Three-Body Unitarity in $\omega/\phi \to \pi^+\pi^-\pi^0$ & C-Violation in $\eta \to \pi^+\pi^-\pi^0$

Hakan Akdag

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HISKP (Theory), University of Bonn

1. Three-Body Unitarity in $\omega/\phi \to \pi^+\pi^-\pi^0$

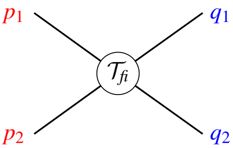
How to resum three-hadron final state interactions?

- Khuri-Treiman equations [*Khuri, Treiman 1960*]
  (e.g. for ! = → 3 [*Niecknig et al. 2012; Danilkin et al. 2015*])
- Three-body unitarity [*Mai et al. 2017; cf. also Mikhasenko et al. 2019*]
  (e.g. for $a_1(1260) \to 3$ [*Sadasivan et al. 2020; Mai et al. 2021*])

Unitarity($\sim$ probability conservation) gives rise to *optical theorem*:

$$\mathcal{T}_{fi} - \mathcal{T}_{if} = i \sum_n \int d\Pi_n (2\;)^4\;{}^4 \sum_{i;n} p_i - k_n \; \mathcal{T}_{ni}\mathcal{T}_{nf}$$

disc (



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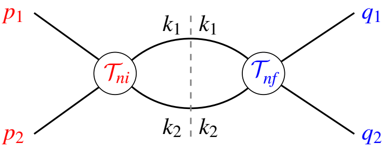
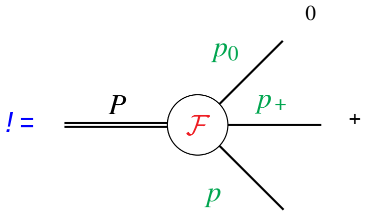


Expect formal equivalence [*Aitchison 2018*], but any quantitative comparison is missing. . .

1.1. Khuri-Treiman Framework

General Amplitude



- Most general amplitude (odd intrinsic parity)

$$\mathcal{M}(s;t;u) = i \qquad " \;\; p_+ p \;\; p_0 \, \mathcal{F}(s;t;u)$$

- Relate to observables

$$\sum_{\text{pol}} |\mathcal{M}(s;t;u)|^2 = \frac{s}{16} \; {}^2(s) \sin^2 \; {}_s |\mathcal{F}(s;t;u)|^2$$

- Idea: derive $V \quad \rightarrow \qquad$ and analytically continue to $V \rightarrow 3$

Partial Wave Expansion

- Consider final state:
  - Total isospin $I = 1$ (odd)
  - Bose: symmetric wave function

  odd partial waves between each pion pair

- Dominant contribution (P-wave):

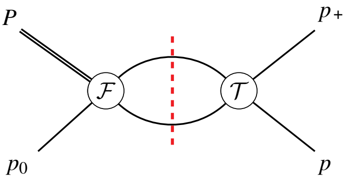
$$f_1(s) = \frac{3}{4} \int_{1}^{1} \mathrm{d}z_s \; 1 - z_s^2 \;\; \mathcal{F}(s; t; u)$$

- Reconstruction theorem (neglect F-wave discontinuities):

$$\mathcal{F}(s; t; u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

- $\mathcal{F}(s)$ only has a right hand cut for $s \geq 4M^2$

Unitarity Relation



Watson-like unitarity relation

$$\mathrm{disc} f_1(s) = 2i f_1(s) \quad (s - 4M^2) \sin \quad (s) e^{\ \ i \ \ (s)}$$

All information about the right hand cut in disc$\mathcal{F}(s)$

$$\mathrm{disc} f_1(s) = \mathrm{disc} \mathcal{F}(s)$$

General solution, add $\hat{\mathcal{F}}(s)$ without disc along the rhc: $f_1(s) = \mathcal{F}(s) + \hat{\mathcal{F}}(s)$

$$\Longrightarrow \mathrm{disc} \mathcal{F}(s) = 2i \quad \mathcal{F}(s) + \hat{\mathcal{F}}(s) \quad (s - 4M^2) \sin \quad (s) e^{\ \ i \ \ (s)}$$

Homogeneous solution ($\hat{\mathcal{F}}(s) = 0$):

$$\mathcal{F}(s) = P(s)\,\Omega(s); \qquad \Omega(s+i\,) = \exp\left(\frac{s}{}\int_{4M^2}^{\infty}\frac{(x)}{x(x-s-i\,)}\,\mathrm{d}x\right)$$

