delta_{ij}\rho_i$

$\mathcal{M}_{ij}(E, L)$ contains the finite volume pieces

Calculations have been done for elastic, coupled channel, and coupled channels with spinning particles.

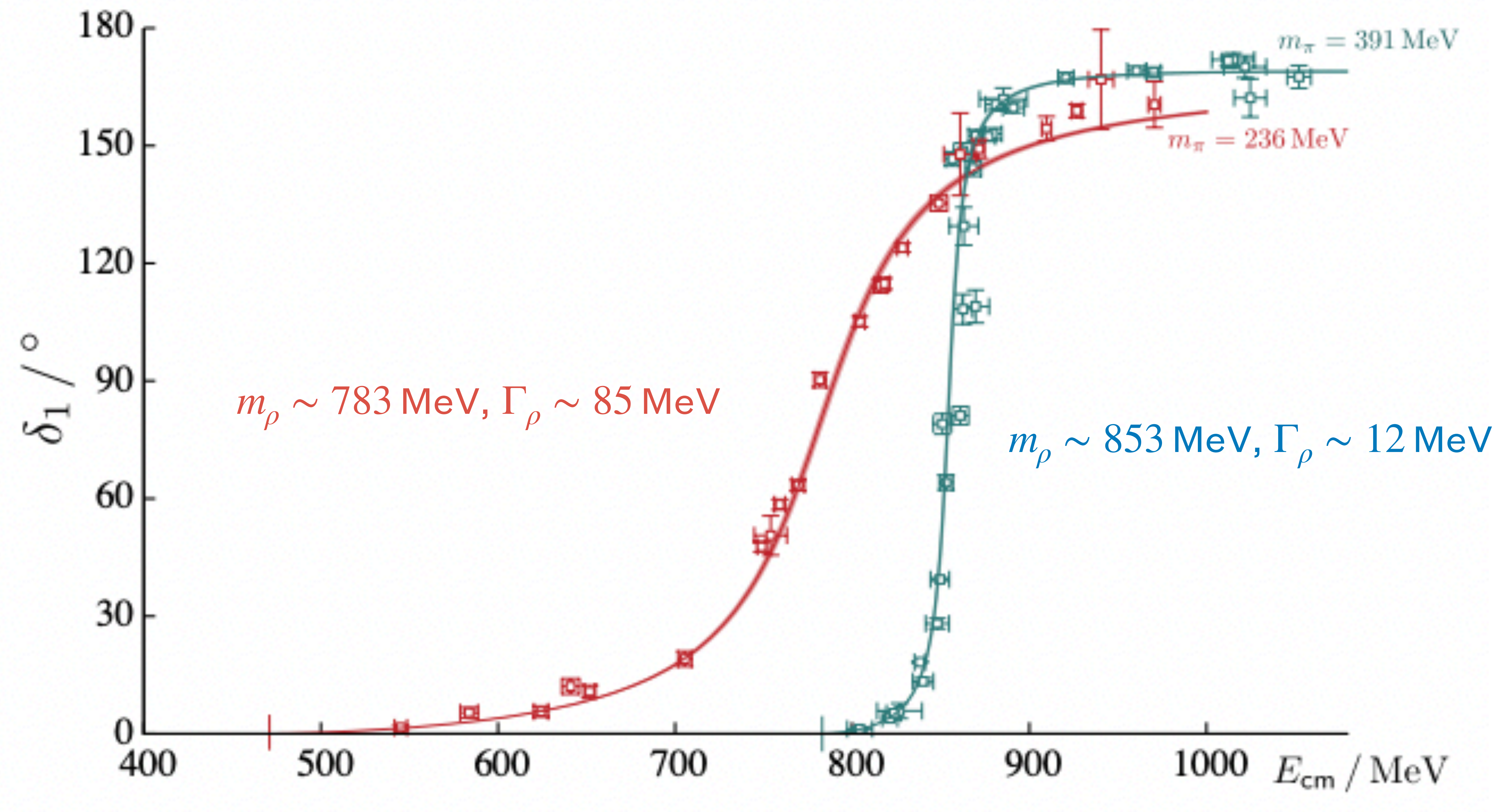
Elastic scattering in a finite volume

$$\det\left[1 + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1} + i\mathbf{M}\right)\right] = 0$$

Can describe the amplitude through a phase shift.

For a single elastic channel (one partial wave) $\Rightarrow t(E) = \dfrac{1}{\rho(\cot\delta(E) - i)}$

Reduces to a single equation $\cot\delta(E) = M(E, L)$



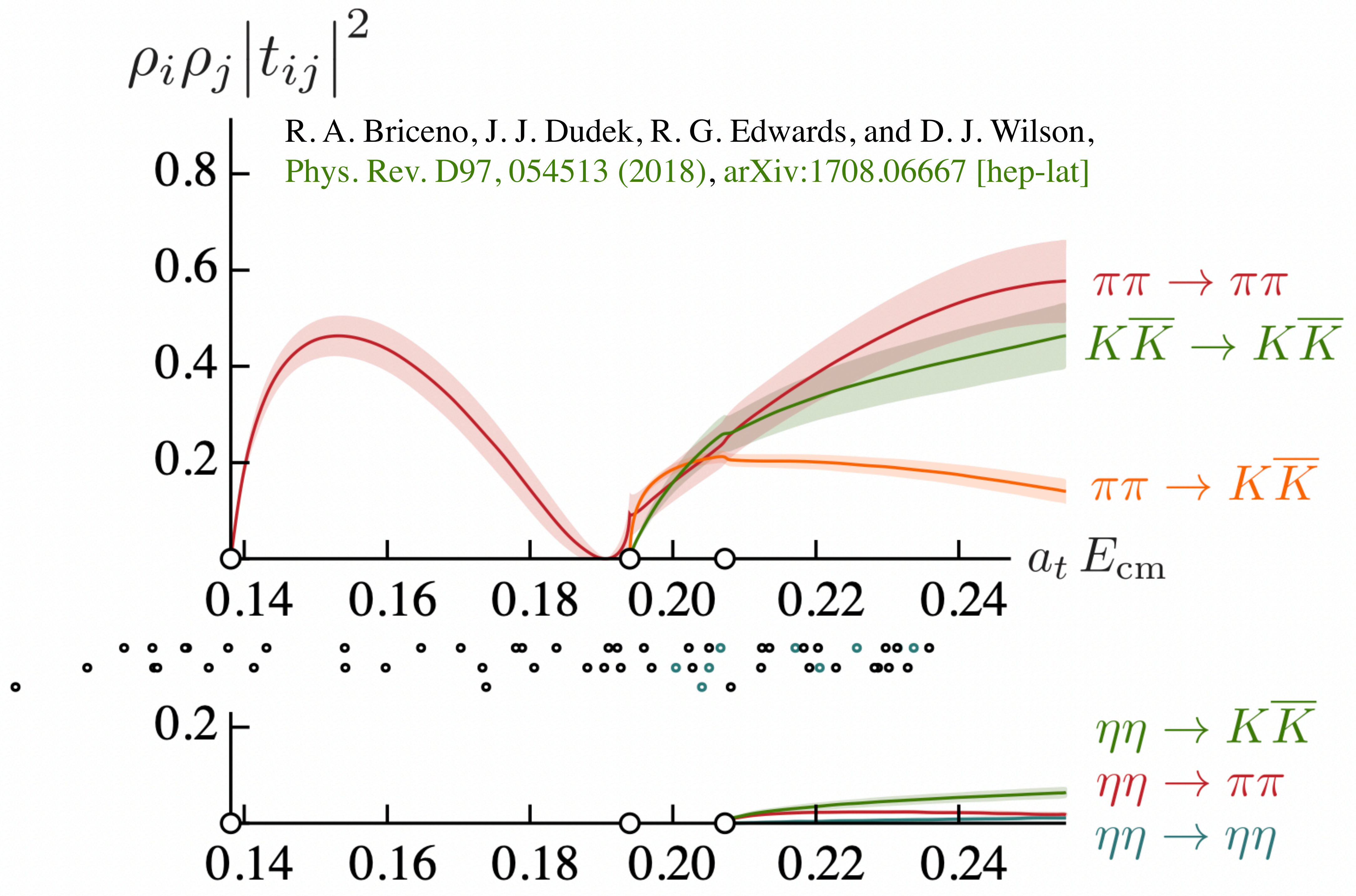
D. J. Wilson, R. A. Briceño, J. J. Dudek, R. G. Edwards, and C. E. Thomas, arXiv:1507.02599v2

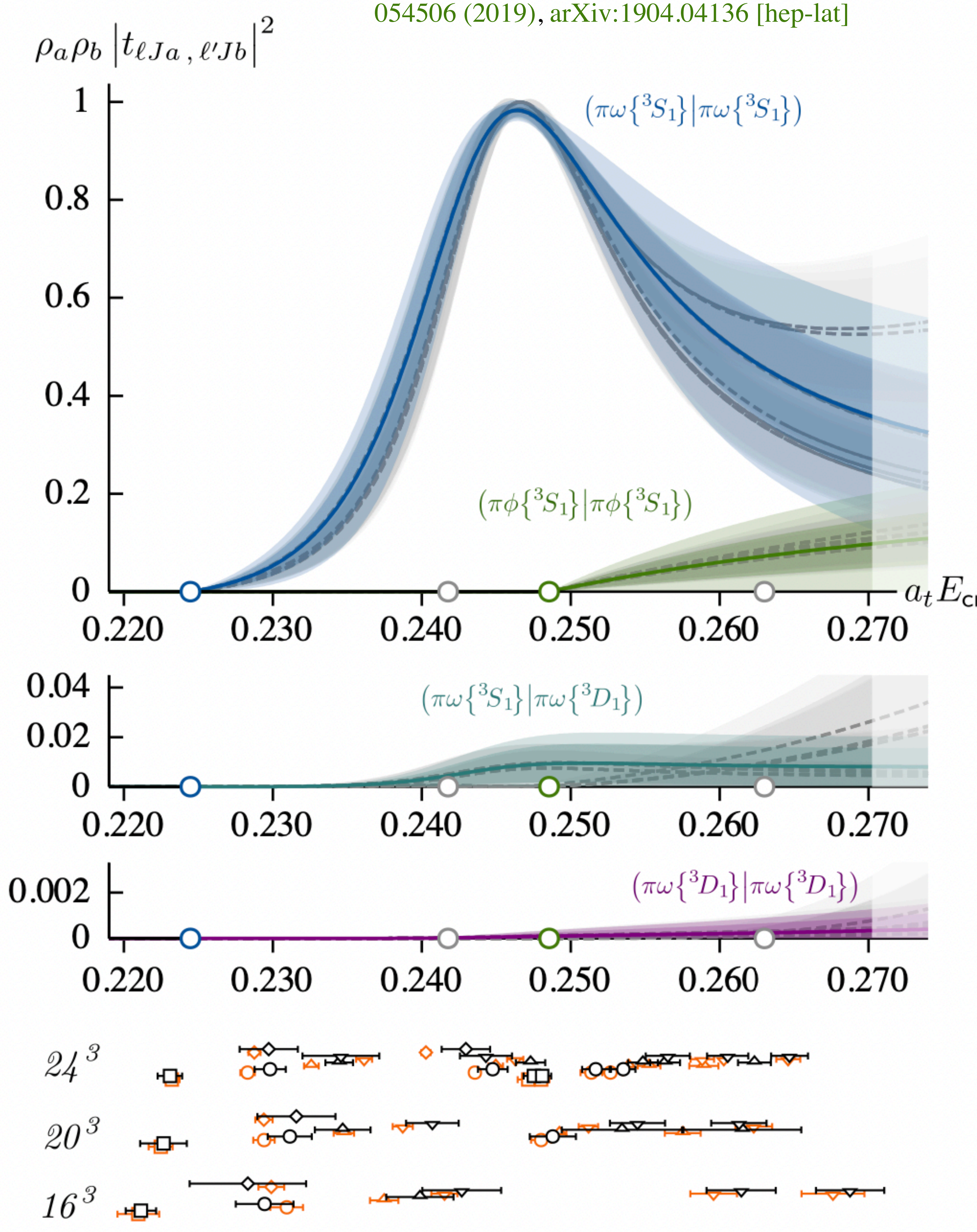
Coupled-channel

$$\det\left[1 + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1}+i\mathbf{M}\right)\right] = 0$$

Solutions follow from K-matrix parameterizations of the amplitude :

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho} \qquad K_{ij}(s) = \sum_\alpha \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_\alpha^2 - s} + \sum_\beta s^\beta \gamma_{ij}$$





This work (excited J^{--} resonances...)

Old:

- Elastic scattering ✓
- Coupled channel ✓
- Spinning hadrons ✓

New:

- Multiple resonances in the same partial waves and irreps ⇒ a proper test of the finite volume formalism

SU(3) Flavor Ensembles

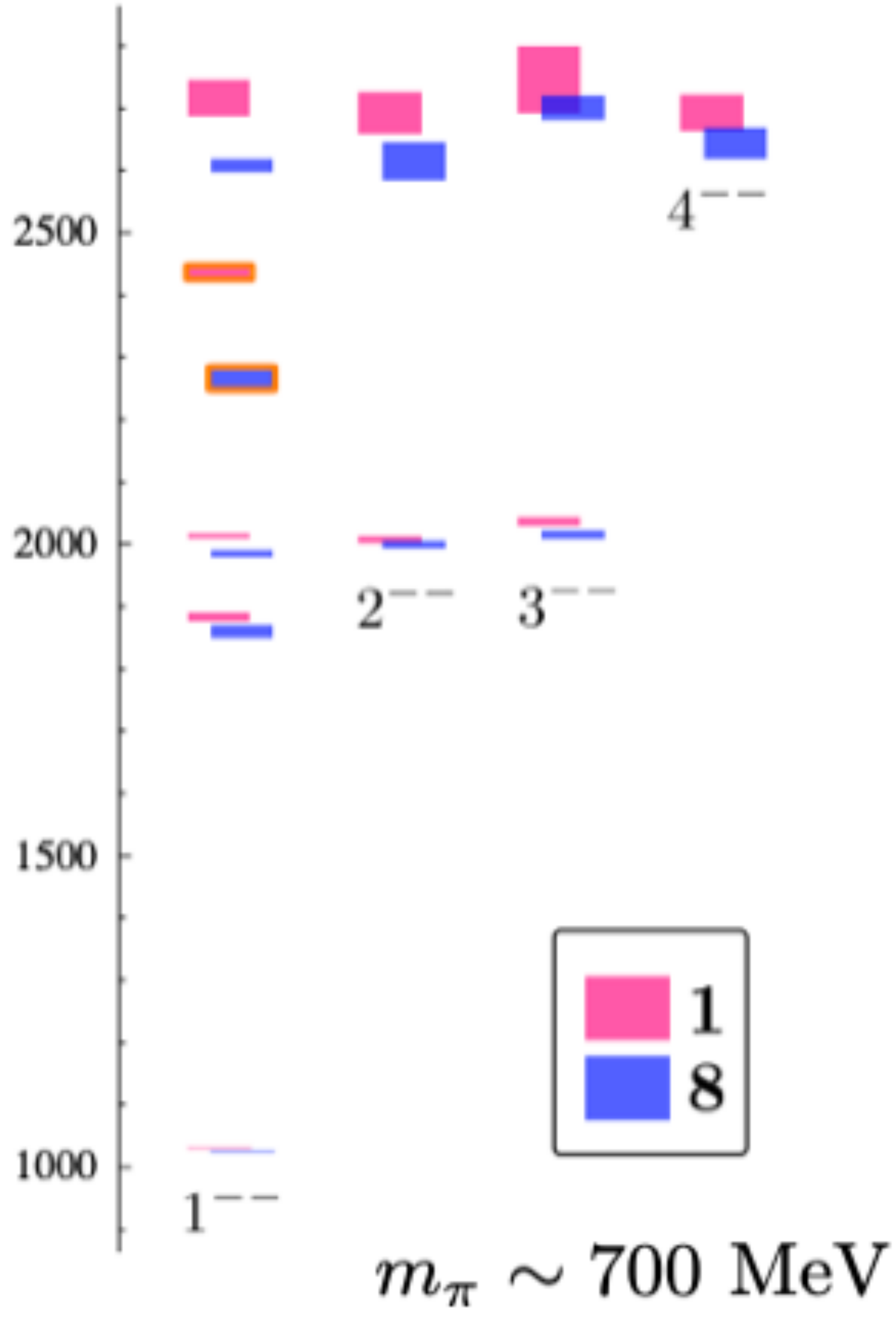
This project studies the isoscalar J^{--}excited mesons at the SU(3) flavor point

⇒Heavier light quark masses allow us to probe higher energy regions:

first three-particle threshold gets moved higher up

resonant states at lighter quark masses feature as stable particles

⇒Fewer channels (ex. $\pi, K, \bar{K}, \eta$ are all just η^8)

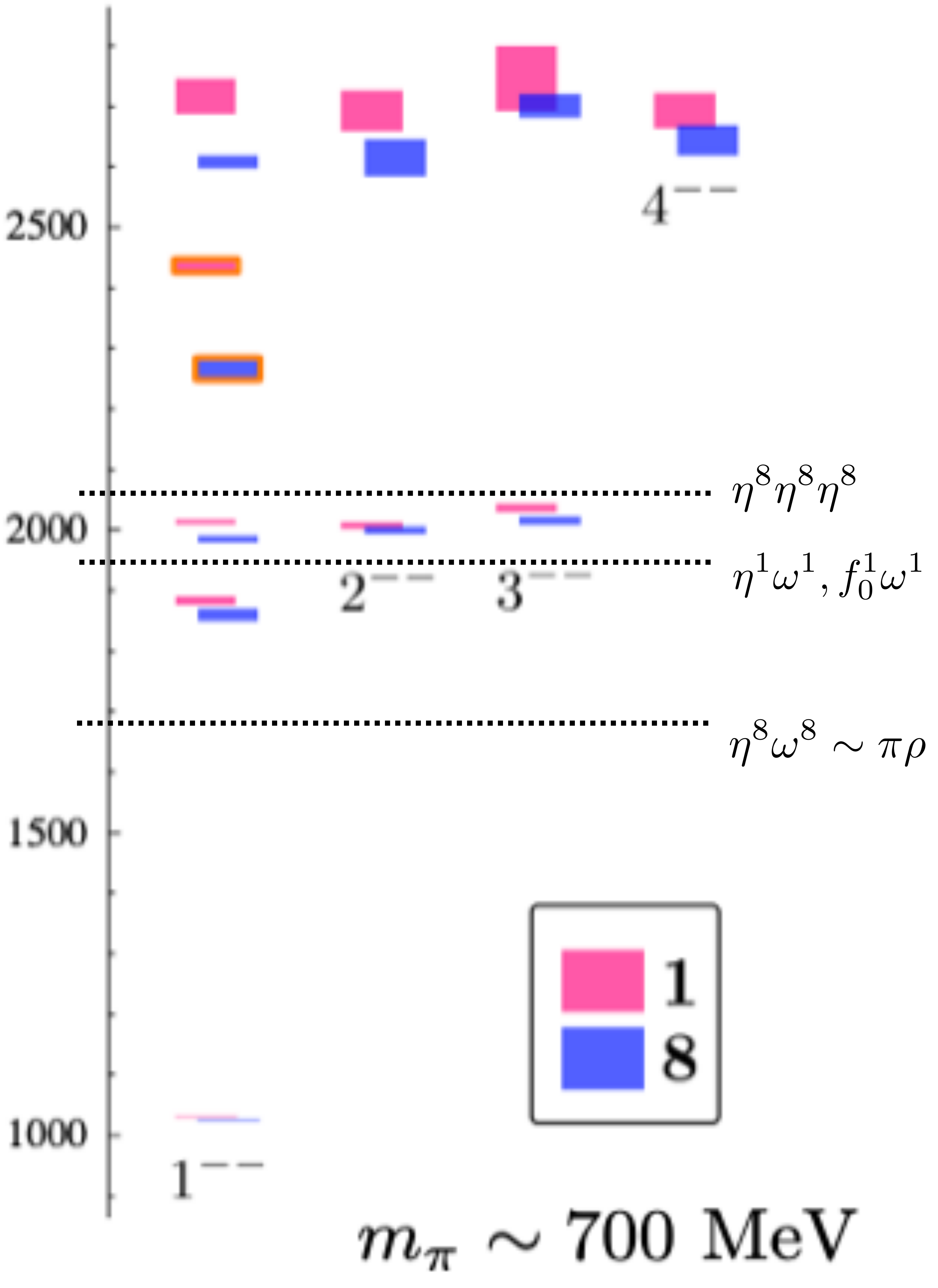


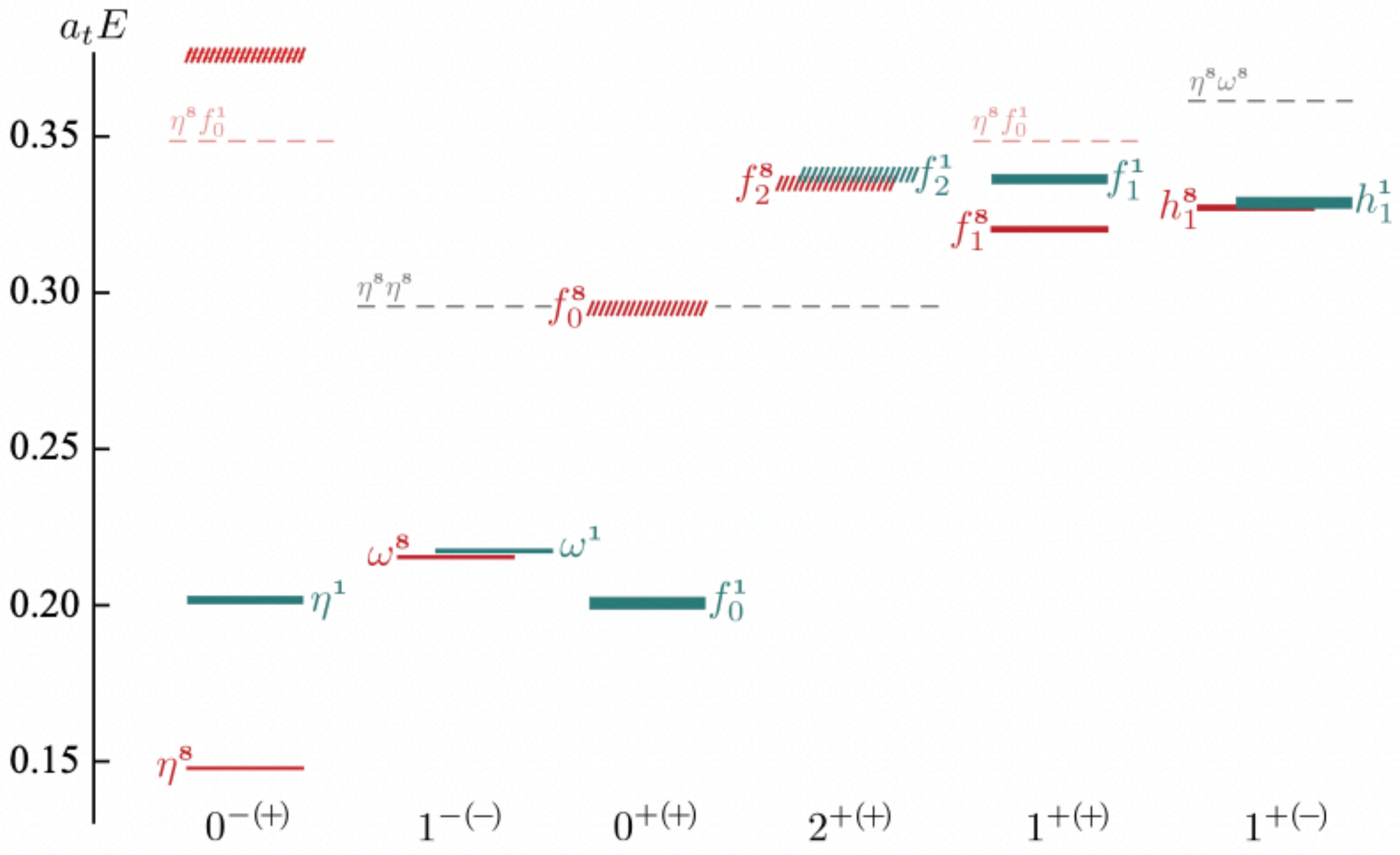
Channels $SU(3)_F$

J=1: $\eta^8\omega^8\{^3P_1\}, f_0^1\omega^1\{^3S_1, {}^3D_1\}, \eta^1\omega^1\{^3P_1\}$

