# Three-Body Unitarity in $\omega/\phi \to \pi^+\pi^-\pi^0$ & *C*-Violation in $\eta \to \pi^+\pi^-\pi^0$

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# 1. Three-Body Unitarity in $\omega/\phi \to \pi^+\pi^-\pi^0$

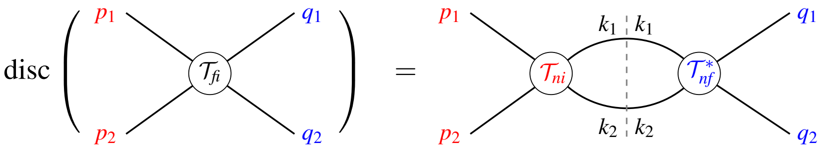
# Introduction

How to resume three-hadron final state interactions?

- Khuri-Treiman equations [*Khuri, Treiman 1960*]
  (e.g. for $\omega/\phi \to 3\pi$ [*Niecknig et al. 2012; Danilkin et al. 2015*])
- Three-body unitarity [*Mai et al. 2017; cf. also Mikhasenko et al. 2019*]
  (e.g. for $a_1(1260) \to 3\pi$ [*Sadasivan et al. 2020; Mai et al. 2021*])

Unitarity($\sim$ probability conservation) gives rise to *optical theorem*:

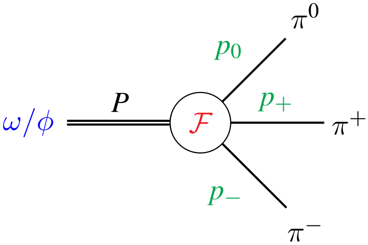
$$\mathcal{T}_{fi} - \mathcal{T}_{if}^* = i \sum_n \int \mathrm{d}\Pi_n (2\pi)^4 \delta^4\Big(\sum_{i,n} p_i - k_n\Big) \mathcal{T}_{ni} \mathcal{T}_{nf}{}^*$$




Expect formal equivalence [*Aitchison 2018*], but any quantitative comparison is missing...

1.1. Khuri-Treiman Framework

# General Amplitude




- Most general amplitude (odd intrinsic parity)

$$\mathcal{M}(s,t,u) = i\,\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{\mu}\,p_+^{\nu}p_-^{\alpha}p_0^{\beta}\,\mathcal{F}(s,t,u)$$

- Relate to observables

$$\sum_{\text{pol}} |\mathcal{M}(s,t,u)|^2 = \frac{s}{16}\kappa^2(s)\sin^2\theta_s|\mathcal{F}(s,t,u)|^2$$

- Idea: derive $V\pi \to \pi\pi$ and analytically continue to $V \to 3\pi$

# Partial Wave Expansion

- Consider $\pi\pi$ final state:
  - Total isospin $I = 1$ (odd)
  - Bose: symmetric $\pi\pi$ wave function

  } odd partial waves between each pion pair

- Dominant contribution (P-wave):

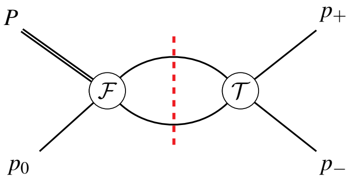
$$f_1(s) = \frac{3}{4} \int_{-1}^{1} \mathrm{d}z_s \left(1 - z_s^2\right) \mathcal{F}(s,t,u)$$

- Reconstruction theorem (neglect F-wave discontinuities):

$$\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

- $\mathcal{F}(s)$ only has a right hand cut for $s \geq 4M_\pi^2$

# Unitarity Relation




Watson-like unitarity relation

$$\mathrm{disc} f_1(s) = 2i f_1(s)\theta(s - 4M_\pi^2)\sin\delta(s) e^{-i\delta(s)}$$