Homogeneous solution ($\hat{\mathcal{F}}(s) = 0$):

$$\mathcal{F}(s) = P(s)\,\Omega(s); \qquad \Omega(s + i\epsilon) = \exp\left(\frac{s}{\pi}\int_{4M^2}^{\infty} \frac{\delta(x)}{x(x - s - i\epsilon)}\,\mathrm{d}x\right)$$

Inhomogeneity as partial wave expansion of crossed channels

$$\hat{\mathcal{F}}(s) = \frac{3}{2}\int_{-1}^{1} \mathrm{d}z_s\,\left(1 - z_s^2\right)\,\mathcal{F}\left(t(z_s)\right)$$

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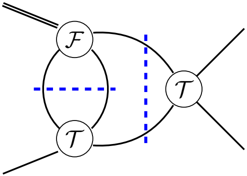
$\Longrightarrow$ $\hat{\mathcal{F}}(s)$ contains left hand cut contributions to the partial wave $f_1(s)$



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Full solution:

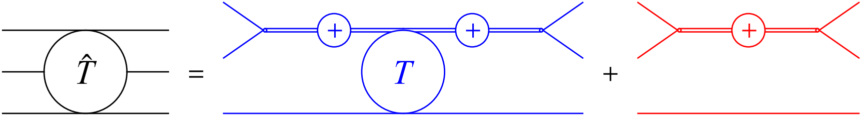
$$\mathcal{F}(s) = \Omega(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M^2}^{\infty} \mathrm{d}s'\,\frac{\sin\delta(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|\,s'^n(s' - s)}\right)$$

1.2. Three-Body Unitarity based on a Bethe Salpeter Ansatz

In collaboration with Michael Döring, Maxim Mai & Daniel Sadasivan

Isobar Formulation

Use isobar picture to split $3 \to 3$ into a *connected* and *disconnected* contribution: [*Mai et al. 2017*]



Building blocks:

1. Vertex v (isobar creation and decay)
2. Isobar propagator S (denoted by "+", includes rescattering)
3. Isobar-spectator interaction T

$$\Longrightarrow \hat{T} = \hat{T}_c + \hat{T}_d = vSTSv + vSv$$

Isobar Picture

- Apply unitarity condition to $\hat{T}$:

$$\hat{T} - \hat{T}^{\dagger} = i \sum_{n} \int \mathrm{d}\Pi_n (2\pi)^4 \delta^4 (P - \sum k) \left(\hat{T}_c + \hat{T}_d \right) \left(\hat{T}_c^{\dagger} + \hat{T}_d^{\dagger} \right)$$

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- Right hand side:
  - 4 combinations: $\hat{T}_d^\dagger \hat{T}_c$, $\hat{T}_d^\dagger \hat{T}_d$, $\hat{T}_c^\dagger \hat{T}_c$, $\hat{T}_c^\dagger \hat{T}_d$
  - 8 topologies: consider same or different spectator in each combination
  - 1 topology is purely disconnected ⇒ only addresses 2-body unitarity

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- Left hand side:
  - Expand: $\hat{T} = vSTSv + vSv$
  - Write in terms of disc$S$ and disc$T$
  - Bethe-Salpeter Equation (BSE):

$$\langle q|T|p\rangle = \langle q|B|p\rangle + \int \frac{d^4k}{(2\pi)^4} \langle q|B|k\rangle \, \tau(k) \, \langle k|T|p\rangle$$

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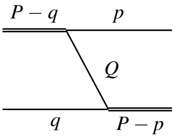
- Match each discontinuity on the left to one topology on the right

Analytic Matching

1. $(p) = (2 \quad) \quad ^{+}(p^2 - m^2)S(p) \Longrightarrow$ set spectator on-shell

2. $\text{disc}S \quad ^{1}(p) = -\frac{i}{2} \; \mathsf{R} \; \frac{\mathrm{d}^4k}{(2 \quad)^2} \quad k^2 - M^2 \qquad Q^2(p) - M^2 \quad v^2\,(k; Q(p))$

3. $B(q;p) = \frac{v(Q;q)v(Q;p)}{m^2 \quad Q^2 \quad i"} \Longrightarrow$ one-meson exchange:



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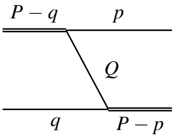
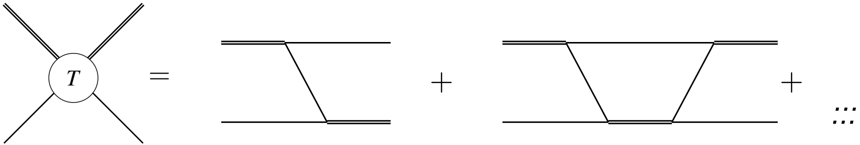