J=2: $\eta^8\omega^8\{^3P_2, {}^3F_2\}, f_0^1\omega^1\{^3D_2\}, \eta^1\omega^1\{^3P_2, {}^3F_2\}$

J=3: $\eta^8\omega^8\{^3F_3\}, f_0^1\omega^1\{^3D_3, {}^3G_3\}, \eta^1\omega^1\{^3F_3\}$





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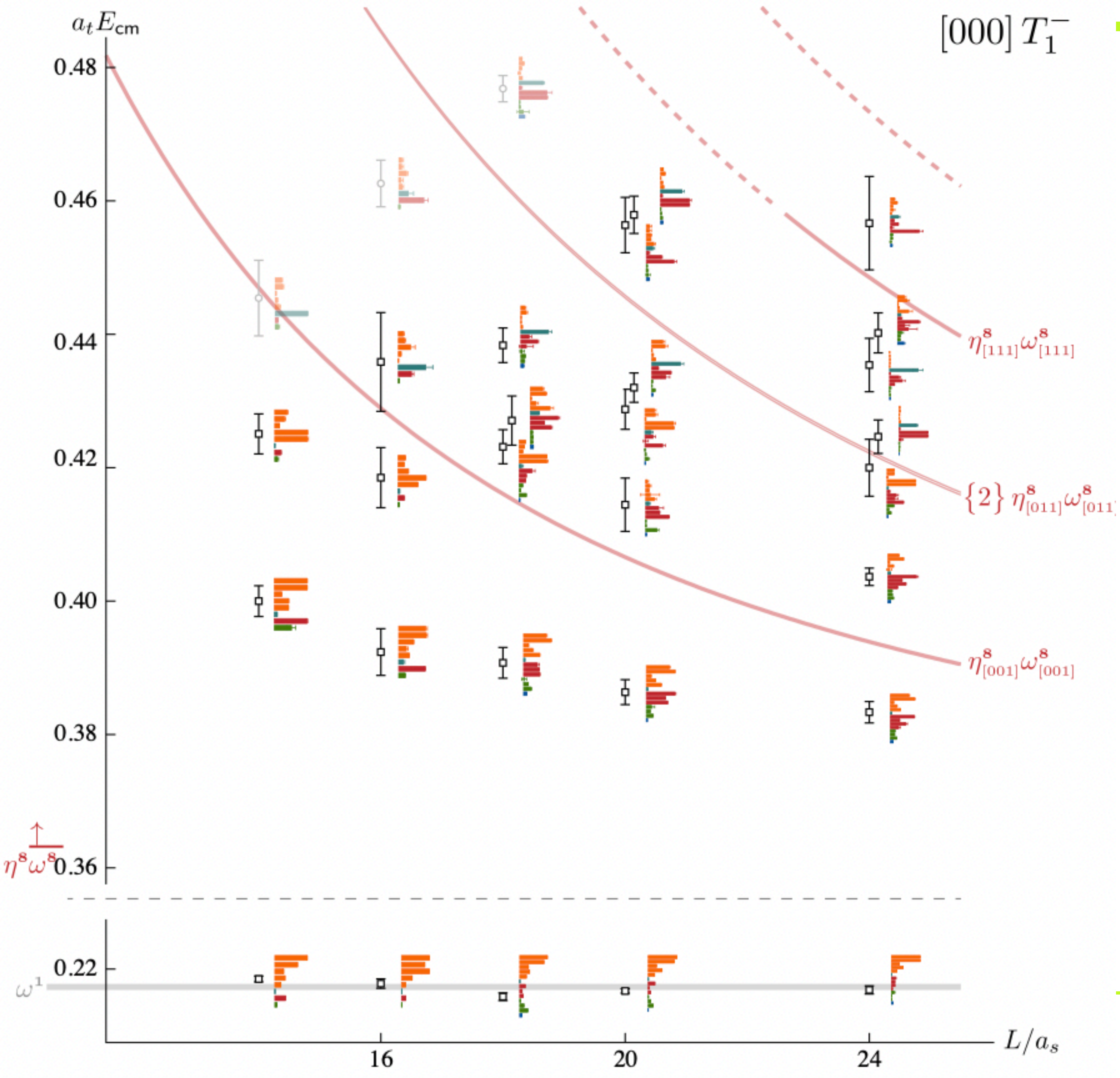
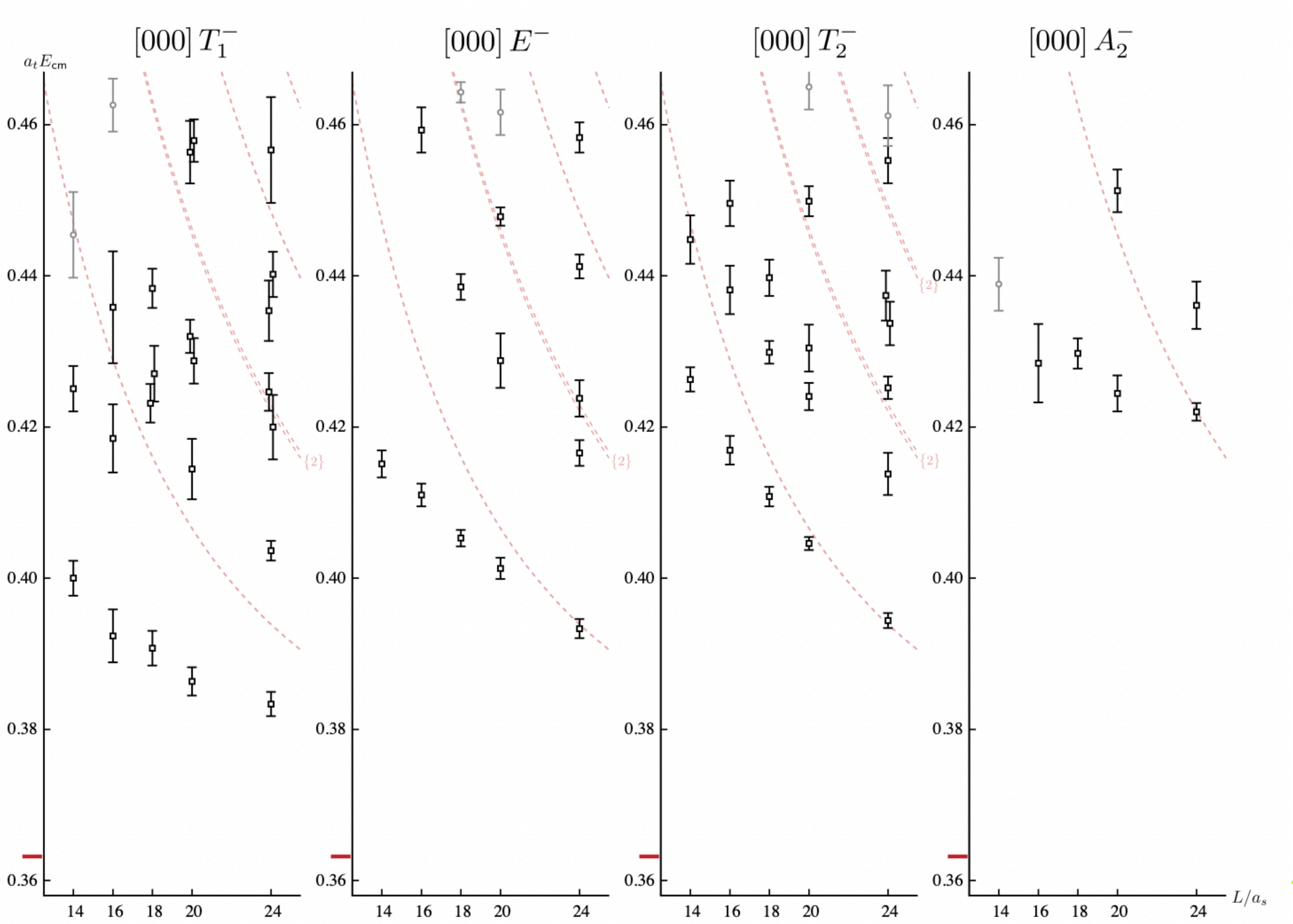


$J^P = (1,3,\ldots)^-$

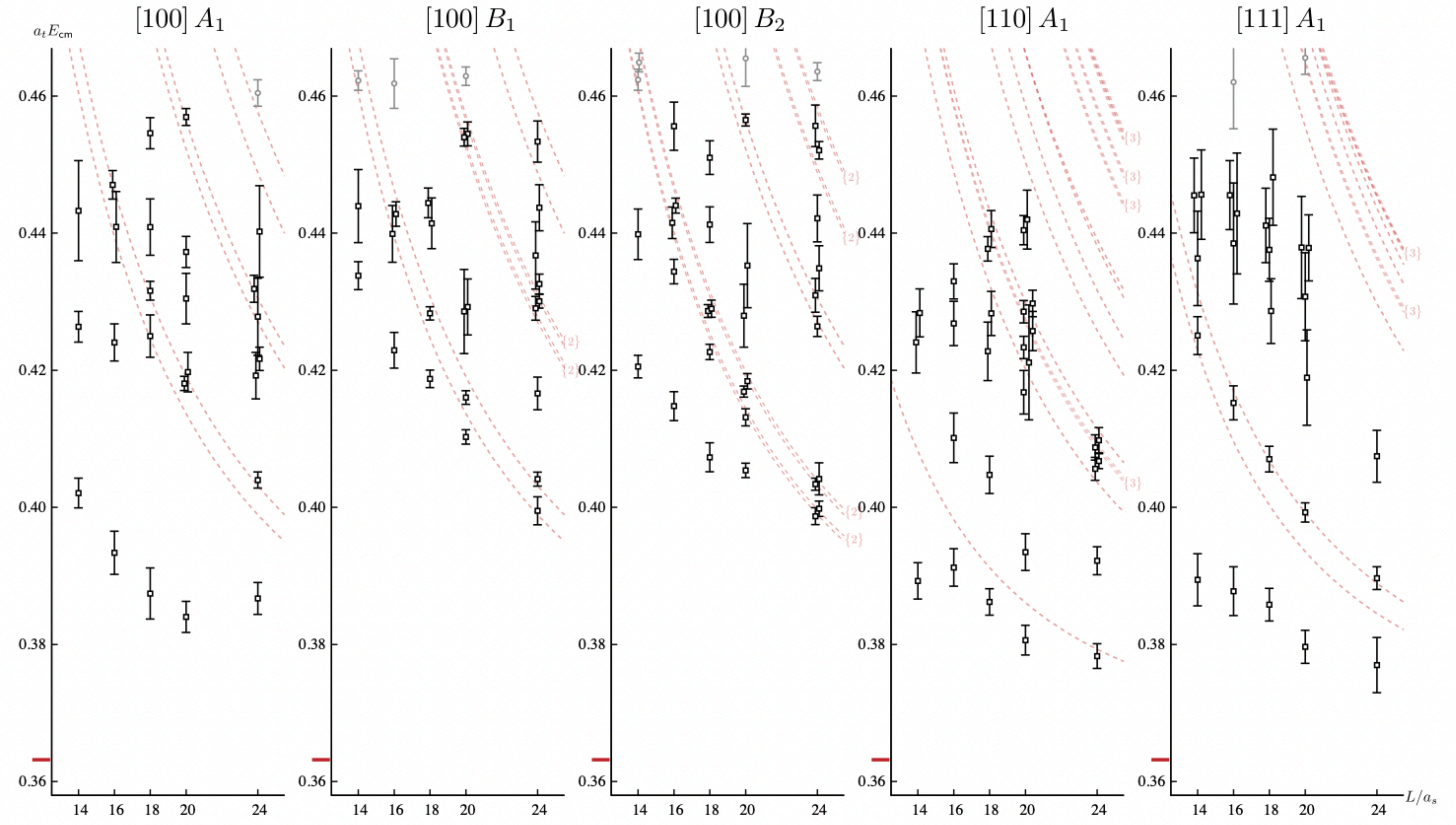
Three resonances in a single irrep.

$\Rightarrow \rho\{^3 2S_1\}, \rho\{^3 D_1\}, \rho\{^3 D_3\}$

Very dense in energy levels.



$J^P = (1, 3, ...)^-$ $J^P = (2, ...)^-$ $J^P = (2, 3, ...)^-$ $J^P = (3, ...)^-$



$J^P = 0^+, 1^-, 2^+, 3^-, \ldots$ $J^P = 2^\pm, 3^\pm, \ldots$ $J^P = 2^\pm, 3^\pm, \ldots$ $J^P = 0^+, 1^-, 2^\pm, 3^\pm, \ldots$ $J^P = 0^+, 1^-, 2^+, 3^\pm, \ldots$

Parameterizations, J=2,3

J=2 dynamically coupled in P- and F-waves

$$K_{ij} \to (2k_i)^{\ell} K_{ij}^{\ell\ell'} (2k_j)^{\ell'}$$

Can handle this with the K-matrix $\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$

$$K_{J=2} = \begin{bmatrix} ({}^3P_2|{}^3P_2) & ({}^3P_2|{}^3F_2) \\ ({}^3P_2|{}^3F_2) & ({}^3F_2|{}^3F_2) \end{bmatrix}$$

	J^P
$\ell = 0$	1^+
$\ell = 1$	$(0, 1, 2)^-$
$\ell = 2$	$(1, 2, 3)^+$
$\ell = 3$	$(2, 3, 4)^-$
$\ldots$	$\ldots$

J=3 Breit-Wigner parameterization

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_{thr}}^{\infty} \frac{\rho(s')}{(s' - s_0)(s' - s - i\epsilon)} ds'$$

$\text{Im}\boldsymbol{I} = -\boldsymbol{\rho}$

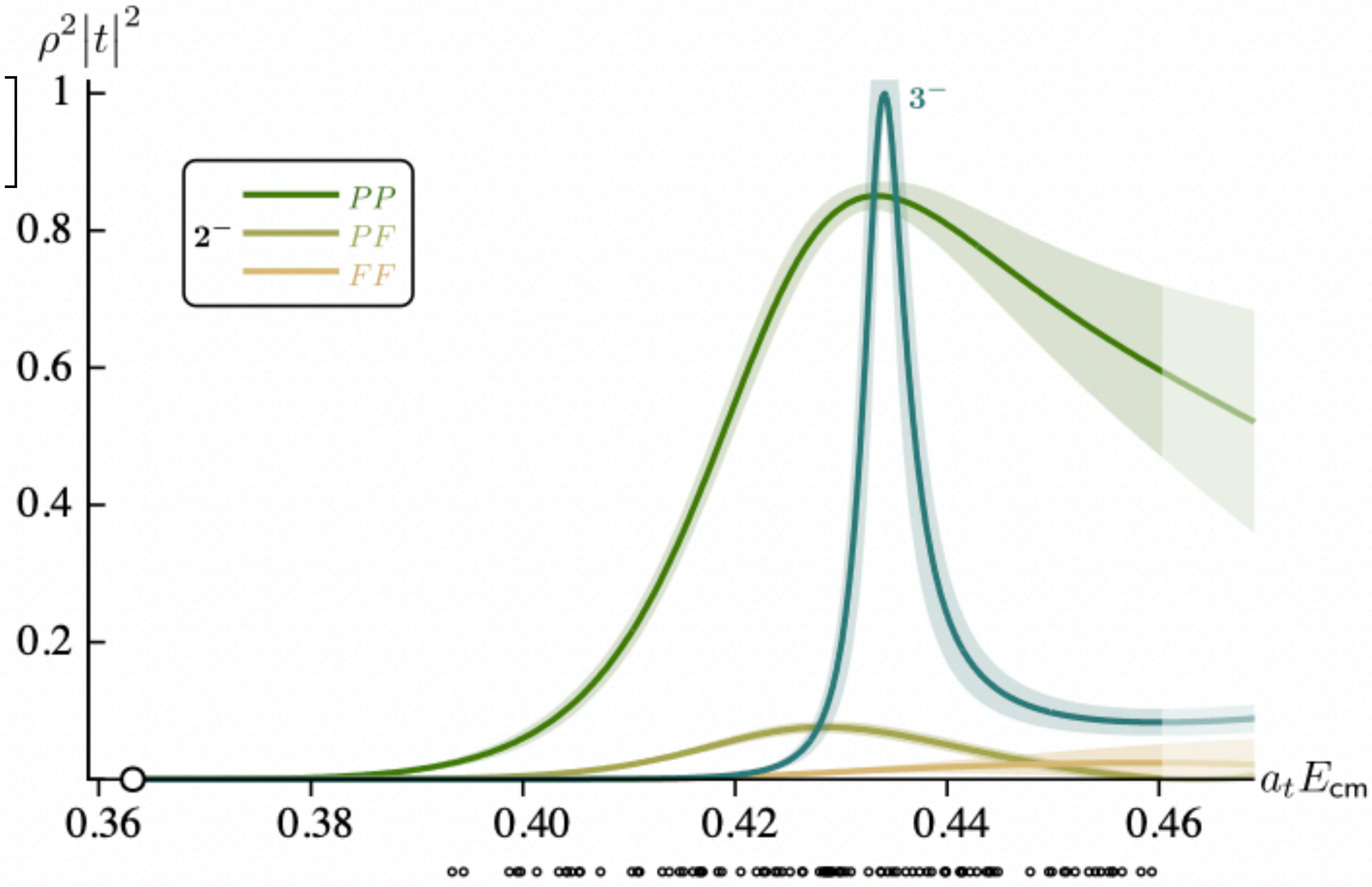
$\eta^8\omega^8$ elastic scattering in $2^{--}, 3^{--}$

$$K_{J=2} = \frac{1}{m_R^2 - s}\begin{bmatrix} g_P^2 & g_P g_F \\ g_P g_F & g_F^2 \end{bmatrix} + \begin{bmatrix} \gamma_{PP} & \gamma_{PF} \\ \gamma_{PF} & \gamma_{FF} \end{bmatrix}$$

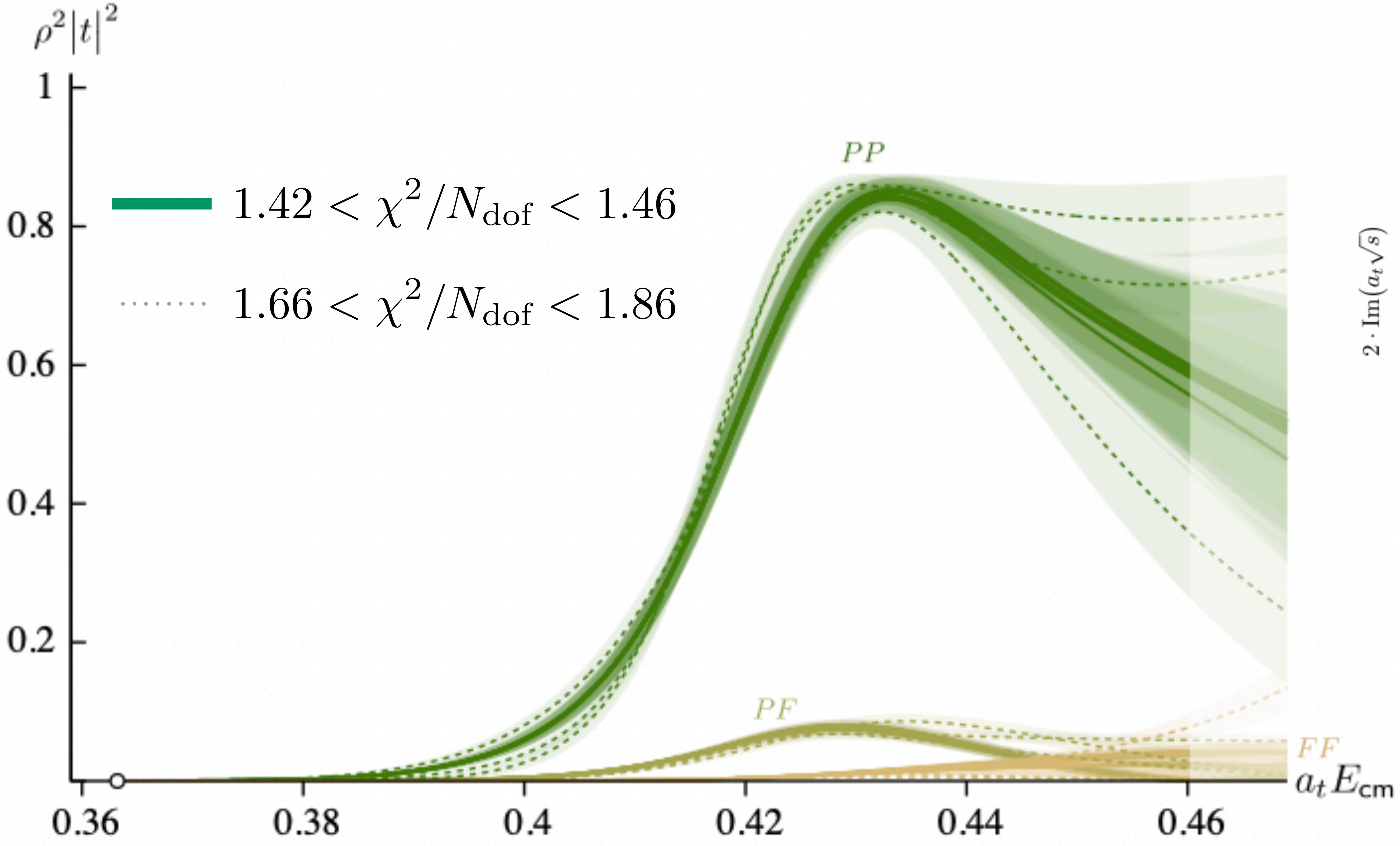
$$K_{J=3} = \frac{g_F^2}{m_R^2 - s}$$

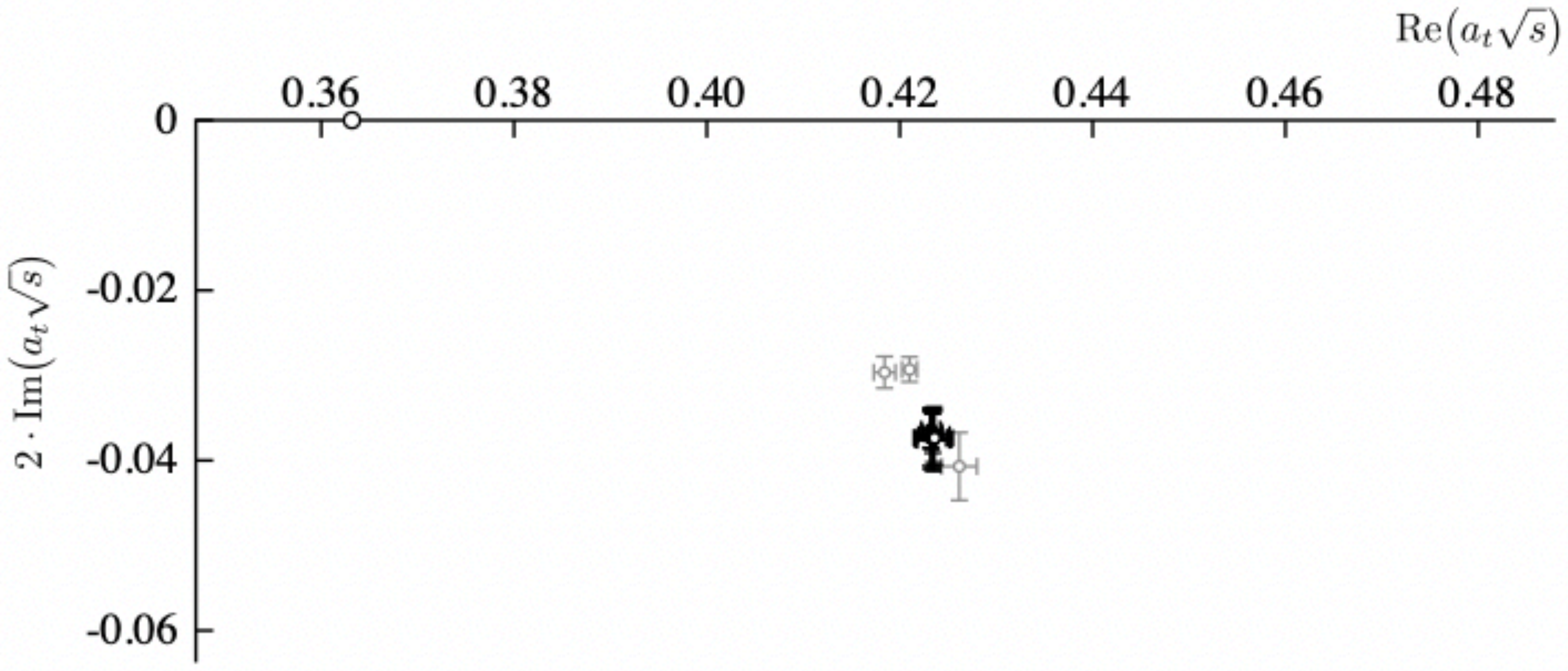
	Parameter								
$J=2$	$m = 0.4322(15)\cdot a_t^{-1}$	1	0.31	0.29	0.13	−0.37	0.31	0.19	0.07
	$g_P = 0.753(37)$		1	−0.08	−0.70	0.04	0.48	0.07	−0.23
	$g_F = -4.13(29)\cdot a_t^2$			1	0.21	−0.15	−0.18	−0.01	−0.12
	$\gamma_{PP} = 0.1(33)\cdot a_t^2$				1	−0.34	−0.34	−0.16	0.23
	$\gamma_{PF} = -110(17)\cdot a_t^4$					1	−0.23	−0.03	−0.05
	$\gamma_{FF} = 143(322)\cdot a_t^6$						1	0.02	0.05
$J=3$	$m = 0.4341(9)\cdot a_t^{-1}$							1	−0.04
	$g = 4.85(28)\cdot a_t^2$								1

