



## The six-pion amplitude

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In low-energy region, we cannot study perturbatively the interactions of hadrons directly from QCD  
↔ alternative approaches → Chiral perturbation theory (ChPT)

*Weinberg, Phys.A 96, (1979), Gasser and Leutwyler, Ann.Ph.158 (1984)*

Many observables are known in ChPT to a high loop order

↔ only recently it has become of interest to calculate the six-pion amplitude at low energies after it has been estimated using lattice QCD

*Blanton et al., PRL 124 (2020), arXiv:2106.05590,  
Fischer et al., EPJC 81 (2021), Hansen et al., PRL 126 (2021),  
Brett et al., PRD 104 (2021)*

The six-pion amplitude at tree level was first done using current algebra methods

e.g. *Osborn, Lett.N.Cim.2 (1969)*

It has been redone with Lagrangian methods many times, not known to one-loop order

e.g. *Low et al., JHEP 11 (2019), Bijmans et al., JHEP 11 (2019)*

We have therefore calculated at NLO the six-pion amplitude,

(as well as the four-pion amplitude, pion mass and decay constant)

↔ within ChPT generalization to the  $O(N+1)/O(N)$  massive nonlinear sigma model

↔ two(-quark)-flavour ChPT equivalent to  $O(4)/O(3)$

The relation to the measurement of the lattice is nontrivial to implement given

↔ complexity of the three-body finite volume calculations

↔ subtraction of the two-body rescatterings involved

*Hansen et al., PRD 90 (2014), PRD 92 (2015), Hammer et al., JHEP 09 (2017), JHEP 10 (2017),  
Mai et al., EPJA 53 (2017), PRL 122 (2019), Romero-López et al., JHEP 02 (2021),  
Blanton et al., PRD 102 (2020)*



Massive  $O(N + 1)/O(N)$  nonlinear sigma model **extended** beyond the LO

$$\mathcal{L} = \frac{F^2}{2} \partial_\mu \Phi^\top \partial^\mu \Phi + F^2 \chi^\top \Phi + l_1 (\partial_\mu \Phi^\top \partial^\mu \Phi) (\partial_\nu \Phi^\top \partial^\nu \Phi) + l_2 (\partial_\mu \Phi^\top \partial_\nu \Phi) (\partial^\mu \Phi^\top \partial^\nu \Phi) + l_3 (\chi^\top \Phi)^2 + l_4 \partial_\mu \chi^\top \partial^\mu \Phi$$

$\Phi$ : real **vector** of  $N + 1$  components,  $\Phi^\top \Phi = 1$

$$\chi^\top = (M^2, \vec{0})$$

$F, M$ : bare pion decay constant and mass

↪ calculate the four-pion and six-pion amplitudes at NLO

External fields can be added as in *Gasser and Leutwyler, Ann.Ph.158 (1984)*

The coefficients (low-energy constants)  $l_i$  are **free** parameters in the theory

↪ both UV divergent and finite parts

$$l_i = (c\mu)^{d-4} \left( \frac{1}{16\pi^2} \frac{1}{d-4} \gamma_i + l_i^r \right), \quad \log c = -\frac{1}{2} (1 - \gamma_E + \log 4\pi)$$



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$$l_i = l_i^r - \frac{1}{16\pi^2} \frac{\gamma_i}{2} \left( \frac{2}{4-d} - \gamma_E + \log 4\pi - \log \mu^2 + 1 \right), \quad l_i^r = \frac{1}{16\pi^2} \frac{\gamma_i}{2} \left( \bar{l}_i + \ln \frac{M_\pi^2}{\mu^2} \right)$$



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From studying the pion mass, decay constant and the four-pion amplitude

$$\gamma_3 = 1 - \frac{N}{2}$$

$$\gamma_1 = \frac{N}{2} - \frac{7}{6}$$

$$\gamma_4 = N - 1$$

$$\gamma_2 = \frac{2}{3}$$



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$$\left. \begin{aligned} \gamma_3 &= 1 - \frac{N}{2} \\ \gamma_4 &= N - 1 \end{aligned} \right\} \begin{aligned} \gamma_1 &= \frac{N}{2} - \frac{7}{6} \\ \gamma_2 &= \frac{2}{3} \end{aligned}$$



