Recent progress from the lattice on the QCD phase diagram

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Wuppertal-Budapest collaboration

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The QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \overline{\psi} \left(\mathrm{i}\gamma_\mu D^\mu - m \right) \psi$$

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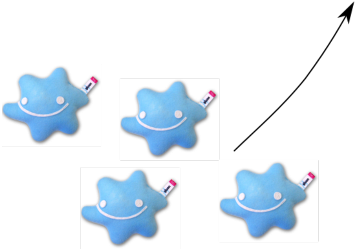

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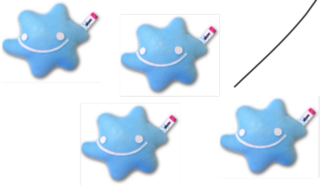

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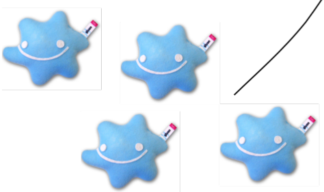
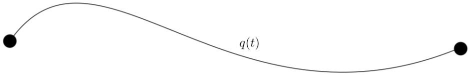


The path integral quantization: from M to QM to QFT

Mechanics:



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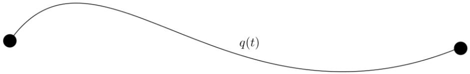
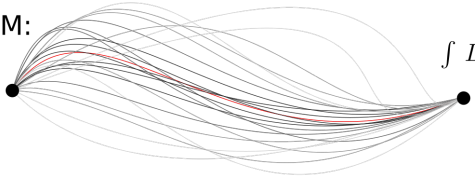


QM:



$$\int Dq(t) e^{\mathrm{i}\int \mathrm{d}t\, L}$$

The path integral quantization: from M to QM to QFT

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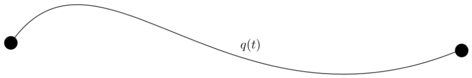

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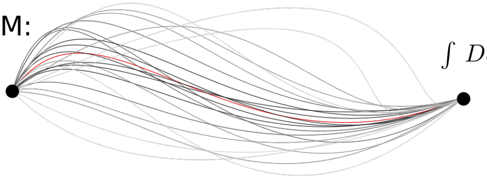

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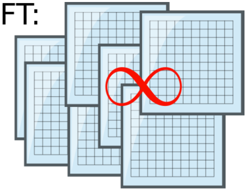
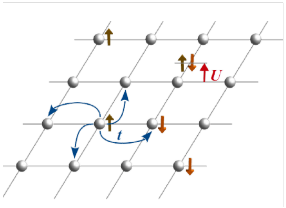
QFT:

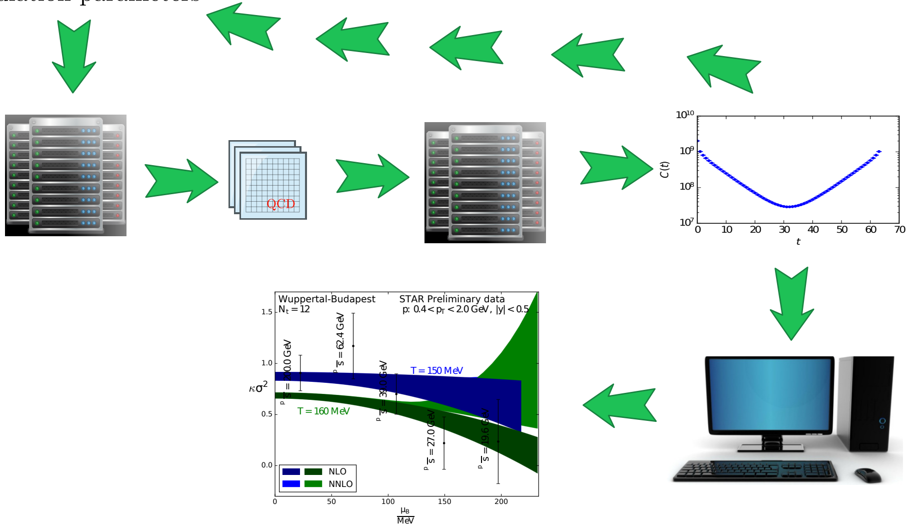


$$\int \mathcal{D}\phi(x)e^{\mathrm{i}\int \mathrm{d}^4x\mathcal{L}}$$



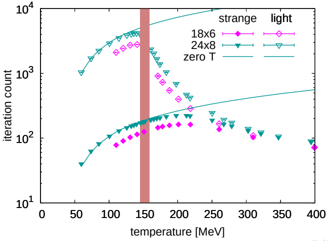
The work flow

simulation parameters



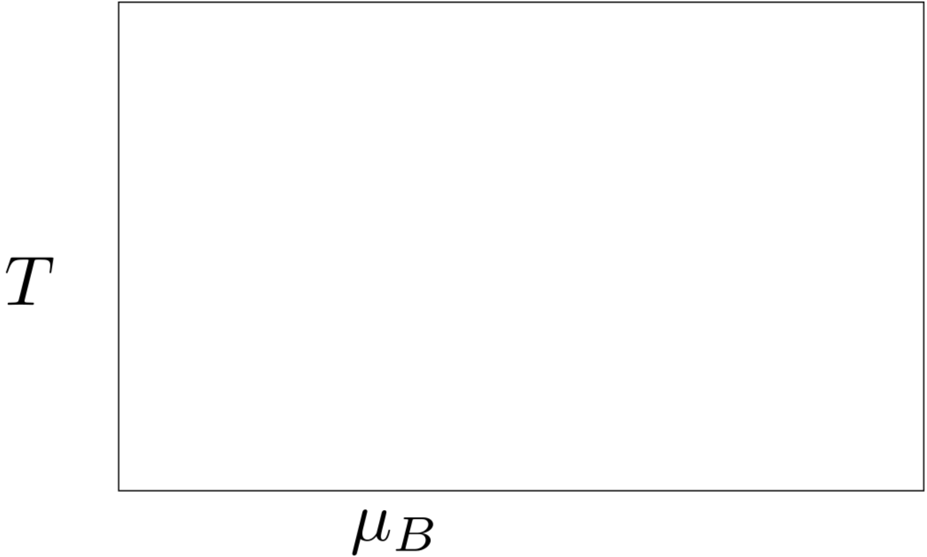
Why aren't we finished yet?

