Quarkonium in a bulk viscous QGP medium

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Outline

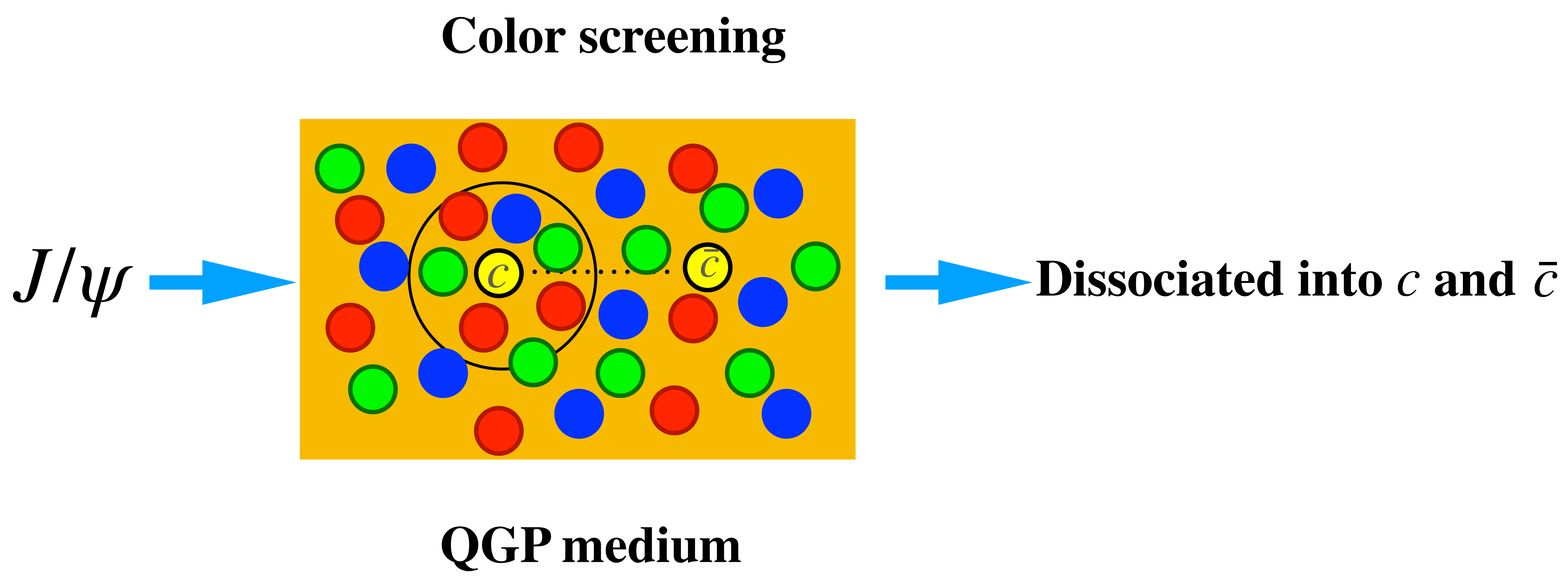
- **Introduction**
  - **Quarkonium-as a probe of QGP**
  - **Bulk viscous QGP medium**
- **Heavy quarkonia in a bulk viscous medium**
  - **Debye masses and color dielectric permittivity**
  - **In-medium heavy quark complex potential**
- **Quarkonium properties**
  - **Binding energies and decay width**
  - **Melting temperatures**
- **Summary**

Introduction

- **Quarkonia** ($Q\bar{Q}$) are the bound state of heavy quarks and its own antiquark

  $\rightarrow$ i.e. Charmonium ($c\bar{c}$) and Bottomonium ($b\bar{b}$).

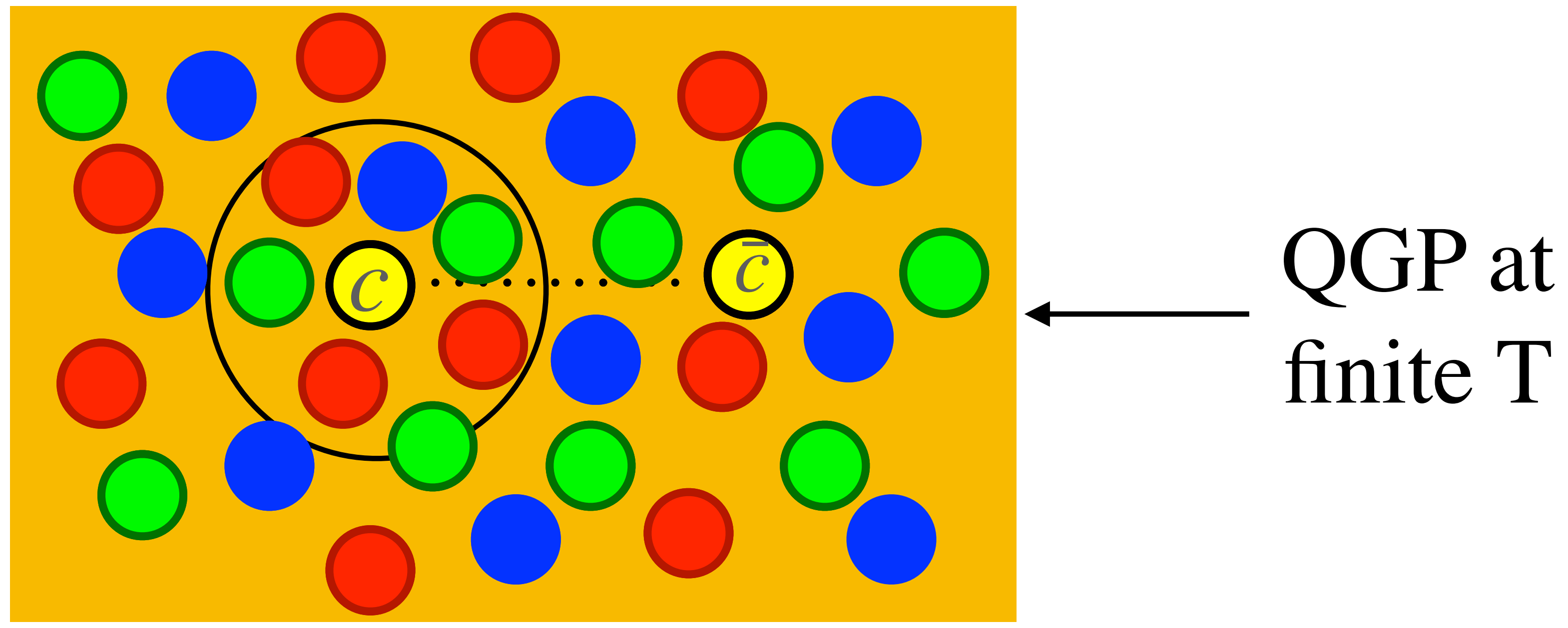
- Quarkonium suppression is one of the first proposed signal of **Quark−Gluon Plasma (QGP)** formation in heavy ion collision.



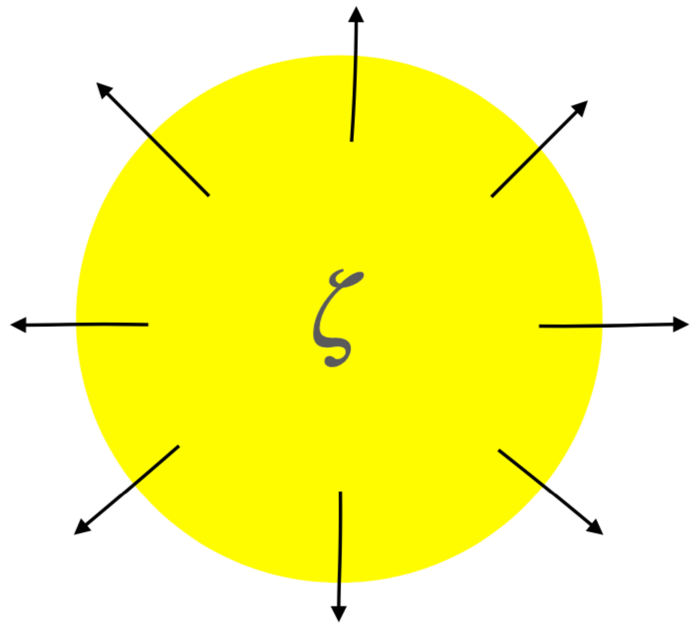
Matsui and Satz, PLB 178 (1986) 416

Non-equilibrium QGP

- QGP has many different **non-equilibrium properties:**
  - Dissipative effects
    - Shear viscosity
    - **Bulk viscosity**
  - Magnetic field
  - …



- **Bulk viscosity** → Fluid's resistance to compression
- QCD matter has non-zero bulk viscosity, which might affect the evolution of the medium and has non-negligible effect on heavy ion observables.