$\chi^2/N_{\text{dof}} = \frac{120.3}{91-8} = 1.45$



$\eta^8\omega^8$ elastic scattering in 2^{--}



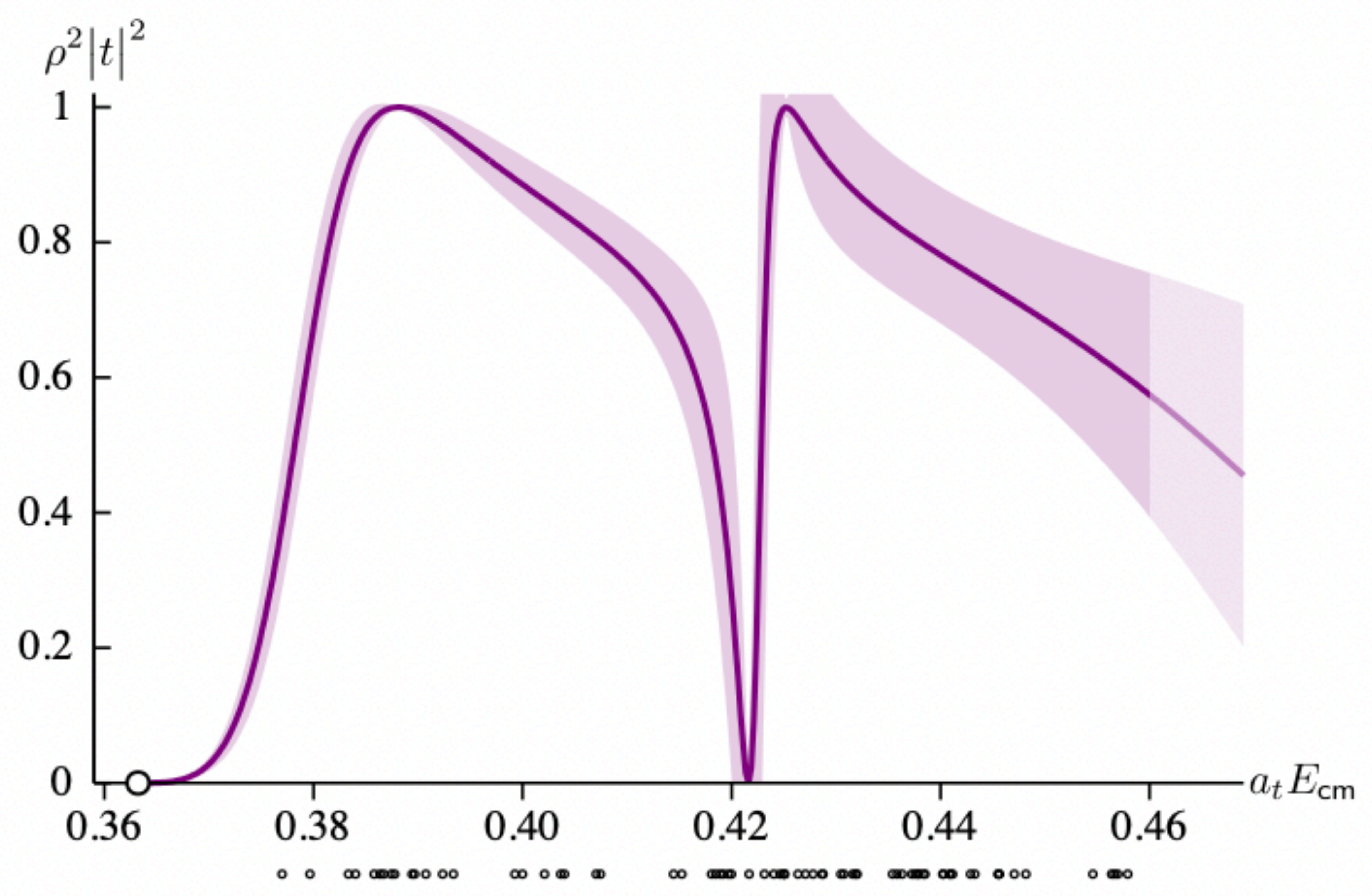


$\eta^8\omega^8$ elastic scattering in 1^{--}

$$K_{J=1} = \frac{g_a^2}{m_a^2 - s} + \frac{g_b^2}{m_b^2 - s} + \gamma$$

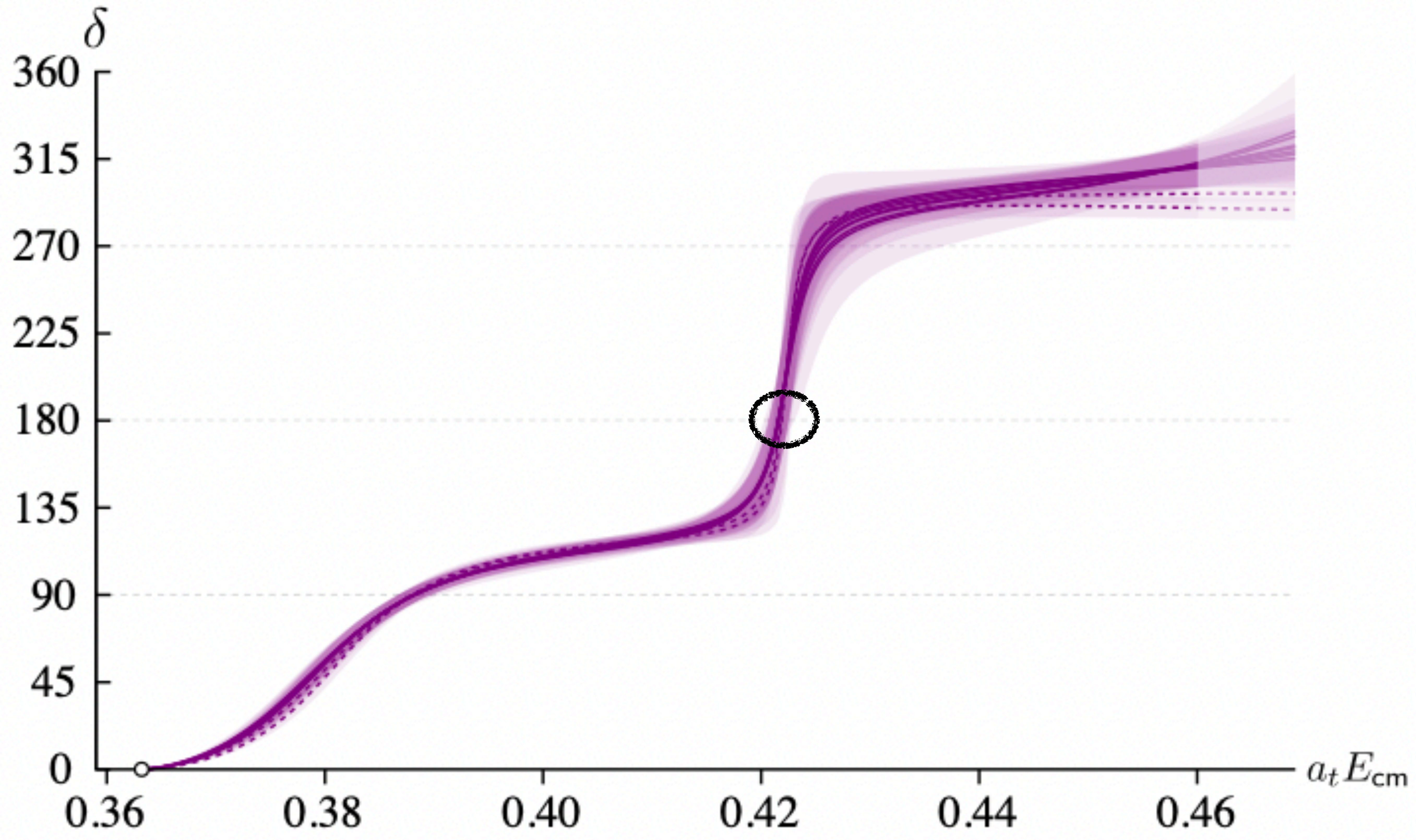
$$\begin{aligned} m_a &= 0.3881(14) \cdot a_t^{-1} \\ g_a &= 1.46(10) \\ m_b &= 0.4242(17) \cdot a_t^{-1} \\ g_b &= -0.36(13) \\ \gamma &= 20.9(86) \cdot a_t^2 \end{aligned} \quad \begin{bmatrix} 1 & 0.08 & 0.43 & -0.33 & 0.19 \\ & 1 & 0.37 & -0.46 & 0.81 \\ & & 1 & -0.86 & 0.49 \\ & & & 1 & -0.57 \\ & & & & 1 \end{bmatrix}$$

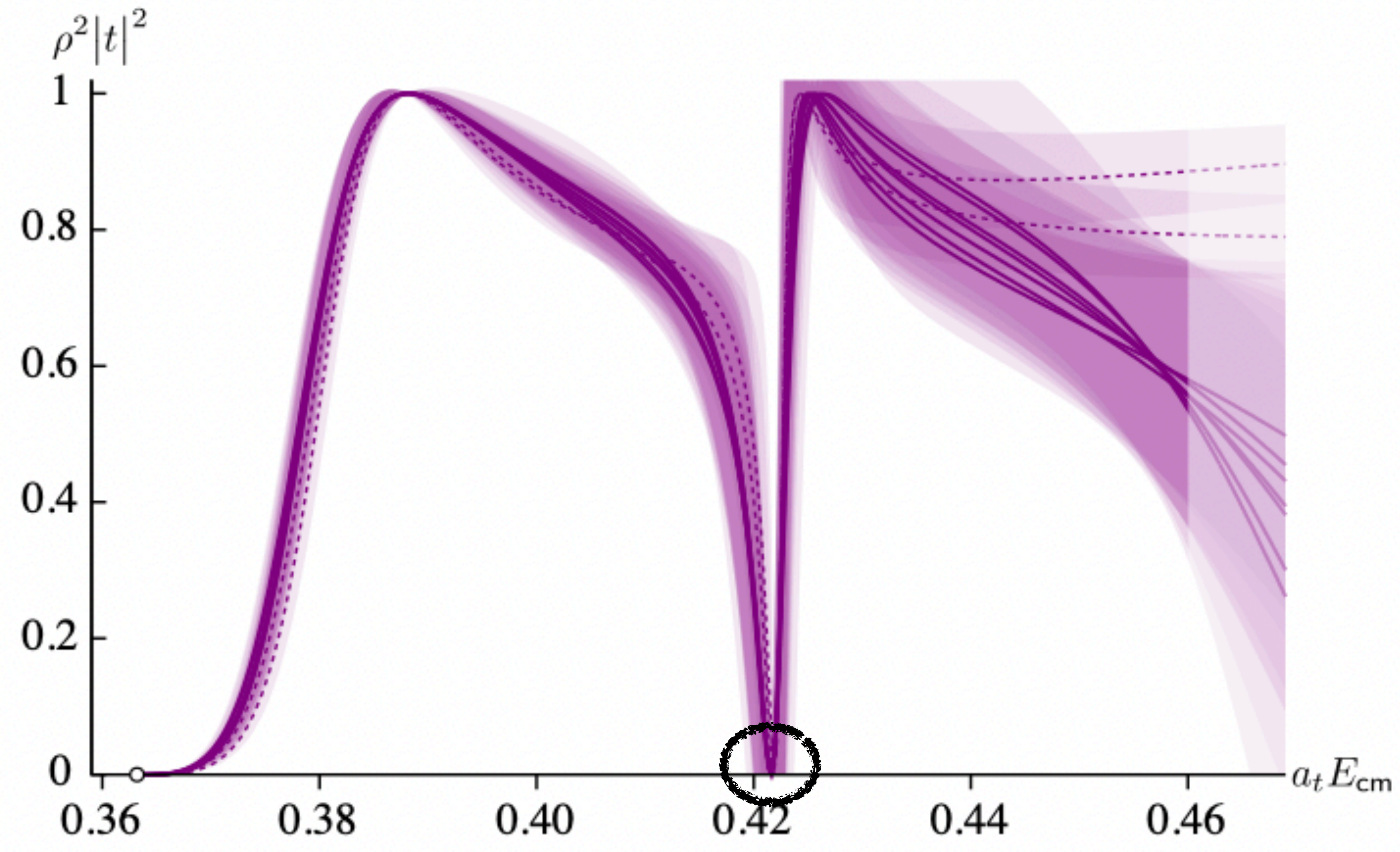
$$\chi^2/N_{\mathrm{dof}} = \frac{91.3}{72-5} = 1.36$$



$\eta^8\omega^8$ elastic scattering in 1^{--}

Elasticity





Zero is a feature of elastic unitarity

$$t = \frac{1}{\rho(\cot\delta - i)}$$

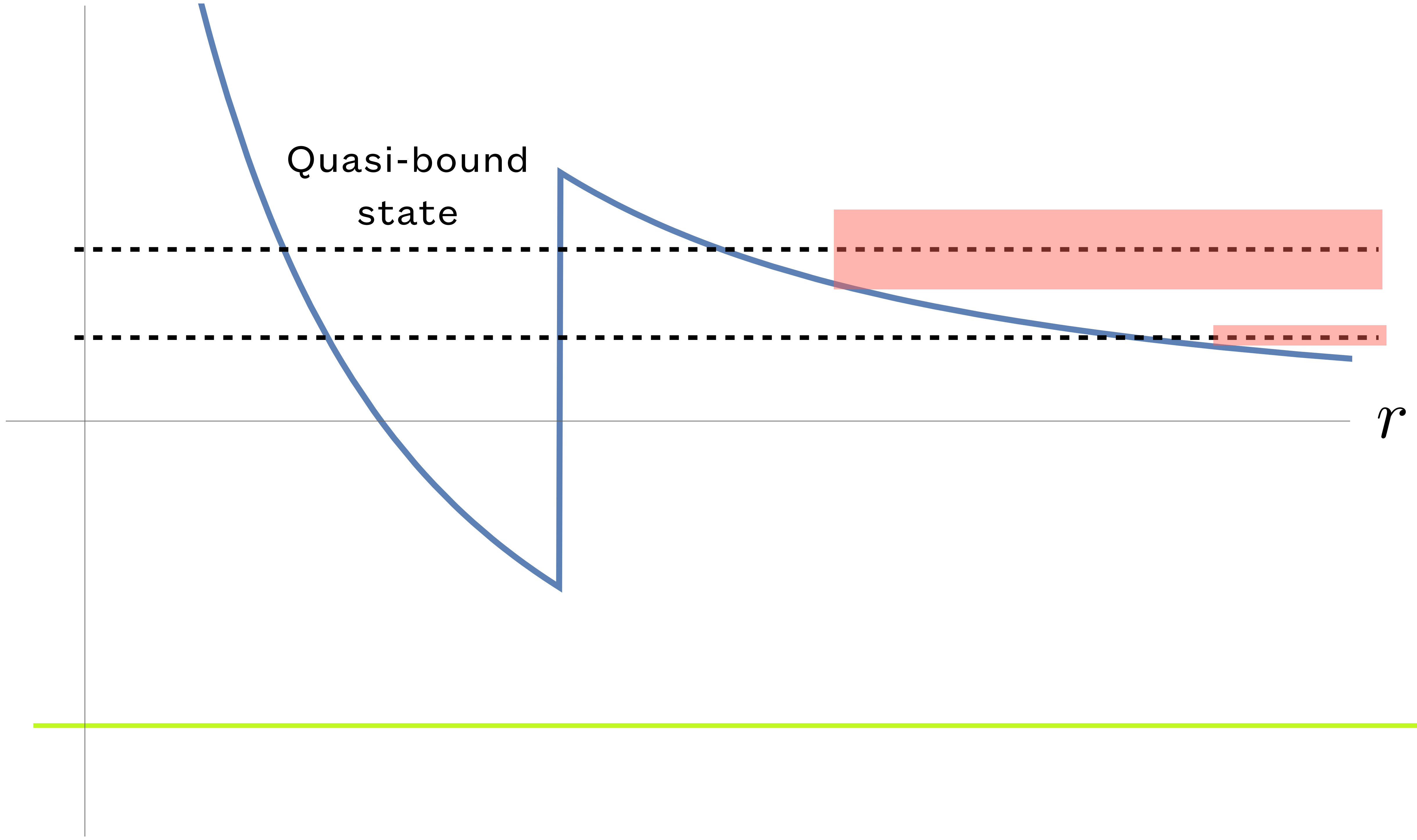
Cannot generate with an effective range

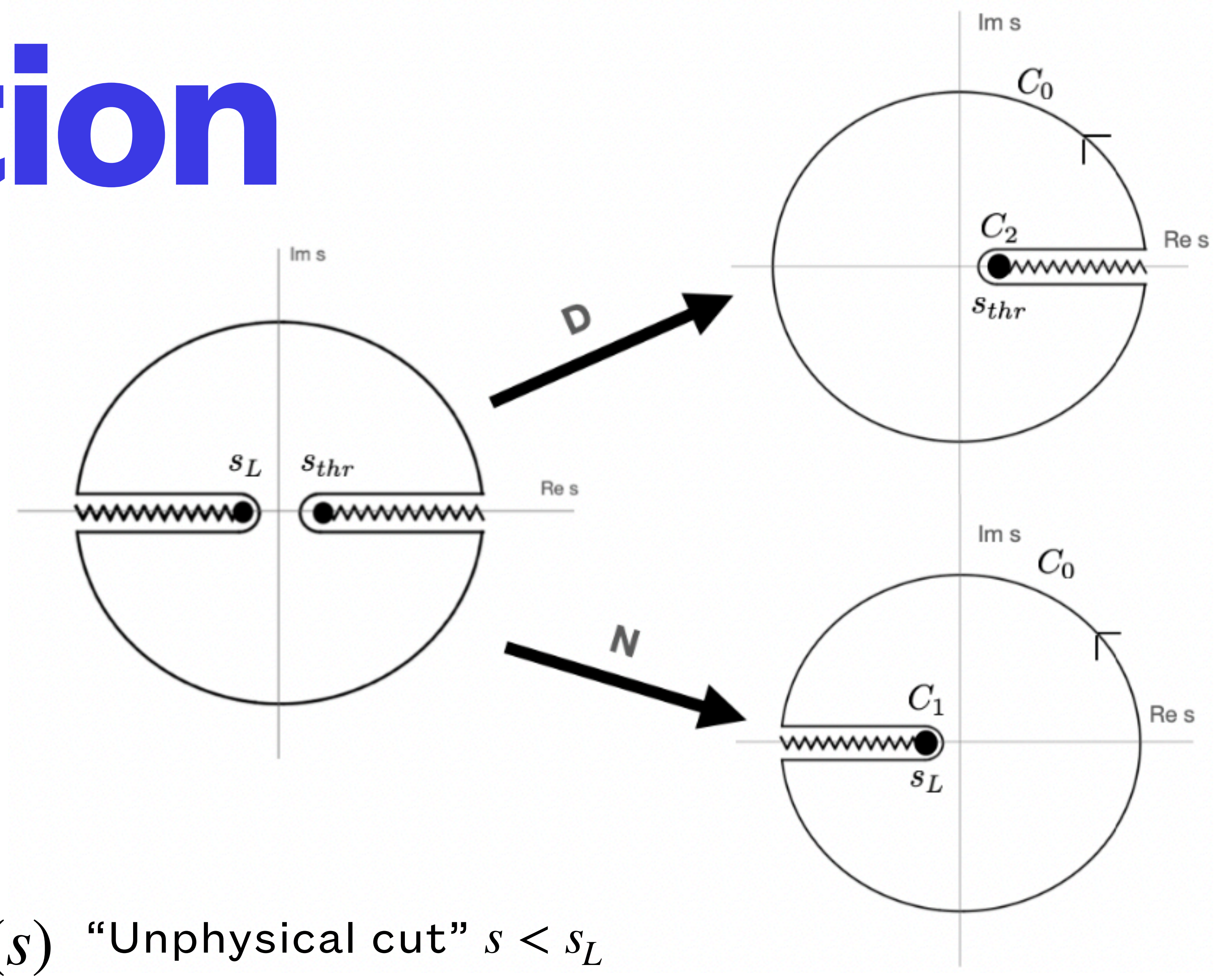
$$k^3 \cot\delta = \frac{1}{a} + \frac{1}{2} r k^2 + \dots$$

Resonance interpretation

In N.R. scattering, the scattering amplitude is completely determined by the potential.

