

Heavy quarks at finite temperature

J. H. Weber¹

¹Humboldt-University of Berlin & IRIS Adlershof, Berlin, Germany



**19th International Conference on Hadron Spectroscopy and
Structure in memoriam Simon Eidelman (HADRON2021)**
07/27/2021

Main (still incomplete) topic of this talk:

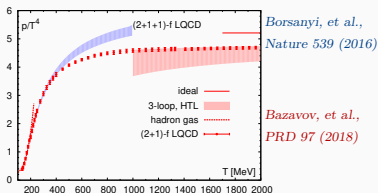
**D. Bala, O. Kaczmarek, R. Larsen, S. Mukherjee,
G. Parkar, P. Petreczky, A. Rothkopf, JHW**

Outline

- 1 **Appetizer:** link to heavy-ion phenomenology
 - Motivation & a glimpse of the future
 - History and how we got where we are
- 2 **Starter:** unbound hard probes and transport coefficients
 - Heavy-quark momentum diffusion coefficient κ
- 3 **Main course:** quarkonium at $T > 0$ beyond screening
 - In-medium quarkonium at weak coupling
 - Static $q\bar{q}$ pair on the lattice
 - Nonrelativistic $q\bar{q}$ pair on the lattice
- 4 **Dessert:** bringing “heavy quarks at finite temperature” full circle

Shift of focus in heavy-ion collision experiments

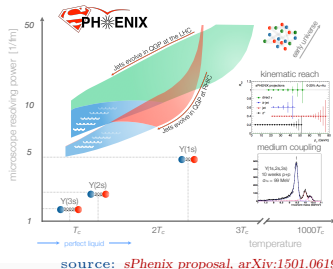
- Fruitful interplay of lattice gauge theory and heavy-ion collisions
- Search for the critical point and scan of the QCD phase diagram
- Or: equation of state and QNS
- But shift of focus in many future heavy-ion collision experiments



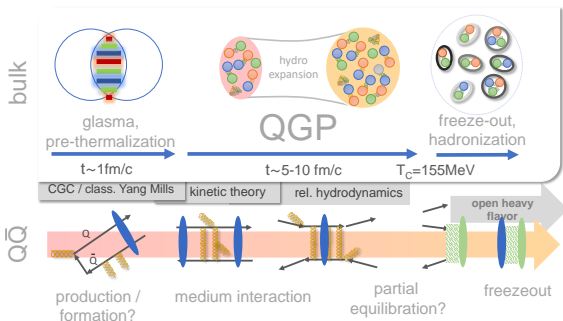
Questions @ ALICE, CMS, sPhenix, ...

- How do **parton showers** develop and propagate in QGP?
- How to reconcile asymptotic freedom and observed strongly-coupled QGP
- Which **dynamical degrees of freedom** play a role in QGP?

Answers in terms of models or PQCD...



Why focus on hard probes in heavy-ion collisions?

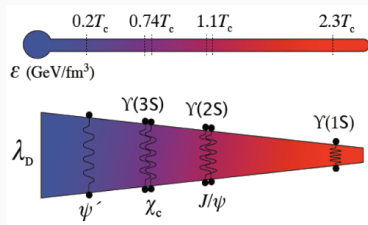


source: Rothkopf, Phys.Rept. 858 (2020) 1-117

- Hard probes are produced in a few **hard processes** in initial collision, neither created or destroyed afterwards, but can alter their nature
- Most important probes: jets, open **heavy flavor** & heavy **quarkonia**

Heavy quarkonia in the hot medium

- Idea to look at **quarkonia** in the QGP is old and famous
Matsui, Satz, PLB 178 (1986)
- **Debye screening** of electric gluons (A_0) dictates a limit of the radius of hadronic bound states
- Consequence: QGP formation \Leftrightarrow **quarkonium suppression**



source: USQCD whitepaper 2018, EPJ A 55 (2019)

At which T are there either bound states or melted $q\bar{q}$ pairs?