# Theoretical setting

Different parameterizations

$$\Phi_1 = \left( \sqrt{1-\varphi}, \frac{\phi^\top}{F} \right)^\top$$

*Gasser and Leutwyler, Ann.Ph.158 (1984)*

$$\Phi_2 = \frac{1}{\sqrt{1+\varphi}} \left( 1, \frac{\phi^\top}{F} \right)^\top$$

simple variation

$$\Phi_3 = \left( 1 - \frac{1}{2}\varphi, \sqrt{1 - \frac{1}{4}\varphi} \frac{\phi^\top}{F} \right)^\top$$

ESB term only gives mass terms of  $\phi_i$ s

$$\Phi_4 = \left( \cos \sqrt{\varphi}, \frac{1}{\sqrt{\varphi}} \sin \sqrt{\varphi} \frac{\phi^\top}{F} \right)^\top$$

follows the general prescription from  
*Coleman, Wess and Zumino, PR 177 (1969)*

$$\Phi_5 = \frac{1}{1 + \frac{1}{4}\varphi} \left( 1 - \frac{1}{4}\varphi, \frac{\phi^\top}{F} \right)^\top$$

*Weinberg, PR 166 (1968)*

$$\varphi \equiv \frac{\phi^\top \phi}{F^2}, \text{ with } \phi^\top = (\phi_1, \dots, \phi_N) \text{ a real vector of } N \text{ components (flavours)}$$

↪ few examples of the **whole class** of parametrizations

$$\Phi = \left( \sqrt{1 - \varphi f^2(\varphi)}, f(\varphi) \frac{\phi^\top}{F} \right)^\top, \text{ with } f(x) \text{ any analytical function satisfying } f(0) = 1$$



# Four-pion amplitude

On-shell amplitude in general

$p_i$ ,  $i = 1, \dots, 4$  pion incoming four-momenta,  $\sum p_i = 0$   
 $i_i$  flavours

Invariance under rotation in the isospin space and crossing symmetry implies

$$A_{4\pi}(p_1, i_1, p_2, i_2, p_3, i_3, i_4) \\ = \delta_{i_1 i_2} \delta_{i_3 i_4} A(p_1, p_2, p_3) + \delta_{i_1 i_3} \delta_{i_2 i_4} A(p_3, p_1, p_2) + \delta_{i_2 i_3} \delta_{i_1 i_4} A(p_2, p_3, p_1)$$

Mandelstam variables

$$s = (p_1 + p_2)^2, t = (p_1 + p_3)^2, u = (p_2 + p_3)^2, s + t + u = 4M^2 \\ \hookrightarrow \text{subamplitude } A(p_1, p_2, p_3) = A(s, t, u)$$

Up-to-and-including  $\mathcal{O}(p^4)$ , order by order

$$A(s, t, u) = A^{(2)}(s, t, u) + A^{(4)}(s, t, u)$$



# Four-pion amplitude

Leading order



The leading-order  $\mathcal{O}(p^2)$  amplitude stems from a **single** diagram



↪ schematically  $A_{4\pi}^{(2)} = \mathcal{M}_{\text{LO}}^{(2)}|_{\text{on-shell}}$

Related LO **subamplitude** (with LO relations  $M \rightarrow M_\pi$  and  $F \rightarrow F_\pi$ )

$$A^{(2)}(s, t, u) = \frac{1}{F_\pi^2} (s - M_\pi^2)$$



# Four-pion amplitude

Next-to-leading order

At NLO, one-loop diagrams (two topologies of 4 one-loop diagrams in total) and a counterterm



(a)  $3\times$



(b)  $1\times$



(c)  $1\times$

+ NLO field renormalization, and mass and decay-constant redefinitions applied to the LO graph

