Nonperturbative uncertainty to the photon PDF in the CT18 global analysis

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The precision requirements

The precision requirements

- The LHC becomes a precision machine.
- Theoretical cross sections have been achieved at NNLO in QCD, $\mathscr{O}(\alpha_s^2)$, for many processes.
- Due to $\alpha_e \sim \alpha_s^2$, we expect the QED corrections are the same level.
- The photon-initiated processes $(\gamma + \gamma, q, g \to X)$ will have observable effects.

Many applications

The SM processes

- Drell-Yan: $\ell^+\ell^-$
- $W^\pm H$
- W^+W^-

BSM scenarios

- Heavy leptons: L^+L^-
- Charged Higgs: $H^\pm, H^{\pm\pm}$

The existing photon PDFs

The first generation

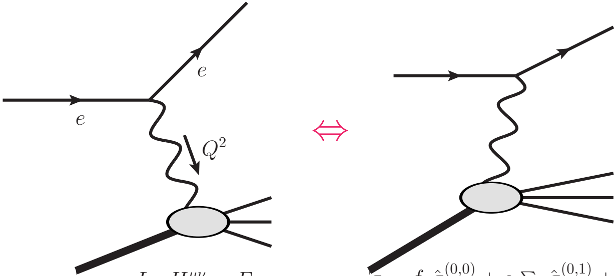
- MRST2004QED [0411040] models the photon PDF with an effective mass scale.
- NNPDF23QED [1308.0598] and NNPDF3.0QED [1410.8849] constrains photon PDF with the LHC Drell-Yan data, $q\bar{q}, \gamma\gamma \to \ell^+\ell^-$
- CT14qed_inc fits the inelastic ZEUS $ep \to e\gamma + X$ data [1509.02905], and include elastic component as well.

The second generation

- Recently, LUXqed directly takes the structure functions $F_{2,L}(x, Q^2)$ to constrain photon PDF uncertainty down to percent level [1607.04266,1708.01256]
- NNPDF3.1luxqed [1712.07053] initializes photon PDF with LUX formula at $\mu_0 = 100\ \ \mathrm{GeV}$ (a high scale) and evolves DGLAP equation both upwardly and downwardly.
- MMHT2015qed [1907.02750] initializes photon at $\mu_0 = 1$ GeV (a low scale) and evolve DGLAP upwardly.
- Our work incorporates the LUX formalism with the CT18 [1912.10053] global analysis.

The LUX formalism [1607.04266,1708.01256]

- The DIS process: $ep \to e + X$



$\sigma \sim L_{\mu\nu} H^{\mu\nu} \sim F_{2,L}$ $\sigma \sim f_\gamma \hat{\sigma}^{(0,0)}_{\ell\gamma} + \alpha \sum_j \hat{\sigma}^{(0,1)}_{\ell j} + \cdots$

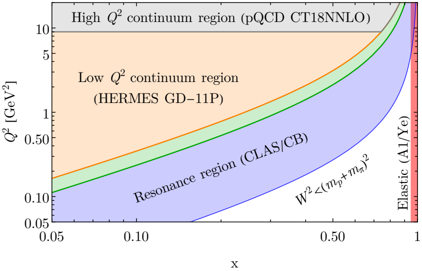
- Matching these two approaches leads to the LUX master formula:

$$x\gamma(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{\mathrm{d}z}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{\mathrm{d}Q^2}{Q^2} \alpha_{\mathrm{ph}}^2(-Q^2) \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_q^2}{Q^2} \right) \times F_2(x/z, Q^2) - z^2 F_L(x/z, Q^2) \right] - \alpha^2(\mu^2) z^2 F_2(x/z, \mu^2) \right\}.$$

The square bracket term corresponds to the "physical factorization" scheme, while the second term is referred as the "$\overline{\mathrm{MS}}$-conversion" term.

- The structure functions $F_{2,L}$ can be directly measured, or calculated through pQCD in the high-energy regime.