Ryu et. al. PRL 115 132301 (2015)

- **Heavy quarkonia** are sensitive to the entire evolution of the QGP.

How sensitive are the quarkonia to the bulk viscous nature of the fluid ?

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Heavy quarkonia in a bulk viscous medium

Quarkonium properties

↑

In-medium heavy quark potential

$$V_{\text{in-medium}}(p) = \varepsilon^{-1}(p)\, V_{\text{Cornell}}(p)$$

↑

Dielectric permittivity $\varepsilon(p)$

$$\varepsilon^{-1}(p) = \lim_{p^0 \to 0} p^2 D^{00}(P)$$

↑

Propagators $D(p)$

$$D^{00}(p) = \frac{1}{2}(D_R + D_A + D_S)$$

$$\mathrm{Re}\, D^{00} = \frac{1}{2}(D_R + D_A)$$

$$\mathrm{Im}\, D^{00} = \frac{1}{2} D_S$$

↑

Self energies $\Pi(p)$

$$D_R(p) = \frac{1}{p^2 - \Pi_R(p)}$$

How to incorporate bulk viscous correction

The quark contribution to the retarded gluon self energy in HTL approximation

$$\Pi_R^{(q)}(P) = \frac{4\pi N_f g^2}{(2\pi)^4} \int kdkd\Omega \left(\frac{f^+(\mathbf{k}) + f^-(\mathbf{k})}{2} \right) \frac{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2}{\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}} + \frac{p^0+i\epsilon}{p}\right)^2}$$

Mrowczynski and Thoma, PRD 62 (2000) 036011

- The modified distribution function of quarks and gluons in the presence of non-equilibrium bulk viscous correction

$$f(\mathbf{k}) = f_0(k) + \delta_{\text{noneq}} f(\mathbf{k})$$

→ **Non-equilibrium corrections from bulk viscosity**

$$\delta_{\text{bulk}} f(k) = \left(\frac{k}{T}\right)^a \Phi \; f_0(k)\,(1 \pm f_0(k))$$

where a and Φ are constants. Φ is proportional to the bulk viscous pressure.

Du, Dumitru, Guo, Strickland, JHEP 01 (2017) 123

Debye masses and dielectric permittivity

Debye mass without correction

$$\lim_{p^0 \to 0} \Pi_R^L(0, p, T) = m_D^2 = \frac{g^2 T^2}{6}\left[2N_c + N_f\left(1 + \frac{3\tilde{\mu}^2}{\pi^2}\right)\right] \quad \textbf{where} \quad \tilde{\mu} = \frac{\mu}{T}$$

Retarded propagator

$$\lim_{p^0 \to 0} D_R(0, p, T) = \frac{1}{p^2 + \widetilde{m}_{D,R}^2}$$

Modified retarded Debye mass

$$\widetilde{m}_{D,R}^2 = m_D^2 + \delta m_{D,R}^2$$

Correction to the Debye mass

$$\delta m_{D,R}^2 = \frac{g^2 T^2}{6}\left[2N_c c_R^g(a)\Phi + N_f\left(1 + \frac{3\tilde{\mu}^2}{\pi^2}\right) c_R^q(a, \tilde{\mu})\Phi\right]$$

Du, Dumitru, Guo, Strickland, JHEP 01 (2017) 123

The quark contribution to the symmetric gluon self energy in HTL approximation

$$\Pi_S^{(q)}(P) = 4iN_f g^2\pi^2 \int \frac{k^2 dk}{(2\pi)^3} \sum_{i=\pm} f^i(k)(f^i(k)-1)\frac{2}{p}\Theta(p^2-p_0^2)$$

Symmetric propagator

$$\lim_{p^0\to 0} D_S(0,p,T) = -\frac{2\pi i T\widetilde{m}_{D,S}^2}{p(p^2+\widetilde{m}_{D,R}^2)^2}$$

Modified symmetric Debye mass

$$\widetilde{m}_{D,S}^2 = m_D^2 + \delta m_{D,S}^2$$

Bulk viscous correction to the symmetric Debye mass

$$\delta m_{D,S}^2 = \frac{g^2T^2}{6}\left(2N_c\, c_S^g(a)\,\Phi + N_f\left(1+\frac{3\tilde{\mu}^2}{\pi^2}\right) c_S^q(a,\tilde{\mu})\,\Phi\right)$$

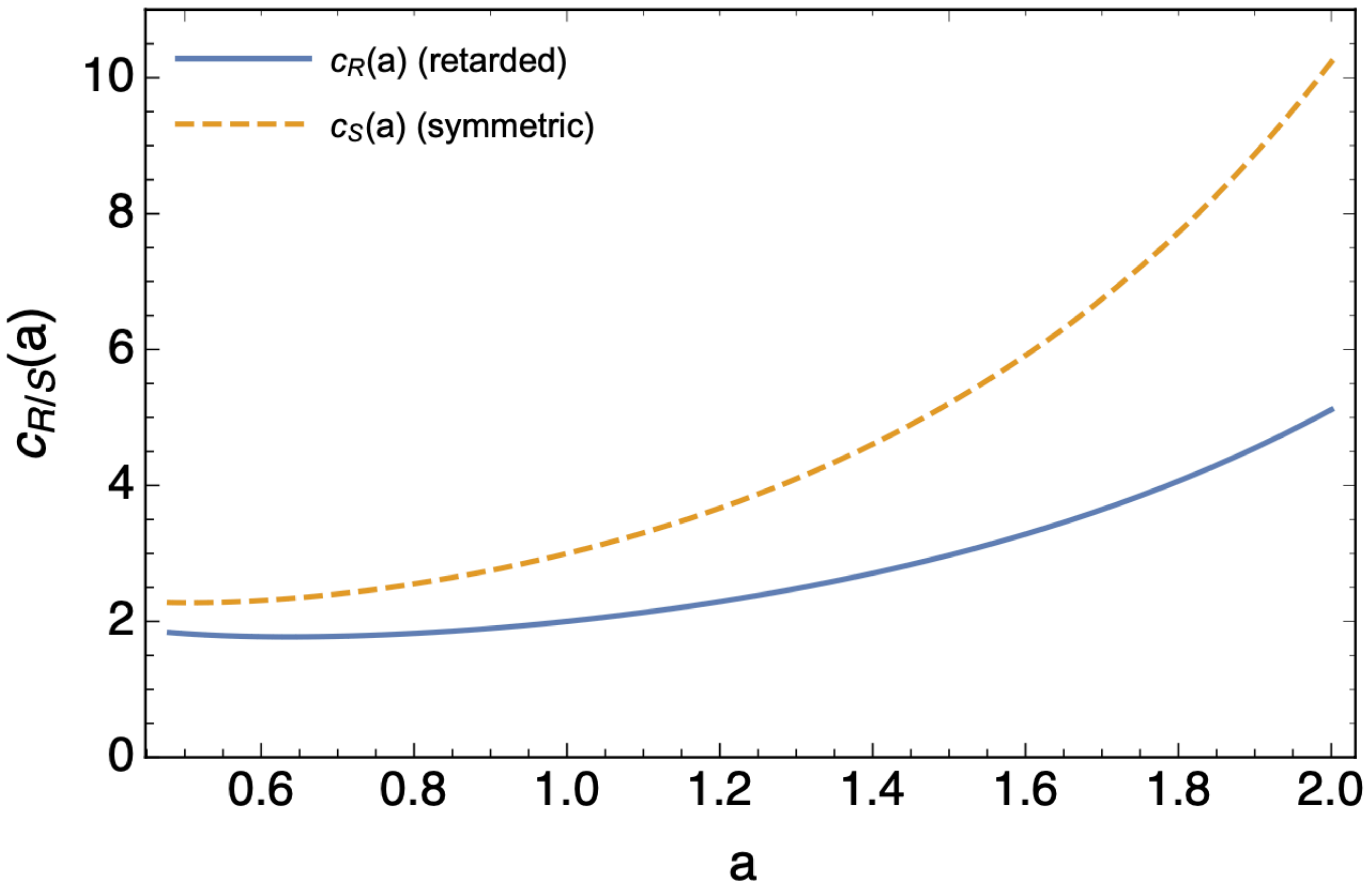
- The modified Debye masses in terms of bulk correction

$$\frac{\widetilde{m}^2_{D,R(S)}}{m^2_D} = 1 + c_{R(S)}(a, \tilde{\mu})\Phi$$

- The linear coefficients $C_{R(S)}(a)$ increases with increase in a, which results in the increase in Debye mass for Φ>0

$$\lambda = \lambda(a, \Phi, \tilde{\mu}) \equiv \frac{\widetilde{m}^2_{D,S}}{\widetilde{m}^2_{D,R}} = \frac{1 + c_S\Phi}{1 + c_R\Phi}$$

- Symmetric Debye mass is larger than the retarded one for $\Phi > 0$, so $\lambda > 1$ in this case.