$$V_{eff}(r) = V(r) + \frac{\ell(\ell+1)}{r^2}$$



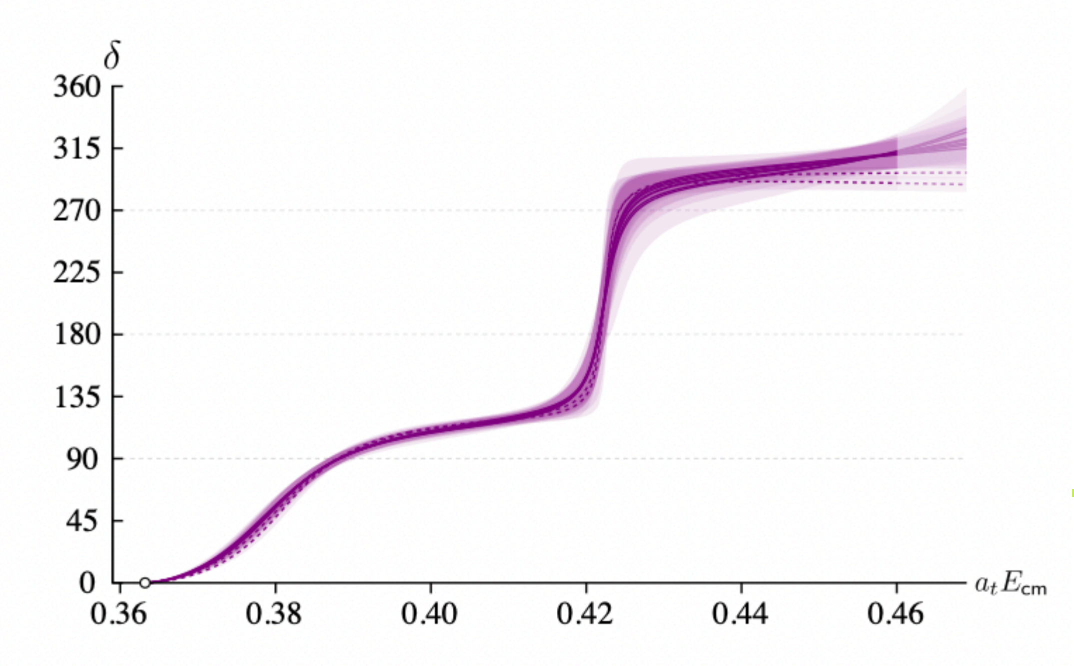


$$t(s) = \frac{N(s)}{D(s)}$$ “Unphysical cut” $s < s_L$

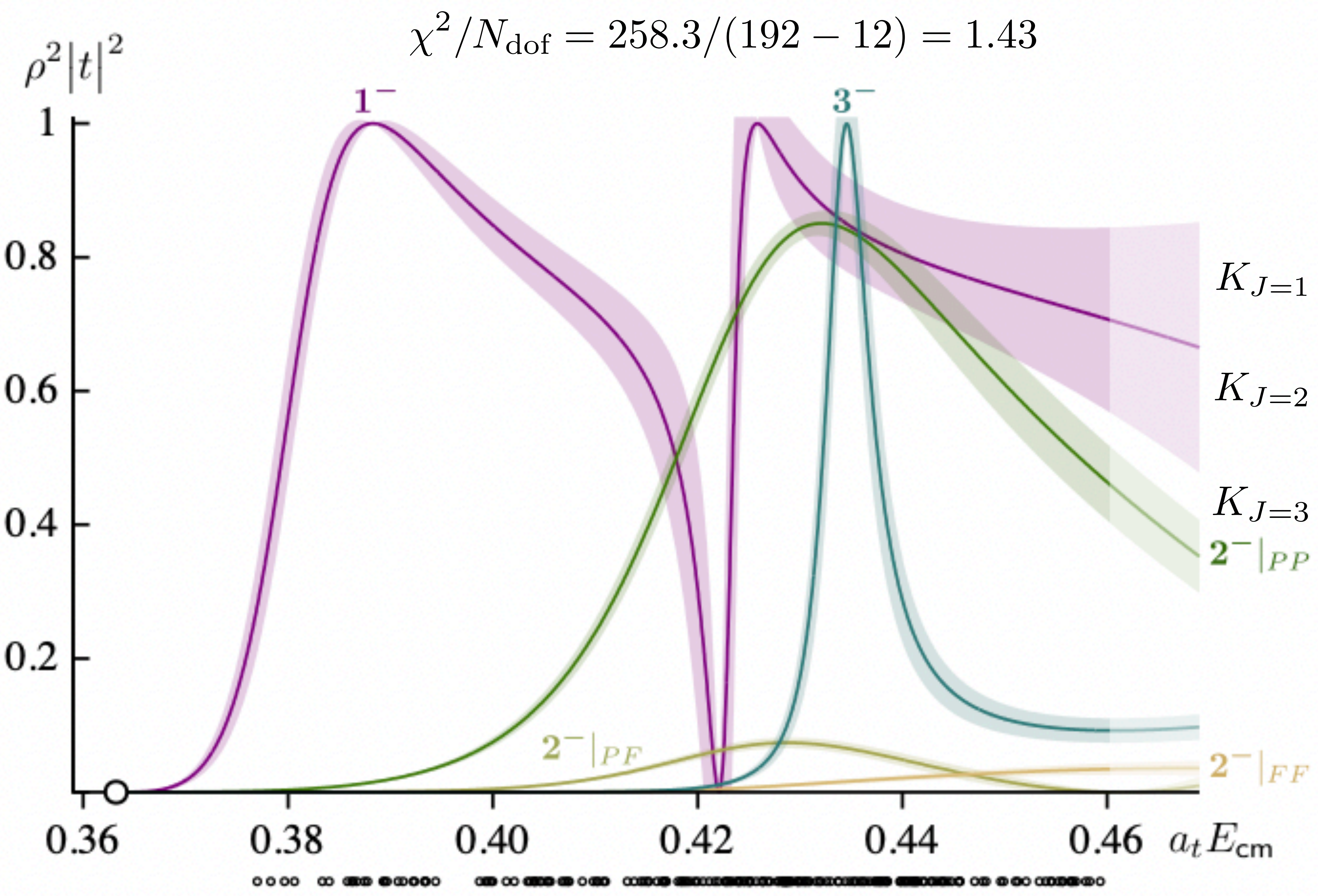
“Physical cut” $s > s_{thr}$

$$D(s) = D(s_0) - \frac{s - s_0}{\pi} \int_{s_{thr}}^{\infty} \frac{N(s')\rho(s')}{(s'-s)(s'-s_0)} ds' + \sum_{\alpha} \frac{g_\alpha^2}{m_\alpha^2 - s}$$

Can add poles to D(s) that produce zeros in t(s)



$1^{--}, 2^{--}, 3^{--}$



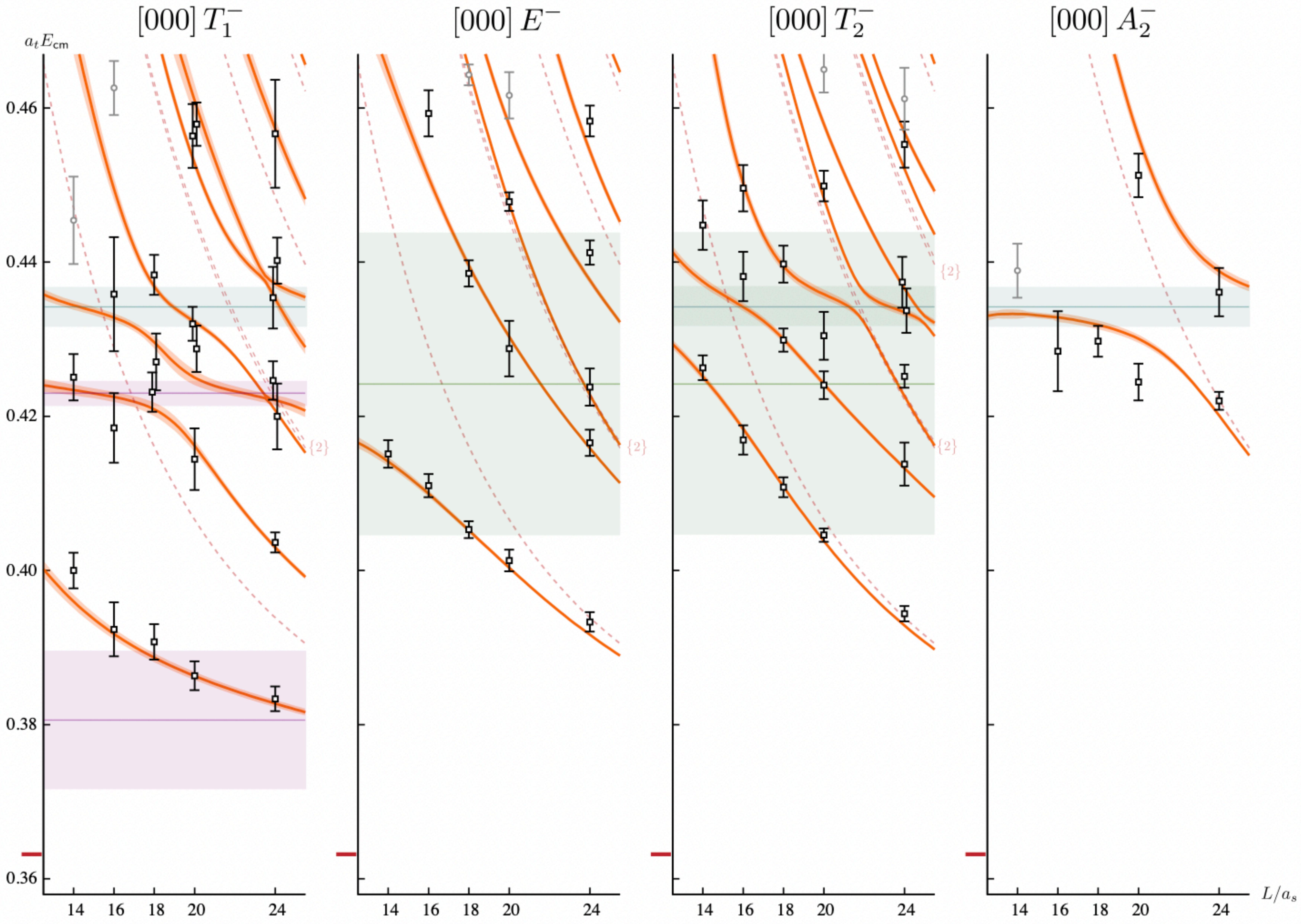
Add the $[011]A_1$ irreps and fit all simultaneously

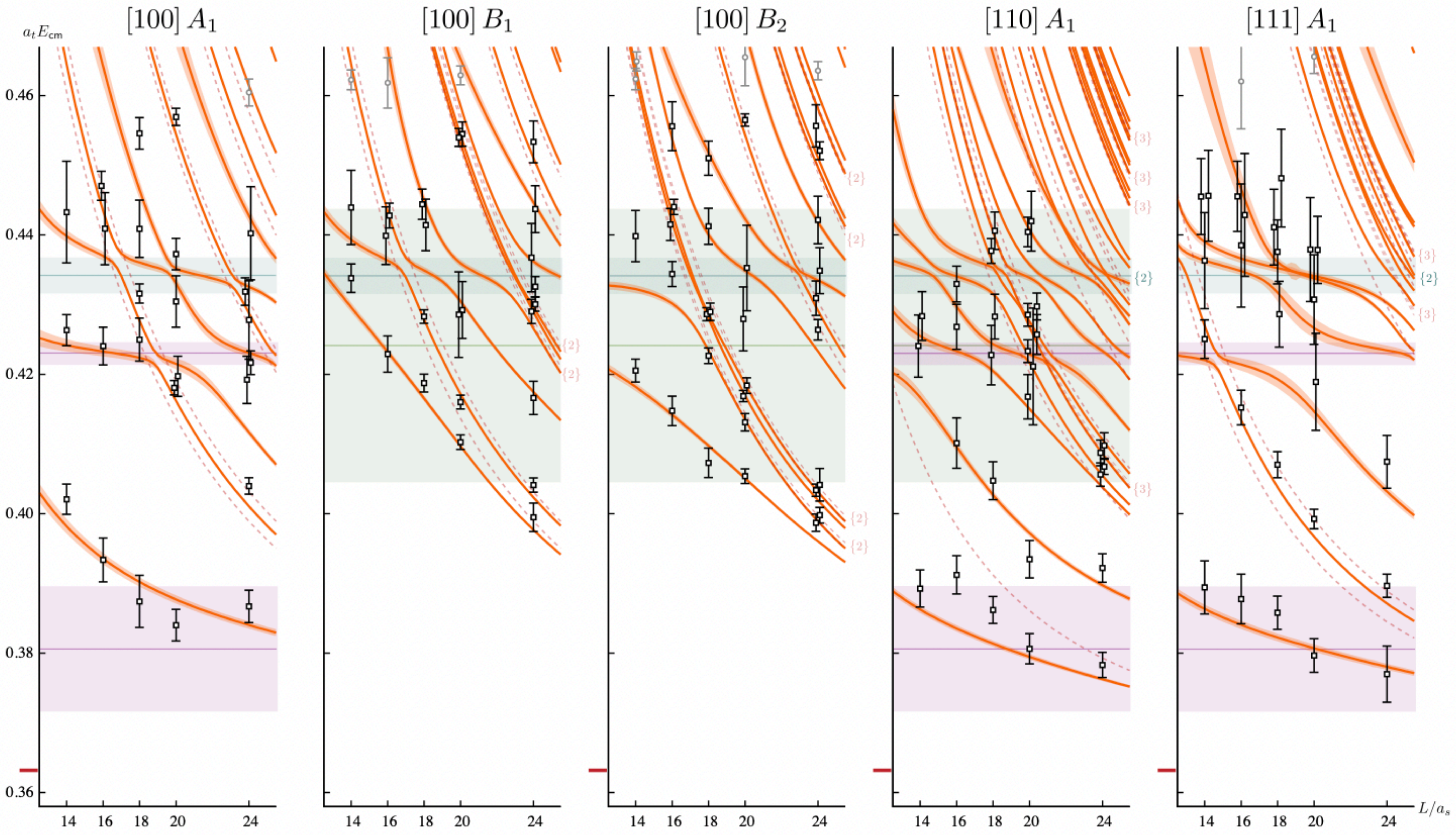
Very good constraint $N_{\mathrm{dof}} = 180$

$$K_{J=1} = \frac{g_{\mathrm{a}}^2}{m_{\mathrm{a}}^2 - s} + \frac{g_{\mathrm{b}}^2}{m_{\mathrm{b}}^2 - s} + \gamma$$

$$K_{J=2} = \frac{1}{m_R^2 - s}\begin{bmatrix} g_P^2 & g_P g_F \\ g_P g_F & g_F^2 \end{bmatrix} + \begin{bmatrix} \gamma_{PP} & \gamma_{PF} \\ \gamma_{PF} & 0 \end{bmatrix}$$

$$K_{J=3} = \frac{g_F^2}{m_R^2 - s}$$


[000] T_1^-
[000] E^-
[000] T_2^-
[000] A_2^-
$a_t E_{cm}$
0.46
0.44
0.42
0.40
0.38
0.36
14 16 18 20 24
L/a_s
{2}



$[100]\ A_1$
$[100]\ B_1$
$[100]\ B_2$
$[110]\ A_1$
$[111]\ A_1$
$a_t E_{\mathsf{cm}}$
0.46
0.44
0.42
0.40
0.38
0.36
14
16
18
20
24
L/a_s
{2}
{3}

A crude extrapolation

$$\omega = \sqrt{\frac{2}{3}}\omega_1 + \sqrt{\frac{1}{3}}\omega_8 \,;\, \phi = \sqrt{\frac{1}{3}}\omega_1 - \sqrt{\frac{2}{3}}\omega_8$$

Assume an exact OZI symmetry to get the couplings to the octet

Assume width scales with the angular momentum $\sim k^\ell$

Octet calculation is underway

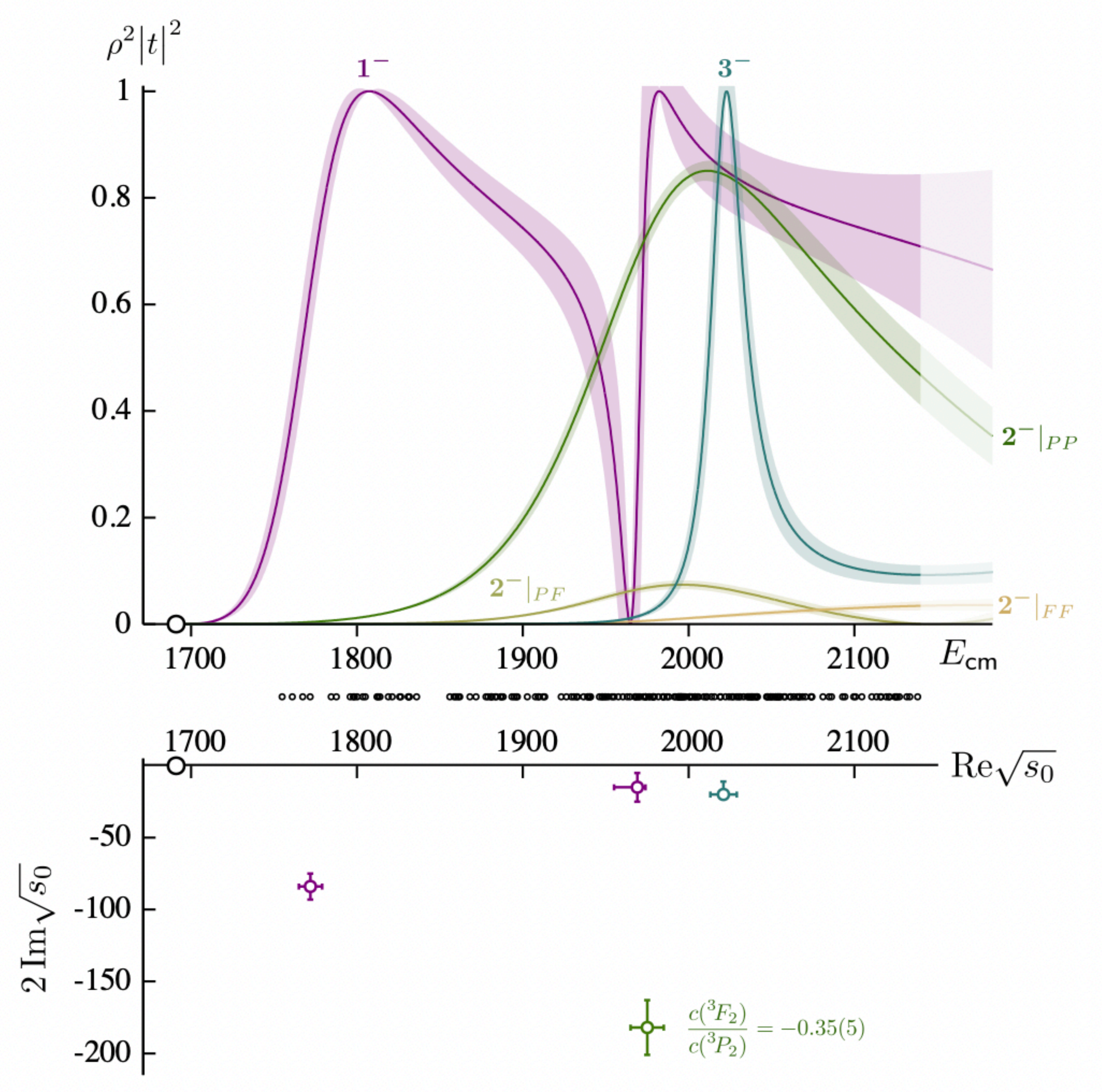
Calculation	PDG
$\Gamma^{\pi\rho}_{\omega_3} \sim 62$ MeV	$\Gamma^{tot}_{\omega_3(1670)} \sim 168(10)$ MeV
$\Gamma^{K\bar{K}^*}_{\omega_3} \sim 2$ MeV	
$\Gamma^{\eta\omega}_{\omega_3} \sim 1$ MeV	
$\Gamma^{K\bar{K}^*}_{\phi_3} \sim 20$ MeV	$\Gamma^{tot}_{\phi_3(1850)} \sim 87(25)$ MeV
$\Gamma^{\eta\phi}_{\phi_3} \sim 3$ MeV	
$\Gamma^{\pi\omega}_{\rho_3} \sim 22$ MeV	$\Gamma^{\pi\omega}_{\rho_3(1690)} \sim 30(10)$ MeV
$\Gamma^{K\bar{K}^*}_{\rho_3} \sim 2$ MeV	$\Gamma^{K\bar{K}\pi}_{\rho_3(1690)} \sim 7$ MeV

Calculation	PDG
$\Gamma^{\pi\rho}_{\omega_a} \sim 384$ MeV	$\Gamma^{\pi\rho}_{\omega(1420)} \sim 240$ MeV
$\Gamma^{K\bar{K}^*}_{\omega_a} \sim 4$ MeV	$\Gamma^{tot}_{\omega(1420)} \sim 290(120)$ MeV
$\Gamma^{\eta\omega}_{\omega_a} \sim 5$ MeV	
$\Gamma^{K\bar{K}^*}_{\phi_a} \sim 154$ MeV	$\Gamma^{tot}_{\phi(1680)} \sim 150(50)$ MeV
$\Gamma^{\eta\omega}_{\phi_a} \sim 25$ MeV	
$\Gamma^{\pi\omega}_{\rho_a} \sim 133$ MeV	$\Gamma^{\pi\omega}_{\rho(1450)} \sim$ 52-78 MeV
$\Gamma^{K\bar{K}^*}_{\rho_a} \sim 9$ MeV	$\Gamma^{tot}_{\rho(1450)} \sim 400(60)$ MeV

Calculation	PDG
$\Gamma^{\pi\rho}_{\omega_b} \sim 25$ MeV	$\Gamma^{\pi\rho}_{\omega(1650)} \sim 84$ MeV
$\Gamma^{K\bar{K}^*}_{\omega_b} \sim 3$ MeV	$\Gamma^{tot}_{\omega(1650)} \sim 315(35)$ MeV
$\Gamma^{\eta\omega}_{\omega_b} \sim 1$ MeV	
$\Gamma^{\pi\omega}_{\rho_b} \sim 9$ MeV	$\Gamma^{\pi\omega}_{\rho(1700)} \sim 0$ MeV
$\Gamma^{K\bar{K}^*}_{\rho_b} \sim 3$ MeV	$\Gamma^{tot}_{\rho(1700)} \sim 250(100)$ MeV

A. B. Clegg and A. Donnachie, Z. Phys. C 62, 455 (1994).

Summary



We extract 4 resonances consistent with the quark model prediction.

1^{--}: broader lighter first resonance and heavier narrower second resonance

2^{--}: broad resonance coupled mostly to P-wave

3^{--}: narrow F-wave resonance

Finite volume formalism can handle multiple resonances in same partial wave and nearly degenerate resonances in same irrep.

Thanks

Future

Calculation of the octet is underway:

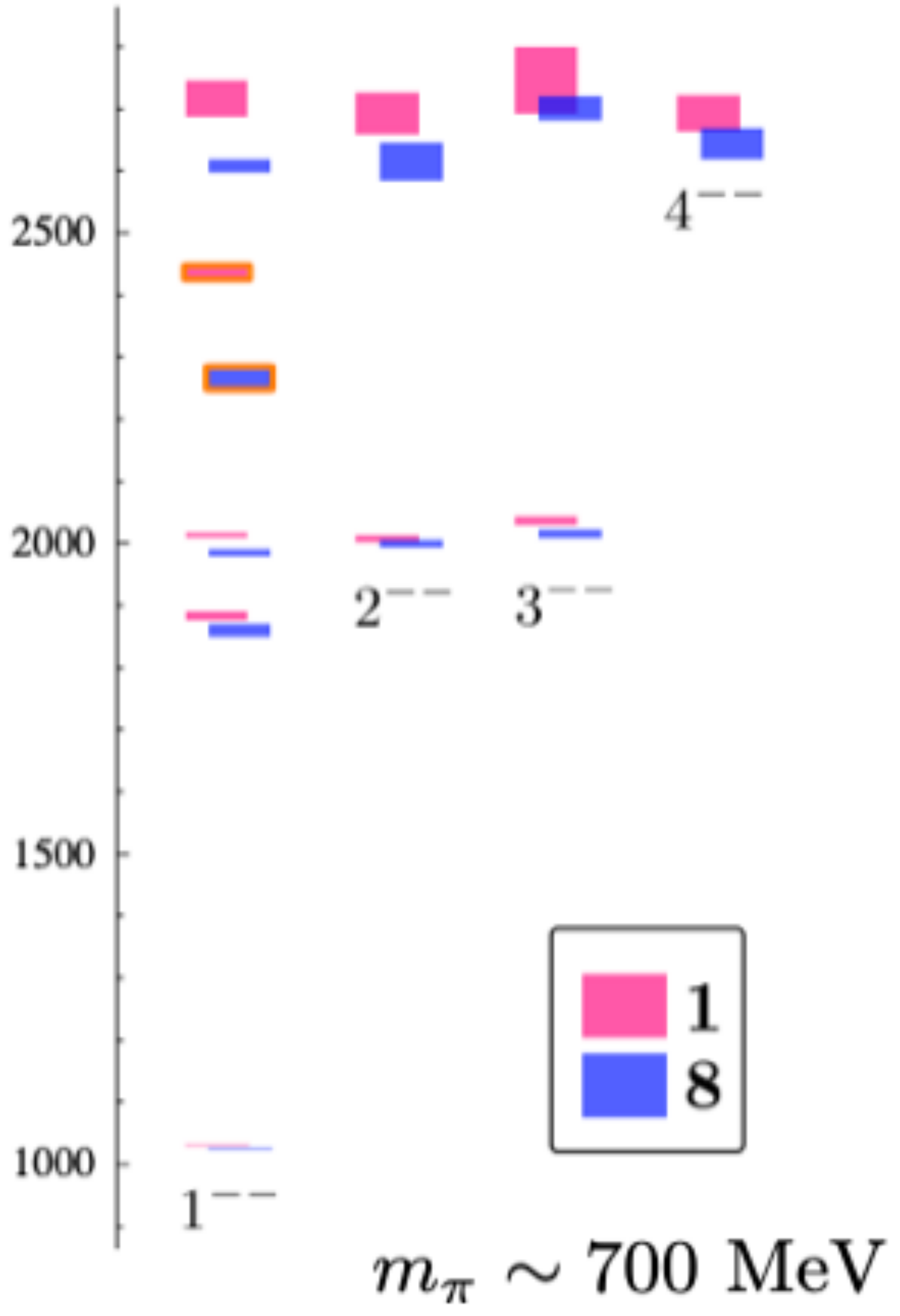
⇒ more channels

⇒ identical particles $\eta^8\eta^8, \omega^8\omega^8$

⇒ nearly degenerate thresholds in $\eta^8\omega^8, \eta^8\omega^1$

Would like to be able to study the hybrid candidate that lies slightly above in 1^{--}

⇒ likely requires three-particle formalism



Extra

Lattice QCD

Optimized operator constructed from applying the eigenvectors extracted from applying the variational method $h^\dagger = \sum_i v_i O_i$

Finite volume spectrum ⇒ $C_{ij}(t) = \sum_\alpha \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$

Single meson operators: $\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi} \overleftrightarrow{D} \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$