Rescattering is already included in T and obtained by iterating the BSE:



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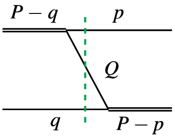
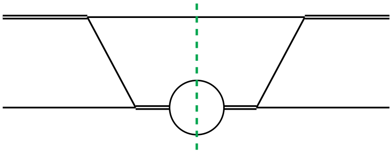
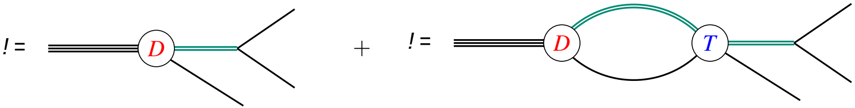


In contrast to other isobaric approaches, three-particle cuts are included:



Embed T in ! = $\to 3$:



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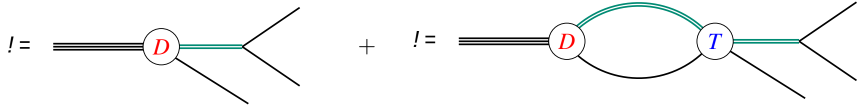

Due to odd intrinsic parity:

$$\mathcal{M}\ (p_0; p_+; p\ ) = \mathcal{F}\ (p_0; p_+; p\ ) - \mathcal{F}\ (p_+; p_0; p\ ) - \mathcal{F}\ (p\ ; p_+; p_0)$$

In angular momentum basis:

$$\mathcal{F}_\ell(p_0; p_+; p\ ) = \sqrt{\frac{3}{4}}\ D_\ell(k) + \int \frac{\mathrm{d}k}{(2\ )^3 2E_k} D_{\ell'}(k)\, S(k)\, T_{\ell'\ell}(k; p_0)\ \ S(p_0)\, v(p_+; p\ )$$

$$\equiv \mathcal{F}(p_0)\, v(p_+; p\ )$$

Use centrifugal barrier to approximate:

$D_\ell(p) \propto p^\ell$ and contact term $C_{\ell'\ell}(k; p_0) \propto k^{\ell'} p_0^{\ell}$ entering $T_{\ell'\ell}$

1.3. Numerical Comparison of the Frameworks

Single Variable Functions

Both single variable amplitudes split into s-channel rescattering and crossed-channel rescattering

$$\text{Khuri-Treiman}: \quad \mathcal{F}(s) = \Omega(s) \left(P_{n-1} + \frac{s^n}{\pi} \int_{4M^2}^{\infty} \mathrm{d}s' \frac{\sin\delta(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|\, s'^n (s'-s)} \right)$$

$$\text{Bethe-Salpeter}: \quad \mathcal{F}(s) = S(s) \left(D(s) + \int \frac{\mathrm{d}k\, k^2}{(2\pi)^3 2E_k} D(k)\, S(k)\, T(k;s) \right)$$

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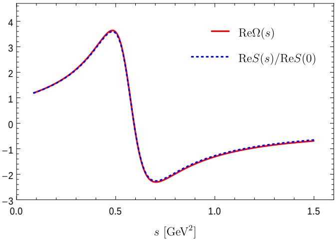
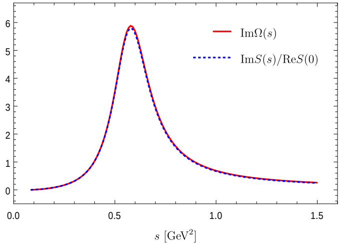
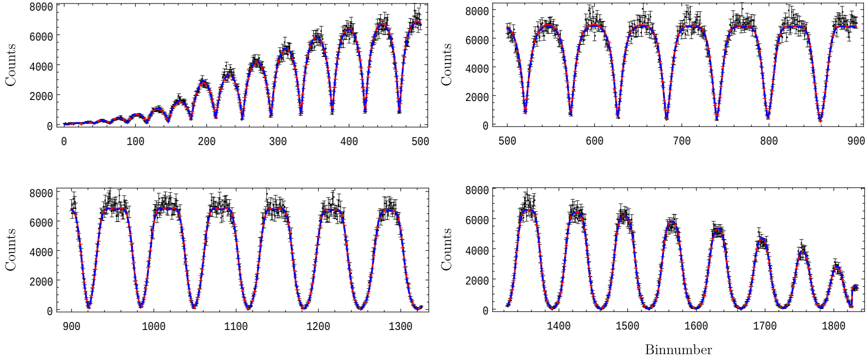
Observe *s*-channel rescattering in nearly perfect agreement





Comparison of crossed-ch. rescattering not possible as BS can not predict it...