Dielectric permittivity in the presence of bulk viscous correction

$$\varepsilon^{-1}(p) = \frac{p^2}{p^2 + \widetilde{m}^2_{D,R}} - i\frac{\pi T p\, \widetilde{m}^2_{D,S}}{(p^2 + \widetilde{m}^2_{D,R})^2}$$

For without bulk viscous correction

$$\widetilde{m}^2_{D,R} = \widetilde{m}^2_{D,S} = m^2_D$$

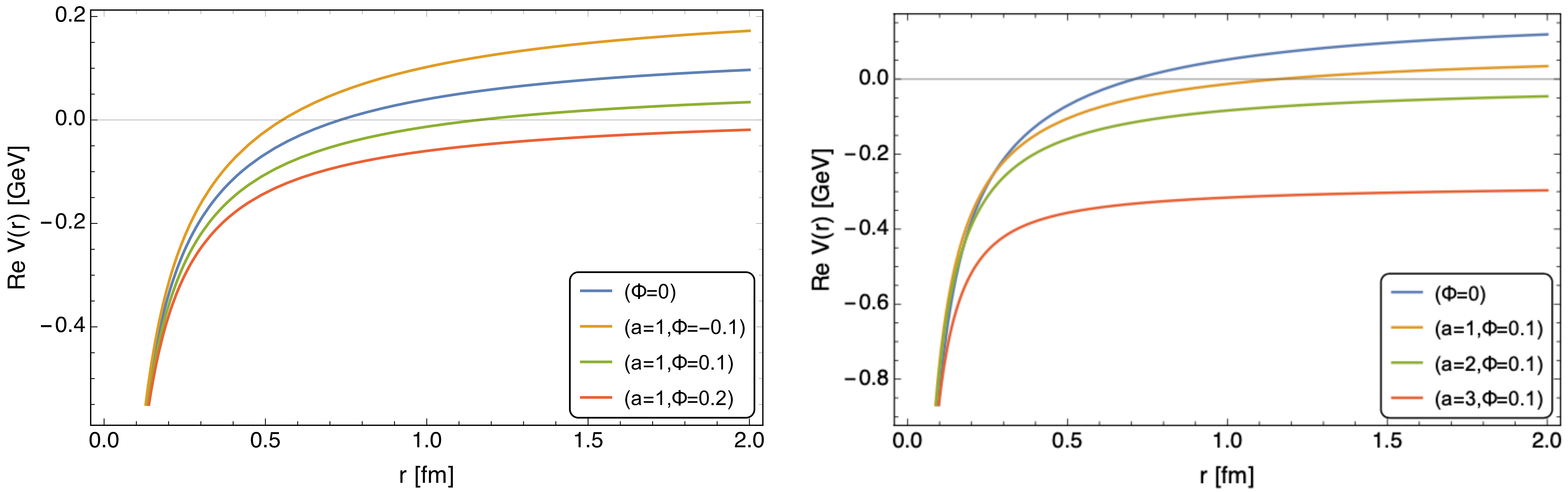
$$\varepsilon^{-1}(p) = \frac{p^2}{p^2 + m^2_D} - i\frac{\pi T p\, m^2_D}{(p^2 + m^2_D)^2}$$

In-medium heavy quark complex potential

Real part of the potential

$$\mathrm{Re}\,V(r) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}}(e^{i\mathbf{p}\cdot\mathbf{r}} - 1)V_{\mathrm{Cornell}}(p)\,\mathrm{Re}\,\varepsilon^{-1}(p)$$

$$= -\alpha\,\widetilde{m}_{D,R}\left(\frac{e^{-\widetilde{m}_{D,R}\,r}}{\widetilde{m}_{D,R}\,r} + 1\right) + \frac{2\sigma}{\widetilde{m}_{D,R}}\left(\frac{e^{-\widetilde{m}_{D,R}\,r} - 1}{\widetilde{m}_{D,R}\,r} + 1\right)$$



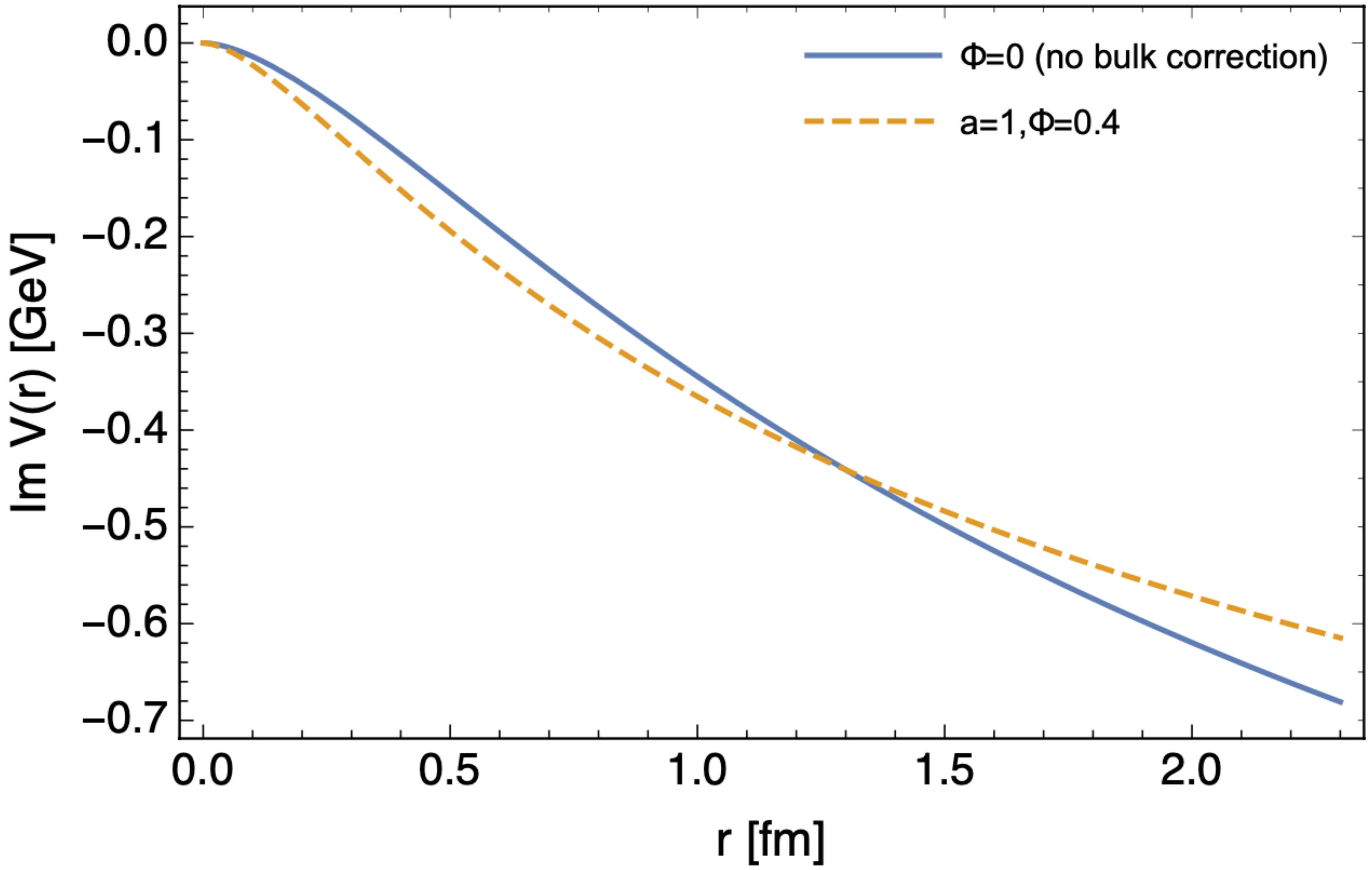
Imaginary part of the potential

$$\mathrm{Im}\, V(r) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) V_{\mathrm{Cornell}}(p)\, \mathrm{Im}\, \varepsilon^{-1}(p)$$
$$= -\alpha\lambda T\, \phi_2(\widetilde{m}_{D,R}\, r) - \frac{2\sigma T\lambda}{\widetilde{m}_{D,R}^2} \chi(\widetilde{m}_{D,R}\, r)$$