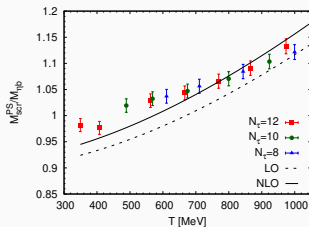
- Spatial $q\bar{q}$ pair correlators are a **model-independent** analysis tool
Bazavov, et al., PRD 91 (2015)

$$G(z, T) = \int_0^{1/\tau} d\tau \int d^2x_{\perp} \langle \mathcal{J}(\tau, \mathbf{x}_{\perp}, z) \mathcal{J}^{\dagger}(0) \rangle$$

$$= \int_0^{\infty} \frac{2d\omega}{\omega} \int_{-\infty}^{\infty} dp_z e^{ip_z z} \rho(\omega, p_z, T)$$

with spectral function $\rho(\omega, p_z, T)$

$$\sim \begin{cases} \delta(\omega^2 - p_z^2 - M_0^2) & \text{bound states} \\ \delta(\omega - \sum_{q_i} \sqrt{m_{q_i}^2 + (\pi T)^2}) & \text{free quarks} \end{cases}, \text{ where } M_0, m_{q_i} \text{ are pole masses}$$



source: *Petreczky, et al., arXiv:2107.11368*

Some drawbacks of analyses of free energies or spatial $q\bar{q}$ pair correlators:

- No insight into the **melting mechanism** at work for intermediate T

Unbound hard probes and heavy-quark transport coefficients

- Thermalized **heavy quarks**: energy $E \sim T$ and thereby $p \sim \sqrt{mT}$, undergo **Langevin-type evolution** with independent kicks of $\Delta p \sim T$
- Heavy-quark momentum diffusion coefficient κ least difficult to obtain

$$\kappa = \lim_{\omega \rightarrow 0} \lim_{m \rightarrow \infty} \kappa^{(m)}(\omega), \quad \kappa^{(m)}(\omega) = \frac{1}{3\chi} \int_{-\infty}^{+\infty} dt e^{i\omega t} \int d^3x \left\langle \frac{1}{2} \left\{ \mathcal{F}^i(\mathbf{x}, t), \mathcal{F}^i(\mathbf{o}, 0) \right\} \right\rangle,$$

$$\text{with } \mathcal{F}^i = m \partial_t \mathcal{J}^i \text{ and } \chi = 1/T \int d^3x \langle \mathcal{J}^0(\mathbf{x}, t) \mathcal{J}^0(\mathbf{o}, 0) \rangle.$$

Caron-Huot, et al., JHEP 04 (2009)

- Integrate out quarks, obtain **Euclidean chromoelectric correlator**

$$G_E(\tau) = \int d^3x \frac{\langle \text{Re tr } U(1/T; \tau) E_i(\tau) U(\tau; 0) E_i(0) \rangle}{3P}$$

- Reconstruct spectral function $\rho(\omega)$ and extract κ nonperturbatively^a

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\omega} \rho(\omega) \frac{\cosh \omega (\tau - 1/2T)}{\sinh \omega / 2T}, \quad \kappa = \lim_{\omega \rightarrow 0} 2T \rho(\omega) / \omega$$

via models of $\rho(\omega)$ that incorporate known limiting behavior

^a Meyer, NJP 13, 035008 (2011); Banerjee, et al., PRD 85, 014510 (2012); Francis, et al., PRD 92, 116003 (2015);

Altenkort, et al., PoS Lat2019 (2019) 204; Brambilla, et al., PRD 102 (2020) 7, 074503

Heavy-quark momentum diffusion coefficient from the future

Use gradient flow to kill many birds with a single stone *Lüscher, JHEP 08(2010)*

$$\frac{dV_{x,\mu}(t)}{dt} = - \left\{ \partial_{x,\mu} S^{\text{flow}} \right\} V_{x,\mu}, \quad V_{x,\mu}(t=0) \equiv U_{x,\mu}$$