Momentum is quantized $\vec{p} = \frac{2\pi}{L}\vec{n}$

Meson-meson operators: $\sum_{\vec{p}_1 + \vec{p}_2 = \vec{P}} C(\vec{p}_1, \vec{p}_2; \vec{P}) h_1^\dagger(\vec{p}_1) h_2^\dagger(\vec{p}_2)$

No interactions

$$E = \sqrt{m_1^2 + \left(\frac{2\pi\vec{n}_1}{L}\right)^2} + \sqrt{m_2^2 + \left(\frac{2\pi\vec{n}_2}{L}\right)^2}$$

Variational Method

Diagonalize matrix of correlation functions to produce the finite volume spectrum:

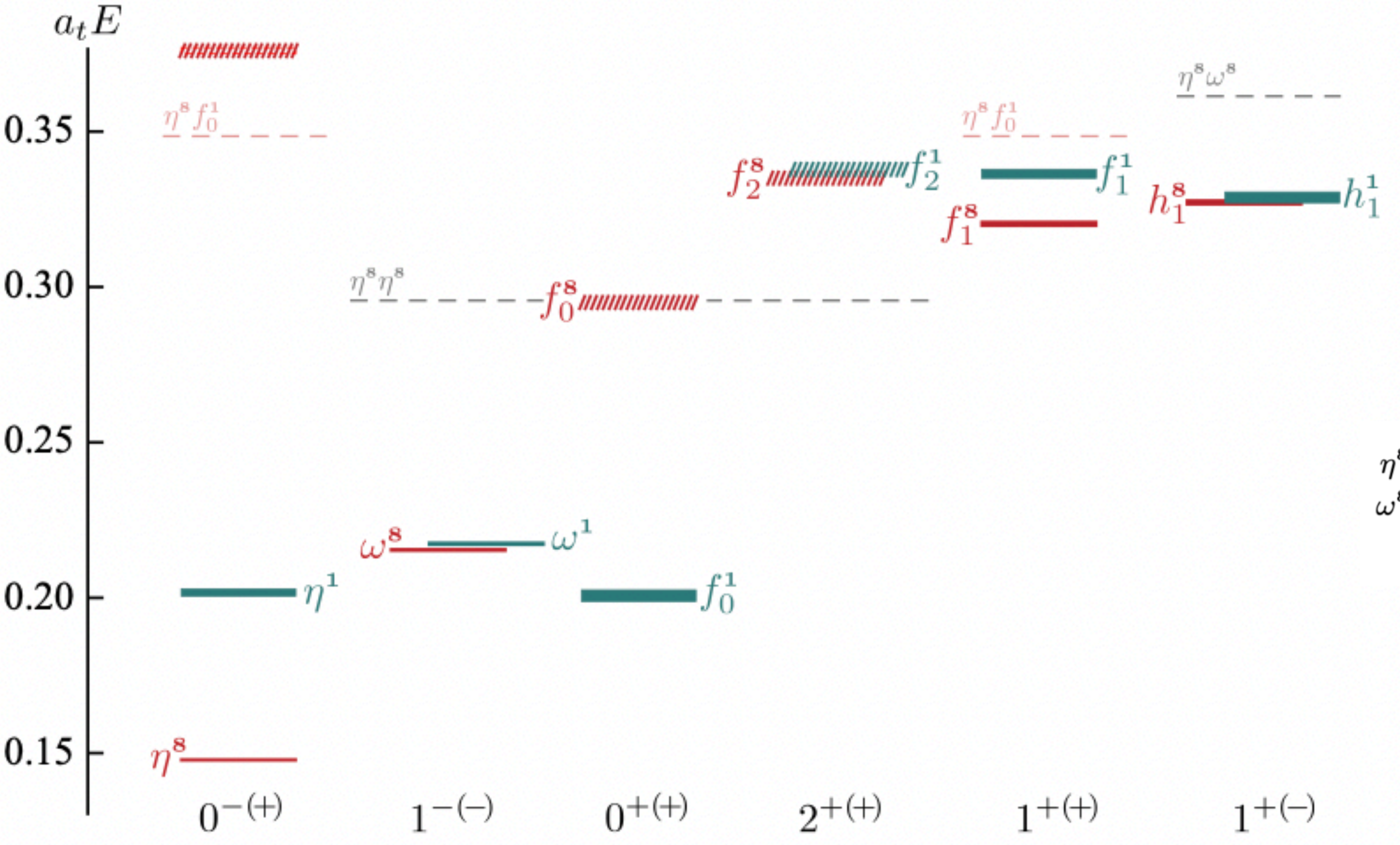
$$C(t)v^{\alpha}(t) = \lambda^{\alpha}(t)C(t_0)v^{\alpha}(t)$$

$$\sim e^{-E^{\alpha}(t-t_0)}$$

$$\langle 0|O_i|\alpha\rangle = (V_i^{\alpha})^{-1}\sqrt{2E^{\alpha}}e^{E^{\alpha}t_0/2}$$

Use the orthonormality of the eigenvectors to distinguish states, and extract energies from the principal correlators $\lambda^{\alpha}(t)$.

SU(3) Flavor



η^8	0.1478(1)	η^1	0.2017(11)
ω^8	0.2154(2)	ω^1	0.2174(3)
		f_0^1	0.2007(18)

Lattice QCD

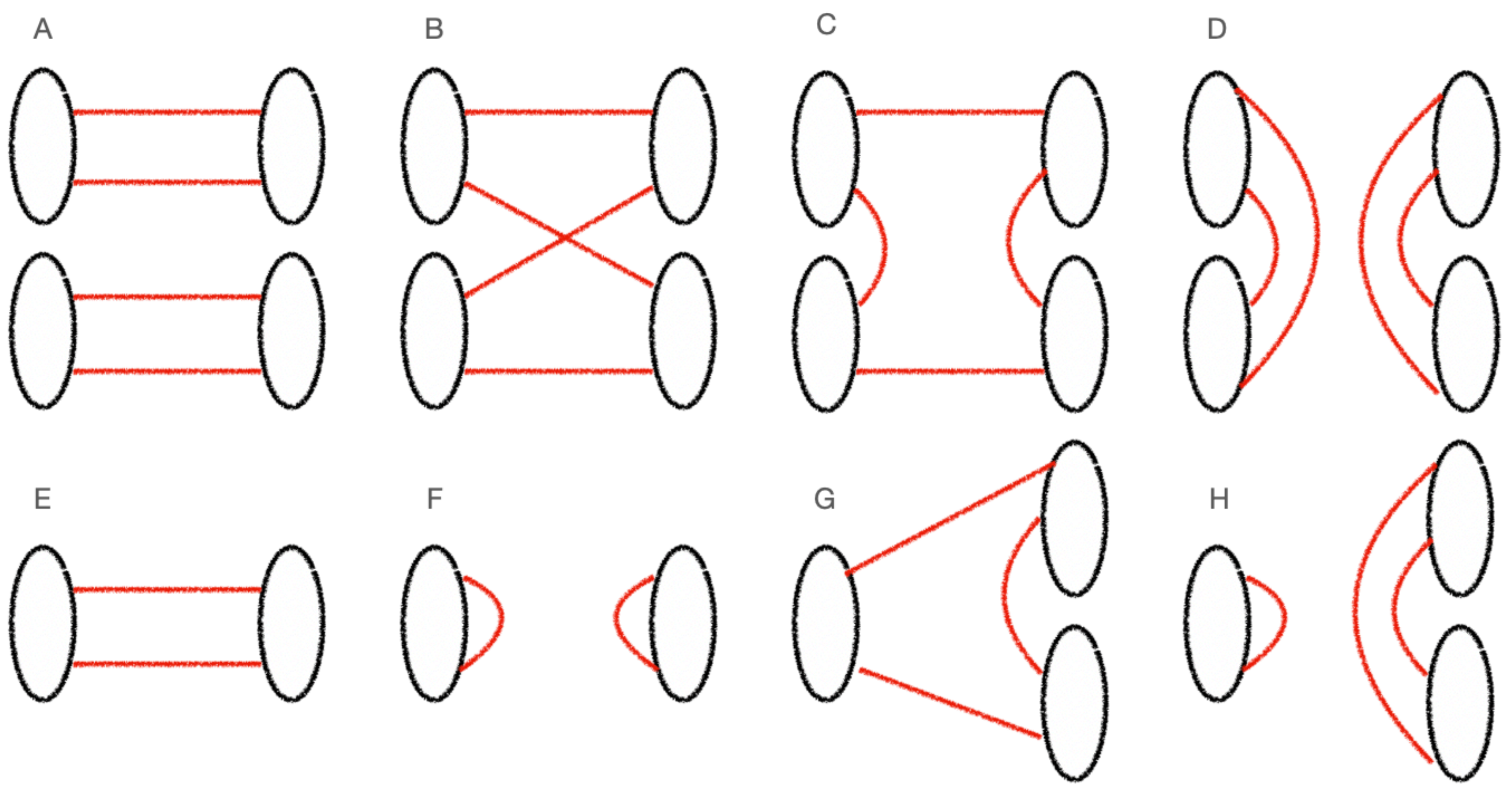
Finite volume spectrum $\Rightarrow C_{ij}(t) = \sum_{\alpha} \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$

Single meson operators: $\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi} \overleftrightarrow{D} \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$

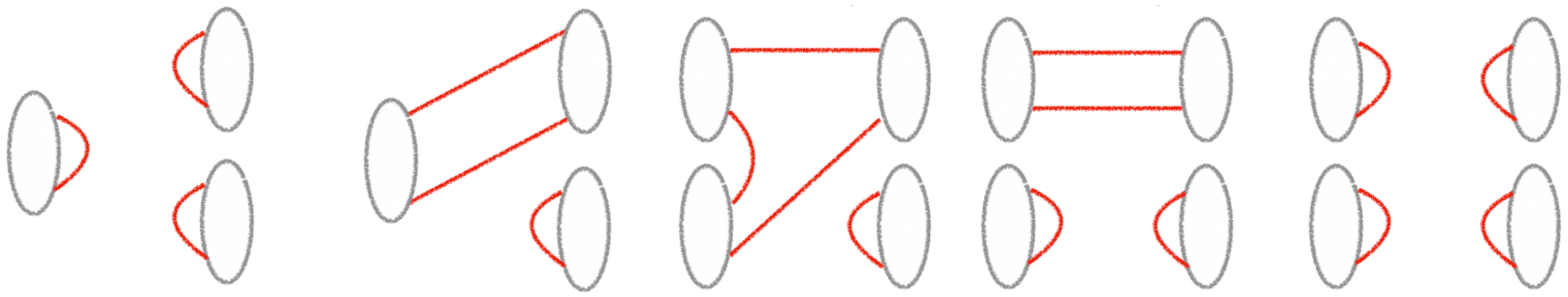
Meson-meson operators: $\sum_{\vec{p}_1 + \vec{p}_2 = \vec{P}} C(\vec{p}_1, \vec{p}_2; \vec{P}) h_1^\dagger(\vec{p}_1) h_2^\dagger(\vec{p}_2)$

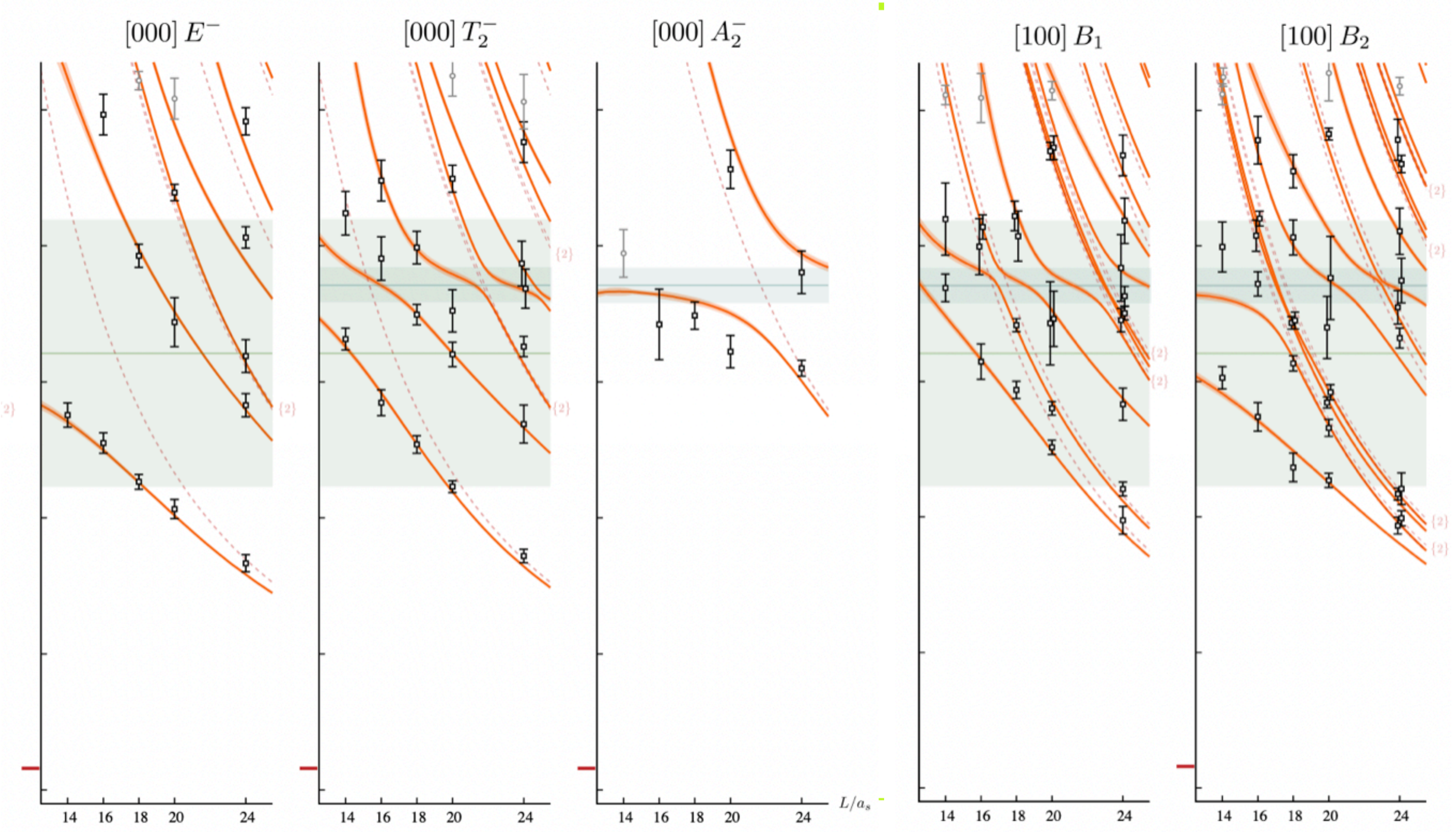
Will include $\eta^8(\vec{p_1})\omega^8(\vec{p_2})$, $\eta^1(\vec{p_1})\omega^1(\vec{p_2})$, $f_0^1(\vec{p_1})\omega^8(\vec{p_2})$

$\eta^8\omega^8$



$\eta^1\omega^1 / f_0^1\omega^1$




[000] E^-
[000] T_2^-
[000] A_2^-
[100] B_1
[100] B_2
{2}
14
16
18
20
24
L/a_s

Comparing to the ω_J^*, ϕ_J^*

Vector states are mixtures of the singlet and octet states

$$\omega = \sqrt{\frac{2}{3}}\omega_1 + \sqrt{\frac{1}{3}}\omega_8\,;\, \phi = \sqrt{\frac{1}{3}}\omega_1 - \sqrt{\frac{2}{3}}\omega_8$$

Pseudoscalar states have little mixing from SU(3) eigenstates $\eta \sim \eta_8, \eta' \sim \eta_1$

If we assume excited J^{--} have the same quark content as the vector states, we need to know the result of the octet couplings to find the partial width of the isoscalar resonances to pseudoscalar-vector final states.

We can still guess what the result of the octet calculation would be by assuming an exact OZI symmetry.

Comparing to the ω_J^*, ϕ_J^*

We first re-write the couplings in the basis of familiar meson states:

$$|\eta^8 \otimes \omega^8 \to \mathbf{1}\rangle = \frac{1}{2\sqrt{2}} \left(K^+\bar{K}^{*-} + K^-\bar{K}^* - K^0\bar{K}^{*0} - \bar{K}^0K^{*0} + \pi^+\rho^- + \pi^-\rho^+ - \pi^0\rho^0 - \eta_8\omega_8\right) : g^1$$

$$|\eta^8 \otimes \omega^8 \to \mathbf{8}\rangle = \sqrt{\frac{1}{20}} \left(K^+K^{*-} + K^-\bar{K}^* - K^0\bar{K}^{*0} - \bar{K}^0K^{*0}\right) - \sqrt{\frac{1}{5}} \left(\pi^+\rho^- + \pi^-\rho^+ - \pi^0\rho^0 - \eta_8\omega_8\right) : g^8$$

$$|\eta^8 \otimes \omega^1 \to \mathbf{8}\rangle = \eta_8\omega_1 = \sqrt{\frac{2}{3}}\eta\omega + \sqrt{\frac{1}{3}}\eta\phi : h^8$$

OZI disallowed decays:

$$\phi^* \to \rho\pi \sim \sqrt{\frac{1}{3}}\frac{1}{2\sqrt{2}}g^1 + \left(-\sqrt{\frac{2}{3}}\right)\left(-\sqrt{\frac{1}{5}}\right)g^8$$

$$\phi^* \to \eta\omega \sim \sqrt{\frac{1}{3}}\left(-\frac{1}{2\sqrt{2}}\right)\sqrt{\frac{1}{3}}g^1 + \left(-\sqrt{\frac{2}{3}}\right)\left(-\sqrt{\frac{1}{5}}\right)\sqrt{\frac{1}{3}}g^8 + \left(-\sqrt{\frac{2}{3}}\right)\sqrt{\frac{2}{3}}h^8$$

Leads to the constraints:

$$g^8 = -\frac{\sqrt{5}}{4}g^1; h^8 = -\frac{1}{2\sqrt{2}}g^1$$

Comparing to the ω_J^*, ϕ_J^*

We write the partial widths as $\Gamma = g^2 \frac{\rho}{M}$

OZI relations together with a sum over the charged states give us the following partial widths:

$$\Gamma(\omega^\star \to \pi\rho) = 3\,\tfrac{\rho}{M}\,\tfrac{3}{16}(g^1)^2$$
$$\Gamma(\omega^\star \to K\overline{K}^*) = 4\,\tfrac{\rho}{M}\,\tfrac{3}{64}(g^1)^2$$
$$\Gamma(\omega^\star \to \eta\omega) = 1\,\tfrac{\rho}{M}\,\tfrac{1}{16}(g^1)^2$$

$$\Gamma(\phi^\star \to K\overline{K}^*) = 4\,\tfrac{\rho}{M}\,\tfrac{3}{32}(g^1)^2$$
$$\Gamma(\phi^\star \to \eta\phi) = 1\,\tfrac{\rho}{M}\,\tfrac{1}{4}(g^1)^2$$

$$\Gamma(\rho^\star \to \pi\omega) = 1\,\tfrac{\rho}{M}\,\tfrac{3}{16}(g^1)^2$$
$$\Gamma(\rho^\star \to K\overline{K}^*) = 2\,\tfrac{\rho}{M}\,\tfrac{3}{32}(g^1)^2\,,$$

We attempt to rescale the angular momentum barrier factors:

$$g^1 = \left|\frac{k^{phys}(M^{phys})}{k(M)}\right|^{\ell} |c_{\eta^8\omega^8}|$$

Comparing to the ω_J^*, ϕ_J^*

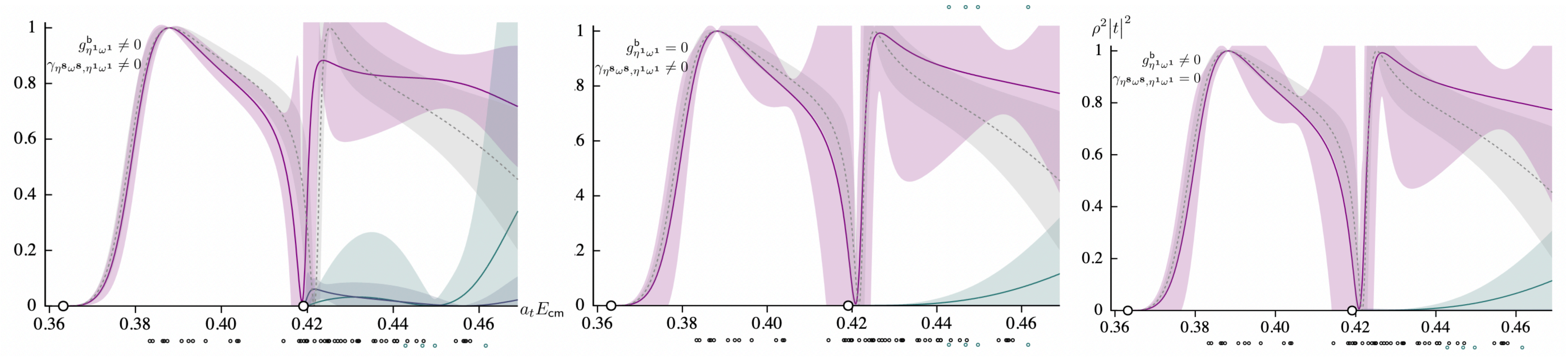
Prediction	Experiment
$\Gamma(\omega_3 \to \pi\rho) = 62$ MeV	$\Gamma^{tot}_{\omega_3(1670)} \sim 168(10)$ MeV
$\Gamma(\omega_3 \to K\bar{K}^*) = 2$ MeV	
$\Gamma(\omega_3 \to \eta\omega) = 1$ MeV	
$\Gamma(\phi_3 \to K\bar{K}^*) = 20$ MeV	$\Gamma^{tot}_{\phi_3(1850)} \sim 87(25)$ MeV
$\Gamma(\phi_3 \to \eta\phi) = 3$ MeV	
$\Gamma(\rho_3 \to \pi\omega) = 22$ MeV	$\Gamma^{\pi\omega}_{\rho_3} \sim 30(10)$ MeV
$\Gamma(\rho_3 \to K\bar{K}^*) = 2$ MeV	$\Gamma^{K\bar{K}\pi}_{\rho_3} \sim 7$ MeV

$$\Gamma(\rho_2 \to \pi\omega, K\overline{K}^*) = 125, 36\,\text{MeV}$$
$$\Gamma(\omega_2 \to \pi\rho, K\overline{K}^*, \eta\omega) = 365, 36, 17\,\text{MeV}$$
$$\Gamma(\phi_2 \to K\overline{K}^*, \eta\phi) = 148, 44\,\text{MeV}\,,$$

Comparing to the ω_J^*, ϕ_J^*

Prediction	Experiment	Prediction	Experiment
$\Gamma(\omega_b \to \pi\rho) = 25$ MeV	$\Gamma^{tot}_{\omega(1650)} \sim 315(35)$ MeV	$\Gamma(\omega_a \to \pi\rho) = 384$ MeV	$\Gamma^{\pi\rho}_{\omega(1420)} \sim 240$ MeV
$\Gamma(\omega_b \to K\bar{K}^*) = 3$ MeV	$\Gamma^{\pi\rho}_{\omega(1650)} \sim 84$ MeV	$\Gamma(\omega_a \to K\bar{K}^*) = 4$ MeV	$\Gamma^{tot}_{\omega(1420)} \sim 290(120)$ MeV
$\Gamma(\omega_b \to \eta\omega) = 1$ MeV		$\Gamma(\omega_a \to \eta\omega) = 5$ MeV	
$\Gamma(\phi_b \to K\bar{K}^*) = 13$ MeV		$\Gamma(\phi_a \to K\bar{K}^*) = 154$ MeV	$\Gamma^{tot}_{\phi(1680)} \sim 150(50)$ MeV
$\Gamma(\phi_b \to \eta\phi) = 5$ MeV		$\Gamma(\phi_a \to \eta\omega) = 25$ MeV	
$\Gamma(\rho_b \to \pi\omega) = 9$ MeV	$\Gamma^{\pi\omega}_{\rho(1700)} \sim 0$ MeV	$\Gamma(\rho_a \to \pi\omega) = 133$ MeV	$\Gamma^{tot}_{\rho(1450)} \sim 400(60)$ MeV
$\Gamma(\rho_b \to K\bar{K}^*) = 3$ MeV	$\Gamma^{tot}_{\rho(1700)} \sim 250(100)$ MeV	$\Gamma(\rho_a \to K\bar{K}^*) = 9$ MeV	$\Gamma^{\pi\omega}_{\rho(1450)} \sim 52 - 78$ MeV