- Simulations take a lot of computer time
- Not everything can be calculated directly. For example:
  - Only thermal equilibrium
  - Only simulations at $\mu_B = 0 \Rightarrow \langle n_B \rangle = 0$ ↯
    heavy ion collision experiments



1000 configurations on a $64^3 \times 16$ lattice cost about 1 million core hours

The (T, μ_B)-phase diagram of QCD



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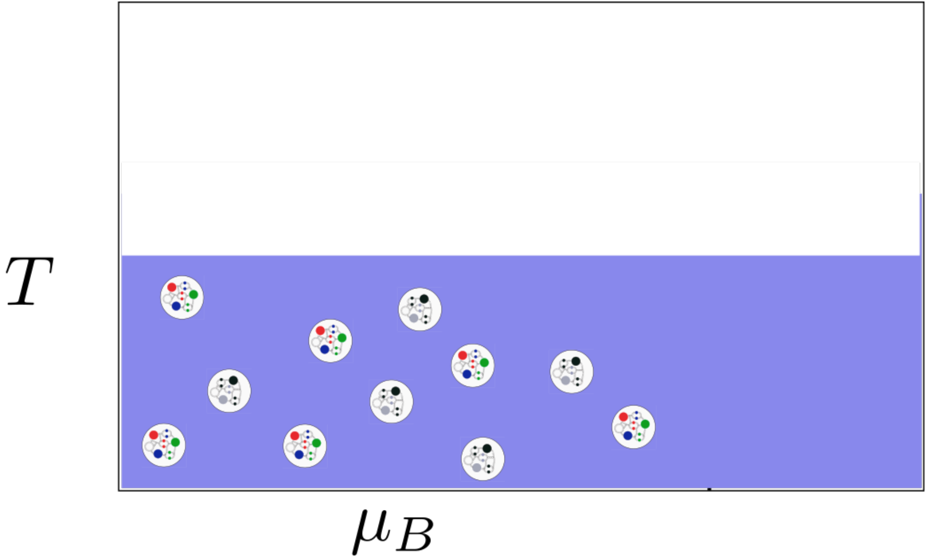

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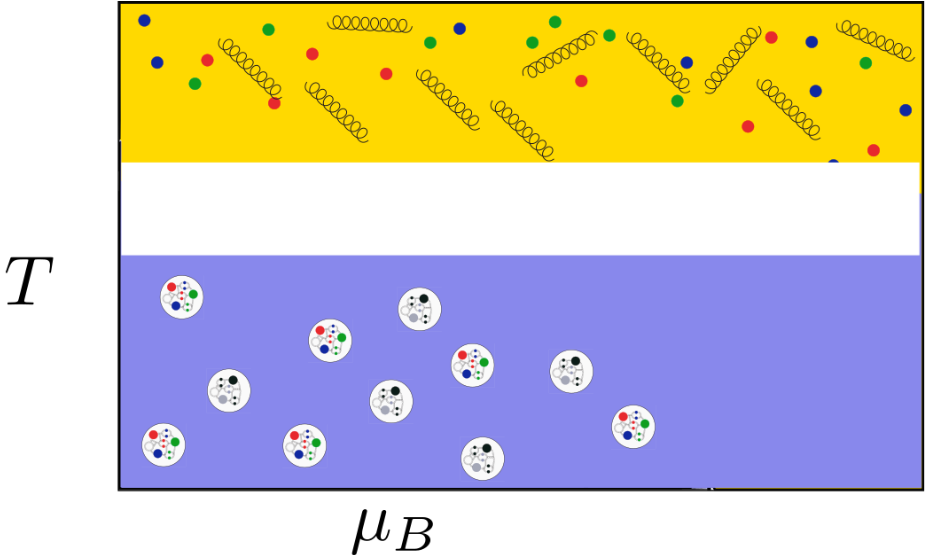

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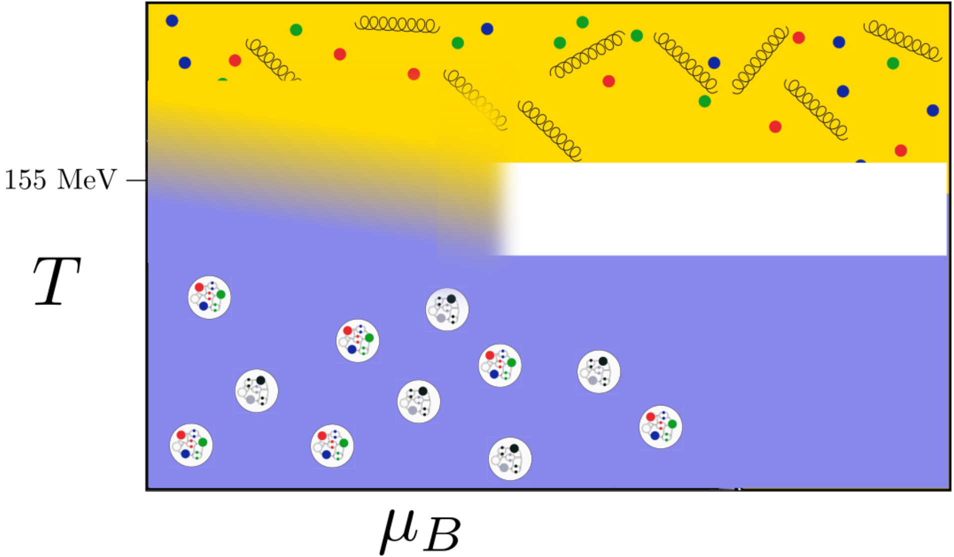

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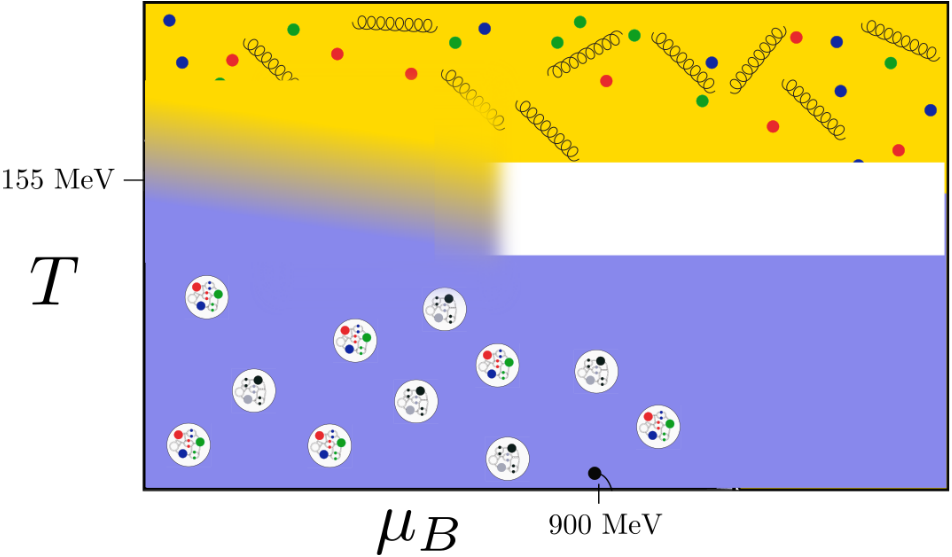