- Extrapolate to continuum limit at fixed t , and only then to the $t \rightarrow 0$ limit in the continuum
- Gradient flow **reduces noise**, consistent with multi-level results

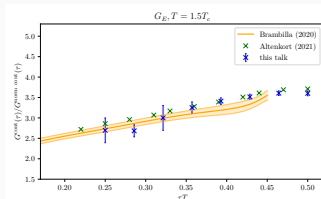
Altenkort, et al., PRD 103 (2021)

- Gradient flow **renormalizes**
- ⇒ opens up window of opportunity for obtaining NLO corrections

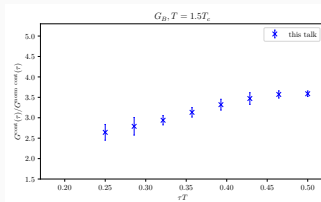
$$G_B(\tau) = \int d^3x \frac{\left\langle \text{tr} U(1/\tau; \tau) B_i(\tau) U(\tau; 0) B_i(0) \right\rangle}{3P}$$

or γ (κ 's dispersive counterpart)

$$G_E^{\text{Im}}(\tau) = \int d^3x \frac{\left\langle \text{Im tr} U(1/\tau; \tau) E_i(\tau) U(\tau; 0) E_i(0) \right\rangle}{3P}$$



source: *Mayer-Stuedte, Lattice 2021 [382]*

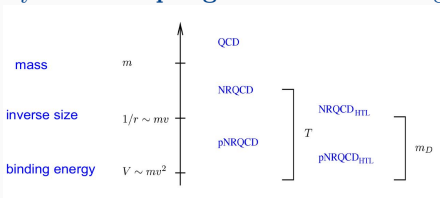


Screening is not the whole story... (at weak coupling)

Matsui & Satz's idea of the **quarkonium suppression mechanism** was turned inside out by **weak-coupling EFT results** emerging 15 years ago

NR hierarchy:

$$V \sim \alpha_s$$



- For $1/r \sim m_D \ll T$: $\text{Re}[V_s] = F_S + \mathcal{O}(g^4)$ and $\text{Im}[V_s] \sim \mathcal{O}(g^2 T)$

$$V_s(T, r) = -C_F \alpha_s \left\{ \frac{e^{-rm_D}}{r} + m_D + iT \phi(rm_D) \right\}, \quad \phi(x) = 2 \int_0^\infty \frac{dz z}{(z^2 + 1)^2} \left\{ 1 - \frac{\sin(zx)}{zx} \right\}$$

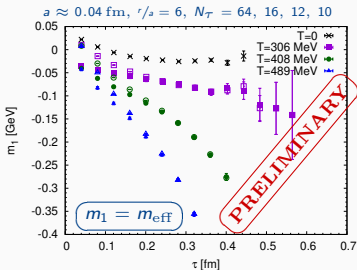
Laine, et al., JHEP 03 (2007)

- For $\Delta V \ll 1/r \ll m_D \ll T$: $\text{Re}[V_s] = V_s + \mathcal{O}(g^4)$ and $\text{Im}[V_s] \sim \mathcal{O}(g^4 r^2 T^3, g^6 T)$

$$V_s(T, r) = \frac{-C_F \alpha_s}{r} + r^2 T^3 \left\{ \mathcal{O}(g^4) + i \mathcal{O} \left(g^4, \frac{g^6}{(rT)^2} \right) \right\}$$

Brambilla, et al., PRD 78 (2008)

Static $q\bar{q}$ pair at $T > 0$ on the lattice



- Static $q\bar{q}$ interaction is encoded in (real-time) **Wilson loops**^a

$$W_{[r, \tau]}(t) = \left\langle e^{ig \oint_{r \times t} dz^\mu A_\mu} \right\rangle_{\text{QCD}, T}$$

- Stable (ground) state Ω_r exists if

$$\Omega_{[r, \tau]} \equiv -i \lim_{t \rightarrow \infty} \partial_t W_{[r, \tau]}(t)$$

^aWe use Wilson line correlators in Coulomb gauge.