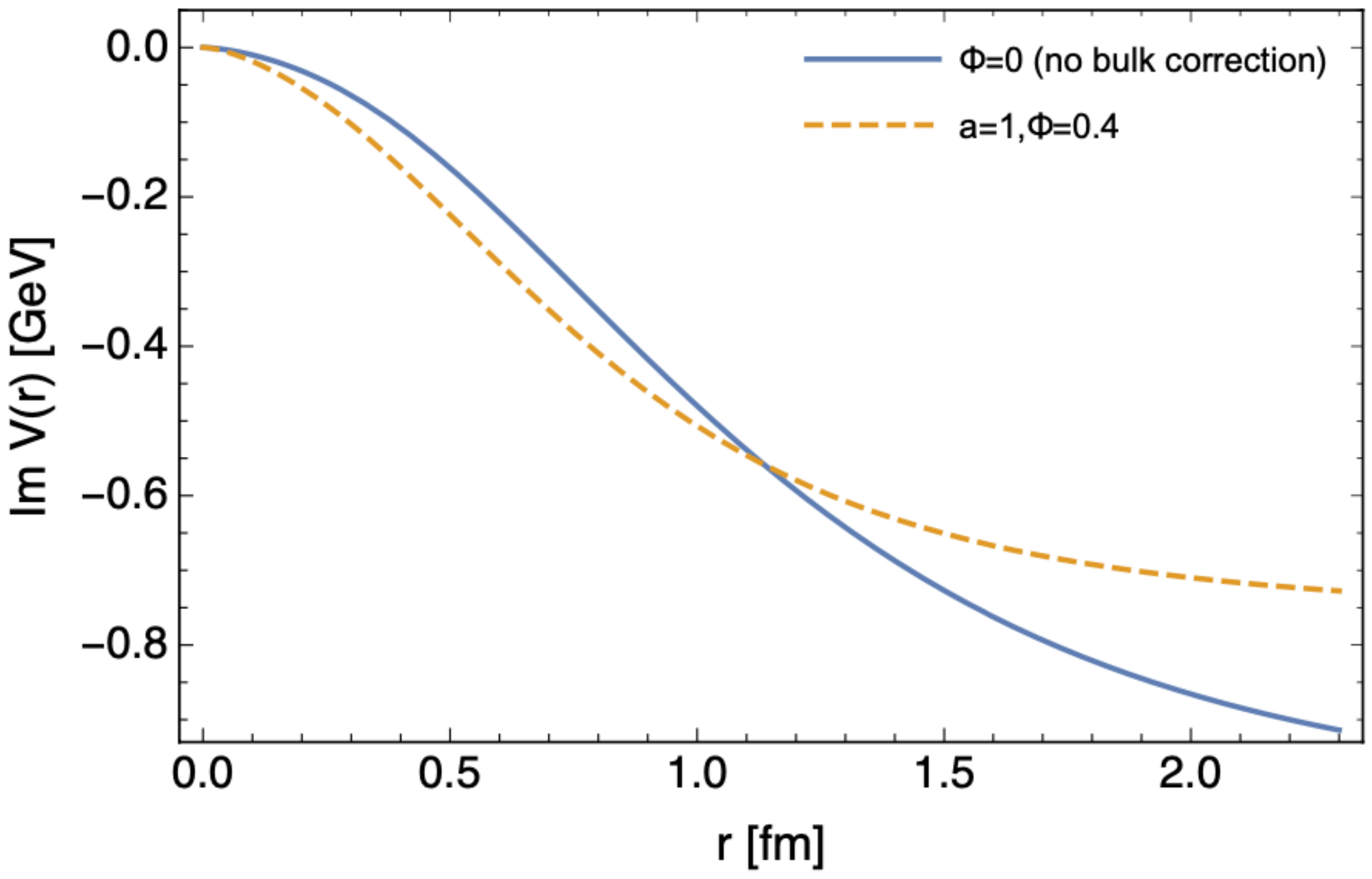
Where

$$\phi_n(x) \equiv 2\int_0^\infty dz \frac{z}{(z^2+1)^n}\left[1 - \frac{\sin(xz)}{xz}\right]$$
$$\chi(x) \equiv 2\int_0^\infty \frac{dz}{z(z^2+1)^2}\left[1 - \frac{\sin(xz)}{xz}\right]$$

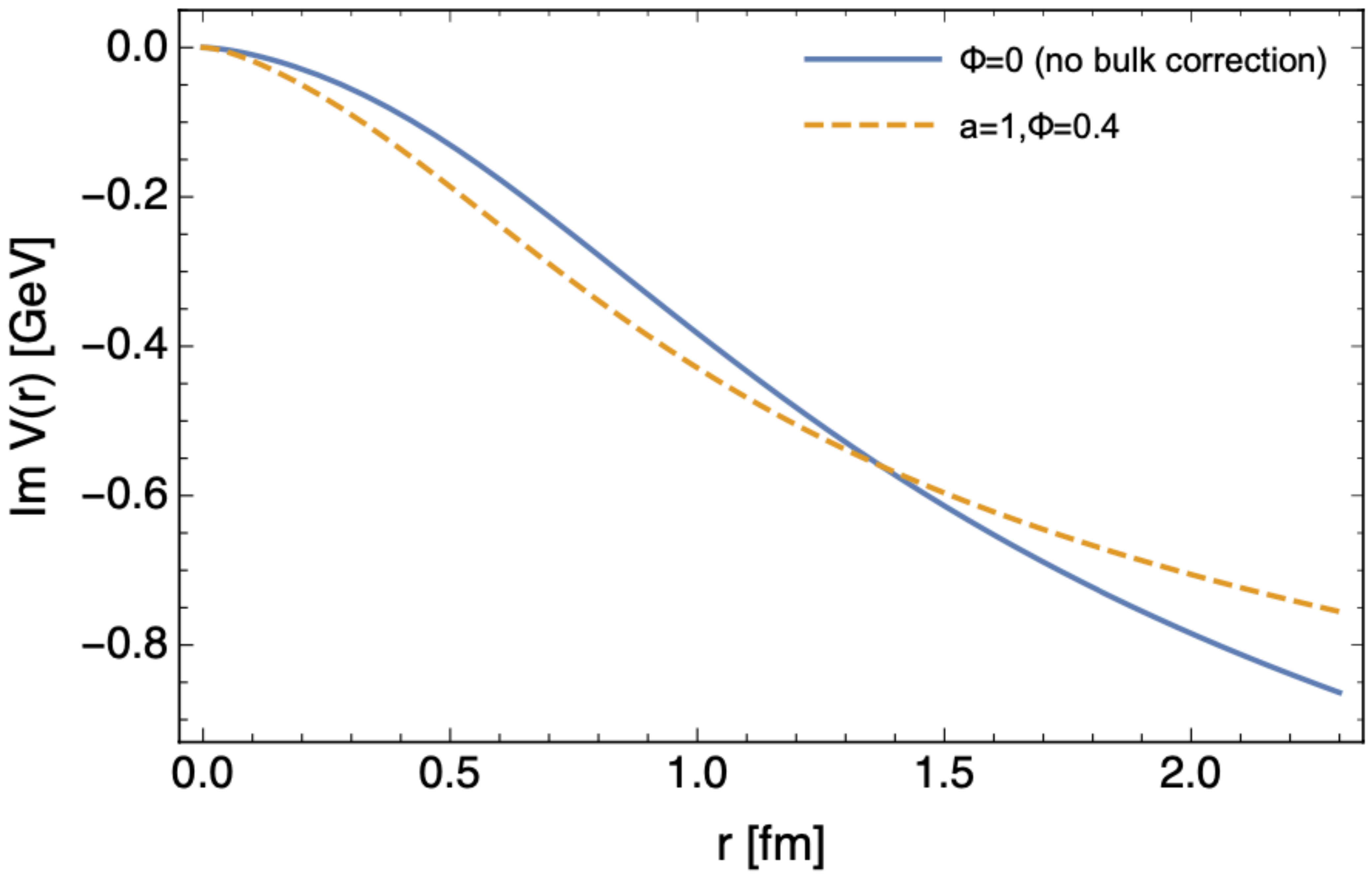
$$\lambda \equiv \frac{\widetilde{m}_{D,S}^2}{\widetilde{m}_{D,R}^2}$$