All information about the right hand cut in disc$\mathcal{F}(s)$

$$\mathrm{disc} f_1(s) = \mathrm{disc}\mathcal{F}(s)$$

General solution, add $\hat{\mathcal{F}}(s)$ without disc along the rhc: $f_1(s) = \mathcal{F}(s) + \hat{\mathcal{F}}(s)$

$$\Longrightarrow \mathrm{disc}\mathcal{F}(s) = 2i\left(\mathcal{F}(s) + \hat{\mathcal{F}}(s)\right)\theta(s - 4M_\pi^2)\sin\delta(s) e^{-i\delta(s)}$$

Homogeneous solution ($\hat{\mathcal{F}}(s) = 0$):

$$\mathcal{F}(s) = P(s)\,\Omega(s), \qquad \Omega(s+i\epsilon) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\delta(x)}{x(x-s-i\epsilon)}\,\mathrm{d}x\right)$$

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Inhomogeneity as partial wave expansion of crossed channels

$$\hat{\mathcal{F}}(s) = \frac{3}{2}\int_{-1}^{1} \mathrm{d}z_s \left(1 - z_s^2\right) \mathcal{F}\left(t(z_s)\right)$$

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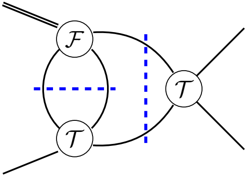
$\Longrightarrow$ $\hat{\mathcal{F}}(s)$ contains left hand cut contributions to the partial wave $f_1(s)$



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Full solution:

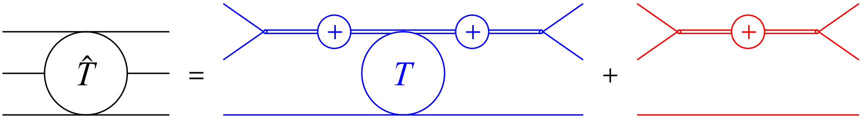
$$\mathcal{F}(s) = \Omega(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty} \mathrm{d}s' \frac{\sin\delta(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|\, s'^n (s'-s)}\right)$$

## 1.2. Three-Body Unitarity based on a Bethe Salpeter Ansatz

In collaboration with Michael Döring, Maxim Mai & Daniel Sadasivan

# Isobar Formulation

Use isobar picture to split $3\pi \to 3\pi$ into a *connected* and *disconnected* contribution: [*Mai et al. 2017*]




Building blocks:

1. Vertex $v$ (isobar creation and decay)
2. Isobar propagator $S$ (denoted by "+", includes $\pi\pi$ rescattering)
3. Isobar-spectator interaction $T$

$$\Longrightarrow \hat{T} = \hat{T}_c + \hat{T}_d = vSTSv + vSv$$

- Apply unitarity condition to $\hat{T}$:

$$\hat{T} - \hat{T}^\dagger = i \sum_n \int \mathrm{d}\Pi_n (2\pi)^4 \delta^4 \left(P - \sum k\right) \left(\hat{T}_c + \hat{T}_d\right) \left(\hat{T}_c^\dagger + \hat{T}_d^\dagger\right)$$

# Isobar Picture

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- Right hand side:
  - 4 combinations: $\hat{T}_d^\dagger \hat{T}_c$, $\hat{T}_d^\dagger \hat{T}_d$, $\hat{T}_c^\dagger \hat{T}_c$, $\hat{T}_c^\dagger \hat{T}_d$
  - 8 topologies: consider same or different spectator in each combination
  - 1 topology is purely disconnected $\Rightarrow$ only addresses 2-body unitarity

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- Left hand side:
  - Expand: $\hat{T} = vSTSv + vSv$
  - Write in terms of disc$S$ and disc$T$
  - Bethe-Salpeter Equation (BSE):

$$\langle q|T|p\rangle = \langle q|B|p\rangle + \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \langle q|B|k\rangle \, \tau(k) \, \langle k|T|p\rangle$$

# Isobar Picture

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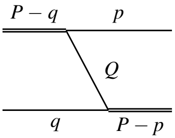
- Match each discontinuity on the left to one topology on the right

# Analytic Matching

1. $\tau(p) = (2\pi)\delta^+(p^2 - m^2)S(p) \Longrightarrow$ set spectator on-shell