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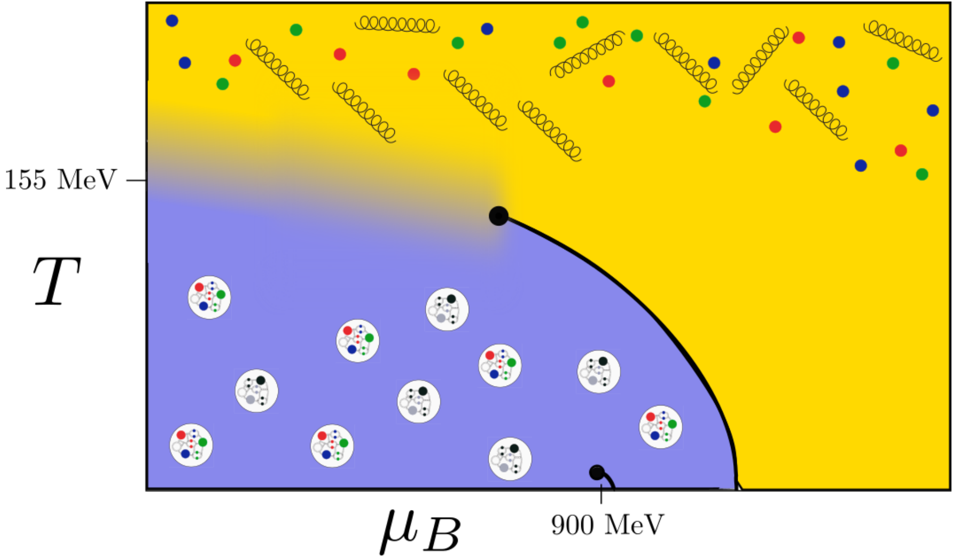

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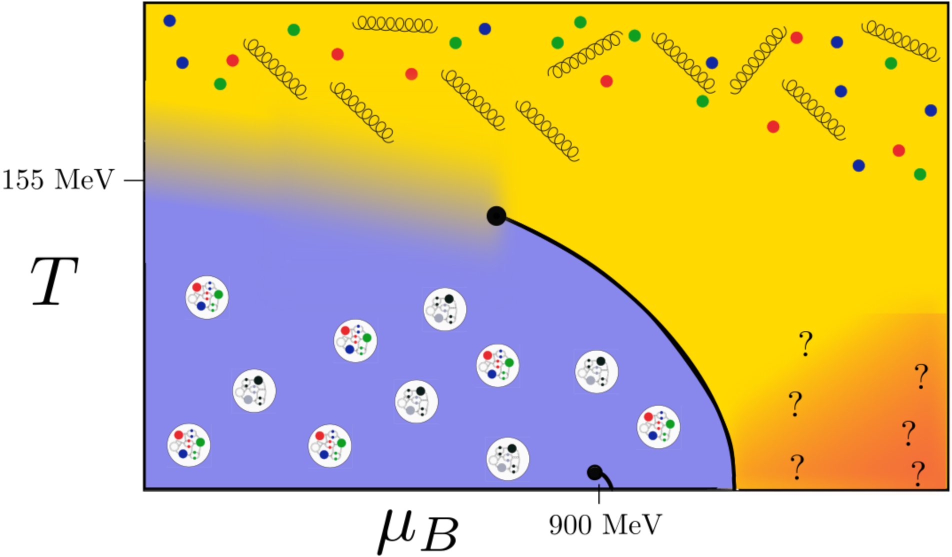

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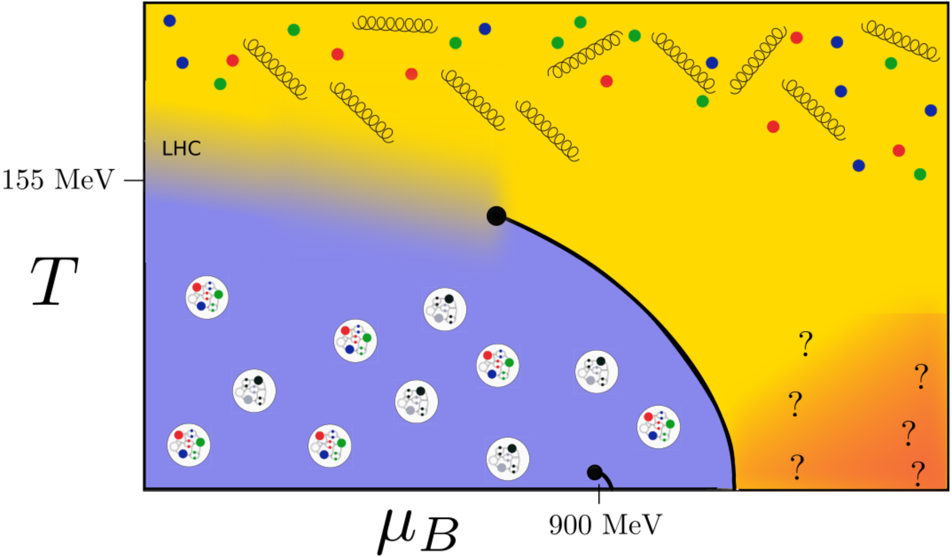

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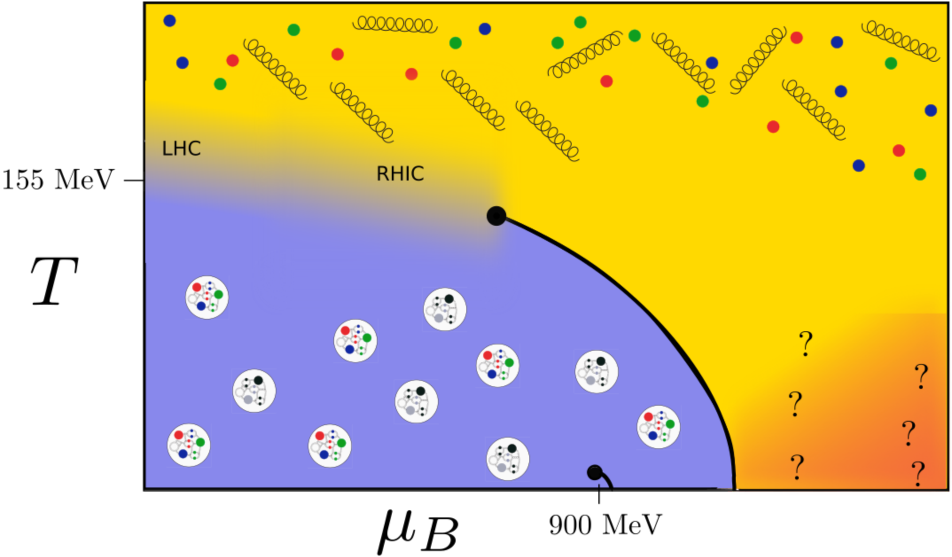