$\hookrightarrow$  schematically  $A_{4\pi}^{(4)} = \mathcal{M}_{1\text{-loop}} + \mathcal{M}_{\text{CT}} + 4(Z^{1/2} - 1)\mathcal{M}_{\text{LO}}^{(2)} + \mathcal{M}_{\text{LO}}^{(4)}$

The  $Z$  factor is related to the pion self-energy  $\Sigma$

$$\frac{1}{Z} = 1 - \left. \frac{\partial \Sigma(p^2)}{\partial p^2} \right|_{p^2=M_\pi^2}$$

Standard relations  $M_\pi^2 = M^2 - \bar{\Sigma}$ ,  $F_\pi = F(1 + \delta F)$  give the substitutions at the given order

$$M^2 \rightarrow M_\pi^2 + \bar{\Sigma}, \quad \bar{\Sigma} = \frac{M_\pi^4}{F_\pi^2} \left[ 2l_3^r + \frac{1}{2}(N-2)L \right] + \mathcal{O}\left(\frac{1}{F_\pi^4}\right)$$

$$\frac{1}{F^2} \rightarrow \frac{1}{F_\pi^2} (1 + 2\delta F), \quad \delta F = \frac{M_\pi^2}{F_\pi^2} \left[ l_4^r - \frac{1}{2}(N-1)L \right] + \mathcal{O}\left(\frac{1}{F_\pi^4}\right)$$



# Four-pion amplitude

Next-to-leading-order result

Parametrization-independent and UV-finite result

$$\begin{aligned}
F_{\pi}^4 A^{(4)}(s, t, u) = & (t-u)^2 \left( -\frac{5}{36} \kappa - \frac{1}{6} L + \frac{1}{2} l_2' \right) \\
& + M_{\pi}^2 s \left[ \left( N - \frac{29}{9} \right) \kappa + \left( N - \frac{11}{3} \right) L - 8l_1' + 2l_4' \right] \\
& + s^2 \left[ \left( \frac{11}{12} - \frac{N}{2} \right) \kappa + \left( 1 - \frac{N}{2} \right) L + 2l_1' + \frac{1}{2} l_2' \right] \\
& + M_{\pi}^4 \left[ \left( \frac{20}{9} - \frac{N}{2} \right) \kappa + \left( \frac{8}{3} - \frac{N}{2} \right) L + 8l_1' + 2l_3' - 2l_4' \right] \\
& + \bar{J}(s) \left[ \left( \frac{N}{2} - 1 \right) s^2 + (3-N) M_{\pi}^2 s + \left( \frac{N}{2} - 2 \right) M_{\pi}^4 \right] \\
& + \left\{ \frac{1}{6} \bar{J}(t) [2t^2 - 10M_{\pi}^2 t - 4M_{\pi}^2 s + st + 14M_{\pi}^4] + (t \leftrightarrow u) \right\}
\end{aligned}$$

Above we used

$$\kappa = \frac{1}{16\pi^2}, \quad L \equiv \kappa \log \frac{M_{\pi}^2}{\mu^2}, \quad \bar{J}(q^2) \equiv \kappa \left( 2 + \sigma \log \frac{\sigma-1}{\sigma+1} \right), \quad \sigma = \sqrt{1 - \frac{4M_{\pi}^2}{q^2}}$$

Form as given in *Bijnens et al.*, **PLB 374 (1996)**, **NPB 508 (1997)**, generalized to  $N \neq 3$

↪ somewhat different from the form given in *Gasser and Leutwyler*, **Ann.Ph.158 (1984)**

↪ equivalent to a given order but different off-shell extrapolations

The expressions agree with the known results

↪ for  $N = 3$ , *Gasser and Leutwyler*, **Ann.Ph.158 (1984)**

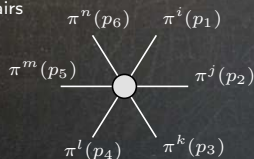
↪ on the  $N$  dependence, e.g. *Dobado and Morales*, **PRD 52 (1995)**,  
*Bijnens and Carloni*, **NPB 827 (2010)**, **NPB 843 (2011)**