2. $\mathrm{disc}S^{-1}(p) = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^2}\,\delta\left(k^2 - M_\pi^2\right)\delta\left(Q^2(p) - M_\pi^2\right)\,v^2\left(k, Q(p)\right)$

3. $B(q,p) = \frac{v(Q,q)v(Q,p)}{m^2 - Q^2 - i\varepsilon} \Longrightarrow$ one-meson exchange:



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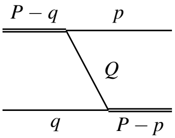
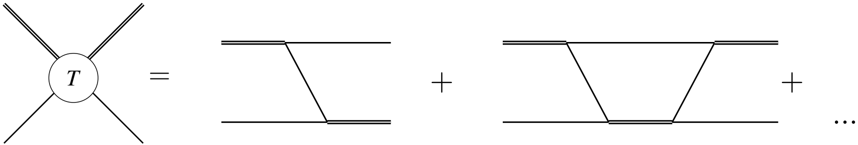



Rescattering is already included in $T$ and obtained by iterating the BSE:



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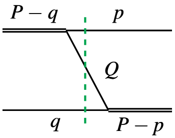
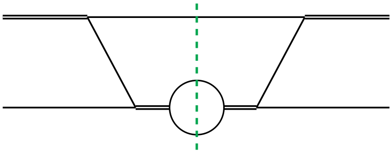
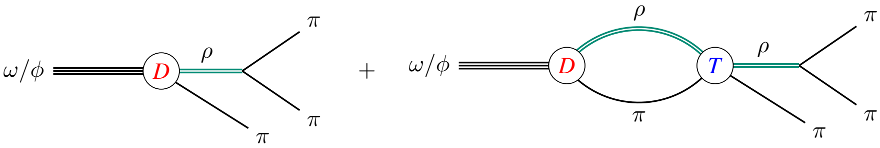



In contrast to other isobaric approaches, three-particle cuts are included:



Embed $T$ in $\omega/\phi \to 3\pi$:



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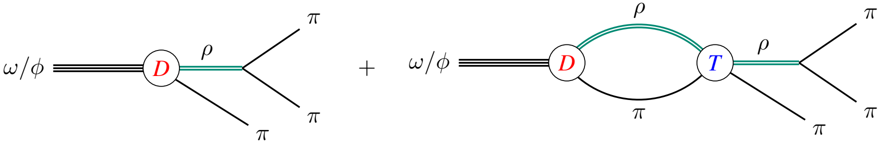


Due to odd intrinsic parity:

$$\mathcal{M}_\Lambda(p_0,p_+,p_-) = \mathcal{F}_\Lambda(p_0,p_+,p_-) - \mathcal{F}_\Lambda(p_+,p_0,p_-) - \mathcal{F}_\Lambda(p_-,p_+,p_0)$$

In angular momentum basis:

$$\mathcal{F}_\ell(p_0,p_+,p_-) = \sqrt{\frac{3}{4\pi}}\left(D_\ell(k) + \int \frac{\mathrm{d}k}{(2\pi)^3 2E_k} D_{\ell'}(k)\, S(k)\, T_{\ell'\ell}(k,p_0)\right) S(p_0)\, v(p_+,p_-)$$
$$\equiv \mathcal{F}(p_0)\, v(p_+,p_-)$$

Use centrifugal barrier to approximate:

$D_\ell(p) \propto p^\ell$ and contact term $C_{\ell'\ell}(k,p_0) \propto k^{\ell'} p_0^\ell$ entering $T_{\ell'\ell}$

## 1.3. Numerical Comparison of the Frameworks

# Single Variable Functions

Both single variable amplitudes split into $s$-channel rescattering and crossed-channel rescattering

$$\text{Khuri-Treiman}: \quad \mathcal{F}(s) = \Omega(s)\left(P_{n-1} + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty} \mathrm{d}s' \frac{\sin\delta(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|\, s'^n (s'-s)}\right)$$

$$\text{Bethe-Salpeter}: \quad \mathcal{F}(s) = S(s)\left(D(s) + \int \frac{\mathrm{d}k\, k^2}{(2\pi)^3 2E_k} D(k)\, S(k)\;\; T(k,s)\right)$$