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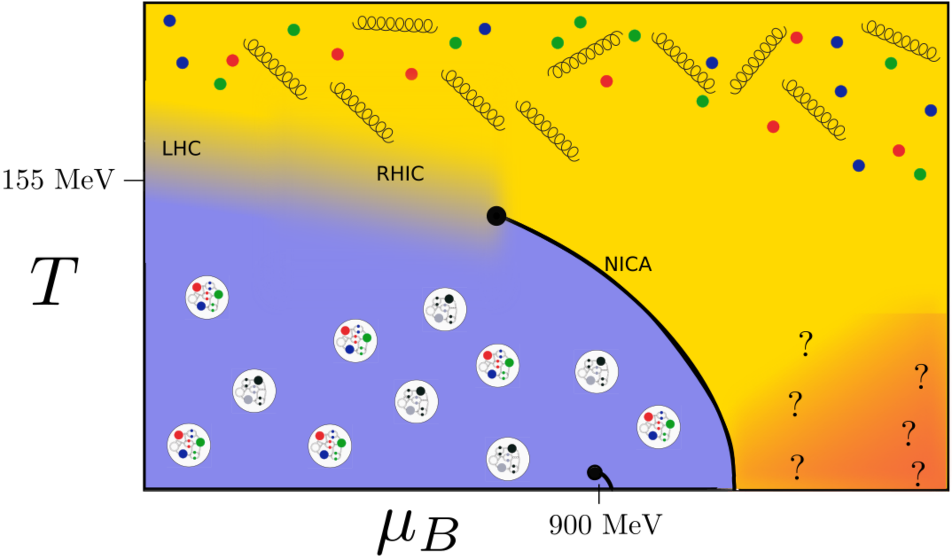

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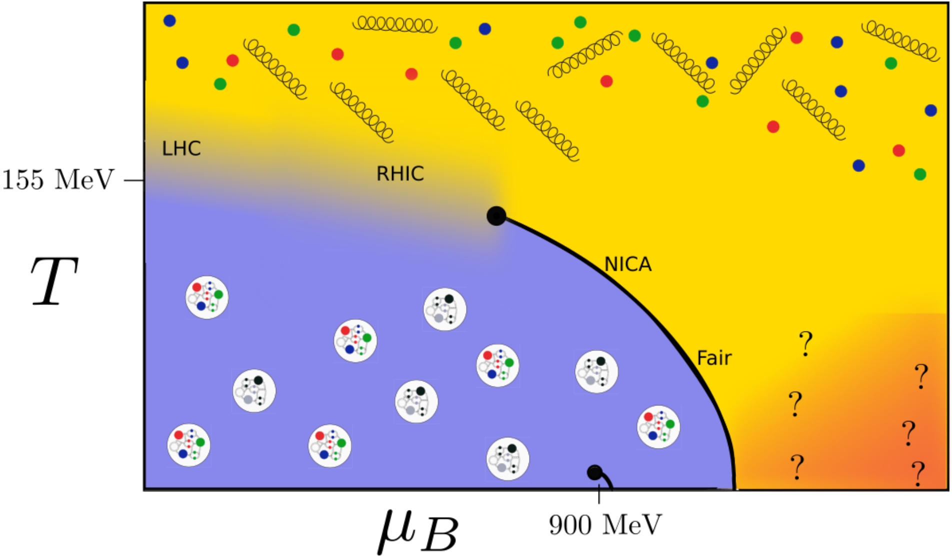

Dealing with the sign problem

- Reweighting techniques
- Canonical ensemble
- Lefshetz Thimble
- Density of state methods
- Dual variables
- Complex Langevin
- ...

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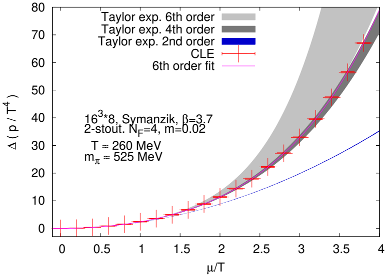

[Sexty:2019vqx]

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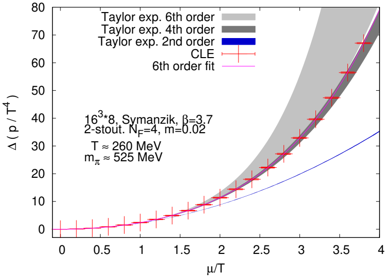
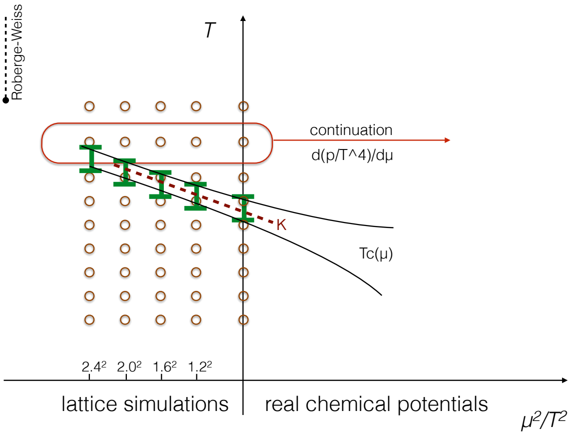


[Sexty:2019vqx]

- (Taylor) expansion
- Imaginary μ

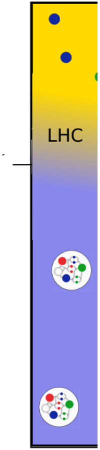
Analytic continuation from imaginary chemical potential



Common technique:

- [deForcrand:2002hgr]
- [Bonati:2015bha]
- [Cea:2015cya]
- [DElia:2016jqh]
- [Bonati:2018nut]
- [Borsanyi:2018grb]
- [Borsanyi:2020fev]
- [Bellwied:2021nrt]
- . . .