Imaginary part of the potentials in the presence of bulk viscosity in other prescriptions



Guo et. al., PRD 100, (2019) 036011



Lafferty and Rothkopf, PRD 101, (2020) 056010

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Quarkonium properties

Time-independent Schrödinger equation for the radial wave function

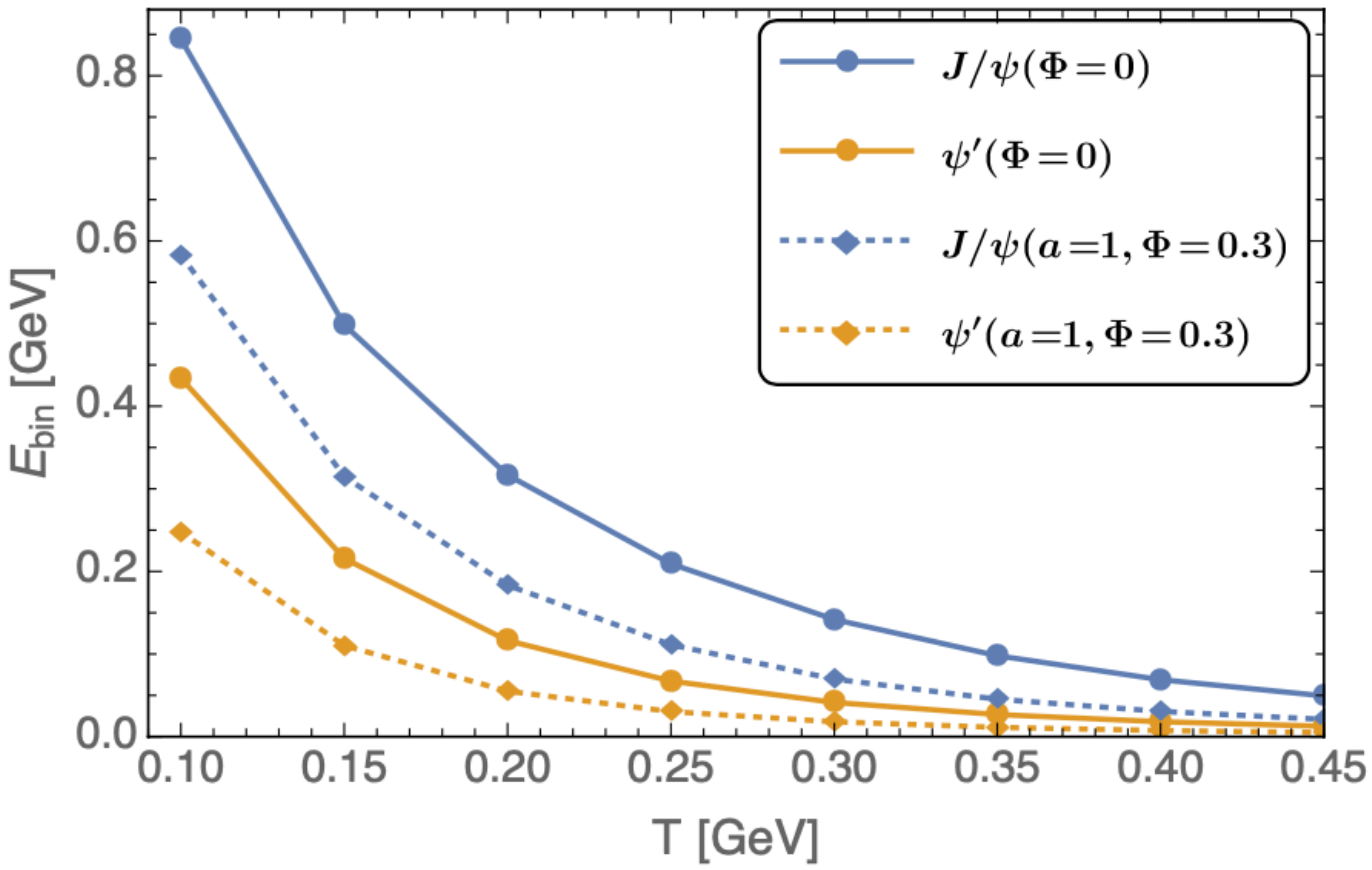
$$-\frac{1}{2m_q}\left(\psi''(r)+\frac{2}{r}\psi'(r)-\frac{\ell(\ell+1)}{r^2}\psi(r)\right)+\mathrm{Re}\,V(r)\,\psi(r)=\epsilon_{n\ell}\,\psi(r)$$

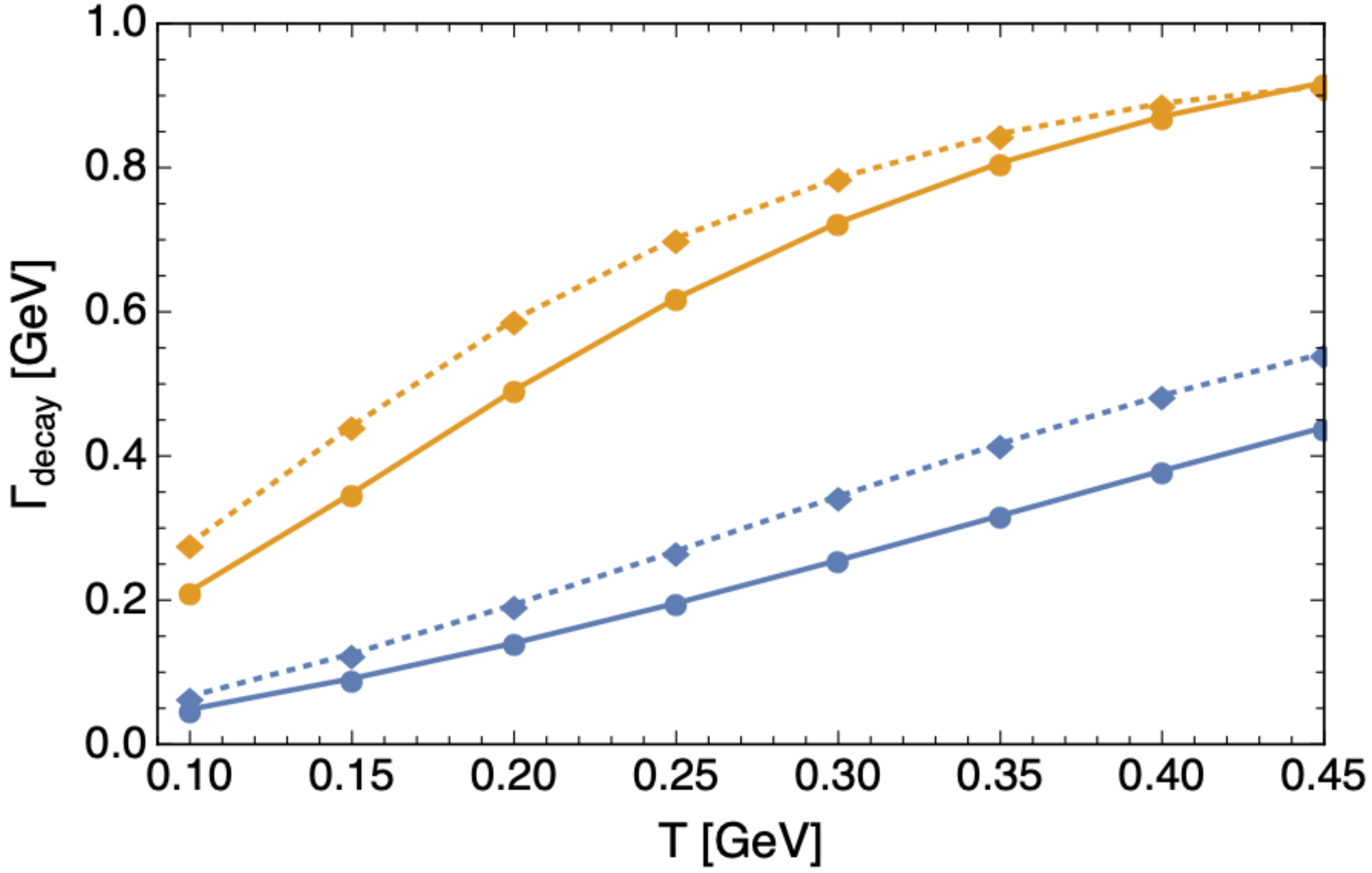
$$\mathrm{Re}\,V(r)=-\alpha\,\widetilde{m}_{D,R}\left(\frac{e^{-\widetilde{m}_{D,R}\,r}}{\widetilde{m}_{D,R}\,r}+1\right)+\frac{2\sigma}{\widetilde{m}_{D,R}}\left(\frac{e^{-\widetilde{m}_{D,R}\,r}-1}{\widetilde{m}_{D,R}\,r}+1\right)$$