- Same spectral functions yield real- or imaginary-time correlators

$$W_{[r, \tau]} \left(\frac{t}{\tau} \right) = \int d\omega \begin{pmatrix} e^{+i\omega\tau} \\ e^{-\omega\tau} \end{pmatrix} \rho_{[r, \tau]}(\omega)$$

- Motivates generic decomposition

$$\rho_{[r, \tau]}(\omega) = \rho_{[r, \tau]}^{\{\Omega; \mathcal{O}(T)\}}(\omega) + \rho_{[r, \tau]}^{\text{tail}}(\omega) + \rho_{[r, \tau]}^{\text{UV}}(\omega)$$

- UV continuum $\rho_{[r, \tau]}^{\text{UV}}(\omega)$ is far above **lowest feature Ω + effects of $\mathcal{O}(T)$**
- \Rightarrow Guess $\rho_{[r, \tau]}^{\text{UV}}(\omega)$ via $\rho_{[r, 0]}^{\text{UV}}(\omega) \Rightarrow$ subtract

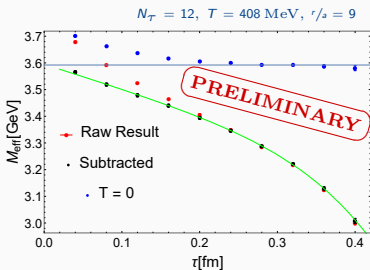
Note: “tail” due to backward propagating UV physics (vacuum excited states) at $\tau \lesssim 1/T$.

Cumulants of spectral functions – what can we expect?

- **Cumulants** of $\rho_{[r, T]}(\omega)$ accessible via τ (**log**) derivatives of $W_{[r, T]}(\tau)$

$$m_1^{[r, T]}(\tau) = -\partial_\tau \ln W_{[r, T]}(\tau) \quad [\equiv m_{\text{eff}}^{[r, T]}(\tau)],$$

$$m_n^{[r, T]}(\tau) = -\partial_\tau m_{n-1}^{[r, T]}(\tau), \quad n > 1$$
- For $N_\tau \leq 16$ obtain up to $m_3^{[r, T]}(\tau)$: supports ≤ 5 parameters for $\rho_{[r, T]}(\omega)$

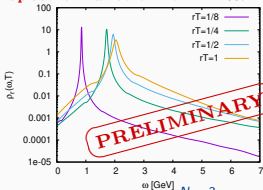


- **Quasiparticles** are represented as **Breit-Wigner** in $\rho_{[r, T]}(\omega)$
- Ansatz: approximate **BW** of $\rho_r^{\{\Omega; \mathcal{O}(T)\}}(\omega)$ locally as **Gaussian**, include **delta function** for $\rho_r^{\text{tail}}(\omega)$

$$W_{[r, T]}(\tau) = A_{[r, T]}^{\{\Omega; \mathcal{O}(T)\}} e^{-\Omega_{[r, T]}\tau + (\Gamma_{[r, T]}^G)^2 \tau^2 / 2} + A_{[r, T]}^{\text{tail}} e^{-\omega_{[r, T]}^{\text{tail}} \tau}, \quad \omega_{[r, T]}^{\text{tail}} \ll \Omega_{[r, T]}$$

Comparison: lattice QCD vs HTL

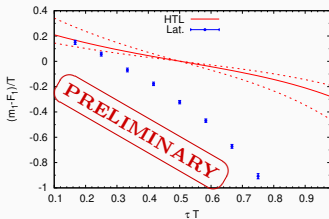
HTL spectral function for $T = 667$ MeV



[NLO, 2-loop $\alpha_s(2\pi T)$, $\Lambda_{\overline{MS}}^{N_f=3} = 332$ MeV]