Regression to the $\to$ ^{0} $^{+}$ Dalitz plot [*KLOE 2005*]:



With 3 free parameters each:

- Khuri-Treiman: $^{2}{=}ndof \approx 1{:}17$
- Bethe-Salpeter: $^{2}{=}ndof \approx 1{:}21$

)

about 3% deviation

2. C–violation in $\eta \to \pi^0\pi^+\pi^-$

In collaboration with Bastian Kubis & Tobias Isken

Motivation

Asymmetry between matter and antimatter

- Origin of matter from baryogenesis presumes C- and CP-violation [*Sakharov 1967*]
- Weak CP-violation not sufficient to create observed asymmetry

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What about strong *CP*-violation?

- Search for strong *P*- and *CP*-violation $\rightarrow 2$ [*Purcell, Ramsey 1950*]
- Theoretical realization: *P*- and *CP*-odd operator in QCD (cf. -term)
- Experimental bounds extremely rigorous (strong *CP*-problem)

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Search for new sources of *CP*-violation:

- Approach mostly neglected so far: *T*-odd *P*-even (TOPE) operators
- Due to *CPT* theorem: *C*- and *CP*-odd
- Consider an eigenstate of *C*, we focus on the meson
- Can investigate *CP*-violation in absence of weak interaction

2.1. Khuri-Treiman Framework

General $\eta \to \pi^+\pi^-\pi^0$ Amplitude

- Consider G-parity breaking decay $\to$ $^{+}$ ^{0}
- In the Standard Model consider isospin breaking with $\Delta I = 1$

$$\mathcal{M}(s;t;u) = \mathcal{M}_1^C(s;t;u)$$

[*Colangelo et al. 2018; Albaladejo et al. 2017; Guo et al. 2017; ...*]

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- Consider G-parity breaking decay $\to \; ^{+} \quad ^{0}$
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- For *C*-violating parts consider $C = -(-1)^{\;\; I}$, i.e. need even isospin [*Gardner, Shi 2020*]

$$\mathcal{M}(s;t;u) = \mathcal{M}_1^{C}(s;t;u) + \mathcal{M}_0^{\not{C}}(s;t;u) + \mathcal{M}_2^{\not{C}}(s;t;u)$$

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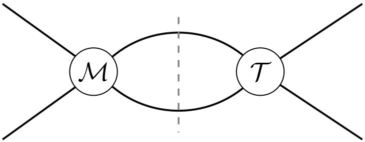
$$\mathcal{M}(s;t;u) = \mathcal{M}_1^{C}(s;t;u) + \mathcal{M}_0^{\not{C}}(s;t;u) + \mathcal{M}_2^{\not{C}}(s;t;u)$$

Evaluate $\mathcal{M}(s;t;u)$ with the Khuri-Treiman framework

- Dispersion relations for scattering process $\to$
- Analytically continue to the realm of the decay $\to 3$



- Bose symmetry:
  odd (even)   -isospin must have odd (even) partial wave

- Reconstruction theorem: expand for fixed isospin and partial wave

$$\mathcal{M}_1^{C}(s;t;u) = \mathcal{F}_0(s) + (s-u)\mathcal{F}_1(t) + (s-t)\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

$$\mathcal{M}_0^{\not{C}}(s;t;u) = (t-u)\mathcal{G}_1(s) + (u-s)\mathcal{G}_1(t) + (s-t)\mathcal{G}_1(u)$$

$$\mathcal{M}_2^{\not{C}}(s;t;u) = 2(u-t)\mathcal{H}_1(s) + (u-s)\mathcal{H}_1(t) + (s-t)\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

[*Gardner, Shi 2020*]

- C-even terms are symmetric and C-odd ones antisymmetric under $t \leftrightarrow u$

- Note: $\mathcal{F}_I$, $\mathcal{G}_I$ and $\mathcal{H}_I$ are completely independent

- Solution analog to $V \to 3$

2.2. Comparison to Experiment

Dalitz Plot

Regression to Dalitz plot [*KLOE, 2016*]

The SM amplitude $\mathcal{M}_1^C$:

- Minimal subtraction scheme 3 dof: ^{2}=dof $\approx$ 1:054
- Observables agree with current literature
  1. Taylor invariants [*Colangelo et al. 2018*]
  2. Branching ratio BR($\to 3$ ^{0})=BR($\to$ ^{0} $^+$) [*PDG 2020*]
  3. Dalitz plot parameters [*Colangelo et al. 2018, PDG 2020*]

$\Longrightarrow$ subtraction scheme justified, apply analogously to $\mathcal{M}_{0;2}^{\not C}$