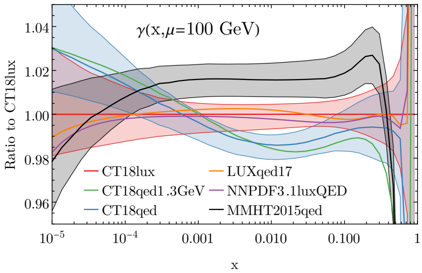
The breakup of (x, Q^2) plane: nonperturbative resources

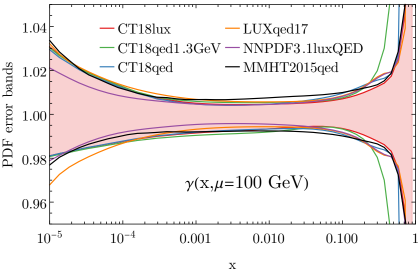


- In the resonance region $W^2 = m_p^2 + Q^2(1/x - 1) < W_{\rm lo}^2$, the structure functions are taken from CLAS [0301204] or Christy-Bosted [0712.3731] fits.
- In the low-Q^2 continuum region $W^2 > W_{\rm hi}^2$ GeV^2, the HERMES GD11-P [1103.5704] fits with ALLM [PLB1991] functional form.
- In the high-Q^2 region ($Q^2 > Q^2_{\rm PDF}$), $F_{2,L}$ are determined through pQCD.
- The elastic form factors are taken from A1 [1307.6227] or Ye [1707.09063] fits of world data.

Two approaches: LUX vs DGLAP

- CT18lux: directly calculate the photon PDF with the LUX formalism
- CT18qed: initialize the inelastic photon PDF with the LUX formalism at low scales, and evolve the $\mathrm{QED}_{\mathrm{NLO}} \otimes \mathrm{QCD}_{\mathrm{NNLO}}$ DGLAP equations up to high scales, similar to MMHT2015qed.





The take-home message:

- In the intermediate-x region, all photon PDFs give similar error bands.
- CT18lux photon PDF is in between LUXqed (also, NNPDF3.1luxQED) and MMHT2015qed, while CT18qed gives a smaller photon PDF.
- In the large-x region, the DGLAP approach (for both MMHT2015qed and CT18qed) gives a smaller photon than the LUX approach.

The difference between LUX and DGLAP

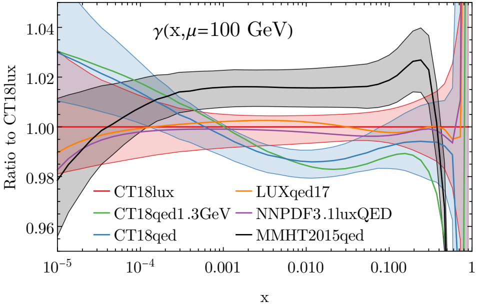
- The DGLAP only evolves the inelastic photon

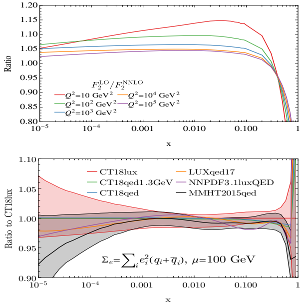
$$\frac{\mathrm{d}x\gamma^{\mathrm{inel}}}{\mathrm{d}\log\mu^2} = \frac{\alpha}{2\pi}\left(xP_{\gamma\gamma}\otimes x\gamma^{\mathrm{inel}} + \sum_i e_i^2 xP_{\gamma q}\otimes xq_i\right)$$

- The first-order solution corresponds to the LO F_2 in LUX formalism

$$x\gamma^{\mathrm{inel}}(x,\mu^2) \sim \int^{\mu^2} \mathrm{d}\log Q^2 \frac{\alpha}{2\pi}\sum_i e_i^2 xP_{\gamma q}\otimes xf_{q_i} \to F_2^{\mathrm{LO}} \text{ in LUX formula}$$