Binding energy $$E_{\mathrm{bin}}=\mathrm{Re}\,V(r\to\infty)-\epsilon_{n\ell}$$

Decay width $$\Gamma=-\langle\psi|\,\mathrm{Im}\;V(r)|\psi\rangle=-\frac{\int dr\,r^2|\psi(r)|^2\,\mathrm{Im}\,V(r)}{\int dr\,r^2|\psi(r)|^2}$$

Binding energies and decay widths in the presence of bulk viscous corrections



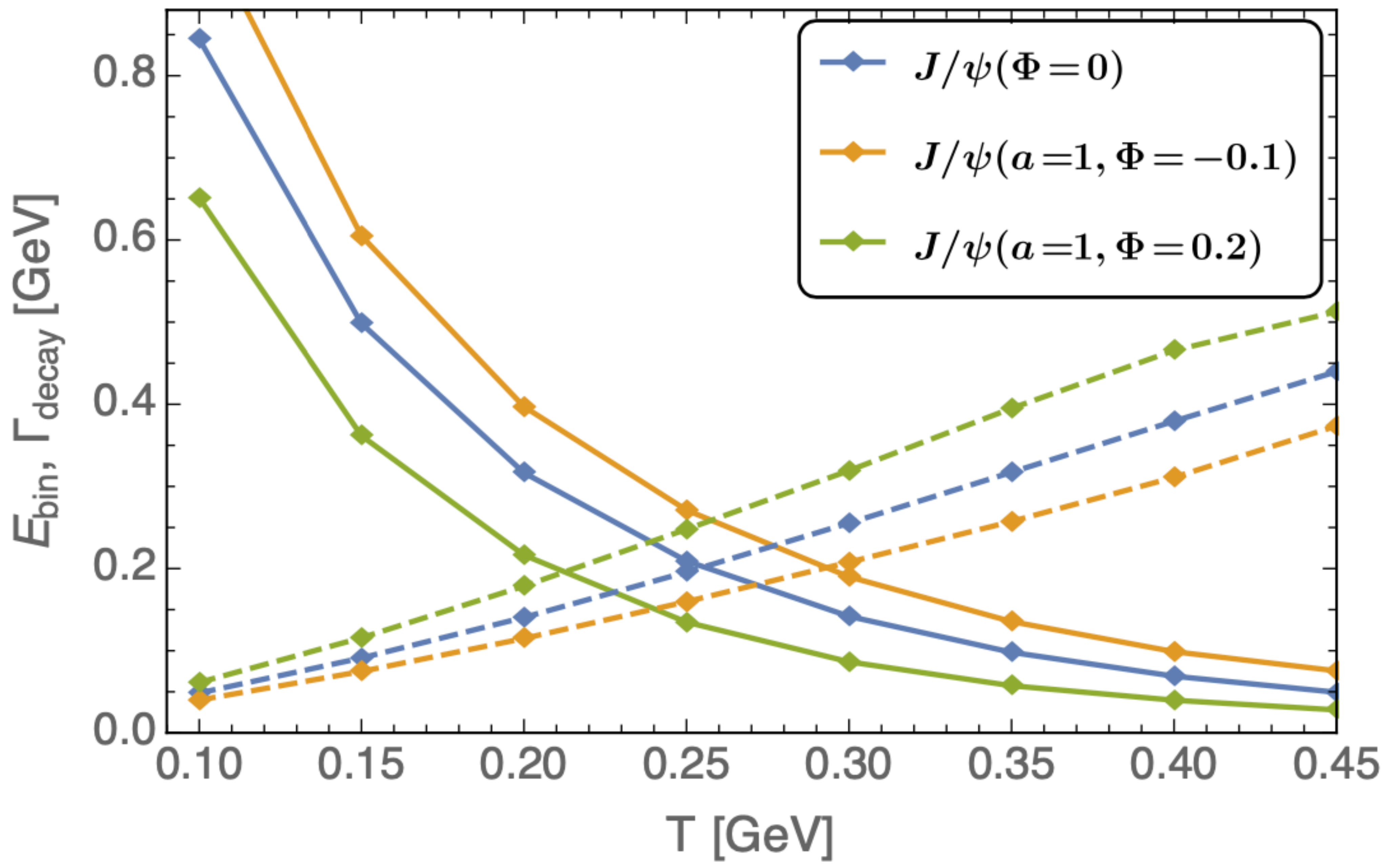


Melting temperatures of quarkonium states

Melting temperature is defined by

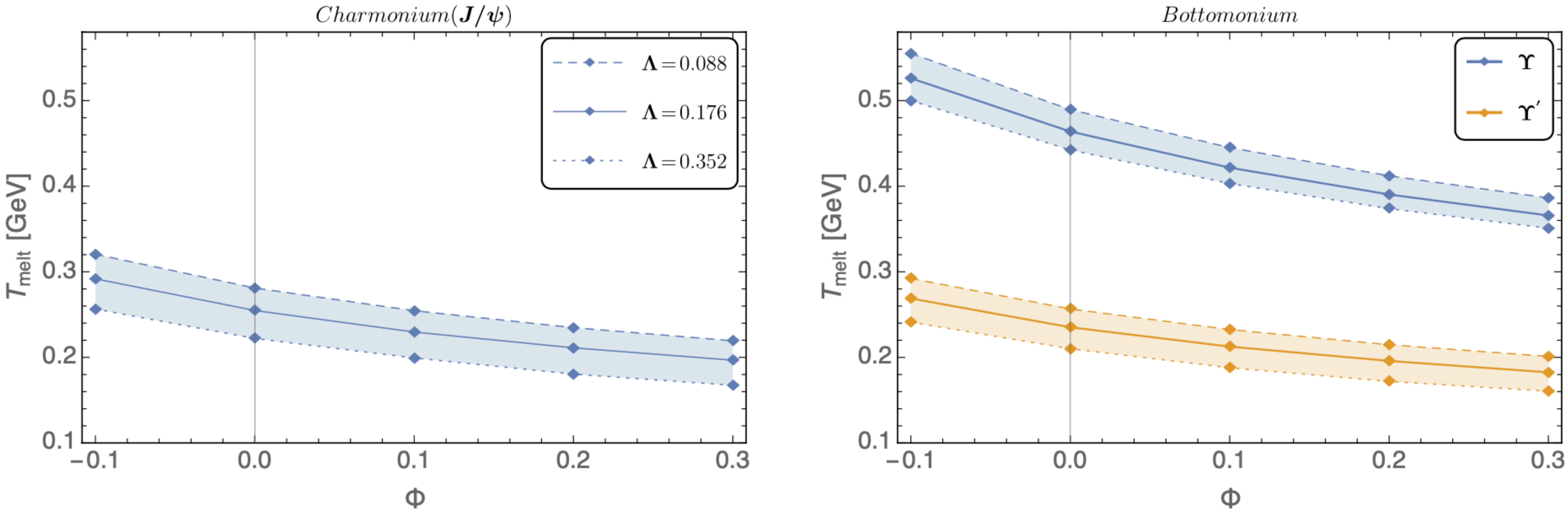
$$\Gamma(T) = E_{bin}(T)$$

Mocsy and Petreczky PRL 99, 211602 (2007)



	$T_{\rm melt}$ [GeV]	**Lattice results** PRD 101 (2020) 056010
J/ψ	$0.254^{+0.027}_{-0.032}$	$0.267^{+0.033}_{-0.036}$
Υ	$0.464^{+0.026}_{-0.022}$	$0.440^{+0.080}_{-0.055}$
Υ'	$0.235^{+0.025}_{-0.022}$	$0.250^{+0.050}_{-0.053}$