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The BSM amplitude $\mathcal{M} = \mathcal{M}_1^C + \mathcal{M}_0^{\not C} + \mathcal{M}_2^{\not C}$:

- Fix $\mathcal{M}_{0;2}^{\not C}$ by just one complex normalization each
- Full amplitude 7 dof: ^{2}=dof $\approx 1{:}048$
- All *C*- and *CP*-violating signals vanish within 1-2

Decompose Dalitz plot:

$$|\mathcal{M}|^2 = |\mathcal{M}_1^{C}|^2 + 2\,\mathrm{Re}\Big[\mathcal{M}_0^{\not C}\,(\mathcal{M}_1^{C})\Big] + 2\,\mathrm{Re}\Big[\mathcal{M}_1^{C}\,(\mathcal{M}_2^{\not C})\Big] + \mathcal{O}(\mathcal{M}^{\not C\,2})$$

Interference terms give rise to asymmetries in $^{+}$ -distribution

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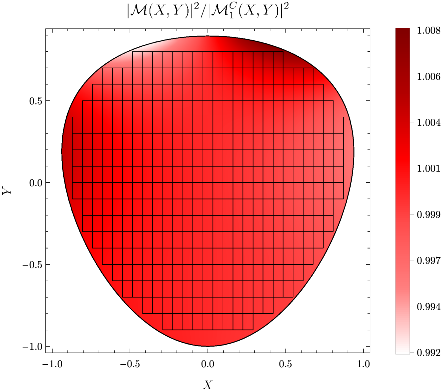

$\Longrightarrow$ relative C-odd terms restricted to the per mille level

Dalitz Plot Asymmetries

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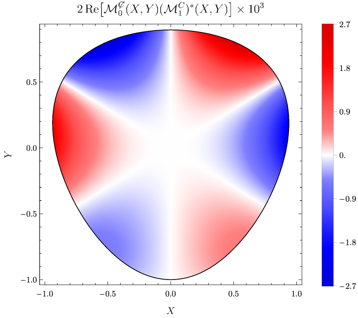
$$|\mathcal{M}|^2 = |\mathcal{M}_1^C|^2 + 2\,\mathrm{Re}\Big[\mathcal{M}_0^{\not C}\,(\mathcal{M}_1^C)\Big] + 2\,\mathrm{Re}\Big[\mathcal{M}_1^C\,(\mathcal{M}_2^{\not C})\Big] + \mathcal{O}(\mathcal{M}^{\not C\,2})$$

Interference terms give rise to asymmetries in $^{+}$ -distribution



$\Longrightarrow$ Interference terms of same order of magnitude

Quantify asymmetries:

$A_{\mathrm{LR}} = -7{:}5(4{:}7) \cdot 10^{\ 4}$ $\quad A_{\mathrm{Q}} = 4{:}1(4{:}3) \cdot 10^{\ 4}$ $\quad A_{\mathrm{S}} = 3{:}8(4{:}3) \cdot 10^{\ 4}$

BSM Couplings

Effective BSM operators

$$X_0^{6} \sim g_0(s-t)(t-u)(u-s) + \mathcal{O}(p^8)$$
$$X_2^{6} \sim g_2(t-u) + \mathcal{O}(p^4)$$

Obtain couplings by a Taylor expansion of $\mathcal{M}_0^{6}$, $\mathcal{M}_2^{6}$:

$$g_0 = (13{:}8(14{:}1) - 28{:}2(53{:}6)i)\ \text{GeV}\ \ ^6$$
$$g_2 = (0{:}004(66) - 0{:}03(18)i)\ \text{GeV}\ \ ^2$$

Relative deviation

$$|g_0{=}g_2| \approx 10^3\,\text{GeV}\ \ ^4$$

Regression to Dalitz plot compensates kinematic suppression of X_0^{6}

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- Based on fundamental principles of analyticity, unitarity and crossing
- Derived *C*- and *CP*-odd contributions driven by $\Delta I = 0; 2$ transitions
- Extracted effective BSM operators $X_0^{\not{C}}$, $X_2^{\not{C}}$
- Current experimental precision
  - $X_0^{\not{C}}$ and $X_2^{\not{C}}$ of same order of magnitude within decay region
  - *C*-odd signals restricted to a relative per mille level (vanish within 1-2)

Summary & Outlook

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C-violation in hadronic and ^{0} decays [*Akdag, Isken, Kubis 2021 (in preparation)*]

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- $^{0} \rightarrow 3$ (X) (does larger phase space show more visible effects?)
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From experimental point of view:

- *JLab Eta Factory* (JEF)
- *Rare Eta Decays with a TPC for Optical Photons* (REDTOP)

Thank you very much for your attention!