Expansion from $\mu = 0$



Taylor expansion

$$\frac{p}{T^4} = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j!k!} \chi_{jk}^{BS} \hat{\mu}_B^j \hat{\mu}_S^k$$

with $\hat{\mu} = \frac{\mu}{T}$

- rapid convergence in Stephan-Boltzmann ($T = \infty$) limit
- expansion coefficients are lattice observables

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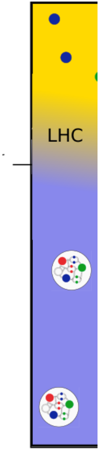

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Fugacity expansion/sector method

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- information about particle content

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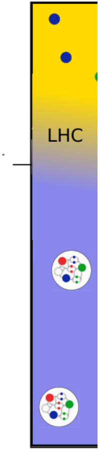

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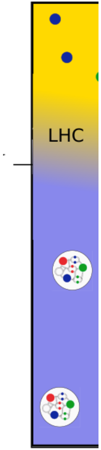
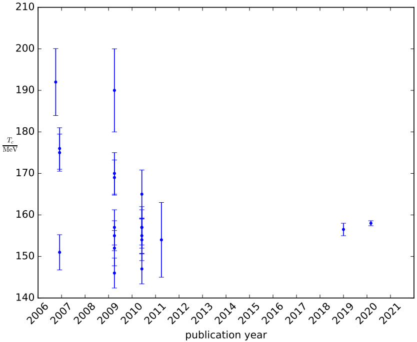
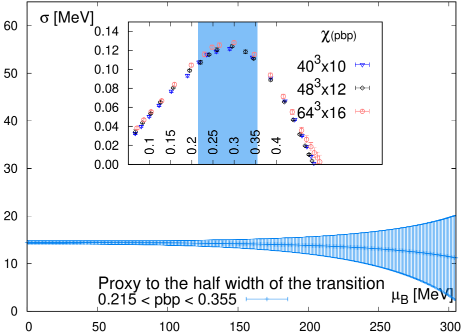
- often the expansion is done for a specific choice of μ_S

The transition temperature





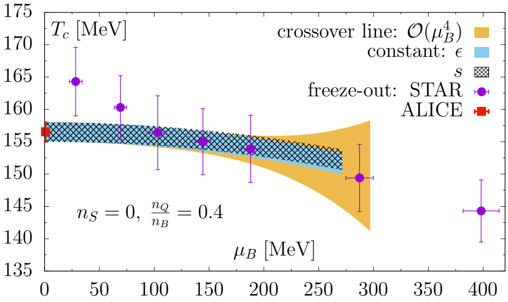
[Cheng:2006qk], [Aoki:2006br], [Aoki:2009sc], [Bazavov:2009zn], [Borsanyi:2010bp], [Bazavov:2011nk], [Bazavov:2018mes], [Borsanyi:2020fev]



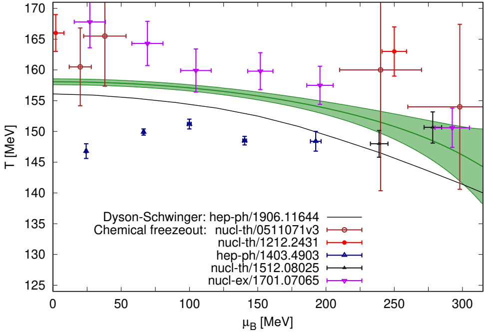
[Borsanyi:2020fev]

Extrapolation of the transition temperature

[Bazavov:2018mes]
Results from the Taylor expansion method
HISQ quarks
Continuum limit from $N_t = 6, 8, 12$

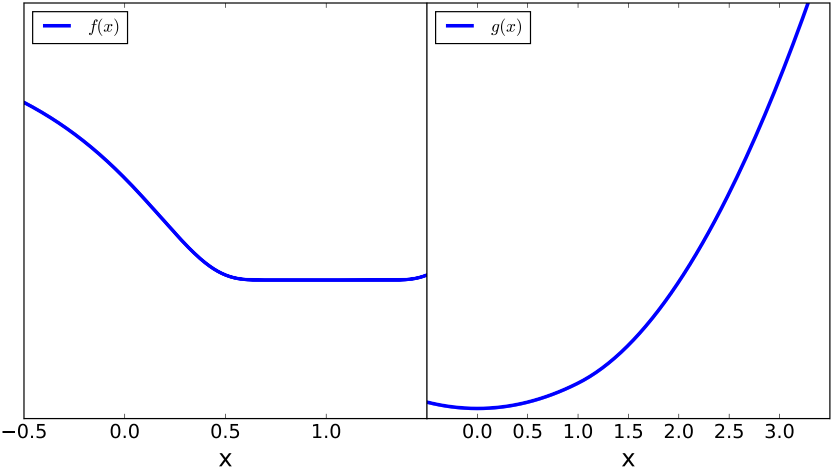


[Borsanyi:2020fev]
Results from the imaginary potential method
staggered quarks
Continuum limit from $N_t = 10, 12, 16$



chemical freezeout: abundancies of hadrons are fixed (frozen-in)
kinetic freezeout: momentum distributions are fixed

Does that mean there is no critical endpoint?



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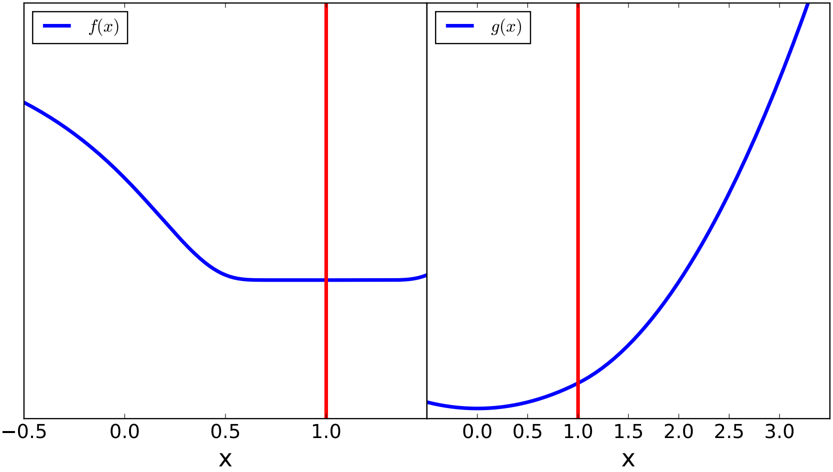