$N_T = 12$, $r/a = 12$, subtracted correlator

$T=667$ MeV, $rT=1$



- **HTL** is an attractive proposition: **motivated & regularized BW**
- **HTL** result is **antisymmetric** around the midpoint $\tau = 1/2T$:

$$\log W_{[r, T]}(\tau) = -\text{Re } V_s(r, T) \times \tau$$

$$+ \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ e^{-\omega\tau} + e^{-\omega(1/\tau - \tau)} \right\}$$

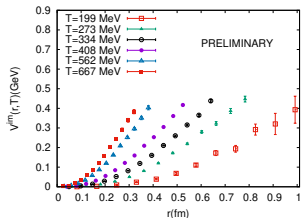
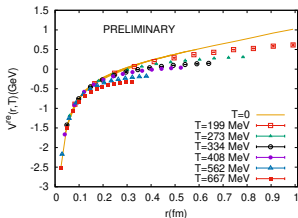
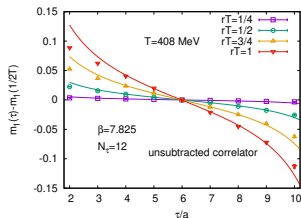
$$\times \{1 + n_B(\omega)\} \sigma_{[r, T]}(\omega)$$

- **No large UV component in HTL**, compare UV-subtracted result
- Subtleties due to renormalons and regulators: consider $(m_1 - F_5)/T$

Complex Potential from HTL motivated approach

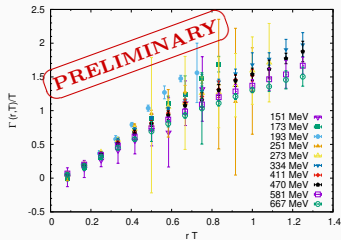
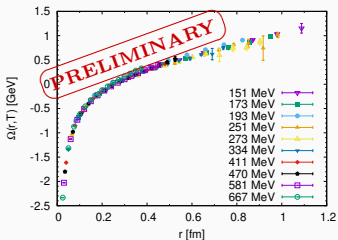
- In HTL, near $\tau \sim \frac{1}{2T}$, (PRD 101, 034507)

$$m_1(r, \tau) = V^{re}(r, T) + V^{im}(r, T) \cot(\pi \tau T)$$
- Real part shows medium modification.
- Imaginary part increases with distance and temperature.



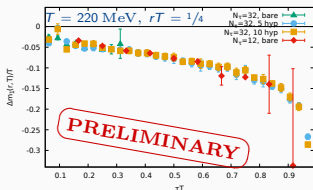
Lowest spectral feature from fits using Gaussian approximation

$N_\tau = 12$, $\Omega(r, T) \equiv \Omega_{[r, T]}$, $\Gamma(r, T) \equiv \sqrt{2 \ln 2} \Gamma_{[r, T]}^G$, subtracted correlators



- Almost no T dependence in $\Omega_{[r, T]}$ (naive correspondence: $\text{Re } V_s(r, T)$)
- Naively expected temperature scaling of $\Gamma(r, T)/T \approx \Gamma(r T)/T$ down to $T \approx T_{pc}$
- Very similar results from Padé interpolation analysis

Fully vacuum-subtracted result



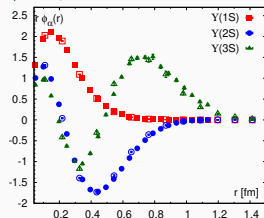
Feasibility study with $N_\tau = 32$: $m_n^{[r, T]}$, $n > 2$?