Coupled-Channel $\eta^8\omega^8 - \eta^1\omega^1$



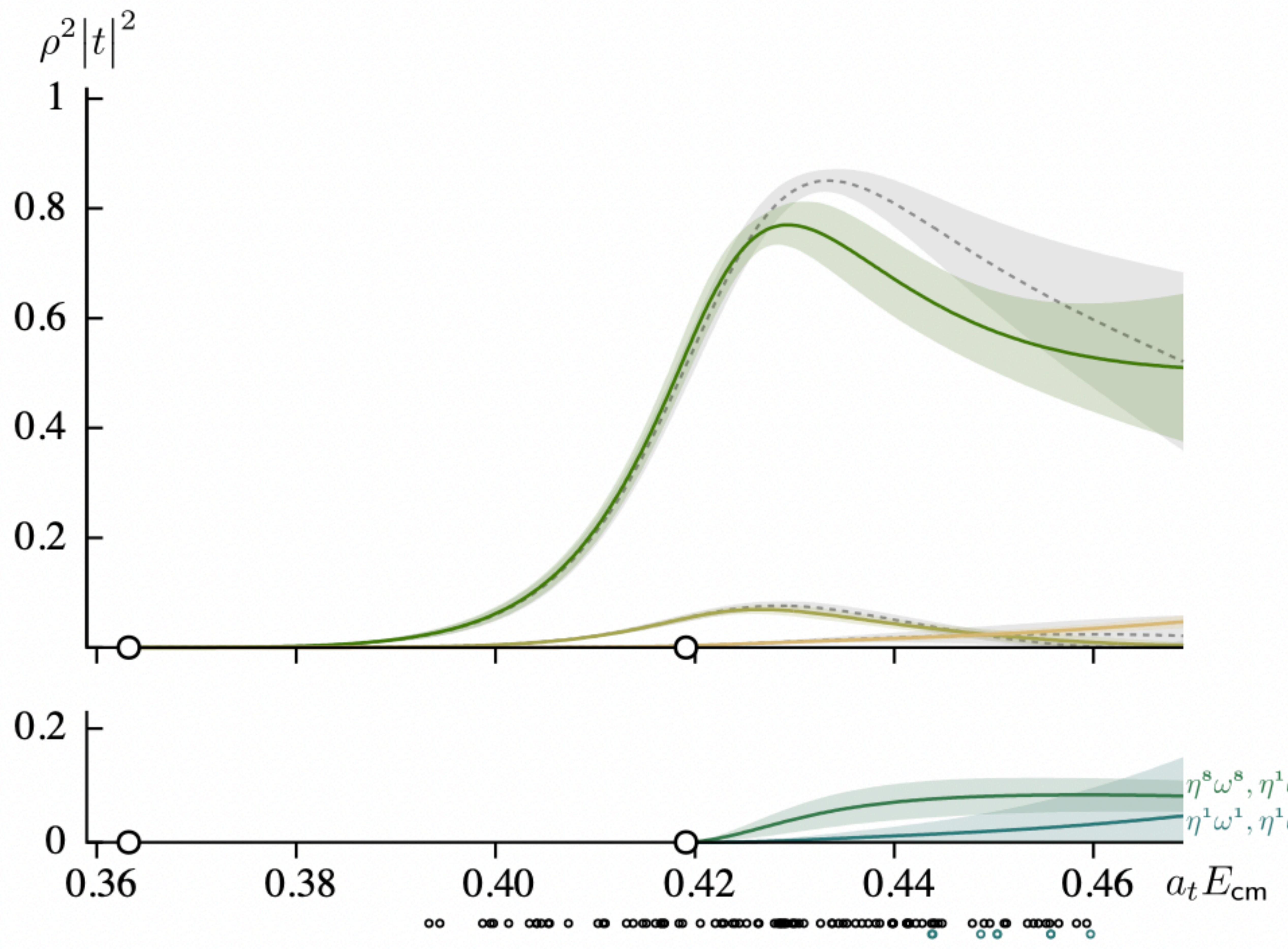
Only 4 levels with large $\eta^1\omega^1$ overlap.

Only real difference in fit-1 which features two $\eta^1\omega^1$ parameters.

Potentially a small coupling $c_{\eta^1\omega^1} \lesssim 0.04$ does not change overall width.

Statistical uncertainties on $f_0^1\omega^1$ energy levels prevent a proper C.C. analysis with this channel.

C.C. $2^{--}, \eta^8\omega^8 - \eta^1\omega^1$



Mild changes in the amplitude.

$a_t|c_{\eta^1\omega^1}| \sim 0.07(2)$ is small and comparable to F-wave coupling.

Additional singularities

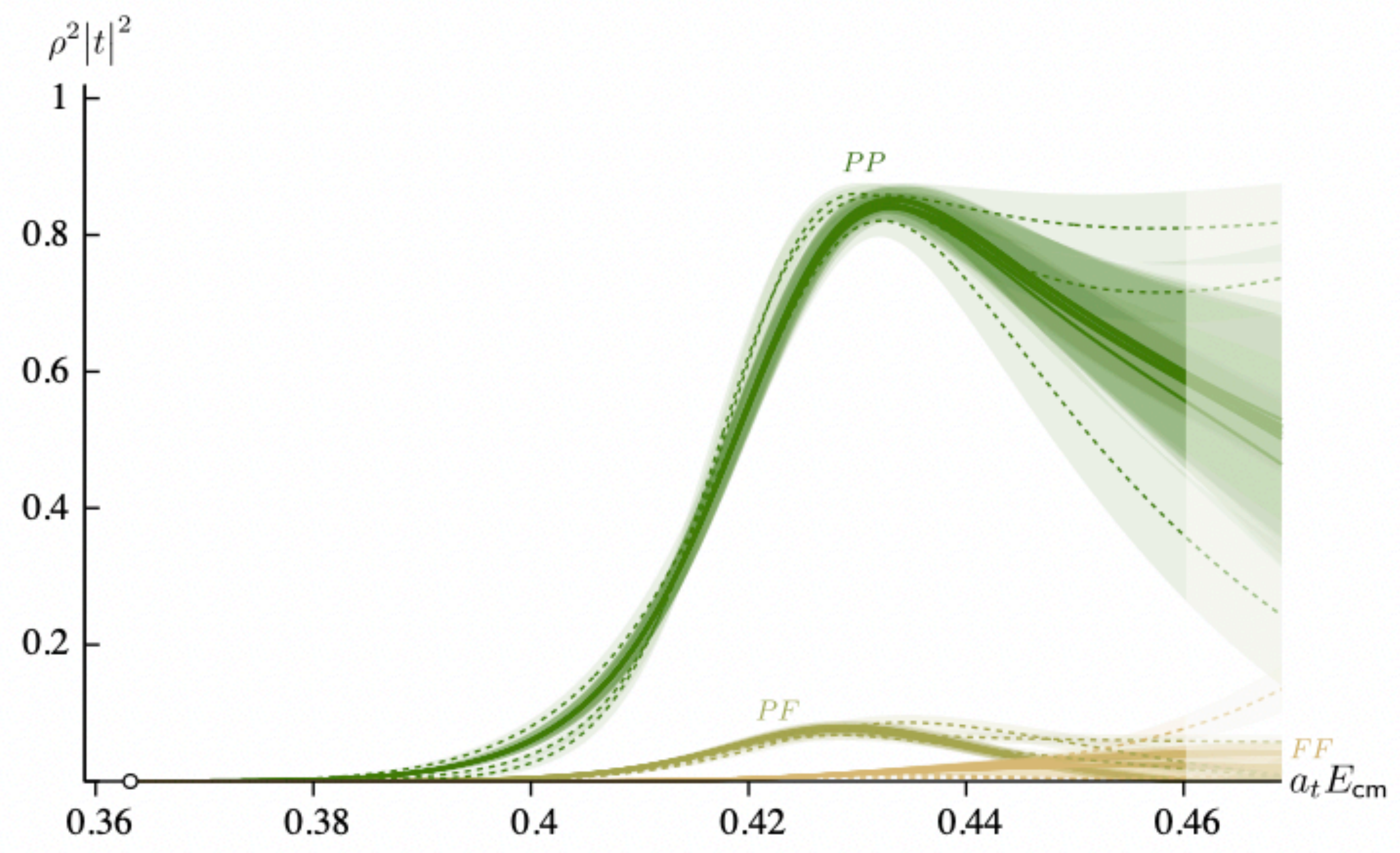
Unphysical sheet real axis pole $a_t\sqrt{s} \sim 0.23$ on many parameterizations

⇒ wanders a bit and remains far from physical scattering

Additional real axis pole $a_t\sqrt{s} \sim 0.24$ for simple phase space parameterization

⇒ not surprising this parameterization has poorer analytic properties

⇒ residue is real, a true p-wave bound state has imaginary coupling



Amplitude analytic structure

The full scattering amplitude T(s,t) relates all scattering channels s,t,u- through an analytic continuation.

s-channel unitarity constrains the “right hand cut” to form $2^{N_{chan}}$ Riemann sheets

⇒ built into our parameterizations

Analyticity requires poles off axis real valued poles be on unphysical sheets.

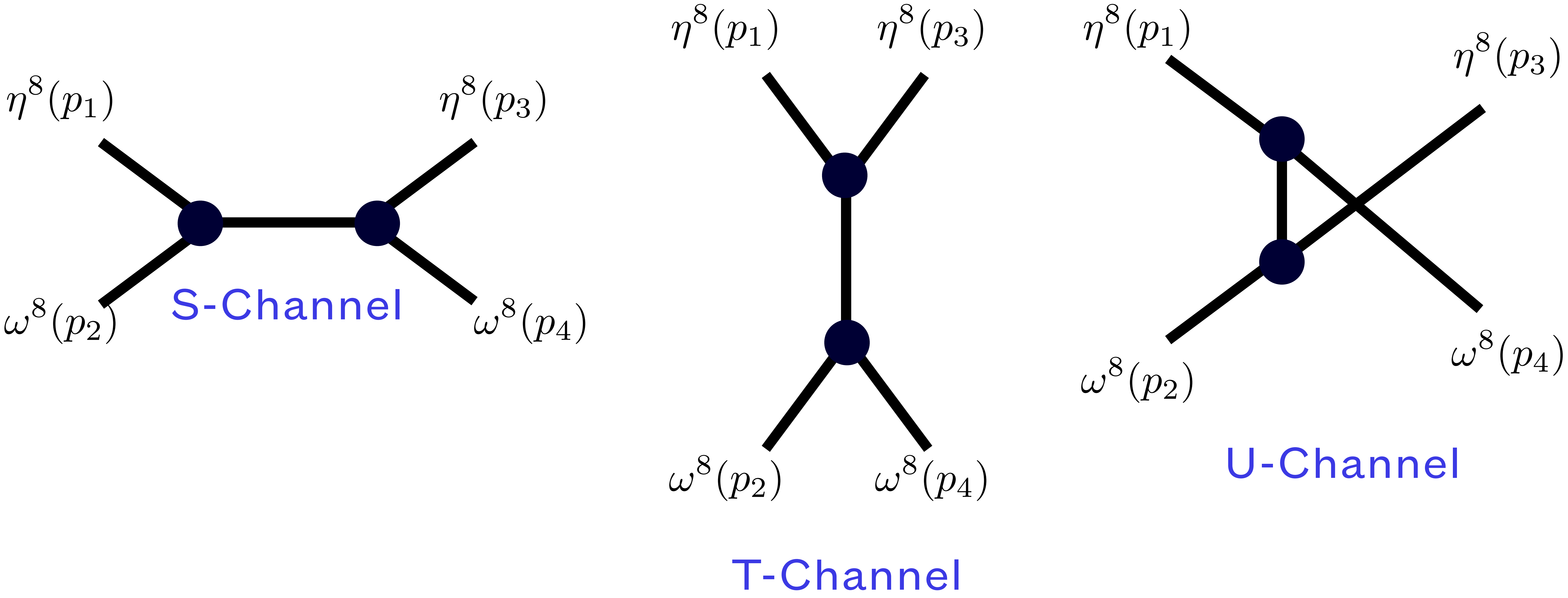
⇒ reject parameterizations that have these

t,u-channel unitarity manifests themselves in the form of a “left hand cut”

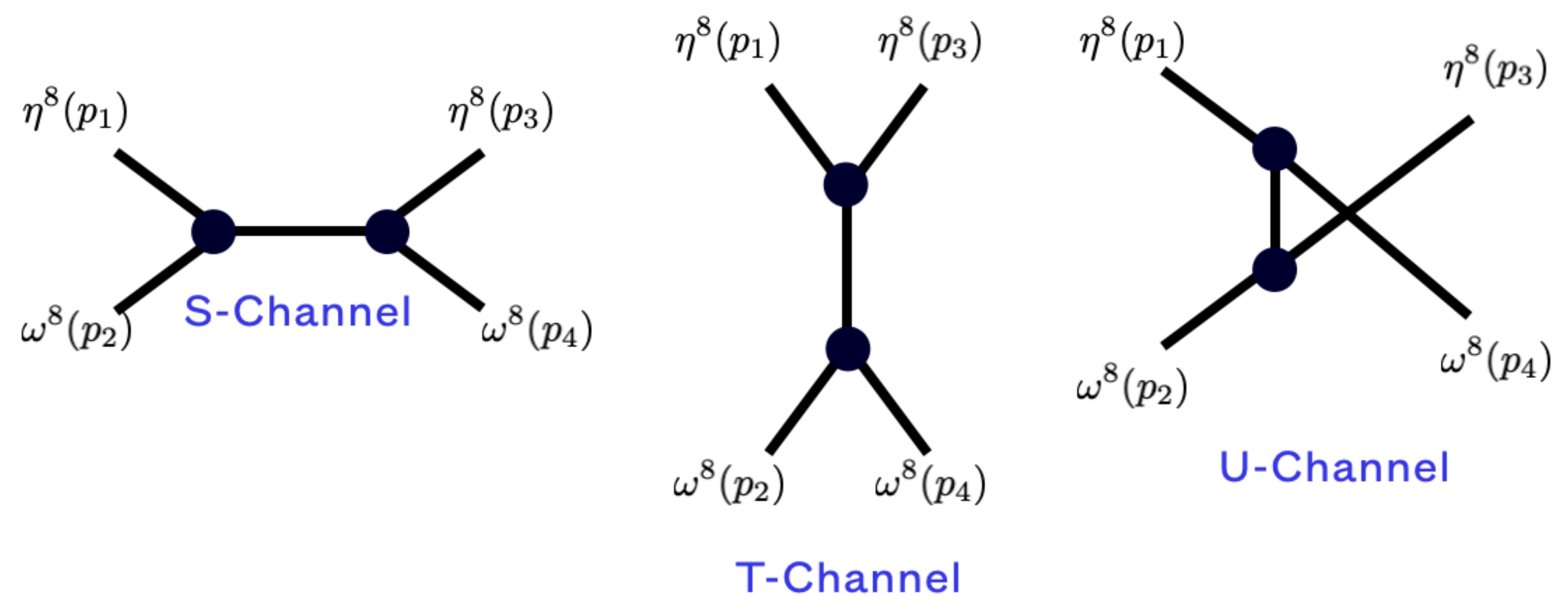
⇒ not described but we know where they are

⇒ hope is we remain far enough away

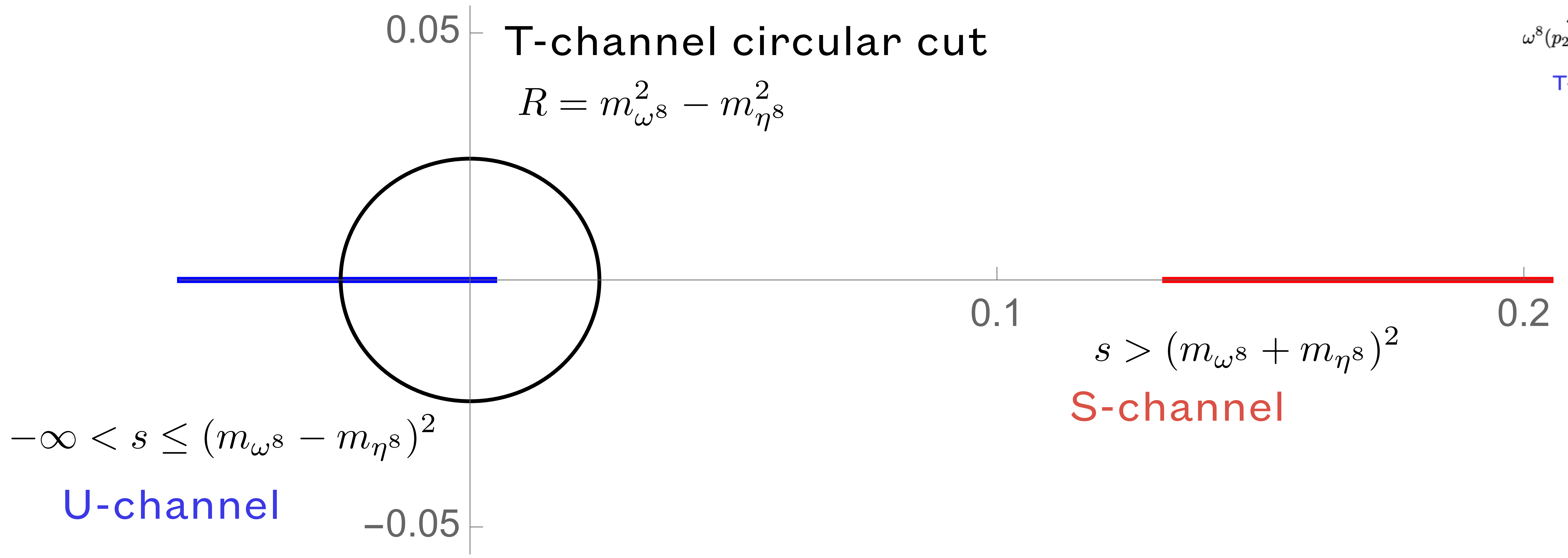
Cross Channels



Cuts

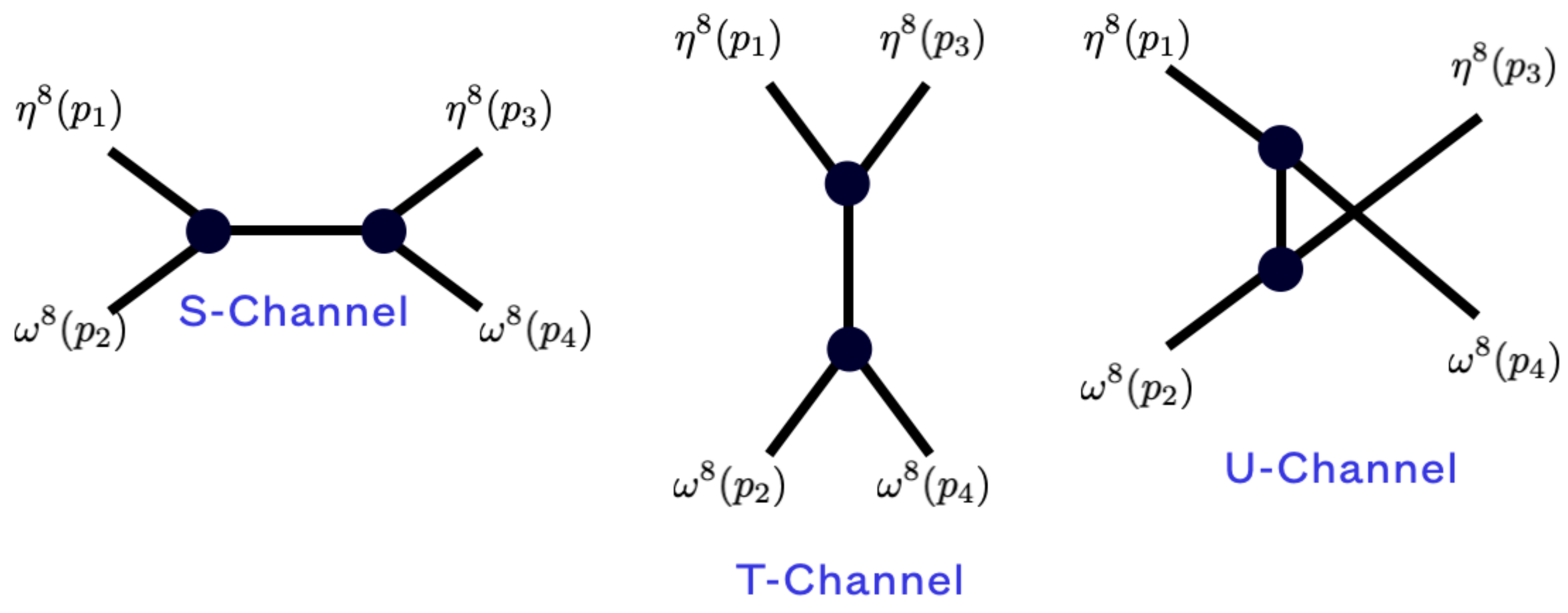


η⁸(p₁)
η⁸(p₃)
ω⁸(p₂)
ω⁸(p₄)
S-Channel
T-Channel
U-Channel

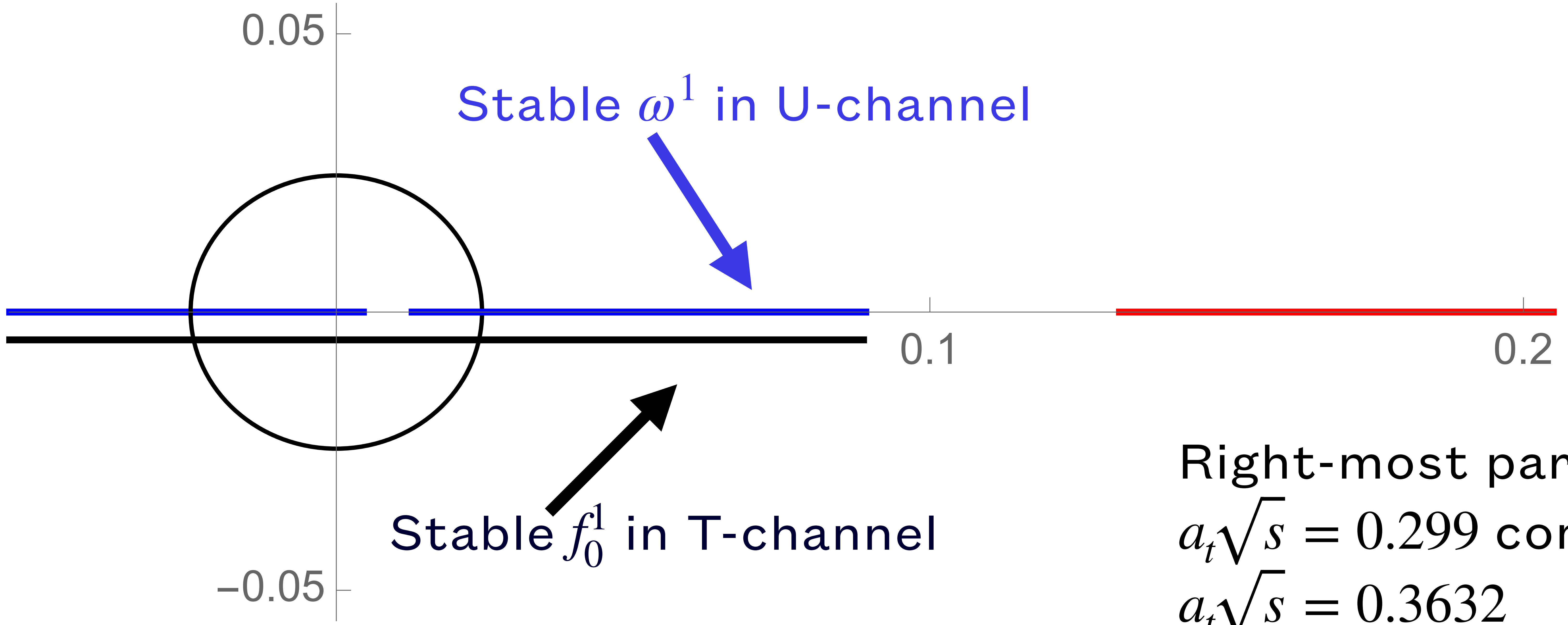


0.05
T-channel circular cut
$R = m_{\omega^8}^2 - m_{\eta^8}^2$
0.1
0.2
$s > (m_{\omega^8} + m_{\eta^8})^2$
S-channel
$-\infty < s \leq (m_{\omega^8} - m_{\eta^8})^2$
U-channel
−0.05