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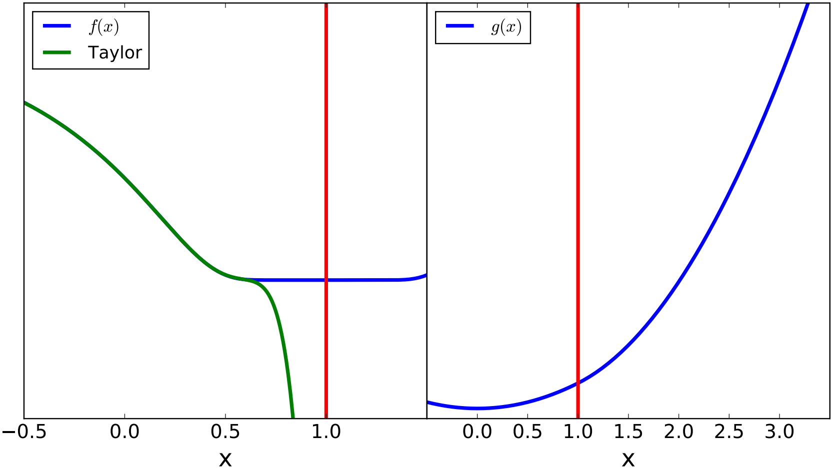

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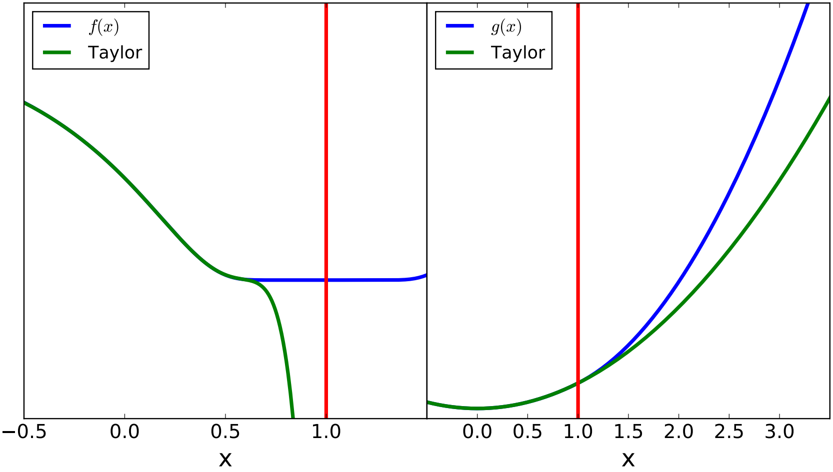
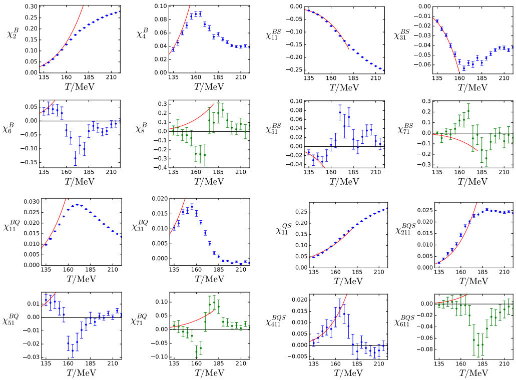


Fluctuations on the lattice

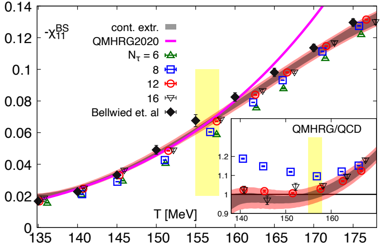
$$\chi_{i,j,k}^{B,Q,S} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}\,, \quad \hat{\mu}_i = \frac{\mu}{T}$$

- can be calculated on the lattice
- can be compared to various models
- can be compared to experiment
- can be used as building blocks for various observables



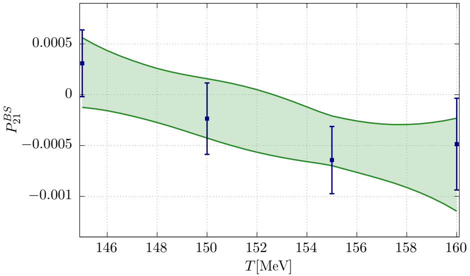
[Borsanyi:2018grb]

Low order fluctuations with high precision



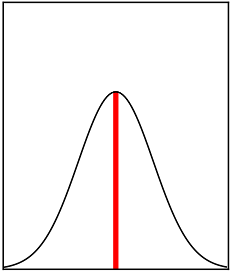
- [Bollweg:2021vqf]
- HISQ
- New continuum extrapolated results ($N_t = 6, 8, 12, 16$) allow for detailed comparisons with various models
- Quark model states are needed for HRG

- [Bellwied:2021nrt]
- continuum estimate from $N_t = 8, 10, 12$
- stout smeared staggered
- contributions vom $N - \Lambda$, $N - \Sigma$ scattering
- negative contribution in the Fugacity expansion indicate repulsive interaction that cannot be described with more resonances



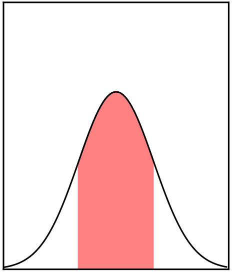
Observables

Cumulants of the net baryon number distributions:



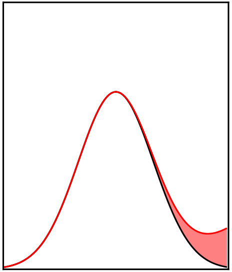
mean

$M_B = \chi_1^B$



variance

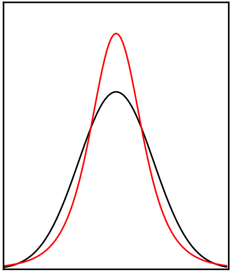
$\sigma_B^2 = \chi_2^B$



skewness

$S_B = \frac{\chi_3^B}{\left(\chi_2^B\right)^{3/2}}$

asymmetry of the distribution



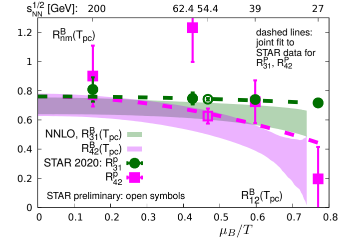
kurtosis

$\kappa_B = \frac{\chi_4^B}{\left(\chi_2^B\right)^2}$

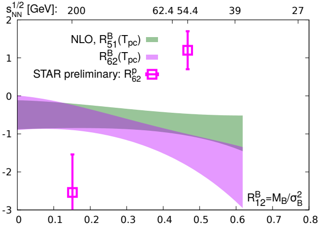
“tailedness” of the distribution

Comparison with heavy ion collision experiments

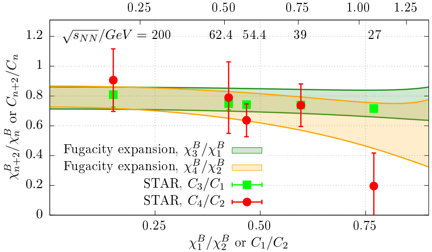
Continuum estimate from $N_t = 8, 12$



$N_t = 8$



- [Bazavov:2020bjn]
- Taylor method
- HISQ



- [Bellwied:2021nrt]
- continuum estimate from $N_t = 8, 10, 12$
- stout smeared staggered

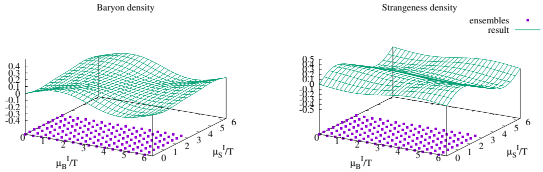
- 2d-extrapolation in μ_B and μ_S
- Fugacity expansion and imaginary chemical potential

Extrapolations are done along the transition line.