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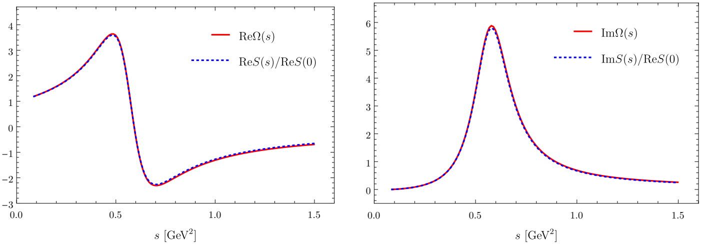
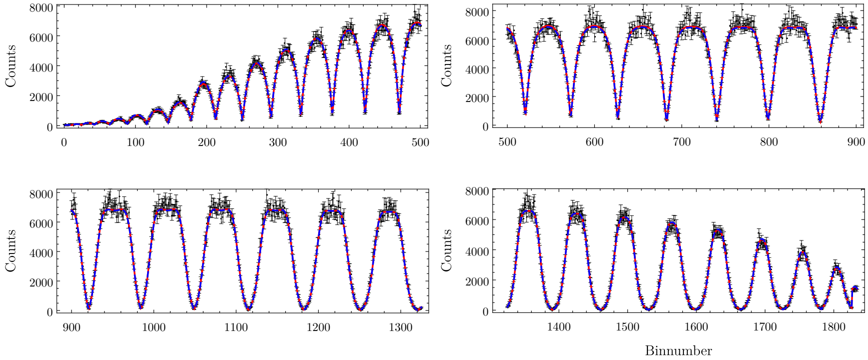
Observe $s$-channel rescattering in nearly perfect agreement




Comparison of crossed-ch. rescattering not possible as BS can not predict it...

# Comparison to Experiment

Regression to the $\phi \to \pi^0\pi^+\pi^-$ Dalitz plot [*KLOE 2005*]:




With 3 free parameters each:

- Khuri-Treiman: $\chi^2/ndof \approx 1.17$
- Bethe-Salpeter: $\chi^2/ndof \approx 1.21$

} about 3% deviation

# 2. *C*–violation in $\eta \to \pi^0\pi^+\pi^-$

In collaboration with Bastian Kubis & Tobias Isken

# Motivation

Asymmetry between matter and antimatter

- Origin of matter from baryogenesis presumes $C$- and $CP$-violation [*Sakharov 1967*]
- Weak $CP$-violation not sufficient to create observed asymmetry

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What about strong $CP$-violation?

- Search for strong $P$- and $CP$-violation $\eta \to 2\pi$ [*Purcell, Ramsey 1950*]
- Theoretical realization: $P$- and $CP$-odd operator in QCD (cf. $\theta$-term)
- Experimental bounds extremely rigorous (strong $CP$-problem)

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Search for new sources of *CP*-violation:

- Approach mostly neglected so far: *T*-odd *P*-even (TOPE) operators
- Due to *CPT* theorem: *C*- and *CP*-odd
- Consider an eigenstate of *C*, we focus on the $\eta$ meson
- Can investigate *CP*-violation in absence of weak interaction

# 2.1. Khuri-Treiman Framework

# General $\eta \to \pi^+\pi^-\pi^0$ Amplitude

- Consider G-parity breaking decay $\eta \to \pi^+\pi^-\pi^0$
- In the Standard Model consider isospin breaking with $\Delta I = 1$

$$\mathcal{M}(s,t,u) = \mathcal{M}_1^C(s,t,u)$$

[*Colangelo et al. 2018; Albaladejo et al. 2017; Guo et al. 2017; ...*]