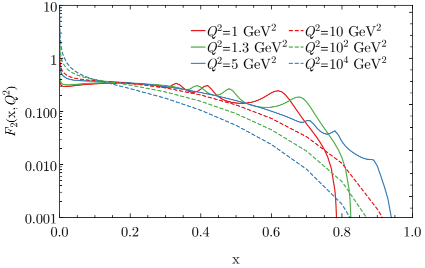
- It explains CT18qed gives larger photon at small x than CT18lux.
- MMHT2015qed gives smaller photon at small x, because the smaller charge-weighted singlet quark distributions.

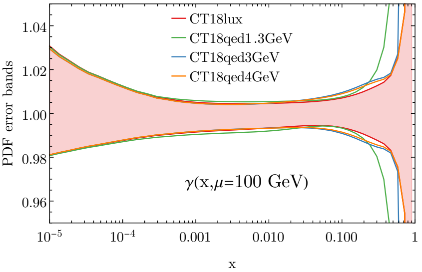




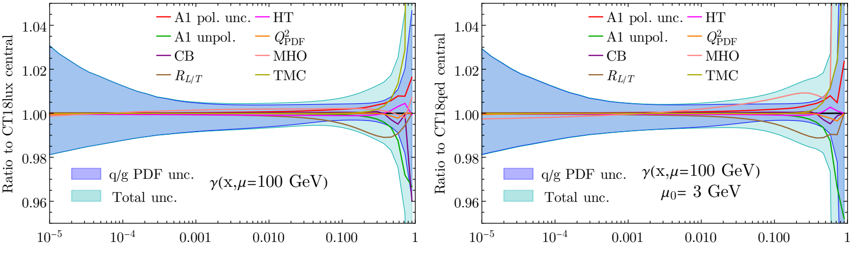
The large x behavior: nonperturbative contribution

- At large x, the LUX approach gives significantly larger PDF than the DGLAP one.
- It is resulted from the non-perturbative F_2 at low energy (resonance and low-Q^2 continuum regions).
- It induces a big uncertainty with the DGLAP low initialization scale approach, just because of scaling violation is not well behaved in the non-perturbative F_2.
- It can be rescued with a slightly higher initialization scale above the pQCD matching scale $Q_{\rm PDF} \sim 3$ GeV.





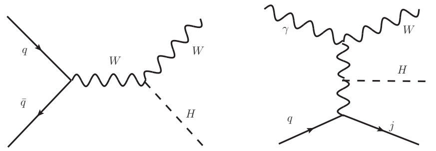
Photon PDF uncertainties

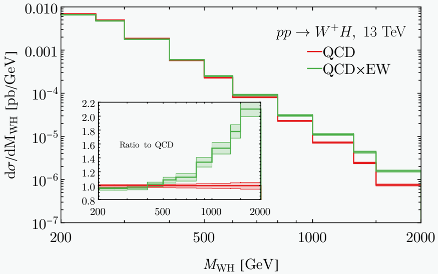


- A1 pol. unc.: the uncertainty of the A1 fit of the world polarized data
- A1 unpol.: Switching to A1 fit of the world unpolarized data
- CB: Changing resonance SF from CLAS to Christy-Bosted fit
- Variations of $R_{L/T} = \sigma_L/\sigma_T$ by 50% [1708.01256]
- HT: Adding higher-twist contribution to F_L [1708.01256] and F_2 [1602.03154].
- $Q^2_{\rm PDF}$: changing the matching scale $9 \to 5\ \ \mathrm{GeV}^2$
- MHO: varying the scale to estimate the missing high-order uncertainty
- TMC: adding the target mass correction to the SFs.