- Fine lattices: $a^{-1} \approx 7 \text{ GeV}$ $m_\pi \approx 0.3 \text{ GeV}$
 - UV filtering (HYP) for noise reduction
- distortions cancel in vacuum subtraction

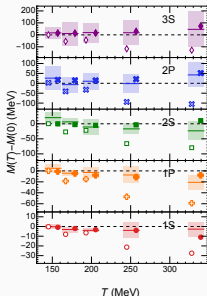
Nonrelativistic bottomonium with extended sources (HotQCD)

- NRQCD correlator study on $N_\tau = 12$ lattices: various **extended sources**
 - Point sources vs Gaussian smearing
 - Cornell pot. eigenstates \rightarrow GEVP
 - BS amplitudes at $T = 0$ and $T > 0$
Larsen, et al., PRD 100 (2019)
+ PLB 800 (2020) + PRD 102 (2020)
 - BS amplitudes in DNN \rightarrow potential
Shi, et al., arXiv:2105.07862

$N_\tau = 12, T = 334 \text{ MeV}$ vs $T = 151 \text{ MeV}$



source: *Larsen, et al., PRD 102, (2020)*

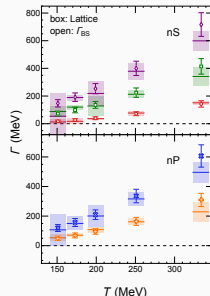


Spectral features of NRQCD bottomonium are similar to **static $q\bar{q}$**

Machine learning yields results compatible with lattice calculation

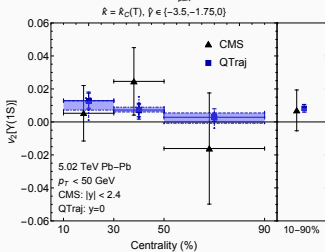
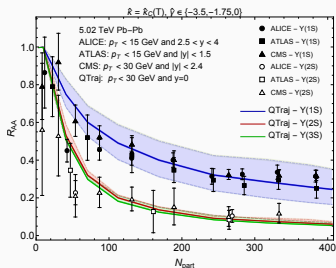
Clearly smaller thermal mass shift and larger width than in HTL

Shi, et al., arXiv:2105.07862



see also \rightarrow Allton, 07/27 18:40 UTC [148]

Modern understanding of quarkonium melting from lattice+EFT



source: [Brambilla, et al., arXiv:2107.06222](#)

- Treat in-medium quarkonium in **OQS+pNRQCD** approach

Brambilla, et al., PRD 96 (2017) + PRD 97 (2018)

→ Vander Griend, 07/26 17:41 UTC [217]

- ⇒ master equation has a Lindblad form, is discretized and solved stochastically^a

→ Strickland, 07/26 17:15 UTC [108]

- Temperature dependence from hydrodynamics evolution using lattice QCD equation of state
- For strongly-coupled plasma: T dependence via **heavy-quark transport coefficients** κ, γ
- Lattice transport coefficients & EoS in OQS+pNRQCD approach: **quarkonium suppression**

^a [Brambilla, et al., JHEP 05 \(2021\) + arXiv:2107.06222](#)

Heavy quarks at finite temperature on the lattice

Major achievements since Lattice 2019

- Use of **gradient flow** for heavy-quark transport coefficients
 - No need for multi-level algorithm creates **option for full QCD** results
 - Access to yet unknown transport coefficients ($\mathcal{O}((T/M)^2)$, dispersive, etc.)
- **Static $q\bar{q}$ pair**
 - Lowest spectral feature $\{\Omega; \mathcal{O}(T)\}$ + tail + UV continuum
 - Model-independent cumulant analysis \rightarrow clear evidence for a large **thermal width being the main cause of quarkonium melting**
 - Consistent with minimal (Gaussian fit) or major (HTL-based fit) medium modification of real part \rightarrow insufficient resolution with $N_\tau \leq 16$
- Nonrelativistic bottomonium
 - Extended sources or BS wave functions boost resolving power of LQCD
 - Spectral features are fully **consistent with static $q\bar{q}$ pair**
- Versatile toolbox for spectral reconstruction and inference

The lattice is in good shape to deliver more accurate and more realistic results needed for HIC phenomenology in the coming years.