Cuts



Stable particles in cross-channels add additional singularities

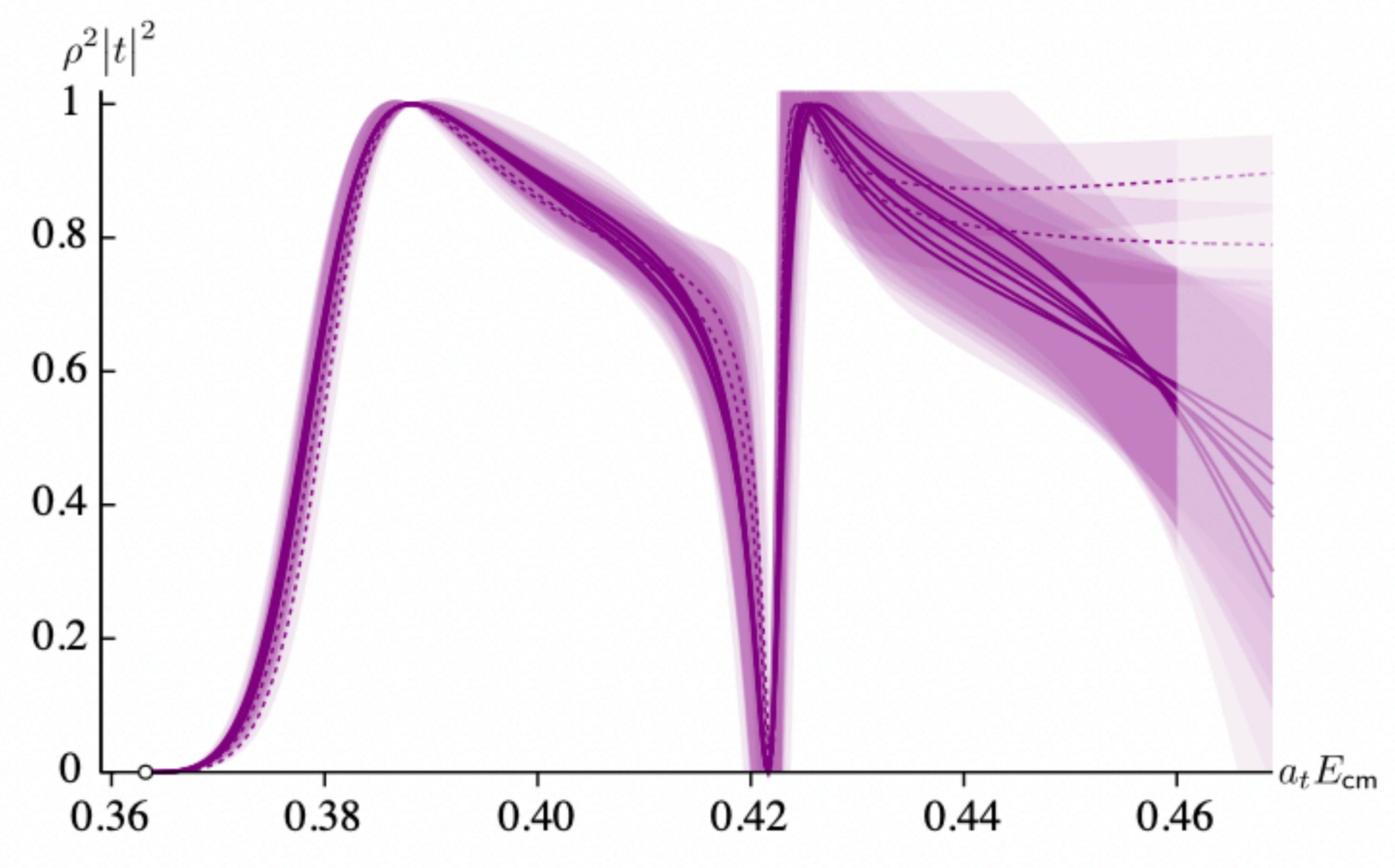


Right-most part of additional cuts at $a_t\sqrt{s} = 0.299$ compared to threshold of $a_t\sqrt{s} = 0.3632$

Additional Singularities

Physical sheet pole at $a_t\sqrt{s} = 0.278(26)$ wrong residue.

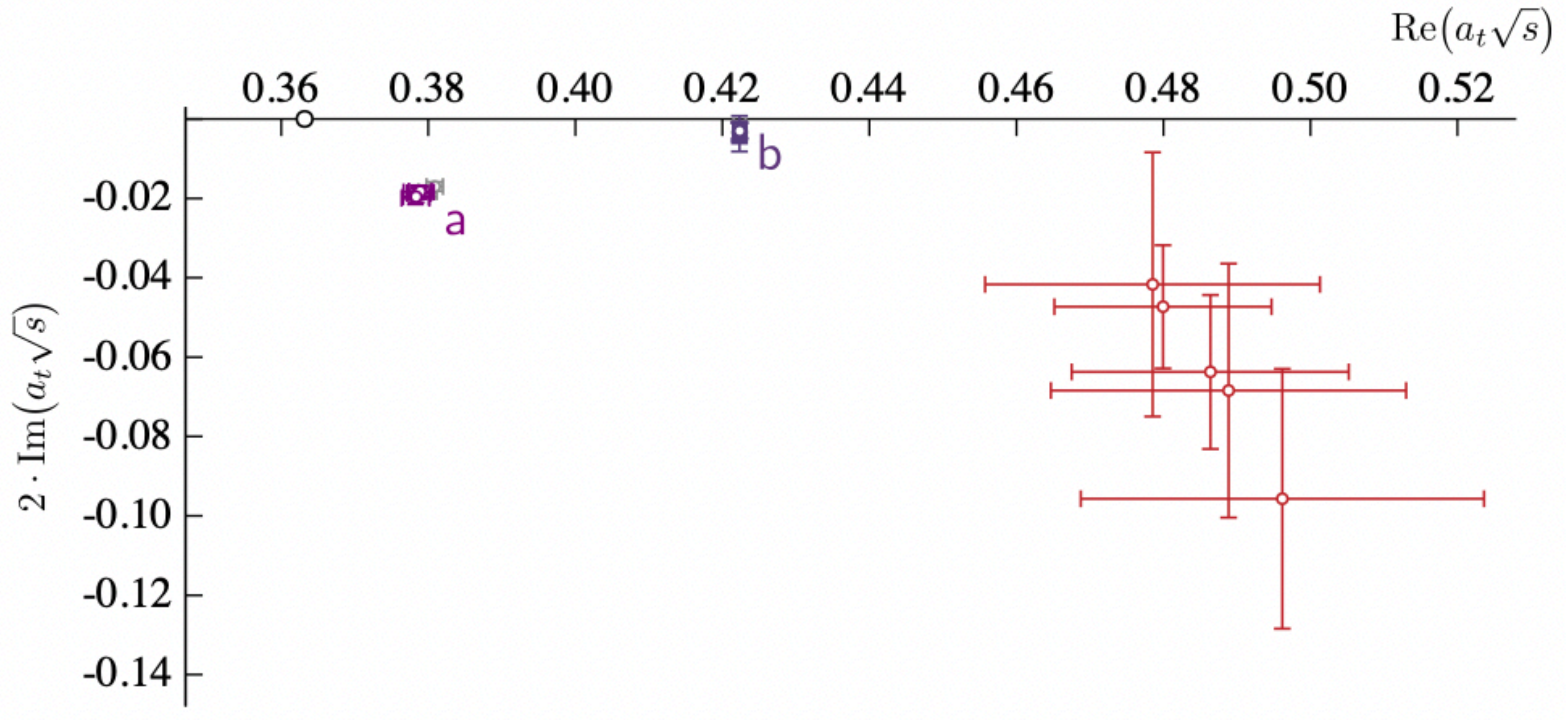
$\Rightarrow$ asses this as a “ghost” occurring from improper treatment of the LHC



Noisy third unphysical sheet pole lies beyond region of constraint $a_t E \sim 0.46$.

$\Rightarrow$ artifact not present in all parameterizations

$\Rightarrow$ could be feeling presence of a hybrid 1^{--} meson we expect in that region



Coupled-channel

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_{thr}}^{\infty} \frac{\rho(s')}{(s' - s_0)(s' - s - i\epsilon)} ds'$$

$$\det\left[1 + i\boldsymbol{\rho} \cdot \mathbf{t} \cdot \left(\mathbf{1} + i\mathbf{M}\right)\right] = 0$$

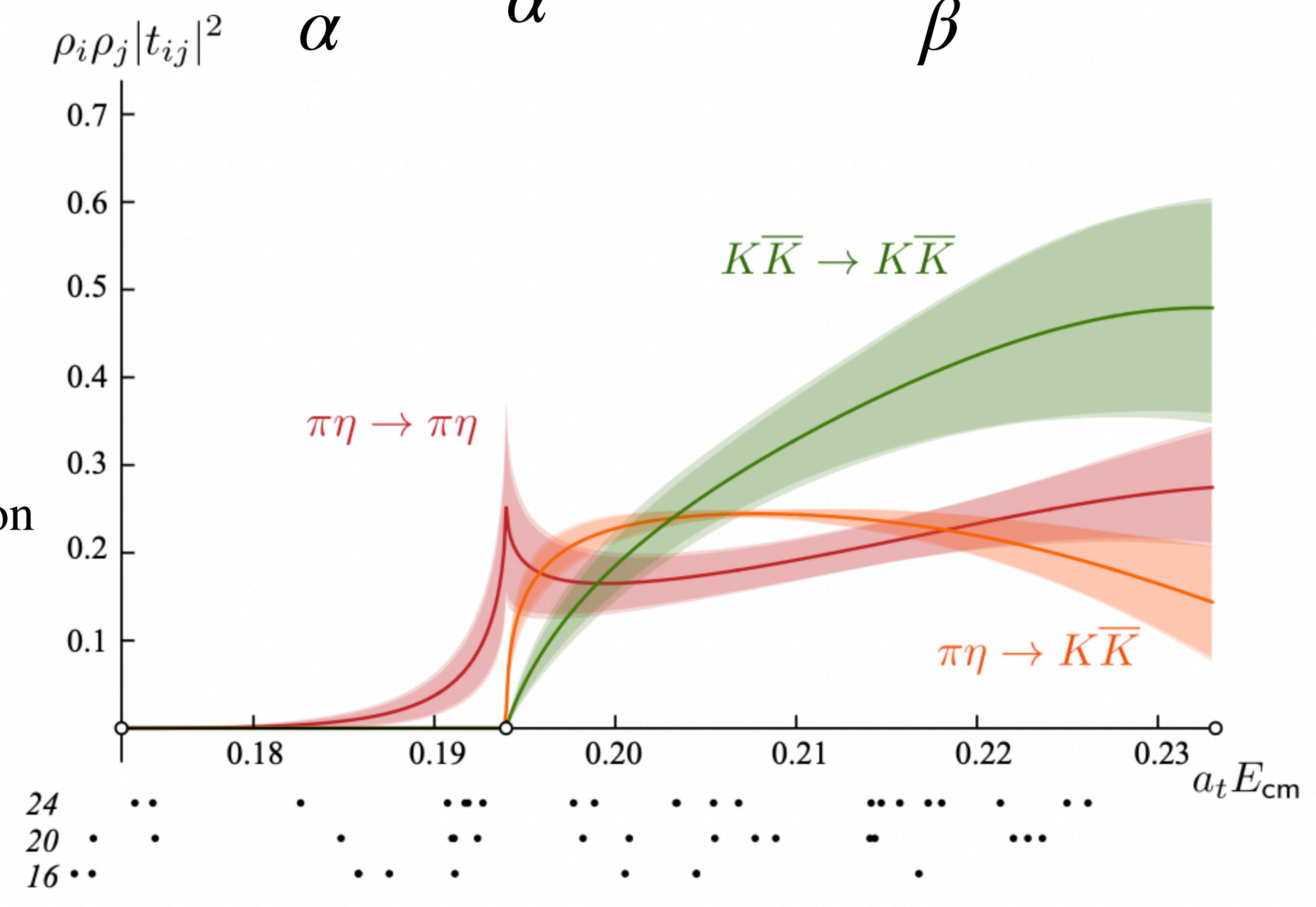
$\mathrm{Im}\boldsymbol{I} = -\boldsymbol{\rho}$

More challenging, can no longer just write in terms of a single phase shift.

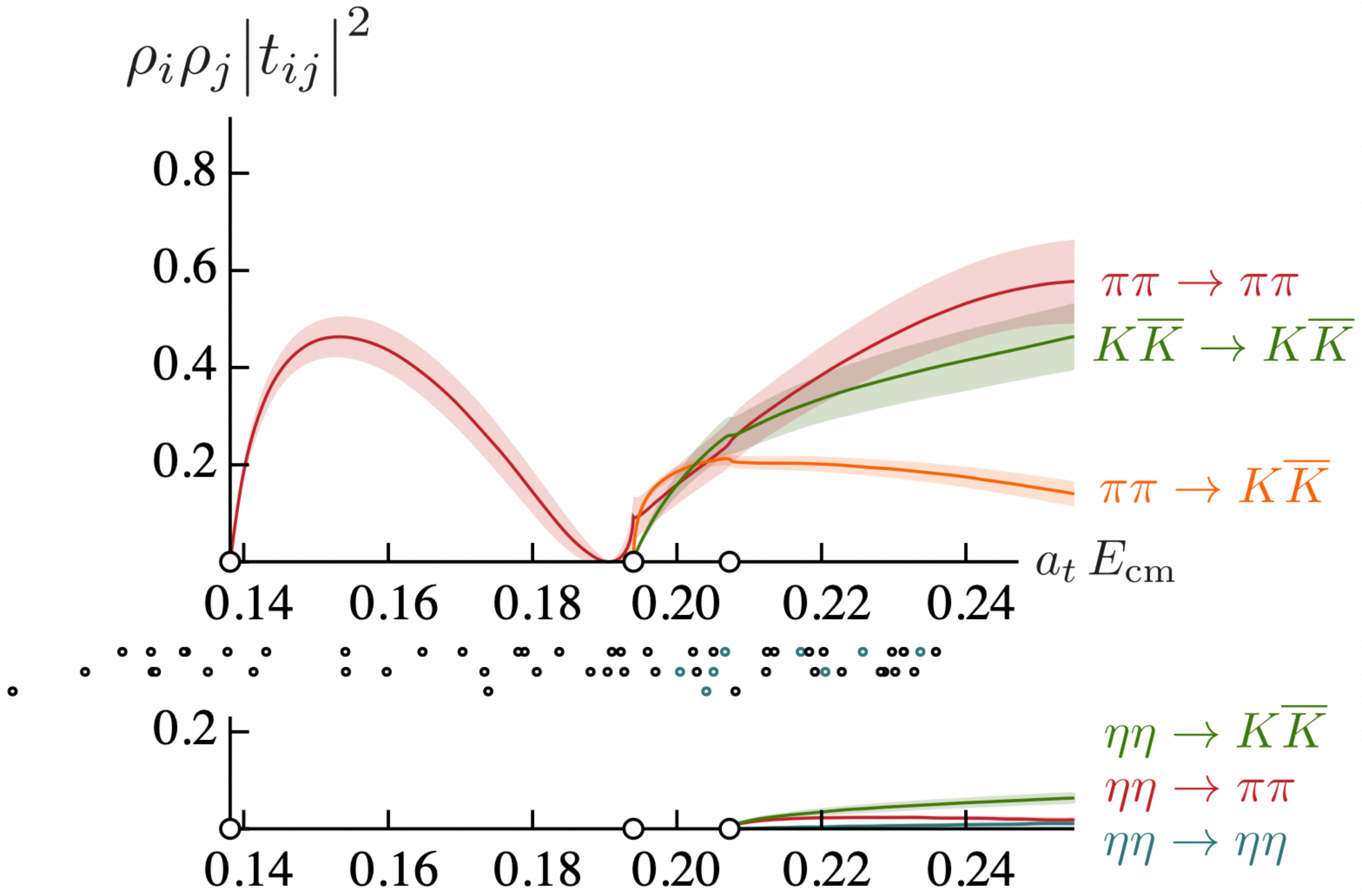
Solutions follow from K-matrix parameterizations of the amplitude : $\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$

K-matrix real and symmetric $K_{ij}(s) = \sum_{\alpha} \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_\alpha^2 - s} + \sum_{\beta} s^{\beta} \gamma_{ij}$

⇒guarantees unitarity



J. J. Dudek, R. G. Edwards, and D. J. Wilson (Hadron Spectrum), Phys. Rev. D93, 094506 (2016), arXiv:1602.05122 [hep-ph]



R. A. Briceno, J. J. Dudek, R. G. Edwards, and D. J. Wilson, Phys. Rev. D97, 054513 (2018), arXiv:1708.06667 [hep-lat]

Coupled channel with nonzero spin

Orbital and angular momentum couple $\boldsymbol{\ell} \otimes \boldsymbol{S} \rightarrow \boldsymbol{J}$

Can use K-matrix to handle this (ex. 0^{-+},1^{--} scattering in $J^P = 1^+$)

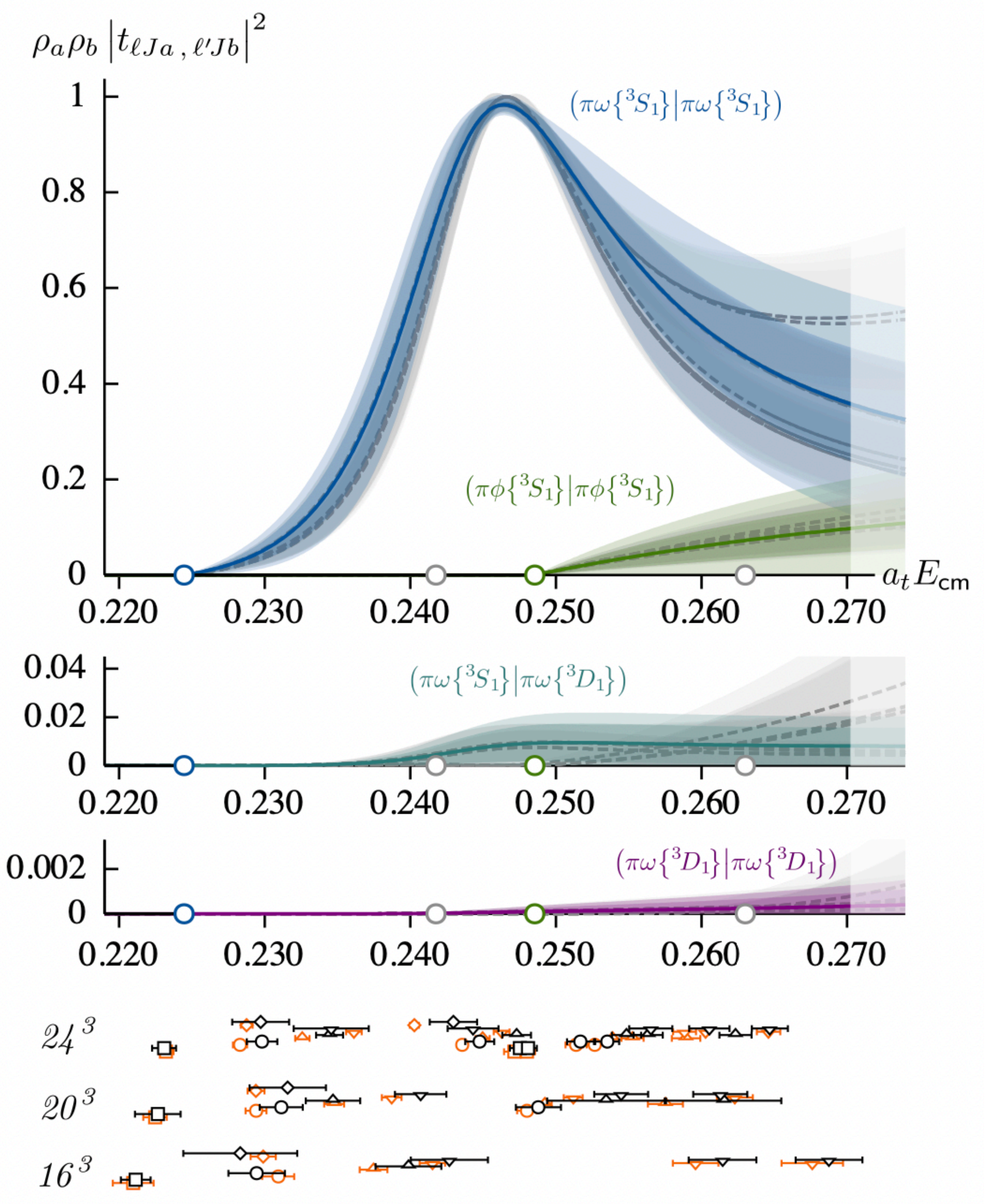
$$K_{1^+} = \begin{pmatrix} \{^3S_1|^3S_1\} & \{^3S_1|^3D_1\} \\ \{^3S_1|^3D_1\} & \{^3D_1|^3D_1\} \end{pmatrix}$$

ℓ	J^P
0	1^+
1	$(0,1,2)^-$
2	$(1,2,3)^+$
3	$(2,3,4)^-$
...	

Done in both non-resonant and resonant systems:

"Dynamically-coupled partial-waves in $\rho\pi$ isospin-2 scattering from lattice QCD"- A. Woss, C. Thomas, J. Dudek, R. Edwards, D. Wilson

"The b_1 resonance in coupled $\pi\omega, \pi\phi$ scattering from lattice QCD"- A. Woss, C. Thomas, J. Dudek, R. Edwards, D. Wilson



Channels in SU(3) Flavor

Conventional $\bar{q}q$ mesons live in either a singlet ($\bar{3} \otimes 3 \to 1$) or octet ($\bar{3} \otimes 3 \to 8$) representations.