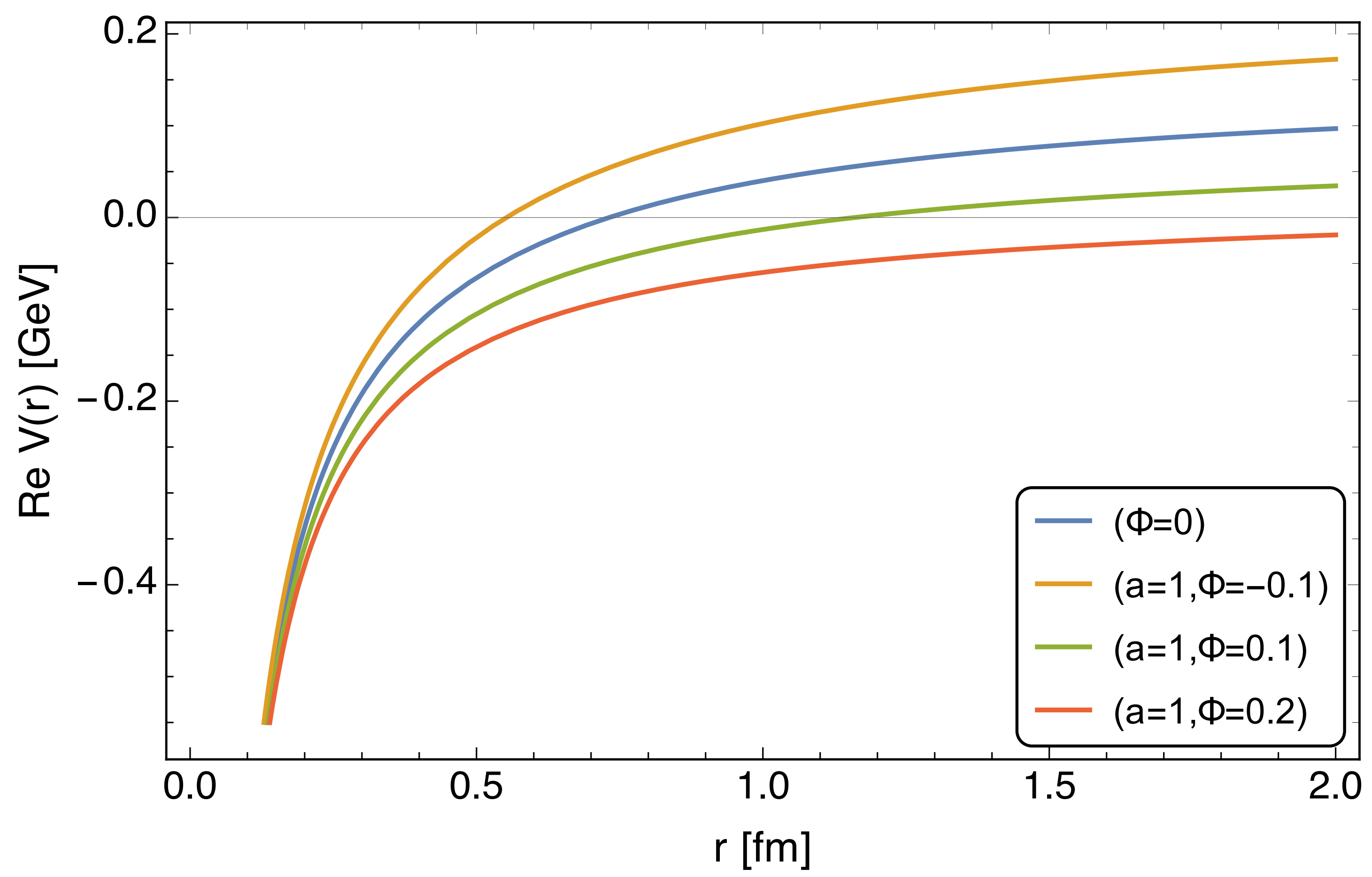
Melting temperatures as a function of bulk viscous corrections