Thank you for your attention!

Jet transport coefficients

- Jet-bulk reaction: copious scattering events – Gaussian approximation
- Jet transport coefficients that can be defined are its first two moments

$$\hat{e} = \langle k^- \rangle / L, \quad \hat{e}_2 = \langle (\Delta k^-)^2 \rangle / L, \quad \left(\begin{array}{c} \text{radiative} \\ \text{energy loss} \end{array} \right) \quad \text{or} \quad \hat{q} = \langle \kappa_\perp^2 \rangle / L \quad \left(\begin{array}{c} \text{transverse} \\ \text{momentum} \\ \text{broadening} \end{array} \right).$$

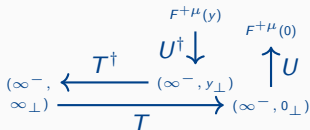
$$\hat{q} \propto \int \frac{dy^- d^2 y_\perp d^2 k_\perp}{(2\pi)^3} e^{i \frac{k_\perp^2 y^-}{2q^-} - i k_\perp \cdot y_\perp} \sum_n \frac{e^{-\beta E_n}}{Z}$$

$$\times \langle n | \text{tr} \left[g F^{+\mu}(y^-, y_\perp) U^\dagger(\infty^-, y_\perp; y^-, y_\perp) \right.$$

$$\times T^\dagger(\infty^-, \infty_\perp; \infty^-, y_\perp) T(\infty^-, \infty_\perp; \infty^-, 0_\perp)$$

$$\left. \times U(\infty^-, 0_\perp; 0^-, 0_\perp) g F_{\mu}^+(\infty^-, 0_\perp) \right] | n \rangle, \quad \mu = \{1, 2\}$$

Infinite Wilson lines in adjoint rep.



Garcia-Echevarria, et al., PRD 84 (2011)

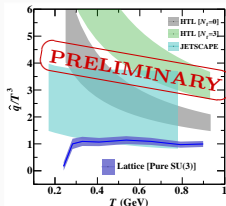
- Avoid Wilson lines via OPE in deep space-like region $q^+ \simeq -q^-$

Majumder, PRC 87 (2013)

- Up to corrections at $\mathcal{O}((k/q^-)^2)$

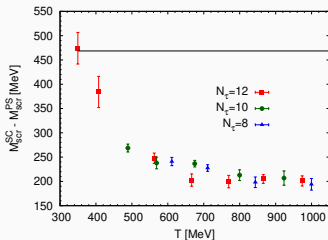
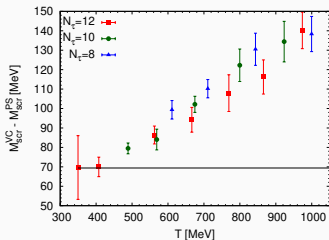
$$\hat{q} \propto g^2 \sum_n \frac{e^{-\beta E_n}}{Z} \langle n | \text{tr} F^{+\mu}(0) F_{,\mu}^+(0) | n \rangle$$

- For a pure gauge plasma: $\hat{q} \propto g^2 s$



In-medium bottomonium splittings

→ Sharma, 07/26 1:45 UTC [476]



● Vector-pseudoscalar splitting

- consistent with vacuum values up to $T \lesssim 350$ MeV
 - after melting due to spin-dependent potential in EQCD
- ⇒ for light quarks $\sim 0.3 T$, for bottom suppressed due to $T/m_b \ll 1$
- $\Upsilon(1S)$ and $\eta_b(1S)$ melt at similar rates, above $T \gtrsim 450$ MeV

● Scalar-pseudoscalar splitting

- similar picture for vector-axialvector splitting
- rapid decrease above $T > 350$ MeV due to melting
- slow decrease at $T > 600$ MeV due to explicit breaking by large mass
- $\chi_{b0}(1P)$ and $\chi_{b1}(1P)$ melt above $T \gtrsim 350$ MeV