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- For $C$-violating parts consider $C = -(-1)^{\Delta I}$, i.e. need even isospin [*Gardner, Shi 2020*]

$$\mathcal{M}(s,t,u) = \mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_0^{\not{C}}(s,t,u) + \mathcal{M}_2^{\not{C}}(s,t,u)$$

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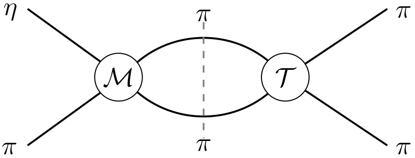
$$\mathcal{M}(s,t,u) = \mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_0^{\not{C}}(s,t,u) + \mathcal{M}_2^{\not{C}}(s,t,u)$$

Evaluate $\mathcal{M}(s,t,u)$ with the Khuri-Treiman framework

- Dispersion relations for scattering process $\eta\pi \to \pi\pi$
- Analytically continue to the realm of the decay $\eta \to 3\pi$



# Amplitude Decomposition

- Bose symmetry:
  odd (even) $\pi\pi$-isospin must have odd (even) partial wave
- Reconstruction theorem: expand for fixed isospin and partial wave

$$\mathcal{M}_1^{C}(s,t,u) = \mathcal{F}_0(s) + (s-u)\mathcal{F}_1(t) + (s-t)\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

$$\mathcal{M}_0^{\not{C}}(s,t,u) = (t-u)\mathcal{G}_1(s) + (u-s)\mathcal{G}_1(t) + (s-t)\mathcal{G}_1(u)$$

$$\mathcal{M}_2^{\not{C}}(s,t,u) = 2(u-t)\mathcal{H}_1(s) + (u-s)\mathcal{H}_1(t) + (s-t)\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

[*Gardner, Shi 2020*]

- $C$-even terms are symmetric and $C$-odd ones antisymmetric under $t \leftrightarrow u$
- Note: $\mathcal{F}_I$, $\mathcal{G}_I$ and $\mathcal{H}_I$ are completely independent
- Solution analog to $V \to 3\pi$

## 2.2. Comparison to Experiment

Regression to Dalitz plot [*KLOE, 2016*]

The SM amplitude $\mathcal{M}_1^C$:

- Minimal subtraction scheme 3 dof: $\chi^2/\text{dof} \approx 1.054$
- Observables agree with current literature
  1. Taylor invariants [*Colangelo et al. 2018*]
  2. Branching ratio $\text{BR}(\eta \to 3\pi^0)/\text{BR}(\eta \to \pi^0\pi^+\pi^-)$ [*PDG 2020*]
  3. Dalitz plot parameters [*Colangelo et al. 2018, PDG 2020*]

$\Longrightarrow$ subtraction scheme justified, apply analogously to $\mathcal{M}_{0,2}^{\not C}$

# Dalitz Plot

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$\Longrightarrow$ subtraction scheme justified, apply analogously to $\mathcal{M}_{0,2}^{\not{C}}$

The BSM amplitude $\mathcal{M} = \mathcal{M}_1^C + \mathcal{M}_0^{\not{C}} + \mathcal{M}_2^{\not{C}}$:

- Fix $\mathcal{M}_{0,2}^{\not{C}}$ by just one complex normalization each
- Full amplitude 7 dof: $\chi^2/\mathrm{dof} \approx 1.048$
- All *C*- and *CP*-violating signals vanish within 1-2$\sigma$

# Dalitz Plot Asymmetries

Decompose Dalitz plot:

$$|\mathcal{M}|^2 = |\mathcal{M}_1^{C}|^2 + 2\,\mathrm{Re}\left[\mathcal{M}_0^{\not C}\,(\mathcal{M}_1^{C})^*\right] + 2\,\mathrm{Re}\left[\mathcal{M}_1^{C}\,(\mathcal{M}_2^{\not C})^*\right] + \mathcal{O}(\mathcal{M}^{\not C\,2})$$