2d-Extrapolation: [Bellwied:2021nrt]

144 ensembles for each temperature and lattice

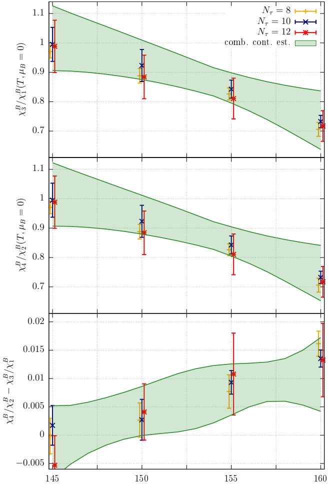
Example at $T = 155$ MeV:



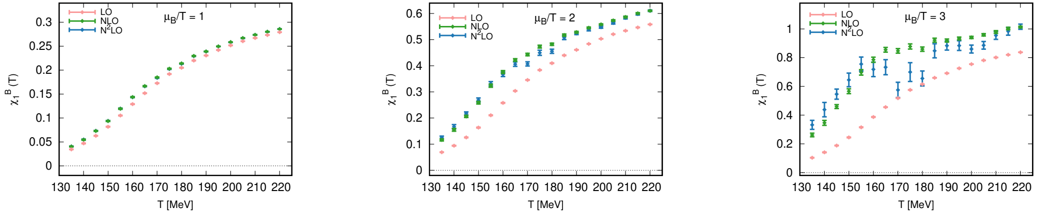
$$P(T, \hat{\mu}_B^I, \hat{\mu}_S^I) = \sum_{j,k} P_{jk}^{BS}(T) \cos(j\hat{\mu}_B^I - k\hat{\mu}_S^I) \ .$$

$$-S = -1, 0, 1, 2, 3; \quad B = 0, 1, 2, 3$$

A surface is fitted on the baryon and strangeness densities, as well as on their susceptibilities.



Trouble with the equation of state



[Borsanyi:2021sxv], [Borsanyi:2018grb], $N_t = 12$

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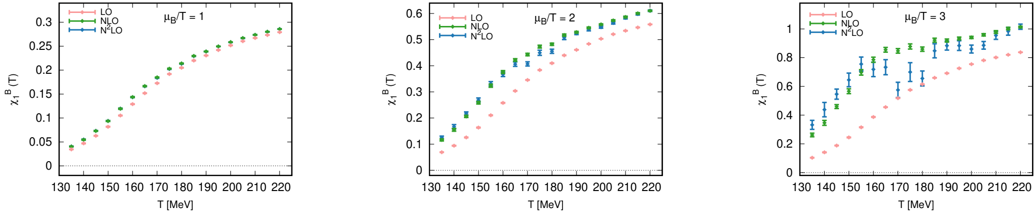

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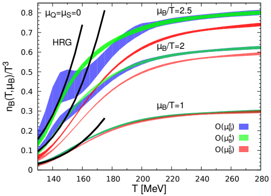
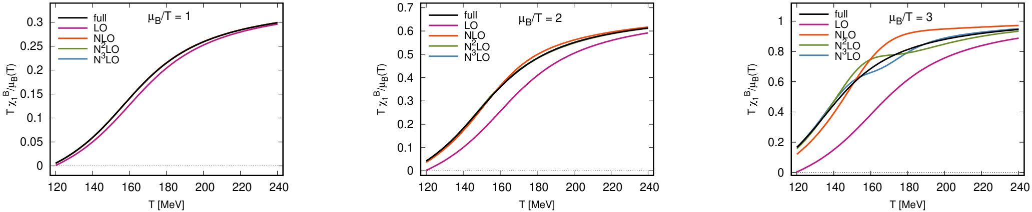
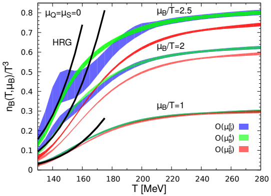


[Bazavov:2017dus]
Taylor method
$N_t = 6, 8, 12, (16)$ (2nd Order)
$N_t = 6, 8$ (4th and 6th Order)

Trouble with the equation of state

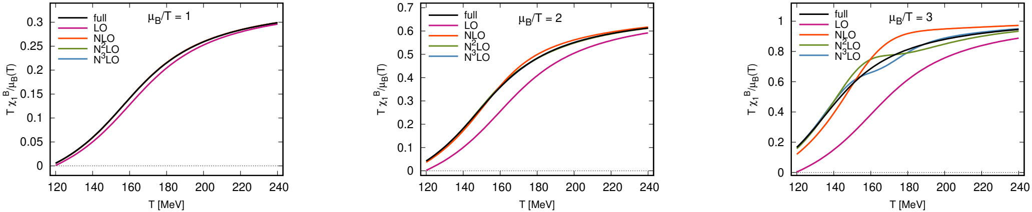


[Borsanyi:2021sxv]

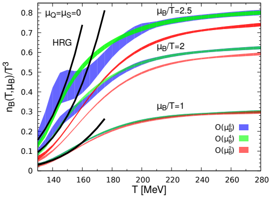


[Bazavov:2017dus]
Taylor method
$N_t = 6, 8, 12, (16)$ (2nd Order)
$N_t = 6, 8$ (4th and 6th Order)

Trouble with the equation of state



[Borsanyi:2021sxv]

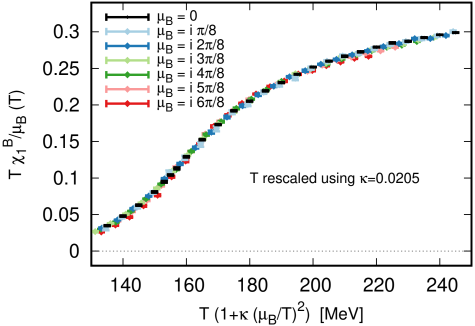


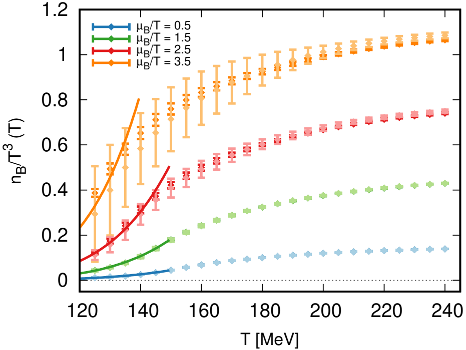
[Bazavov:2017dus]
Taylor method
$N_t = 6, 8, 12, (16)$ (2nd Order)
$N_t = 6, 8$ (4th and 6th Order)

- extrapolation at fixed T cross the transition line
- bad convergence with low order Taylor coefficients

Equation of state

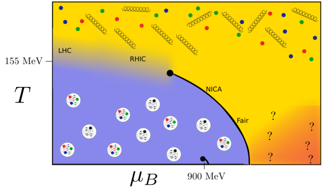
Find a different extrapolation scheme for extrapolating to higher μ_B.