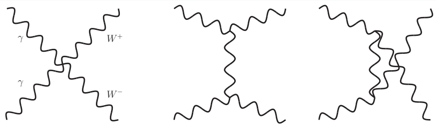
The applications

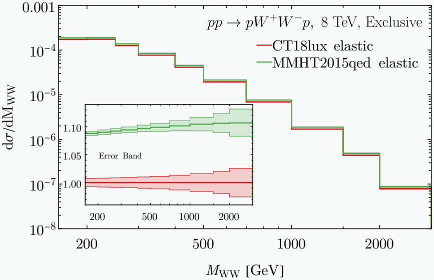
W^+H production





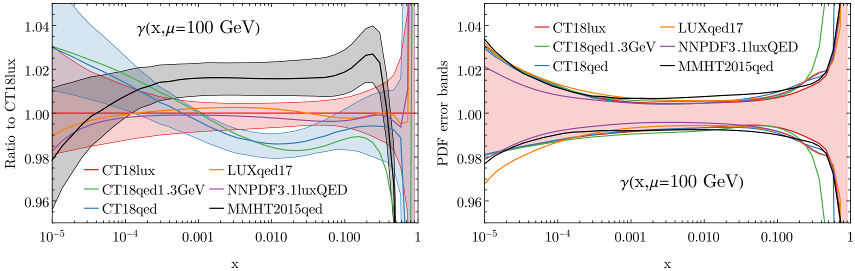
Exclusive $\gamma\gamma \to W^+W^-$





- At a large invariant mass, the photon initiated processes make a significant contribution
- CT18lux elastic photon (including both quarks and leptons) is smaller than MMHT2015qed one (only including quarks).

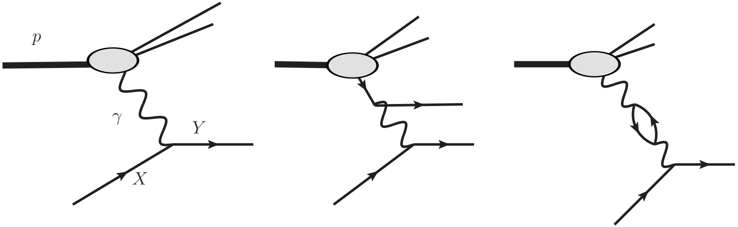
Summary and conclusions



- We have two photon PDF sets, CT18lux and CT18qed, based on the LUX and DGLAP approach, respectively.
- The overall uncertainties agree with the LUXqed(also NNPDF3.1luxQED) and MMHT2015qed.
- In the intermediate-x region, CT18lux is in between the LUXqed(also NNPDF3.1luxQED) and MMHT2015qed, while CT18qed is smaller.
- In the small-x region, the CT18qed is lager than CT18lux, due to the equivalent LO SF. The MMHT2015qed becomes smaller because of the smaller singlet PDFs Σ_e.
- In the large-x region, the DGLAP approach (MMHT2015qed and CT18qed) give smaller PDFs due to the non-perturbative SFs.
- The low-μ_0 DGLAP approach gives larger uncertainty at large x, due to non-perturbative SFs at low scales.

The cancellation in a higher order calculation

- Suppose we want to calculate a process $\gamma + X \to Y$.



- At one order higher, both photon and quark parton will participate.
- The PDFs are related with the DGLAP evolution, with divergence properly canceled.
- This can be also achieved in the LUX approach, with proper $\overline{\mathrm{MS}}$ conversion terms order by order.