We observe the resonances in the singlet representation of meson-meson scattering:

$$8 \otimes 8 \to 1$$

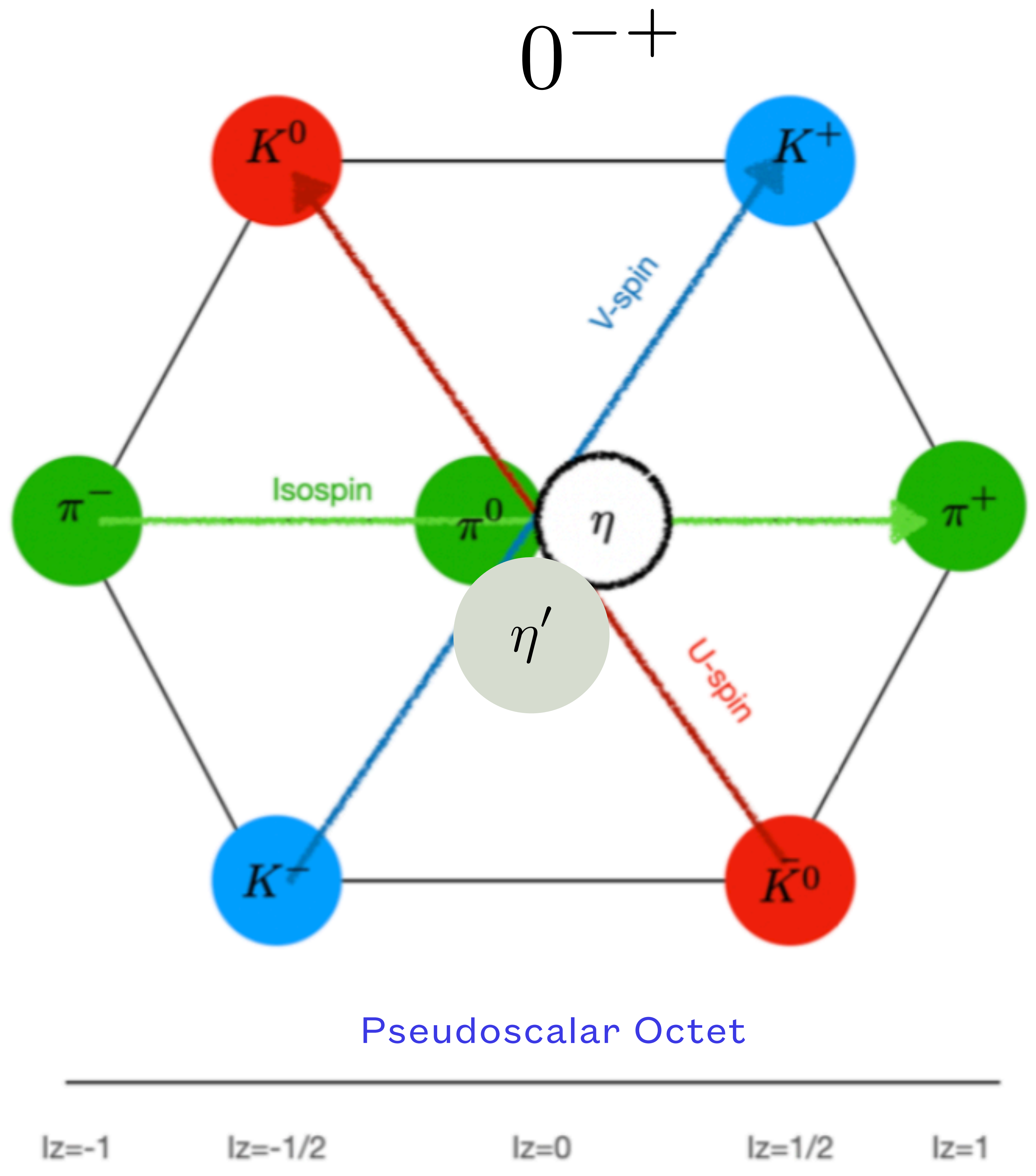
$$1 \otimes 1 \to 1$$

Charge conjugation in neutral member of the octet $|I = I_z = Y = 0\rangle$ for $8 \otimes 8 \to 1$:

$$\hat{C}(|8_1, C_1\rangle \otimes |8_2, C_2\rangle) \to C_1 C_2(|8_1, C_1\rangle \otimes |8_2, C_2\rangle)$$

$\Rightarrow$ channels with C=- : $\eta^8(0^{-+})\omega^8(1^{--}), f_0^1(0^{++})\omega^1(1^{--}), \eta^1(0^{-+})\omega^1(1^{--})$

$\Rightarrow$ can't have identical particles with C=-



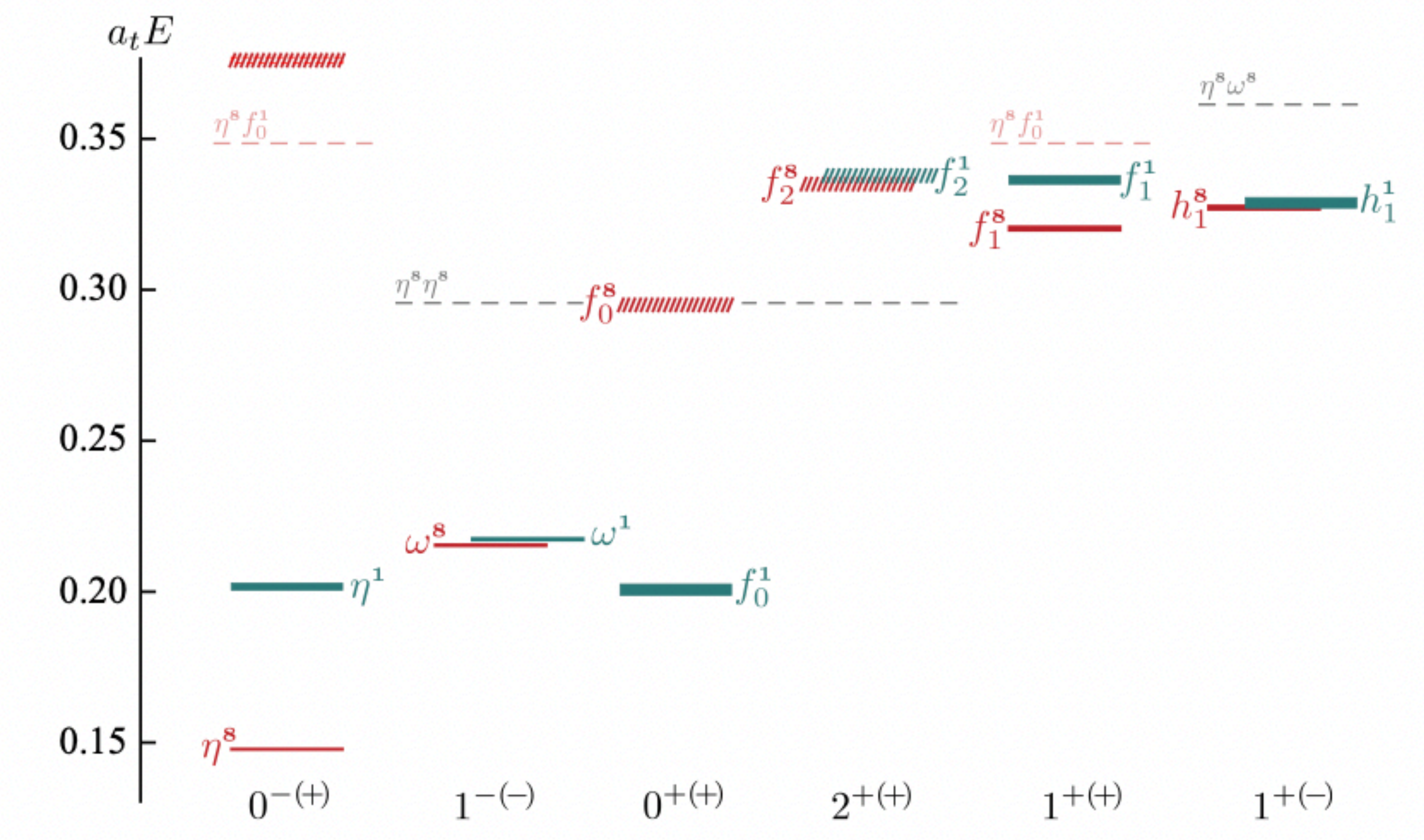
Channels

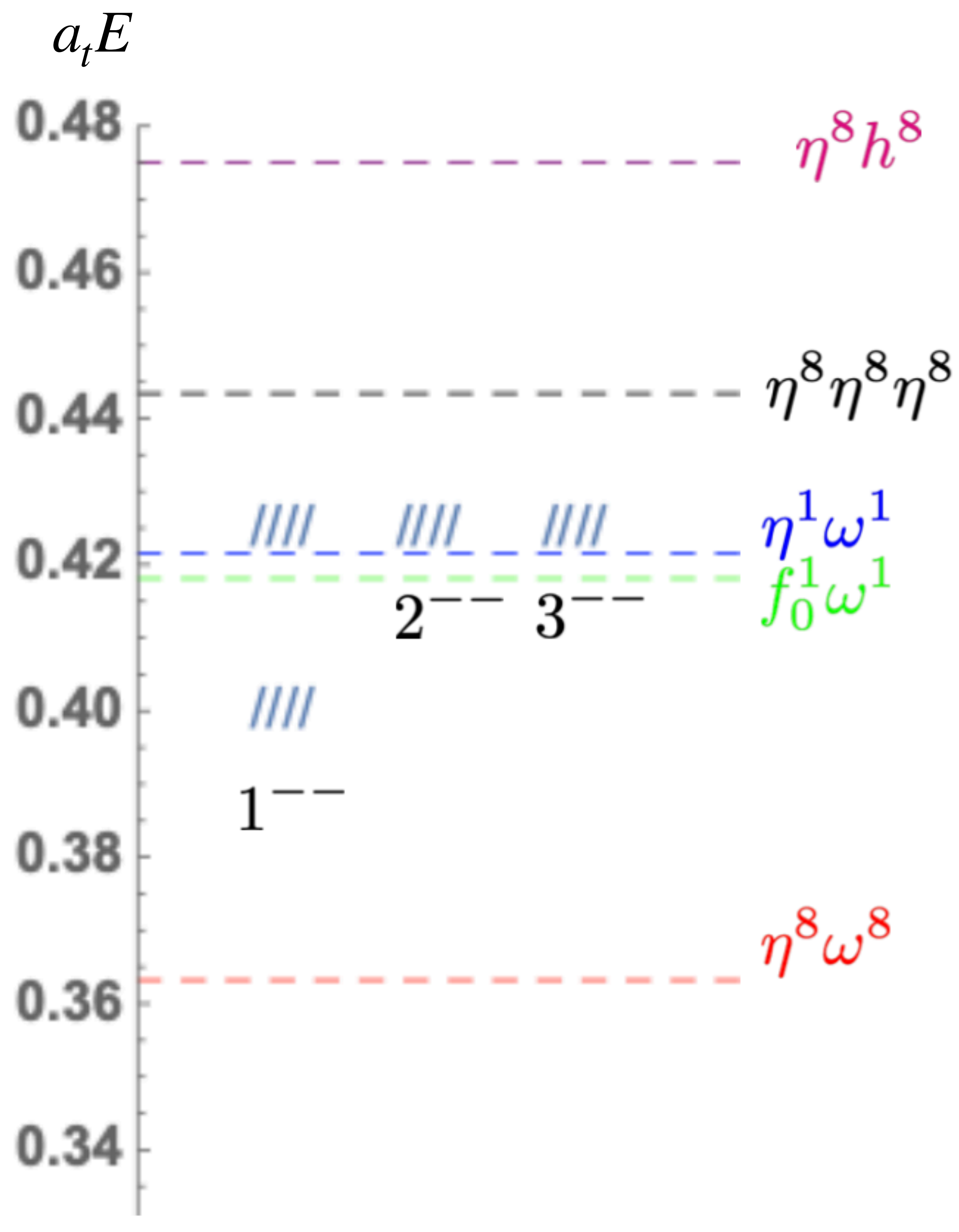
η^8	0.1478(1)	η^1	0.2017(11)
ω^8	0.2154(2)	ω^1	0.2174(3)
		f_0^1	0.2007(18)

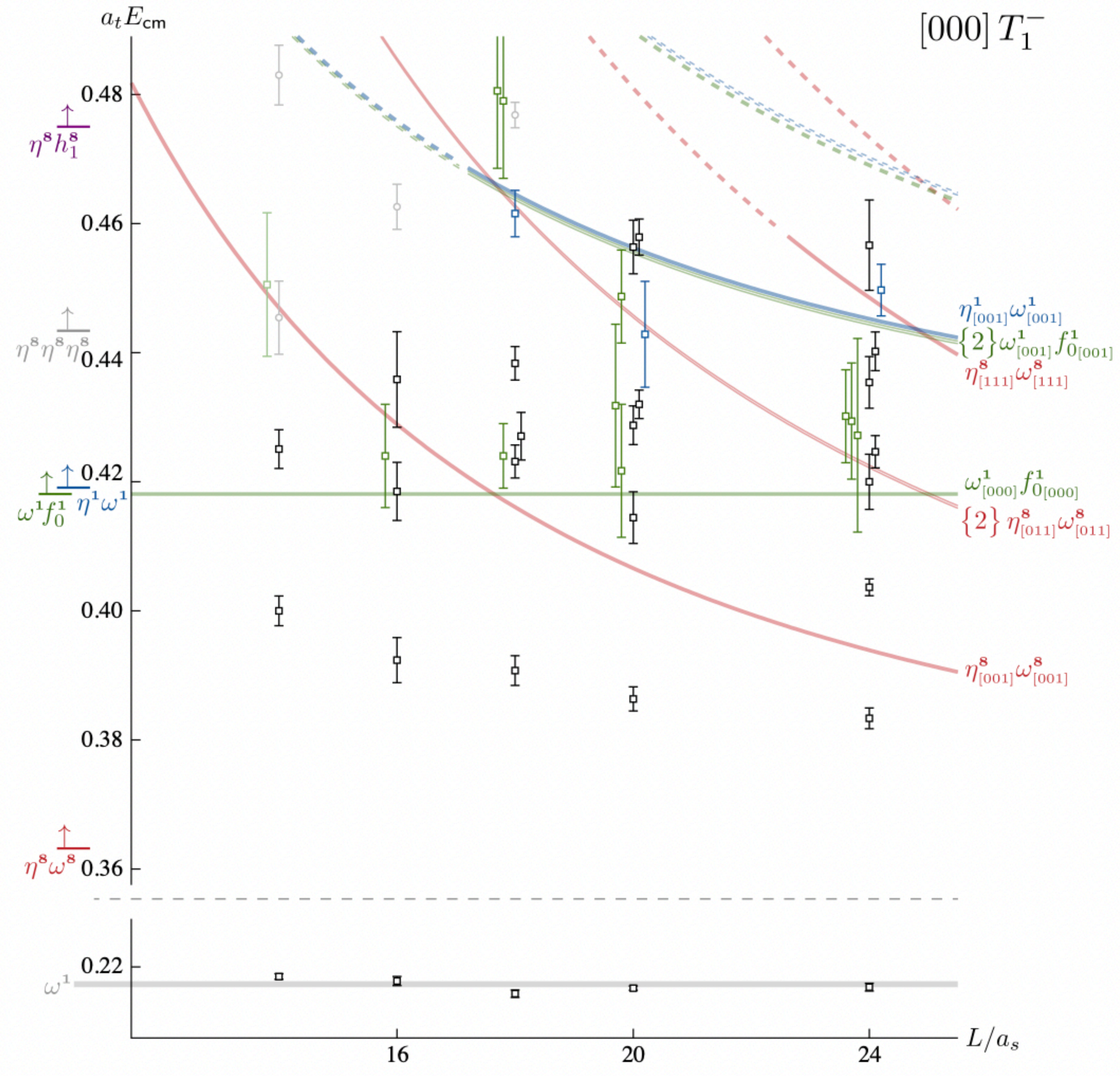
J=1: $\eta^8\omega^8\{^3P_1\}, f_0^1\omega^1\{^3S_1, ^3D_1\}, \eta^1\omega^1\{^3P_1\}$

J=2: $\eta^8\omega^8\{^3P_2, ^3F_2\}, f_0^1\omega^1\{^3D_2\}, \eta^1\omega^1\{^3P_2, ^3F_2\}$

J=3: $\eta^8\omega^8\{^3F_3\}, f_0^1\omega^1\{^3D_3, ^3G_3\}, \eta^1\omega^1\{^3F_3\}$







$J^P = (1,3,...)^-$

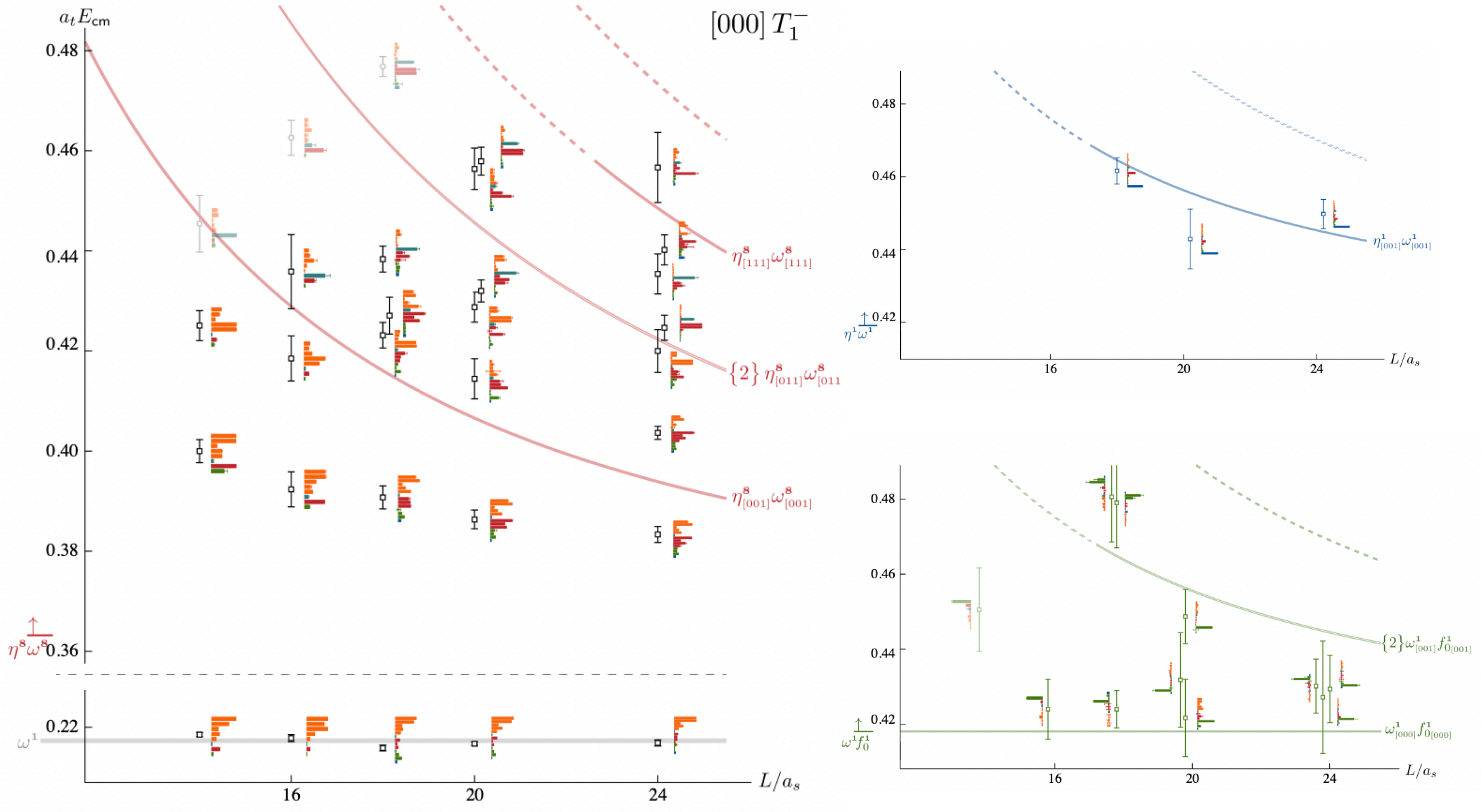
Three resonances in a single irrep.

$$\Rightarrow \rho\{^3 2S_1\}, \rho\{^3 D_1\}, \rho\{^3 D_3\}$$

Very dense in energy levels.

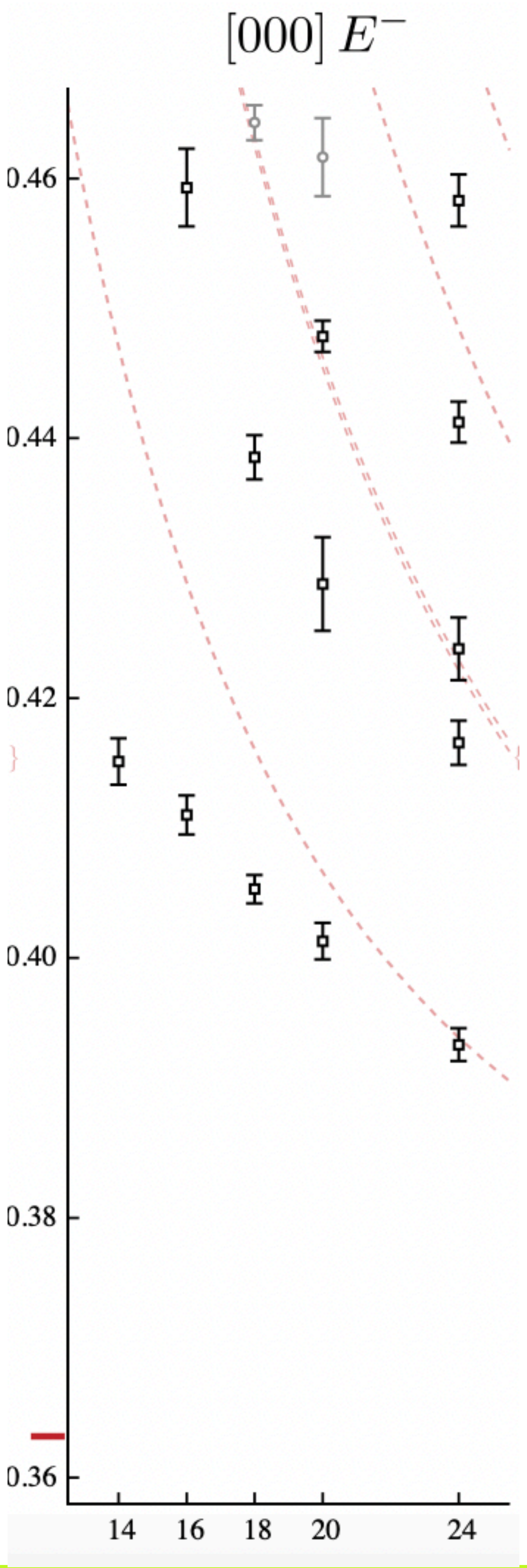
Appears to be a decoupling within the heavier channels $f_0^1\omega^1, \eta^1\omega^1$

$$C_{ij}(t) = \sum_\alpha \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$$

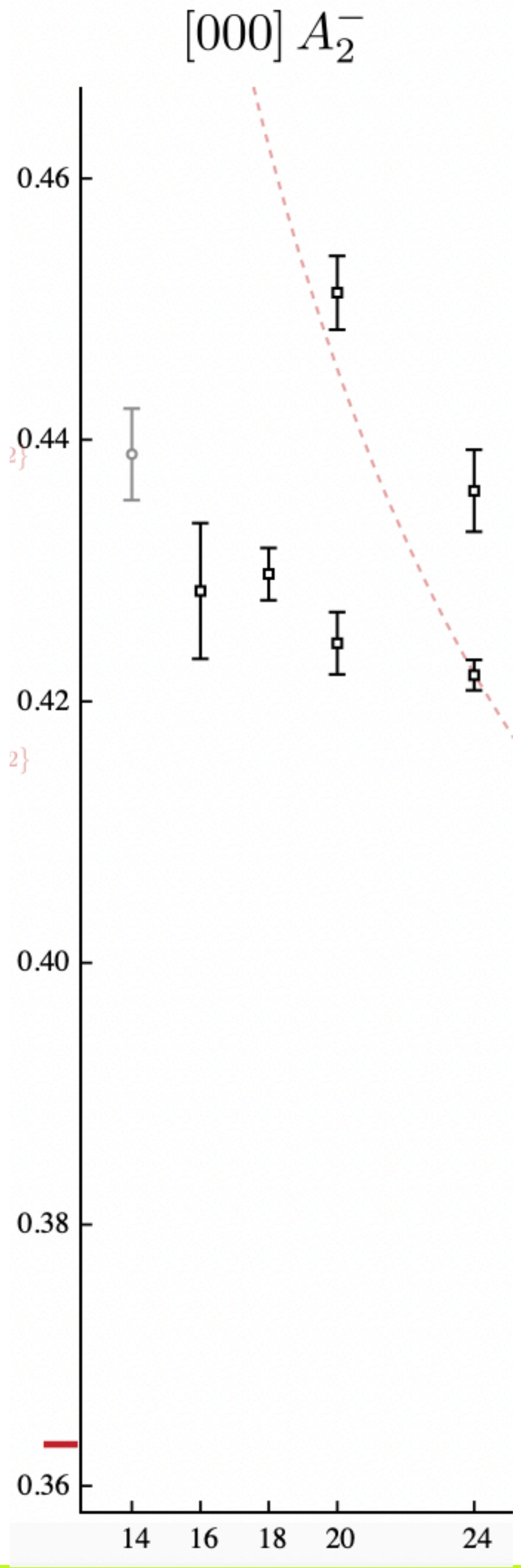


[000] T_1^-
$a_t E_{\text{cm}}$
0.48
0.46
0.44
0.42
0.40
0.38
0.36
0.22
$\eta^8\omega^8$
ω^1
$\eta^8_{[111]}\omega^8_{[111]}$
$\{2\}\,\eta^8_{[011]}\omega^8_{[011]}$
$\eta^8_{[001]}\omega^8_{[001]}$
16
20
24
L/a_s
$\eta^1\omega^1$
$\eta^1_{[001]}\omega^1_{[001]}$
$\omega^1 f_0^1$
$\{2\}\omega^1_{[001]} f^1_{0[001]}$
$\omega^1_{[000]} f^1_{0[000]}$

How do we solve this?



$J^P = (2, \dots)^-$



$J^P = (3, \dots)^-$

Broken rotational symmetry of the lattice causes different resonances to be in the same representation.

Only two systems that isolate a single resonance

All other irreps will feature a minimum of TWO resonances

$^3D_{1,2,3}$ states are expected to be nearly degenerate.

Plan of attack

Carry forward with elastic scattering in $\eta^8\omega^8$

⇒ fit to amplitudes of J=2,3 simultaneously $(T_2^-[000], E^-[000], A_2^-[000], B_1[001], B_2[001])$

⇒ fix J=3 amplitude and fit for the J=1 amplitude $(T_1^-[000], A_1[001], A_1[111])$

⇒ fit to all amplitudes for J=1,2,3 together