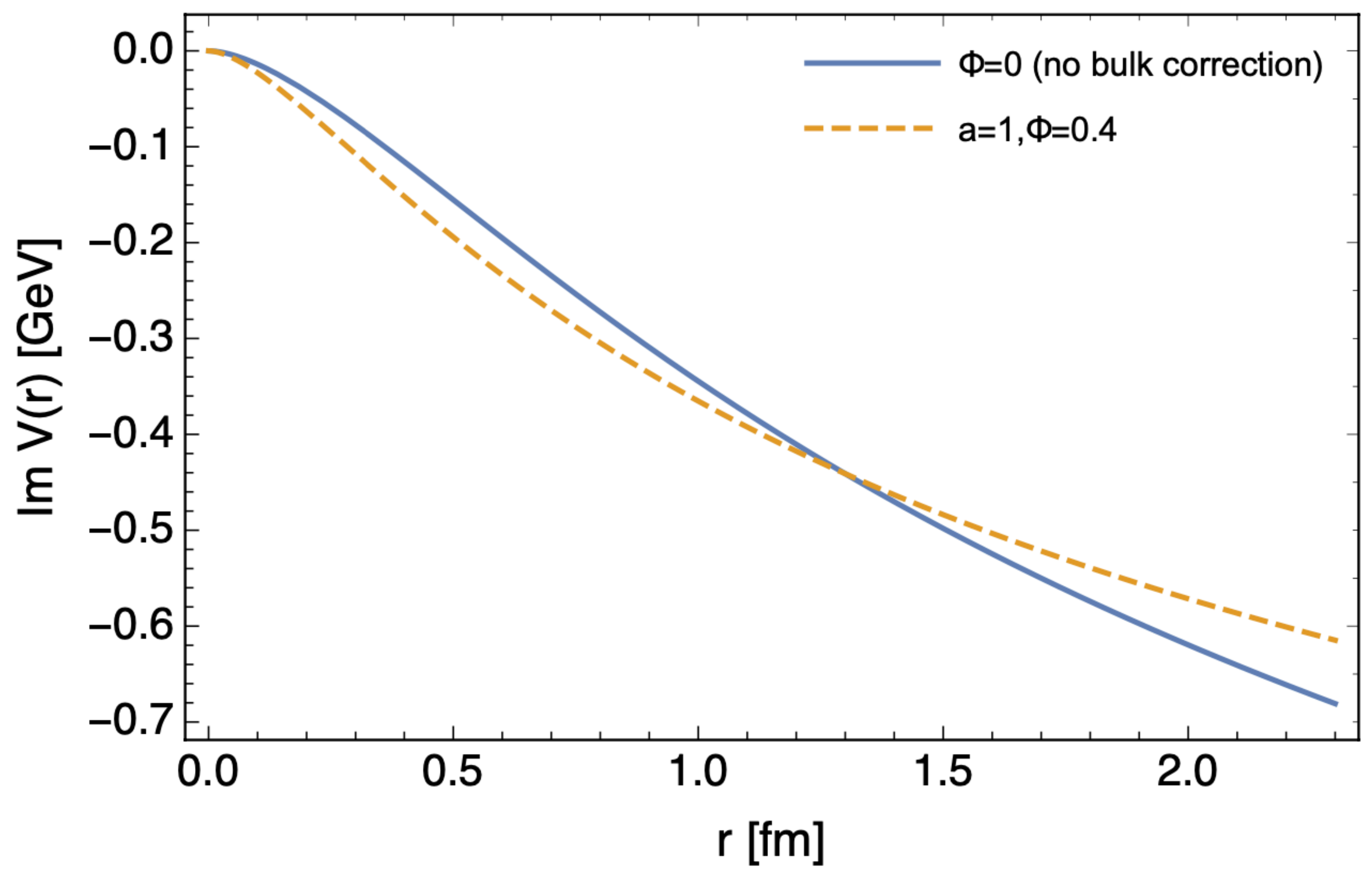


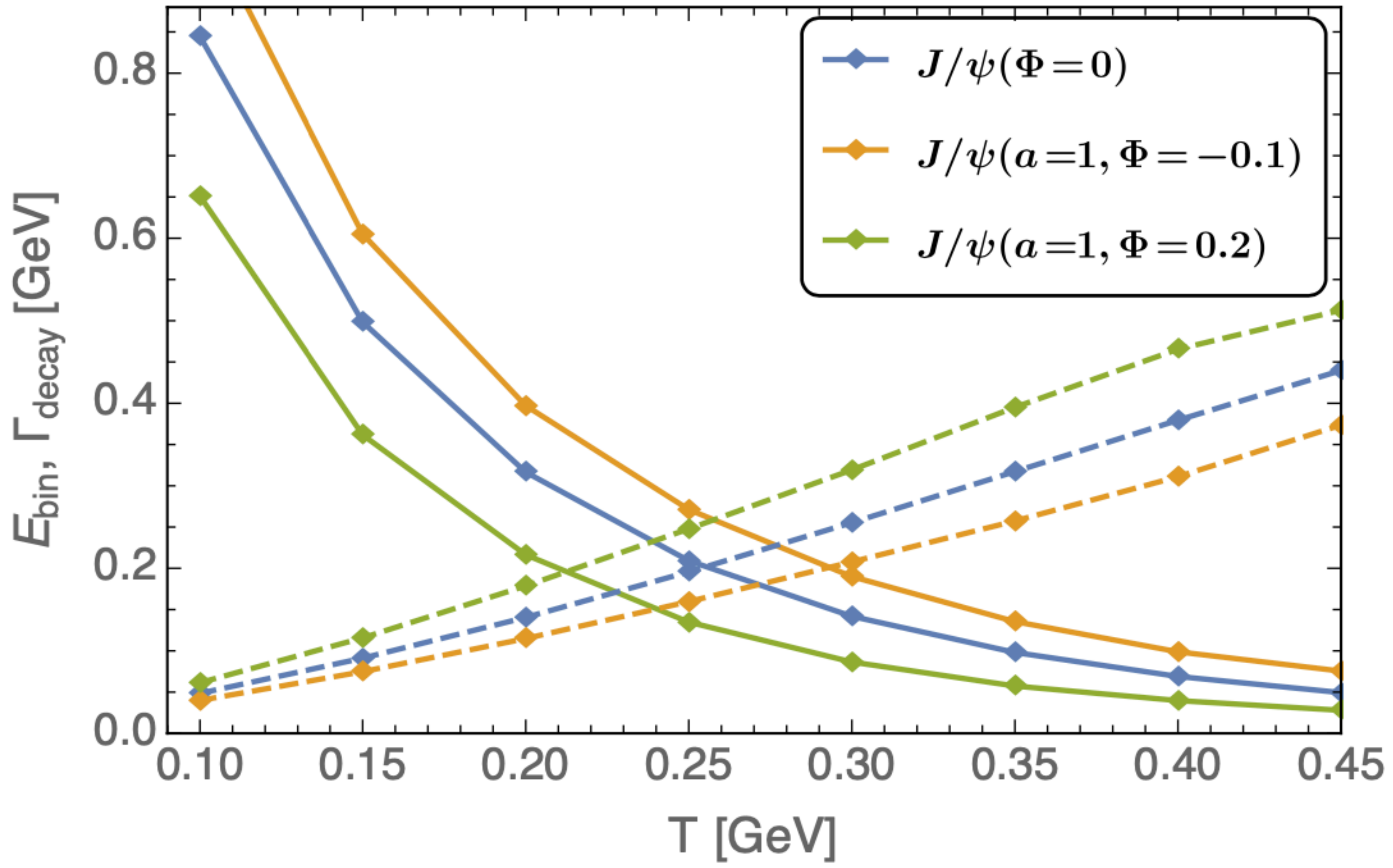
Thakur, Haque and Hirono, JHEP 06 (2020) 071

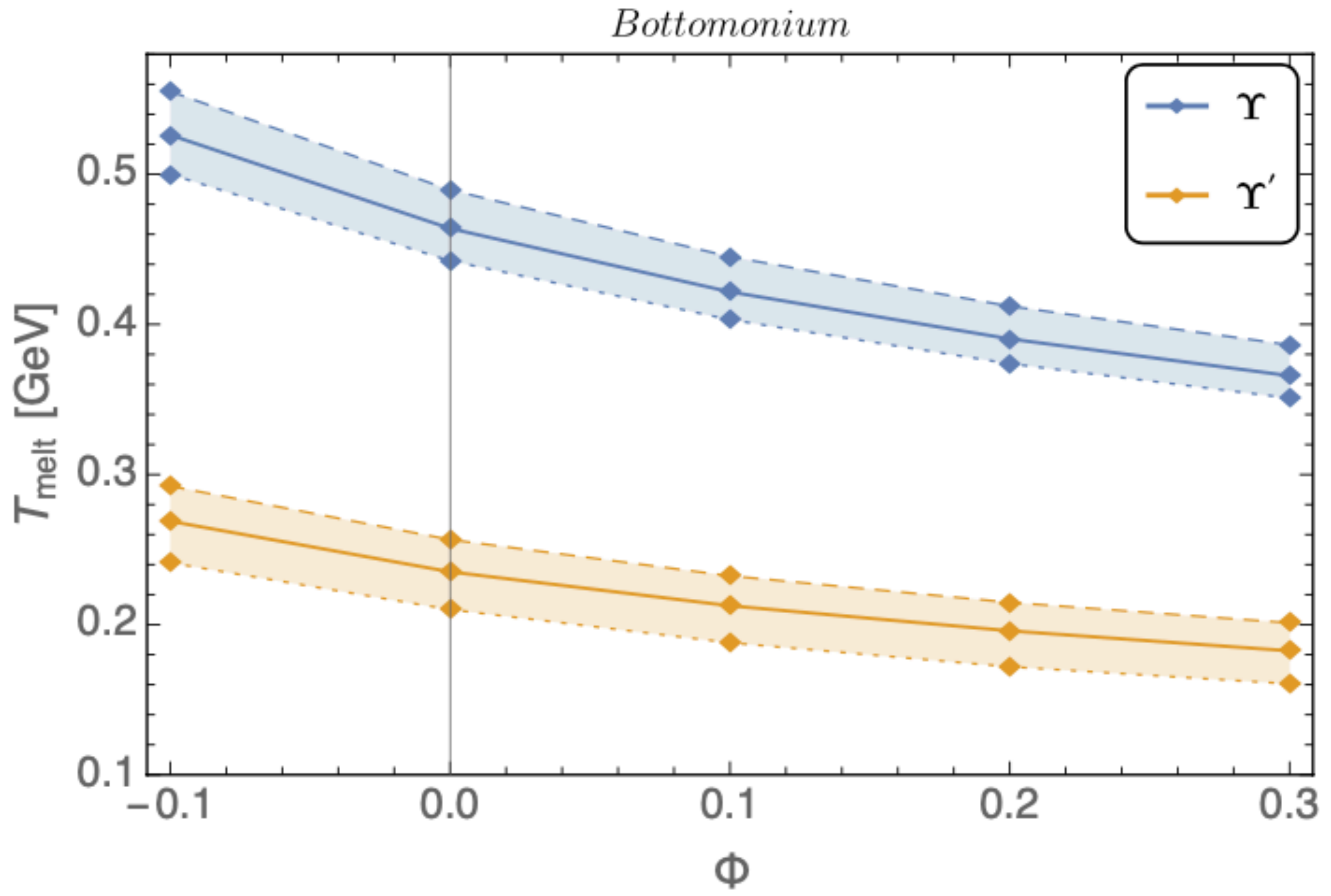
Summary

- **Heavy quarkonia are sensitive to the bulk viscous QGP medium**









Thank you !

Backup slides

Parameters

One-loop coupling constant

$$\alpha_s = \frac{g^2}{4\pi} = \frac{6\pi}{(11N_c - 2N_f)\log\left(2\pi\sqrt{T^2 + \mu^2/\pi^2}/\Lambda\right)}$$

$$\alpha = C_F\,\alpha_s$$

$$\sigma = (0.44\ \mathrm{GeV})^2$$

$$m_b = 4.66\ \mathrm{GeV}$$

$$m_c = 1.25\ \mathrm{GeV}$$

$$\Lambda = 0.176\ GeV$$

$$c_S^{q,g} = \frac{1}{\Phi} \frac{\int dk\ k^2 \delta_{\text{bulk}} f(k)(1 \pm 2 f_0(k))}{\int dk\ k^2 f_0(k)(1 \pm f_0(k))}$$

$$c_R^{q,g} = \frac{1}{\Phi} \frac{\int kdk\ \delta_{\text{bulk}} f(k)}{\int kdk\ f_0(k)}$$

$$c_R^q(a, \tilde{\mu}) = -\frac{6}{\pi^2 + 3\tilde{\mu}^2}\ \Gamma(a+2) \left[\mathrm{Li}_{a+1}(-e^{\tilde{\mu}}) + \mathrm{Li}_{a+1}(-e^{-\tilde{\mu}})\right]$$

$$c_R^g(a) = \frac{6}{\pi^2} \Gamma(a+2) \zeta(a+1)$$

Comparison with different potential models in equilibrium case