The scale variation of the $\overline{\mathrm{MS}}$ conversion term

- In the default scale choice $\mu^2/(1-z)$, the $\overline{\mathrm{MS}}$-conversion term is
$$x\gamma^{\rm con} \sim (-z^2)F_2(x/z,\mu^2),$$
which is negative
- When varying the scale as μ^2, the conversion term should be change as well,
$$x\gamma^{\rm con}([M]) = x\gamma^{\rm con} + \frac{1}{2\pi\alpha}\int_x^1 \frac{\mathrm{d}z}{z}\int_{M^2[z]}^{\frac{\mu^2}{1-z}}\frac{\mathrm{d}Q^2}{Q^2}\alpha^2 z p_{\gamma q}(z)F_2(x/z,Q^2).$$
With $M^2[z]=\mu^2$, we have $\int_{\mu^2}^{\frac{\mu^2}{1-z}}\frac{\mathrm{d}Q^2}{Q^2} = \log\frac{1}{1-z}$.
- The central MMHT2015qed corresponds to $M^2[z]=\mu^2$ choice at low scale $\mu_0 = 1~~\mathrm{GeV}$.
- The DGLAP approach at low scale DOES give larger uncertainty due to the large non-perturbative contributions to structure functions.
- One method to avoid it is to start γ PDF at a higher scale in the pQCD region, i.e., $\mu_0^2 > Q^2_{\rm PDF}$.

The DGLAP approach gives smaller PDFs at large x

- MMHT2015qed divides the integration into two regions:

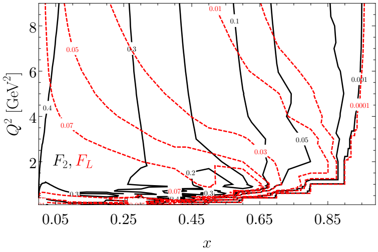
$$\left(\int_{\frac{x^2 m_p^2}{1-z}}^{\mu_0^2} + \int_{\mu_0^2}^{\frac{\mu_0^2}{1-z}}\right)[\cdots]$$

The second part is integrated semi-analytically:

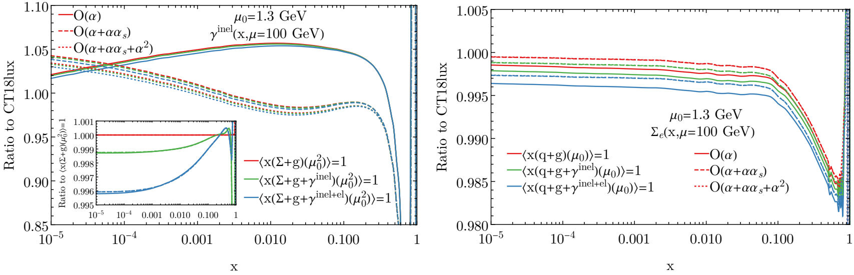
$$\int_{\mu_0^2}^{\frac{\mu_0^2}{1-z}} \frac{\mathrm{d}Q^2}{Q^2}\alpha^2\left(zp_{\gamma q} + \frac{2x^2 m_p^2}{Q^2}\right)F_2(x/z,\mu_0^2) = \alpha^2(\mu_0^2)\left(zp_{\gamma q}\log\frac{1}{1-z} + \frac{2x^2 m_p^2 z}{\mu_0^2}\right)F_2\left(\frac{x}{z},\mu_0^2\right)$$

The F_L is dropped because $F_L \sim \mathscr{O}(\alpha_s) \ll F_2$.

- In contrast, we integrate over $F_2(x/z, Q^2)$ rather than $F_2(x/z,\mu_0^2)$.
- It explains the MMHT2015qed gives smaller photon at large x than CT18qed.
- MMHT15 does not include the uncertainty induced by μ_0 variation.



The NLO QED evolution and momentum sum rules



- The NLO QED corrections to splitting functions

$$P_{ij} = \frac{\alpha}{2\pi} P_{ij}^{(0,1)} + \frac{\alpha}{2\pi}\frac{\alpha_S}{2\pi} P_{i,j}^{(1,1)} + \left(\frac{\alpha}{2\pi}\right)^2 P_{ij}^{(0,2)} + \cdots$$

- The NLO QED correction is negative.
- The momentum sum rules: the impact is $\mathscr{O}(0.1\%)$, negligible compared with higher order QED evolution.

$$\langle x(\Sigma + g + \gamma^{\text{inel+el}}\rangle = 1$$