# Six-pion amplitude

4 pions  $\rightarrow$  3 channels/permutations/ways to distribute 4 pions in 2 pairs

6 pions  $\rightarrow$  10 ways in 2 groups of three ( $P_{10}$ )

$\hookrightarrow$  15 ways in 3 pairs ( $P_{15}$ )



The full six-pion amplitude at  $\mathcal{O}(p^6)$

$$A_{6\pi} = A_{6\pi}^{(4\pi)} + A_{6\pi}^{(6\pi)}$$

$A_{6\pi}^{(4\pi)}$  can be written in terms of the four-pion amplitude and  $A_{6\pi}^{(6\pi)}$  is the remainder



(a)  $1 \times$



(b)  $10 \times$

# Six-pion amplitude

Feynman diagrams of relevant topologies



(a)  $1\times$



(b)  $1\times$



(c)  $15\times$



(d)  $20\times$



(e)  $20\times$



(f)  $60\times$



(g)  $10\times$



(h)  $10\times$



(i)  $15\times$



# Six-pion amplitude

The pole part

$$A_{6\pi}^{(4\pi)} \equiv \sum_{P_{10}, o} A_{4\pi}(p_i, i_i, p_j, i_j, p_k, i_k, o) \frac{(-1)}{p_{ijk}^2 - M_\pi^2} A_{4\pi}(p_l, i_l, p_m, i_m, p_n, i_n, o)$$

↪ residue at the pole unique, off-shell extrapolation away from  $p_{ijk}^2 \equiv (p_i + p_j + p_k)^2 = M_\pi^2$  **not**  
 $A_{4\pi}(p_i, i_i, p_j, i_j, p_k, i_k, o)$  is the four-pion amplitude with one leg off-shell

$$\begin{aligned} & A_{4\pi}(p_i, i_i, p_j, i_j, p_k, i_k, o) \\ &= \delta_{i_i i_j} \delta_{i_k o} A(p_i, p_j, p_k) + \delta_{i_i i_k} \delta_{i_j o} A(p_k, p_i, p_j) + \delta_{i_j i_k} \delta_{i_i o} A(p_j, p_k, p_i) \end{aligned}$$

The (four-pion) subamplitude  $A(p_i, p_j, p_k) = A(s, t, u)$  is defined as usual

↪  $s = (p_i + p_j)^2$ ,  $t = (p_i + p_k)^2$  and  $u = (p_j + p_k)^2$ , although now  $s + t + u = 3M_\pi^2 + p_{ijk}^2$

We have **chosen** a particular form for the **off-shell** four-pion subamplitude  $A(s, t, u)$

↪ other off-shell extrapolations are possible and will lead to a different  $A_{6\pi}^{(6\pi)}$

↪ independent of the parametrization used

↪ also  $A_{4\pi}$  and, consequently, the respective parts  $A_{6\pi}^{(4\pi)}$  and  $A_{6\pi}^{(6\pi)}$  by definition

↪ the way how the contributions from the one-particle irreducible and reducible diagrams are distributed within the final result is parametrization dependent

# Six-pion amplitude

The 1PI part



$$A_{6\pi}^{(6\pi)} \equiv \sum_{P_{15}} \delta_{i_i i_j} \delta_{i_k i_l} \delta_{i_m i_n} A(p_i, p_j, p_k, p_l, p_m, p_n)$$

The (six-pion) **subamplitude**  $A(p_1, p_2, p_3, p_4, p_5, p_6)$

↪ no poles, only cuts (however, the imaginary part of the triangle integrals can contain poles)

↪ function of **three pairs** of momenta

↪ fully symmetric under the interchange of any of the pairs

↪ symmetric for the interchange within a pair

The six-pion subamplitude respecting orders in the expansion

$$A(p_1, p_2, p_3, p_4, p_5, p_6) = A^{(2)}(p_1, p_2, p_3, p_4, p_5, p_6) + A^{(4)}(p_1, p_2, p_3, p_4, p_5, p_6)$$