Interference terms give rise to asymmetries in $\pi^+\pi^-$-distribution

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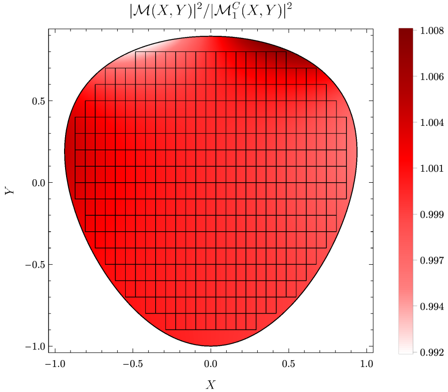


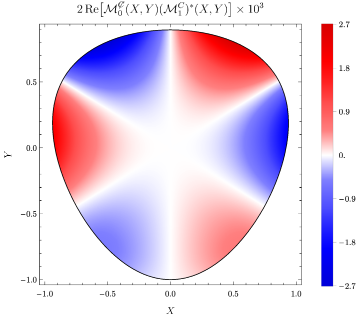

$\Longrightarrow$ relative $C$-odd terms restricted to the per mille level

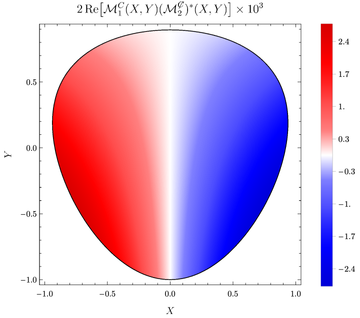
# Dalitz Plot Asymmetries

Decompose Dalitz plot:

$$|\mathcal{M}|^2 = |\mathcal{M}_1^C|^2 + 2\,\mathrm{Re}\left[\mathcal{M}_0^{\not C}\,(\mathcal{M}_1^C)^*\right] + 2\,\mathrm{Re}\left[\mathcal{M}_1^C\,(\mathcal{M}_2^{\not C})^*\right] + \mathcal{O}(\mathcal{M}^{\not C\,2})$$

Interference terms give rise to asymmetries in $\pi^+\pi^-$-distribution




$\Longrightarrow$ Interference terms of same order of magnitude

Quantify asymmetries:

$A_{\mathrm{LR}} = -7.5(4.7)\cdot 10^{-4} \quad A_{\mathrm{Q}} = 4.1(4.3)\cdot 10^{-4} \quad A_{\mathrm{S}} = 3.8(4.3)\cdot 10^{-4}$

# BSM Couplings

Effective BSM operators

$$X_0^{\not C} \sim g_0(s-t)(t-u)(u-s) + \mathcal{O}(p^8)$$
$$X_2^{\not C} \sim g_2(t-u) + \mathcal{O}(p^4)$$

Obtain couplings by a Taylor expansion of $\mathcal{M}_0^{\not C}$, $\mathcal{M}_2^{\not C}$:

$$g_0 = (13.8(14.1) - 28.2(53.6)i)\ \text{GeV}^{-6}$$
$$g_2 = (0.004(66) - 0.03(18)i)\ \text{GeV}^{-2}$$

Relative deviation

$$|g_0/g_2| \approx 10^3\,\text{GeV}^{-4}$$

Regression to Dalitz plot compensates kinematic suppression of $X_0^{\not C}$

# Summary & Outlook

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- Based on fundamental principles of analyticity, unitarity and crossing
- Derived $C$- and $CP$-odd contributions driven by $\Delta I = 0, 2$ transitions
- Extracted effective BSM operators $X_0^{\not{C}}, X_2^{\not{C}}$
- Current experimental precision
  - $X_0^{\not{C}}$ and $X_2^{\not{C}}$ of same order of magnitude within decay region
  - $C$-odd signals restricted to a relative per mille level (vanish within 1-2$\sigma$)

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- $\eta' \to 3\pi$ (✓) (does larger phase space show more visible effects?)
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From experimental point of view:

- *JLab Eta Factory* (JEF)
- *Rare Eta Decays with a TPC for Optical Photons* (REDTOP)

Thank you very much for your attention!