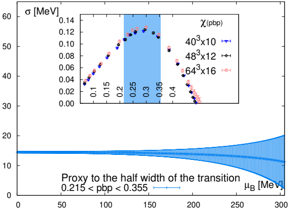


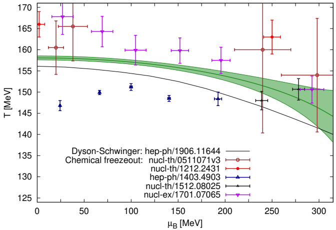


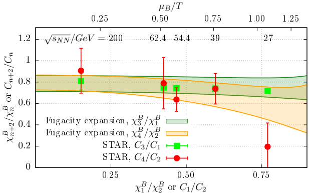
- [Borsanyi:2021sxv]
- $N_t = 10, 12, 16$

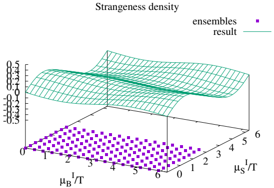
Summary

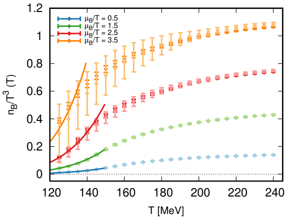

155 MeV
T
LHC
RHIC
NICA
Fair
?
μ_B
900 MeV


σ [MeV]
χ(pbp)
40^3x10
48^3x12
64^3x16
Proxy to the half width of the transition
0.215 < pbp < 0.355
μ_B [MeV]


T [MeV]
μ_B [MeV]
Dyson-Schwinger: hep-ph/1906.11644
Chemical freezeout: nucl-th/0511071v3
nucl-th/1212.2431
hep-ph/1403.4903
nucl-th/1512.08025
nucl-ex/1701.07065


μ_B/T
$\sqrt{s_{NN}}/GeV$ = 200
62.4
54.4
39
27
χ^B_{n+2}/χ^B_n or C_{n+2}/C_n
Fugacity expansion, χ^B_3/χ^B_1
Fugacity expansion, χ^B_4/χ^B_2
STAR, C_3/C_1
STAR, C_4/C_2
χ^B_1/χ^B_2 or C_1/C_2


Strangeness density
ensembles
result
$\mu_B{}^I/T$
$\mu_S{}^I/T$


μ_B/T = 0.5
μ_B/T = 1.5
μ_B/T = 2.5
μ_B/T = 3.5
n_B/T^3 (T)
T [MeV]

The sign problem

The QCD partition function:

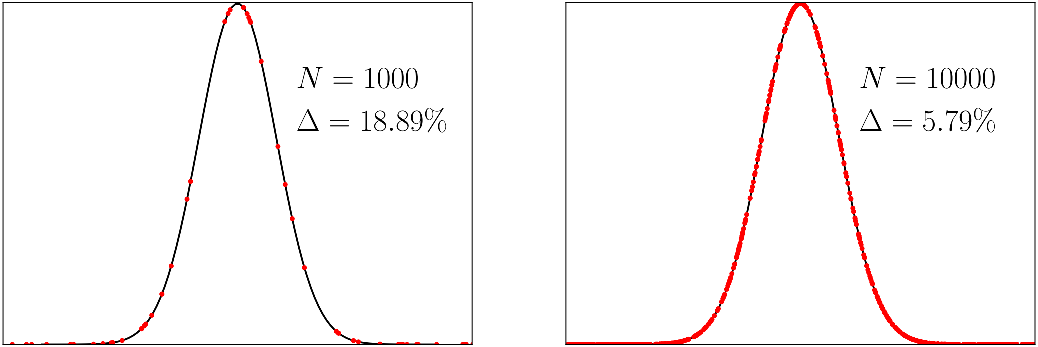
$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \;\det M(U) e^{-\beta S_G(U)}$$

- For Monte Carlo simulations $\det M(U) e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

The sign problem

$$\int_{-\infty}^{\infty} \mathrm{d}x\ (100 - x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \approx \int_{-100}^{100} \mathrm{d}x\ (100 - x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100 - x_i^2)\frac{e^{-\frac{1}{2}x_i^2}}{\sqrt{2\pi}} \cdot \frac{200}{N}$$

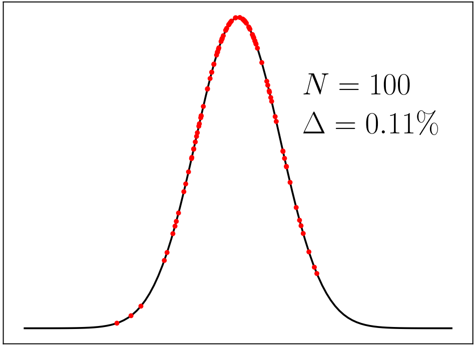
The x_i are drawn from a uniform distribution in the interval $[-100, 100]$

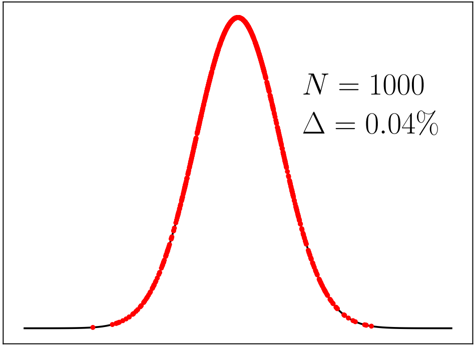


Importance sampling

$$\int_{-\infty}^{\infty} \mathrm{d}x\ (100 - x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100 - x_i^2)\cdot\frac{1}{N}$$

The x_i are drawn from a normal distribution





The sign problem

$$\int_{-\infty}^{\infty} \mathrm{d}x (100 - x^2) \frac{e^{-\frac{\mathrm{i}}{2}x^2}}{\sqrt{2\pi}}$$