# Six-pion amplitude

Leading order



(a) 1×



(b) 10×

The full six-pion amplitude at  $\mathcal{O}(p^6)$

$$A_{6\pi} = A_{6\pi}^{(4\pi)} + A_{6\pi}^{(6\pi)}$$

↪ (a) **only** contributes to  $A_{6\pi}^{(6\pi)}$

↪ (b) contributes to **both** the pole and non-pole parts  $A_{6\pi}^{(4\pi)}$  and  $A_{6\pi}^{(6\pi)}$

At LO a simple expression

$$A^{(2)}(p_1, p_2, p_3, p_4, p_5, p_6) = \frac{1}{F_\pi^4} (2p_1 \cdot p_2 + 2p_3 \cdot p_4 + 2p_5 \cdot p_6 + 3M_\pi^2)$$

↪ dependence on momenta is the **only one** at this order compatible with the symmetries



# Six-pion amplitude

Next-to-leading order



The **main new result** is the next-order six-pion subamplitude

↪ split it up into numerous parts:

$$F_\pi^6 A^{(4)}(p_1, p_2, \dots, p_6) = A_{C_3} + A_{C_{21}}^{(1)} + A_{C_{21}}^{(2)} + A_{C_{11}} + A_C^{(1)} + A_C^{(2)} + A_C^{(3)} \\ + A_J^{(1)} + A_J^{(2)} + A_\pi + A_L + A_l$$

(suppressed the arguments  $(p_1, p_2, \dots, p_6) \equiv (p_1, p_2, p_3, p_4, p_5, p_6)$ )

↪ each of the terms has the required symmetries under interchange of momenta

Large number of kinematic invariants → reduction to master integrals (scalar triangle integrals)  
leads to an enormous expression

↪ we have chosen a redundant basis of integrals that have good symmetry properties

Results are rather **lengthy**, but can be written in a relatively compact way

↪ see paper [arXiv:2107.06291](https://arxiv.org/abs/2107.06291)

# Six-pion amplitude

Particular kinematical setting



We choose a **symmetric 3 → 3 scattering** configuration given by

$$p_1 = \left( E_p, p, 0, 0 \right)$$

$$p_4 = \left( -E_p, 0, 0, p \right)$$

$$p_2 = \left( E_p, -\frac{1}{2}p, \frac{\sqrt{3}}{2}p, 0 \right)$$

$$p_5 = \left( -E_p, \frac{\sqrt{3}}{2}p, 0, -\frac{1}{2}p \right)$$

$$p_3 = \left( E_p, -\frac{1}{2}p, -\frac{\sqrt{3}}{2}p, 0 \right)$$

$$p_6 = \left( -E_p, -\frac{\sqrt{3}}{2}p, 0, -\frac{1}{2}p \right)$$

We use following **numerical** inputs:

$$M_\pi = 0.139570 \text{ GeV}$$

$$\bar{l}_1 = -0.4$$

$$F_\pi = 0.0927 \text{ GeV}$$

$$\bar{l}_2 = 4.3$$

$$\mu = 0.77 \text{ GeV}$$

$$\bar{l}_3 = 3.41$$

$$N = 3$$

$$\bar{l}_4 = 4.51$$

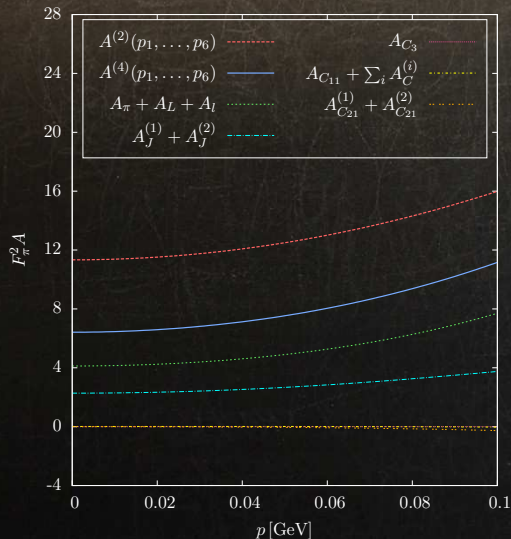
*Bijnens, Ecker, ARNPS 64 (2014)*

*Colangelo, Gasser, Leutwyler, NPB 603 (2001)*

*Aoki et al., EPJC 77 (2017)*

# Six-pion amplitude

Numerical results



$F_\pi^2 A$		$F_\pi^2 A$	
$A_{6\pi}^{(4\pi)}$ (LO)	-319.00	$A^{(2)}(p_1, \dots, p_6)$	<b>15.99</b>
$A_{6\pi}^{(4\pi)}$ (NLO)	-28.54	$A^{(4)}(p_1, \dots, p_6)$	<b>11.16</b>
$F_\pi^2 \times A/F_\pi^6$		$F_\pi^2 \times A/F_\pi^6$	
$A_{C_3}$	<b>0.002</b>	$A_J^{(1)}$	<b>1.917</b>
$A_{C_{21}}^{(1)}$	-0.948	$A_J^{(2)}$	<b>1.835</b>
$A_{C_{21}}^{(2)}$	<b>0.682</b>	$A_\pi$	-2.488
$A_{C_{11}}$	0.090	$A_L$	8.985
$A_C^{(1)}$	-0.026	$A_t$	1.209
$A_C^{(2)}$	0.890		
$A_C^{(3)}$	-0.984		

Real parts of the amplitudes for  
 $p = 0.1 \text{ GeV}$

# Six-pion amplitude

in the limit of vanishing momentum



In the limit  $p \rightarrow 0$ , we find the following analytical expressions:

$$F_\pi^2 A^{(2)}(p_1, p_2, \dots, p_6) \Big|_{p \rightarrow 0} = 5 \frac{M_\pi^2}{F_\pi^2} \approx 11.33$$

$$\begin{aligned} & F_\pi^2 \operatorname{Re} A^{(4)}(p_1, p_2, \dots, p_6) \Big|_{p \rightarrow 0} \\ &= \frac{M_\pi^4}{F_\pi^4} \left\{ \underbrace{\frac{1}{18}(-2 - 225N)\kappa}_{A_\pi} + \underbrace{\frac{1}{6}(-14 - 75N)L}_{A_L} + \underbrace{(16l_1' + 56l_2' + 6l_3' + 20l_4')}_{A_I} \right. \\ & \quad \left. + \underbrace{(-44 + 30N)\kappa}_{A_J^{(1)}} + \underbrace{(24)\kappa}_{A_J^{(2)}} + \frac{1}{2}\kappa \left[ \underbrace{- (30 - 9N)}_{A_C^{(1)}} + \underbrace{(20)}_{A_C^{(2)}} + \underbrace{(-16)}_{A_C^{(3)}} \right] \right\} \approx 6.416 \end{aligned}$$



# Summary

We calculated the pion mass, decay constant, the four-pion and six-pion amplitude to NLO in the massive  $O(N)$  nonlinear sigma model

- ↔ relevant NLO Lagrangian constructed in analogy with two(-quark)-flavour ChPT Lagrangian
- ↔ our results agree with previous results for  $N = 3$  and general- $N$  behaviour

Our main result is the six-pion amplitude

- ↔ split in one-particle reducible and irreducible parts

The reducible part employs the off-shell four-pion amplitude generalizing (beyond  $N = 3$ ) the amplitude given by *Bijnens et al.*, PLB 374 (1996), NPB 508 (1997)

The irreducible part can be divided in a large number of subparts

- ↔ each subpart satisfies the expected permutation symmetries
- ↔ the choice of triangle loop integrals with high symmetry allows for a fairly compact expression
- ↔ NLO correction is sizable but not very large

## Outlook

Work in progress

- ↔ combine our results with the methods for extracting three-body scattering from finite volume in lattice QCD

Might be of interest for the amplitude community

More details in [arXiv:2107.06291](https://arxiv.org/abs/2107.06291)